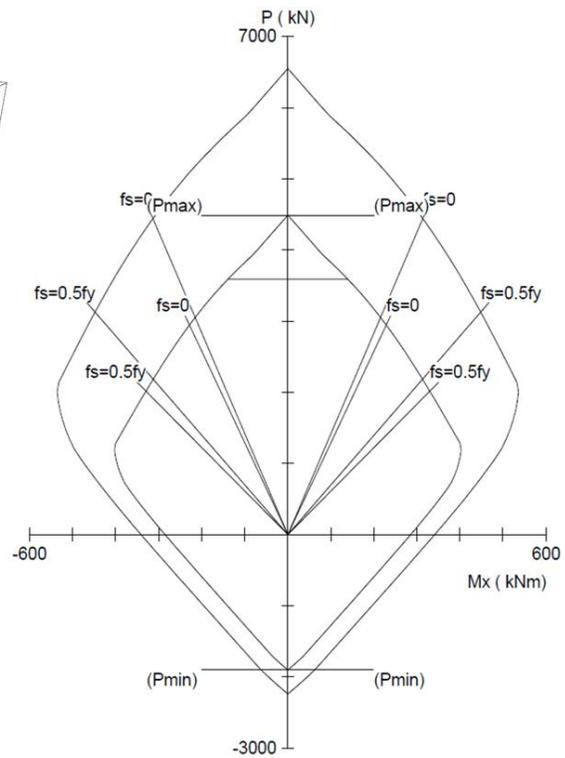
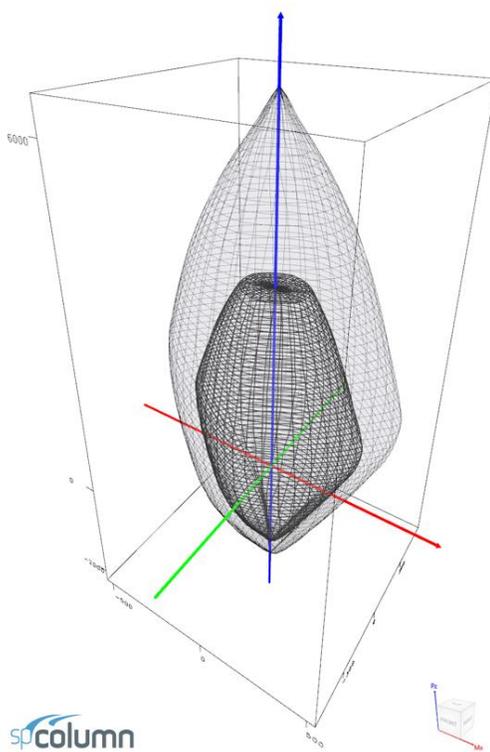
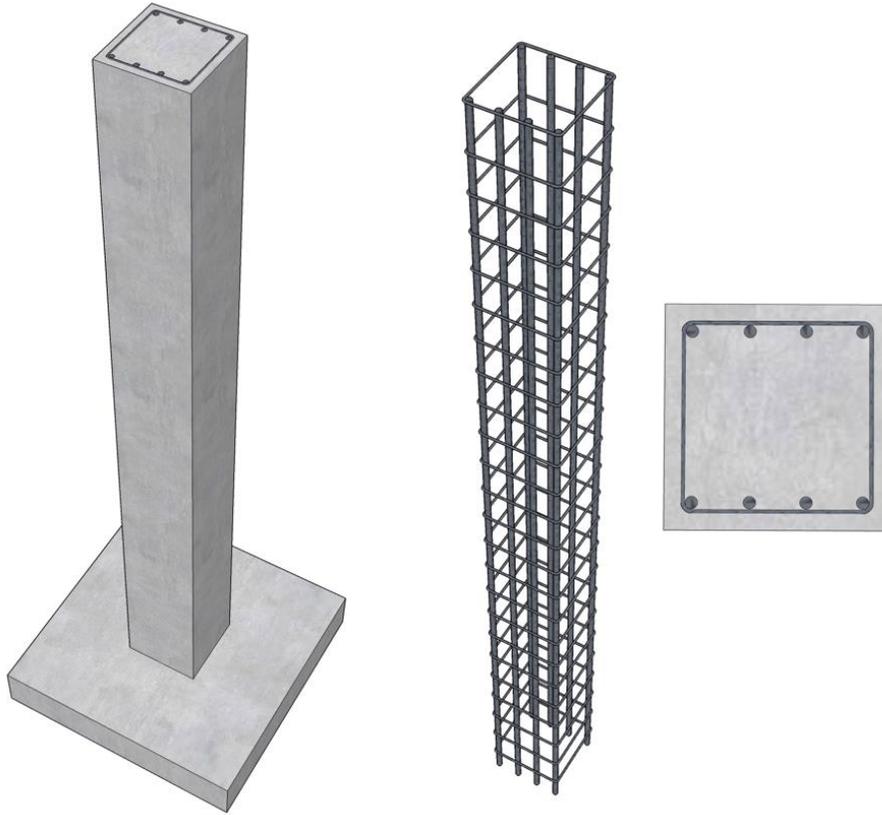


Interaction Diagram - Tied Reinforced Concrete Column (Using CSA A23.3-94)



Interaction Diagram - Tied Reinforced Concrete Column

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using CSA A23.3-14 provisions. Determine six control points on the interaction diagram and compare the calculated values in the Reference and with exact values from the complete interaction diagram generated by [spColumn](#) engineering software program from [StructurePoint](#).

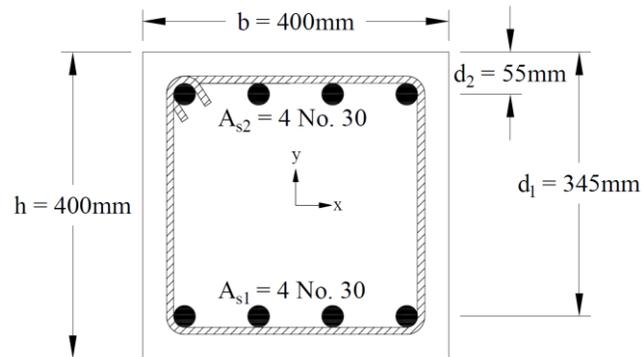


Figure 1 – Reinforced Concrete Column Cross-Section

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Code

Design of Concrete Structures (CSA A23.3-94)

Reference

Reinforced Concrete Mechanics and Design, 1st Canadian Edition, 2000, James MacGregor and Fred Michael
Bratlett, Prentice Hall Canada Inc.

Design Data

$$f_c' = 35 \text{ MPa}$$

$$f_y = 400 \text{ MPa}$$

Cover = 55 mm to the center of the reinforcement

Column 400 mm x 400 mm

Top reinforcement = 4 No. 30

Bottom reinforcement = 4 No. 30

Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following six control points:

Point 1: Pure compression

Point 2: Bar stress near tension face of member equal to zero, ($f_s = 0$)

Point 3: Bar stress near tension face of member equal to $0.5 f_y$ ($f_s = -0.5 f_y$)

Point 4: Bar stress near tension face of member equal to f_y ($f_s = -f_y$)

Point 5: Pure bending

Point 6: Pure tension

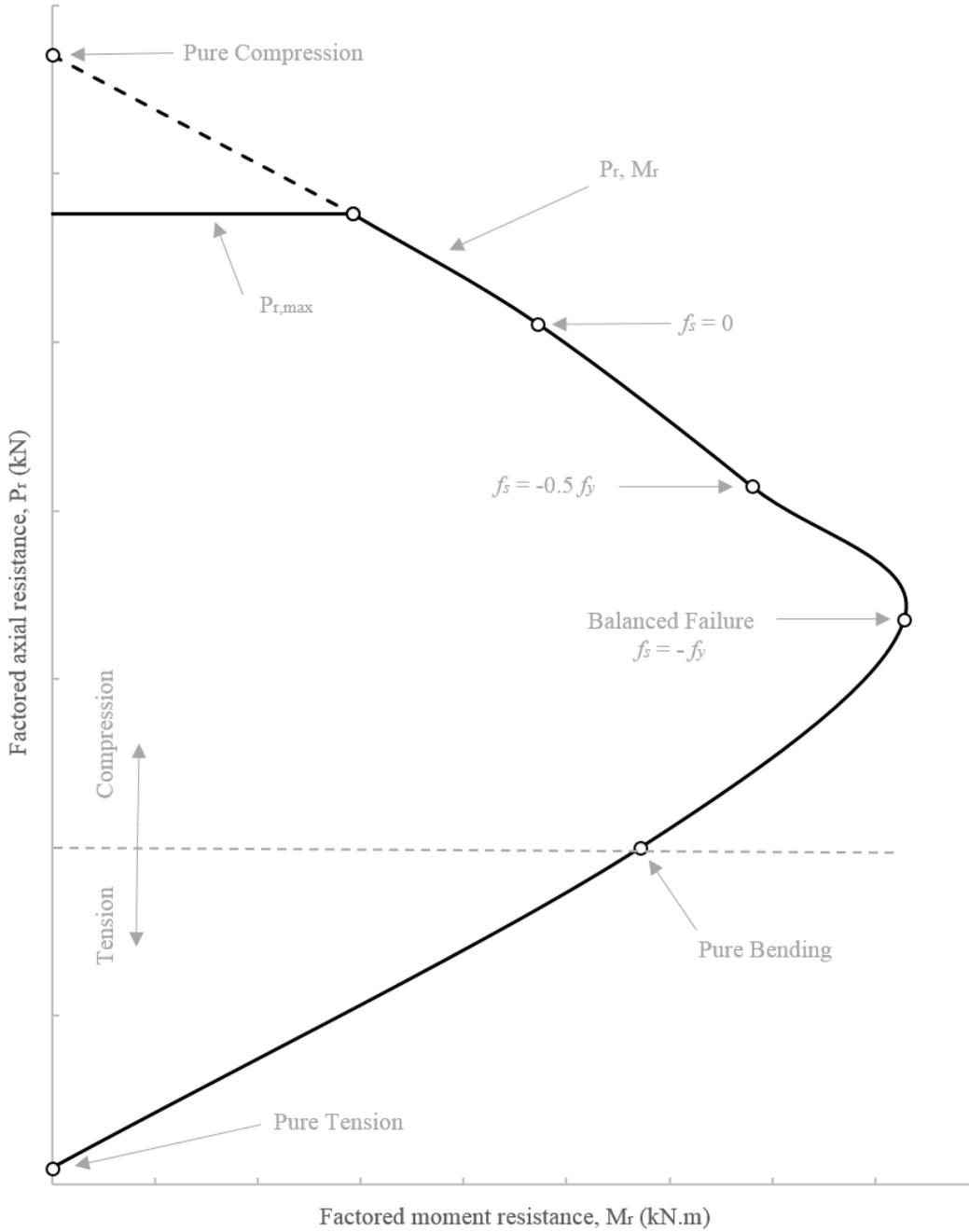


Figure 2 – Control Points

1. Pure Compression

1.1. Nominal axial resistance at zero eccentricity

$$P_o = \alpha_1 f'_c (A_g - A_{st}) + f_y A_{st}$$

$$P_o = 0.798 \times 35 \times (400 \times 400 - 8 \times 700) + 400 \times 8 \times 700 = 6550 \text{ kN}$$

$$\text{Where } \alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67$$

CSA A23.3-94 (Equation 10-1)

$$\alpha_1 = 0.85 - 0.0015 \times 35 = 0.798 \geq 0.67$$

1.2. Factored axial load resistance at zero eccentricity

Since this column is a tied column with steel strain in compression:

$$P_{ro} = \alpha_1 \phi_c f'_c (A_g - A_{st}) + \phi_s f_y A_{st}$$

CSA A23.3-94 (Equation 10-10)

$$P_{ro} = 0.798 \times 0.6 \times 35 \times (400 \times 400 - 8 \times 700) + 0.85 \times 400 \times 8 \times 700 = 4490 \text{ kN}$$

Where:

$$\phi_c = 0.6$$

CSA A23.3-94 (8.4.2)

$$\phi_s = 0.85$$

CSA A23.3-94 (8.4.3)

1.3. Maximum factored axial load resistance

$$P_{r,max} = 0.80 P_{ro}$$

CSA A23.3-94 (Equation 10-9)

$$P_{r,max} = (0.2 + 0.002 \times 400) \times 4490 = 4490 \text{ kN} \leq 0.80 P_{ro} = 0.80 \times 4490 = 3592 \text{ kN}$$

$$P_{r,max} = 3592 \text{ kN}$$

2. Bar Stress Near Tension Face of Member Equal to Zero, ($\epsilon_s = f_s = 0$)

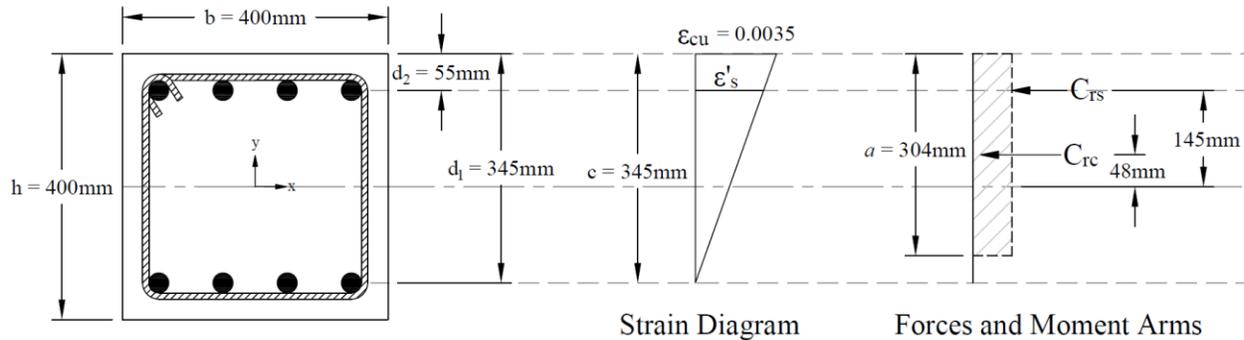


Figure 3 – Strains, Forces, and Moment Arms ($\epsilon_t = f_s = 0$)

Strain ϵ_s is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. CSA A23.3-94 (12.15 and 16)

2.1. c, a, and strains in the reinforcement

$$c = d_1 = 345 \text{ mm}$$

Where c is distance from extreme compression fiber to neutral axis.

CSA A23.3-94 (10.0)

$$a = \beta_1 \times c = 0.883 \times 345 = 304 \text{ mm}$$

CSA A23.3-94 (10.1.7)

Where:

a = Depth of equivalent rectangular stress block

CSA A23.3-94 (10.0)

$$\beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67$$

CSA A23.3-94 (Equation 10-2)

$$\epsilon_s = 0$$

$$\phi_c = 0.6$$

CSA A23.3-94 (8.4.2)

$$\phi_s = 0.85$$

CSA A23.3-94 (8.4.3)

$$\epsilon_{cu} = 0.0035$$

CSA A23.3-94 (10.1.3)

$$\epsilon'_s = (c - d_2) \times \frac{\epsilon_{cu}}{c} = (345 - 55) \times \frac{0.0035}{345} = 0.00294 \text{ (Compression)} > \epsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002$$

2.2. Forces in the concrete and steel

$$C_{rc} = \alpha_1 \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 304 \times 400 = 2040 \text{ kN}$$

CSA A23.3-94 (10.1.7)

$$f_s = 0 \text{ kN} \rightarrow T_{rs} = \phi_s \times f_s \times A_{s1} = 0 \text{ kN}$$

Since $\varepsilon_s' > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$$\therefore f_s' = f_y = 400 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c . As a result, it is necessary to subtract $\alpha_1 \phi_c f_c'$ from $\phi_s f_s'$ before computing C_{rs} :

$$C_{rs} = (\phi_s \times f_s' - \alpha_1 \times \phi_c \times f_c') \times A_{s2} = (0.85 \times 400 - 0.798 \times 0.6 \times 35) \times 2800 = 905 \text{ kN}$$

2.3. P_r and M_r

$$P_r = C_{rc} + C_{rs} - T_{rs} = 2040 + 905 - 0 = 2945 \text{ kN}$$

$$M_r = C_{rc} \times \left(\frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left(\frac{h}{2} - d_2 \right) + T_{rs} \times \left(d_1 - \frac{h}{2} \right)$$

$$M_r = 2040 \times \left(\frac{400}{2} - \frac{304}{2} \right) + 905 \times \left(\frac{400}{2} - 55 \right) + 0 \times \left(345 - \frac{400}{2} \right) = 229 \text{ kN.m}$$

3. Bar Stress Near Tension Face of Member Equal to $0.5 f_y$, ($f_s = -0.5 f_y$)

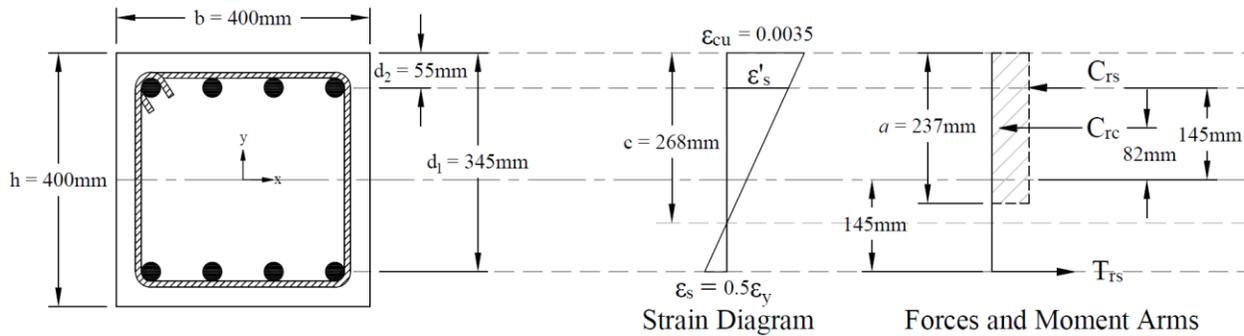


Figure 4 – Strains, Forces, and Moment Arms ($f_s = -0.5 f_y$)

3.1. c, a, and strains in the reinforcement

$$\varepsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.002}{2} = 0.001 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$$\phi_c = 0.6$$

CSA A23.3-94 (8.4.2)

$$\phi_s = 0.85$$

CSA A23.3-94 (8.4.3)

$$\varepsilon_{cu} = 0.0035$$

CSA A23.3-94 (10.1.3)

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{345}{0.001 + 0.0035} \times 0.0035 = 268 \text{ mm}$$

Where c is depth of the neutral axis measured from the compression edge of the column section.

CSA A23.3-94 (10.0)

$$a = \beta_1 \times c = 0.883 \times 268 = 237 \text{ mm}$$

CSA A23.3-94 (10.1.7)

Where:

a = Depth of equivalent rectangular stress block

CSA A23.3-94 (10.0)

$$\beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67$$

CSA A23.3-94 (Equation 10-2)

$$\varepsilon'_s = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (268 - 55) \times \frac{0.0035}{268} = 0.00278 \text{ (Compression)} > \varepsilon_y$$

3.2. Forces in the concrete and steel

$$C_{rc} = \alpha_1 \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 237 \times 400 = 1586 \text{ kN}$$

CSA A23.3-94 (10.1.7)

$$f_s = \varepsilon_s \times E_s = 0.001 \times 200,000 = 200 \text{ MPa}$$

$$T_{rs} = \phi_s \times f_s \times A_{s1} = 0.85 \times 200 \times 2800 = 476 \text{ kN}$$

Since $\varepsilon_s' > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$$\therefore f_s' = f_y = 400 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c . As a result, it is necessary to subtract $\alpha_1 \phi_c f_c'$ from $\phi_s f_s'$ before computing C_{rs} :

$$C_{rs} = (\phi_s \times f_s' - \alpha_1 \times \phi_c \times f_c') \times A_{s2} = (0.85 \times 400 - 0.798 \times 0.6 \times 35) \times 2800 = 905 \text{ kN}$$

3.3. P_r and M_r

$$P_r = C_{rc} + C_{rs} - T_{rs} = 1586 + 905 - 476 = 2015 \text{ kN}$$

$$M_r = C_{rc} \times \left(\frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left(\frac{h}{2} - d_2 \right) + T_{rs} \times \left(d_1 - \frac{h}{2} \right)$$

$$M_r = 1586 \times \left(\frac{400}{2} - \frac{237}{2} \right) + 905 \times \left(\frac{400}{2} - 55 \right) + 476 \times \left(345 - \frac{400}{2} \right) = 330 \text{ kN.m}$$

4. Bar Stress Near Tension Face of Member Equal to f_y , ($f_s = -f_y$)

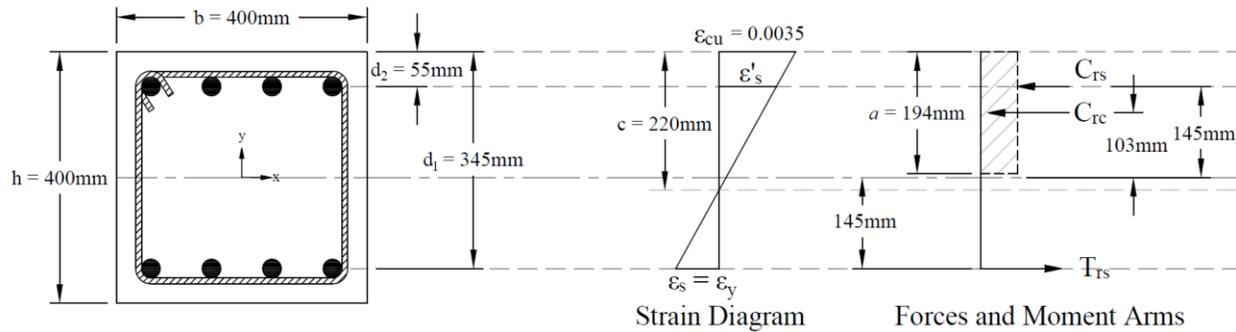


Figure 5 – Strains, Forces, and Moment Arms ($f_s = -f_y$)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement.

4.1. c , a , and strains in the reinforcement

$$\epsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002$$

$$\epsilon_s = \epsilon_y = 0.002 \rightarrow \text{tension reinforcement has yielded}$$

$$\phi_c = 0.6$$

CSA A23.3-94 (8.4.2)

$$\phi_s = 0.85$$

CSA A23.3-94 (8.4.3)

$$\epsilon_{cu} = 0.0035$$

CSA A23.3-94 (10.1.3)

$$c = \frac{d_1}{\epsilon_s + \epsilon_{cu}} \times \epsilon_{cu} = \frac{345}{0.002 + 0.0035} \times 0.0035 = 220 \text{ mm}$$

Where c is depth of the neutral axis measured from the compression edge of the column section.

CSA A23.3-94 (10.0)

$$a = \beta_1 \times c = 0.883 \times 220 = 194 \text{ mm}$$

CSA A23.3-94 (10.1.7)

Where:

a = Depth of equivalent rectangular stress block

CSA A23.3-94 (10.0)

$$\beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67$$

CSA A23.3-94 (Equation 10-2)

$$\epsilon'_s = (c - d_2) \times \frac{\epsilon_{cu}}{c} = (220 - 55) \times \frac{0.0035}{220} = 0.00262 \text{ (Compression)} > \epsilon_y$$

4.2. Forces in the concrete and steel

$$C_{rc} = \alpha_1 \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 194 \times 400 = 1298 \text{ kN}$$

CSA A23.3-94 (10.1.7)

$$f_s = f_y = 400 \text{ MPa}$$

$$T_{rs} = \phi_s \times f_s \times A_{s1} = 0.85 \times 400 \times 2800 = 952 \text{ kN}$$

Since $\epsilon'_s > \epsilon_y \rightarrow$ compression reinforcement has yielded

$$\therefore f'_s = f_y = 400 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c . As a result, it is necessary to subtract $\alpha_1 \phi_c f'_c$ from $\phi_s f'_s$ before computing C_{rs} :

$$C_{rs} = (\phi_s \times f'_s - \alpha_1 \times \phi_c \times f'_c) \times A_{s2} = (0.85 \times 400 - 0.798 \times 0.6 \times 35) \times 2800 = 905 \text{ kN}$$

4.3. P_r and M_r

$$P_r = C_{rc} + C_{rs} - T_{rs} = 1406 + 901 - 952 = 1355 \text{ kN}$$

$$M_r = C_{rc} \times \left(\frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left(\frac{h}{2} - d_2 \right) + T_{rs} \times \left(d_1 - \frac{h}{2} \right)$$

$$M_r = 1298 \times \left(\frac{400}{2} - \frac{194}{2} \right) + 905 \times \left(\frac{400}{2} - 55 \right) + 952 \times \left(345 - \frac{400}{2} \right) = 403 \text{ kN.m}$$

5. Pure Bending

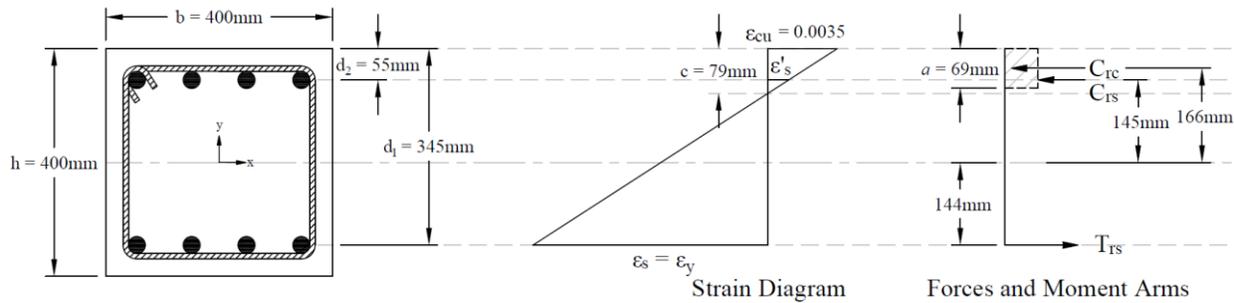


Figure 6 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the factored axial load resistance, P_r , is equal to zero. Iterative procedure is used to determine the factored moment resistance as follows:

5.1. c , a , and strains in the reinforcement

Try $c = 78.55$ mm

Where c is depth of the neutral axis measured from the compression edge of the column section.

$$\underline{CSA A23.3-94 (10.0)}$$

$$a = \beta_1 \times c = 0.883 \times 78.55 = 69 \text{ mm}$$

$$\underline{CSA A23.3-94 (10.1.7)}$$

Where:

$$\beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67$$

$$\underline{CSA A23.3-94 (Equation 10-2)}$$

$$\varepsilon_{cu} = 0.0035$$

$$\underline{CSA A23.3-94 (10.1.3)}$$

$$\varepsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002$$

$$\varepsilon_s = (d_1 - c) \times \frac{\varepsilon_{cu}}{c} = (345 - 78.55) \times \frac{0.0035}{78.55} = 0.01187 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\phi_c = 0.6$$

$$\underline{CSA A23.3-94 (8.4.2)}$$

$$\phi_s = 0.85$$

$$\underline{CSA A23.3-94 (8.4.3)}$$

$$\varepsilon'_s = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (78.55 - 55) \times \frac{0.0035}{78.55} = 0.00105 \text{ (Compression)} < \varepsilon_y$$

5.2. Forces in the concrete and steel

$$C_{rc} = \alpha_1 \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 69 \times 400 = 474 \text{ kN}$$

$$\underline{CSA A23.3-94 (10.1.7)}$$

$$f_s = f_y = 400 \text{ MPa}$$

$$T_{rs} = \phi_s \times f_s \times A_{s1} = 0.85 \times 400 \times 2800 = 952 \text{ kN}$$

Since $\varepsilon_s' < \varepsilon_y$ \rightarrow compression reinforcement has not yielded

$$\therefore f_s' = \varepsilon_s' \times E_s = 0.00105 \times 200,000 = 210 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c . As a result, it is necessary to subtract $\alpha_1 \phi_c f_c'$ from $\phi_s f_s'$ before computing C_{rs} :

$$C_{rs} = (\phi_s \times f_s' - \alpha_1 \times \phi_c \times f_c') \times A_{s2} = (0.85 \times 210 - 0.798 \times 0.6 \times 35) \times 2800 = 477 \text{ kN}$$

5.3. P_r and M_r

$$P_r = C_{rc} + C_{rs} - T_{rs} = 474 + 477 - 952 \approx 0 \text{ kN}$$

The assumption that $c = 78.55 \text{ mm}$ is correct

$$M_r = C_{rc} \times \left(\frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left(\frac{h}{2} - d_2 \right) + T_{rs} \times \left(d_1 - \frac{h}{2} \right)$$

$$M_r = 474 \times \left(\frac{400}{2} - \frac{69}{2} \right) + 477 \times \left(\frac{400}{2} - 55 \right) + 952 \times \left(345 - \frac{400}{2} \right) = 285 \text{ kN.m}$$

6. Pure Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

6.1. Strength under pure axial tension (P_{rt})

$$P_{rt} = \phi_s \times f_y \times (A_{s1} + A_{s2}) = 0.85 \times 400 \times (2800 + 2800) = 1904 \text{ kN}$$

6.2. Corresponding Moment (M_{rt})

Since the section is symmetrical

$$M_{rt} = 0 \text{ kN.m}$$

7. Column Interaction Diagram - spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with control points using the CSA A23.3-94. In lieu of using program shortcuts, spSection (Figure 9) was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

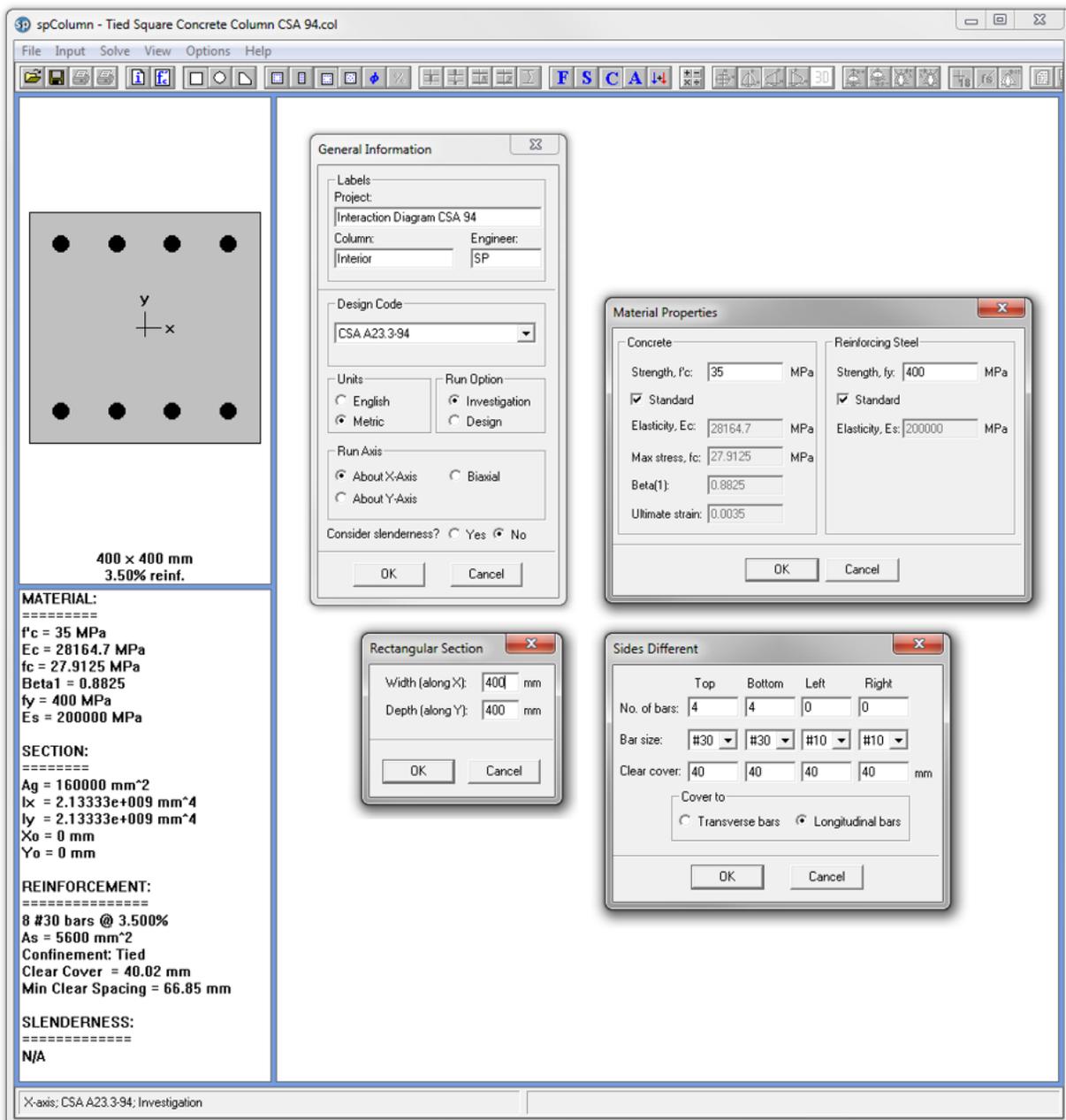


Figure 7 – Generating spColumn Model

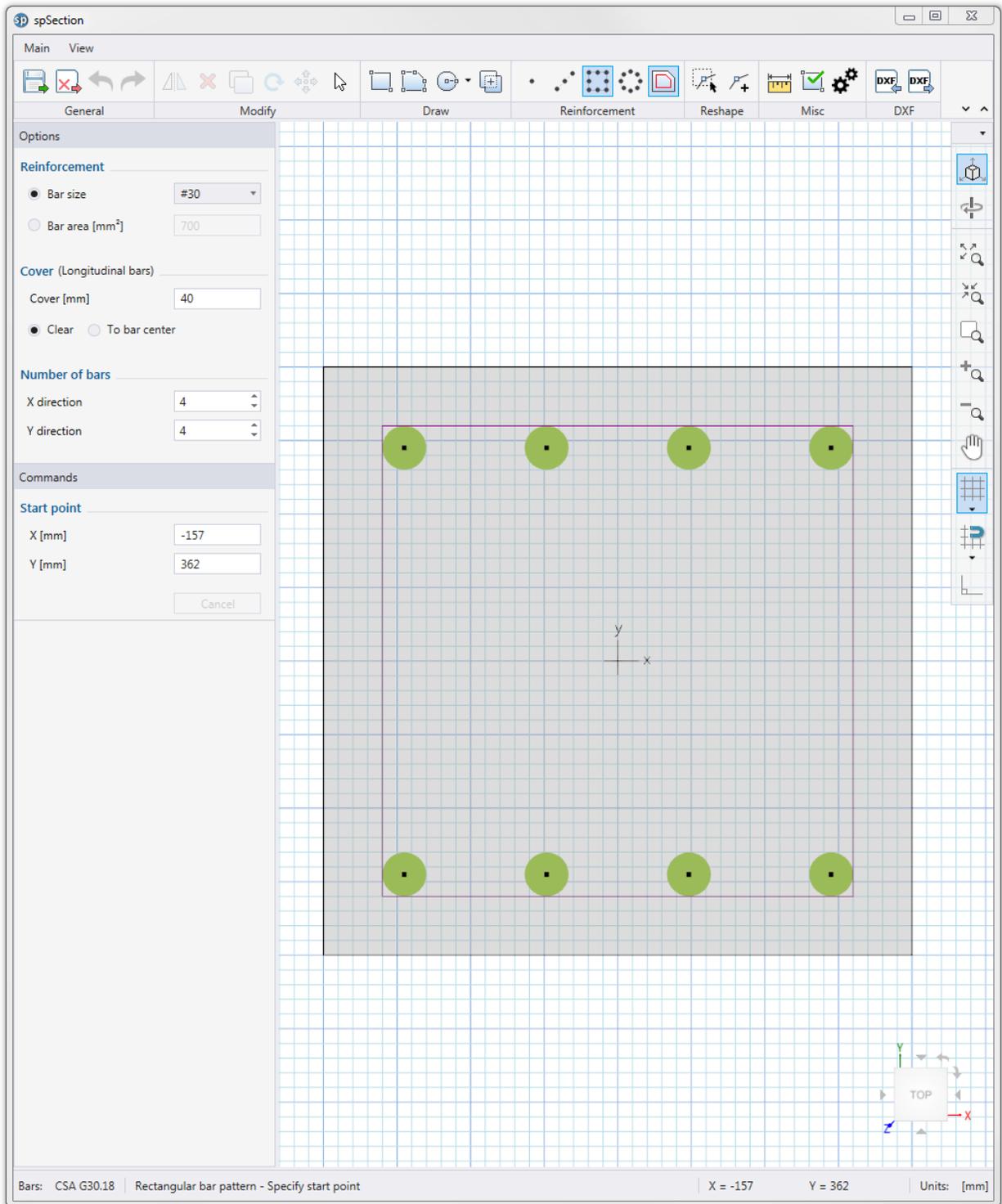


Figure 8 – spColumn Model Editor (spSection)

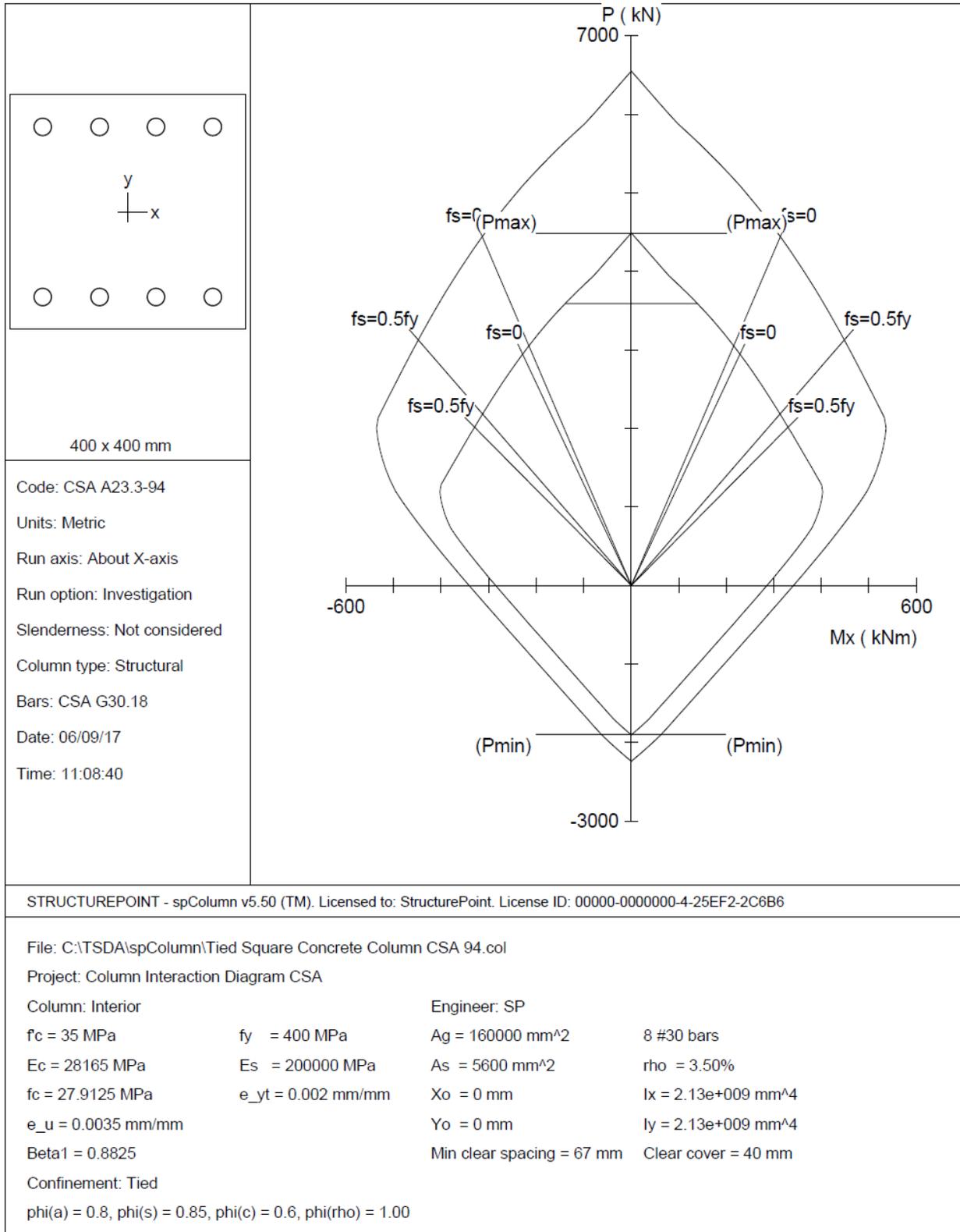


Figure 9 – Column Section Interaction Diagram about the X-Axis (spColumn)

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                oo   oo          oo
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                        spColumn v5.50 (TM)
Computer program for the Strength Design of Reinforced Concrete Sections
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General Information:

```

=====
File Name: C:\TSDA\spColumn\Tied Square Concrete Column CSA 94.col
Project: Column Interaction Diagram CSA
Column: Interior Engineer: SP
Code: CSA A23.3-94 Units: Metric

Run Option: Investigation Slenderness: Not considered
Run Axis: X-axis Column Type: Structural
    
```

Material Properties:

```

=====
Concrete: Standard Steel: Standard
f'c = 35 MPa fy = 400 MPa
Ec = 28164.7 MPa Es = 200000 MPa
fc = 27.9125 MPa Eps_yt = 0.002 mm/mm
Eps_u = 0.0035 mm/mm
Beta1 = 0.8825
    
```

Section:

```

=====
Rectangular: Width = 400 mm Depth = 400 mm

Gross section area, Ag = 160000 mm^2
Ix = 2.13333e+009 mm^4 Iy = 2.13333e+009 mm^4
rx = 115.47 mm ry = 115.47 mm
Xo = 0 mm Yo = 0 mm
    
```

Reinforcement:

```

=====
Bar Set: CSA G30.18
Size Diam (mm) Area (mm^2) Size Diam (mm) Area (mm^2) Size Diam (mm) Area (mm^2)
-----
# 10 11 100 # 15 16 200 # 20 20 300
# 25 25 500 # 30 30 700 # 35 36 1000
# 45 44 1500 # 55 56 2500
    
```

Confinement: Tied; #10 ties with #55 bars, #15 with larger bars.
phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.6, phi(rho) = 1.00

Layout: Rectangular
Pattern: Sides Different (Cover to longitudinal reinforcement)
Total steel area: As = 5600 mm^2 at rho = 3.50%
Minimum clear spacing = 67 mm

	Top	Bottom	Left	Right
Bars	4 #30	4 #30	0 #10	0 #10
Cover (mm)	40	40	40	40

Control Points:

```

=====
Bending about Axial Load P X-Moment Y-Moment NA depth Dt depth eps_t
              kN kNm kNm mm mm
-----
X @ Max compression 4489.8 -0.00 0.00 805 345 -0.00200
@ Allowable comp. 3591.9 138.87 0.00 415 345 -0.00059
@ fs = 0.0 2945.0 228.68 0.00 345 345 -0.00000
@ fs = 0.5*fy 2015.7 329.76 0.00 268 345 0.00100
@ Balanced point 1251.2 403.22 0.00 220 345 0.00200
@ Pure bending 0.0 285.45 0.00 80 345 0.01156
@ Max tension -1904.0 -0.00 -0.00 0 345 9.99999
    
```

STRUCTUREPOINT - spColumn v5.50 (TM)
Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-2C6B6
C:\TSDA\spColumn\Fied Square Concrete Column CSA 94.col

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-X @ Max compression	4489.8	-0.00	0.00	805	345	-0.00200
@ Allowable comp.	3591.9	-138.87	-0.00	415	345	-0.00059
@ fs = 0.0	2945.0	-228.68	-0.00	345	345	0.00000
@ fs = 0.5*fy	2019.7	-329.76	-0.00	268	345	0.00100
@ Balanced point	1251.2	-403.22	-0.00	220	345	0.00200
@ Pure bending	0.0	-285.45	-0.00	80	345	0.01156
@ Max tension	-1904.0	-0.00	-0.00	0	345	9.99999

*** End of output ***

8. Summary and Comparison of Design Results

Table 1 - Comparison of Results						
Support	P_r , kN			M_r , kN.m		
	Hand	Reference*	spColumn	Hand	Reference*	spColumn
Max compression	4490	4490	4490	0	0	0
Allowable compression	3592	3592	3592	---	---	---
$f_s = 0.0$	2945	2945	2945	229	229	229
$f_s = 0.5 f_y$	2015	2015	2016	330	330	330
Balanced point	1251	1253	1251	403	403	403
Pure bending	0	0	0	285	285	285
Max tension	1904	1904	1904	0	0	0

* Reinforced Concrete Mechanic and Design, 1st Canadian Edition, James MacGregor and Fred Bartlett – Example 11-1

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the [spColumn](#) program.

9. Conclusions & Observations

The analysis of the reinforced concrete section performed by [spColumn](#) conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where $f_s = f_y$).

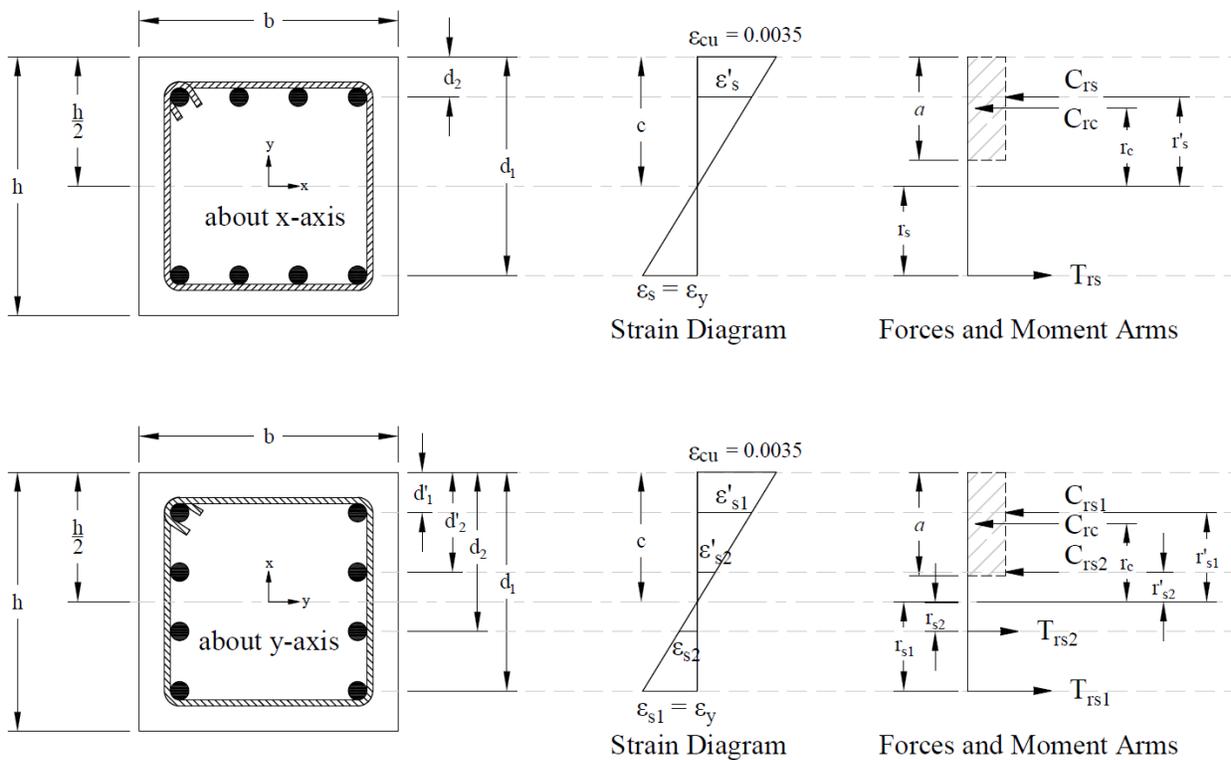


Figure 10 – Strains, Forces, and Moment Arms ($f_s = - f_y$ Moments About x- and y-axis)

When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure.

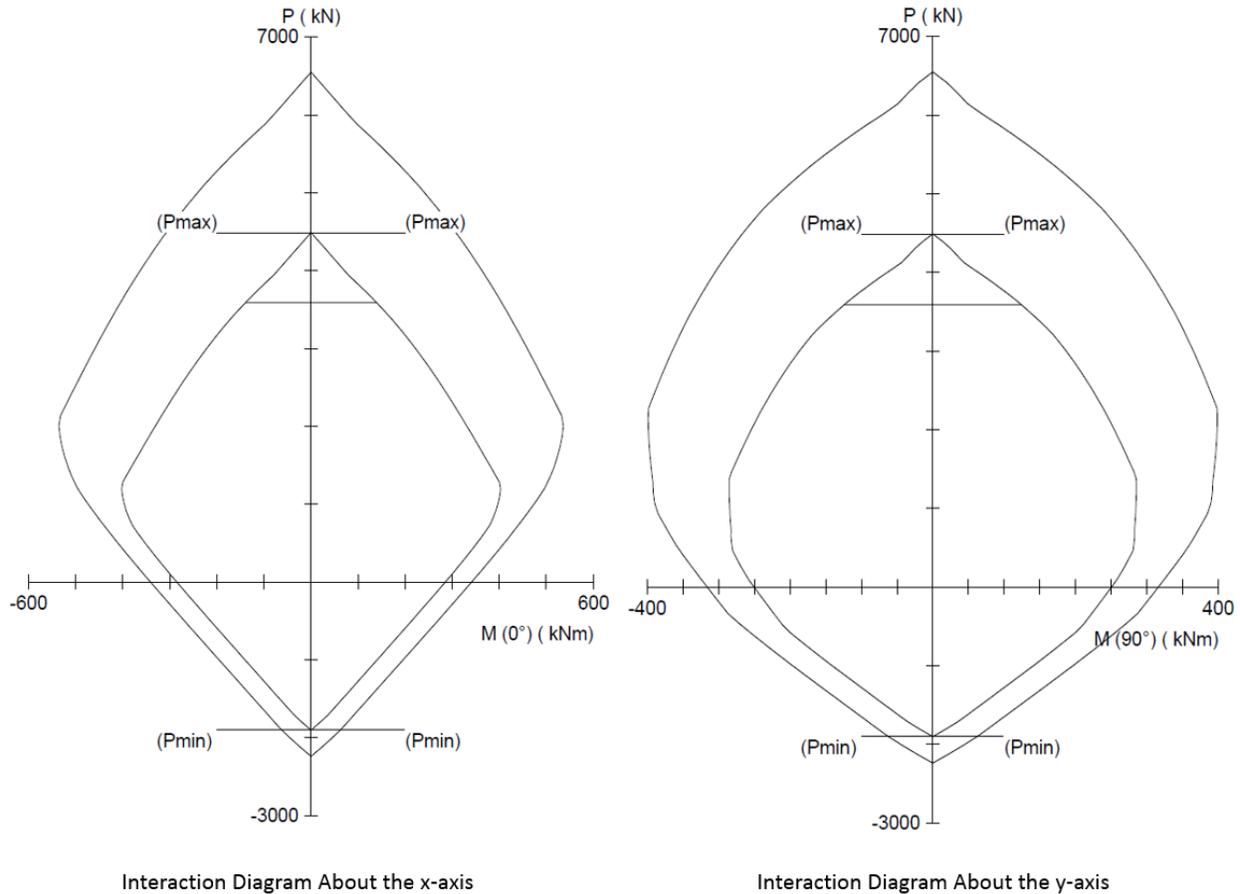


Figure 11 – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)

Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

In most building design calculations, such as the examples shown for [flat plate](#) or [flat slab](#) concrete floor systems, all building columns are subjected to M_x and M_y due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StructurePoint's [spColumn](#) program can also evaluate column sections in biaxial mode to produce the results shown in the following Figure for the column section in this example.

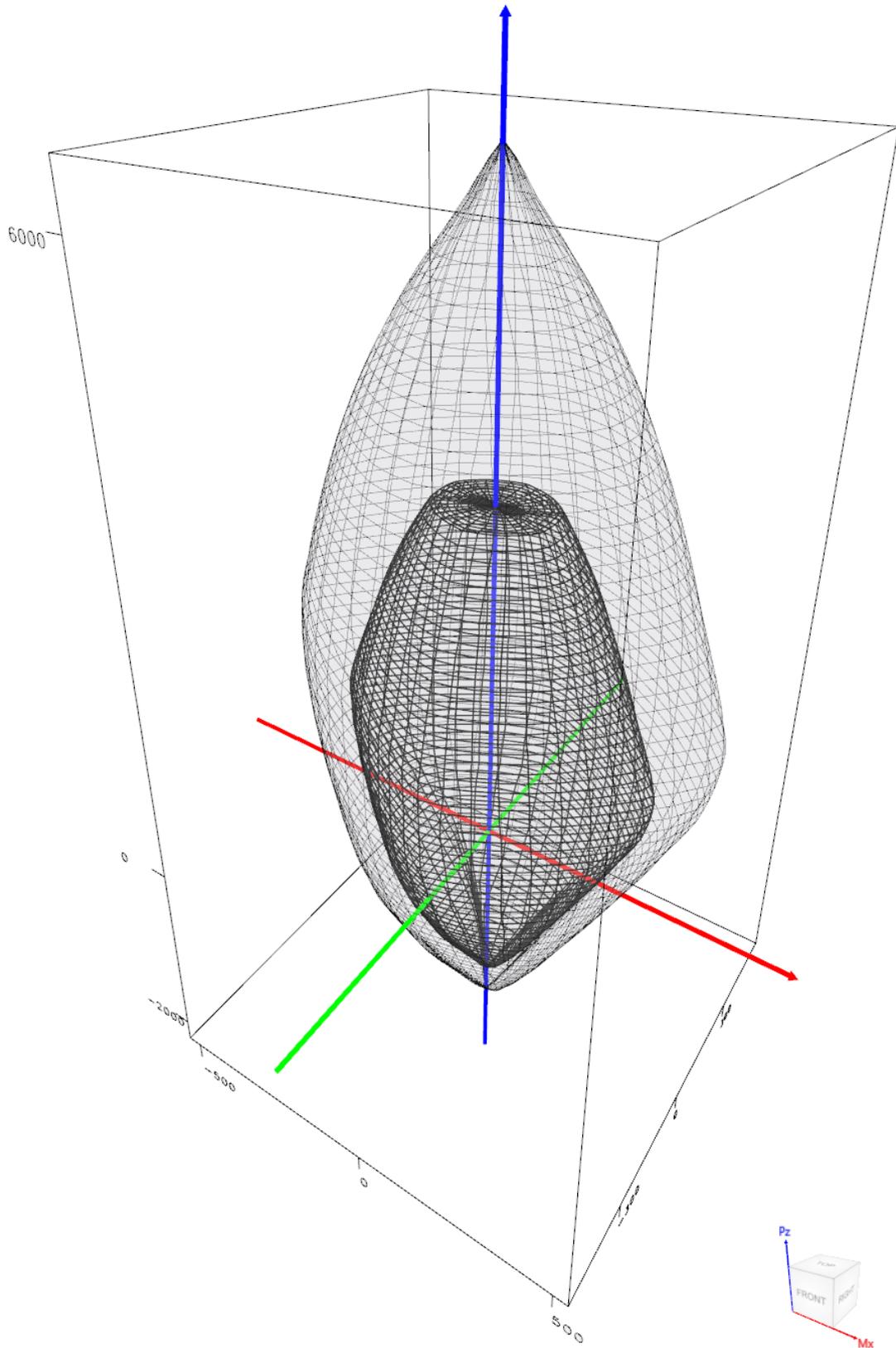


Figure 12 – Nominal & Design Interaction Diagram in Two Directions (Biaxial) (spColumn)