Interaction Diagram for a Tied Square Concrete Column

Develop an interaction diagram for the concrete column shown in figure below. Determine 5 control points on the interaction diagram and compare of calculated values with exact values from the complete interaction diagram generated by spColumn software program.

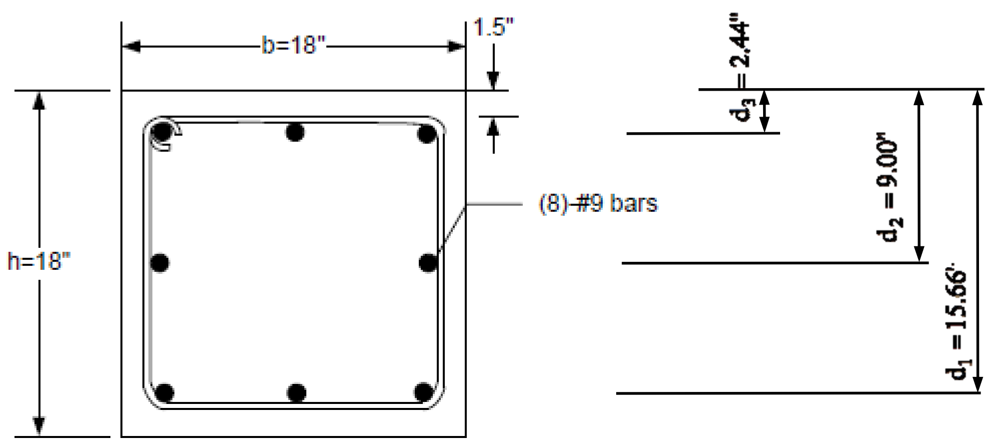
Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Design Data

$f_c' = 4{,}000\ \text{psi}$

$f_y = 60{,}000\ \text{psi}$



Solution

Use the traditional approach hand calculations to establish the interaction diagram for the column section shown in above by determining the following 5 points:

Point 1: Pure compression

Point 2: Bar stress near tension face of member equal to zero, ($f_s = 0$)

Point 3: Bar stress near tension face of member equal to $0.5f_y$ ($f_s = -0.5f_y$)

Point 4: Bar stress near tension face of member equal to f_y ($f_s = -f_y$)

Point 5: Pure bending

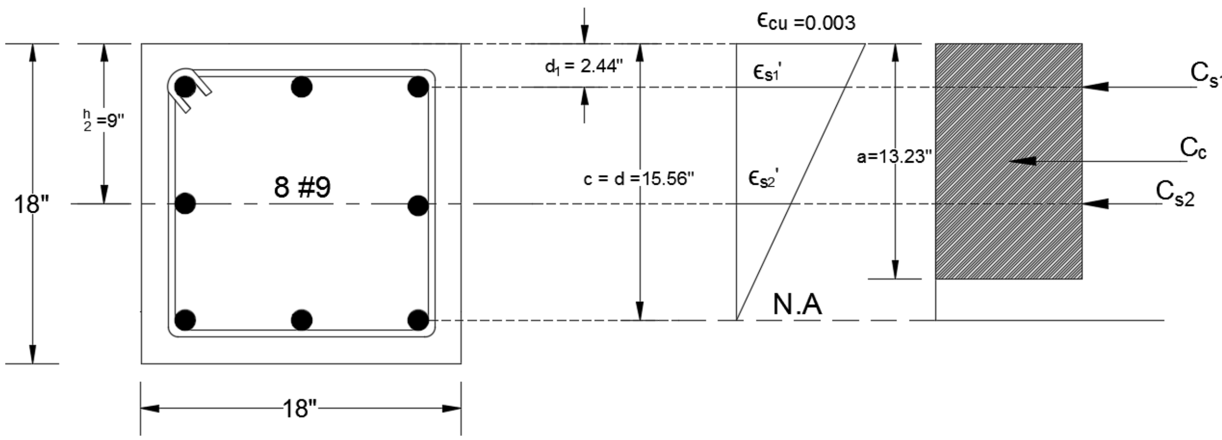
Point 1 (Pure compression):

$$\phi P_{n(max)} = 0.80\phi A_g[0.85f_c^{'} + \rho_g(f_y - 0.85f_c^{'})]$$

$$= 0.80 \times 0.65 \times 324[(0.85 \times 4) + 0.0247 \times (60 - (0.85 \times 4))]$$

$$= 808.3 \text{ kips}$$

Point 2 ($f_s = 0$):



Determine the Neutral Axis Depth, c

Given: $\varepsilon_s = 0$

Therefore, neutral axis depth, c is equal to effective depth, d and the tensile Force, T = 0

c = 15.56 in

a = 0.85 x 15.56 = 13.23 in

Concrete compressive block force

$C_c = 0.85f_c'ab$

$C_c = 0.85 \times 4 \times 13.23 \times 18 = 809.68$ kips

Check whether compression reinforcement has yielded:

Strain in first top layer:

$$\frac{\varepsilon_{s1}'}{(c - d_1)} = \frac{0.003}{c}$$

$$\frac{\varepsilon_{s1}'}{(15.56 - 2.44)} = \frac{0.003}{15.56}$$

$\varepsilon_{s1}' = 0.00253 > \varepsilon_y = 0.00207 > \varepsilon_t = 0.002$

Therefore, section is compression control. Use $\phi = 0.65$

First/top layer of compression reinforcement has yielded.

Compressive force, $C_{s1} = A_{s1}' \times (f_y - 0.85 f_c')$

$C_{s1} = 3 \times 1 \times (60 - 0.85 \times 4) = 169.8$ kips

Strain in Second layer of compression reinforcement;

$$\frac{\varepsilon_{s2}'}{(c - 9)} = \frac{0.003}{c}$$

$$\frac{\varepsilon_{s2}'}{(15.56 - 9)} = \frac{0.003}{15.56}$$

$\varepsilon_{s2}' = 0.00126 < \varepsilon_y = 0.00207$

Therefore, second layer of compression reinforcement has not yielded.

Compressive force, $C_{s2} = A_{s2}'\left(\varepsilon_{s2}' Es - 0.85 f_c'\right)$

$C_{s2} = 2 \times 1 \times (0.00126 \times 29000 - 0.85 \times 4) = 66.28$ kips

Strain on tension side

$\varepsilon_s = 0$

Tensile force, $T = 0$

$P_n = C_c + C_{s1} + C_{s2} - T$

$P_n = 809.68 + 169.8 + 66.28 - 0$

$P_n = 1045.71$ kips

$\phi P_n = 0.65 \text{ x } 1045.71 = 679.71$ kips

Moment Capacity

Summing moments around the centroidal axis;

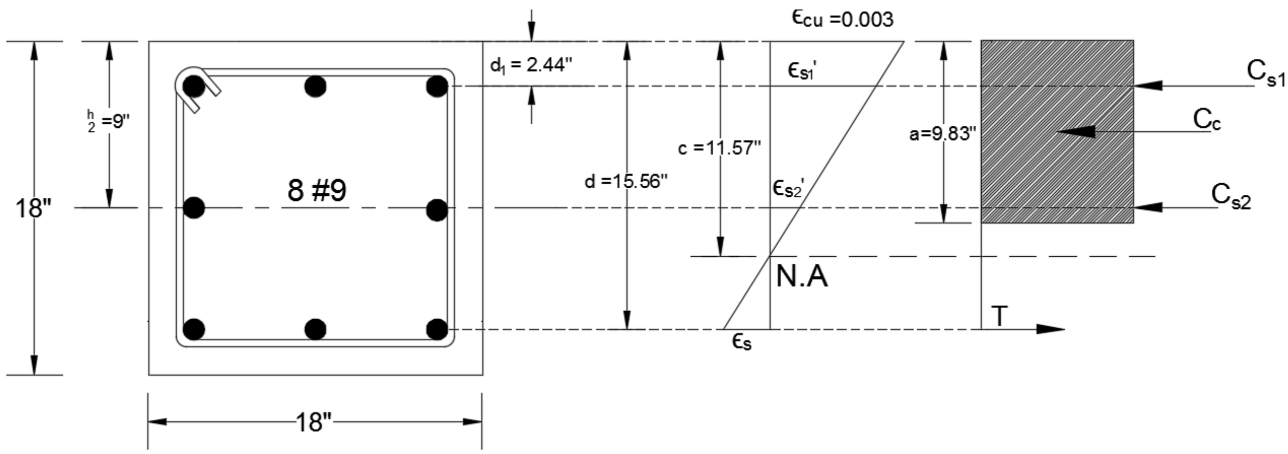
$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_{s1} \times \left(\frac{h}{2} - d_1\right) + T \times \left(\frac{h}{2} - d_1\right)$$

$$M_n = 809.68 \times \left(\frac{18}{2} - \frac{13.23}{2}\right) + 169.8 \times \left(\frac{18}{2} - 2.44\right)$$

$$M_n = 3045 \text{ k-in} = 253.75 \text{ k-ft}$$

$$\phi M_n = 0.65 \text{ x } 253.75 = 164.94 \text{ k-ft}$$

Point 3 ($f_s = -0.5f_y$):



Determine the Neutral Axis Depth, c

Given: $0.5\,\varepsilon_y = \varepsilon_s$

$\varepsilon_s = 0.5 \text{ x } 0.00207 = 0.001035$

Using similar triangle;

$$\frac{\varepsilon_s}{(d-c)} = \frac{\varepsilon_{cu}}{c}$$

$$\frac{0.001305}{(15.56-c)} = \frac{0.003}{c}$$

$$0.001035\,c = (15.56 - c) \times 0.003$$

$$0.001035\,c = 0.04668 - 0.003c$$

$c = 11.57$ in

$a = 0.85 \text{ x } 11.57 = 9.834$ in

Concrete compressive block force

$C_c = 0.85 f_c' ab$

$C_c = 0.85 \times 4 \times 9.834 \times 18 = 601.84$ kips

Checking whether compression reinforcement has yielded:

Strain in first top layer:

$$\frac{\varepsilon_{s1}'}{(c - d_1)} = \frac{0.003}{c}$$

$$\frac{\varepsilon_{s1}'}{(11.57 - 2.44)} = \frac{0.003}{11.57}$$

$\varepsilon_{s1}' = 0.00237 > \varepsilon_y = 0.00207 > \varepsilon_t = 0.002$

Therefore, section is compression control. Use $\phi = 0.65$

First/top layer of compression reinforcement has yielded.

Compressive force, $C_{s1} = A_{s1}'(f_y - 0.85 f_c')$

$C_{s1} = 3 \times 1 \times (60 - 0.85 \times 4) = 169.8$ kips

Strain in second layer of compression reinforcement;

$$\frac{\varepsilon_{s2}'}{(c - 9)} = \frac{0.003}{c}$$

$$\frac{\varepsilon_{s2}'}{(11.57 - 9)} = \frac{0.003}{11.57}$$

$\varepsilon_{s2}' = 0.00067 < \varepsilon_y = 0.00207$

Therefore, second layer of compression reinforcement has not yielded.

Compressive force, $C_{s2} = A_{s2}'\left(\varepsilon_{s2}' Es - 0.85 f_c'\right)$

$C_{s2} = 2 \times 1 \times (0.00067 \times 29000 - 0.85 \times 4) = 32.06$ kips

Strain on tension side

$\varepsilon_s = 0.001035$ *(Calculated previously)*

Tensile force, $T = A_s \varepsilon_s E_s$

$$T = 3 \times 1 \times 0.001035 \times 29000 = 90 \text{ kips}$$

$$P_n = C_c + C_{s1} + C_{s2} - T$$

$$P_n = 601.84 + 32.06 + 169.8 - 90$$

$$P_n = 713.7 \text{ kips}$$

$$\phi P_n = 0.65 \text{ x } 713.7 = 463.91 \text{ kips}$$

Moment Capacity

Summing moments around the centroidal axis;

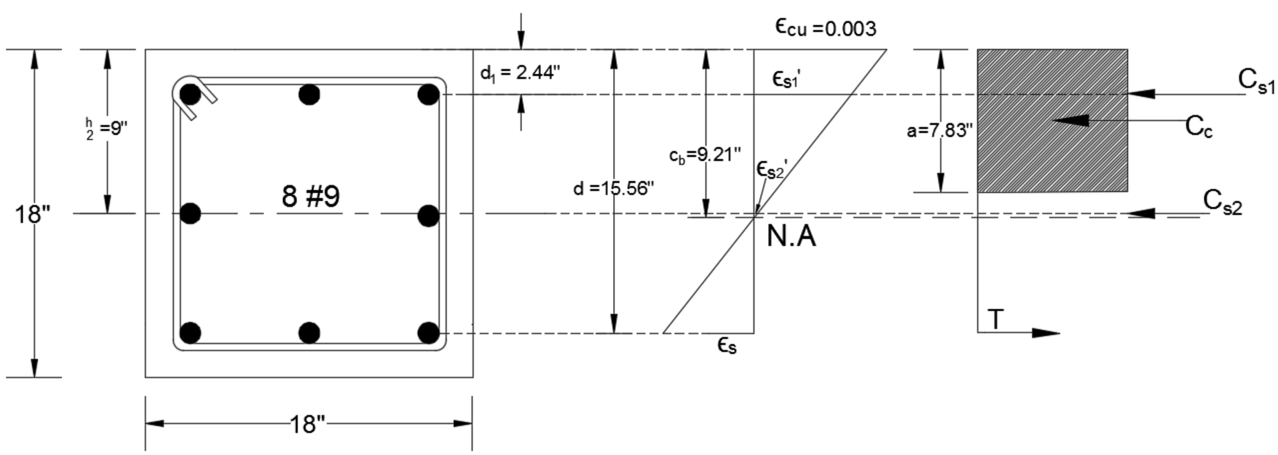
$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_{s1} \times \left(\frac{h}{2} - d_1\right) + T \times \left(\frac{h}{2} - d_1\right)$$

$$M_n = 601.84 \times \left(\frac{18}{2} - \frac{9.834}{2}\right) + 169.8 \times \left(\frac{18}{2} - 2.44\right) + 90 \times \left(\frac{18}{2} - 2.44\right)$$

$$M_n = 4161.6 \text{ k-in} = 346.75 \text{ k-ft}$$

$$\phi M_n = 0.65 \text{ x } 346.75 = 225.39 \text{ k-ft}$$

Point 4 ($f_s = -f_y$):



Determine the Neutral Axis Depth, c

Given: $\varepsilon_y = \varepsilon_s$ (Balanced condition)

At balanced condition, neutral axis depth is given by;

$$c_b = \frac{0.003}{0.003+\frac{f_y}{E_s}} = = \frac{0.003}{0.003+0.00207}$$

$$c_b = 9.21 \text{ in}$$

$$a_b = 0.85 \text{ x } 9.21 = 7.83 \text{ in}$$

Concrete compressive block force

$$C_c = 0.85 f_c' ab$$

$$C_c = 0.85 \times 4 \times 7.83 \times 18 = 479.2 \text{ kips}$$

Checking whether compression reinforcement has yielded:

Strain in first top layer:

$$\frac{\varepsilon_{s1}'}{(c - d_1)} = \frac{0.003}{c}$$

$$\frac{\varepsilon_{s1}'}{(9.21 - 2.44)} = \frac{0.003}{9.21}$$

$$\varepsilon_{s1}' = 0.00221 > \varepsilon_y = 0.00207 > \varepsilon_t = 0.002$$

Therefore, section is compression control. Use $\phi = 0.65$

First/top layer of compression reinforcement has yielded.

Compressive force, $C_{s1} = A_{s1}'(f_y - 0.85 f_c')$

$$C_{s1} = 3 \times 1 \times (60 - 0.85 \times 4) = 169.8 \text{ kips}$$

Strain in second layer of compression reinforcement;

$$\frac{\varepsilon_{s2}'}{(c - 9)} = \frac{0.003}{c}$$

$$\frac{\varepsilon_{s2}'}{(9.21 - 9)} = \frac{0.003}{9.21}$$

$$\varepsilon_{s2}' = 0.000068 < \varepsilon_y = 0.00207$$

Therefore, second layer of compression reinforcement has not yielded.

Compressive force, $C_{s2} = A_{s2}'(\varepsilon_{s2}' Es - 0.85 f_c')$

Since a = 7.83 in, second layer of reinforcement does not lie within the compression block zone. Therefore, above equation changes to the following:

$$C_{s2} = A_{s2}'(\varepsilon_{s2}' Es)$$

$$C_{s2} = 2 \times 1 \times (0.00068 \times 29000) = 3.94 \text{ kips}$$

Strain on tension side

Since, $\varepsilon_s = \varepsilon_s$ (Balanced condition)

Therefore, tensile reinforcement has yielded.

Tensile force, $T = A_s f_y$

$$T = 3 \times 1 \times 60 = 180 \text{ kips}$$

$$P_n = C_c + C_{s1} + C_{s2} - T$$

$$P_n = 479.2 + 169.8 + 3.94 - 180 = 472.94$$

$$\phi P_n = 0.65 \text{ x } 472.94 = 307.41 \text{ kips}$$

Moment Capacity

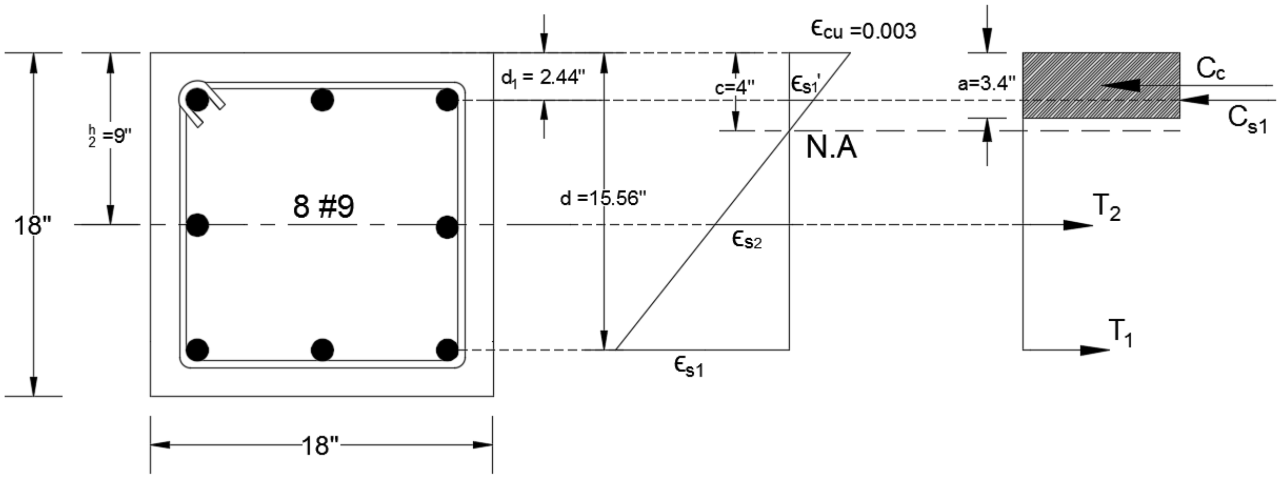
Summing moments around the centroidal axis;

$$M_n = 479.2 \times \left(\frac{18}{2} - \frac{7.83}{2}\right) + 169.8 \times \left(\frac{18}{2} - 2.44\right) + 180 \times \left(\frac{18}{2} - 2.44\right)$$

$$M_n = 4731.42 \text{ k-in} = 394.29 \text{ k-ft}$$

$$\phi M_n = 0.65 \text{ x } 394.29 = 256.29 \text{ k-ft}$$

Point 5 (Pure Bending):



Use iterative procedure to determine ϕM_n

Try $c = 4.0$ in.

a = 0.85 x 4 = 3.4 in

Concrete compressive block force

$C_c = 0.85 f_c' ab$

$C_c = 0.85 \times 4 \times 3.4 \times 18 = 208.08$ kips

Strain on the tension side

Bottom layer in the tension zone

$$\varepsilon_{s1} = 0.003 \times \left(\frac{d - c}{c}\right) = 0.003 \times \left(\frac{15.56 - 4}{4}\right) = 0.00867$$

$$\varepsilon_y = 0.00207 < \varepsilon_{s1} = = 0.00867 > \varepsilon_t = 0.005$$

Therefore, section is tension control. Use $\phi = 0.9$

Therefore, bottom layer of tensile reinforcement has yielded.

$T_1 = 3 \times 1 \times 60 = 180$ kips

Middle layer in the tension zone

$$\varepsilon_{s2} = 0.003 \times \left(\frac{h/2 - c}{c}\right) = 0.003 \times \left(\frac{9 - 4}{4}\right) = 0.00375$$

$$\varepsilon_y = 0.00207 < \varepsilon_{s2} = = 0.00867$$

Therefore, middle layer of tensile reinforcement has yielded.

$$T_2 = 2 \times 1 \times 60 = 120 \text{ kips}$$

Strain on the compression side

$$\varepsilon_{s1}' = 0.003 \times \left(\frac{c - d_1}{c} \right) = 0.003 \times \left(\frac{4 - 2.44}{4} \right) = 0.0017$$

$$\varepsilon_{s1}' = 0.0017 < \varepsilon_y = 0.00207$$

Therefore, compression reinforcement has not yielded.

$$C_{s1} = A_{s1}'(\varepsilon_{s1}' Es - 0.85 f_c')$$

$$C_{s1} = 2 \times 1 \times (0.0017 \times 29000 - 0.85x4) = 91.8 \text{ kips}$$

Total tensile force, $T = T_1 + T_2$

$$T = 180 + 120 = 300 \text{ kips}$$

Total compressive force, $C = C_c + C_{s1}$

$$C = 208.08 + 91.8 = 299.88 \text{ kips} \approx 300 \text{ kips}$$

Since $T \approx C$, therefore, use $c = 4.0$ in

$$M_n = 208.08 \times \left(\frac{18}{2} - \frac{3.4}{2} \right) + 91.8 \times \left(\frac{18}{2} - 2.44 \right) + 180 \times \left(\frac{18}{2} - 2.44 \right)$$

$$M_n = 3302 \text{ k-in} = 275.17 \text{ k-ft}$$

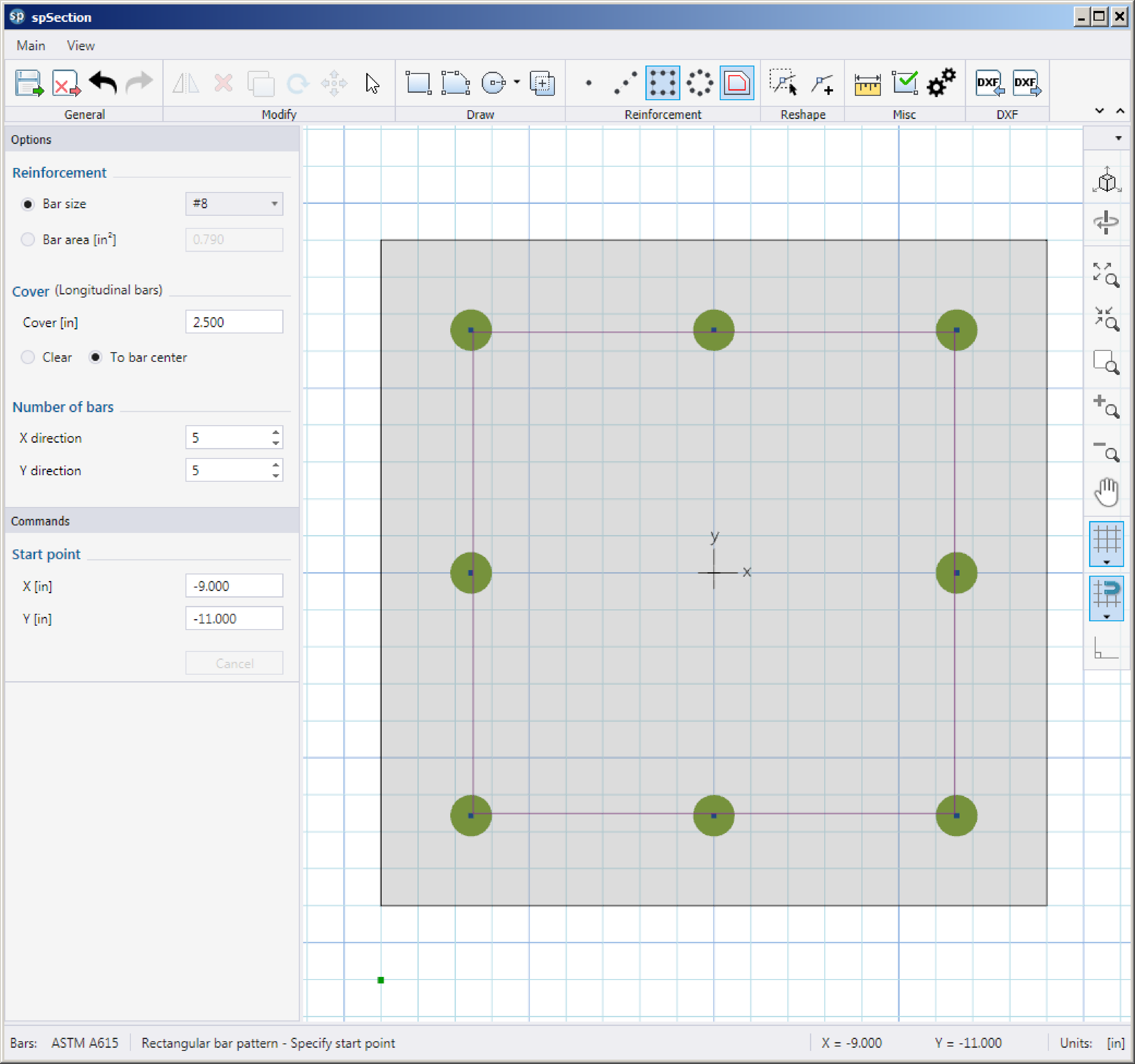
$$\phi M_n = 0.9 \text{ x } 275.17 = 247.65 \text{ k-ft}$$

Conclusions and Observations

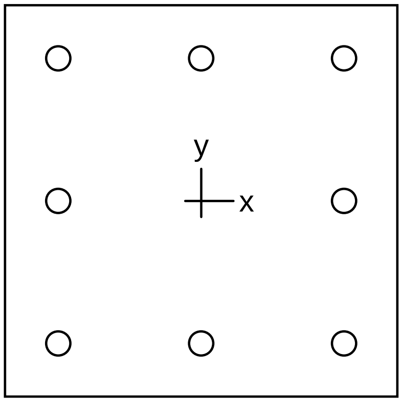
As summarized in Table 1 below and plotted within the spColumn interaction diagram output, the hand-calculated control point values (red line) of the interaction diagram are in very good agreement with the control point values of the interaction diagram generated by spColumn Program (black line).

Table 1 – Comparison of spColumn and hand-calculated interaction diagram values

Point	Parameter	Hand Calculated Values	spColumn Values (exact)
1: Pure Compression	ϕP_n (kips)	808.3	808.3
2: $f_s = 0$	ϕP_n (kips)	679.71	679.8
	ϕM_n (ft-kips)	164.94	164.99
3: $f_s = -0.5f_y$	ϕP_n (kips)	463.91	463.8
	ϕM_n (ft-kips)	225.39	225.43
4: $f_s = -f_y$	ϕP_n (kips)	307.41	307.3
	ϕM_n (ft-kips)	259.21	259.2
5: Pure Bending	ϕM_n (ft-kips)	247.65	247.72



spSection
Main
View
General
Modify
Draw
Reinforcement
Reshape
Misc
DXF
Options
Reinforcement
Bar size
#8
Bar area [in²]
0.790
Cover (Longitudinal bars)
Cover [in]
2.500
Clear
To bar center
Number of bars
X direction
5
Y direction
5
Commands
Start point
X [in]
-9.000
Y [in]
-11.000
Cancel
y
x
Bars: ASTM A615
Rectangular bar pattern - Specify start point
X = -9.000
Y = -11.000
Units: [in]



18 x 18 in

Code: ACI 318-14

Units: English

Run axis: About X-axis

Run option: Investigation

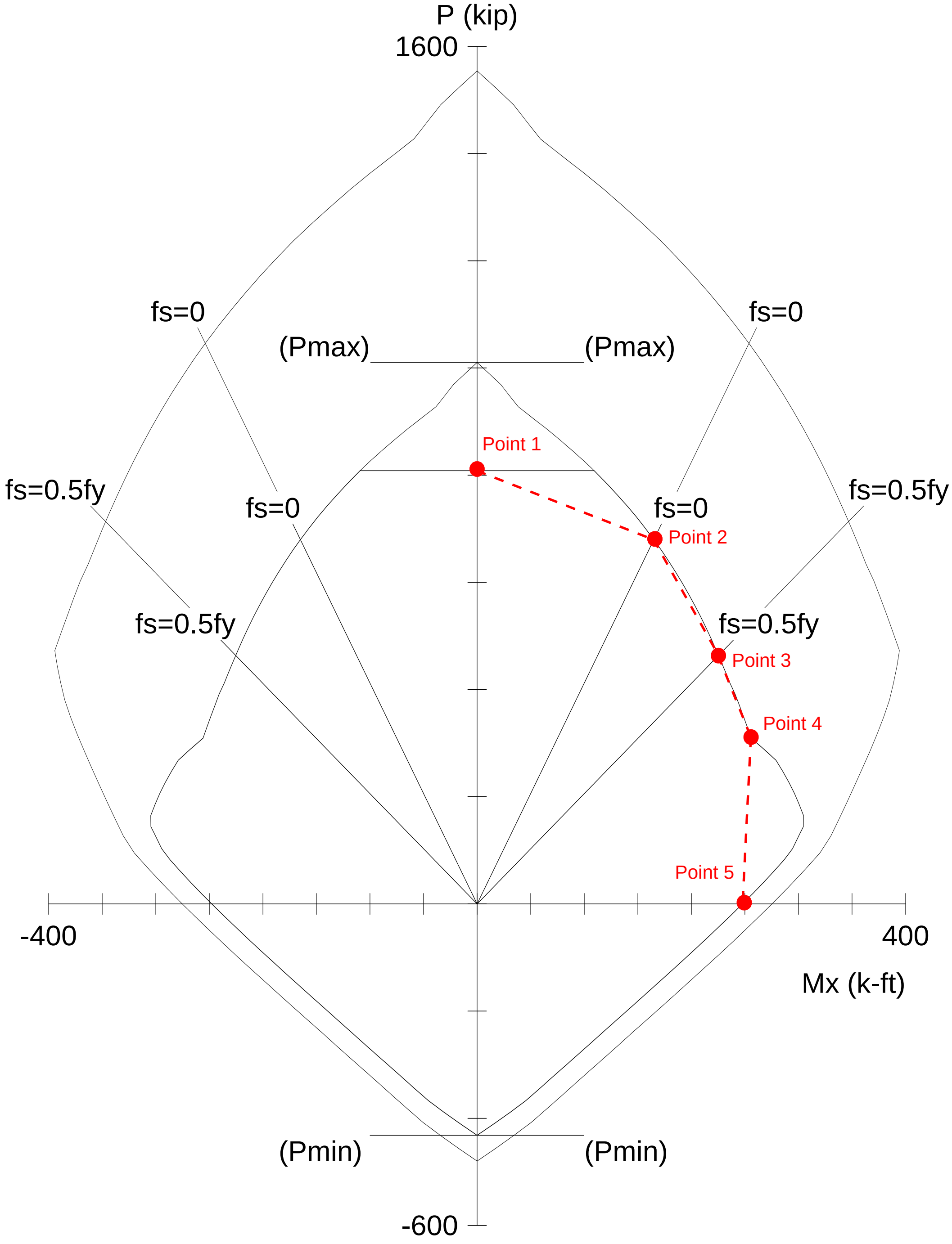
Slenderness: Not considered

Column type: Structural

Bars: ASTM A615

Date: 07/20/16

Time: 14:42:41



STRUCTUREPOINT - spColumn v5.10 (TM). Licensed to: StructurePoint. License ID: 00000-0000000-4-2A05D-2471B

File: C:\TSDA-spColumn-Interaction Diagram-Control Points.col

Project: Interaction Dia Control Points

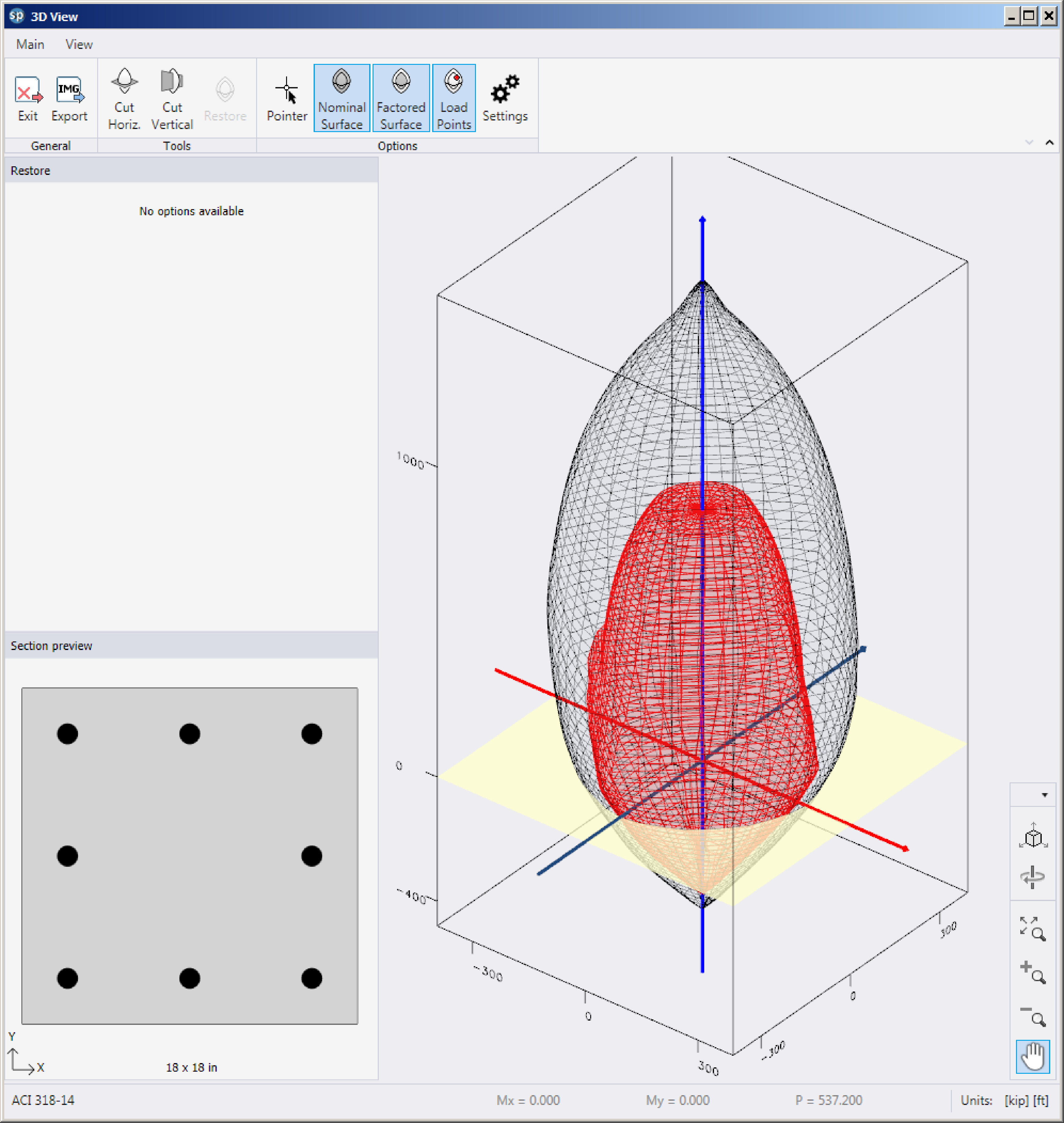
Column: Interior Col

Engineer: SP

f'c = 4 ksi	fy = 60 ksi	Ag = 324 in^2	8 #9 bars
Ec = 3605 ksi	Es = 29000 ksi	As = 8.00 in^2	rho = 2.47%
fc = 3.4 ksi	e_yt = 0.00206897 in/in	Xo = 0.00 in	Ix = 8748 in^4
e_u = 0.003 in/in		Yo = 0.00 in	Iy = 8748 in^4
Beta1 = 0.85		Min clear spacing = 5.43 in	Clear cover = 1.88 in

Confinement: Tied

phi(a) = 0.8, phi(b) = 0.9, phi(c) = 0.65


3D View
Main
View
Exit
Export
General
Cut Horiz.
Cut Vertical
Restore
Tools
Pointer
Nominal Surface
Factored Surface
Load Points
Settings
Options
Restore
No options available
Section preview
Y
X
18 x 18 in
1000
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-400
-300
0
300
300
0
-300
ACI 318-14
Mx = 0.000
My = 0.000
P = 537.200
Units: [kip] [ft]

General Information:

File Name: C:\TSDA-spColumn-Interaction Diagram-Control Points.col
Project: Interaction Dia Control Points
Column: Interior Col — Engineer: SP
Code: ACI 318-14 — Units: English

Run Option: Investigation — Slenderness: Not considered
Run Axis: X-axis — Column Type: Structural

Material Properties:

Concrete: Standard — Steel: Standard
f'c = 4 ksi — fy = 60 ksi
Ec = 3605 ksi — Es = 29000 ksi
fc = 3.4 ksi — Eps_yt = 0.00206897 in/in
Eps_u = 0.003 in/in
Beta1 = 0.85

Section:

Rectangular: Width = 18 in — Depth = 18 in

Gross section area, Ag = 324 in^2
Ix = 8748 in^4 — Iy = 8748 in^4
rx = 5.19615 in — ry = 5.19615 in
Xo = 0 in — Yo = 0 in

Reinforcement:

Bar Set: ASTM A615

Size	Diam (in)	Area (in^2)	Size	Diam (in)	Area (in^2)	Size	Diam (in)	Area (in^2)
# 3	0.38	0.11	# 4	0.50	0.20	# 5	0.63	0.31
# 6	0.75	0.44	# 7	0.88	0.60	# 8	1.00	0.79
# 9	1.13	1.00	# 10	1.27	1.27	# 11	1.41	1.56
# 14	1.69	2.25	# 18	2.26	4.00			

Confinement: Tied; #3 ties with #10 bars, #4 with larger bars.
phi(a) = 0.8, phi(b) = 0.9, phi(c) = 0.65

Layout: Rectangular
Pattern: All Sides Equal (Cover to transverse reinforcement)
Total steel area: As = 8.00 in^2 at rho = 2.47%
Minimum clear spacing = 5.43 in

8 #9 Cover = 1.5 in

Control Points:

Bending about	Axial Load P kip	X-Moment k-ft	Y-Moment k-ft	NA depth in	Dt depth in	eps_t	Phi
X @ Max compression	1010.4	-0.00	0.00	50.14	15.56	-0.00207	0.650
@ Allowable comp.	808.3	109.38	0.00	18.46	15.56	-0.00047	0.650
@ fs = 0.0	679.8	164.99	0.00	15.56	15.56	0.00000	0.650
@ fs = 0.5*fy	463.8	225.43	0.00	11.57	15.56	0.00103	0.650
@ Balanced point	307.3	256.30	0.00	9.21	15.56	0.00207	0.650
@ Tension control	153.8	306.74	0.00	5.84	15.56	0.00500	0.900
@ Pure bending	0.0	247.72	0.00	4.00	15.56	0.00866	0.900
@ Max tension	-432.0	-0.00	0.00	0.00	15.56	9.99999	0.900
-X @ Max compression	1010.4	-0.00	-0.00	50.14	15.56	-0.00207	0.650
@ Allowable comp.	808.3	-109.38	-0.00	18.46	15.56	-0.00047	0.650
@ fs = 0.0	679.8	-164.99	-0.00	15.56	15.56	-0.00000	0.650
@ fs = 0.5*fy	463.8	-225.43	-0.00	11.57	15.56	0.00103	0.650
@ Balanced point	307.3	-256.30	-0.00	9.21	15.56	0.00207	0.650
@ Tension control	153.8	-306.74	-0.00	5.84	15.56	0.00500	0.900
@ Pure bending	0.0	-247.72	-0.00	4.00	15.56	0.00866	0.900
@ Max tension	-432.0	-0.00	0.00	0.00	15.56	9.99999	0.900

*** End of output ***