

### Shear and Moment Diagrams for a Continuous Beam

The slope-deflection method is used to determine the shear and moment diagram for the beam shown below. A comparison between results obtained from the hand solution and spSlab/spBeam software is provided to illustrate the features and capabilities of the spBeam and spSlab software programs.

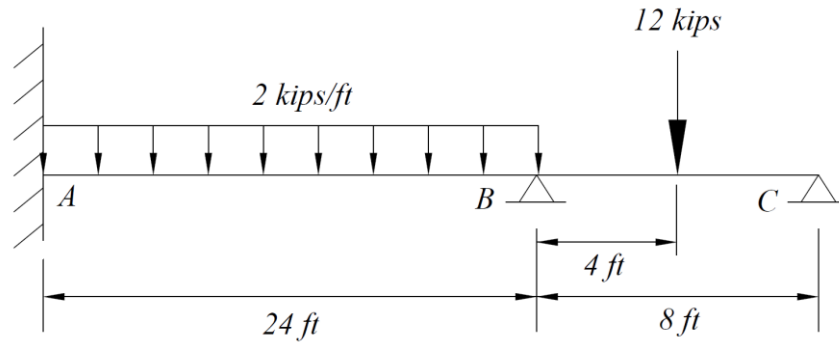


Figure 1 – Continuous Beam

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**Scope**

|  |   |
|--|---|
| 1. Determine the Fixed-End Moments (FEM) .....         | 1 |
| 2. Slope-Deflection Equations.....                     | 2 |
| 2.1. Span AB (End Span with Far End Fixed) .....       | 2 |
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## Reference

Structural Analysis. Eighth Edition, 2012 R. C. Hibbeler, Example 11.2

## Analysis Data

$EI$  is considered constant

$$L_{AB} = 24 \text{ ft}$$

$$L_{BC} = 8 \text{ ft}$$

Support  $A$  is fixed

Supports  $B$  and  $C$  are pinned

$w_u = 2$  kips/ft between supports  $A$  and  $B$

$P_u = 12$  kips at 4 ft away from support  $B$

## Solution

The slope-deflection technique is used to analyze indeterminate beams and framed structures along with the moment distribution technique, this method was originally developed in the 1915 by G. Manderla and O Mohr to investigate the secondary stresses in trusses. G. A. Maney developed this technique and applied it to the analysis of indeterminate beams and framed structures. The following shows a detailed analysis of two-span beam using slope-deflection technique. The results are compared with values obtained from spSlab software.

### 1. Determine the Fixed-End Moments (FEM)

$$FEM_{AB} = -\frac{wL^2}{12} = -\frac{2 \times 24^2}{12} = -96 \text{ kips} - \text{ft}$$

$$FEM_{BA} = \frac{wL^2}{12} = \frac{2 \times 24^2}{12} = 96 \text{ kips} - \text{ft}$$

$$FEM'_{BC} = -\frac{3PL}{16} = -\frac{3 \times 12 \times 8}{16} = -18 \text{ kips} - \text{ft}$$

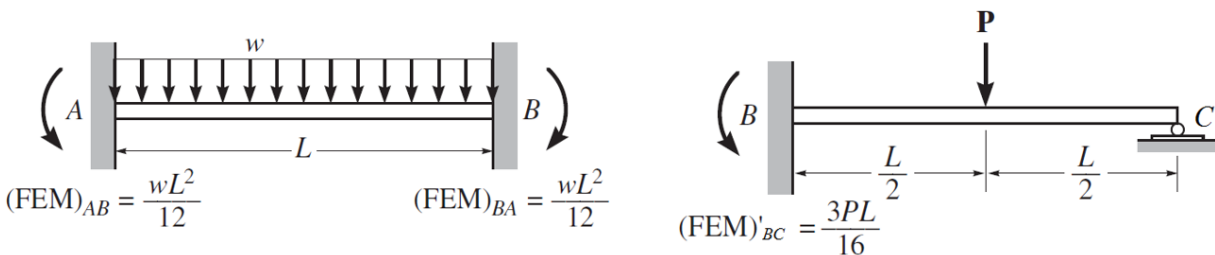


Figure 2 – Fixed-End Moments Equations

## 2. Slope-Deflection Equations

### 2.1. Span AB (End Span with Far End Fixed)

Span AB is end span with far end fixed, the general slope-deflection equation should be used:

$$M_N = 2Ek(2\theta_N + \theta_F - 3\Psi) + FEM_N$$

Where:

$M_N$  = internal moment in the near end of the span; *kips-ft*

This moment is positive clockwise when acting on the span.

$E$  = modulus of elasticity; *ksi*

$k$  = span stiffness =  $I/L$ ; *in<sup>3</sup>*

$\theta_N$  = near-end slope or angular displacement of the span at the support; *rad positive clockwise*

$\theta_F$  = far-end slope or angular displacement of the span at the support; *rad positive clockwise*

$\Psi$  = span rotation of its cord due to a linear displacement =  $\Delta/L$ ; *rad positive clockwise*

Since the supports do not settle,  $\Psi_{AB} = \Psi_{BC} = 0$ .

$FEM_N$  = fixed-end moment at the near-end support; *kips-ft positive clockwise when acting on the span*

Note that by inspection,  $\theta_A = 0$ , for  $M_{AB}$ :

$$M_{AB} = 2E\left(\frac{I}{L}\right)(2\theta_A + \theta_B - 3(0)) + FEM_{AB} = 2E\left(\frac{I}{24}\right)(2(0) + \theta_B - 3(0)) - 96$$

$$M_{AB} = 0.08333EI\theta_B - 96 \quad \text{Eq. (1)}$$

For  $M_{BA}$ :

$$M_{BA} = 2E\left(\frac{I}{L}\right)(2\theta_B + \theta_A - 3(0)) + FEM_{BA} = 2E\left(\frac{I}{24}\right)(2\theta_B + 0 - 3(0)) + 96$$

$$M_{BA} = 0.1667EI\theta_B + 96 \quad \text{Eq. (2)}$$

### 2.2. Span BC (End Span with Far End Pinned)

Span BC is end span with far end pinned, the following slope-deflection equation can be used so that it has to be applied only once to the span rather than twice:

$$M_N = 3Ek(\theta_N - \Psi) + FEM_N$$

Applying this equation for span BC with B as the near end and C as the far end:

$$M_{BC} = 3E\left(\frac{I}{L}\right)(\theta_B - (0)) + FEM_{BC} = 3E\left(\frac{I}{8}\right)(\theta_B - (0)) - 18$$

$$M_{BC} = 0.375EI\theta_B - 18 \quad \text{Eq. (3)}$$

### 3. Equilibrium Equations

The conditions of equilibrium at the support B is used to find the fourth equation that is necessary to calculate the four unknowns from the above three equations.

From the free body diagram for support B shown in Figure 3:

$$M_{BA} + M_{BC} = 0 \quad \text{Eq. (4)}$$

Where counterclockwise is positive.

By substituting Eq. (2) and Eq. (3) into Eq. (4)

$$(0.1667EI\theta_B + 96) + (0.375EI\theta_B - 18) = 0$$

$$\theta_B = -\frac{144}{EI}$$

By substituting  $\theta_B$  into Eq. (1), Eq. (2), and Eq. (3):

$$M_{AB} = -108.0 \text{ kips} - \text{ft}$$

$$M_{BA} = +72.0 \text{ kips} - \text{ft}$$

$$M_{BC} = -72.0 \text{ kips} - \text{ft}$$

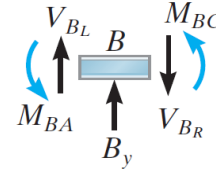


Figure 3 – Free Body Diagram for Support B

### 4. Shear and Moment Diagrams

Using these moments, the shear reactions at the ends of the beam spans can be found as shown in Figure 4. The shear and moment diagrams are plotted in Figure 5.

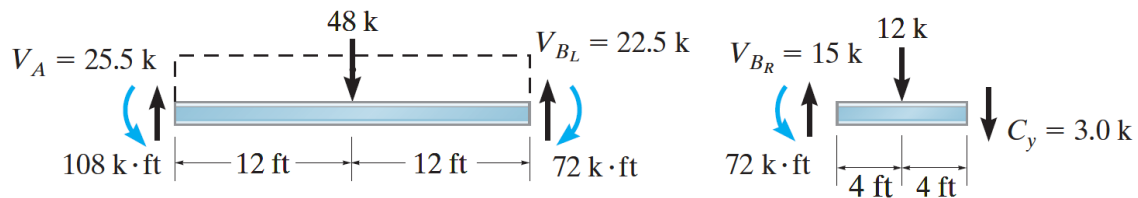


Figure 4 – Moments and Shear Reactions at the Ends of the Beam Spans

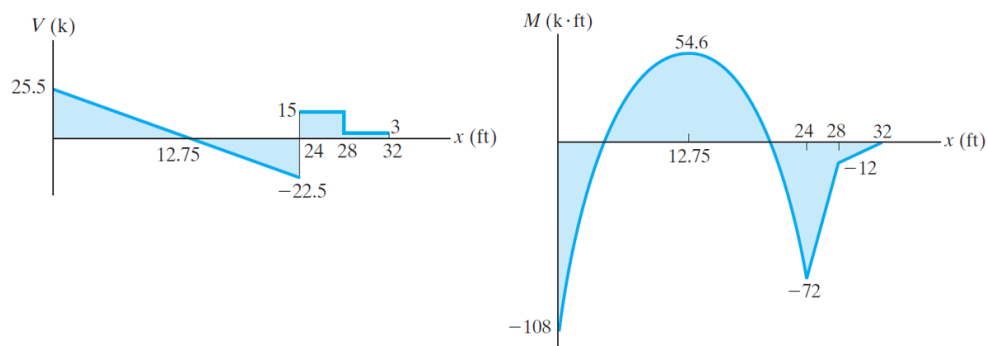


Figure 5 – Shear and Moment Diagrams

**5. spSlab/spBeam Software Program Model Solution**

spSlab/spBeam is utilized to analyze continuous beams and one-way slabs using the stiffness method. The software calculates the internal forces (shear forces and bending moments), moment and shear capacities, immediate and long-term deflection results, and required flexural and shear reinforcement. The goal of this example is to show how spSlab/spBeam software calculates moment and shear values that are used to complete the design of beams and slabs in concrete floor systems.

The graphical and text results are provided in Appendix A for both input and output of the spSlab/spBeam model.

**6. Summary and Comparison of Results**

The moment diagram from the hand calculation shows positive moments on the compression side. While the positive moments from spSlab/spBeam is drawn on the tension side.

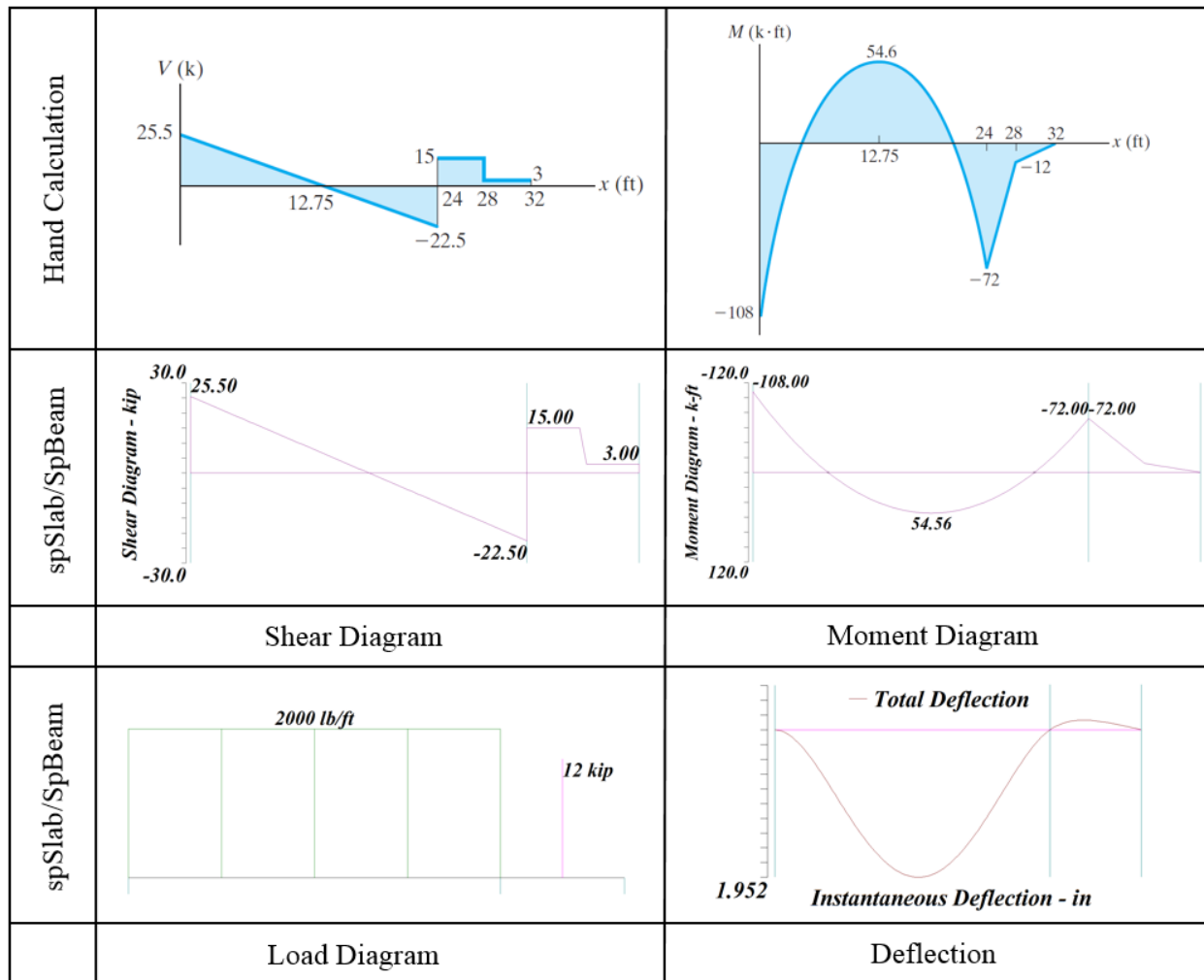
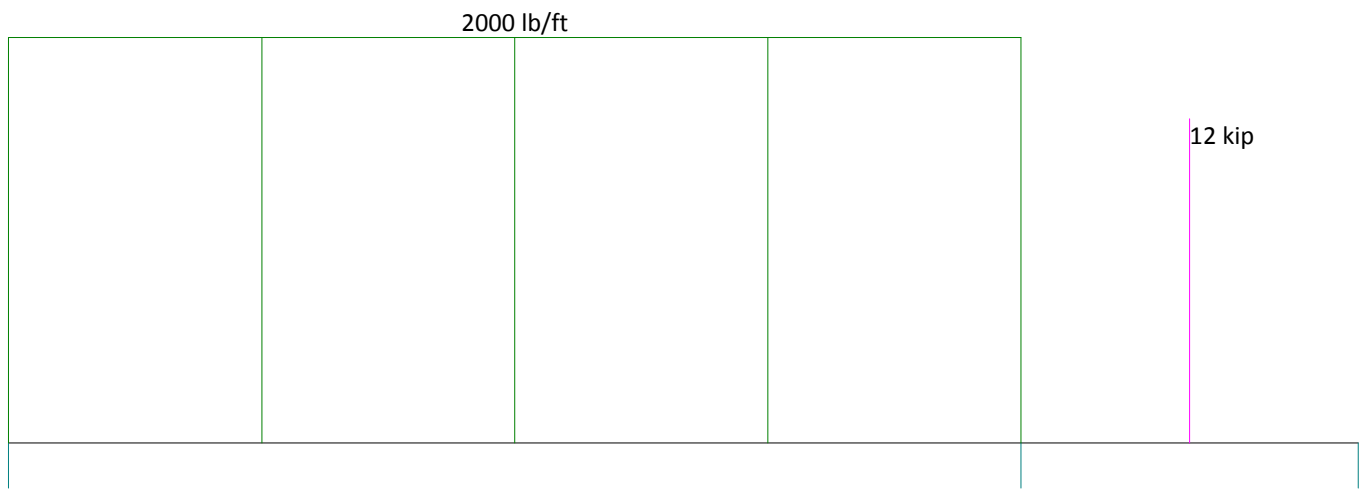


Figure 6 – Shear and Moment Diagrams

## 7. Conclusions & Observations

The results from hand solution using slope-deflection and spSlab/spBeam software using the stiffness method are exactly identical. spBeam/spSlab output provides a host of other detailed output such as deflections and reactions. The output is also used to determine required flexural, shear, and torsion reinforcement for concrete floor systems. The software can be used for up to 20 spans including various load type and conditions and considers live load patterning and moment redistribution for up to 255 load combinations.



CASE: Dead

spSlab v5.00. Licensed to: StructurePoint. License ID: 66184-1055152-4-2C6B6-2C6B6

File: C:\Continuous Beam\Continuous\_Beam\_Slope\_Deflection.slb

Project: Continuous Beam

Frame: Continuous Beam

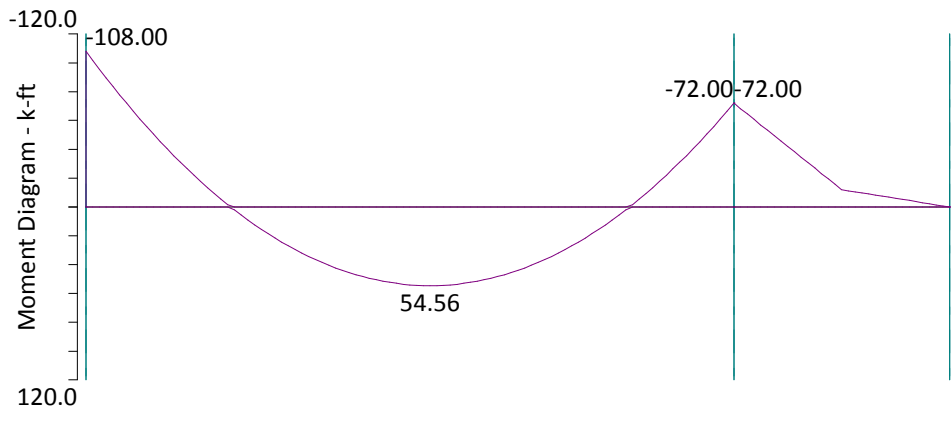
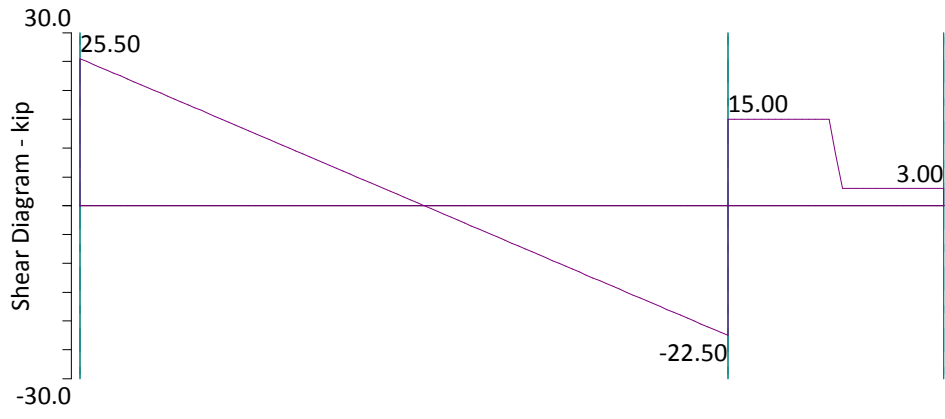
Engineer: SP

Code: ACI 318-14

Date: 12/19/16

Time: 15:09:14





LEGEND:  
— Envelope

spSlab v5.00. Licensed to: StructurePoint. License ID: 66184-1055152-4-2C6B6-2C6B6

File: C:\Continuous Beam\Continuous\_Beam\_Slope\_Deflection.slb

Project: Continuous Beam

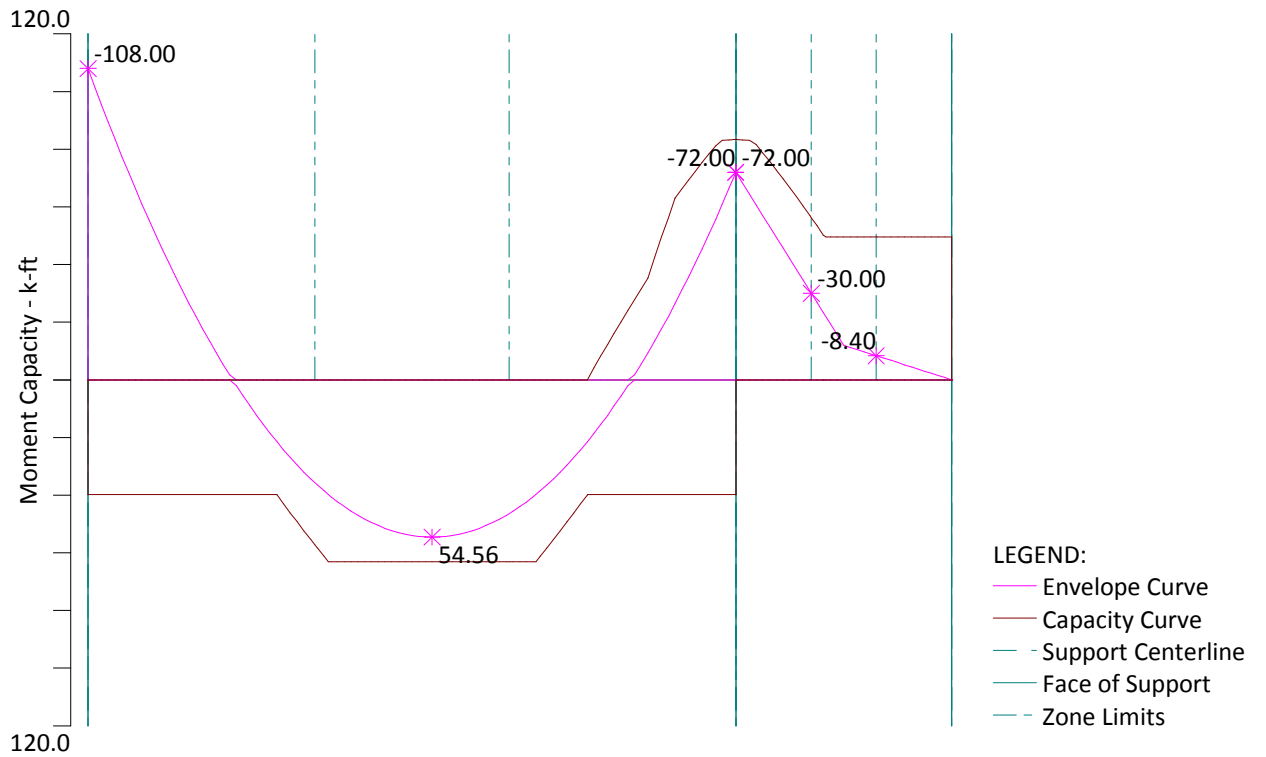
Frame: Continuous Beam

Engineer: SP

Code: ACI 318-14

Date: 12/19/16

Time: 15:10:01



spSlab v5.00. Licensed to: StructurePoint. License ID: 66184-1055152-4-2C6B6-2C6B6

File: C:\Continuous Beam\Continuous\_Beam\_Slope\_Deflection.slb

Project: Continuous Beam

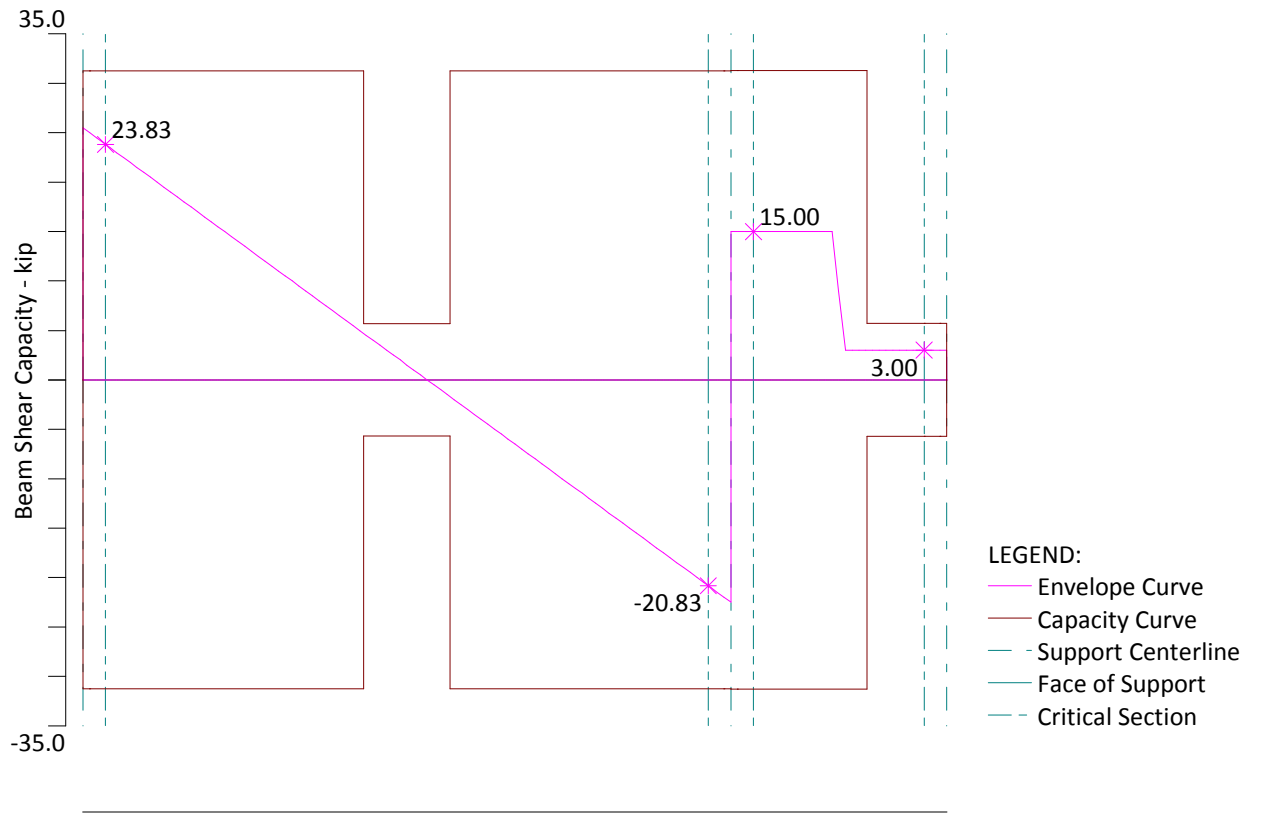
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Engineer: SP

Code: ACI 318-14

Date: 12/19/16

Time: 15:10:38



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Project: Continuous Beam

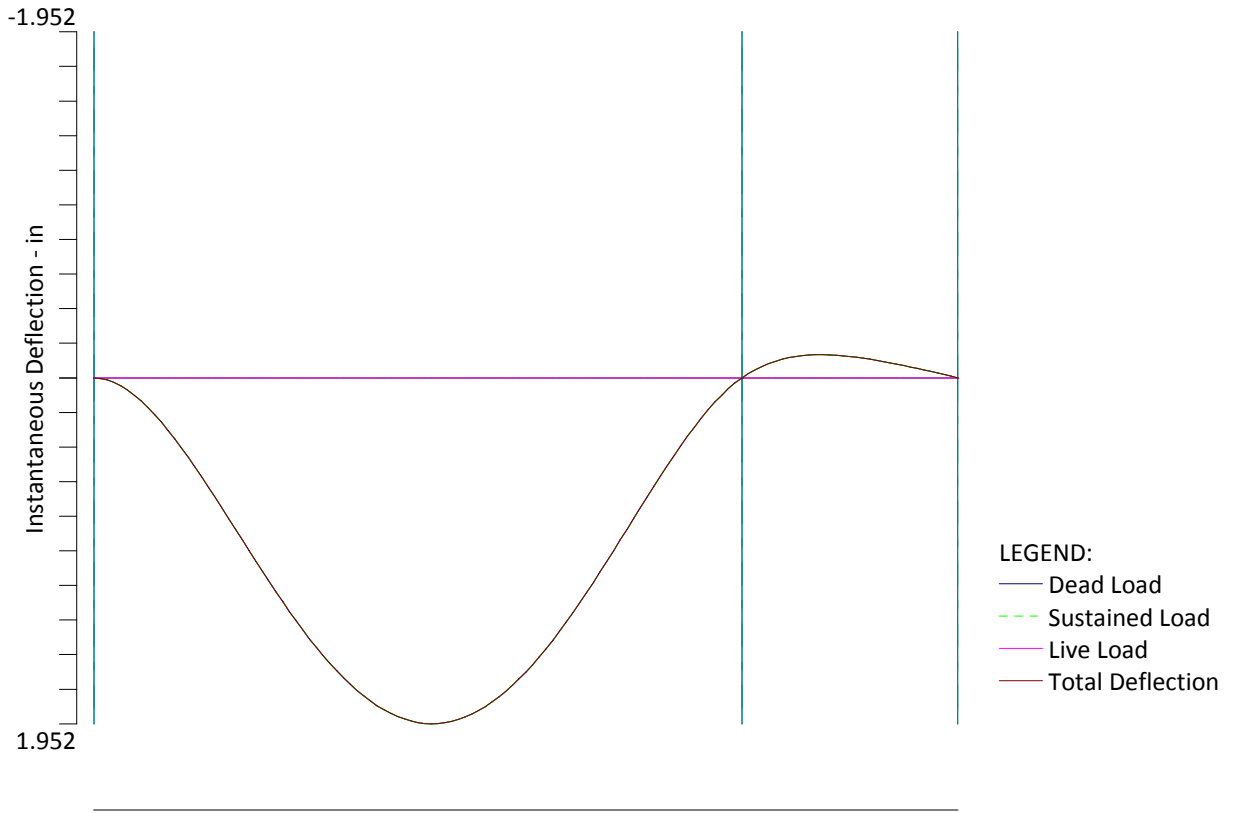
Frame: Continuous Beam

Engineer: SP

Code: ACI 318-14

Date: 12/19/16

Time: 15:11:11



spSlab v5.00. Licensed to: StructurePoint. License ID: 66184-1055152-4-2C6B6-2C6B6

File: C:\Continuous Beam\Continuous\_Beam\_Slope\_Deflection.slb

Project: Continuous Beam

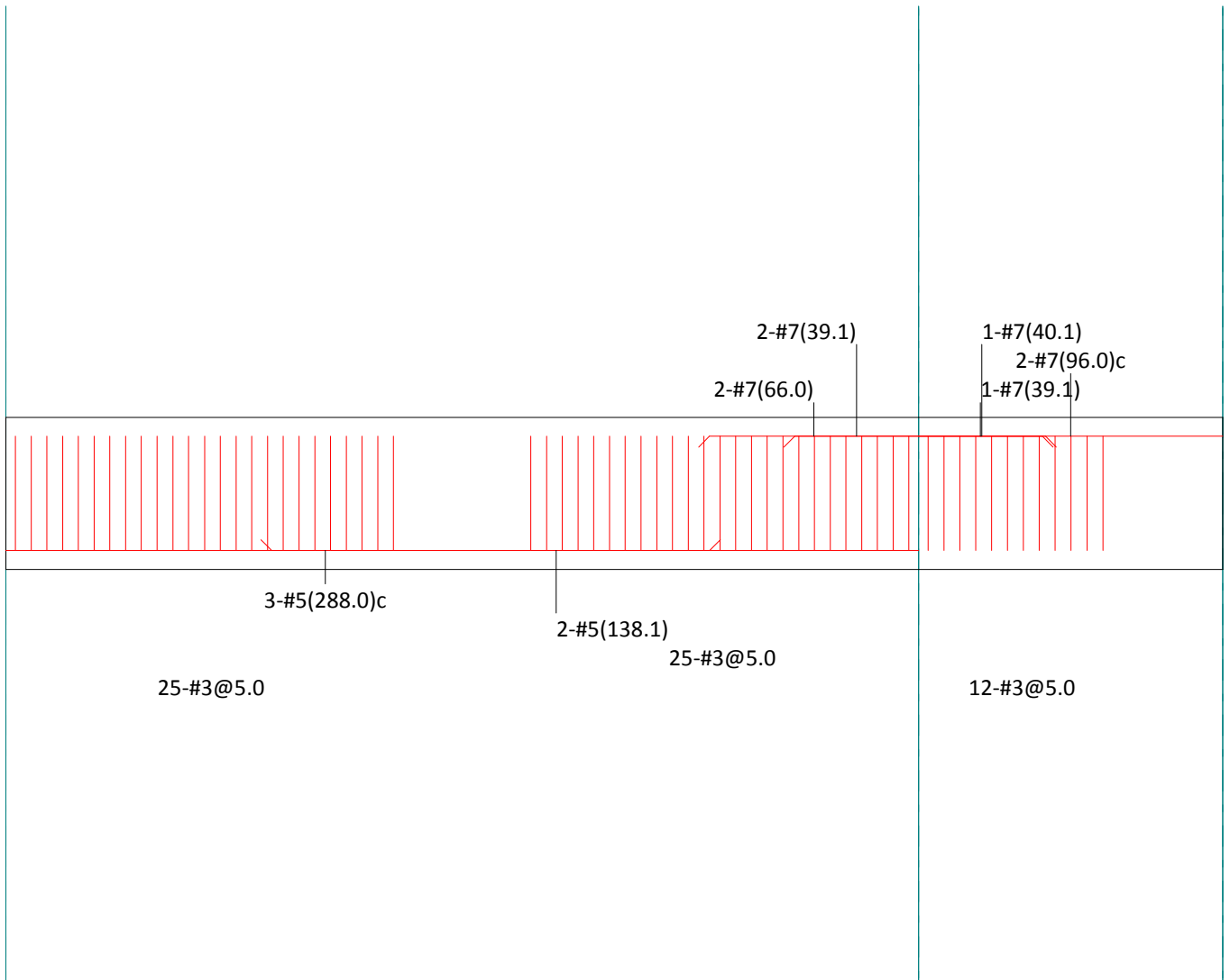
Frame: Continuous Beam

Engineer: SP

Code: ACI 318-14

Date: 12/19/16

Time: 15:11:27



Flexural and Transverse Reinforcement

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File: C:\Continuous Beam\Continuous\_Beam\_Slope\_Deflection.slb

Project: Continuous Beam

Frame: Continuous Beam

Engineer: SP

Code: ACI 318-14

Date: 12/19/16

Time: 15:13:22

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    oooooo   oo   oooooo   ooo   oooooo   o   oooooo   (TM)
  
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[1] INPUT ECHO

General Information

```

=====
File name: C:\Continuous Beam\Continuous_Beam_Slope_Deflection.slb
Project: Continuous Beam
Frame: Continuous Beam
Engineer: SP
Code: ACI 318-14
Reinforcement Database: ASTM A615
Mode: Design
Number of supports = 3
Floor System: One-Way/Beam
  
```

```

Live load pattern ratio = 100%
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Moment redistribution NOT selected.
Effective flange width calculations NOT selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.
  
```

Material Properties

```

=====
                Slabs|Beams          Columns
                -----
wc   =          150                150 lb/ft3
f'c  =           4                  4 ksi
Ec   =        3834.3              3834.3 ksi
fr   =        0.47434            0.47434 ksi

fy   =          60 ksi, Bars are not epoxy-coated
fyt  =          60 ksi
Es   =        29000 ksi
  
```

Reinforcement Database

```

=====
Units: Db (in), Ab (in^2), Wb (lb/ft)
Size   Db   Ab   Wb   Size   Db   Ab   Wb
-----
#3     0.38  0.11  0.38  #4     0.50  0.20  0.67
#5     0.63  0.31  1.04  #6     0.75  0.44  1.50
#7     0.88  0.60  2.04  #8     1.00  0.79  2.67
#9     1.13  1.00  3.40  #10    1.27  1.27  4.30
#11    1.41  1.56  5.31  #14    1.69  2.25  7.65
#18    2.26  4.00  13.60
  
```

Span Data

=====

Slabs

-----

Units: L1, wL, wR (ft); t, Hmin (in)

| Span | Loc | L1     | t    | wL    | wR    | Hmin |
|------|-----|--------|------|-------|-------|------|
| 1    | Int | 24.000 | 0.00 | 0.500 | 0.500 | 0.00 |
| 2    | Int | 8.000  | 0.00 | 0.500 | 0.500 | 0.00 |

Ribs and Longitudinal Beams

-----

Units: b, h, Sp (in)

| Span | Ribs |      |      | Beams |       | Span     |
|------|------|------|------|-------|-------|----------|
|      | b    | h    | Sp   | b     | h     | Hmin     |
| 1    | 0.00 | 0.00 | 0.00 | 12.00 | 12.00 | 15.57 *b |
| 2    | 0.00 | 0.00 | 0.00 | 12.00 | 12.00 | 5.19     |

NOTES:

\*b - Span depth is less than minimum. Deflection check required.

Support Data

=====

Columns

-----

Units: c1a, c2a, c1b, c2b (in); Ha, Hb (ft)

| Supp | c1a  | c2a  | Ha    | c1b  | c2b  | Hb    | Red% |
|------|------|------|-------|------|------|-------|------|
| 1    | 0.00 | 0.00 | 0.000 | 0.00 | 0.00 | 0.000 | 999  |
| 2    | 0.00 | 0.00 | 0.000 | 0.00 | 0.00 | 0.000 | 0    |
| 3    | 0.00 | 0.00 | 0.000 | 0.00 | 0.00 | 0.000 | 0    |

Boundary Conditions

-----

Units: Kz (kip/in); Kry (kip-in/rad)

| Supp | Spring Kz | Spring Kry | Far End A | Far End B |
|------|-----------|------------|-----------|-----------|
| 1    | 0         | 0          | Fixed     | Fixed     |
| 2    | 0         | 0          | Fixed     | Fixed     |
| 3    | 0         | 0          | Fixed     | Fixed     |

Load Data

=====

Load Cases and Combinations

-----

| Case Type | Dead  |
|-----------|-------|
| U1        | 1.000 |

Line Loads

-----

Units: Wa, Wb (lb/ft), La, Lb (ft)

| Case/Patt | Span | Wa      | La    | Wb      | Lb     |
|-----------|------|---------|-------|---------|--------|
| Dead      | 1    | 2000.00 | 0.000 | 2000.00 | 24.000 |

Point Forces

-----

Units: Wa (kip), La (ft)

| Case/Patt | Span | Wa    | La    |
|-----------|------|-------|-------|
| Dead      | 2    | 12.00 | 4.000 |

Reinforcement Criteria

=====

Slabs and Ribs

-----

|             | Top bars |       | Bottom bars |          |
|-------------|----------|-------|-------------|----------|
|             | Min      | Max   | Min         | Max      |
| Bar Size    | #4       | #4    | #4          | #4       |
| Bar spacing | 1.00     | 18.00 | 1.00        | 18.00 in |
| Reinf ratio | 0.18     | 2.00  | 0.18        | 2.00 %   |
| Cover       | 1.00     |       | 1.00        | in       |

There is NOT more than 12 in of concrete below top bars.

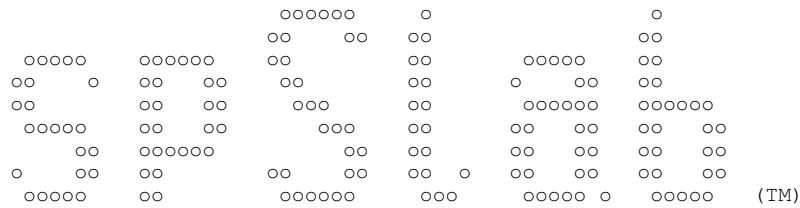
Beams

-----

|             | Top bars |       | Bottom bars |        | Stirrups |          |
|-------------|----------|-------|-------------|--------|----------|----------|
|             | Min      | Max   | Min         | Max    | Min      | Max      |
| Bar Size    | #5       | #8    | #5          | #8     | #3       | #5       |
| Bar spacing | 1.00     | 18.00 | 1.00        | 18.00  | 6.00     | 18.00 in |
| Reinf ratio | 0.14     | 5.00  | 0.14        | 5.00 % |          |          |
| Cover       | 1.50     |       | 1.50        | in     |          |          |
| Layer dist. | 1.00     |       | 1.00        | in     |          |          |
| No. of legs |          |       |             |        | 2        | 6        |
| Side cover  |          |       |             |        | 1.50     | in       |

1st Stirrup 3.00 in  
There is NOT more than 12 in of concrete below top bars.





=====  
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=====  
 [2] DESIGN RESULTS  
 =====

Top Reinforcement

| Units: Width (ft), Mmax (k-ft), Xmax (ft), As (in^2), Sp (in) |       |        |        |       |       |       |        |          |  |
|---|-------|--------|--------|-------|-------|-------|--------|----------|--|
| Span Zone   | Width | Mmax   | Xmax   | AsMin | AsMax | AsReq | SpProv | Bars     |  |
| 1 Left  | 1.00  | 108.00 | 0.000  | 0.408 | 2.208 | 3.010 | 0.000  | > MAX *2 |  |
| Midspan   | 1.00  | 0.00   | 12.000 | 0.000 | 2.208 | 0.000 | 0.000  | ---      |  |
| Right   | 1.00  | 72.00  | 24.000 | 0.403 | 2.181 | 1.837 | 2.133  | 4-#7     |  |
| 2 Left  | 1.00  | 72.00  | 0.000  | 0.403 | 2.181 | 1.837 | 2.133  | 4-#7     |  |
| Midspan   | 1.00  | 30.00  | 2.800  | 0.403 | 2.181 | 0.698 | 6.399  | 2-#7     |  |
| Right   | 1.00  | 8.40   | 5.200  | 0.250 | 2.181 | 0.188 | 6.399  | 2-#7 *3  |  |

NOTES:  
 \*2 - Reinforcement exceeds maximum allowable value.  
 \*3 - Design governed by minimum reinforcement.

Top Bar Details

| Units: Length (ft) |       |        |      |        |            |        |       |        |      |        |  |
|--------------------|-------|--------|------|--------|------------|--------|-------|--------|------|--------|--|
| Span               | Left  |        |      |        | Continuous |        | Right |        |      |        |  |
|                    | Bars  | Length | Bars | Length | Bars       | Length | Bars  | Length | Bars | Length |  |
| 1                  | ERROR |        |      |        | ---        |        | 2-#7  | 5.50   | 2-#7 | 3.26   |  |
| 2                  | 1-#7  | 3.34   | 1-#7 | 3.26   | 2-#7       | 8.00   | ---   |        | ---  |        |  |

Top Bar Development Lengths

| Units: Length (in) |      |        |      |        |            |        |       |        |      |        |  |
|--------------------|------|--------|------|--------|------------|--------|-------|--------|------|--------|--|
| Span               | Left |        |      |        | Continuous |        | Right |        |      |        |  |
|                    | Bars | Length | Bars | DevLen | Bars       | DevLen | Bars  | DevLen | Bars | DevLen |  |
| 1                  | ---  |        |      |        | ---        |        | 2-#7  | 39.09  | 2-#7 | 39.09  |  |
| 2                  | 1-#7 | 39.09  | 1-#7 | 39.09  | 2-#7       | 16.36  | ---   |        | ---  |        |  |

Bottom Reinforcement

| Units: Width (ft), Mmax (k-ft), Xmax (ft), As (in^2), Sp (in) |       |       |        |       |       |       |        |      |  |
|---|-------|-------|--------|-------|-------|-------|--------|------|--|
| Span  | Width | Mmax  | Xmax   | AsMin | AsMax | AsReq | SpProv | Bars |  |
| 1   | 1.00  | 54.56 | 12.750 | 0.408 | 2.208 | 1.315 | 1.644  | 5-#5 |  |
| 2   | 1.00  | 0.00  | 4.000  | 0.000 | 2.208 | 0.000 | 0.000  | ---  |  |

Bottom Bar Details

| Units: Start (ft), Length (ft) |           |       |        |      |            |        |  |
|--------------------------------|-----------|-------|--------|------|------------|--------|--|
| Span                           | Long Bars |       |        |      | Short Bars |        |  |
|                                | Bars      | Start | Length | Bars | Start      | Length |  |
| -----                          |           |       |        |      |            |        |  |

|   |      |      |       |      |      |       |
|---|------|------|-------|------|------|-------|
| 1 | 3-#5 | 0.00 | 24.00 | 2-#5 | 6.99 | 11.51 |
| 2 | ---  |      |       | ---  |      |       |

Bottom Bar Development Lengths

Units: DevLen (in)

| Span | Long Bars |        | Short Bars |        |
|------|-----------|--------|------------|--------|
|      | Bars      | DevLen | Bars       | DevLen |
| 1    | 3-#5      | 22.95  | 2-#5       | 22.95  |
| 2    | ---       |        | ---        |        |

Flexural Capacity

Units: x (ft), As (in^2), PhiMn, Mu (k-ft)

| Span   | x      | Top    |        |         |      |     |           | Bottom |        |       |      |     |        |
|--------|--------|--------|--------|---------|------|-----|-----------|--------|--------|-------|------|-----|--------|
|        |        | AsTop  | PhiMn- | Mu-     | Comb | Pat | Status    | AsBot  | PhiMn+ | Mu+   | Comb | Pat | Status |
| 1      | 0.000  | 0.00   | 0.00   | -108.00 | U1   | All | *EXCEEDED | 0.93   | 39.77  | 0.00  | U1   | All | OK     |
|        | 5.250  | 0.00   | 0.00   | -1.69   | U1   | All | *EXCEEDED | 0.93   | 39.77  | 0.00  | U1   | All | OK     |
|        | 6.994  | 0.00   | 0.00   | 0.00    | U1   | All | OK        | 0.93   | 39.77  | 21.43 | U1   | All | OK     |
|        | 8.400  | 0.00   | 0.00   | 0.00    | U1   | All | OK        | 1.39   | 57.18  | 35.63 | U1   | All | OK     |
|        | 8.906  | 0.00   | 0.00   | 0.00    | U1   | All | OK        | 1.55   | 63.11  | 39.77 | U1   | All | OK     |
|        | 12.000 | 0.00   | 0.00   | 0.00    | U1   | All | OK        | 1.55   | 63.11  | 54.00 | U1   | All | OK     |
|        | 12.750 | 0.00   | 0.00   | 0.00    | U1   | All | OK        | 1.55   | 63.11  | 54.56 | U1   | All | OK     |
|        | 15.600 | 0.00   | 0.00   | 0.00    | U1   | All | OK        | 1.55   | 63.11  | 46.43 | U1   | All | OK     |
|        | 16.594 | 0.00   | 0.00   | 0.00    | U1   | All | OK        | 1.55   | 63.11  | 39.77 | U1   | All | OK     |
|        | 18.500 | 0.00   | 0.00   | 0.00    | U1   | All | OK        | 0.93   | 39.85  | 21.50 | U1   | All | OK     |
|        | 18.506 | 0.00   | -0.10  | 0.00    | U1   | All | OK        | 0.93   | 39.77  | 21.43 | U1   | All | OK     |
|        | 20.743 | 0.83   | -35.16 | -9.32   | U1   | All | OK        | 0.93   | 39.77  | 0.00  | U1   | All | OK     |
| 21.757 | 1.57   | -63.06 | -26.57 | U1      | All  | OK  | 0.93      | 39.77  | 0.00   | U1    | All  | OK  |        |
| 24.000 | 2.40   | -83.42 | -72.00 | U1      | All  | OK  | 0.93      | 39.77  | 0.00   | U1    | All  | OK  |        |
| 2      | 0.000  | 2.40   | -83.42 | -72.00  | U1   | All | OK        | 0.00   | 0.00   | 0.00  | U1   | All | OK     |
|        | 0.081  | 2.39   | -83.39 | -70.79  | U1   | All | OK        | 0.00   | 0.00   | 0.00  | U1   | All | OK     |
|        | 2.800  | 1.38   | -56.31 | -30.00  | U1   | All | OK        | 0.00   | 0.00   | 0.00  | U1   | All | OK     |
|        | 3.257  | 1.21   | -50.13 | -23.14  | U1   | All | OK        | 0.00   | 0.00   | 0.00  | U1   | All | OK     |
|        | 3.338  | 1.20   | -49.57 | -21.93  | U1   | All | OK        | 0.00   | 0.00   | 0.00  | U1   | All | OK     |
|        | 4.000  | 1.20   | -49.57 | -12.00  | U1   | All | OK        | 0.00   | 0.00   | 0.00  | U1   | All | OK     |
|        | 5.200  | 1.20   | -49.57 | -8.40   | U1   | All | OK        | 0.00   | 0.00   | 0.00  | U1   | All | OK     |
|        | 8.000  | 1.20   | -49.57 | 0.00    | U1   | All | OK        | 0.00   | 0.00   | 0.00  | U1   | All | OK     |

Longitudinal Beam Transverse Reinforcement Demand and Capacity

Section Properties

Units: d (in), Av/s (in^2/in), PhiVc (kip)

| Span | d (Av/s)min | PhiVc  |
|------|-------------|--------|
| 1    | 10.00       | 0.0100 |
| 2    | 10.06       | 0.0100 |

Beam Transverse Reinforcement Demand

Units: Start, End, Xu (in), Vu (ft), Av/s (kip/in^2)

| Span | Start  | End    | Required |       |           | Av/s   | Demand    |
|------|--------|--------|----------|-------|-----------|--------|-----------|
|      |        |        | Xu       | Vu    | Comb/Patt |        | Av/s      |
| 1    | 0.250  | 4.024  | 0.833    | 23.83 | U1/All    | 0.0277 | 0.0277    |
|      | 4.024  | 7.214  | 4.024    | 17.45 | U1/All    | 0.0135 | 0.0135    |
|      | 7.214  | 10.405 | 7.214    | 11.07 | U1/All    | 0.0000 | 0.0100 *8 |
|      | 10.405 | 13.595 | 10.405   | 4.69  | U1/All    | 0.0000 | 0.0000    |
|      | 13.595 | 16.786 | 16.786   | 8.07  | U1/All    | 0.0000 | 0.0100 *8 |
|      | 16.786 | 19.976 | 19.976   | 14.45 | U1/All    | 0.0068 | 0.0100 *8 |
| 2    | 19.976 | 23.750 | 23.167   | 20.83 | U1/All    | 0.0210 | 0.0210    |
|      | 0.250  | 1.892  | 0.839    | 15.00 | U1/All    | 0.0078 | 0.0100 *8 |
|      | 1.892  | 2.946  | 1.892    | 15.00 | U1/All    | 0.0078 | 0.0100 *8 |
|      | 2.946  | 4.000  | 2.946    | 15.00 | U1/All    | 0.0078 | 0.0100 *8 |
|      | 4.000  | 5.054  | 4.000    | 9.00  | U1/All    | 0.0000 | 0.0100 *8 |
|      | 5.054  | 6.108  | 5.054    | 3.00  | U1/All    | 0.0000 | 0.0000    |
|      | 6.108  | 7.750  | 6.108    | 3.00  | U1/All    | 0.0000 | 0.0000    |

NOTES:

\*8 - Minimum transverse (stirrup) reinforcement governs.

Beam Transverse Reinforcement Details

Units: spacing & distance (in).

Span Size Stirrups (2 legs each unless otherwise noted)

|   |                                       |
|---|---------------------------------------|
| 1 | #3 25 @ 5.0 + <-- 38.3 --> + 25 @ 5.0 |
| 2 | #3 12 @ 5.0 + <-- 32.4 -->            |

Beam Transverse Reinforcement Capacity

| Units: Start, End, Xu (ft), Vu, PhiVn (kip), Av/s (in <sup>2</sup> /in), Av (in <sup>2</sup> ), Sp (in) |        |        |          |       |           |        |          |       |        |          |
|---|--------|--------|----------|-------|-----------|--------|----------|-------|--------|----------|
| Span  | Start  | End    | Required |       |           |        | Provided |       |        |          |
|   |        |        | Xu       | Vu    | Comb/Patt | Av/s   | Av       | Sp    | Av/s   | PhiVn    |
| 1   | 0.000  | 0.250  | 0.833    | 23.83 | U1/All    | -----  | -----    | ----- | -----  | -----    |
|   | 0.250  | 10.405 | 0.833    | 23.83 | U1/All    | 0.0277 | 0.22     | 5.0   | 0.0442 | 31.29    |
|   | 10.405 | 13.595 | 10.405   | 4.69  | U1/All    | 0.0000 | -----    | ----- | -----  | 5.69     |
|   | 13.595 | 23.750 | 23.167   | 20.83 | U1/All    | 0.0210 | 0.22     | 5.0   | 0.0442 | 31.29    |
|   | 23.750 | 24.000 | 23.167   | 20.83 | U1/All    | -----  | -----    | ----- | -----  | -----    |
| 2   | 0.000  | 0.250  | 0.839    | 15.00 | U1/All    | -----  | -----    | ----- | -----  | -----    |
|   | 0.250  | 5.054  | 0.839    | 15.00 | U1/All    | 0.0078 | 0.22     | 5.0   | 0.0439 | 31.33 *8 |
|   | 5.054  | 7.161  | 5.054    | 3.00  | U1/All    | 0.0000 | -----    | ----- | -----  | 5.73     |
|   | 7.161  | 7.750  | 7.161    | 3.00  | U1/All    | 0.0000 | -----    | ----- | -----  | 5.73     |
|   | 7.750  | 8.000  | 7.161    | 3.00  | U1/All    | -----  | -----    | ----- | -----  | -----    |

NOTES:  
 \*8 - Minimum transverse (stirrup) reinforcement governs.

Slab Shear Capacity

| Units: b, d (in), Xu (ft), PhiVc, Vu(kip) |     |     |             |       |     |     |
|---|-----|-----|-------------|-------|-----|-----|
| Span                                      | b   | d   | Vratio      | PhiVc | Vu  | Xu  |
| 1   | --- | --- | Not checked | ---   | --- | --- |
| 2   | --- | --- | Not checked | ---   | --- | --- |

Material Takeoff

| Reinforcement in the Direction of Analysis |                      |     |                          |     |  |
|--|----------------------|-----|--------------------------|-----|--|
| Top Bars:                                  | 82.0 lb              | <=> | 2.56 lb/ft               | <=> | 2.562 lb/ft <sup>2</sup>               |
| Bottom Bars:                               | 99.1 lb              | <=> | 3.10 lb/ft               | <=> | 3.097 lb/ft <sup>2</sup>               |
| Stirrups:                                  | 73.8 lb              | <=> | 2.31 lb/ft               | <=> | 2.307 lb/ft <sup>2</sup>               |
| Total Steel:                               | 254.9 lb             | <=> | 7.97 lb/ft               | <=> | 7.966 lb/ft <sup>2</sup>               |
| Concrete:                                  | 32.0 ft <sup>3</sup> | <=> | 1.00 ft <sup>3</sup> /ft | <=> | 1.000 ft <sup>3</sup> /ft <sup>2</sup> |

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 spSlab v5.00 (TM)  
 A Computer Program for Analysis, Design, and Investigation of  
 Reinforced Concrete Beams, One-way and Two-way Slab Systems  
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=====  
 [3] DEFLECTION RESULTS  
 =====

Section Properties

Frame Section Properties

Units: Ig, Icr (in^4), Mcr (k-ft)

| Span Zone | M+ve |     |       | M-ve |     |        |
|-----------|------|-----|-------|------|-----|--------|
|           | Ig   | Icr | Mcr   | Ig   | Icr | Mcr    |
| 1 Left    | 1728 | 471 | 11.38 | 1728 | 0   | -11.38 |
| Midspan   | 1728 | 471 | 11.38 | 1728 | 0   | -11.38 |
| Right     | 1728 | 471 | 11.38 | 1728 | 920 | -11.38 |
| 2 Left    | 1728 | 0   | 11.38 | 1728 | 920 | -11.38 |
| Midspan   | 1728 | 0   | 11.38 | 1728 | 558 | -11.38 |
| Right     | 1728 | 0   | 11.38 | 1728 | 558 | -11.38 |

NOTES: M+ve values are for positive moments (tension at bottom face).  
 M-ve values are for negative moments (tension at top face).

Frame Effective Section Properties

Units: Ie, Ie,avg (in^4), Mmax (k-ft)

| Span Zone | Weight | Load Level |     |           |     |           |     |
|-----------|--------|------------|-----|-----------|-----|-----------|-----|
|           |        | Dead       |     | Sustained |     | Dead+Live |     |
|           |        | Mmax       | Ie  | Mmax      | Ie  | Mmax      | Ie  |
| 1 Left    | 0.150  | -108.00    | 2   | -108.00   | 2   | -108.00   | 2   |
| Middle    | 0.700  | 54.56      | 483 | 54.56     | 483 | 54.56     | 483 |
| Right     | 0.150  | -72.00     | 923 | -72.00    | 923 | -72.00    | 923 |
| Span Avg  | ----   | ----       | 477 | ----      | 477 | ----      | 477 |
| 2 Left    | 0.150  | -72.00     | 923 | -72.00    | 923 | -72.00    | 923 |
| Middle    | 0.850  | -27.00     | 646 | -27.00    | 646 | -27.00    | 646 |
| Span Avg  | ----   | ----       | 688 | ----      | 688 | ----      | 688 |

Instantaneous Deflections

Extreme Instantaneous Frame Deflections and Corresponding Locations

Units: Def (in), Loc (ft)

| Span Direction | Value | Dead | Live      |             |       | Total     |           |
|----------------|-------|------|-----------|-------------|-------|-----------|-----------|
|                |       |      | Sustained | Unsustained | Total | Sustained | Dead+Live |
| 1              | Down  | Def  | 1.952     | ---         | ---   | ---       | 1.952     |
|                |       | Loc  | 12.500    | ---         | ---   | ---       | 12.500    |
|                | Up    | Def  | ---       | ---         | ---   | ---       | ---       |
|                |       | Loc  | ---       | ---         | ---   | ---       | ---       |
| 2              | Down  | Def  | ---       | ---         | ---   | ---       | ---       |
|                |       | Loc  | ---       | ---         | ---   | ---       | ---       |
|                | Up    | Def  | -0.130    | ---         | ---   | ---       | -0.130    |
|                |       | Loc  | 3.000     | ---         | ---   | ---       | 3.000     |

Long-term Deflections

=====  
 Long-term Deflection Factors

Time dependant factor for sustained loads = 2.000

Units: Astop, Asbot (in<sup>2</sup>), b, d (in), Rho' (%), Lambda (-)

| Span | Zone    | M+ve  |      |      |       |        | M-ve  |      |      |       |        |
|------|---------|-------|------|------|-------|--------|-------|------|------|-------|--------|
|      |         | Astop | b    | d    | Rho'  | Lambda | Asbot | b    | d    | Rho'  | Lambda |
| 1    | Midspan | ----  | ---- | ---- | 0.000 | 2.000  | ----  | ---- | ---- | 0.000 | 2.000  |
| 2    | Midspan | ----  | ---- | ---- | 0.000 | 2.000  | ----  | ---- | ---- | 0.000 | 2.000  |

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.  
 Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Extreme Long-term Frame Deflections and Corresponding Locations

Units: Def (in), Loc (ft)

| Span | Direction | Value | cs     | cs+lu  | cs+l   | Total  |
|------|-----------|-------|--------|--------|--------|--------|
| 1    | Down      | Def   | 3.904  | 3.904  | 3.904  | 5.856  |
|      |           | Loc   | 12.500 | 12.500 | 12.500 | 12.500 |
|      | Up        | Def   | ---    | ---    | ---    | ---    |
|      |           | Loc   | ---    | ---    | ---    | ---    |
| 2    | Down      | Def   | ---    | ---    | ---    | ---    |
|      |           | Loc   | ---    | ---    | ---    | ---    |
|      | Up        | Def   | -0.260 | -0.260 | -0.260 | -0.390 |
|      |           | Loc   | 3.000  | 3.000  | 3.000  | 3.000  |

NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.  
 Incremental deflections after partitions are installed can be estimated by deflections due to:  
 - creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,  
 - creep and shrinkage plus live load (cs+l), if live load applied after partitions.  
 Total deflections consist of dead, live, and creep and shrinkage deflections.