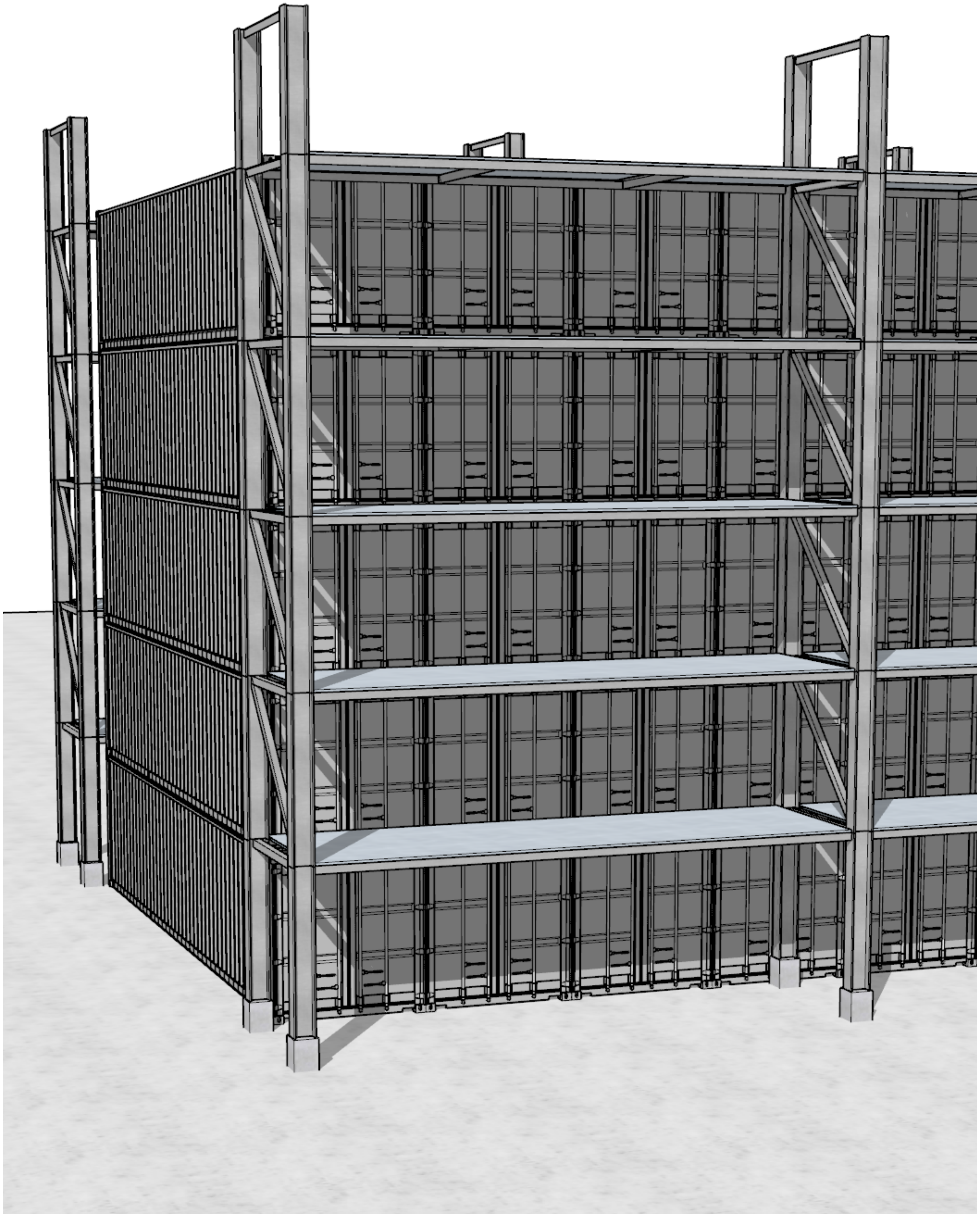
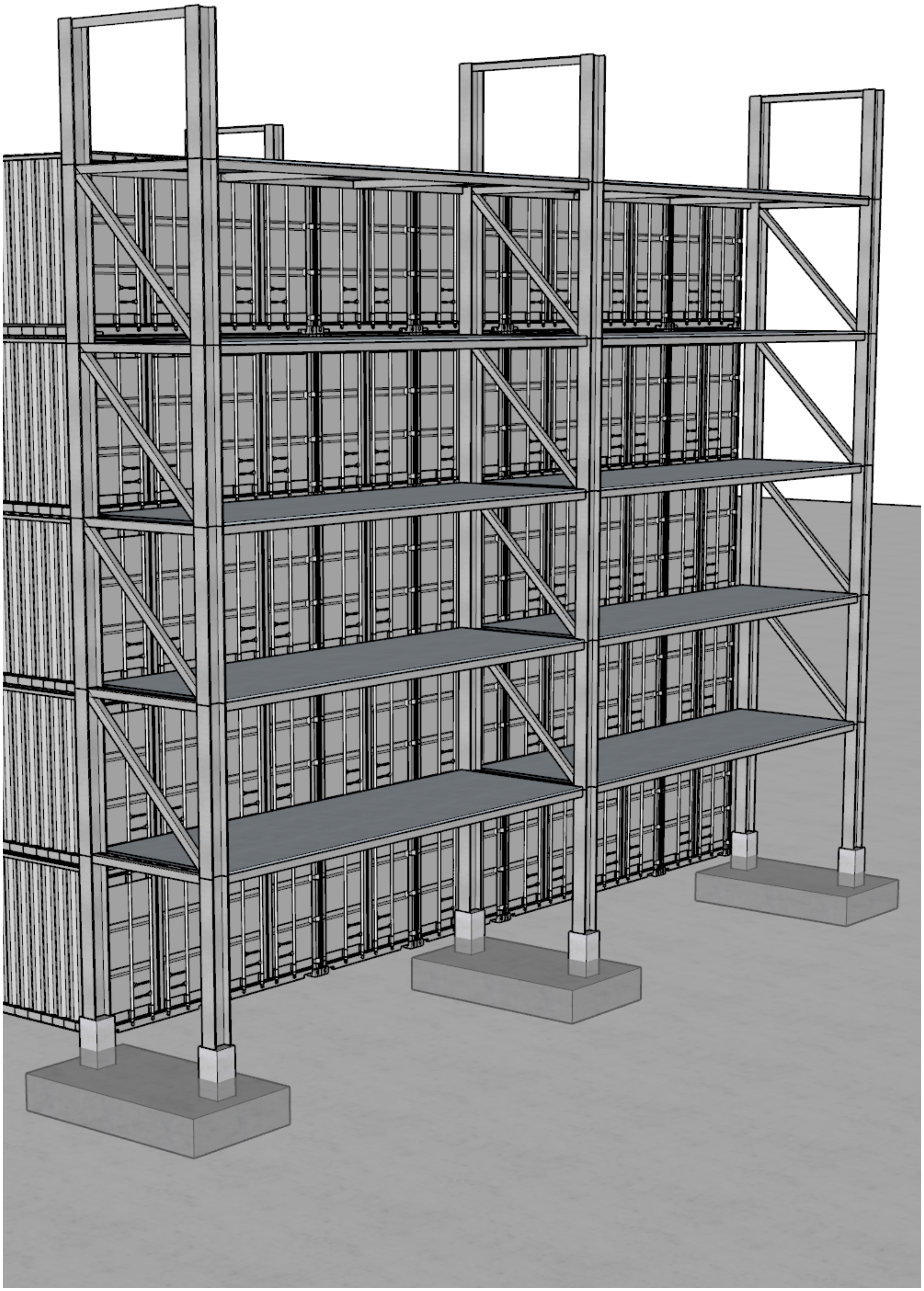
Combined Footing – Finite Element Analysis vs Beam-on-Elastic-Foundation





Combined Footing – Finite Element Analysis vs Beam-on-Elastic-Foundation

Combined footings are very commonly used in construction to support building and non-building structures alike. They are also particularly common foundation system for numerous equipment, tanks, silos, cranes and many other industrial applications like refrigerated container racks known as reefer racks.

There are many ways to accurately analyze and design soil supported combined footings for gravity and lateral applied loads. Most commonly equations for Beam-on-Elastic-Foundation (BOEF) are used to arrive at internal forces and then design to an applicable concrete standard such as ACI 318. Such equations are restrictive to specific shapes, geometry and load patterns and consider one-way behavior as the dominant direction of analysis. In construction projects, however, there are many variations that make such equations impractical and engineers may utilize the Finite Element Analysis (FEA) methods to consider any variation to address project needs.

This case study aims to show an evaluation of a standard combined footing using Beam-on-Elastic-Foundation and the Finite Element Analysis methods and to report on the results of each using two programs, spBeam and spMats, from the StructurePoint Software Suite.

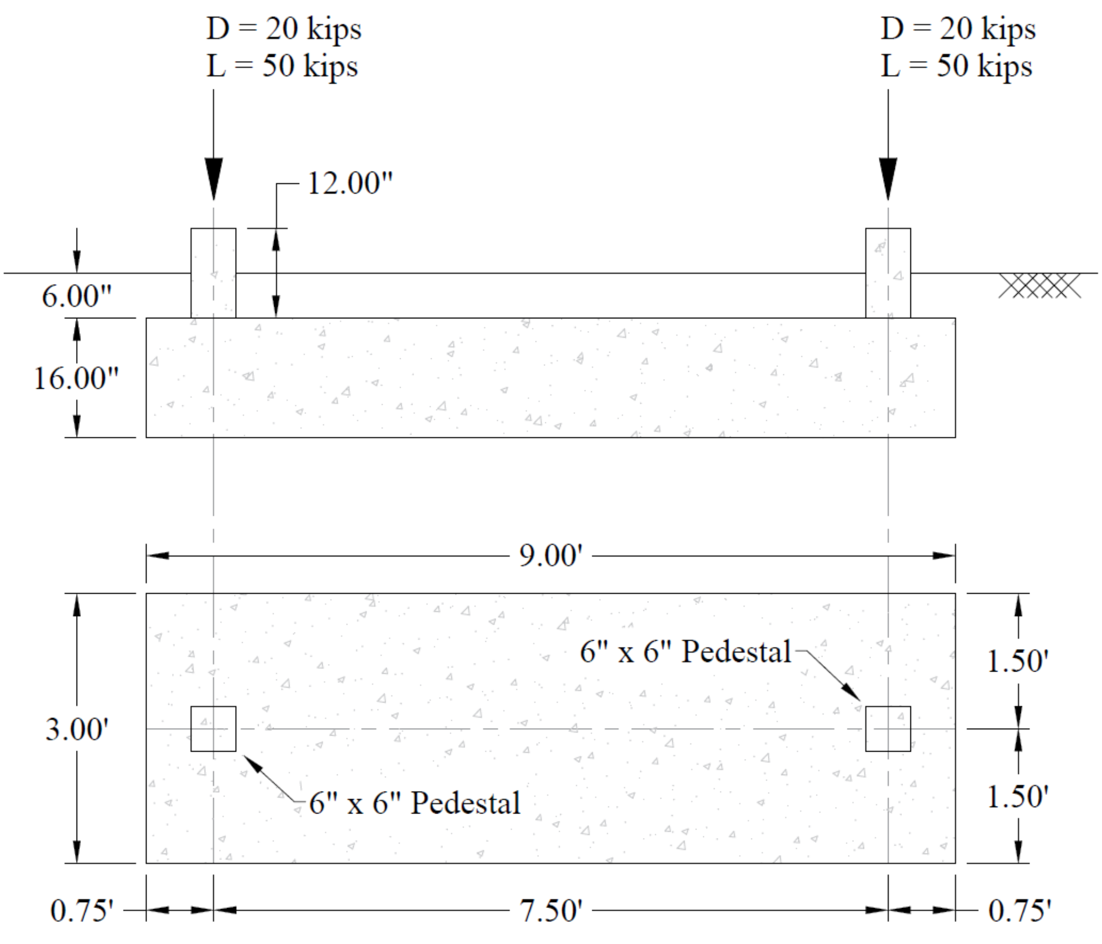


Figure 1 – Reinforced Concrete Combined Footing Geometry

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

- spMats Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2020
- spBeam Engineering Software Program Manual v5.50, STRUCTUREPOINT, 2018
- Contact Support@StructurePoint.org to obtain supplementary materials (spMats and spBeam models: Case-Study.matx and Case-Study.slb)

Design Data

Foundation Geometry:

Width = 3'-0"

Length = 9'-0"

Thickness = 16 in.

Column reactions:

Service dead load = 20 kips

Service live load = 50 kips

Soil properties:

Allowable bearing capacity = 6 ksf

Subgrade modulus, k_s = 100 kcf

Material Properties:

f'_c = 4 ksi

f_y = 60 ksi

Contents

1. Method of Solution

In spMats, the combined footing will be analyzed by Finite Element Analysis Method where the soil support is modeled as a group of linear uncoupled springs (Winkler type) concentrated at the nodes. spMats calculates soil spring stiffness automatically.

In spBeam, on the other hand, the supports can be modeled by a series of vertical support springs to be used in the stiffness analysis method. The vertical support spring constant, *kz*, input in spBeam is calculated manually and equals to the soil subgrade modulus, *ks*, multiplied by the tributary area of around the support.

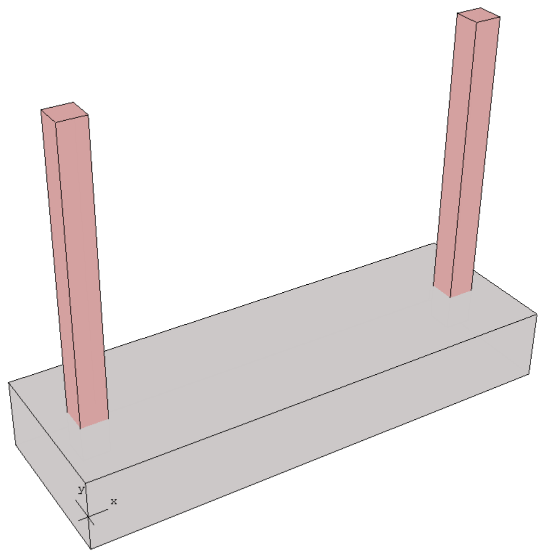


Figure 2 – 3D View Combined Footing (spMats)

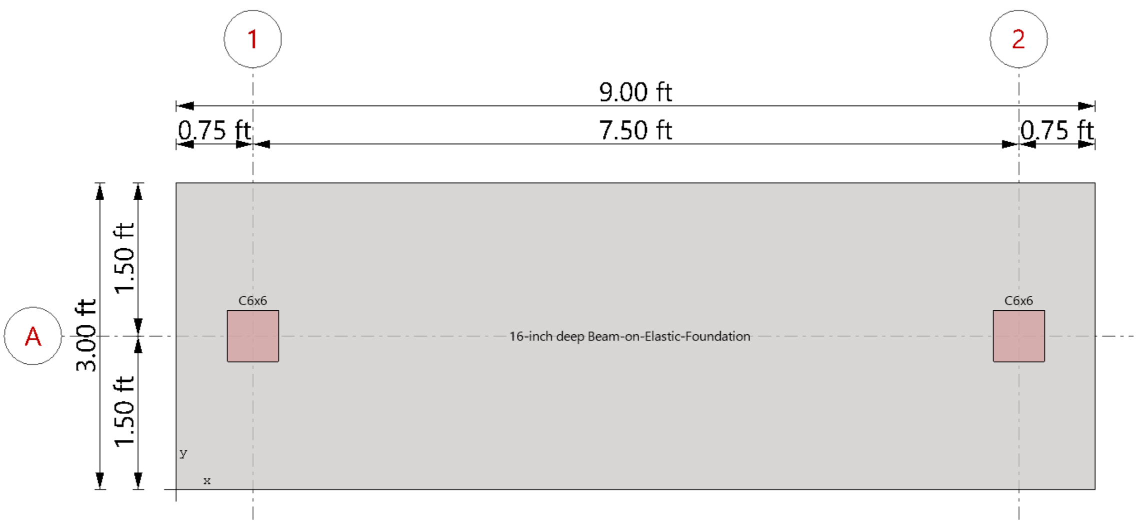


Figure 3 – Plan View Combined Footing (spMats)

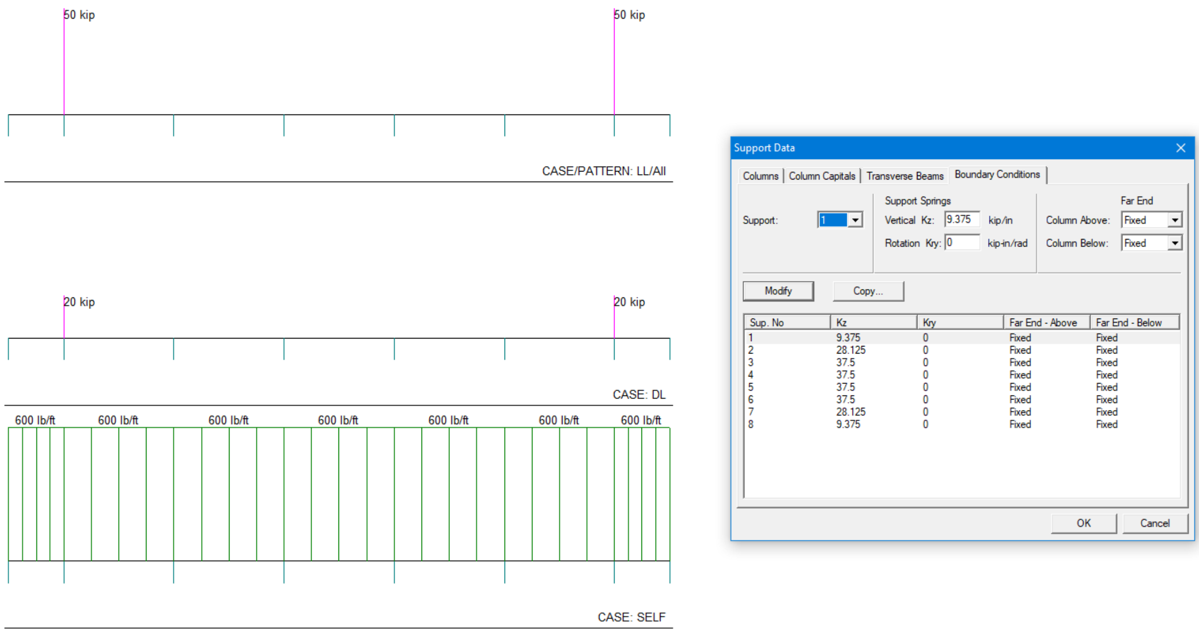


Figure 4 – The 7-Span Model – Loading and Soil Spring Support Data (spBeam)

2. Design Bending Moments and Reinforcement

2.1. Using spMats

2.1.1. At the Top (Along X-Direction)

The design moment at top (midspan) along X-Direction, M_{ux} = 3 ft × 51.483 kip-ft/ft = 154.45 kip-ft.

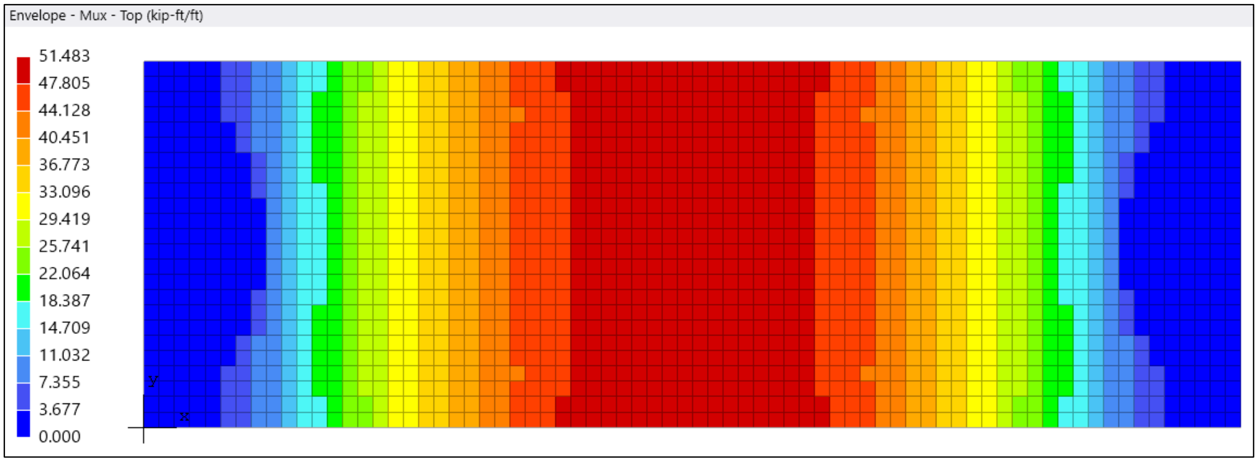


Figure 5 – Envelope – Element Design Moment, M_{ux} – Top, along X-Direction (spMats)

The reinforcement requirement at top (midspan) along X-Direction, A_{sx} = 3 ft × 0.877 in^2/ft = 2.631 in^2.

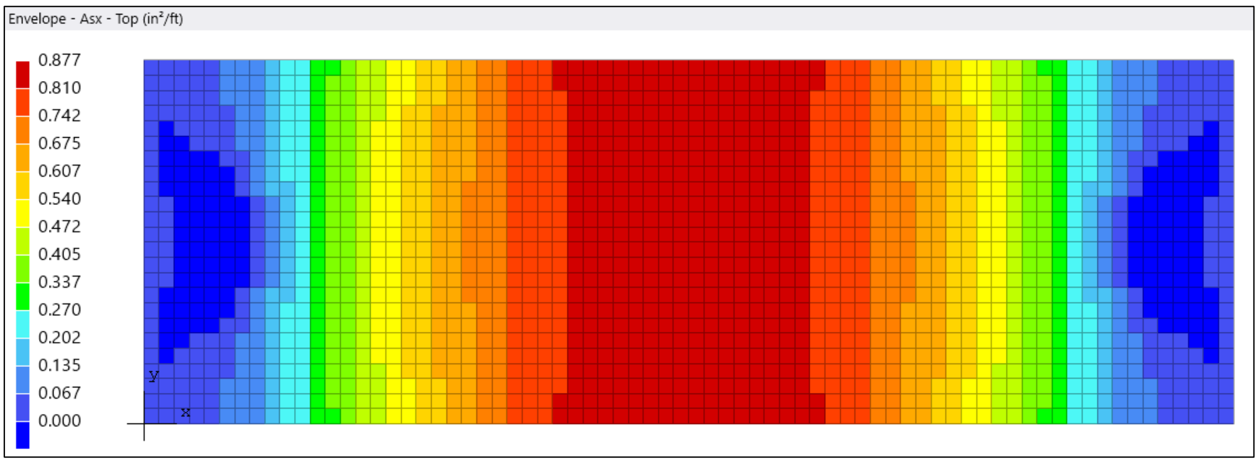


Figure 6 – Envelope – Required Reinforcement, A_{sx} -Top along X-Direction (spMats)

2.1.2. At the Top (Along Y-Direction)

The design moments at top along Y-Direction are negligible.

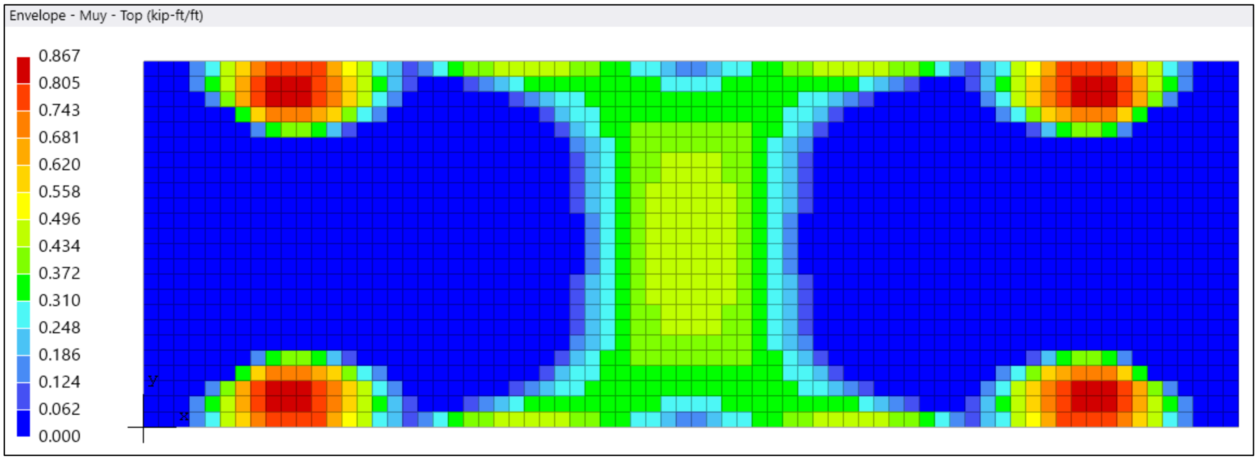


Figure 7 – Envelope – Element Design Moment, M_{uy} – Top, along Y-Direction (spMats)

The reinforcement requirement at top along Y-Direction is negligible.

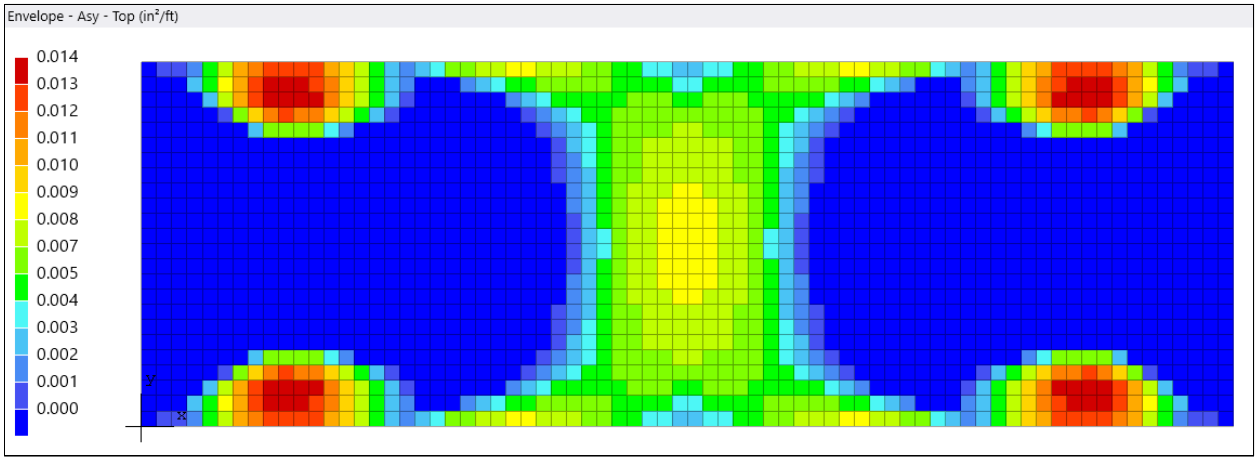


Figure 8 – Envelope – Required Reinforcement, A_{sy} -Top along Y-Direction (spMats)

2.1.3. At the Bottom (Along X-Direction)

The design moments at bottom along X-Direction dissipate to zero towards the edges.

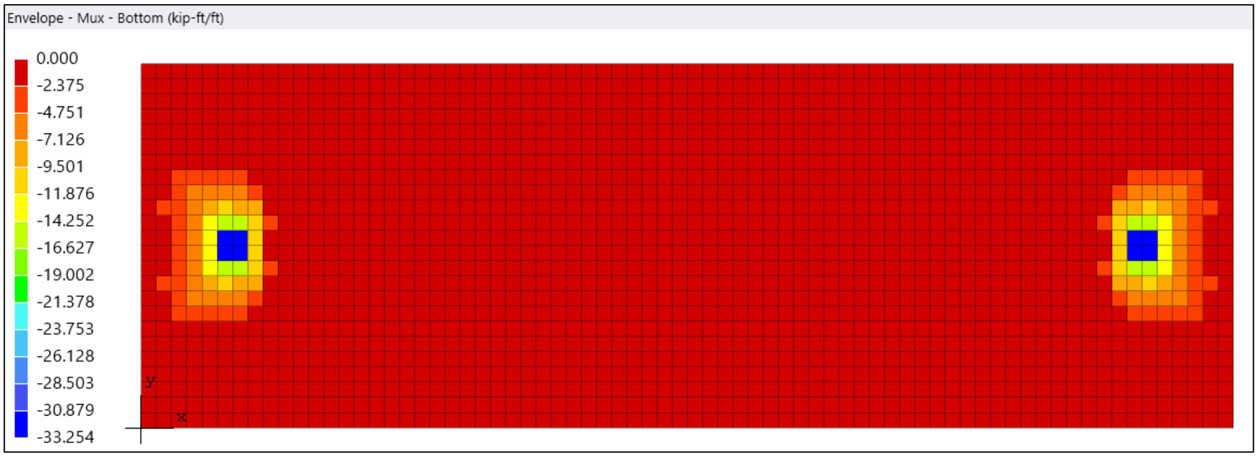


Figure 9 – Envelope – Element Design Moment, M_{ux} – Bottom, along X-Direction (spMats)

The reinforcement requirement at bottom along X-Direction dissipates to zero towards the edges.

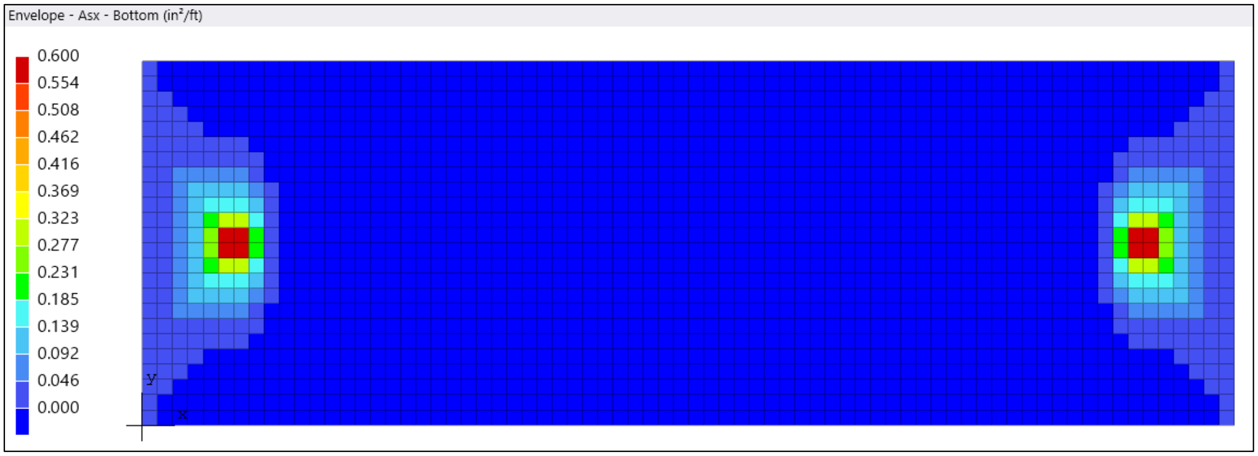


Figure 10 – Envelope – Required Reinforcement, A_{sx} -Bottom along X-Direction (spMats)

2.1.4. At the Bottom (Along Y-Direction)

The design moment contour view at bottom along Y-Direction indicates a majority demand within a band width of approximately 2'-6" under the column.

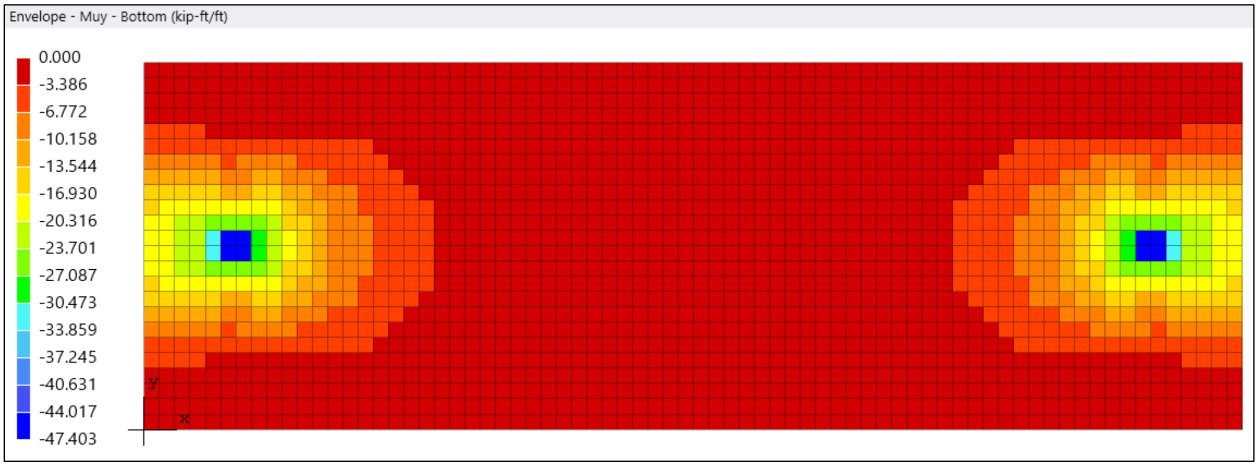


Figure 11 – Envelope – Element Design Moment, M_{uy} – Bottom, along Y-Direction (spMats)

The reinforcement requirement contour view at bottom along Y-Direction indicates a majority demand within a band width of approximately 2'-6" under the column.

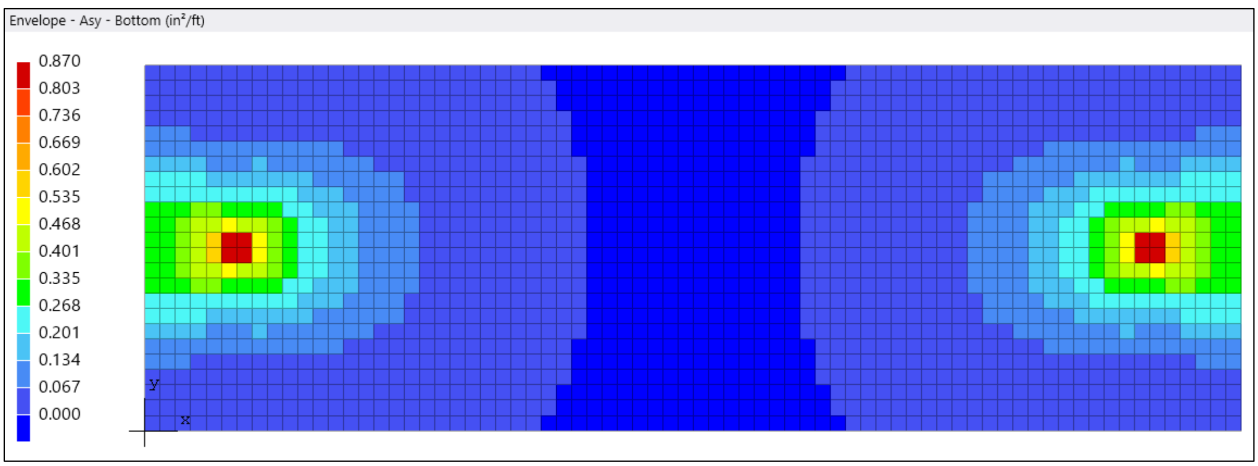


Figure 12 – Envelope – Required Reinforcement, A_{sy} -Bottom along Y-Direction (spMats)

2.2. Using spBeam

The design moment at top (midspan), M_u equals to 147.31 kip-ft. As the spacing between soil spring supports is being reduced, this value will get closer to the 154.45 kip-ft value obtained from spMats run. In this spBeam model, the span length is taken as 1.5 ft except end spans which are 0.75 ft long.

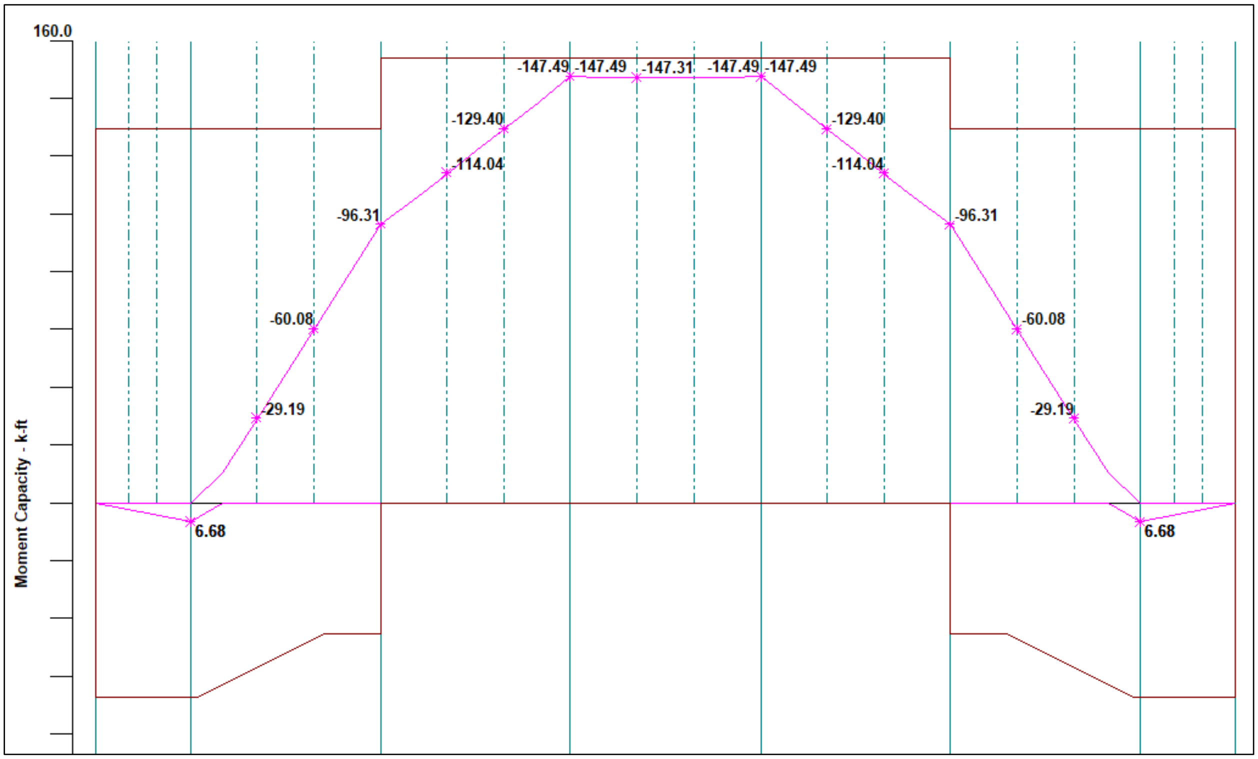


Figure 13 – Bending Moment Demand and Capacity Diagram for the Beam-on-Elastic Foundations Model (spBeam)

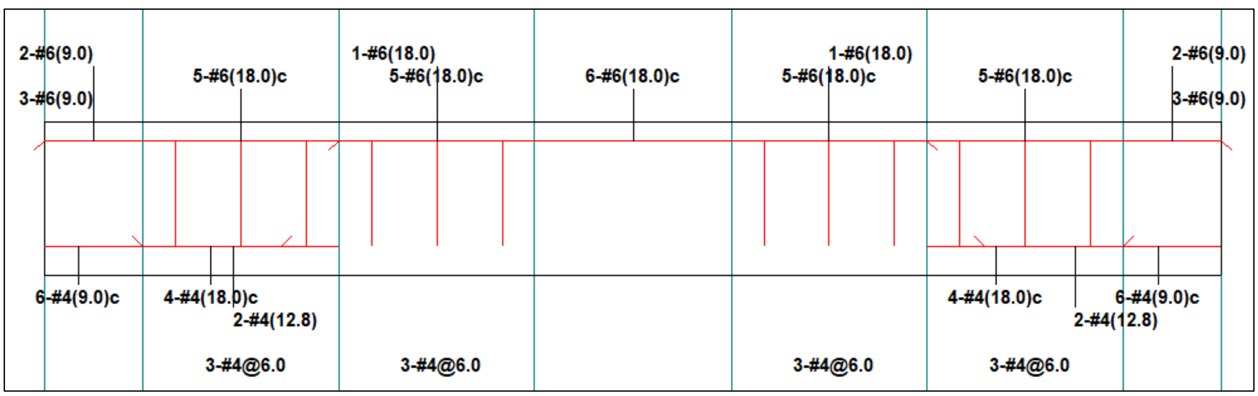


Figure 14 – Reinforcement (Flexural and Transverse) Diagram for the Beam-on-Elastic Foundations Model (spBeam)

The reinforcement requirement at top (Span 4) along X-Direction, A_{sx} = 2.52 in.^{2}.

Design Results - Top Reinforcement

NOTE: *3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Span	Zone	Width	Mmax	Xmax	As,min	As,max	As,req	SpProv	Bars
		ft	k-ft	ft	in^2	in^2	in^2	in	
4	Left	3.00	147.49	0.000	1.635	8.860	2.520	6.177	6-#6
	Midspan	3.00	147.31	0.525	1.635	8.860	2.516	6.177	6-#6
	Right	3.00	147.49	1.500	1.635	8.860	2.520	6.177	6-#6

Figure 15 – Top Reinforcement (Flexural) for the Beam-on-Elastic Foundations Model (spBeam)

The reinforcement requirement at bottom (Spans 1, 2 & 6, 7) along X-Direction, A_{sx} = 0.12 in.^{2}.

Design Results - Bottom Reinforcement

NOTE: *3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Span	Width	Mmax	Xmax	As,min	As,max	As,req	SpProv	Bars	
	ft	k-ft	ft	in^2	in^2	in^2	in		
1	3.00	6.68	0.750	0.643	8.291	0.117	6.212	6-#4	*3 *5
2	3.00	6.68	0.000	0.643	8.291	0.117	6.212	6-#4	*3 *5

Figure 16 – Bottom Reinforcement (Flexural) for the Beam-on-Elastic Foundations Model (spBeam)

3. Soil Pressures

3.1. Using spMats

The minimum soil pressure = 5.291 ksf at midspan & maximum soil pressure = 5.540 ksf at the ends per spMats. Since these values are lower than the allowable bearing pressure of 6 ksf, the design is adequate.

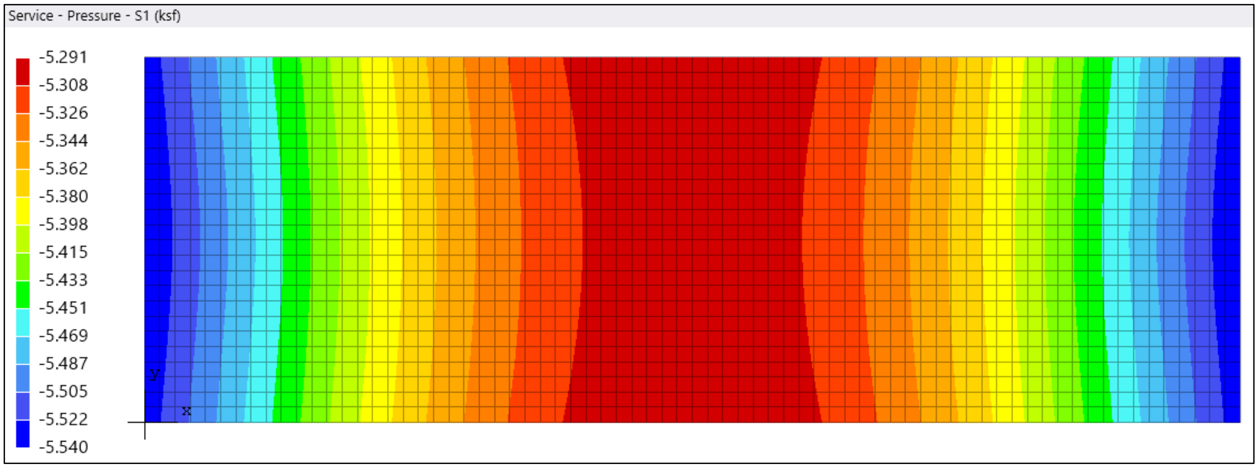


Figure 17 – Service Load Combination S1 – Soil Pressure (spMats)

3.2. Using spBeam

The spring reaction is divided by the tributary area to indicate the soil pressure as follows:

The service-level spring reaction at support no. 4 = 0.90 + 6.56 + 16.40 = 23.86 kip

The tributary area at support no. 4 = 1.5 × 3.0 = 4.5 ft

The soil pressure in the vicinity of support 4 = 23.86 / 4.5 = 5.30 ksf (a good match with at midspan pressure of 5.291 ksf)

4. One-Way Shear Design and Reinforcement

4.1. Using spMats

One way shear demand can be calculated by adding ultimate soil reactions at each node along an X-grid. For example, at x = 0 ft, the sum of the ultimate soil reactions equal to 1.5 kip value of shear and it increases by 3.0 kip at every 0.125 ft interval (mesh spacing) along X-Direction. At x = 0.75 ft (location of first column), the cumulative sum of the ultimate soil reactions will be approximately equal to 16.5 kip of shear value (1.5 + 5 × 3.0 = 16.5 kip). The average shear value shall be equal to 9.0 [(1.5 + 16.5)/2] kip over the 0.75 ft length from the edge of foundation to the location of the column in X-Direction. (a good match with an average shear value of 8.9 kip [(9.17 + 8.63)/2]) over the same length in spBeam.

4.2. Using spBeam

The minimum transverse (stirrup) reinforcement governs the design at spans 2, 3 & 5, 6 which amounts to 2-leg No. 4 stirrups at 6 in. spacing as shown above.

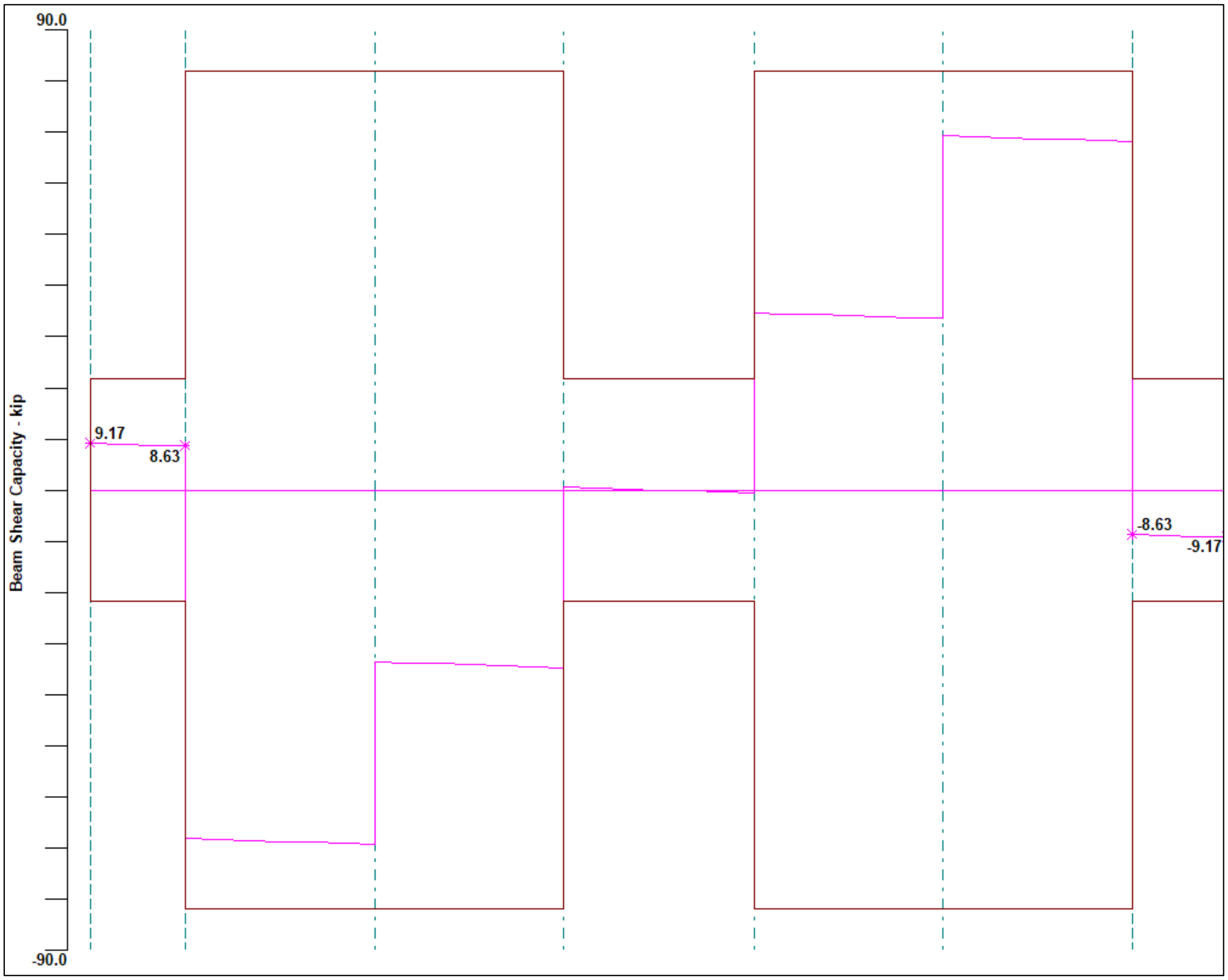


Figure 18 – Shear Demand and Capacity Diagram for the Beam-on-Elastic-Foundation (spBeam)

Design Results - Longitudinal Beam Transverse Reinforcement Demand and Capacity

NOTE: *8 - Minimum transverse (stirrup) reinforcement governs.

			Required				Provided				
Span	Start	End	Xu	Vu	Comb/Patt	Av/s	Av	Sp	Av/s	ΦVn	
	ft	ft	ft	kip		in^2/in	in^2	in	in^2/in	kip	
1	0.000	0.750	0.000	9.17	U1/All	0.0000	-----	-----	-----	21.77	
2	0.000	0.250	0.750	68.66	U1/All	-----	-----	-----	-----	-----	
	0.250	1.250	0.750	68.66	U1/All	0.0438	0.40	6.0	0.0667	81.79	
	1.250	1.500	0.750	68.66	U1/All	-----	-----	-----	-----	-----	
3	0.000	0.250	0.750	34.12	U1/All	-----	-----	-----	-----	-----	
	0.250	1.250	0.750	34.12	U1/All	0.0000	0.40	6.0	0.0667	81.79	*8
	1.250	1.500	0.750	34.12	U1/All	-----	-----	-----	-----	-----	
4	0.000	1.500	0.750	0.00	U1/All	0.0000	-----	-----	-----	21.77	

Figure 19 – Shear Demand and Capacity Results Table for the Beam-on-Elastic-Foundation (spBeam)

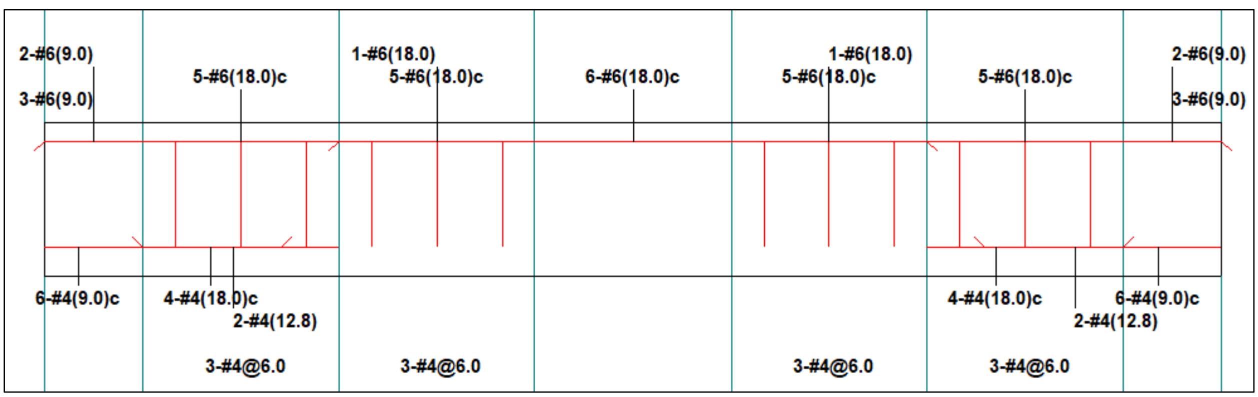


Figure 20 – Provided Shear and Flexural Reinforcement for the Beam-on-Elastic-Foundation (spBeam)

5. Displacements

5.1. Using spMats

In spMats, the minimum displacement is 0.635 in. (downward) and it occurs near midspan. The maximum displacement is 0.665 in. (downward) and it occurs near the ends. The deformed shape confirms the dominant one-way behavior in this system.

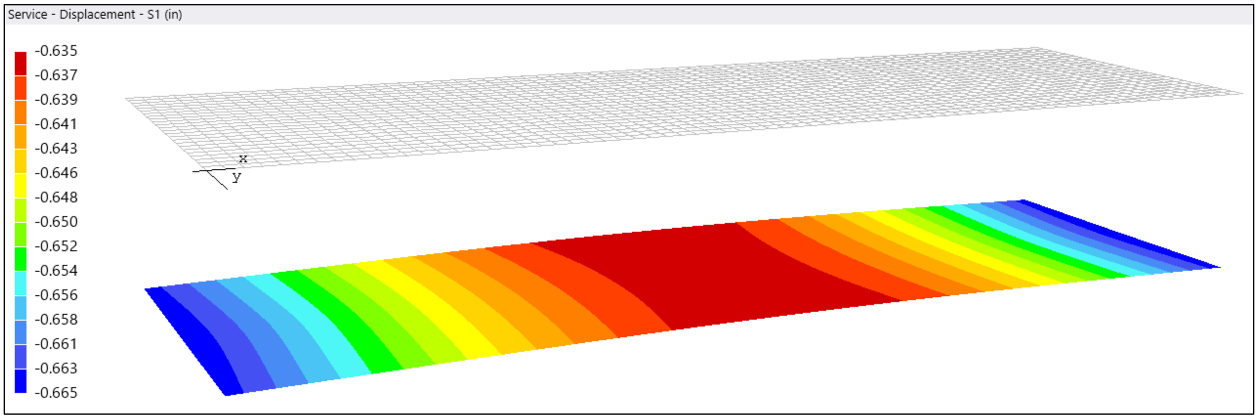


Figure 21 – Service Load Combination S1 – Displacement (spMats)

5.2. Using Beam

In spBeam, the minimum displacement is 0.636 in. (downward) and it occurs at span 4. The maximum displacement is 0.663 in. (downward) and it occurs at spans 1 and 7.

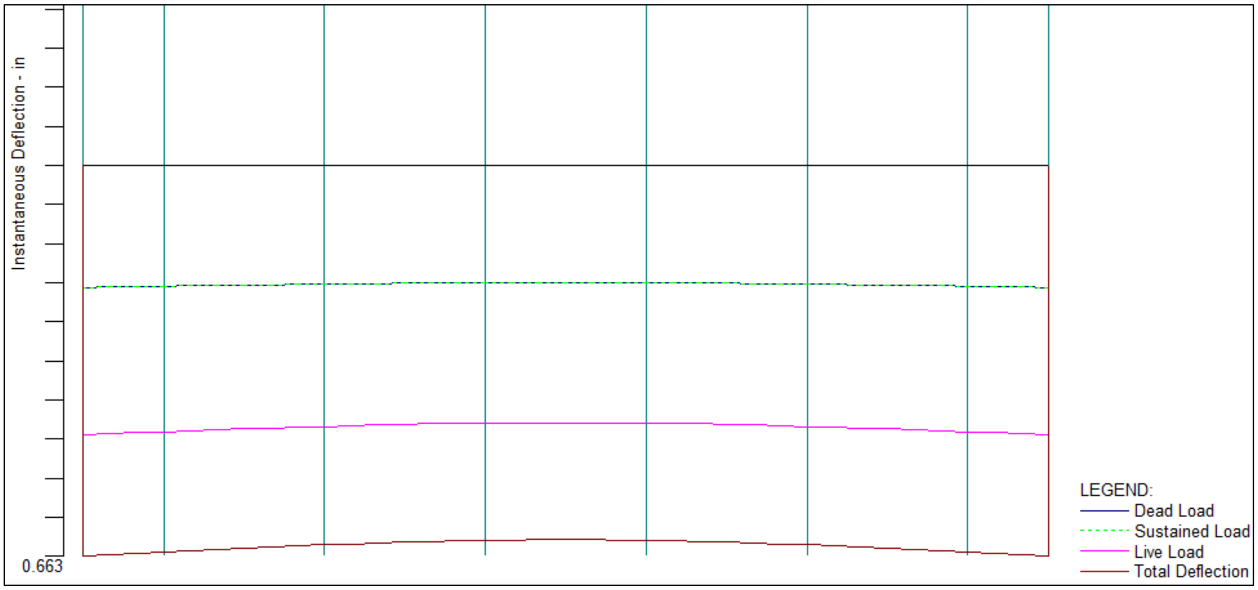


Figure 22 – Instantaneous Deflection for the Beam-on-Elastic-Foundation (spBeam)

Deflection Results: Summary - Instantaneous Deflections - Extreme Instantaneous Frame Deflections

					Live			Total	
Span	Direction	Value	Units	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	in	0.207	---	0.457	0.457	0.207	0.663
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
2	Down	Def	in	0.205	---	0.452	0.452	0.205	0.657
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.201	---	0.443	0.443	0.201	0.644
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
4	Down	Def	in	0.199	---	0.437	0.437	0.199	0.636
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
	Up	Def	in	---	---	---	---	---	---

Figure 23 – Instantaneous Deflections (Summary Results) for the Beam-on-Elastic-Foundation (spBeam)

6. Conclusions and Observations

The results obtained from the finite element analysis from spMats and the beam-on-elastic-foundation using spBeam Programs as discussed above are in good agreement. The observations are listed as follows:

- The beam-on-elastic-foundation modeling of the combined footing in spBeam focuses on the one-way behavior. Therefore, it is not feasible to capture two-way behavior and its influence on the flexural design in Y-Direction as with the FEA in spMats.
- As the length (between columns or loading points) to width ratio of the combined footing increases, the effect of two-way behavior diminishes, the combined footing analysis can utilize one-way BOEF analysis.
- Unlike spMats, spBeam does not provide an output for soil pressures. Instead, spBeam output contains spring support reactions which can be divided by the tributary area around the support in order to obtain soil pressures. Reasonably closely spaced soil spring supports lead to comparable FEA results in spMats.
- spBeam features one-way shear analysis and design which is not performed in spMats as it utilizes the thin plate theory, which makes use of the following Kirchhoff hypotheses. The one-way shear values may be obtained in spMats by utilizing pressure values as shown in the manual calculation presented.