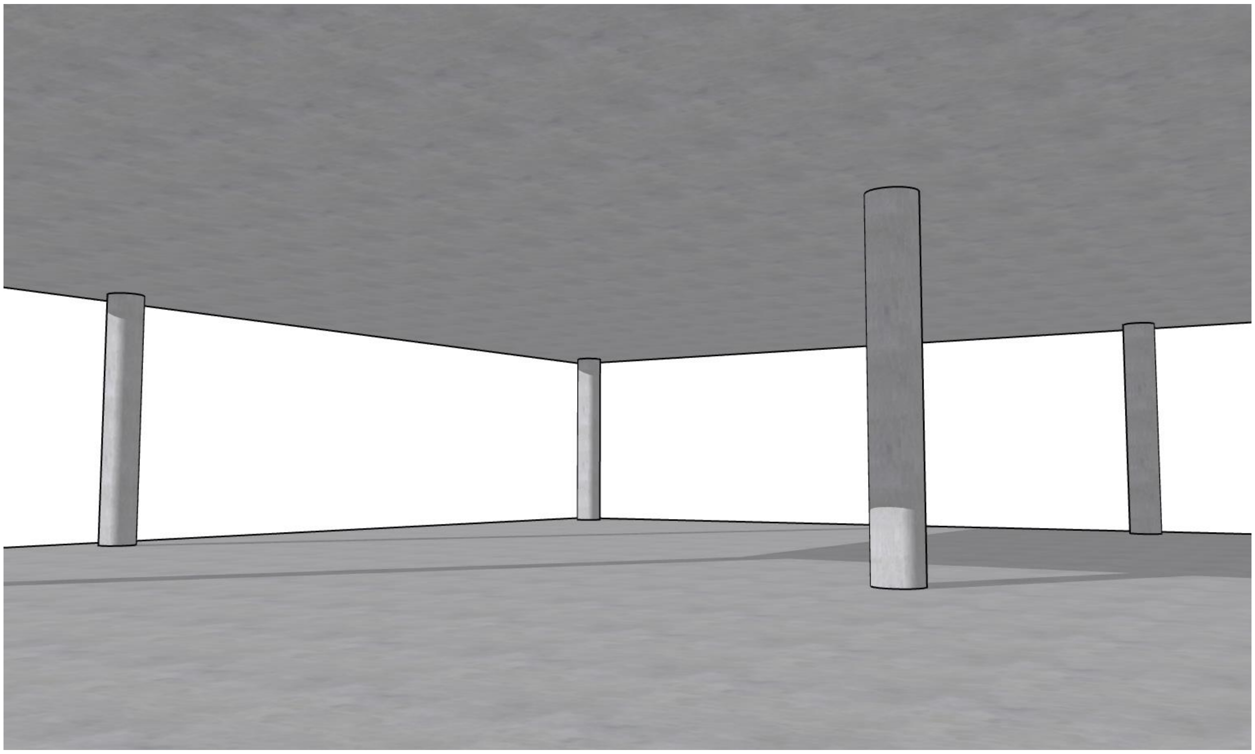
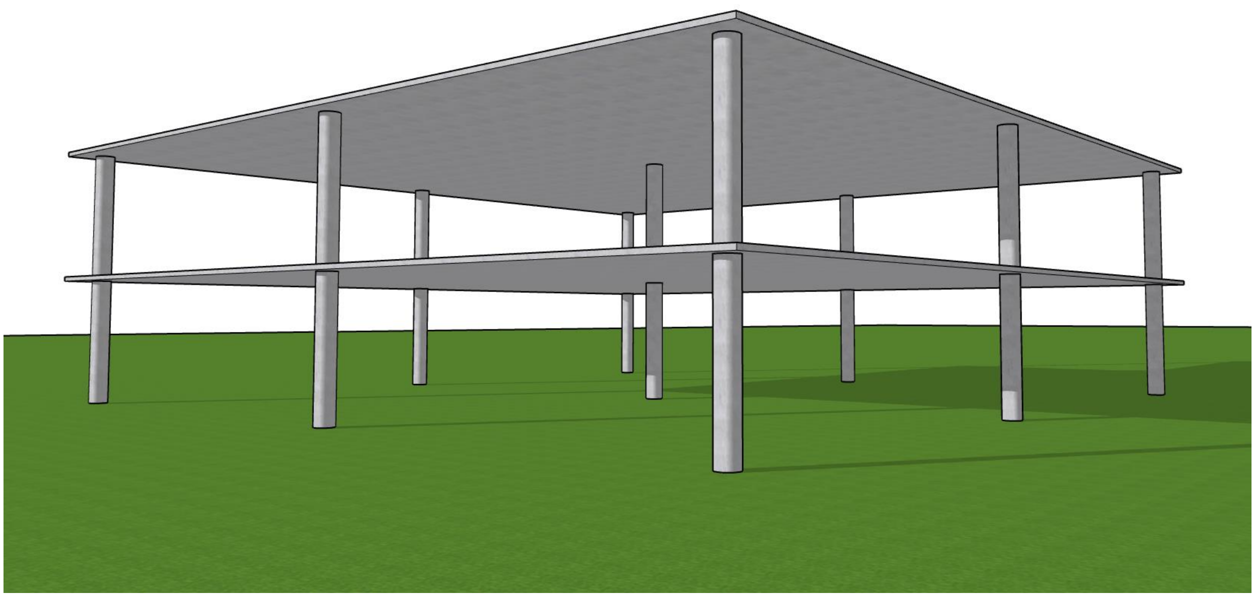
Two-way (Punching) Shear Calculations for Concrete Slab Supported by Circular Columns





Objective

Perform the two-way (punching) shear calculations around the exterior and interior circular columns supporting a two-way flat plate concrete slab. These calculations are widely published in text books for square and rectangular shapes but rarely are discussed in detail for circular columns or column capitals. This design example provides step-by-step hand-calculations and compares various CSA methodologies to determine the critical shear perimeter of circular columns.

Codes

Design of Concrete Structures (CSA A23.3-14) and Explanatory Notes on CSA Group standard A23.3-14 "Design of Concrete Structures"

Design of Concrete Structures (CSA A23.3-19) and Explanatory Notes on CSA Group standard A23.3-19 "Design of Concrete Structures"

References

[1] MacGregor J.G., Bartlett F.M., Reinforced Concrete – Mechanics and Design, First Canadian Edition, Prentice Hall Canada Inc., 2000

[2] spSlab Engineering Software Program Manual v5.50, StructurePoint LLC., 2018.

[3] "Two-way (Punching) Shear Calculations for Concrete Slab Supported by Circular Columns – ACI 318" Design Example, STRUCTUREPOINT, 2019

Analysis and Design Data

f_c' = 35 MPa and f_y = 400 MPa

l_1 = 6.0 m and l_2 = 5.5 m

Uniformly distributed factored load, w_u = 47.00 kN/m^2 (includes self-weight of slab, live load, and superimposed dead load)

Supporting circular column diameter, h_c = 850 mm

Effective depth for two-way shear, d = 260 mm (average of effective depths in two orthogonal directions per ACI 318)

Strength concrete reduction factor, ϕ_c, for shear equals to 0.65 per CSA A23.3-14.

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Two-way Slab Model

The isometric and plan views below are produced by the spSlab Program.

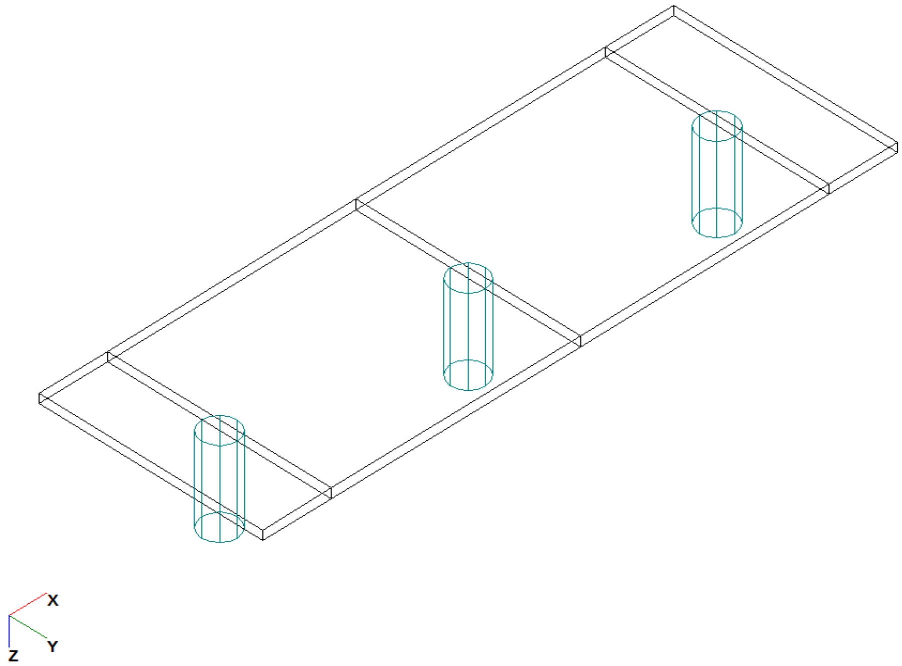

Figure 1 – Isometric view of two-way plate concrete floor slab supported by circular columns

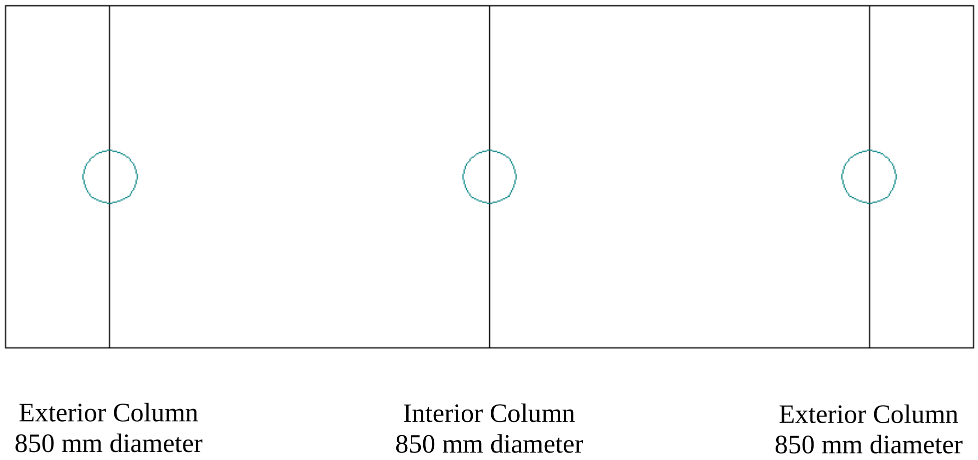

Figure 2 – Plan view of two-way plate concrete floor slab supported by circular columns

Internal Force (Shear Force & Bending Moment) Diagrams

The shear force and bending moment diagrams below are produced by the spSlab Program and will be used for two-way (punching) shear calculations.

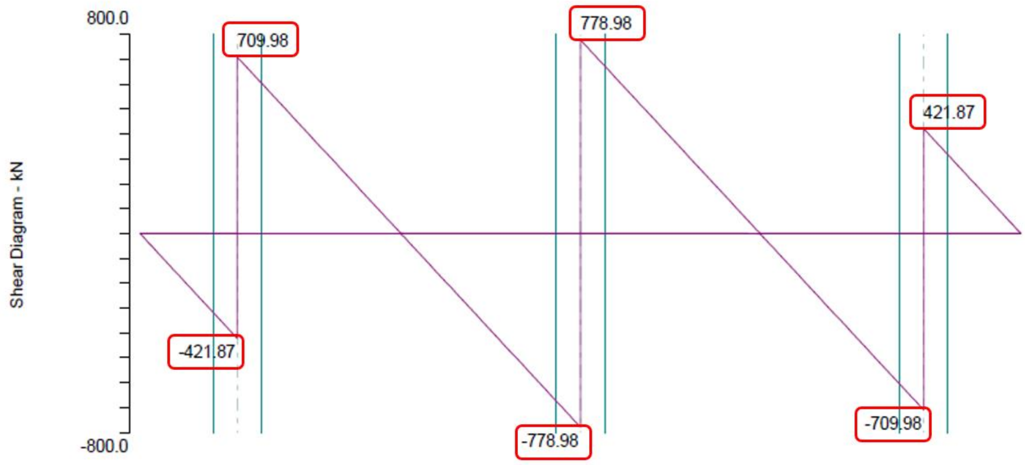

Figure 3 – Shear Force Diagram from spSlab

From the shear force diagram, the reactions at the centroid of the critical section for exterior and interior columns are:

At exterior supporting column, the reaction, $R = [421.87 + 709.98] = 1131.85$ kN

At interior supporting column, the reaction, $R = [778.98 + 778.98] = 1557.96$ kN

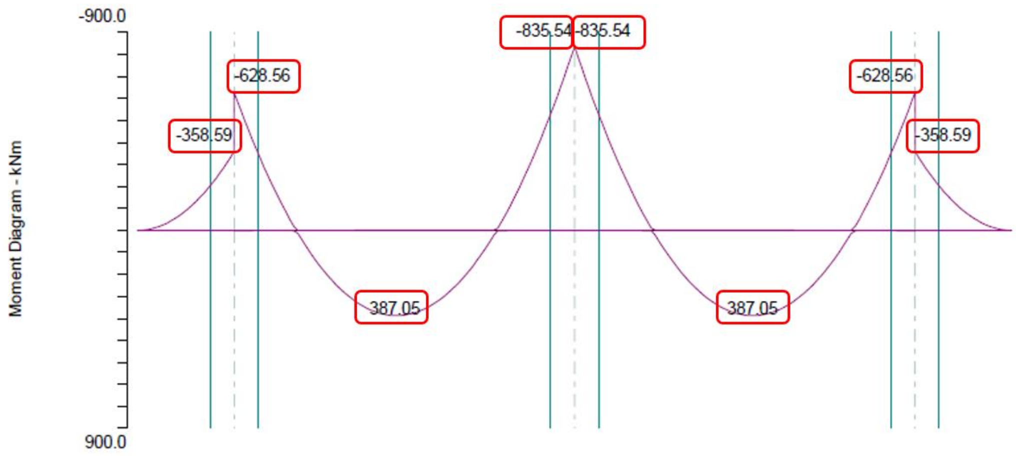

Figure 4 – Bending Moment Diagram from spSlab

From the bending moment diagram, the factored unbalanced moment used for shear transfer, M_{unb}, for exterior and interior columns are:

At exterior supporting column, the unbalanced moment, $M_f = [628.56 - 358.59] = 269.97$ kN-m

At interior supporting column, the unbalanced moment, $M_f = [835.54 - 835.54] = 0.00$ kN-m

Two-way (punching) Shear Calculations

Two-way (punching) shear calculations are performed to ensure that the concrete slab design shear strength, $v_r = v_c$, shall be greater than or equal to the factored shear stress, v_f.

$$v_r = v_c \geq v_f$$

The combined two-way (punching) shear stress, v_f, is calculated as the summation of direct shear alone and direct shear transfer resulting from the unbalanced moment:

$$v_f = \frac{V_f}{b_0 \times d} + \left(\frac{\gamma_v \times M_f \times e}{J}\right)_x + \left(\frac{\gamma_v \times M_f \times e}{J}\right)_y$$

CSA A23.3-14 (Eq. 13.9)

The factored shear force, V_f, at the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section perimeter.

The factored unbalanced moment used for shear transfer, M_f, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

Without shear reinforcement in the slab, the equivalent concrete stress corresponding to nominal two-way shear strength of slab, v_r, equals to the stress corresponding to nominal two-way shear strength provided by concrete, v_c.

$$v_r = v_c = \min \left\{ \begin{array}{l} 0.38 \times \lambda \times \phi_c \times \sqrt{f_c'} \\ \left(1 + \frac{2}{\beta_c}\right) \times 0.19 \times \lambda \times \phi_c \times \sqrt{f_c'} \\ \left(\frac{\alpha_s d}{b_o} + 0.19\right) \times \lambda \times \phi_c \times \sqrt{f_c'} \end{array} \right\}$$

CSA A23.3-14 (13.3.4.1)

β_c = the ratio of the long side to the short side of column, concentrated load, or reaction area

α_s = a constant dependent on supporting column location (equals to 4 for an interior effective critical area)

λ = 1.0 (normal density concrete)

b_0 = the perimeter of the critical section for two-way shear. The critical section shall be located so that the perimeter, b_0, is a minimum but need not be closer than d/2 to the perimeter of the supporting column.

CSA A23.3-14 (13.3.3.1)

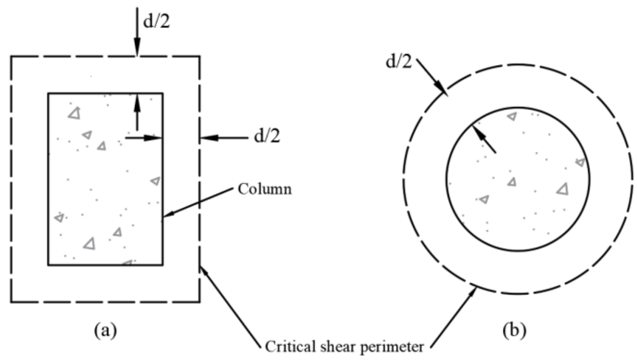

Figure 5 – Location of critical shear perimeter

The two-way shear calculations based on an exact circular critical shear perimeter of circular interior and exterior supporting column as shown above along with two approximate critical shear perimeter methods will be discussed.

Method 1: Exact Circular Critical Shear Perimeter – CSA A23.3-14

This method uses the exact circular critical shear perimeter located d/2 away from circular column perimeter for two-way (punching) shear calculations.

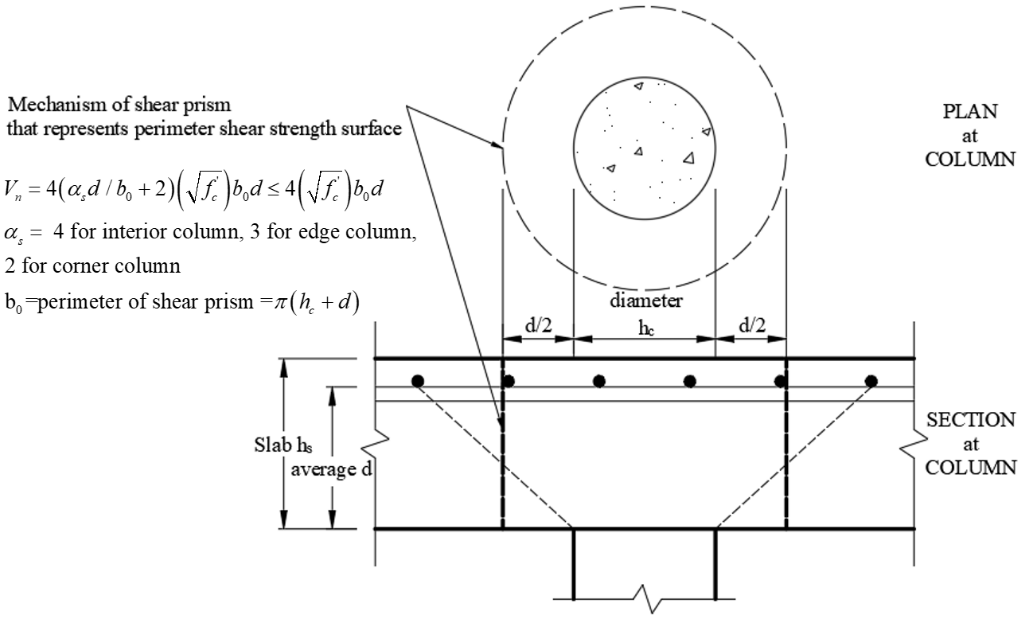

Figure 6 – Mechanism of shear prism that represents perimeter shear strength surface

Circular critical section properties for two-way shear stress calculations for circular column are shown below. Critical section for circular column, b_0 is based on circular perimeter.

$$b_0 = \pi \times (h_c + d)$$

Area of circular concrete section resisting shear transfer, A_c, equals to circular perimeter of critical section, b_0, multiplied by the effective depth, d.

$$A_c = b_0 \times d = \pi(h_c + d) \times d$$

$$c = c' = \frac{(h_c + d)}{2}$$

$$\frac{J}{c} = \pi \times d \times \left[\frac{(h_c + d)}{2}\right]^2 + \frac{d^3}{3}$$

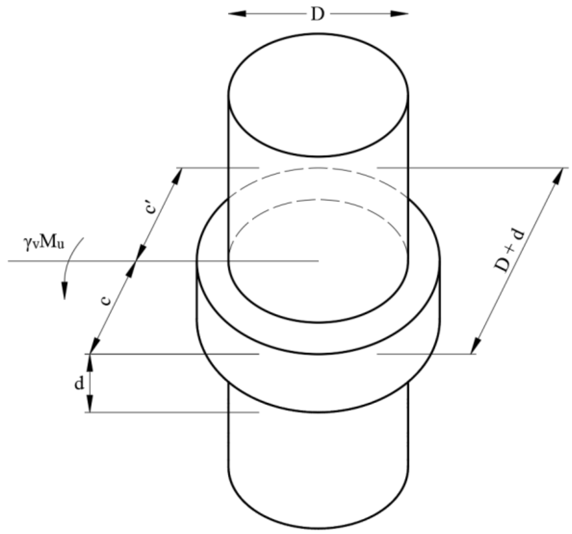

Figure 7 – Section Properties for Shear Stress Computation – Circular Interior Column

For the combined two-way (punching) shear stress, v_f, calculations, circular critical section properties of exterior and interior columns are identical due to the existence of 1.7 m cantilever span at exterior columns.

$$b_0 = \pi \times (h_c + d) = \pi \times (850 + 260) = 3,487 \text{ mm}$$

$$\text{A}_c = b_0 \times d = \pi \times (h_c + d) \times d = \pi \times (850 + 260) \times 260 = 906,664 \text{ mm}^2 = 0.907 \text{ m}^2$$

$$\text{c} = c' = \frac{(h_c + d)}{2} = \frac{850 + 260}{2} = 555 \text{ mm} = 0.555 \text{ m}$$

$$\frac{J}{c} = \pi \times d \times \left[\frac{(h_c + d)}{2}\right]^2 + \frac{d^3}{3} = \pi \times 260 \times \left[\frac{(850 + 260)}{2}\right]^2 + \frac{260^3}{3} = 257,457,827 \text{ mm}^3 = 0.257 \text{ m}^3$$

Sum of the self-weight and superimposed surface dead and live load acting within the critical section perimeter is:

$$w_u \left(\frac{\pi \times (h_c + d)^2}{4}\right) = \frac{47.00}{1000^2} \times \left(\frac{\pi \times (850 + 260)^2}{4}\right) = 45.48 \text{ kN}$$

Exterior supporting column:

The two-way combined shear stress, v_f, can be calculated as:

$$v_f = \frac{\text{V}_\text{f}}{\text{b}_0 \times \text{d}} + \left(\frac{\gamma_\text{v} \times \text{M}_\text{f} \times e}{\text{J}}\right)_\text{x} + \left(\frac{\gamma_\text{v} \times \text{M}_\text{f} \times e}{\text{J}}\right)_\text{y} \quad \text{where:}$$

$$\gamma_v = 1 - \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{h_c}{h_c}}} = 1 - \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{850}{850}}} = 0.40$$ ***CSA A23.3-14 (Eq. 13.8)***

The reaction, R = 1131.85 kN from shear diagram and the unbalanced moment, M_f = 269.97 kN-m, from bending moment diagram.

$$v_f = \frac{(1131.85-45.48)}{3487\times 260} + \frac{0.4\times[269.97\times 1000\times 1000]}{257,457,827}$$

$$v_f = 1.20 + 0.42 = 1.62 \text{ MPa}$$

The two-way design shear strength without shear reinforcement, v_c, can be calculated as:

$$v_r = v_c = \min\begin{Bmatrix} 0.38\times\lambda\times\phi_c\times\sqrt{f_c'} \\ \left(1+\frac{2}{\beta_c}\right)\times 0.19\times\lambda\times\phi_c\times\sqrt{f_c'} \\ \left(\frac{\alpha_s d}{b_o}+0.19\right)\times\lambda\times\phi_c\times\sqrt{f_c'} \end{Bmatrix} = \min\begin{Bmatrix} 0.38\times 1.00\times 0.65\times\sqrt{35} \\ \left(1+\frac{2}{1}\right)\times 0.19\times 1.00\times 0.65\times\sqrt{35} \\ \left(\frac{4\times 260}{3,487}+0.19\right)\times 1.00\times 0.65\times\sqrt{35} \end{Bmatrix}$$

$$v_r = v_c = \min\begin{Bmatrix} 1.46 \\ 2.19 \\ 1.88 \end{Bmatrix} = 1.46 \text{ MPa}$$

$$v_f / v_c = 1.62/1.46 = 1.11 > 1.0 \text{ N.G.}$$

Since $v_c < v_f$ at the critical section, the slab has **inadequate** two-way shear strength at the exterior column.

Interior supporting column:

The two-way combined shear stress, v_f, can be calculated as:

$$v_f = \frac{V_f}{b_0\times d} + \left(\frac{\gamma_v\times M_f\times e}{J}\right)_x + \left(\frac{\gamma_v\times M_f\times e}{J}\right)_y \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 1557.96 kN from shear diagram and the unbalanced moment, M_f = 0.00 kN-m from bending moment diagram.

$$v_f = \frac{(1557.96-45.48)}{3487\times 260} + \frac{0.4\times[0.00\times 1000\times 1000]}{257,457,827}$$

$$v_f = 1.67 + 0.00 = 1.67 \text{ MPa}$$

The two-way design shear strength without shear reinforcement, v_c, can be calculated as:

$$v_r = v_c = \min \begin{cases} 0.38 \times \lambda \times \phi_c \times \sqrt{f_c'} \\ \left(1 + \frac{2}{\beta_c}\right) \times 0.19 \times \lambda \times \phi_c \times \sqrt{f_c'} \\ \left(\frac{\alpha_s d}{b_o} + 0.19\right) \times \lambda \times \phi_c \times \sqrt{f_c'} \end{cases} = \min \begin{cases} 0.38 \times 1.00 \times 0.65 \times \sqrt{35} \\ \left(1 + \frac{2}{1}\right) \times 0.19 \times 1.00 \times 0.65 \times \sqrt{35} \\ \left(\frac{4 \times 260}{3,487} + 0.19\right) \times 1.00 \times 0.65 \times \sqrt{35} \end{cases}$$

$$v_r = v_c = \min \begin{cases} 1.46 \\ 2.19 \\ 1.88 \end{cases} = 1.46 \text{ MPa}$$

$$v_f / v_c = 1.67 / 1.46 = 1.14 > 1.0 \text{ N.G.}$$

Since $v_c < v_f$ at the critical section, the slab has **inadequate** two-way shear strength at the interior column.

Method 2: Approximate Square Critical Shear Perimeter based on an Equivalent Square Column Perimeter that is equal to Circular Supporting Column Perimeter

The perimeter of critical section for circular column, b_0, is based on an equivalent square column with the same centroid and the same length of perimeter. $b_0 = 4\times\left(\frac{\pi}{4}\times h_c + d\right) = \left(\pi\times h_c + 4\times d\right)$

Area of square concrete section resisting shear transfer, A_c, equals to square perimeter of critical section, b_0, multiplied by the effective depth, d.

$$A_c = b_0 \times d = 4\times\left(\frac{\pi}{4}\times h_c + d\right)\times d$$

$$c = c' = \frac{\left(\frac{\pi}{4}\times h_c + d\right)}{2}$$

$$\frac{J}{c} = \frac{4\times\left[\left(\frac{\pi}{4}\times h_c + d\right)^2 \times d\right] + d^3}{3}$$

In Method 2, the side dimension for the equivalent square column would be $\frac{\pi}{4}\times h_c$ (i.e. $0.785\times h_c$ as it appears at Fig. 13-57 of Reference [1])

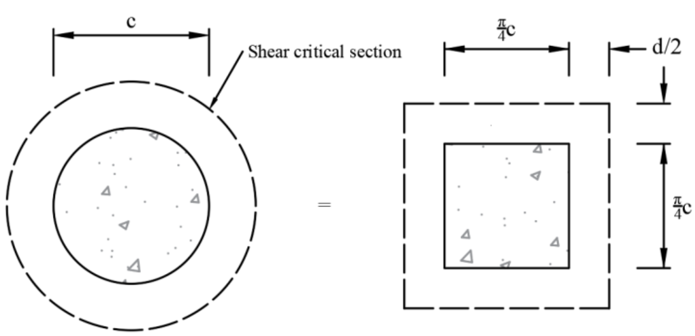

Figure 8 – Approximate Square Critical Shear Perimeter

From the equations above:

$$b_0 = 4\times\left(\frac{\pi}{4}\times h_c + d\right) = 4\times\left(\frac{\pi}{4}\times 850 + 260\right) = 3710 \text{ mm}$$

$$A_c = b_0 \times d = 4\times\left(\frac{\pi}{4}\times h_c + d\right)\times d = 4\times\left(\frac{\pi}{4}\times 850 + 260\right)\times 260 = 964,692 \text{ mm}^2 = 0.965 \text{ m}^2$$

$$c = c' = \frac{\left(\frac{\pi}{4}\times h_c + d\right)}{2} = \frac{\left(\frac{\pi}{4}\times 850 + 260\right)}{2} = 464 \text{ mm}$$

$$\frac{J}{c} = \frac{4\times\left[\left(\frac{\pi}{4}\times h_c + d\right)^2 \times d\right] + d^3}{3} = \frac{4\times\left[\left(\frac{\pi}{4}\times 850 + 260\right)^2 \times 260\right] + \left(260\right)^3}{3} = 304,137,708 \text{ mm}^3 = 0.304 \text{ m}^3$$

Sum of the self-weight and superimposed surface dead and live load acting within the critical section perimeter is:

$$w_u\left(\frac{\pi}{4}\times h_c+d\right)^2=\frac{47.00}{1000^2}\times\left(\frac{\pi}{4}\times 850+260\right)^2=40.44\text{ kN}$$

Exterior supporting column:

The two-way combined shear stress, v_f, can be calculated as:

$$v_f=\frac{\mathrm{V_f}}{\mathrm{b_0}\times\mathrm{d}}+\left(\frac{\gamma_\mathrm{v}\times\mathrm{M_f}\times e}{\mathrm{J}}\right)_\mathrm{x}+\left(\frac{\gamma_\mathrm{v}\times\mathrm{M_f}\times e}{\mathrm{J}}\right)_\mathrm{y}\quad\text{where }\gamma_\mathrm{v}=0.40$$

The reaction, R = 1131.85 kN from shear diagram and the unbalanced moment, M_f = 269.97 kN-m from bending moment diagram.

$$v_f=\frac{(1131.85-40.44)}{3710\times 260}+\frac{0.4\times[269.97\times 1000\times 1000]}{304{,}137{,}708}$$

$$v_f=1.13+0.36=1.49\text{ MPa}$$

For this exterior column, the liberalization attributed to this method leads to 5.83% lower direct shear stress [1.13 MPa (Method 2) vs. 1.20 MPa (Method 1)], 14.3% lower shear stress due to unbalanced moment [0.36 MPa (Method 2) vs. 0.42 Mpa (Method 1)]. Therefore, the two-way combined shear stress, v_f, value of 1.49 MPa from Method 2 is 8.0% lesser in magnitude and still acceptable (the liberalization) as compared to the value of 1.62 MPa from the exact circular critical shear perimeter method (Method 1).

The two-way design shear strength without shear reinforcement, v_c, can be calculated as:

$$v_r=v_c=\min\begin{Bmatrix}0.38\times\lambda\times\phi_c\times\sqrt{f_c'}\\ \left(1+\frac{2}{\beta_c}\right)\times 0.19\times\lambda\times\phi_c\times\sqrt{f_c'}\\ \left(\frac{\alpha_s d}{b_o}+0.19\right)\times\lambda\times\phi_c\times\sqrt{f_c'}\end{Bmatrix}=\min\begin{Bmatrix}0.38\times 1.00\times 0.65\times\sqrt{35}\\ \left(1+\frac{2}{1}\right)\times 0.19\times 1.00\times 0.65\times\sqrt{35}\\ \left(\frac{4\times 260}{3{,}487}+0.19\right)\times 1.00\times 0.65\times\sqrt{35}\end{Bmatrix}$$

$$v_r=v_c=\min\begin{Bmatrix}1.46\\ 2.19\\ 1.88\end{Bmatrix}=1.46\text{ MPa}$$

$$v_f/v_c=1.49/1.46=1.02>1.0\ \text{ N.G.}$$

Since $v_c<v_f$ at the critical section, the slab has **inadequate** two-way shear strength at the exterior column.

Interior supporting column:

The two-way combined shear stress, v_f, can be calculated as:

$$v_f = \frac{V_f}{b_0 \times d} + \left(\frac{\gamma_v \times M_f \times e}{J}\right)_x + \left(\frac{\gamma_v \times M_f \times e}{J}\right)_y \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 1557.96 kN from shear diagram and the unbalanced moment, M_f = 0.00 kN-m from bending moment diagram.

$$v_f = \frac{(1557.96 - 40.44)}{3710 \times 260} + \frac{0.4 \times [0.00 \times 1000 \times 1000]}{304,137,708}$$

$$v_f = 1.57 + 0.00 = 1.57 \text{ MPa}$$

For this interior column with zero unbalanced moment, the liberalization attributed to this method leads to 5.9% lower direct shear stress [1.57 MPa (Method 2) vs. 1.67 MPa (Method 1)]. Therefore, the two-way combined shear stress, v_u, value of 1.57 MPa from Method 2 is 5.9% lesser in magnitude and still acceptable (the liberalization) as compared to the value of 1.67 MPa from the exact circular perimeter method (Method 1).

The two-way design shear strength without shear reinforcement, v_c, can be calculated as:

$$v_r = v_c = \min\begin{Bmatrix} 0.38 \times \lambda \times \phi_c \times \sqrt{f_c'} \\ \left(1 + \frac{2}{\beta_c}\right) \times 0.19 \times \lambda \times \phi_c \times \sqrt{f_c'} \\ \left(\frac{\alpha_s d}{b_o} + 0.19\right) \times \lambda \times \phi_c \times \sqrt{f_c'} \end{Bmatrix} = \min\begin{Bmatrix} 0.38 \times 1.00 \times 0.65 \times \sqrt{35} \\ \left(1 + \frac{2}{1}\right) \times 0.19 \times 1.00 \times 0.65 \times \sqrt{35} \\ \left(\frac{4 \times 260}{3,487} + 0.19\right) \times 1.00 \times 0.65 \times \sqrt{35} \end{Bmatrix}$$

$$v_r = v_c = \min\begin{Bmatrix} 1.46 \\ 2.19 \\ 1.88 \end{Bmatrix} = 1.46 \text{ MPa}$$

$$v_f / v_c = 1.57 / 1.46 = 1.08 > 1.0 \text{ N.G.}$$

Since $v_c < v_f$ at the critical section, the slab has **inadequate** two-way shear strength at the interior column.

Method 3: Approximate Square Critical Shear Perimeter based on an Equivalent Square Column Area that is equal to Circular Supporting Column Area - CSA A23.3-19

The perimeter of critical section for circular column, b_0, is based on a square column with the same centroid and the same area.

$$b_0 = 4\times\left(\sqrt{\frac{\pi}{4}}\times h_c + d\right)$$

Area of square concrete section resisting shear transfer, A_c, equals to square perimeter of critical section, b_0, multiplied by the effective depth, d.

$$A_c = b_0\times d = 4\times\left(\sqrt{\frac{\pi}{4}}\times h_c + d\right)\times d$$

$$c = c' = \frac{\left(\sqrt{\frac{\pi}{4}}\times h_c + d\right)}{2}$$

$$\frac{J}{c} = \frac{4\times\left[\left(\sqrt{\frac{\pi}{4}}\times h_c + d\right)^2\times d\right] + d^3}{3}$$

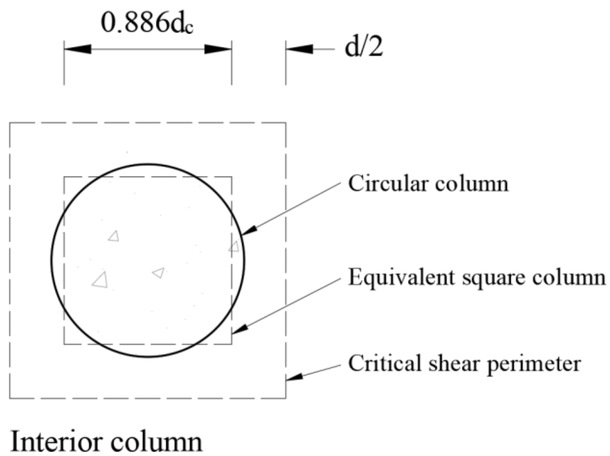

Figure 9 – Critical Shear Perimeter for Moments and Shear Transfer at Circular Columns

In Method 3, the side dimension for the equivalent square column would be $\sqrt{\frac{\pi}{4}}\times h_c$ (i.e. $0.886h_c$)

From the equations above:

$$b_0 = 4\times\left(\sqrt{\frac{\pi}{4}}\times h_c + d\right) = 4\times\left(\sqrt{\frac{\pi}{4}}\times 850 + 260\right) = 4053 \text{ mm}$$

$$A_c = b_0\times d = 4\times\left(\sqrt{\frac{\pi}{4}}\times h_c + d\right)\times d = 4\times\left(\sqrt{\frac{\pi}{4}}\times 850 + 260\right)\times 260 = 1{,}053{,}825 \text{ mm}^2 = 1.054 \text{ m}^2$$

$$c = c' = \frac{\left(\sqrt{\frac{\pi}{4}} \times h_c + d\right)}{2} = \frac{\left(\sqrt{\frac{\pi}{4}} \times 850 + 260\right)}{2} = 507 \text{ mm}$$

$$\frac{J}{c} = \frac{4 \times \left[\left(\sqrt{\frac{\pi}{4}} \times h_c + d\right)^2 \times d\right] + d^3}{3} = \frac{4 \times \left[\left(\sqrt{\frac{\pi}{4}} \times 850 + 260\right)^2 \times 260\right] + (260)^3}{3} = 361{,}802{,}991 \text{ mm}^3 = 0.362 \text{ m}^3$$

Sum of the self-weight and superimposed surface dead and live load acting within the critical section perimeter is:

$$w_u \times \left(\sqrt{\frac{\pi}{4}} \times h_c + d\right)^2 = \frac{47.00}{1000^2}\left(\sqrt{\frac{\pi}{4}} \times 850 + 260\right)^2 = 48.26 \text{ kN}$$

Exterior supporting column:

The two-way combined shear stress, v_f, can be calculated as:

$$v_f = \frac{V_f}{b_0 \times d} + \left(\frac{\gamma_v \times M_f \times e}{J}\right)_x + \left(\frac{\gamma_v \times M_f \times e}{J}\right)_y \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 1131.85 kN from shear diagram and the unbalanced moment, M_f = 269.97 kN-m from bending moment diagram.

$$v_f = \frac{(1131.85 - 48.26)}{4053 \times 260} + \frac{0.4 \times [269.97 \times 1000 \times 1000]}{361{,}802{,}991}$$

$$v_f = 1.03 + 0.30 = 1.33 \text{ MPa}$$

For this exterior column, this method per CSA A23.3-14 leads to 14.2% lower direct shear stress [1.03 MPa (Method 3) vs. 1.20 MPa (Method 1)], 28.6% lower shear stress due to unbalanced moment [0.30 MPa (Method 3) vs. 0.42 MPa (Method 1)]. Therefore, the two-way combined shear stress, v_u, value of 1.33 MPa from Method 3 is 17.9% lesser in magnitude as compared to the value of 1.62 MPa from the exact circular critical shear perimeter method (Method 1).

The two-way design shear strength without shear reinforcement, v_c, can be calculated as:

$$v_r = v_c = \min\begin{Bmatrix} 0.38 \times \lambda \times \phi_c \times \sqrt{f_c'} \\ \left(1 + \frac{2}{\beta_c}\right) \times 0.19 \times \lambda \times \phi_c \times \sqrt{f_c'} \\ \left(\frac{\alpha_s d}{b_o} + 0.19\right) \times \lambda \times \phi_c \times \sqrt{f_c'} \end{Bmatrix} = \min\begin{Bmatrix} 0.38 \times 1.00 \times 0.65 \times \sqrt{35} \\ \left(1 + \frac{2}{1}\right) \times 0.19 \times 1.00 \times 0.65 \times \sqrt{35} \\ \left(\frac{4 \times 260}{3{,}487} + 0.19\right) \times 1.00 \times 0.65 \times \sqrt{35} \end{Bmatrix}$$

$$v_r = v_c = \min\begin{Bmatrix}1.46\\2.19\\1.88\end{Bmatrix} = 1.46 \text{ MPa}$$

$$v_f / v_c = 1.33/1.46 = 0.91 > 1.0 \text{ O.K.}$$

Since $v_c > v_f$ at the critical section, the slab has **adequate** two-way (punching) shear strength at the exterior column. It is important to point out the slab at exterior column that was deemed inadequate for two-way shear strength per Methods 1 and 2, appears not only to be adequate but to have extra 9.0% reserve capacity per the Method 3.

Interior supporting column:

The two-way combined shear stress, v_f, can be calculated as:

$$v_f = \frac{V_f}{b_0 \times d} + \left(\frac{\gamma_v \times M_f \times e}{J}\right)_x + \left(\frac{\gamma_v \times M_f \times e}{J}\right)_y \quad \text{where } \gamma_v = 0.40$$

The reaction, R = 1557.96 kN from shear diagram and the unbalanced moment, M_f = 0.00 kN-m from bending moment diagram.

$$v_f = \frac{(1557.96 - 48.26)}{4053 \times 260} + \frac{0.4 \times [0.00 \times 1000 \times 1000]}{361{,}802{,}991}$$

$$v_f = 1.43 + 0.00 = 1.43 \text{ MPa}$$

For this interior column with zero unbalanced moment, this method per CSA A23.3-19 leads to 14.4% lower direct shear stress [1.43 MPa (Method 3) vs. 1.67 MPa (Method 1)]. Therefore, the two-way combined shear stress, v_u ,value of 1.43 MPa from Method 3 is 14.4% lesser in magnitude as compared to the value of 1.67 MPa from the exact circular critical shear perimeter method (Method 1).

The two-way design shear strength without shear reinforcement, v_c, can be calculated as:

$$v_r = v_c = \min\begin{Bmatrix}0.38 \times \lambda \times \phi_c \times \sqrt{f_c'}\\ \left(1 + \frac{2}{\beta_c}\right) \times 0.19 \times \lambda \times \phi_c \times \sqrt{f_c'}\\ \left(\frac{\alpha_s d}{b_o} + 0.19\right) \times \lambda \times \phi_c \times \sqrt{f_c'}\end{Bmatrix} = \min\begin{Bmatrix}0.38 \times 1.00 \times 0.65 \times \sqrt{35}\\ \left(1 + \frac{2}{1}\right) \times 0.19 \times 1.00 \times 0.65 \times \sqrt{35}\\ \left(\frac{4 \times 260}{3{,}487} + 0.19\right) \times 1.00 \times 0.65 \times \sqrt{35}\end{Bmatrix}$$

$$v_r = v_c = \min\begin{Bmatrix}1.46\\2.19\\1.88\end{Bmatrix} = 1.46 \text{ MPa}$$

$v_f / v_c = 1.43 / 1.46 = 0.98 > 1.0$ O.K.

Since $v_c > v_f$ at the critical section, the slab has **adequate** two-way (punching) shear strength at the interior column. It is important to point out the slab at interior column that was deemed inadequate for two-way shear strength per Methods 1 and 2, appears not only to be adequate but to have extra 2.0% reserve capacity per the Method 3.

Summary of Results

The results related to two-way (punching) shear calculations per three methods are summarized in table below:

Method Used	**Method 1**	**Method 2**	Diff.	**Method 3**	Diff.
Critical Shear Perimeter	Exact Circular	Approximate Square		Approximate Square	
Column Geometry utilized for Critical Shear Perimeter Calculation	Circular Supporting Column	Square Column with Same Perimeter as the Circular Supporting Column		Square Column with Same Area as the Circular Supporting Column	
Side Length of Square Column where h_c is the diameter of circular column	N/A	$\frac{\pi}{4}h_c$ $(0.785h_c)$		$\sqrt{\frac{\pi}{4}}h_c$ $(0.886h_c)$	
The perimeter of the critical section, b_0 (mm)	3,487	3,710	+6.4%	4,053	+16.2%
Area of circular concrete section resisting shear transfer, A_c (mm^2)	906,664	964,692	+6.4%	1,053,825	+16.2%
J/c (mm^3)	257,457,827	304,137,708	+18.1%	361,802,991	+40.5%
Exterior Column					
The two-way combined shear stress, v_f (MPa)	1.62	1.49	-8.0%	1.33	-17.9%
The two-way design shear strength without shear reinforcement, v_c (MPa)	1.46	1.46	0%	1.46	0%
Capacity Ratio v_f / v_c (≤1.00 is Safe)	1.11	1.02		0.91	
Status	**Inadequate**	**Inadequate**		**Adequate**	
Interior Column					
The two-way combined shear stress, v_f (MPa)	1.67	1.57	-5.9%	1.43	-14.4%
The two-way design shear strength without shear reinforcement, v_c (MPa)	1.46	1.46	0%	1.46	0%
Capacity Ratio v_f / v_c (≤1.00 is Safe)	1.14	1.08		0.98	
Status	**Inadequate**	**Inadequate**		**Adequate**	

Conclusions:

The calculations shown in this article using the CSA standard provisions is based on a more detailed article based on ACI standard provisions and references. The reader is advised to read and review the references given in the ACI version of this example for a complete and detailed background. The ACI article is found here:
Two-way (Punching) Shear Calculations for Concrete Slab Supported by Circular Columns – ACI 318