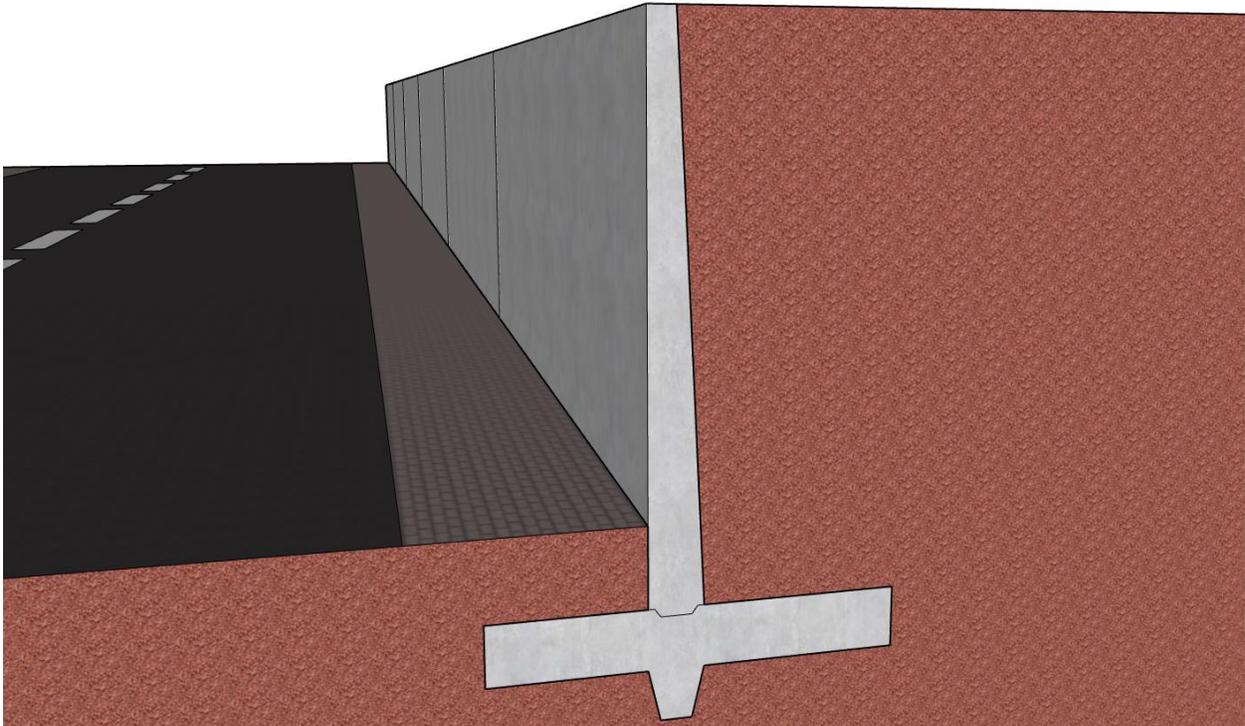
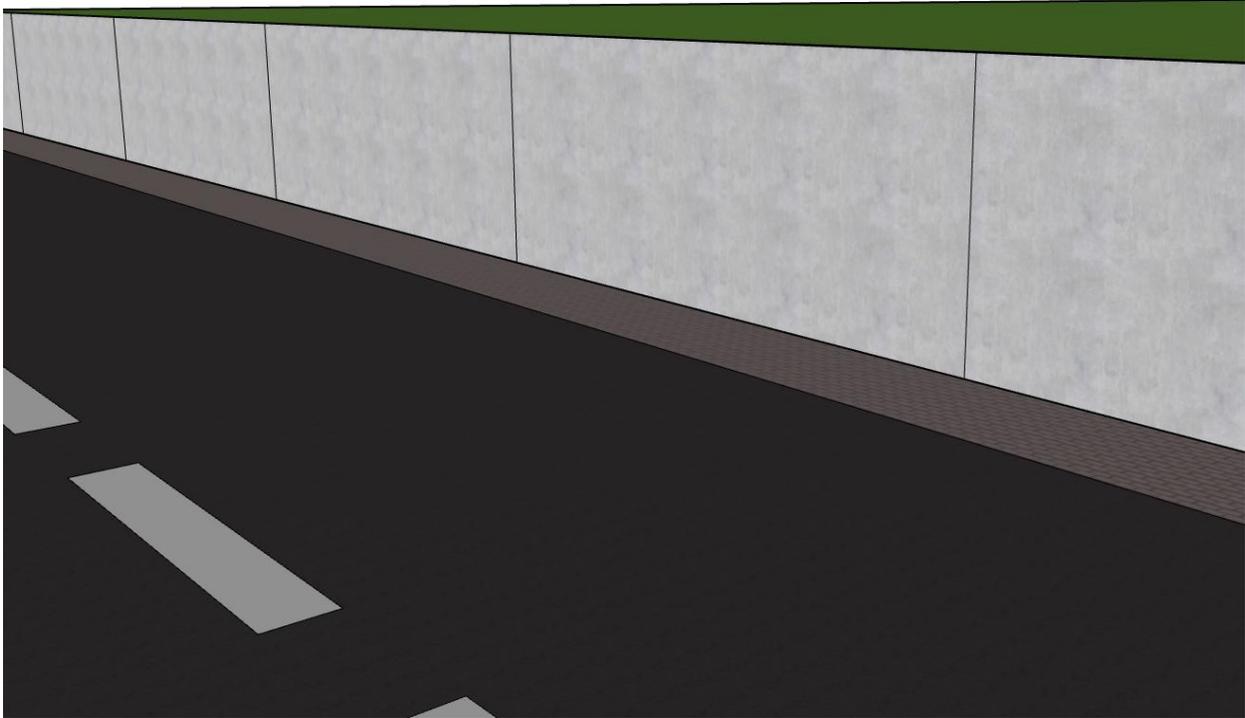


Reinforced Concrete Cantilever Retaining Wall Analysis and Design (ACI 318-19)



Reinforced Concrete Cantilever Retaining Wall Analysis and Design (ACI 318-19)

Reinforced concrete cantilever retaining walls consist of a relatively thin stem and a base slab. The stem may have constant thickness along the length or may be tapered based on economic and construction criteria. The base is divided into two parts, the heel and toe. The heel is the part of the base under the backfill. This system uses much less concrete than monolithic gravity walls, but require more design and careful construction. Cantilever retaining walls can be precast in a factory or formed on site and considered economical up to about 25 ft in height. This design example focuses on the analysis and design of a tapered cantilever retaining wall including a comparison with model results from the engineering software programs [spWall](#) and [spMats](#). The retaining wall is fixed to the reinforced concrete slab foundation with a shear key for sliding resistance. The following figure and design data section will serve as input for detailed analysis and design.

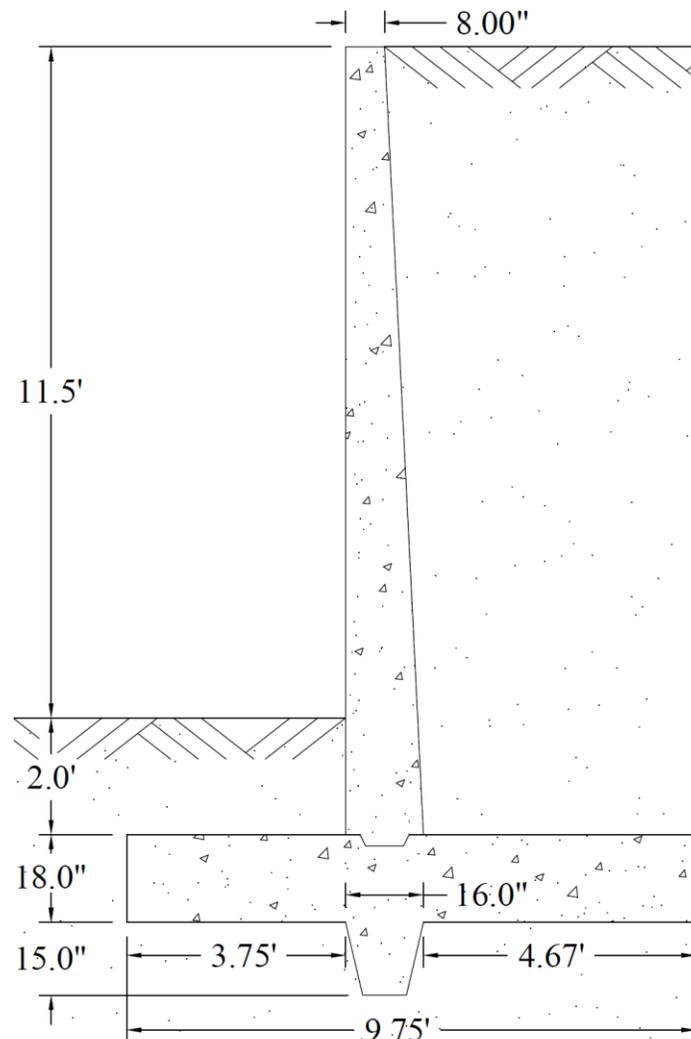


Figure 1 – Cantilever Retaining Wall Dimensions

Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

- Design of Concrete Structures, 16th Edition, 2021, Darwin et. al., McGraw-Hill Education, Example 16.8
- [spWall Engineering Software Program Manual v10.50](#), STRUCTUREPOINT, 2026
- [spMats Engineering Software Program Manual v10.50](#), STRUCTUREPOINT, 2025
- Contact Support@StructurePoint.org to obtain supplementary materials ([spWall](#) model: DE-Cantilever-Retaining-Wall-ACI-19.walx)
- Contact Support@StructurePoint.org to obtain supplementary materials ([spMats](#) model: DE-Cantilever-Retaining-Wall-Foundation-ACI-19.matx)

Design Data

Wall Stem Materials

$$f_c' = 4,500 \text{ psi}$$

$$f_y = 60,000 \text{ psi}$$

$$\gamma_c = 150 \text{ pcf}$$

Wall Foundation Materials

$$f_c' = 4,500 \text{ psi}$$

$$f_y = 60,000 \text{ psi}$$

$$\gamma_c = 150 \text{ pcf}$$

Wall Stem Dimensions

$$\text{Width} = 1.00 \text{ ft strip}$$

$$\text{Height} = 13.50 \text{ ft}$$

$$\text{Thickness} = 8 \text{ in. top}$$
$$= 16 \text{ in. bottom}$$

Wall Foundation Dimensions

$$\text{Width} = 1.00 \text{ ft strip}$$

$$\text{Length} = 9.75 \text{ ft}$$

$$\text{Thickness} = 18 \text{ in.}$$

Retaining Wall Loads

The following figure shows all the loads applied to the cantilever retaining wall where:

$$W_1 = 0.67 \times 13.5 \times 150 = 1,360 \text{ lb}$$

$$W_2 = 0.67 \times 0.5 \times 13.5 \times 150 = 680 \text{ lb}$$

$$W_3 = 9.75 \times 1.5 \times 150 = 2,190 \text{ lb}$$

$$W_4 = 1.33 \times 1.25 \times 150 = 250 \text{ lb}$$

$$W_5 = 3.75 \times 2 \times 120 = 900 \text{ lb}$$

$$W_6 = 0.67 \times 0.5 \times 13.5 \times 120 = 540 \text{ lb}$$

$$W_7 = 4.67 \times 13.5 \times 120 = 7,570 \text{ lb}$$

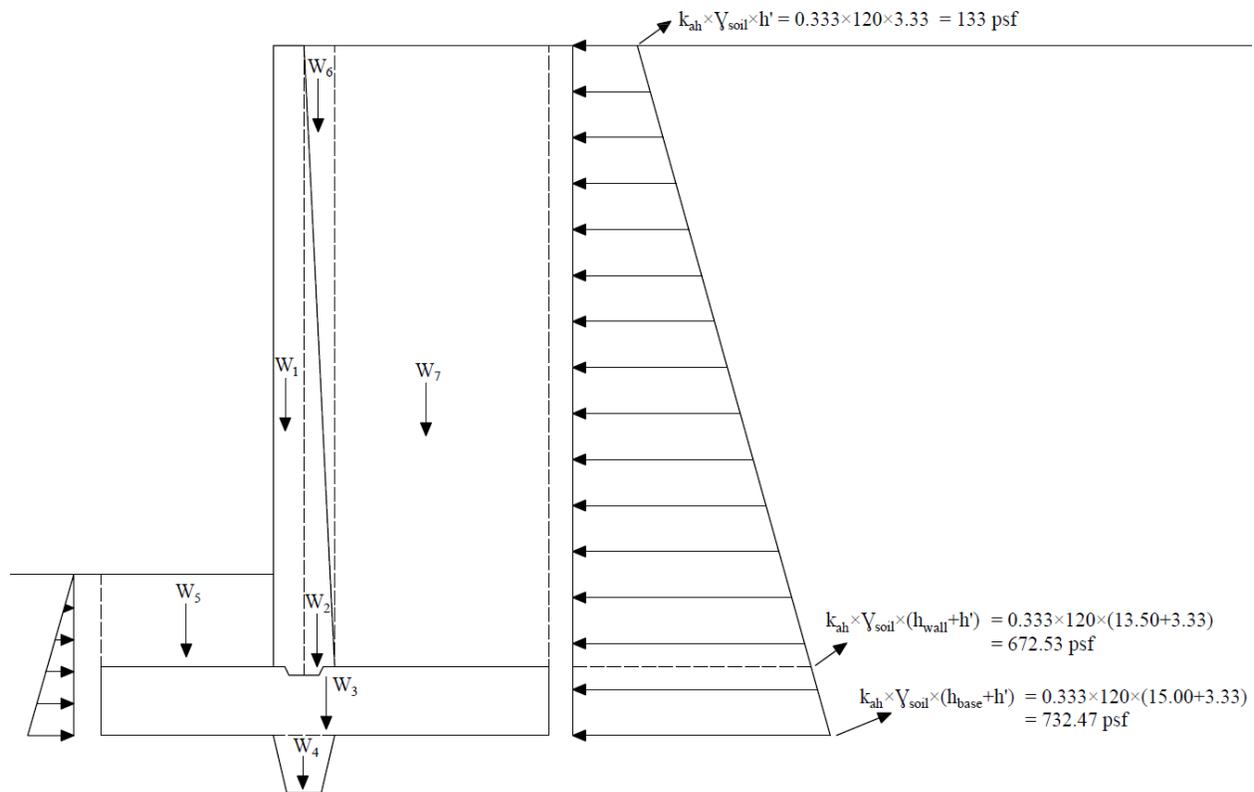


Figure 2 – Applied Loads and Soil Pressure at Critical Sections

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1. Notations

This section (based on ACI 318-19 provisions) defines notation and terminology used in this design example:

a = depth of equivalent rectangular stress block, in.

A_g = gross area of concrete section, in.² For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s)

A_s = area of nonprestressed longitudinal tension reinforcement, in.²

$A_{s,min}$ = minimum area of flexural reinforcement, in.²

b_w = web width or diameter of circular section, in.

c = distance from extreme compression fiber to neutral axis, in.

c_c = clear cover of reinforcement, in.

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

d_b = nominal diameter of bar, wire, or prestressing strand, in.

E_c = modulus of elasticity of concrete, psi

E_s = modulus of elasticity of reinforcement and structural steel, excluding prestressing reinforcement, psi

f_c' = specified compressive strength of concrete, psi

f_s = tensile stress in reinforcement at service loads, excluding prestressed reinforcement, psi

f_y = specified yield strength for nonprestressed reinforcement, psi

M_u = factored moment at section, in.-lb

N_u = factored axial force normal to cross section occurring simultaneously with V_u or T_u ; to be taken as positive for compression and negative for tension, lb

P_u = factored axial force; to be taken as positive for compression and negative for tension, lb

s = center-to-center spacing of items, such as longitudinal reinforcement, transverse reinforcement, tendons, or anchors, in.

V_c = nominal shear strength provided by concrete, lb

V_u = factored shear force at section, lb

w_c = density, unit weight, of normalweight concrete or equilibrium density of lightweight concrete, lb/ft³

β_1 = factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis

- ϵ_t = net tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength, excluding strains due to effective prestress, creep, shrinkage, and temperature
- ϵ_{ty} = value of net tensile strain in the extreme layer of longitudinal tension reinforcement used to define a compression-controlled section
- λ = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal weight concrete of the same compressive strength
- λ_s = factor used to modify shear strength based on the effects of member depth, commonly referred to as the size effect factor
- ρ_w = ratio of A_s to $b_w d$
- ϕ = strength reduction factor

2. Preliminary Design

The thickness of the footing is roughly estimated to calculate the required thickness of the stem at the critical section (stem bottom). With the bottom of the footing at 3.50 ft below grade and an estimated footing thickness of 1.50 ft, the free height of the stem is 13.50 ft. using the information provided in [Figure 1](#) and [Figure 2](#):

$$P = 0.5 \times 0.333 \times 120 \times 13.5 \times (13.5 + 2 \times 3.33) = 5,440 \text{ lb (at the stem bottom)}$$

$$y = \frac{13.5^2 + 3 \times 13.5 \times 3.33}{3 \times (13.5 + 2 \times 3.33)} = 5.25 \text{ ft}$$

$$M_u = P_u \times y = 1.6 \times 5,440 \times 5.25 = 45.7 \text{ ft-kip}$$

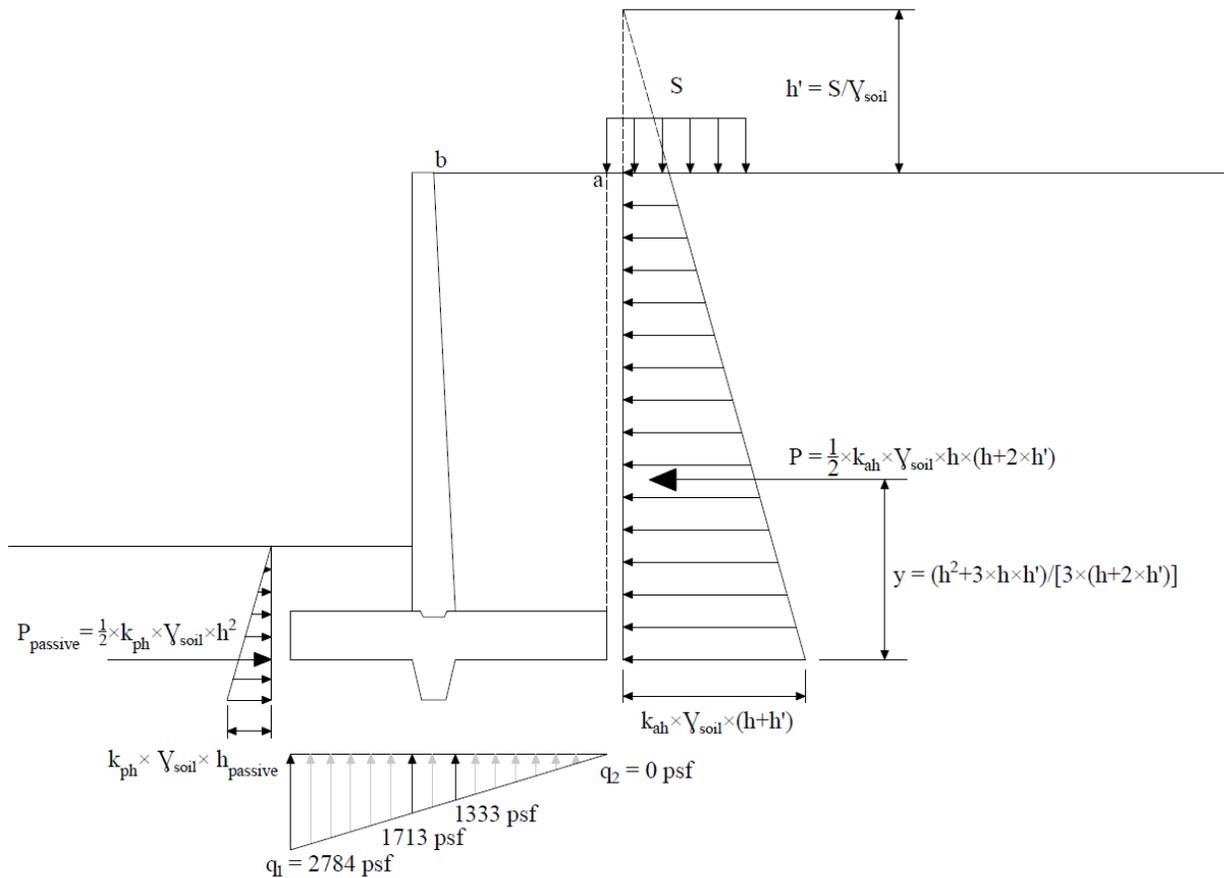


Figure 3 – Bearing Pressure, Overturning and Sliding Loads

The preliminary dimensions are selected using design aids from the reference Appendix A.

$$\rho_{max} = 0.85 \times \beta_1 \times \frac{f'_c}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y + 0.003}$$

Reference 1 (Table A.4)

$$\rho_{max} = 0.85 \times 0.825 \times \frac{4,500}{60,000} \times \frac{0.003}{0.003 + 0.002 + 0.003} = 0.0197$$

The reference recommends the use of a ratio of about 40% of the maximum ($\rho = 0.008$) for economy and ease of bar placement.

$$\frac{M_u}{\phi \times b \times d^2} = 430$$

Reference 1 (Graph A.1b)

$$d = \sqrt{\frac{45,700 \times 12}{0.9 \times 12 \times 430}} = 10.9 \text{ in.}$$

Using cover of 2 in. for members exposed to weather or in contact with ground. **ACI 318-19 (Table 20.5.1.3.1)**

And #8 bars ($d_b = 1$ in.), the minimum required thickness of the stem at the base equals:

$$\text{minimum } t_{stem,base} = d_{min} + \text{cover} + \frac{d_b}{2} = 10.9 + 2 + \frac{1}{2} = 13.4 \text{ in.}$$

Use $t_{stem,base} = 16.00$ in.

For Shear Check:

Following the approach adopted by the reference, the wall arm is checked for shear at the base of the wall at its intersection with the supporting slab, in accordance with **ACI 318-19 (13.3.6.3)** (or at a distance of 13.50 ft below the top of the wall):

$$P = 0.5 \times 0.333 \times 120 \times 13.5 \times (12.5 + 2 \times 3.33) = 5,170 \text{ lb}$$

$$V_u = 1.6 \times P = 1.6 \times 5,170 = 8,270 \text{ lb}$$

The factored shear force is approximately equal to $\phi 1.0 \sqrt{f'_c} b_w d$ as indicated by the reference, which is sufficiently small to allow the design to proceed. The author further notes that, because the shear strength depends on the flexural reinforcement ratio and the size effect factor, the final shear strength verification is performed after completion of the flexural design (see [Section 5](#)).

The thickness of the foundation (base) is the same as or slightly larger than that at the bottom of the stem. Thus, the 18 in. selected earlier need not be revised. The stem thickness can be reduced by tapering one side only up to 8 in. at the top since the bending moment decreases with increasing distance from the wall base to zero at the top of the wall.

3. Wall Stability Checks

The wall has two failure modes: 1) Wall parts may not be strong enough to resist the acting forces, 2) the wall as a rigid body may be displaced or overturned by the earth pressure acting on it. The latter will be discussed in this section to ensure that the retaining wall is stable by checking stability against overturning, sliding, and allowable soil bearing pressure.

Note: two cases are being examined. Case 1 where surcharge load is applied to point a (see [Figure 3](#)), and Case 2 where surcharge load is applied to point b.

3.1. Wall Overturning Check

Case 1 governs for wall overturning since it generated the highest overturning with the least resistance.

Weights and moments about the front edge of the wall are shown in the following table (See [Figure 2](#) and design data section):

Table 1 – Weights and Moments about the Front Edge			
Component Weights	<i>W</i> (kips)	<i>x</i> (ft)	<i>M_r</i> (ft-kip)
<i>W₁</i>	1.36	4.08	5.55
<i>W₂</i>	0.68	4.67	3.18
<i>W₃</i>	2.19	4.88	10.69
<i>W₄</i>	0.25	4.42	1.11
<i>W₅</i>	0.90	1.88	1.69
<i>W₆</i>	0.54	4.86	2.62
<i>W₇</i>	7.57	7.42	56.17
Total	13.49		81.00

$$P = 0.5 \times 0.333 \times 120 \times 15 \times (15 + 2 \times 3.33) = 6.49 \text{ kips}$$

$$y = \frac{15^2 + 3 \times 15 \times 3.33}{3 \times (15 + 2 \times 3.33)} = 5.77 \text{ ft}$$

The overturning moment is equal to:

$$M_o = P \times y = 6,492 \times 5.77 = 37.46 \text{ ft-kip}$$

Factor of Safety against overturning:

$$FOS_{\text{overturning}} = \frac{81.00}{37.46} = 2.16 > 1.5 \text{ (o.k.)}$$

3.2. Soil Bearing Pressure

The distance of the resultant force from the base slab front edge is:

$$a = \frac{81.00 - 37.46}{13.49} = 3.23 \text{ ft} \approx \frac{9.75}{3} = 3.25 \text{ ft}$$

The resultant is barely outside the middle third of the foundation (it is assumed that the bearing pressure becomes zero exactly at the edge of the heel as shown in [Figure 2](#)). The maximum soil pressure at the toe is calculated as follows:

$$q_1 = \frac{2 \times R_v}{3 \times a} \quad \text{Reference 1 (Figure 16.5c)}$$

$$q_1 = \frac{2 \times 13.49 \times 1,000}{3 \times 3.23} = 2,784 \text{ psf} < q_{allowable} = 8,000 \text{ psf (o.k.)}$$

$$q_2 = 0 \quad \text{Reference 1 (Figure 16.5c)}$$

The soil pressure values calculated for Case 1. The soil pressure values for Case 2 do not govern for overturning and sliding. However, values calculated from Case 2 are needed for foundation flexural design as follows:

$$q_1 = (4 \times l - 6 \times a) \frac{R_v}{l^2} \quad \text{Reference 1 (Figure 16.5a)}$$

$$q_1 = 2,710 \text{ psf} < q_{allowable} = 8,000 \text{ psf (o.k.)}$$

$$q_2 = (6 \times a - 2 \times l) \frac{R_v}{l^2} \quad \text{Reference 1 (Figure 16.5a)}$$

$$q_2 = 492 \text{ psf} < q_{allowable} = 8,000 \text{ psf (o.k.)}$$

3.3. Wall Sliding Check

Case 1 also governs for sliding since it produces the least pressure and corresponding friction resistance.

The coefficient of friction that applies for the length along the heel and key is 0.5, while the coefficient of friction for the length in front of the key is equal to the internal soil friction, that is, $\tan 30^\circ = 0.577$. More information about selecting the friction coefficient can be found in the reference in chapter 16 section 4. (for case where surcharge load is applied to point a):

Friction, toe:

$$F_{\text{toe}} = 0.5 \times (2,784 + 1,713) \times 3.75 \times 0.577 = 4.87 \text{ kips}$$

Friction, heel and key:

$$F_{\text{heel and key}} = 0.5 \times 1,713 \times 6 \times 0.5 = 2.57 \text{ kips}$$

Passive earth pressure:

$$P_{\text{passive}} = 0.5 \times 3.0 \times 120 \times (4.75 - 1.5)^2 = 1.90 \text{ kips}$$

Note that the top 1.5 ft layer of soil is discounted in this check as unreliable.

Total:

$$F_{\text{total}} = 4.87 + 2.57 + 1.90 = 9.34 \text{ kips}$$

Factor of Safety against sliding:

$$FOS_{\text{sliding}} = \frac{9.34}{6.49} = 1.44 \approx 1.5 \text{ (can be regarded as adequate)}$$

Thus, the retaining wall with the selected geometry is externally stable.

4. Flexural Reinforcement Requirements

The required flexural reinforcement is traditionally calculated at three critical sections: at the stem base, the toe and heel at the face of the stem.

Calculate the required reinforcement to resist the moment at the stem base:

$$M_u = 45.7 \text{ kip-ft}$$

Use 2.0 in. concrete cover per ACI 318-19 (Table 20.5.1.3.1). The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, d , is calculated below:

$$d = 16 - (2 + 0.5 \times 1) = 13.50 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the beam section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.95d$. The assumptions will be verified once the area of steel is finalized.

$$jd = 0.95 \times d = 0.95 \times 13.5 = 12.83 \text{ in.}$$

$$b = 12.00 \text{ in.}$$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{45.7 \times 12,000}{0.9 \times 60,000 \times 12.83} = 0.79 \text{ in.}^2$$

$$\text{Recalculate 'a' for the actual } A_s = 0.79 \text{ in.}^2: a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{0.79 \times 60,000}{0.85 \times 4,500 \times 12} = 1.04 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{1.04}{0.83} = 1.25 \text{ in.}$$

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4,000)}{1,000} = 0.85 - \frac{0.05 \times (4,500 - 4,000)}{1,000} = 0.83 \quad \text{ACI 318-19 (Table 22.2.2.4.3)}$$

$$\varepsilon_{ty} = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207 \quad \text{ACI 318-19 (21.2.2.1)}$$

$$\varepsilon_t = \left(\frac{0.003}{c} \right) \times d_t - 0.003 = \left(\frac{0.003}{1.25} \right) \times 13.50 - 0.003 = 0.0293 > 0.003 + \varepsilon_{ty} = 0.00507$$

Therefore, section is tension controlled

ACI 318-19 (Table 21.2.2)

$$\phi = 0.90$$

ACI 318-19 (Table 21.2.2)

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times (d - a/2)} = \frac{45.7 \times 12,000}{0.9 \times 60,000 \times (13.5 - 1.04/2)} = 0.78 \text{ in.}^2$$

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{3 \times \sqrt{f'_c}}{f_y} \times b \times d = \frac{3\sqrt{4,500}}{60,000} \times 12 \times 13.5 = 0.54 \text{ in.}^2$$

ACI 318-19 (9.6.1.2(a))

And not less than

$$A_{s,\min} = \frac{200}{f_y} \times b \times d = \frac{200}{60,000} \times 12 \times 13.5 = 0.54 \text{ in.}^2$$

ACI 318-19 (9.6.1.2(b))

$$\therefore A_{s,\min} = 0.54 \text{ in.}^2$$

Maximum spacing allowed:

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

$$s = 15 \left(\frac{40,000}{f_s} \right) - 2.5c_c \leq 12 \left(\frac{40,000}{f_s} \right)$$

ACI 318-19 (Table 24.3.2)

$$c_c = 2.00 \text{ in.}$$

$$\text{Use } f_s = \frac{2}{3} f_y = 40,000 \text{ psi}$$

ACI 318-19 (24.3.2.1)

$$s = 15 \times \left(\frac{40,000}{40,000} \right) - 2.5 \times 2.0 = 10 \text{ in. (Governs)}$$

$$s = 12 \times \left(\frac{40,000}{40,000} \right) = 12 \text{ in.}$$

Provide #7 bars at 8 in. on centers.

Note that the stem bending moment decreases rapidly with increasing distance from the bottom. For this reason, only part of the main reinforcement is needed at higher elevations and alternate bars can be discontinued where no longer needed. More information about cutting bars in the stem are provided in the reference. All the values in the following table are calculated based on the procedure outlined above for the stem.

Table 2 – Reinforcing Design Summary			
Critical Section	Stem Base	Toe	Heel
Design Moment, M_u (ft-kips)	45.7	24.3	29.9
Effective depth, d (in.)	13.5	14.5	14.5
$A_{s,req}$ (in. ²)	0.78	0.38	0.47
$A_{s,min}$ (in. ²)	0.54	0.39	0.39
Reinforcement	#7 @ 8 in.	#7 @ 12 in.	#7 @ 12 in.

5. Wall Shear Stress Check

The nominal shear strength at the base of the wall is:

$$\phi V_c = \phi \left[8\lambda_s \lambda (\rho_w)^{1/3} \sqrt{f'_c} + \frac{N_u}{6A_g} \right] b_w d \quad \text{ACI 318-19 (Table 22.5.5.1(c))}$$

$$\phi V_c = 0.75 \times \left[8 \times 0.92 \times 1 \times (0.0056)^{1/3} \sqrt{4,500} + \frac{3,078.00}{6 \times (16.00 \times 12.00)} \right] \times 12.00 \times 13.50$$

$$\phi V_c = 0.75 \times 14.64 = 10.98 \text{ kips}$$

Where:

The depth of the arm exceeds 10 in. and requires an adjustment for size effects.

$$\lambda_s = \sqrt{\frac{2}{1 + \frac{d}{10}}} = \sqrt{\frac{2}{1 + \frac{13.50}{10}}} = 0.92 \leq 1 \quad \text{ACI 318-19 (22.5.5.1.3)}$$

$$\lambda = 1 \text{ for normal weight concrete} \quad \text{ACI 318-19 (19.2.4.1)}$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{0.60 \times \frac{12.00}{8.00}}{12.00 \times 13.50} = 0.0056$$

$$N_u = 1.2 \times \left[\left(\frac{8.00 + 16.00}{2 \times 12.00} \right) \times 13.50 \times 1.00 \times 150 + 540 \right] = 3,078.00 \text{ lb}$$

$$\phi = 0.75 \text{ for shear} \quad \text{ACI 318-19 (Table 21.2.1)}$$

$$\phi V_c = 10.98 \text{ kips} \geq V_u = 8.27 \text{ kips}$$

6. Cantilever Retaining Wall Analysis and Design – spWall Software

[spWall](#) is a program for the analysis and design of reinforced concrete shear walls, tilt-up walls, precast walls, retaining walls, tank walls, and Insulate Concrete Form (ICF) walls. It uses a graphical interface that enables the user to easily generate complex wall models. Graphical user interface is provided for:

- Wall geometry (including any number of openings and stiffeners)
- Material properties including cracking coefficients
- Wall loads (point, line, and area loads)
- Support conditions (including translational and rotational spring supports)

[spWall](#) uses the Finite Element Method for the structural modeling, analysis, and design of slender and non-slender reinforced concrete walls subject to static loading conditions. The wall is idealized as a mesh of rectangular plate elements and straight line stiffener elements. Walls of any geometry are idealized to conform to geometry with rectangular boundaries. Plate and stiffener properties can vary from one element to another but are assumed by the program to be uniform within each element.

Six degrees of freedom exist at each node: three translations and three rotations relating to the three Cartesian axes. An external load can exist in the direction of each of the degrees of freedom. Sufficient number of nodal degrees of freedom should be restrained in order to achieve stability of the model. The program assembles the global stiffness matrix and load vectors for the finite element model. Then, it solves the equilibrium equations to obtain deflections and rotations at each node. Finally, the program calculates the internal forces and internal moments in each element. At the user's option, the program can perform second order analysis. In this case, the program considers the effect of in-plane forces on the out-of-plane deflection with any number of openings and stiffeners.

In [spWall](#), the required flexural reinforcement is computed based on the selected design standard (ACI 318-19 is used in this example), and the user can specify one or two layers of wall reinforcement. In stiffeners and boundary elements, [spWall](#) calculates the required shear and torsion steel reinforcement. Wall concrete strength (in-plane and out-of-plane) is calculated for the applied loads and compared with the code permissible shear capacity.

For illustration purposes, the following figures provide a sample of the input modules and results obtained from an [spWall](#) model created for the cantilever retaining wall in this design example.

6.1. Cantilever Retaining Wall Model Input

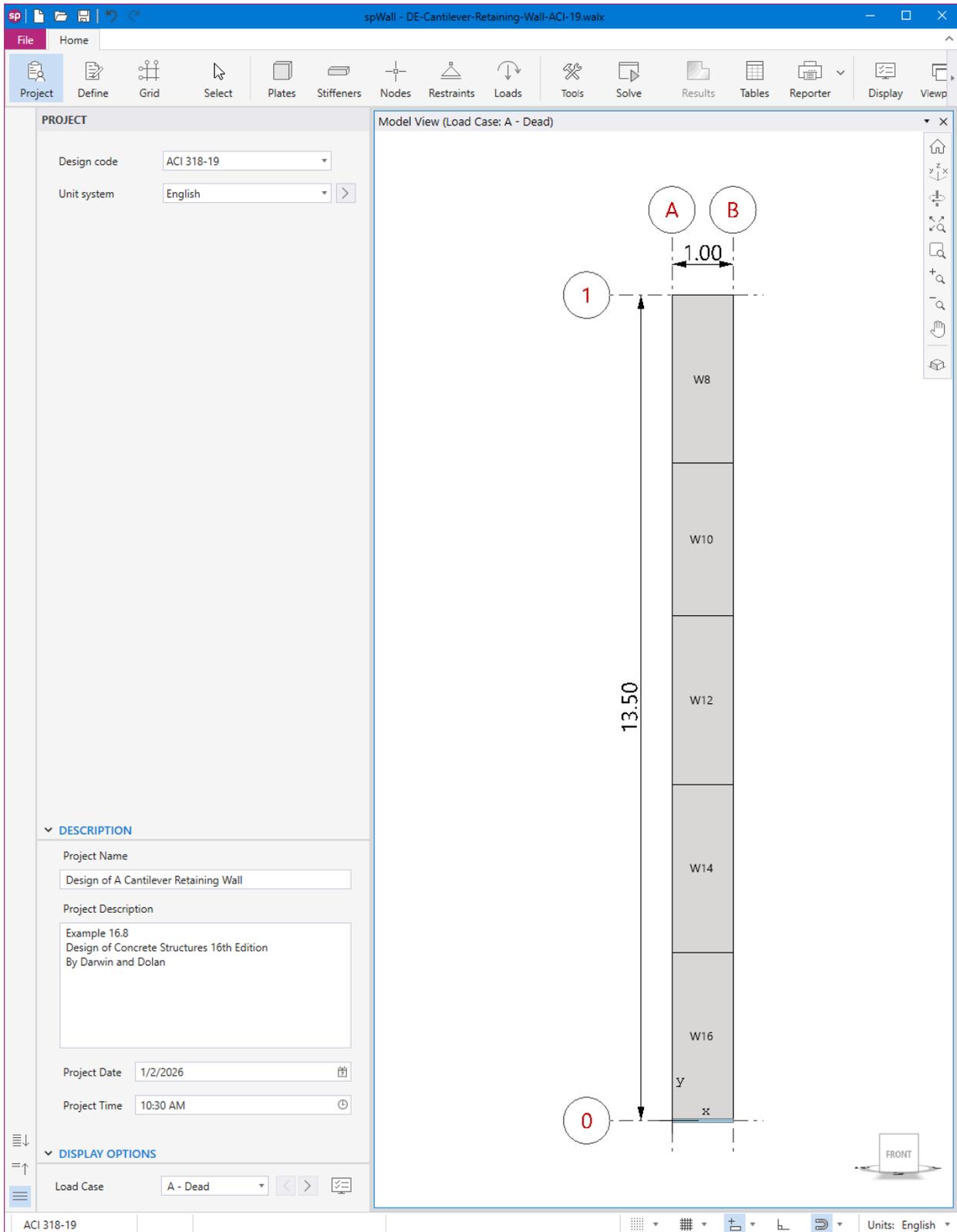


Figure 4 – spWall Interface

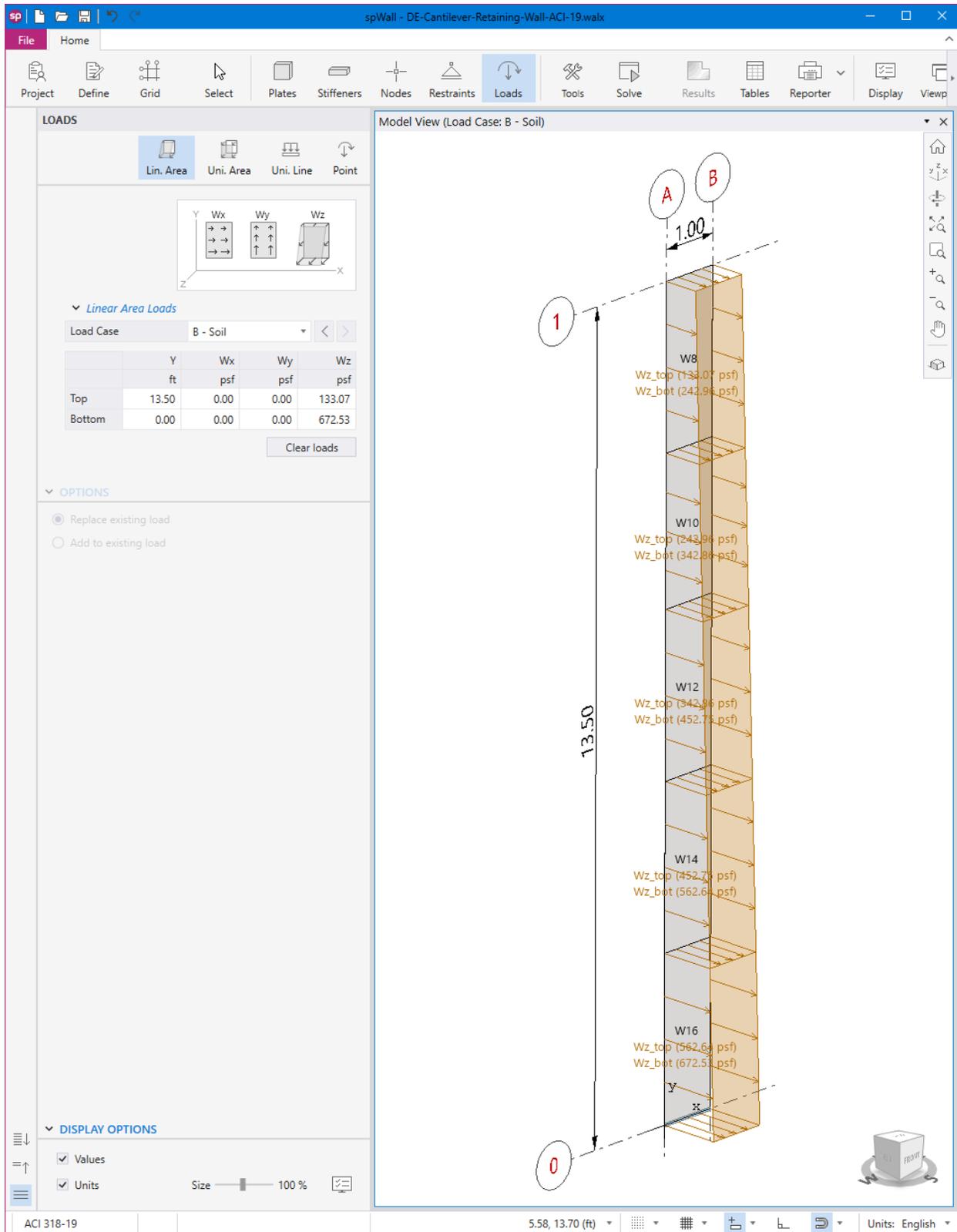


Figure 5 – Assigning Soil Loads for Cantilever Retaining Wall (spWall)

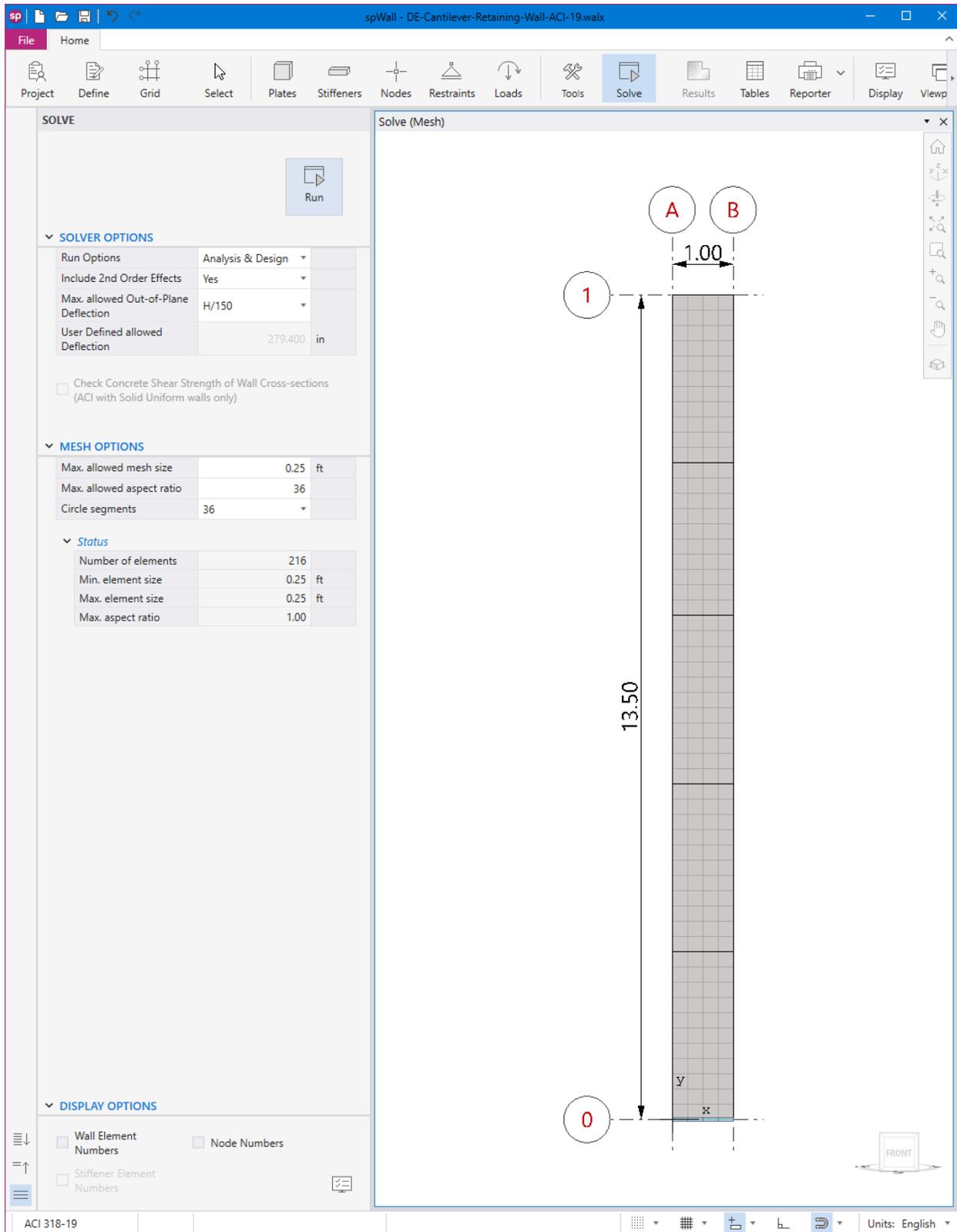


Figure 6 – Solve and Mesh Options (spWall)

6.2. Cantilever Retaining Wall Result Contours

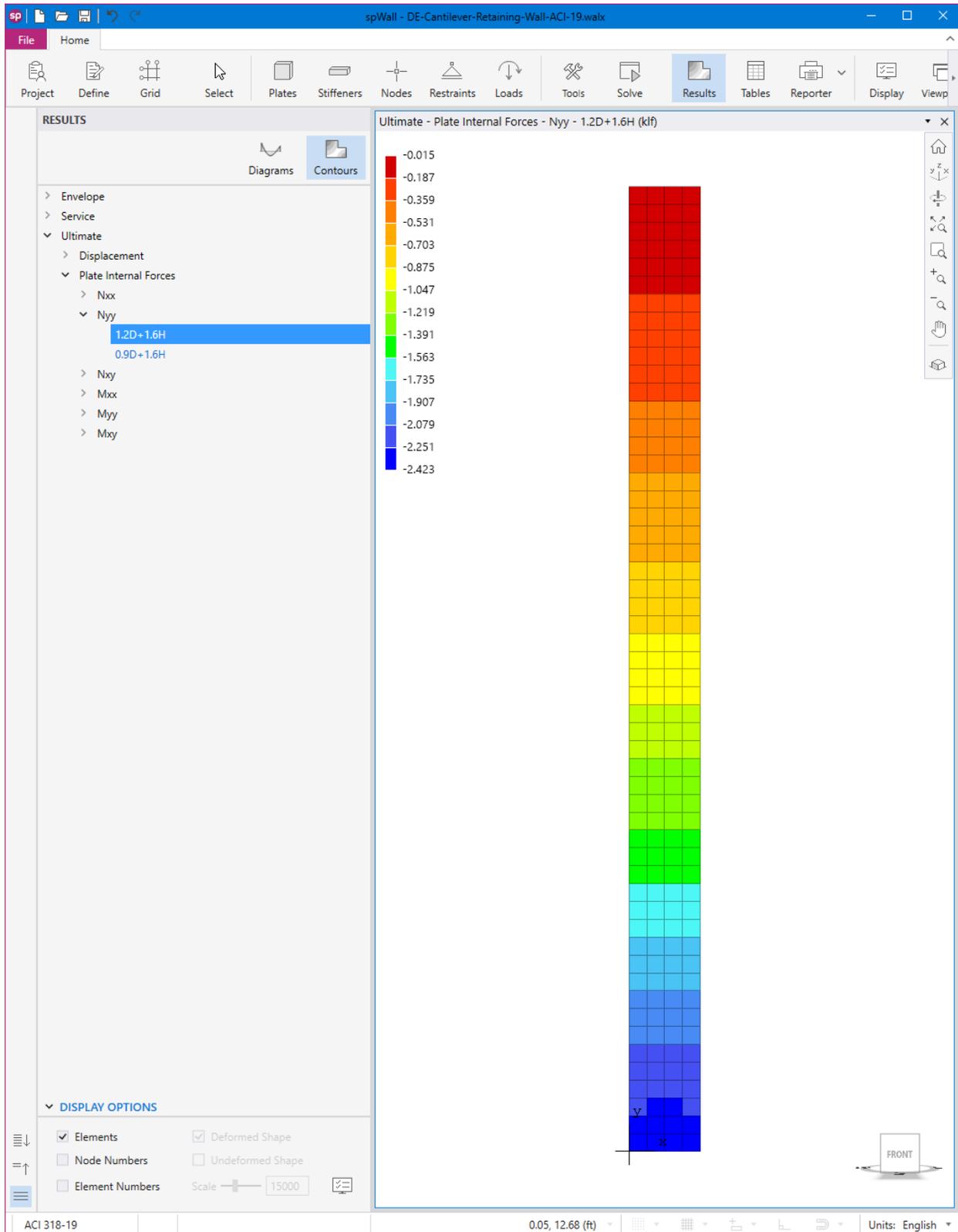


Figure 7 – Factored Axial Force Contour (spWall)

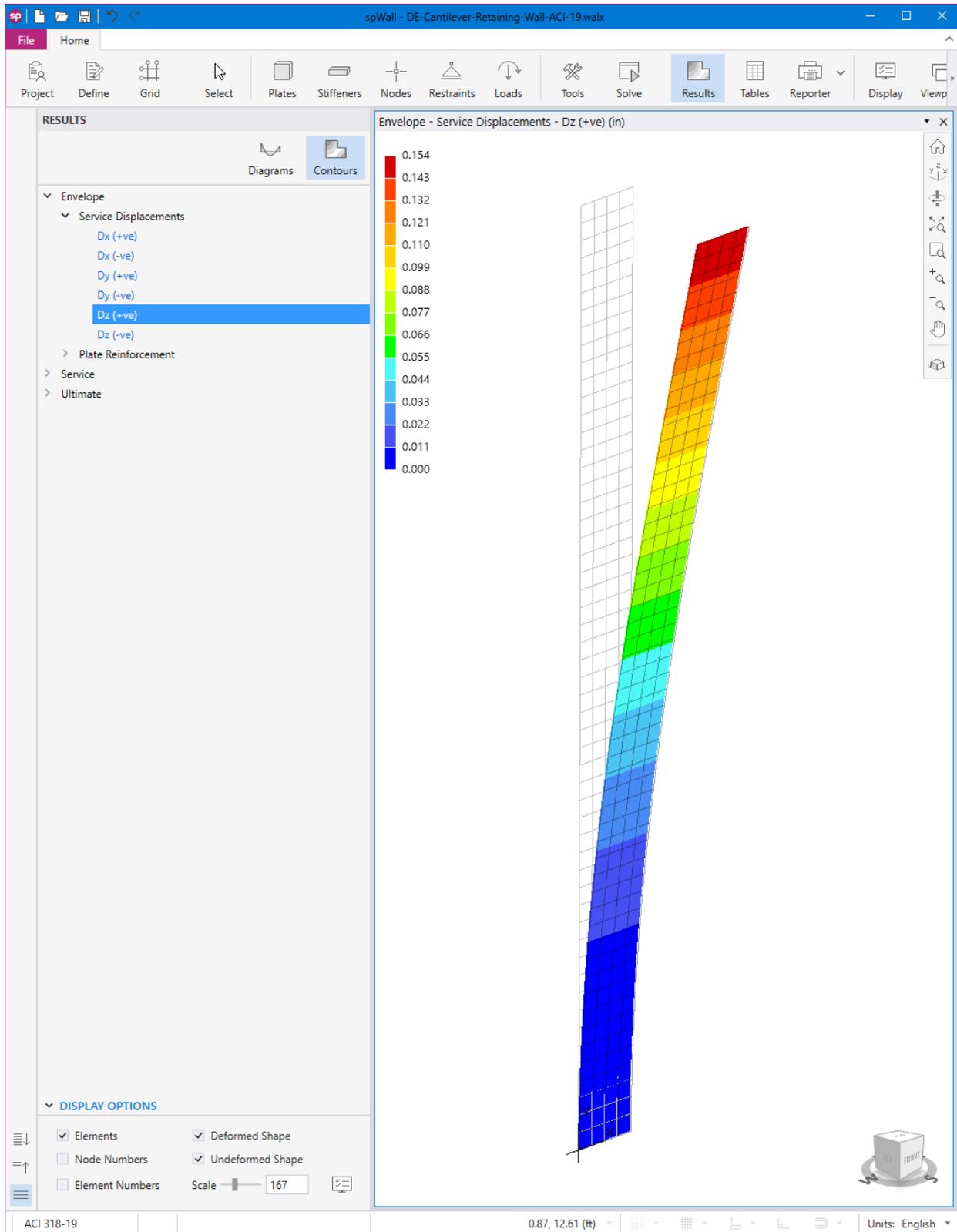


Figure 8 – Lateral Displacement Contour (Out-of-Plane) (spWall)

6.3. Cantilever Retaining Wall Cross-Sectional Forces

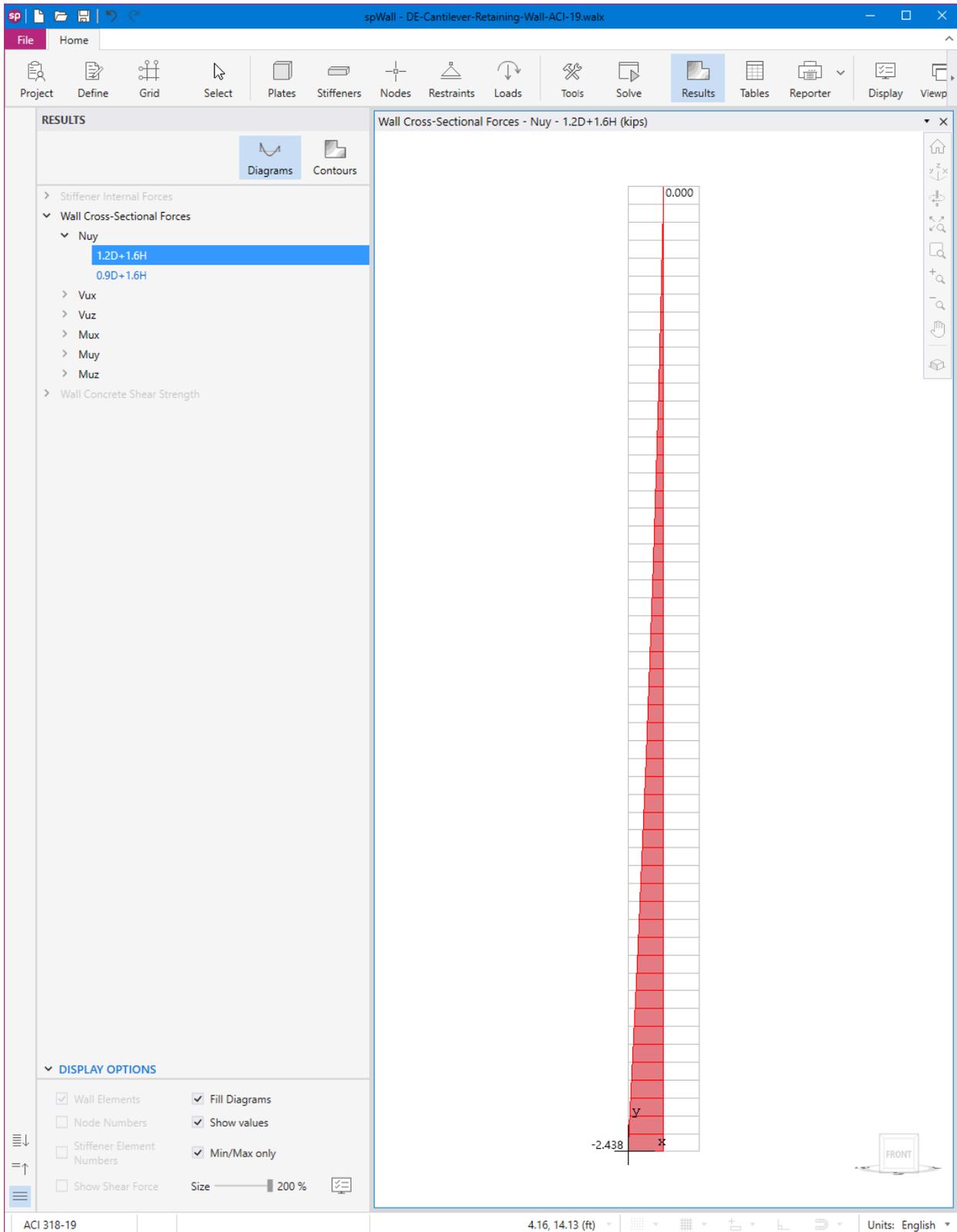


Figure 9 – Axial Load Diagram (spWall)

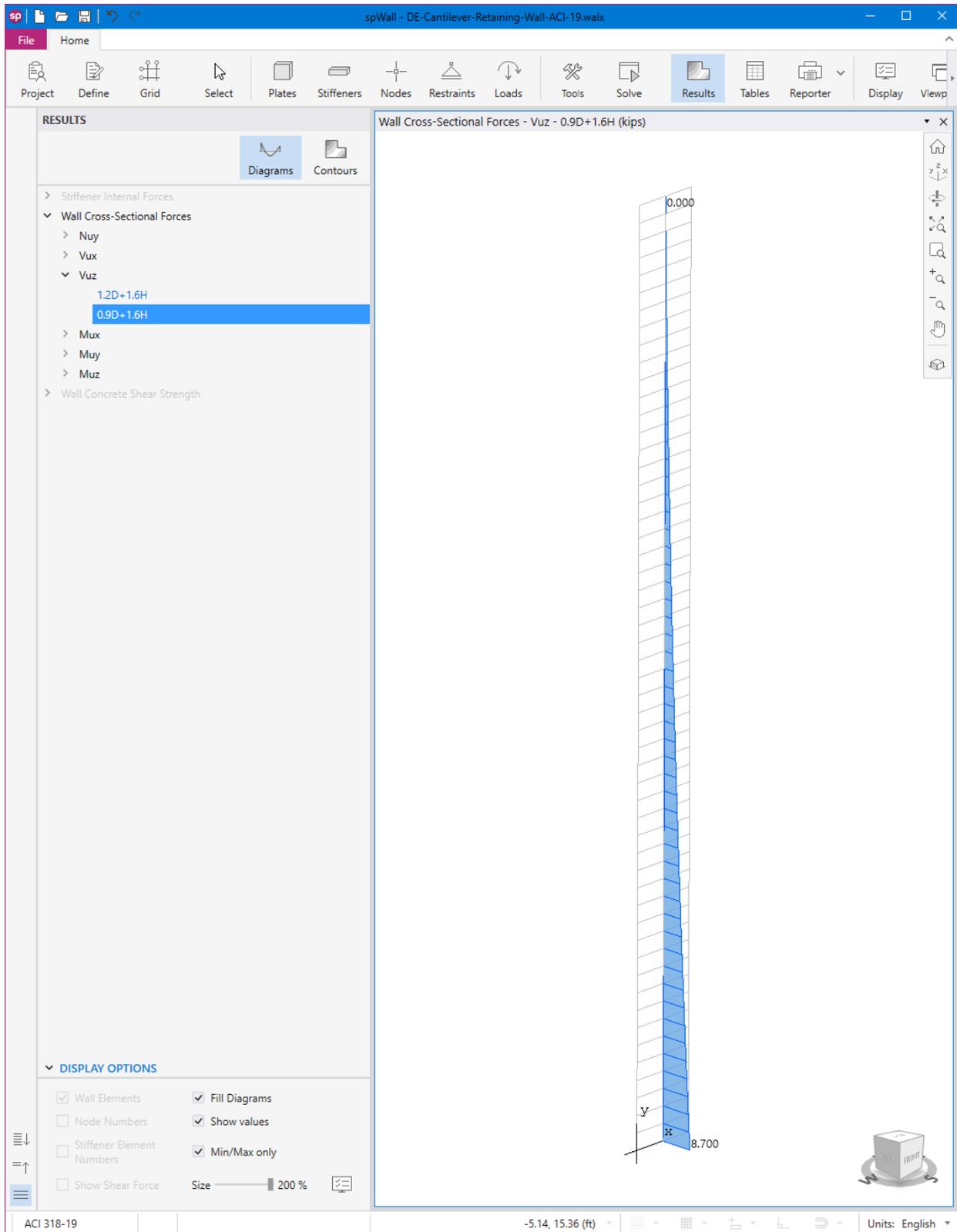


Figure 10 – Out-of-Plane Shear Diagram (spWall)

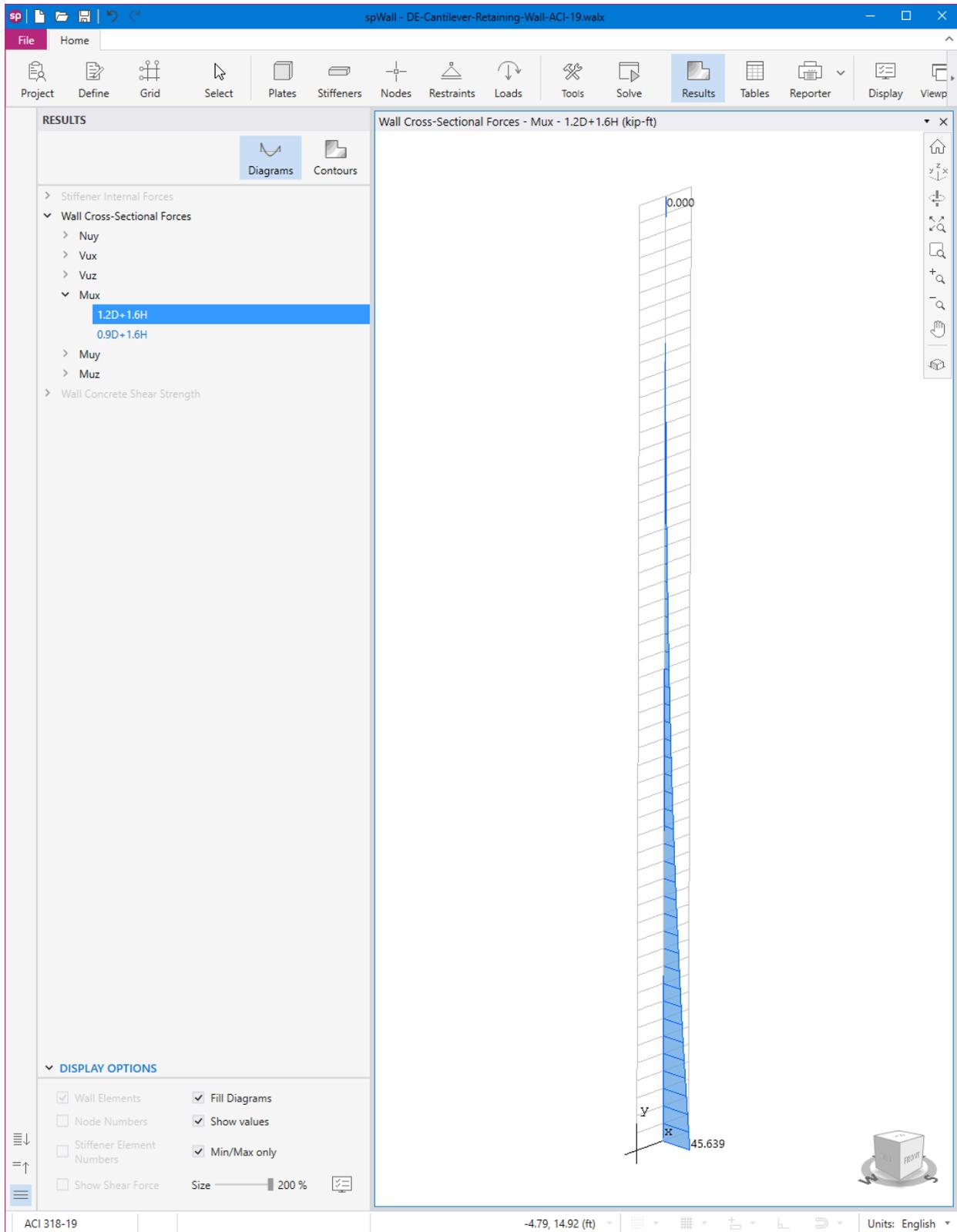


Figure 11 – Bending Moment Diagram (spWall)

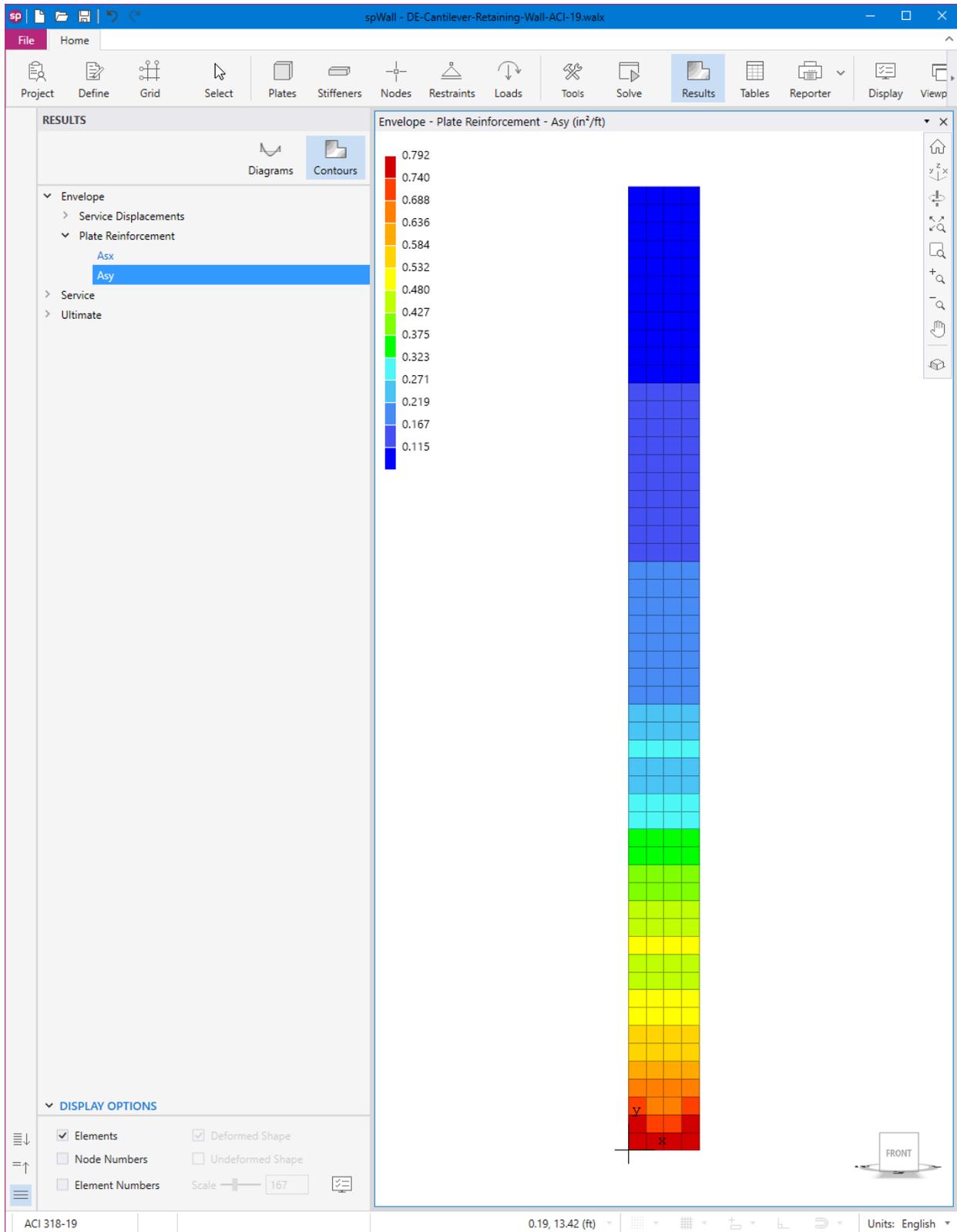


Figure 12 – Required Vertical Reinforcement (spWall)

(Note: Minimum reinforcement value shown is based on the top wall stem thickness of 8” while the hand calculations show the minimum required at the wall stem base with 16” thickness)

6.4. Cantilever Retaining Wall Maximum Displacement

1. Results

1.1. Service

1.1.1. Nodal Displacements

1.1.1.1. 1.0D+1.0H

Coordinate System: Global

Node	Dx in	Dy in	Dz in
271	0.000	0.000	0.154
272	0.000	0.000	0.154
273	0.000	0.000	0.154
274	0.000	0.000	0.154
275	0.000	0.000	0.154

Figure 13 – Displacement at Critical Section (Service Combinations) ([spWall](#))

1.2. Ultimate

1.2.1. Nodal Displacements

1.2.1.1. 1.2D+1.6H

Coordinate System: Global

Node	Dx in	Dy in	Dz in
271	0.000	0.000	0.246
272	0.000	0.000	0.246
273	0.000	0.000	0.246
274	0.000	0.000	0.246
275	0.000	0.000	0.246

Figure 14 – Displacement at Critical Section (Ultimate Combinations) ([spWall](#))

6.5. Cantilever Retaining Wall Cross-Sectional Forces at Stem Base

1.2.2. Wall Cross-Sectional Forces

1.2.2.1. 1.2D+1.6H

Coordinate System: Global

(+) Horizontal cross-section above Y-coordinate

(-) Horizontal cross-section below Y-coordinate

No.	Wall Crosssection		In-Plane Forces			Out-Of-Plane Forces		
	Y coordinate ft	X-Centroid ft	Vux kips	Nuy kips	Muz kip-ft	Vuz kips	Mux kip-ft	Muy kip-ft
1+	0.00	0.50	0.00	-2.44	0.00	8.70	45.64	0.00

Figure 15 – Wall Cross-Sectional Forces ([spWall](#))

6.6. Cantilever Retaining Wall Reinforcement

1.3. Envelope

1.3.1. Plate Flexure Reinforcement

$$A_{s,avg} = 0.785 \text{ in.}^2$$

Coordinate System: Global

Element	Curtains	Direction	Mu (x/y) kip-ft/ft	Nu (x/y) Ld Comb. klf	ϵ_t -	ϕ	As (x/y) in ² /ft	Rho Tie %
1	1	Horizontal	8.00	-0.40 0.9D+1.6H	0.0623	0.90	0.384	0.20
		Vertical	46.16	-1.80 0.9D+1.6H	0.0285	0.90	0.777	0.40
2	1	Horizontal	9.34	-0.24 0.9D+1.6H	0.0628	0.90	0.384	0.20
		Vertical	47.07	-1.78 0.9D+1.6H	0.0280	0.90	0.792	0.41
3	1	Horizontal	9.34	-0.24 0.9D+1.6H	0.0628	0.90	0.384	0.20
		Vertical	47.07	-1.78 0.9D+1.6H	0.0280	0.90	0.792	0.41
4	1	Horizontal	8.00	-0.40 0.9D+1.6H	0.0623	0.90	0.384	0.20
		Vertical	46.16	-1.80 0.9D+1.6H	0.0285	0.90	0.777	0.40

Elements along the wall base

Figure 16 – Required Vertical Reinforcement (spWall)

7. Cantilever Retaining Wall Foundation Analysis and Design – [spMats](#) Software

[spMats](#) uses the Finite Element Method for the structural modeling, analysis and design of reinforced concrete slab systems or mat foundations subject to static loading conditions.

The slab, mat, or footing is idealized as a mesh of rectangular elements interconnected at the corner nodes. The same mesh applies to the underlying soil with the soil stiffness concentrated at the nodes. Slabs of irregular geometry can be idealized to conform to geometry with rectangular boundaries. Even though slab and soil properties can vary between elements, they are assumed uniform within each element. Piles and/or supporting soil are modeled as springs connected to the nodes of the finite element model.

For illustration purposes, the following figures provide a sample of the input modules and results obtained from an [spMats](#) model created for the cantilever retaining wall foundation in this design example (For simplicity, the foundation shear check is not included in this example).

7.1. Cantilever Retaining Wall Foundation Model Input

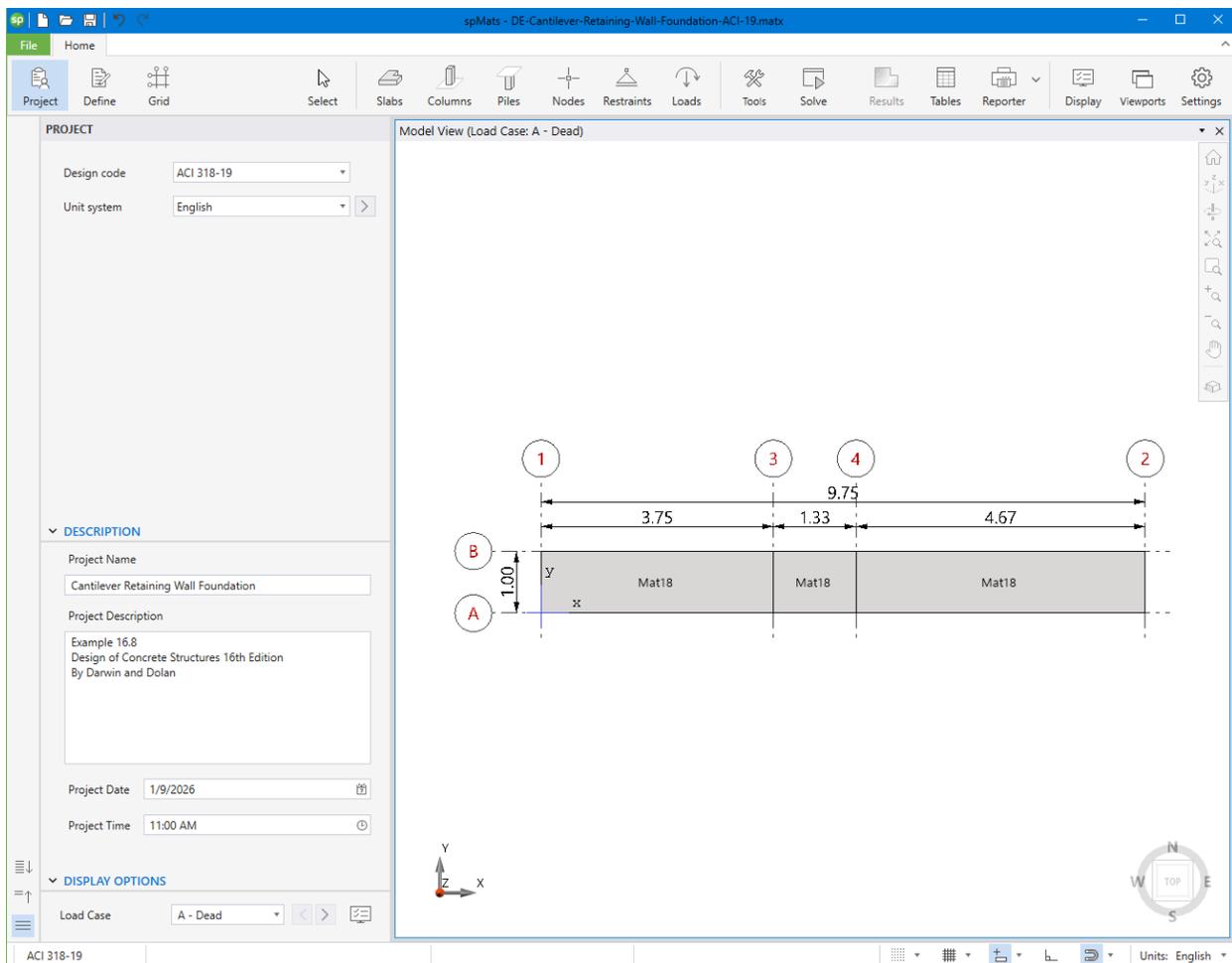


Figure 17 – [spMats](#) Interface

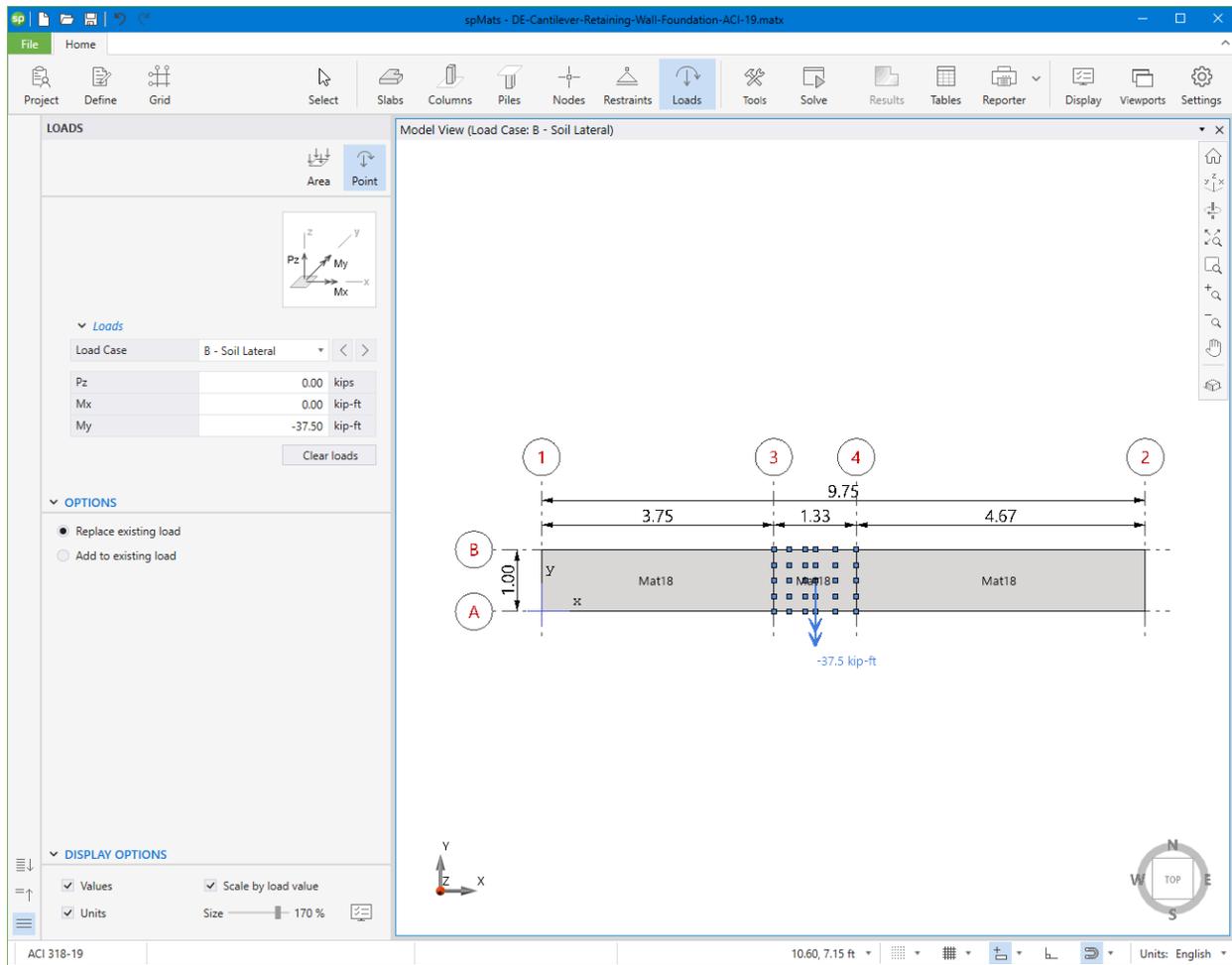


Figure 18 – Assigning Soil Lateral Moment for Cantilever Retaining Wall Foundation (spMats)

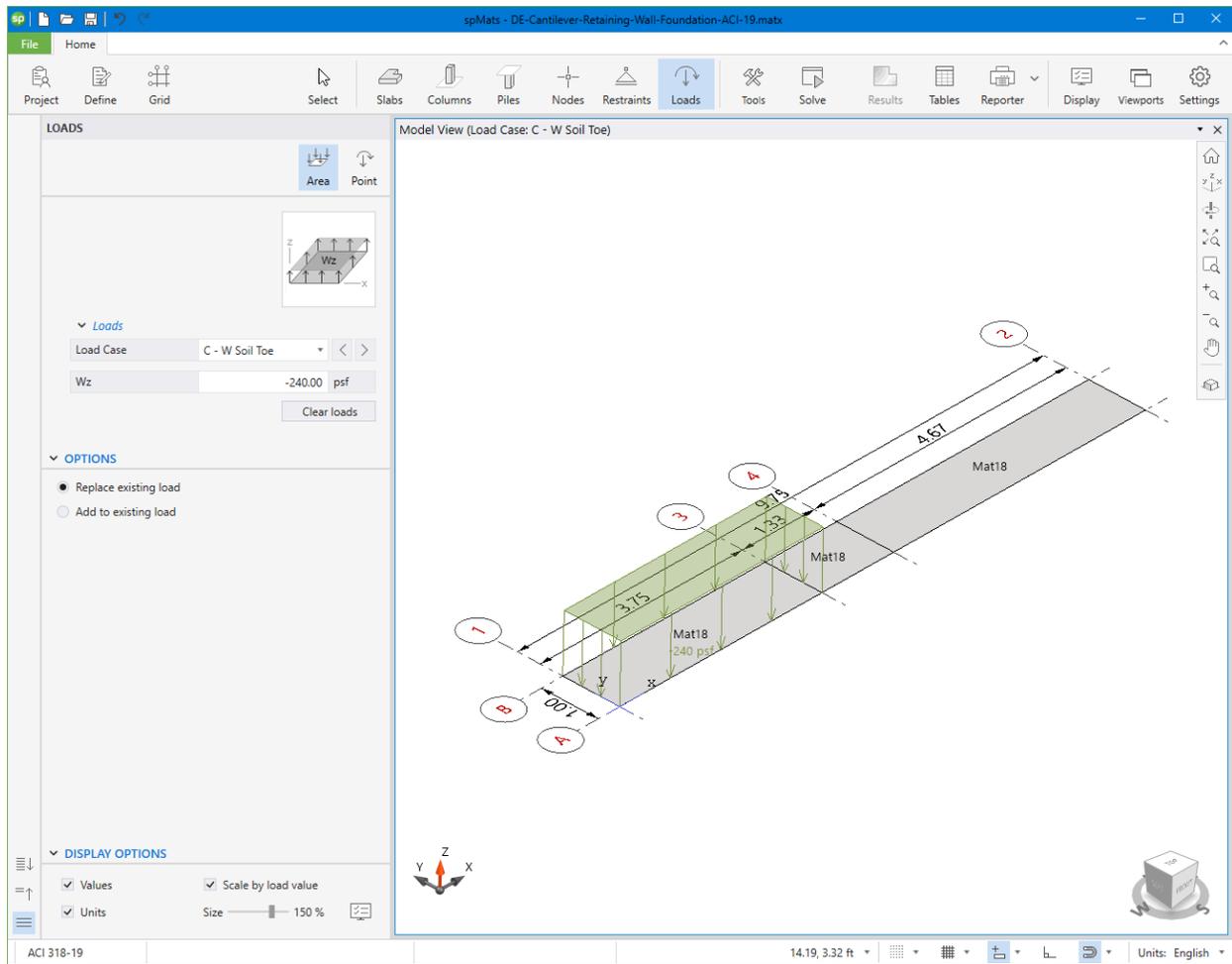


Figure 19 – Assigning Soil Toe Load for Cantilever Retaining Wall Foundation (spMats)

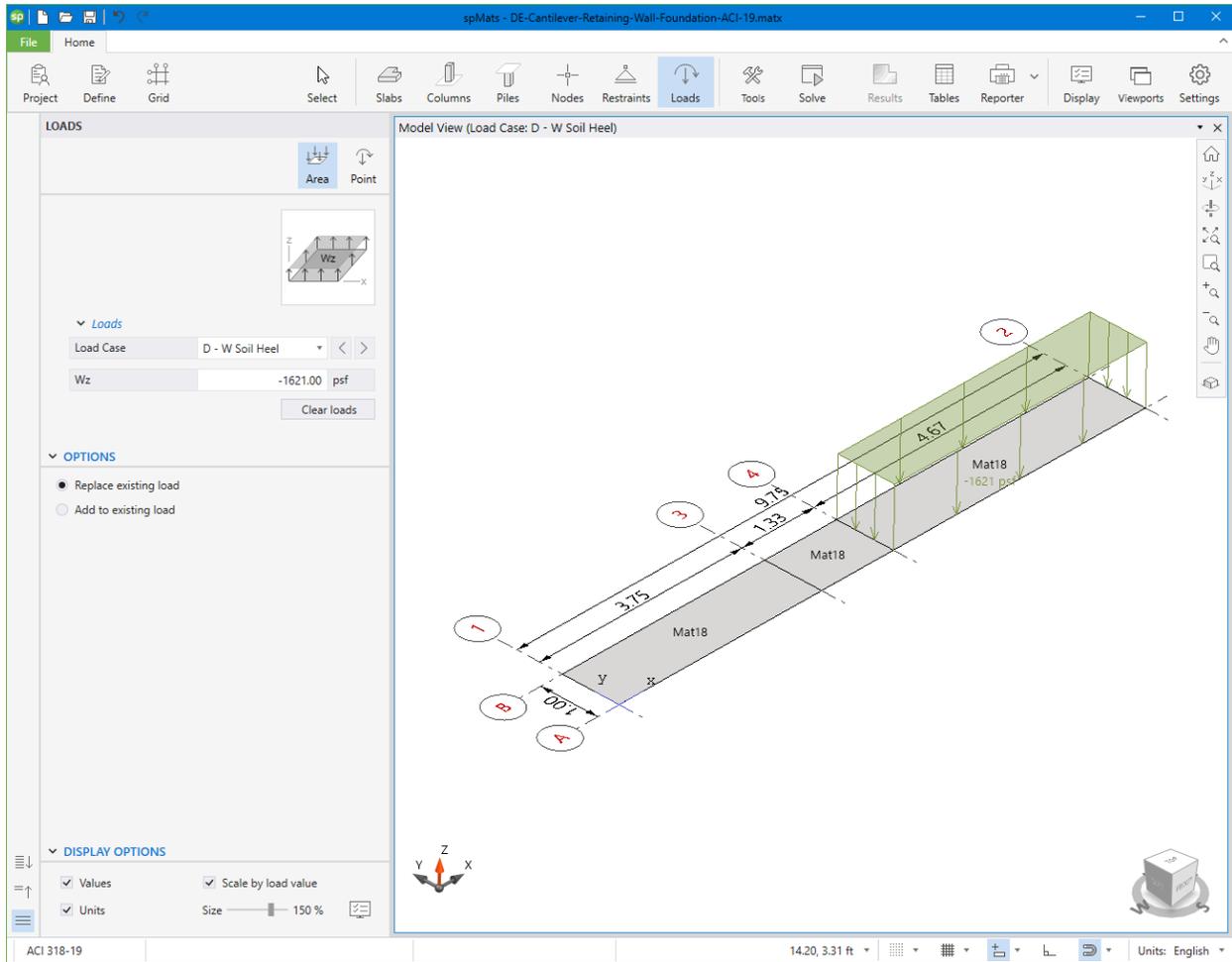


Figure 20 – Assigning Soil Heel Load for Cantilever Retaining Wall Foundation (spMats)

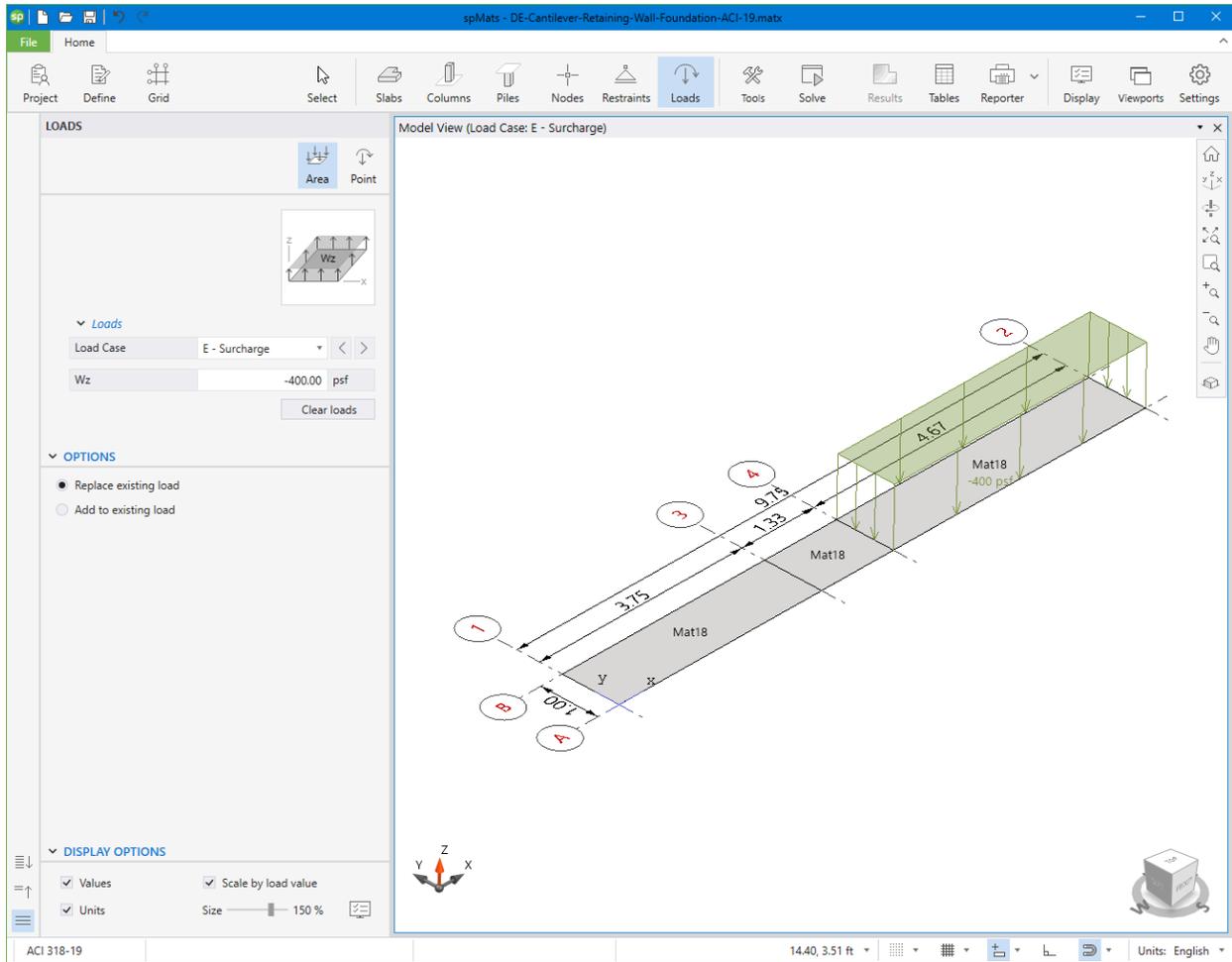


Figure 21 – Assigning Surcharge Load for Cantilever Retaining Wall Foundation (spMats)

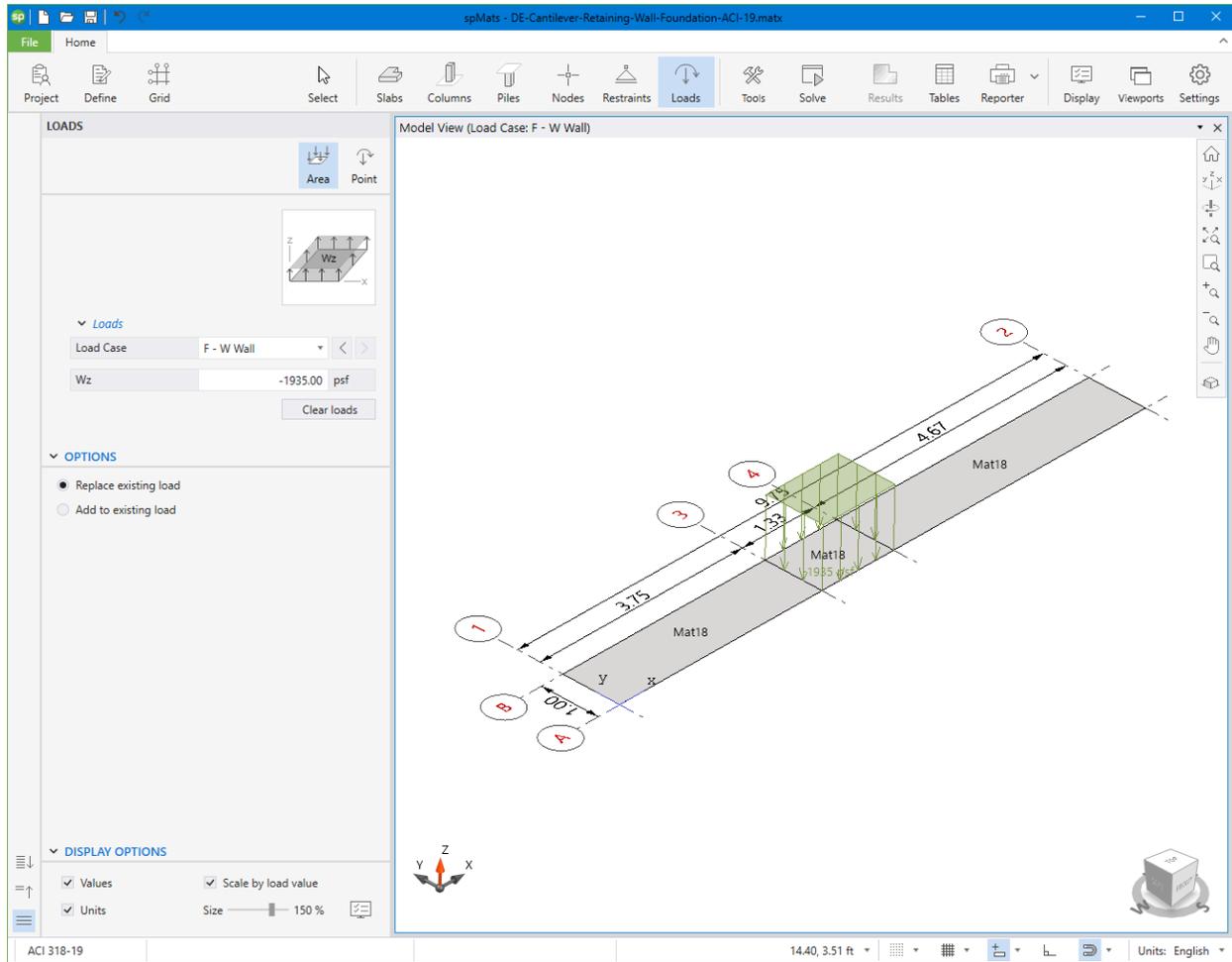


Figure 22 – Assigning Wall Load for Cantilever Retaining Wall Foundation (spMats)

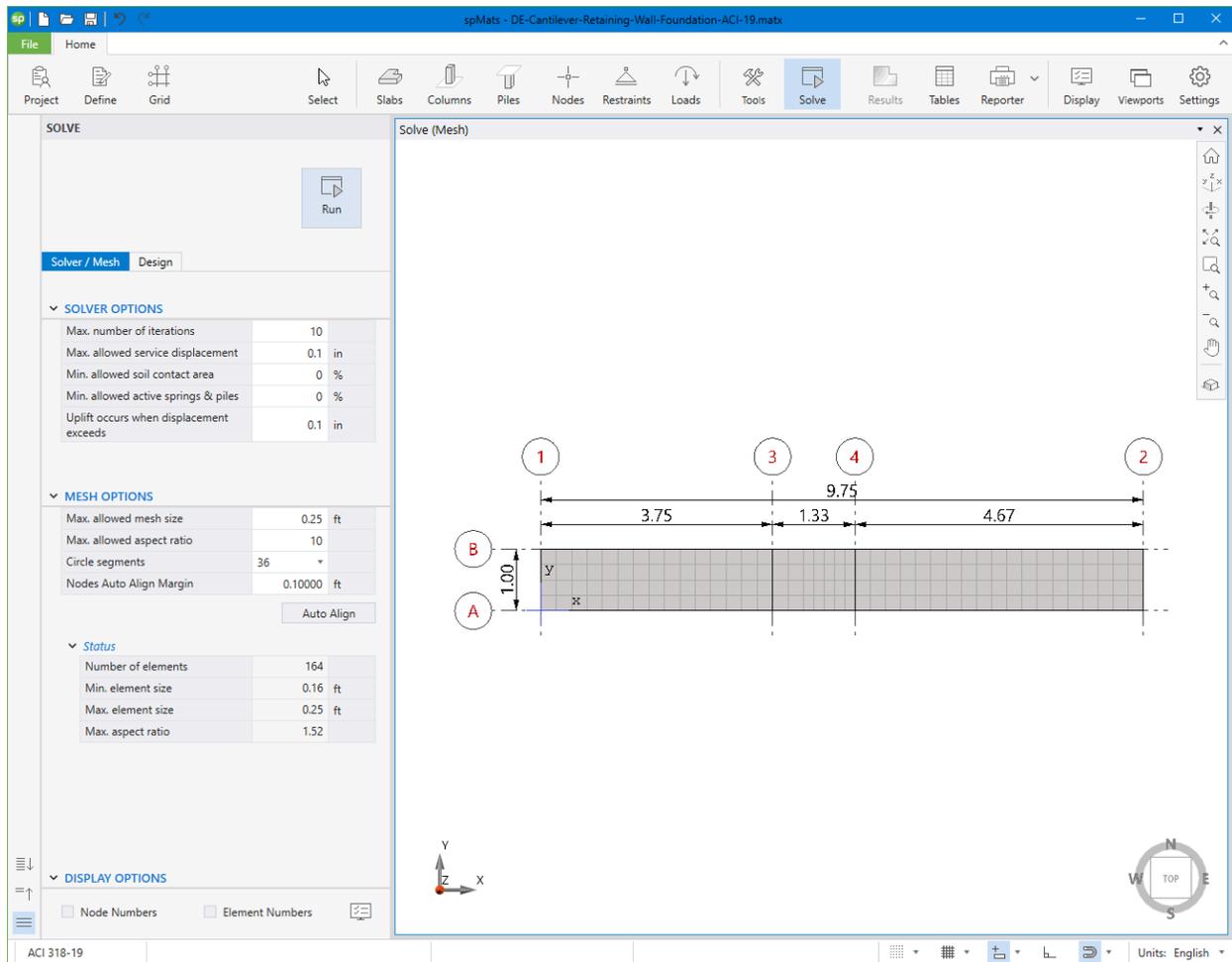


Figure 23 – Solve and Mesh Options (spMats)

7.2. Cantilever Retaining Wall Foundation Result Contours

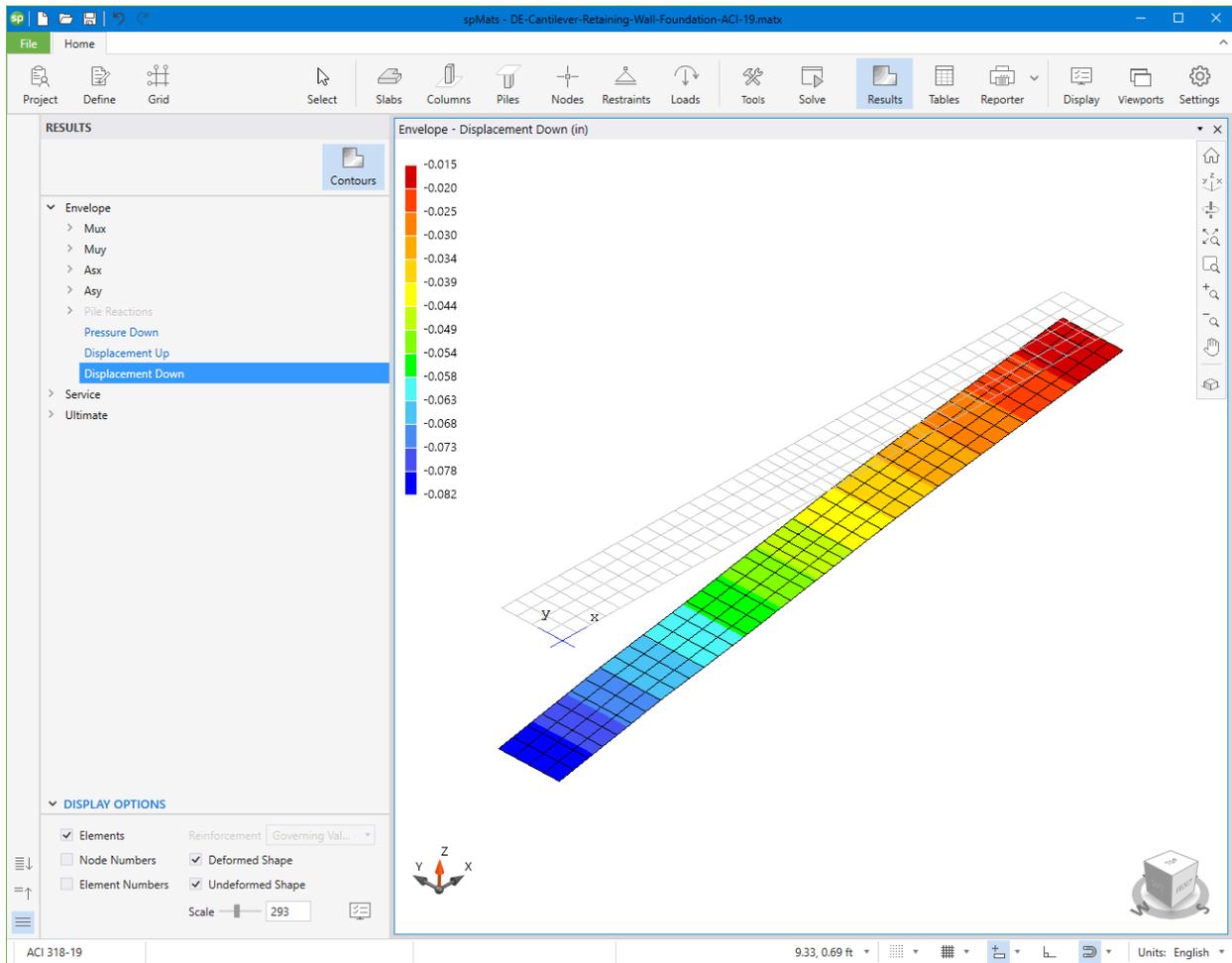


Figure 24 – Vertical (Down) Displacement Contour (spMats)

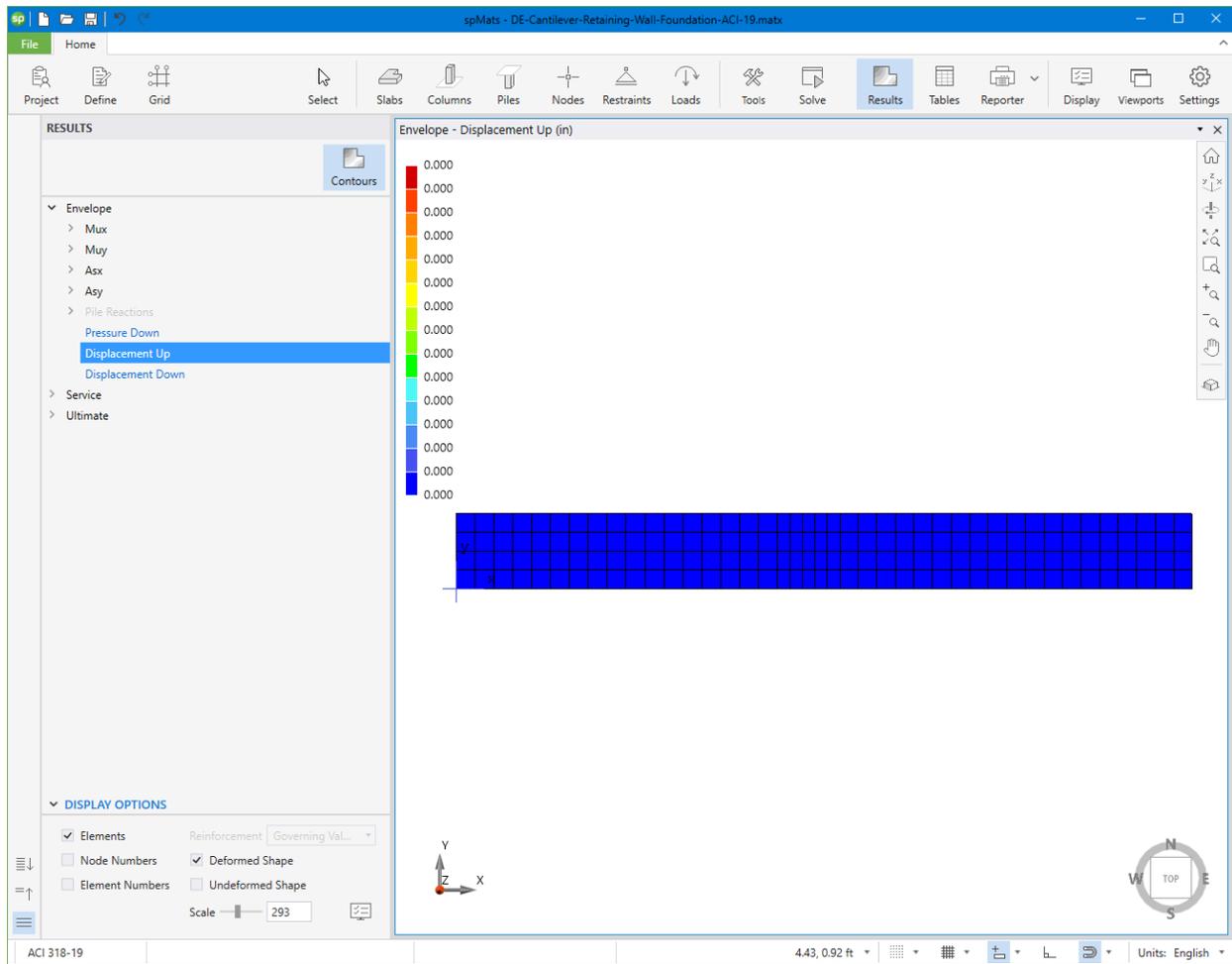


Figure 25 – Vertical (Up) Displacement Contour (spMats)
(Note: figure indicates no uplift in the wall base)

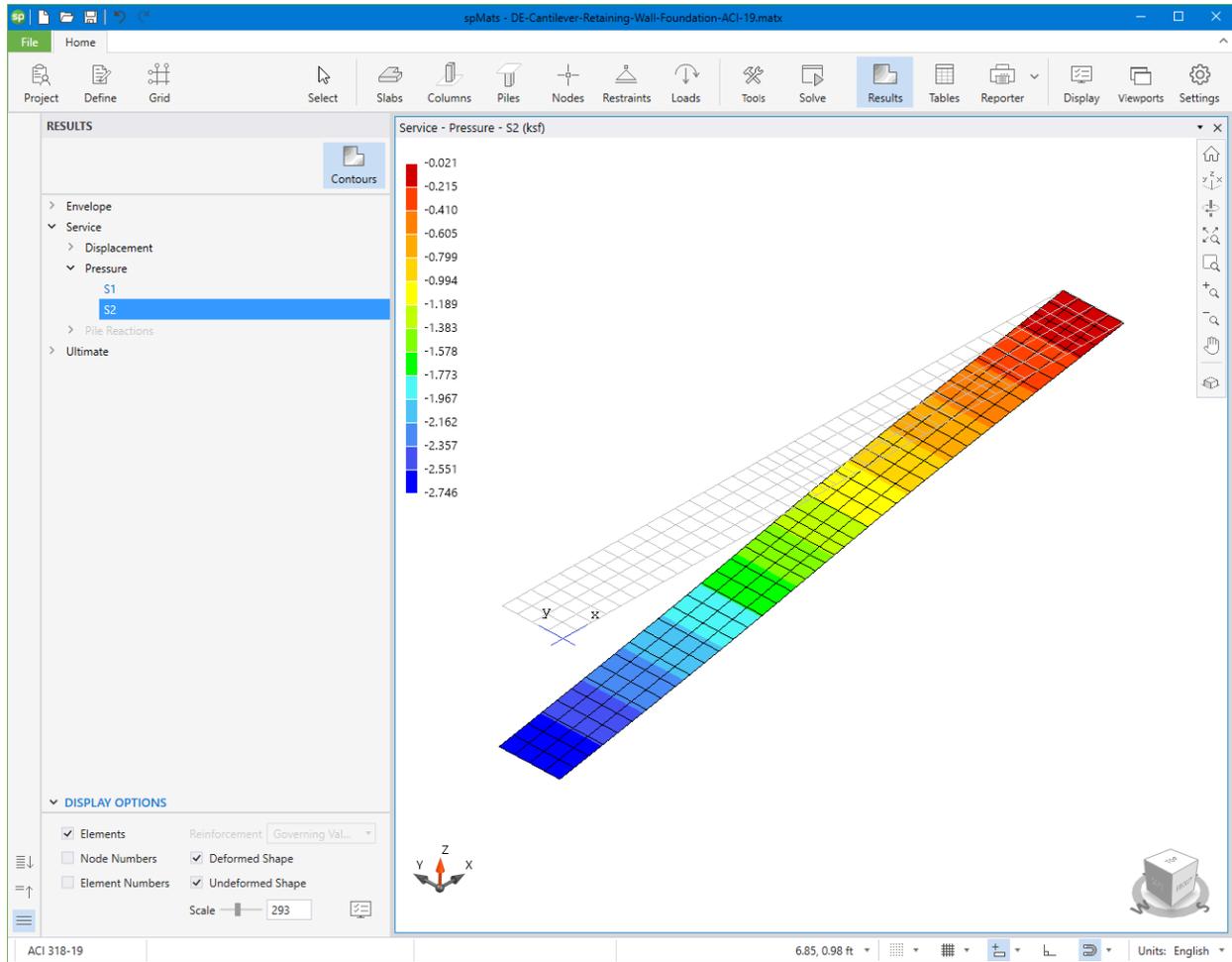


Figure 26 – Soil Bearing Pressure Contour for Case 1 (spMats)

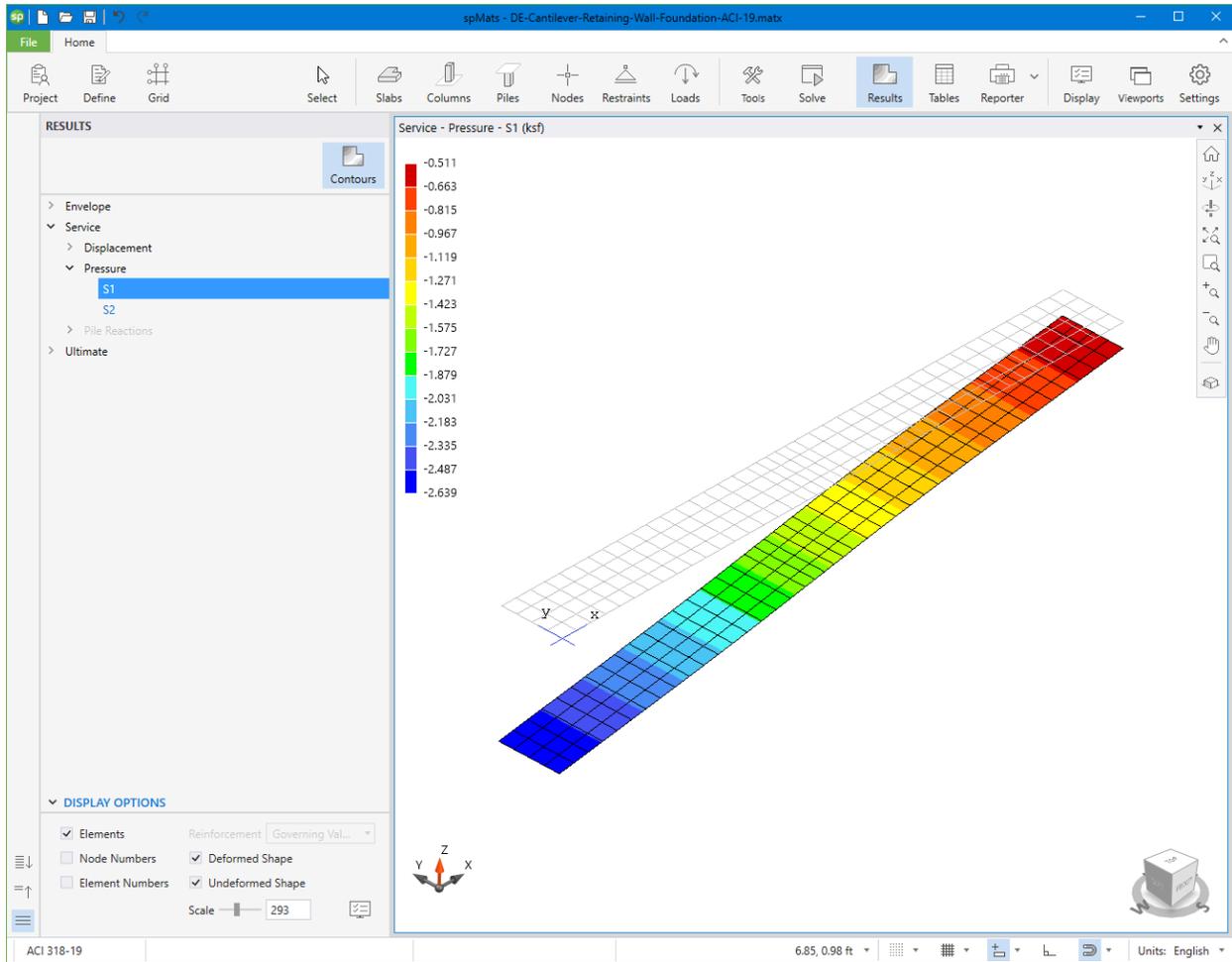


Figure 27 – Soil Bearing Pressure Contour for Case 2 (spMats)

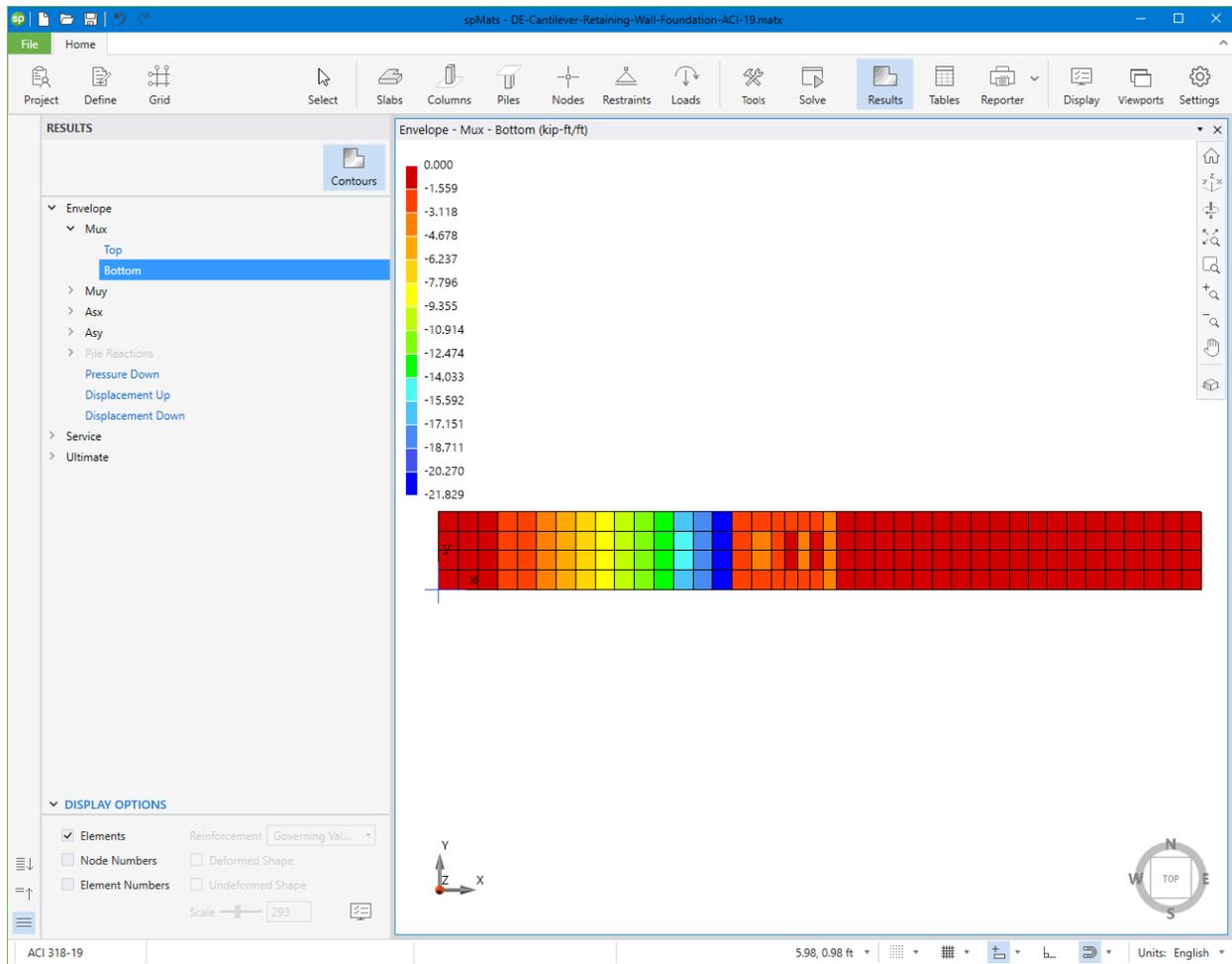


Figure 28 – Moment Contour along X-Axis (Max for Toe) (spMats)

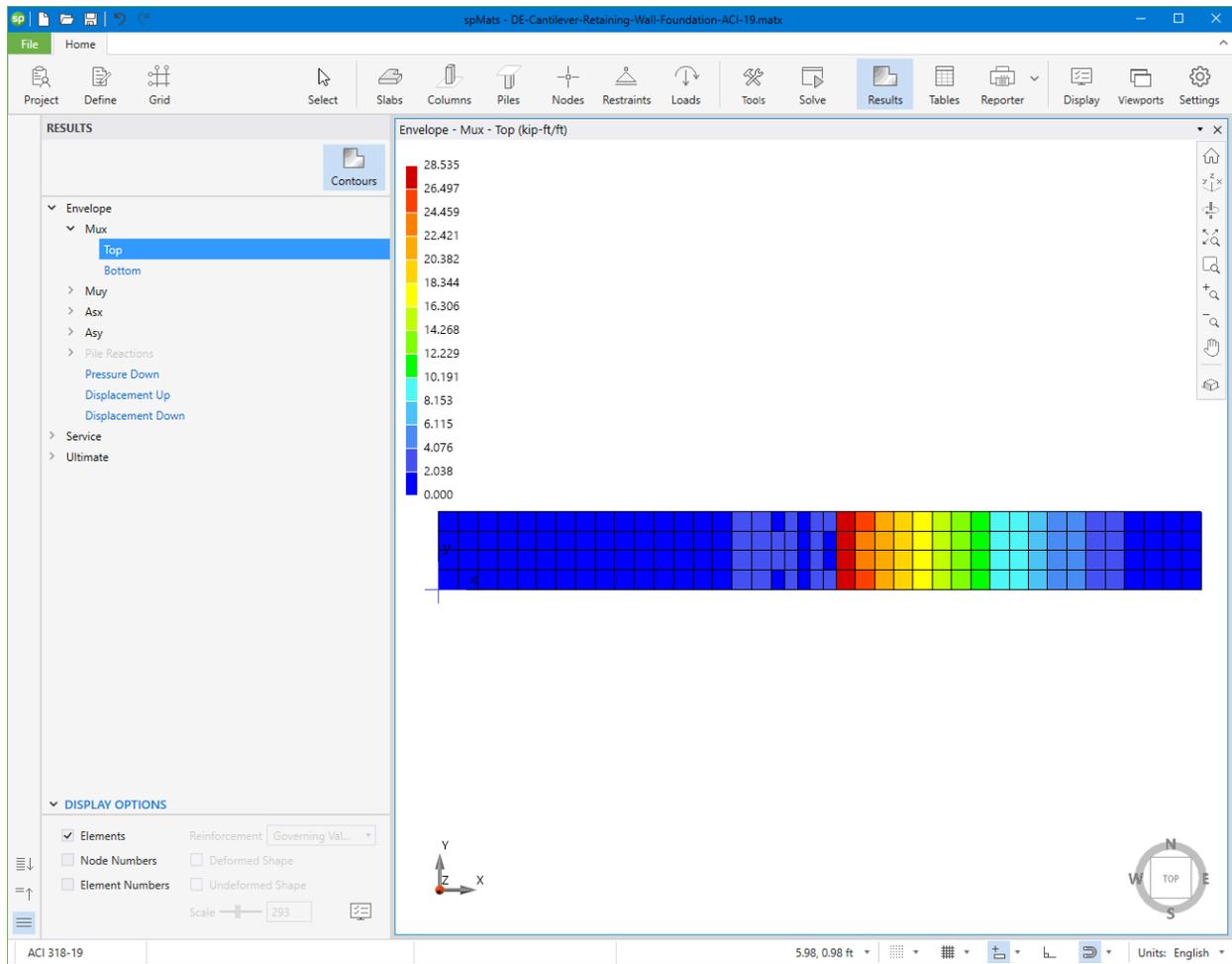


Figure 29 – Moment Contour along X-Axis (Max for Heel) (spMats)

7.3. Cantilever Retaining Wall Foundation Required Reinforcement

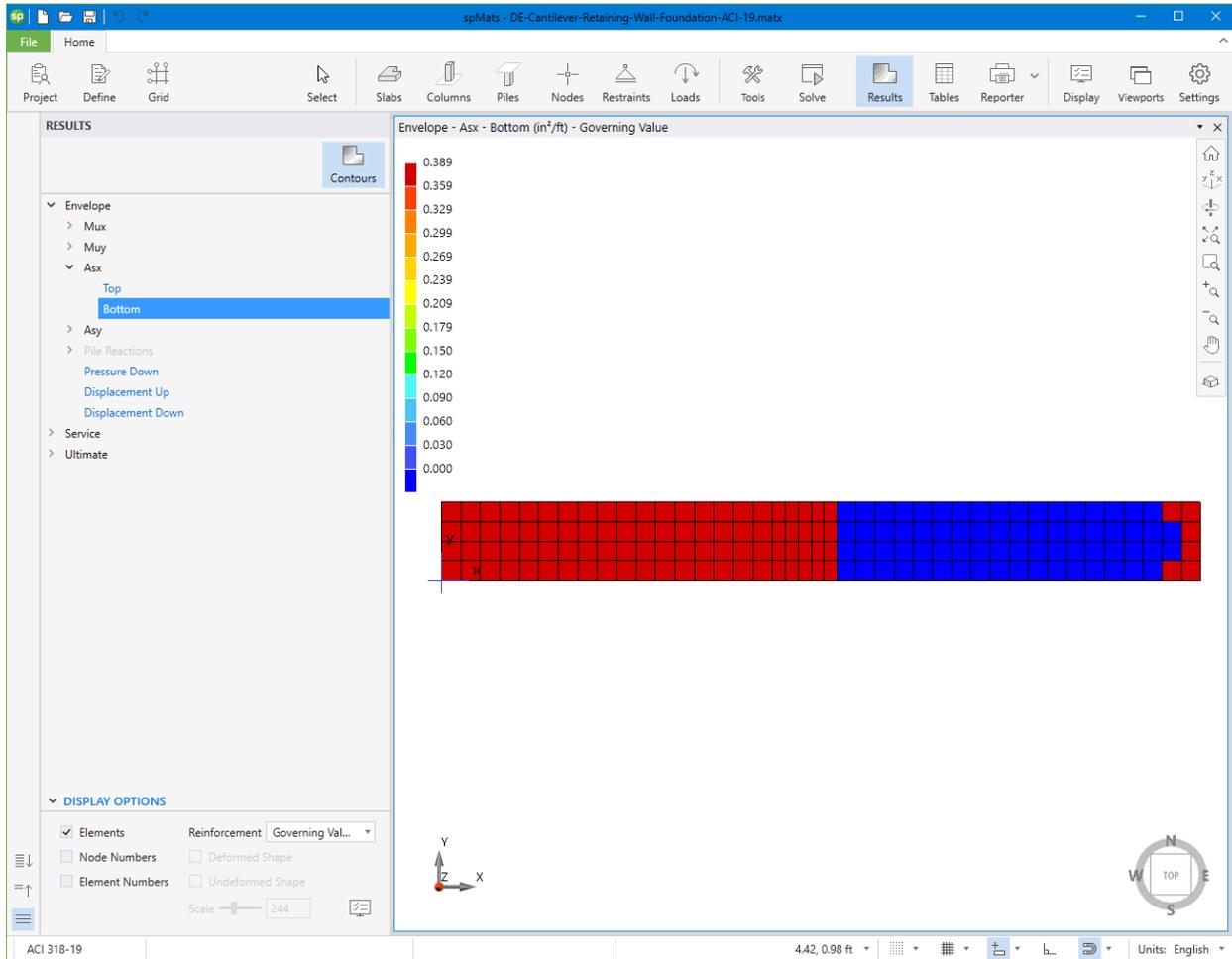


Figure 30 – Required Reinforcement Contour along X Direction (Bottom – Toe Design) (spMats)
(Note: minimum reinforcement governs)

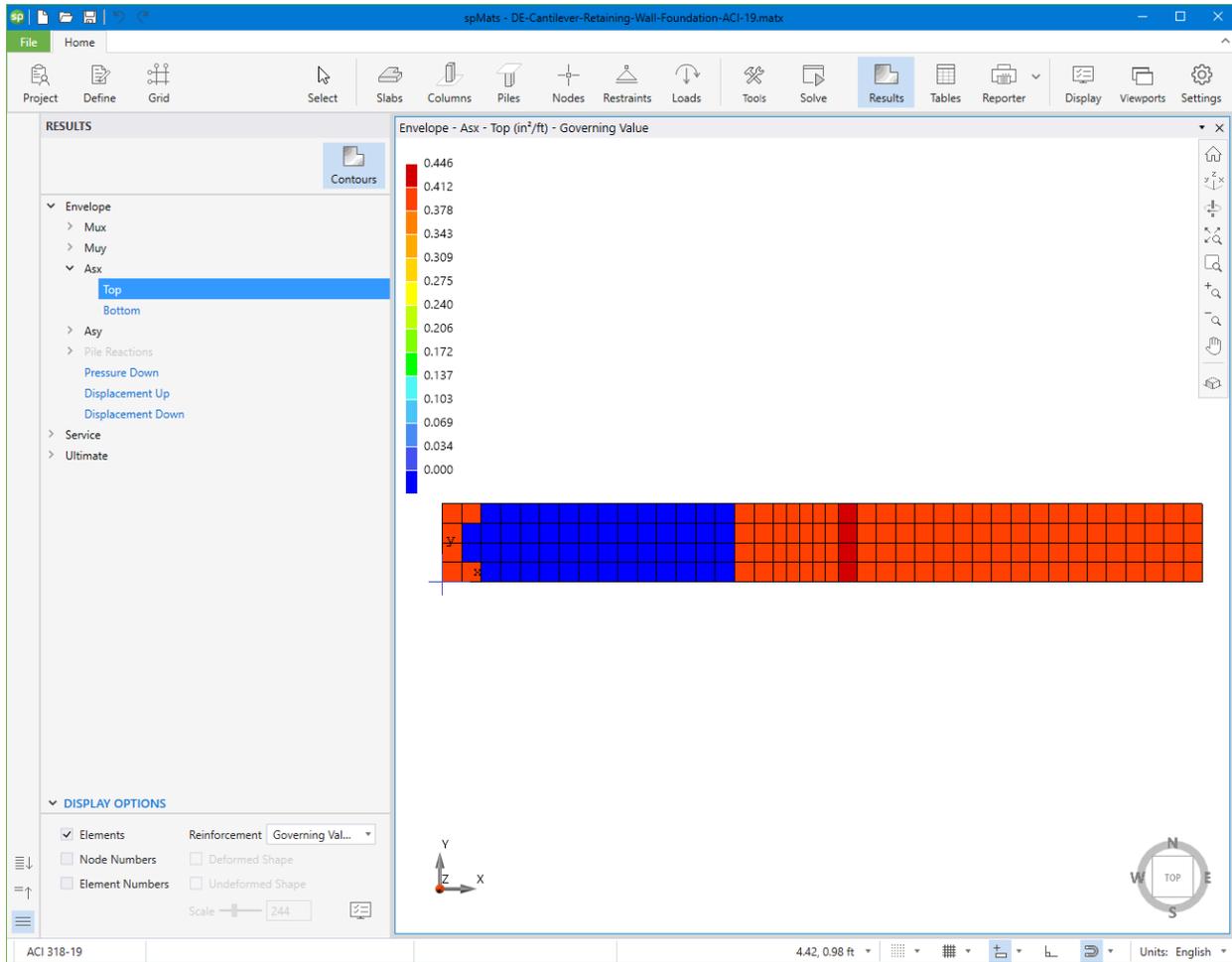


Figure 31 – Required Reinforcement Contour along X Direction (Top – Heel Design) (spMats)
(Note: minimum reinforcement governs)

7.4. Soil Reactions / Pressure

1. Results

1.1. Service

1.1.1. Sum of Reactions

1.1.1.1. S2

NOTES:

Sum of all forces and moments with respect to center of gravity (X,Y) = (4.88, 0.50) ft

Sum of Reactions	Fz kips	Mx kip-ft	My kip-ft
Soil	13.24	0.00	22.16
Springs	-	-	-
Piles	-	-	-
Restraints	-	-	-
Slaved nodes	0.00	0.00	0.00
Total Reactions	13.24	0.00	22.16
Total loads	-13.24	0.00	-22.16

Figure 32 – Soil Service Reactions

1.1.2. Soil Disp. & Pressure

1.1.2.1. S2

Case 1

NOTES:

[x] Indicates allowable pressure is exceeded.

Element	Node	Disp, Dz in	Pressure, Qz ksf	Node	Disp, Dz in	Pressure, Qz ksf
83	128	-0.080	-2.678	86	-0.080	-2.678
	127	-0.082	-2.746	85	-0.082	-2.746
123	168	-0.001	-0.021	126	-0.001	-0.021
	167	-0.002	-0.083	125	-0.002	-0.083

1.1.2. Soil Disp. & Pressure

1.1.2.2. S1

Case 2

NOTES:

[x] Indicates allowable pressure is exceeded.

Element	Node	Disp, Dz in	Pressure, Qz ksf	Node	Disp, Dz in	Pressure, Qz ksf
83	128	-0.078	-2.587	86	-0.078	-2.587
	127	-0.079	-2.639	85	-0.079	-2.639
123	168	-0.015	-0.511	126	-0.015	-0.511
	167	-0.017	-0.558	125	-0.017	-0.558

Figure 33 – Soil Bearing Pressure

7.5. Cantilever Retaining Wall Foundation Mesh Status

Since [spMats](#) is utilizing finite element analysis to model and design the foundation. It is useful to track the number of elements used in the model to optimize the model results (accuracy) and running time (processing stage). [spMats](#) provides mesh status to keep tracking the mesh sizing as a function of the number of elements, minimum and maximum element sizes, and maximum aspect ratio.



A screenshot of a software interface showing a table titled "Status" with a dropdown arrow. The table contains four rows of mesh data: Number of elements (164), Min. element size (0.16 ft), Max. element size (0.25 ft), and Max. aspect ratio (1.52).

▼ Status		
Number of elements	164	
Min. element size	0.16	ft
Max. element size	0.25	ft
Max. aspect ratio	1.52	

Figure 34 – Mesh Status

8. Cantilever Retaining Wall Analysis and Design Results Comparison & Conclusions

Table 3 – Cantilever Retaining Wall Flexural Results		
Method of Solution	M_u (kip-ft/ft)	$A_{s,req}$ (in. ² /ft)
Reference	45.70	0.79
Hand	45.70	0.78
spWall	45.64	0.79

Table 4 – Cantilever Retaining Wall Foundation Soil Bearing Pressure				
Method of Solution	Case 1		Case 2	
	q_1 (psf)	q_2 (psf)	q_1 (psf)	q_2 (psf)
Reference	2,780	0	2,710	492
Hand	2,784	0	2,715	496
spMats	2,746	21	2,639	511

Table 5 – Cantilever Retaining Wall Foundation Results				
Method of Solution	Toe		Heel	
	M_u (kip-ft/ft)	$A_{s,req}$ (in. ² /ft)	M_u (kip-ft/ft)	$A_{s,req}$ (in. ² /ft)
Reference	25.8*	0.39	38.2**	0.61
Hand	24.3	0.39	29.9	0.47
spMats	21.8	0.39	28.5	0.45

* the downward load of the earth fill over the toe is neglected by the reference
 ** the upward reaction of the soil is neglected by the reference

The results of all the hand calculations and the reference used illustrated above are in agreement with the automated exact results obtained from the [spWall](#) and [spMats](#) programs.

Note that the hand and reference considered the toe and heel as cantilever projecting outward and inward from the face of the stem, respectively. [spMats](#) provides the flexibility of modeling the foundation with the exact geometry and boundary conditions to achieve more accurate results leading to potential savings in the reinforcement required.

Some load cases were neglected by the reference for simplicity and to achieve a more conservative design. On the other hand, [spMats](#) take into account all the applied load cases and include them in the calculations of the required reinforcement for the toe and heel. Additional load combination can be easily employed in [spMats](#) to explore more loading scenarios to meet project criteria.