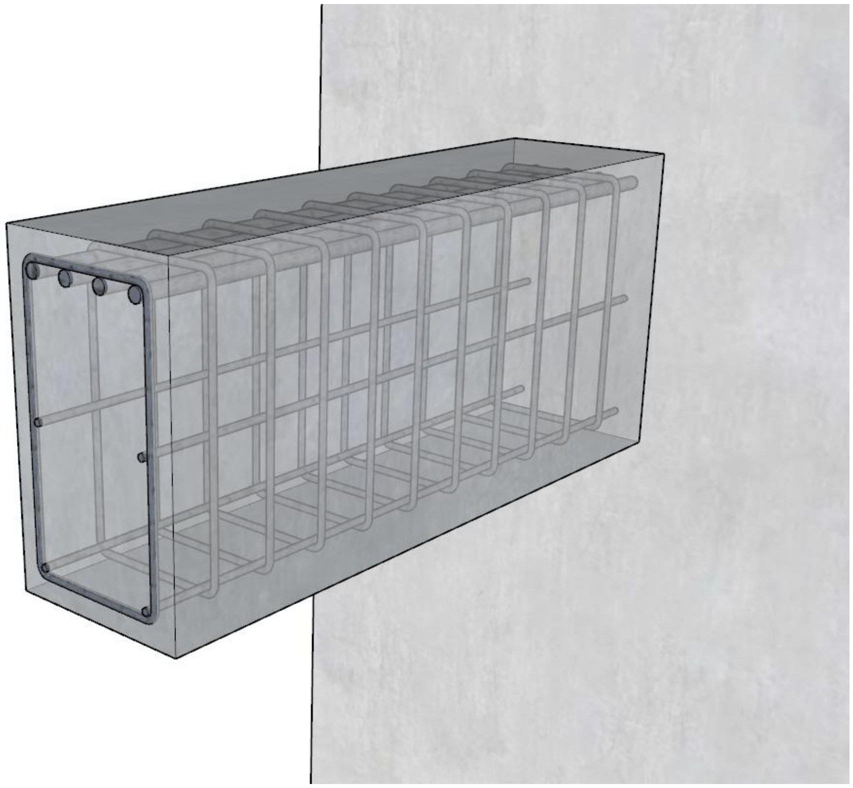
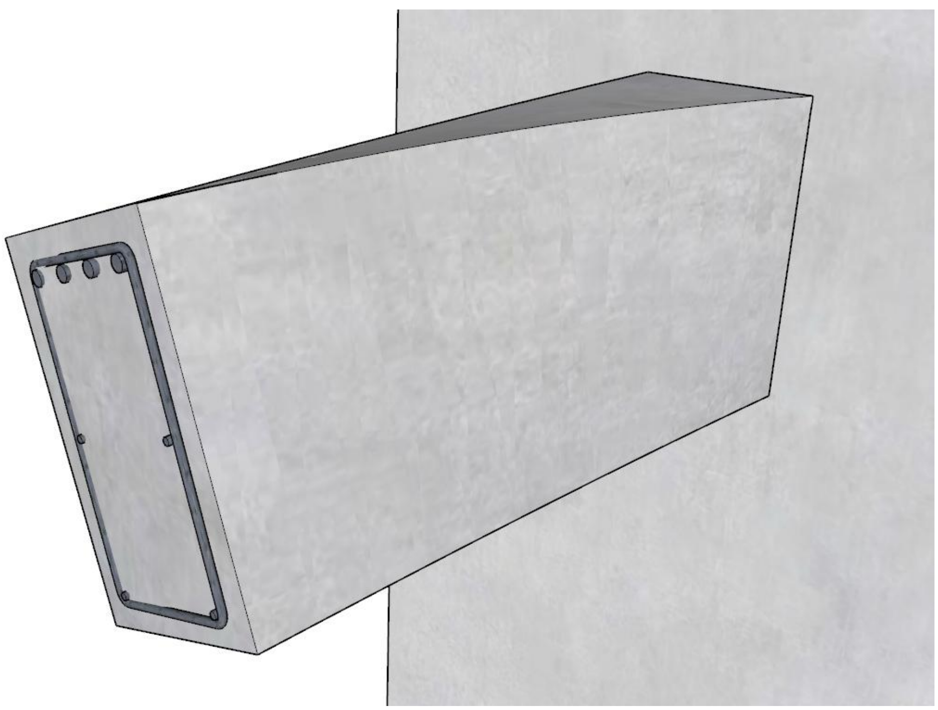
Equilibrium Torsion in Beams (ACI 318-14)





Equilibrium Torsion in Beams (ACI 318-14)

The cantilever beam shown supports its self-weight in addition to a concentrated load applied with an eccentricity from the centroidal axis, resulting in combined flexure, shear, and equilibrium torsion. This example demonstrates the structural analysis and design of the rectangular reinforced concrete cantilever beam in accordance with ACI 318-14 provisions, with particular emphasis on equilibrium torsion behavior and its impact on detailing requirements. Hand calculations are presented and compared with numerical analysis and design results obtained from the spBeam engineering software program by StructurePoint.

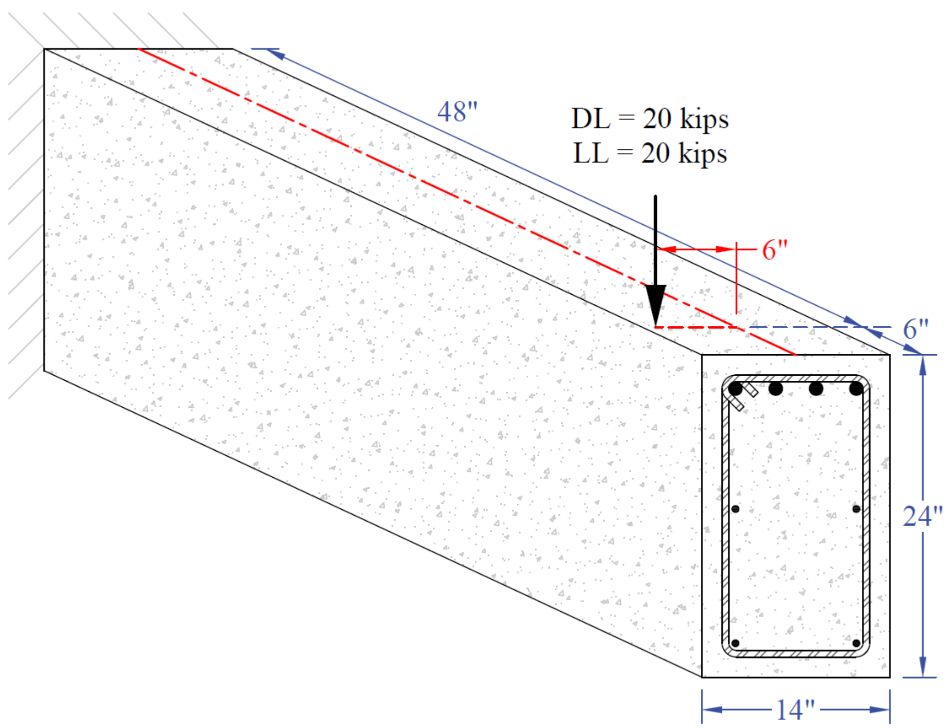


Figure 1 – Rectangular Reinforced Concrete Cantilever Beam

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

References

- Reinforced Concrete Mechanics and Design, 7th Edition, 2016, James Wight, Pearson, Example 7-2
- spBeam Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2024
- Contact Support@StructurePoint.org to obtain supplementary materials (spBeam model: DE-Equilibrium-Torsion-ACI-14.slbx)

Design Data

f_c' = 4 ksi normal weight concrete (w_c = 150.00 lb/ft^3)

f_y = 60 ksi

Dead load, DL = 20.00 kip applied at 6 in. from the end of the beam and 6 in. away from the centroidal axis.

Live load, LL = 20.00 kip applied at 6 in. from the end of the beam and 6 in. away from the centroidal axis.

Beam span length, L = 54.00 in. = 4.50 ft

Use #8 bars for longitudinal reinforcement (A_s = 0.79 in.^{2}, d_b = 1.00 in.)

Use #4 bars for stirrups (A_s = 0.20 in.^{2}, d_b = 0.50 in.)

Clear cover = 1.50 in. ***ACI 318-14 (Table 20.6.1.3.1)***

1. Notations

This section (based on ACI 318-14 provisions) defines notation and terminology used in this design example:

a = depth of equivalent rectangular stress block, in.

A_{cp} = area enclosed by outside perimeter of concrete cross section, in.^{2}

A_l = total area of longitudinal reinforcement to resist torsion, in.^{2}

$A_{l,min}$ = minimum area of longitudinal reinforcement to resist torsion, in.^{2}

A_o = gross area enclosed by torsional shear flow path, in.^{2}

A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement, in.^{2}

A_s = area of nonprestressed longitudinal tension reinforcement, in.^{2}

$A_{s,min}$ = minimum area of flexural reinforcement, in.^{2}

A_t = area of one leg of a closed stirrup, hoop, or tie resisting torsion within spacing s, in.^{2}

A_v = area of shear reinforcement within spacing s, in.^{2}

b = width of compression face of member, in.

b_w = web width or diameter of circular section, in.

c = distance from extreme compression fiber to neutral axis, in.

c_c = clear cover of reinforcement, in.

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

d_b = nominal diameter of bar, wire, or prestressing strand, in.

f_c' = specified compressive strength of concrete, psi

f_y = specified yield strength for nonprestressed reinforcement, psi

f_{yt} = specified yield strength of transverse reinforcement, psi

h = overall thickness, height, or depth of member, in.

l = span length of beam or one-way slab; clear projection of cantilever, in.

l_n = length of clear span measured face-to-face of supports, in.

M_u = factored moment at section, in.-lb

p_{cp} = outside perimeter of concrete cross section, in.

p_h = perimeter of centerline of outermost closed transverse torsional reinforcement, in.

P_u = factored axial force; to be taken as positive for compression and negative for tension, lb

T_{cr} = cracking torsional moment, in.-lb

T_{th} = threshold torsional moment, in.-lb

T_u = factored torsional moment at section, in.-lb

s = center-to-center spacing of items, such as longitudinal reinforcement, transverse reinforcement, tendons, or anchors, in.

V_c = nominal shear strength provided by concrete, lb

V_n = nominal shear strength, lb

V_s = nominal shear strength provided by shear reinforcement, lb

V_u = factored shear force at section, lb

w_u = factored load per unit length of beam or one-way slab, lb/in.

w_c = density, unit weight, of normalweight concrete, lb/ft^3

β_1 = factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis

ε_t = net tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength, excluding strains due to effective prestress, creep, shrinkage, and temperature

θ = angle between axis of strut, compression diagonal, or compression field and the tension chord of the members

λ = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normalweight concrete of the same compressive strength

ϕ = strength reduction factor

2. Preliminary Member Sizing

Check the minimum beam depth requirement of ***ACI 318-14 (Table 9.3.1.1)*** to waive deflection computations.

Using the minimum depth for non-prestressed beams in ***Table 9.3.1.1***.

$$h_{\min} = \frac{l_n}{8} = \frac{54.00}{8} = 6.75 \text{ in. (For cantilever beams)}$$ ***ACI 318-14 (Table 9.3.1.1)***

Therefore, since $h_{min} = 6.75$ in. $< h = 24.00$ in. the preliminary beam depth satisfies the minimum depth requirement, and the beam deflection computations are not required.

The width of the rectangular section (b) may be chosen in the following range:

$$\left(\frac{1}{2} \times h = 12.00 \text{ in.}\right) \le b = 14.00 \text{ in.} \le \left(\frac{2}{3} \times h = 16.00 \text{ in.}\right)$$ ***o.k.***

3. Load and Load combination

The governing load combination:

$$U = 1.20 \times D + 1.60 \times L$$ ***ACI 318-14 (Eq. 5.3.1b)***

For the Factored Uniform Load

$$\text{Self-weight} = \frac{24.00}{12.00} \times \frac{14.00}{12.00} \times 0.15 = 0.35 \ \frac{\text{kips}}{\text{ft}}$$

$$w_u = 1.20 \times 0.35 = 0.42 \ \frac{\text{kips}}{\text{ft}}$$

For the Factored Concentrated Load

$$P_u = 1.20 \times 20.00 + 1.60 \times 20.00 = 56.00 \text{ kips}$$

For the Factored Torsional Moment

The moment arm is the distance from the midspan to the centerline of the exterior beam section = 6.00 in. = 0.50 ft.

$$T_u = (1.20 \times 20.00 + 1.60 \times 20.00) \times \frac{6.00}{12.00} = 28.00 \text{ ft-kips}$$

4. Structural Analysis

Cantilever beams can be analyzed by calculating shear, moment and torsion diagrams or using Design Aid tables as shown below:

Shear, Moment and Torsion Diagrams:

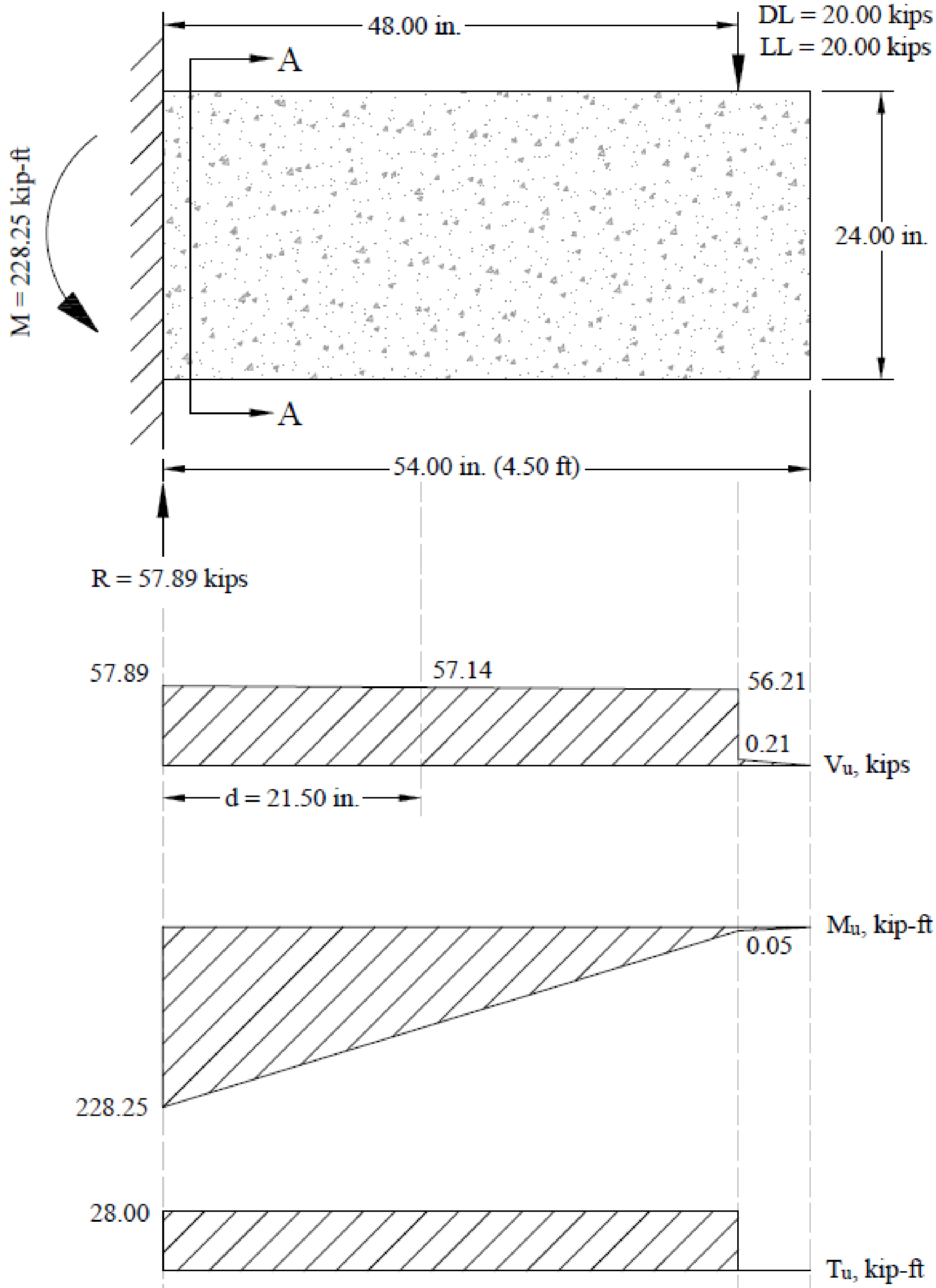


Figure 2 – Moment, Shear and Torsion Diagrams

Using Design Aid Tables:

$$V_u = P_u + w_u \times l = 56.00 + 0.42 \times \frac{54.00}{12.00} = 57.89 \text{ kips}$$

$$M_u = P_u \times l_p + \frac{w_u \times l^2}{2} = 56.00 \times \frac{48.00}{12.00} + \frac{0.42 \times 54.00^2}{2 \times 144.00} = 228.25 \text{ kip-ft}$$

Where:

$l = 54.00$ in.

$l_p = l - 6.00 = 54.00 - 6.00 = 48.00$ in.

5A.2 CANTILEVER BEAM – CONCENTRATED LOAD AT ANY POINT

$R = V$ .... $= P$

M_{max} (at fixed end) .... $= Pb$

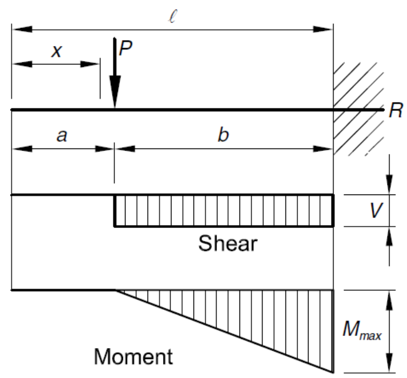
M_x (when $x > a$) .... $= P(x - a)$

Δ_{max} (at free end) .... $= \frac{Pb^2}{6EI}(3\ell - b)$

Δ_a (at point of load) .... $= \frac{Pb^3}{3EI}$

Δ_x (when $x < a$) .... $= \frac{Pb^2}{6EI}(3\ell - 3x - b)$

Δ_x (when $x > a$) .... $= \frac{P(\ell - x)^2}{6EI}(3b - \ell + x)$



5B.1 CANTILEVER BEAM – UNIFORMLY DISTRIBUTED LOAD

$R = V$ .... $= w\ell$

V_x .... $= wx$

M_{max} (at fixed end) .... $= \frac{w\ell^2}{2}$

M_x .... $= \frac{wx^2}{2}$

Δ_{max} (at free end) .... $= \frac{w\ell^4}{8EI}$

Δ_x .... $= \frac{w}{24EI}(x^4 - 4\ell^3 x + 3\ell^4)$

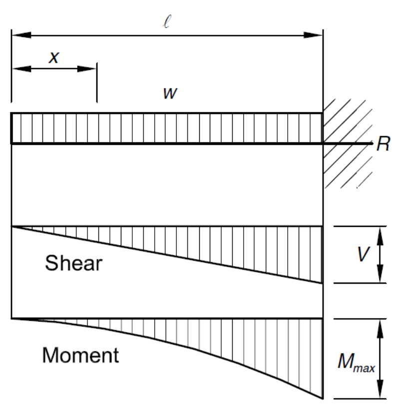


Figure 3 - Design Aid Tables (Beam Design Equations and Diagrams) – PCI Design Handbook

5. Flexural, Shear, and Torsion Design

5.1. Flexural Design

For this beam, the moment at the fixed end governs the design as shown previously.

M_u = 228.25 kip-ft

Use #8 bars with 1.50 in. concrete cover per ***ACI 318-14 (Table 20.6.1.3.1)*** and #4 stirrups. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, d, is calculated below:

$$d = h - \left(\text{clear cover} + d_{b,stirrups} + \frac{d_{Longitudinal\ bar}}{2} \right)$$

$$d = 24.00 - \left(1.50 + 0.50 + \frac{1.00}{2} \right) = 21.50 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the beam section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to 0.9253 × d. The assumptions will be verified once the area of steel is finalized.

Assume jd = 0.9253 × d = 0.9253 × 21.50 = 19.89 in.

Beam width, b = 14.00 in.

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{228.25 \times 12{,}000}{0.90 \times 60{,}000 \times 19.89} = 2.550 \text{ in.}^2$$

Recalculate 'a' for the actual A_s = 2.550 in.^{2}:

$$a = \frac{A_s \times f_y}{0.85 \times f_c' \times b} = \frac{2.550 \times 60{,}000}{0.85 \times 4{,}000 \times 14.00} = 3.214 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{3.21}{0.85} = 3.781 \text{ in.}$$

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4{,}000)}{1{,}000}$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$\beta_1 = 0.85 - \frac{0.05 \times (4,000 - 4,000)}{1,000} = 0.85$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) \times d_t - 0.003 = \left(\frac{0.003}{3.781}\right) \times 21.50 - 0.003 = 0.0141 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left(d - \frac{a}{2}\right)} = \frac{228.25 \times 12,000}{0.90 \times 60,000 \times \left(21.50 - \frac{3.214}{2}\right)} = 2.550 \text{ in.}^2$$

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{3 \times \sqrt{f_c'}}{f_y} \times b_w \times d = \frac{3 \times \sqrt{4,000}}{60,000} \times 14.00 \times 21.50 = 0.952 \text{ in.}^2$$ ***ACI 318-14 (9.6.1.2(a))***

And not less than

$$A_{s,\min} = \frac{200}{f_y} \times b_w \times d = \frac{200}{60,000} \times 14.00 \times 21.50 = 1.003 \text{ in.}^2$$ ***ACI 318-14 (9.6.1.2(b))***

$$\therefore A_{s,\min} = 1.003 \text{ in.}^2$$

$$A_s = 2.550 \text{ in.}^2 \geq A_{s,min} = 1.003 \text{ in.}^2$$

Torsion requirements for longitudinal steel have to be determined and combined with reinforcement area required for flexure.

5.2. Torsion Design

Check if torsional effects can be neglected:

If $T_u < \phi T_{th}$, it shall be permitted to neglect torsional effects. ***ACI 318-14 (22.7.1.1)***

Where:

T_u = 28.00 kip-ft (Figure 2)

ϕT_{th} = 5.87 kip-ft = Threshold torsion (the calculation of ϕT_{th} is shown in the next section)

Since $T_u > \phi T_{th}$, the torsional effects must be considered.

Check if the factored design torsion can be reduced:

It is permitted to reduce T_u to ϕT_{cr}; due to redistribution of internal forces after torsional cracking; if the exterior continuous beam meet the following requirements: ***ACI 318-14 (22.7.3.2)***

1. The beam is statically indeterminate (continuous beam).

2. $T_u \geq \phi T_{cr}$.

The Torsion is needed for equilibrium; therefore, design for:

T_u = 28.00 kip-ft (Figure 2)

The torsional properties of the beam are the following:

$$A_{cp} = (14.00 \times 24.00) = 336.00 \text{ in.}^2$$

$$p_{cp} = 2 \times (14.00 + 24.00) = 76.00 \text{ in.}$$

$$\frac{A_{cp}^{\ 2}}{p_{cp}} = \frac{(336.00)^2}{76.00} = 1,485.47 \text{ in.}^3$$

$$\phi T_{cr} = \phi \times 4 \times \lambda \times \sqrt{f_c'} \times \left(\frac{A_{cp}^{\ 2}}{p_{cp}} \right)$$ ***ACI 318-14 (Table 22.7.5.1(a))***

$$\phi T_{cr} = 0.75 \times 4 \times 1.00 \times \sqrt{4,000} \times 1485.47 = 281,848.81 \text{ lb-in.} = 23.49 \text{ kip-ft}$$

$$\phi T_{th} = \phi \times \lambda \times \sqrt{f_c'} \times \left(\frac{A_{cp}^{\ 2}}{p_{cp}} \right) = \frac{\phi T_{cr}}{4} = \frac{23.49}{4} = 5.87 \text{ kip-ft}$$

5.2.1. Adequacy of Cross-Sectional Dimensions for Torsion

For solid sections, the limit on shear and torsion is given by:

$$\sqrt{\left(\frac{V_u}{b_w \times d}\right)^2 + \left(\frac{T_u \times p_h}{1.70 \times A_{oh}{}^2}\right)^2} \leq \phi \times \left(\frac{V_c}{b_w \times d} + 8 \times \sqrt{f'_c}\right)$$ **_ACI 318-14 (22.7.7.1a)_**

Using d = 21.50 in., the factored shear force at the critical section located at a distance d from the face of the support is:

V_u = 57.14 kips (Figure 2)

Also, the nominal shear strength provided by the concrete is:

$$V_c = 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d$$ **_ACI 318-14 (Eq. 22.5.5.1)_**

Using a 1.50 in. clear cover to # 4 closed stirrups.

$$A_{oh} = \left[24.00 - (1.50 + 1.50) - 0.50\right] \times \left[14.00 - (1.50 + 1.50) - 0.50\right] = 215.25 \text{ in.}^2$$

$$p_h = 2 \times \left\{\left[24.00 - (1.50 + 1.50) - 0.50\right] + \left[14.00 - (1.50 + 1.50) - 0.50\right]\right\} = 62.00 \text{ in.}$$

$$\sqrt{\left(\frac{57.14 \times 1{,}000}{14.00 \times 21.50}\right)^2 + \left(\frac{28.00 \times 12{,}000 \times 62.00}{1.70 \times 215.25^2}\right)^2} = 325.55 \text{ psi}$$

$$< 0.75 \times \left[\frac{2 \times 1 \times \sqrt{4{,}000} \times 14.00 \times 21.50}{14.00 \times 21.50} + 8 \times \sqrt{4{,}000}\right] = 474.34 \text{ psi}$$

Therefore, the section is adequate.

5.2.2. Transverse Reinforcement Required for Torsion

$$\frac{A_t}{s}=\frac{T_u}{\phi\times 2\times A_o\times f_{yt}\times\cot(\theta)}$$ ***ACI 318-14 (Eq. 22.7.6.1a)***

Where:

A_o = 0.85 × A_{oh} = 0.85 × 215.25 = 182.96 in.^{2} ***ACI 318-14 (22.7.6.1.1)***

$\theta = 45^\circ$ ***ACI 318-14 (22.7.6.1.2(a))***

Therefore,

$$\frac{A_t}{s}=\frac{28.00\times 12,000}{0.75\times 2\times 182.96\times 60,000\times\cot(45^\circ)}=0.0204\text{ in.}^2/\text{ in. per leg}$$

5.2.3. Transverse Reinforcement Required for Shear

V_u = 57.14 kips (Figure 2)

Shear strength provided by concrete

$$\phi V_c=\phi\times\left(2\times\lambda\times\sqrt{f'_c}\times b_w\times d\right)$$ ***ACI 318-14 (Eq. 22.5.5.1)***

$$\phi V_c=0.75\times\left(2.00\times 1.00\times\sqrt{4,000}\times 14.00\times 21.50\right)=28,555.37\text{ lb}=28.56\text{ kips}$$

Since $V_u>\frac{\phi V_c}{2}$, shear reinforcement is required.

The nominal shear strength required to be provided by shear reinforcement is

$$V_s=V_n-V_c=\frac{V_u}{\phi}-V_c=\left(\frac{57.14}{0.75}\right)-38.07=38.11\text{ kips}$$

Check whether V_s is less than $8\times\sqrt{f'_c}\times b_w\times d$

If V_s is greater than $8\times\sqrt{f'_c}\times b_w\times d$, then the cross-section has to be revised as ***ACI 318-14*** limits the shear capacity to be provided by stirrups to $8\times\sqrt{f'_c}\times b_w\times d$ ***ACI 318-14 (22.5.1.2)***

$$8\times\sqrt{f'_c}\times b_w\times d\;=8\times\sqrt{4,000}\times 14.00\times 21.50=152,295.29\text{ lb}=152.30\text{ kips}$$

Since V_s does not exceed $8\times\sqrt{f'_c}\times b_w\times d$, the cross section is adequate.

Calculate the required transverse reinforcement for shear as

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi \times f_{yt} \times d} = \frac{57.14 - 28.56}{0.75 \times 60.000 \times 21.50} = 0.0295 \ \frac{\text{in.}^2}{\text{in.}}$$ ***ACI 318-14 (22.5.10.5.3)***

5.2.4. Total Required Transverse Reinforcement for Combined Shear and Torsion

$$\frac{A_v}{s} + 2 \times \frac{A_t}{s} = 0.0295 + 2 \times 0.0204 \ \frac{\text{in.}^2/\text{in.}}{\text{leg}} = 0.0704 \ \text{in.}^2/\text{in.}$$

Minimum transverse reinforcement for shear and torsion is calculated as follows:

$$\frac{(A_v + 2 \times A_t)_{min}}{s} = \text{greater of} \begin{bmatrix} 0.75 \times \sqrt{f'_c} \times \left(\dfrac{b_w}{f_{yt}}\right) \\ 50 \times \left(\dfrac{b_w}{f_{yt}}\right) \end{bmatrix}$$ ***ACI 318-14 (9.6.4.2)***

$$\frac{(A_v + 2 \times A_t)_{min}}{s} = \text{greater of} \begin{bmatrix} 0.75 \times \sqrt{4,000} \times \left(\dfrac{14.00}{60,000}\right) \\ 50 \times \left(\dfrac{14.00}{60,000}\right) \end{bmatrix} = \begin{bmatrix} 0.0111 \\ 0.0117 \end{bmatrix} = 0.0117 \ \text{in.}^2/\text{in.} < 0.0704 \ \text{in.}^2/\text{in.}$$

Then, provide $\frac{A_v}{s} + 2 \times \frac{A_t}{s} = 0.0704 \ \text{in.}^2/\text{in.}$

<u>Calculate the required spacing:</u>

Maximum spacing of transverse torsion reinforcement:

$$s_{max} = \text{lesser of} \begin{bmatrix} p_h / 8 \\ 12.00 \text{ in.} \end{bmatrix}$$ ***ACI 318-14 (9.7.6.3.3)***

$$s_{max} = \text{lesser of} \begin{bmatrix} 62.00 / 8 \\ 12.00 \text{ in.} \end{bmatrix} = \text{lesser of} \begin{bmatrix} 7.75 \text{ in.} \\ 12.00 \text{ in.} \end{bmatrix} = 7.75 \text{ in.}$$

Maximum spacing of transverse shear reinforcement:

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ***ACI 318-14 (9.7.6.2.2)***.

Check whether V_s is less than $4 \times \sqrt{f'_c} \times b_w \times d$

$$4 \times \sqrt{f'_c} \times b_w \times d = 4 \times \sqrt{4,000} \times 14.00 \times 21.50 = 76,147.65 \text{ lb} = 76.15 \text{ kips} > V_s = 38.11 \text{ kips}$$

Therefore, maximum stirrup spacing shall be the smallest of d/2 and 24.00 in.

$$s_{max} = \text{lesser of} \begin{bmatrix} d/2 \\ 24.00 \text{ in.} \end{bmatrix}$$ ***ACI 318-14 (Table 9.7.6.2.2)***

$$s_{max} = \text{lesser of} \begin{bmatrix} 21.50/2 \\ 24.00 \text{ in.} \end{bmatrix} = \text{lesser of} \begin{bmatrix} 10.75 \text{ in.} \\ 24.00 \text{ in.} \end{bmatrix} = 10.75 \text{ in. (governs)}$$

For #3 stirrups, A_{v+t} (two legs) = 0.22 in.^{2}, and the required s = 0.22 / 0.0704 = 3.13 in. < 10.75 in.

For #4 stirrups, A_{v+t} (two legs) = 0.40 in.^{2}, and the required s = 0.40 / 0.0704 = 5.69 in. < 10.75 in.

Use No. 4 closed stirrups at 5.00 in. on centers, placing the first stirrup at 2.50 in. from the face of the support.

5.2.5. Additional Required Longitudinal Reinforcement for Torsion

$$A_l = \frac{T_u \times p_h}{\phi \times 2 \times A_o \times f_y \times \cot(\theta)}$$ ***ACI 318-14 (Eq. 22.7.6.1b)***

Where:

$A_o = 0.85 \times A_{oh} = 0.85 \times 215.25 = 182.96$ in.^{2} ***ACI 318-14 (22.7.6.1.1)***

$\theta = 45^\circ$ ***ACI 318-14 (22.7.6.1.2(a))***

Therefore,

$$A_l = \frac{28.00 \times 12,000 \times 62.00}{0.75 \times 2 \times 182.96 \times 60,000 \times \cot(45^\circ)} = 1.265 \text{ in.}^2$$

The minimum total area of longitudinal torsional reinforcement:

$$A_{l,min} = \text{lesser of} \begin{bmatrix} \dfrac{5 \times \sqrt{f'_c} \times A_{cp}}{f_y} - \left(\dfrac{A_t}{s}\right) \times p_h \times \dfrac{f_{yt}}{f_y} \\ \dfrac{5 \times \sqrt{f'_c} \times A_{cp}}{f_y} - \left(\dfrac{25 \times b_w}{f_{yt}}\right) \times p_h \times \dfrac{f_{yt}}{f_y} \end{bmatrix}$$ ***ACI 318-14 (9.6.4.3)***

$$A_{l,min} = \text{lesser of} \begin{bmatrix} \dfrac{5 \times \sqrt{4,000} \times 336.00}{60,000} - \left(0.0204 \times 62.00 \times \dfrac{60,000}{60,000}\right) \\ \dfrac{5 \times \sqrt{4,000} \times 336.00}{60,000} - \left(\dfrac{25 \times 14}{60,000} \times 62.00 \times \dfrac{60,000}{60,000}\right) \end{bmatrix} = \begin{bmatrix} 0.506 \\ 1.409 \end{bmatrix} = 0.506 \text{ in.}^2$$

Since $A_l > A_{l,min}$, use A_l = 1.265 in.^{2}

The longitudinal reinforcement is to be distributed around the perimeter of the stirrups, with a maximum spacing of 12.00 in. There shall be at least one longitudinal bar in each corner of the stirrups.

ACI 318-14 (9.7.5.1)

Longitudinal bars shall have a diameter at least 0.042 times the stirrup spacing, but not less than 3/8 in.

ACI 318-14 (9.7.5.2)

Provide 4 - #4 bars in the lower portion of the beam (two at mid-depth and two at bottom) and add 1.265 in.^{2} (Section 5.2.5) - (4 × 0.20 in.^{2}) = 0.465 in.^{2} to the flexural steel.

The total area of steel required in the top face of the beam is 2.550 in.^{2} (Section 5.1) + 0.465 in.^{2} = 3.015 in.^{2} Thus, we have:

6 - #7 bars: A_s = 3.600 in.^{2}, will not fit in one layer.

4 - #8 bars = A_s = 3.160 in.^{2}, will fit in one layer.

Provide 4 - #8 bars at the top of the beam (see Figure 4).

ACI 318-14 (9.5.4.5) allows a designer to subtract M_u / (0.90 × d × f_y) from the area of longitudinal steel in the flexural compression zone. We shall not do this, for three reasons:

1. M_u varies along the length of the beam.
2. We need two bars of minimum diameter 0.21 in. in the corners of the stirrups.
3. We need bars to support the corners of the stirrups.

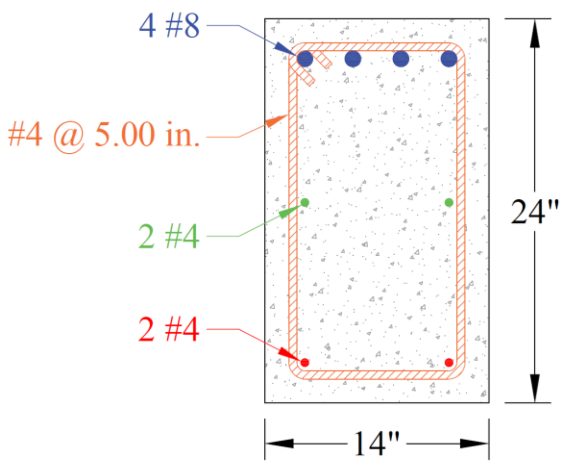


Figure 4 – Reinforced Concrete Cantilever Beam Cross Section

6. Equilibrium Torsion in Beams – spBeam Software

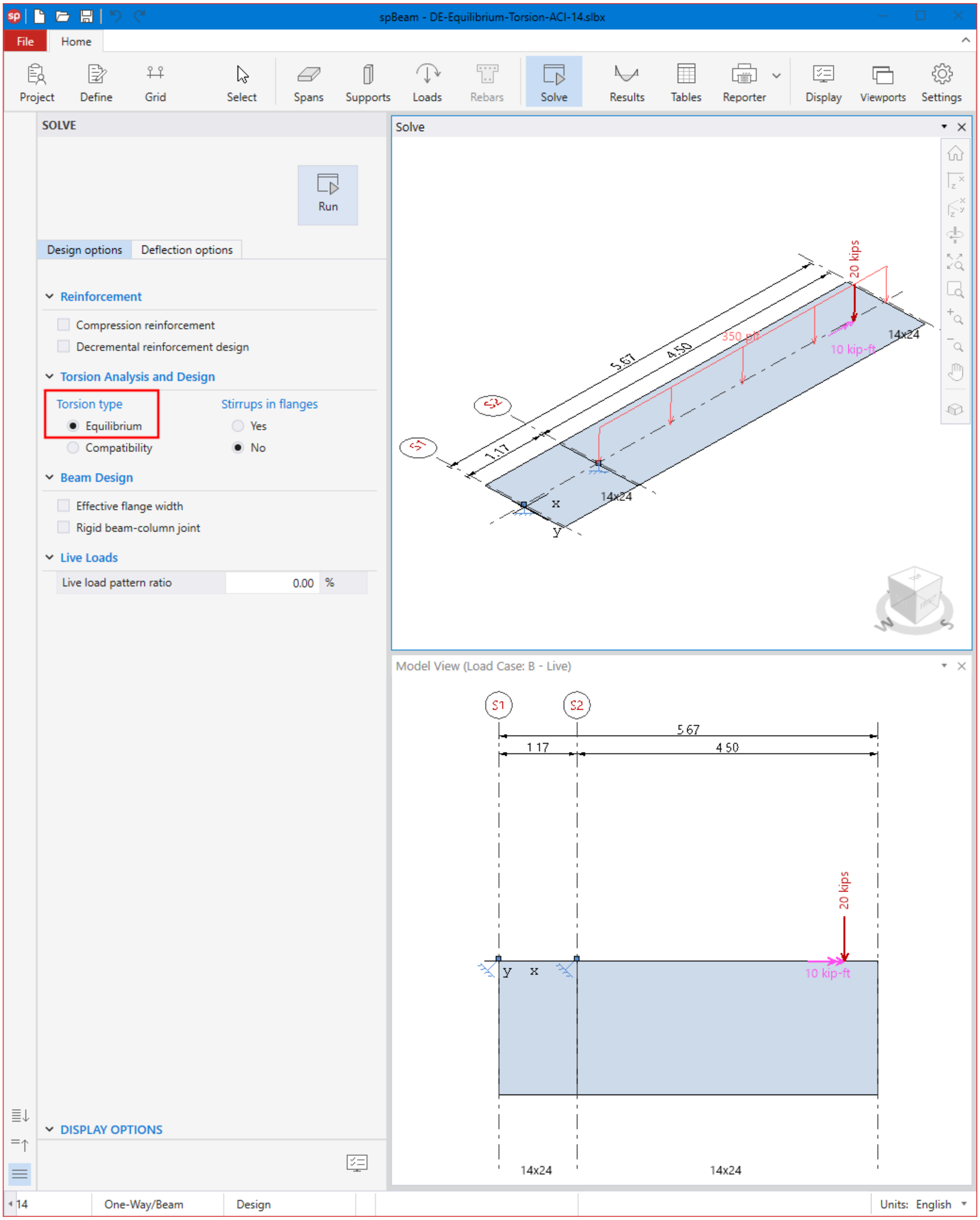
spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318-14) and Canadian (CSA A23.3-14) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

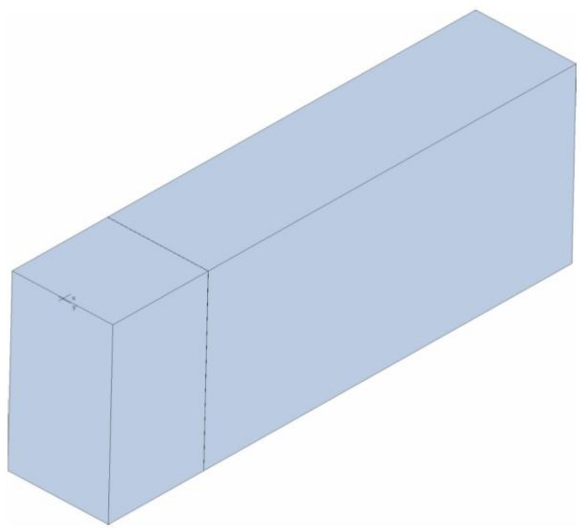
Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or T-shaped sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, spBeam provide users with options and flexibility to:

1. Design the beam as a rectangular cross-section or a T-shaped section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.
4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the equilibrium torsion in beams discussed in this example.


spBeam - DE-Equilibrium-Torsion-ACI-14.slbx
File
Home
Project
Define
Grid
Select
Spans
Supports
Loads
Rebars
Solve
Results
Tables
Reporter
Display
Viewports
Settings
SOLVE
Run
Design options
Deflection options
Reinforcement
Compression reinforcement
Decremental reinforcement design
Torsion Analysis and Design
Torsion type
Equilibrium
Compatibility
Stirrups in flanges
Yes
No
Beam Design
Effective flange width
Rigid beam-column joint
Live Loads
Live load pattern ratio
0.00
%
DISPLAY OPTIONS
Solve
20 kips
350 plf
10 kip-ft
14x24
5.67
4.50
1.17
S1
S2
Model View (Load Case: B - Live)
14
One-Way/Beam
Design
Units: English

spBeam v10.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams and One-way Slab Systems



Contents

1. Input Echo

1.1. General Information

File Name	C:\Structur...\DE-Equilibrium-Torsion-ACI-14.slbx
Project	Equilibrium Torsion
Frame	Cantilever Beam
Engineer	StructurePoint
Code	ACI 318-14
Units	English
Reinforcement Database	ASTM A615
Mode	Design
Number of supports =	2 + Right Cantilever
Floor System	One-Way/Beam

1.2. Solve Options

Live load pattern ratio = 0%
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Moment redistribution NOT selected.
Effective flange width calculations NOT selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design selected.
Stirrups in flanges (if available) NOT selected.
Compatibility torsion NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150	pcf
f'_c	4	ksi
E_c	3834.25	ksi
f_r	0.474342	ksi

1.3.2. Concrete: Columns

w_c	150	pcf
f'_c	4	ksi
E_c	3834.25	ksi
f_r	0.474342	ksi

1.3.3. Reinforcing Steel

f_y	60	ksi
f_{yt}	60	ksi
E_s	29000	ksi
Epoxy coated bars	No	

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

1.5. Span Data

1.5.1. Slabs

Notes:

Span	Loc	L1	t	wL	wR	H_{min}	
		ft	in	ft	ft	in	
1	Int	1.166	0.00	0.583	0.583	0.00	---
2	Int	4.500	0.00	0.583	0.583	0.00	RC

1.5.2. Ribs and Longitudinal Beams

Notes:
*c - Deep beam. Additional design and bar detailing required.

Span	Ribs			Beams		Span	
	b	h	Sp	b	h	H_{min}	
	in	in	in	in	in	in	
1	0.00	0.00	0.00	14.00	24.00	0.87	*c
2	0.00	0.00	0.00	14.00	24.00	6.75	*c

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	0.00	0.00	0.000	0.00	0.00	0.000	999
2	0.00	0.00	0.000	0.00	0.00	0.000	999

1.6.2. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kips/in	kip-in/rad		
1	0.00	0.00	Fixed	Fixed
2	0.00	0.00	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.200	1.600

1.7.2. Line Loads

Case/Patt	Span	Wa	La	Wb	Lb
		plf	ft	plf	ft
Dead	2	350.00	0.000	350.00	4.500

1.7.3. Point Forces

Case/Patt	Span	Wa	La
		kips	ft
Dead	2	20.00	4.000
Live	2	20.00	4.000

1.7.4. Point Torques

Case/Patt	Span	Wa	La
		kip-ft	ft
Dead	2	10.00	4.000
Live	2	10.00	4.000

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#5	#8	#5	#8
Bar spacing	in	1.00	18.00	1.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	in	1.50		1.50	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#8	#8	#4	#4	#4	#4
Bar spacing	in	1.00	18.00	1.00	18.00	5.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	2.00		2.00			
Layer dist.	in	1.00		1.00			
No. of legs						2	2
Side cover	in					1.50	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

2. Design Results

2.1. Top Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.

Span	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$\boxed{A_{s,req}}$	Sp_{Prov}	Bars
		ft	kip-ft	ft	in²	in²	in²	in	
1	Left	1.17	0.00	0.000	0.000	5.437	0.000	0.000	---
	Midspan	1.17	0.00	0.408	0.000	5.437	0.000	0.000	---
	Right	1.17	0.00	1.166	0.421	5.437	0.000	2.902	4-#8 *3
2	Left	1.17	228.25	0.000	1.003	5.437	$\boxed{2.550}$	2.902	4-#8
	Midspan	1.17	137.60	1.575	1.003	5.437	1.487	2.902	4-#8

Span	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
		ft	kip-ft	ft	in²	in²	in²	in	
	Right	1.17	60.72	2.925	0.851	5.437	0.640	2.902	4-#8 *3

2.2. Top Bar Details

Span	Left				Continuous		Right			
	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
		ft		ft		ft		ft		ft
1	---		---		---		2-#8	1.17	2-#8	1.17
2	---		---		4-#8	4.50	---		---	

2.3. Top Bar Development Lengths

Span	Left				Continuous		Right			
	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
		in		in		in		in		in
1	---		---		---		2-#8	12.00	2-#8	12.00
2	---		---		4-#8	23.07	---		---	

2.4. Bottom Reinforcement

Span	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
	ft	kip-ft	ft	in²	in²	in²	in	
1	1.17	0.00	1.166	0.000	5.500	0.000	0.000	---
2	1.17	0.00	2.250	0.000	5.500	0.000	0.000	---

2.5. Bottom Bar Details

Span	Long Bars			Short Bars		
	Bars	Start	Length	Bars	Start	Length
		ft	ft		ft	ft
1	---			---		
2	---			---		

2.6. Bottom Bar Development Lengths

Span	Long Bars		Short Bars	
	Bars	DevLen	Bars	DevLen
		in		in
1	---		---	
2	---		---	

2.7. Flexural Capacity

Span	Top						Bottom				
	x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
	ft	in²	kip-ft	kip-ft			in²	kip-ft	kip-ft		
1	0.000	3.16	-277.41	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.408	3.16	-277.41	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.583	3.16	-277.41	0.00	U1 Odd	OK	0.00	0.00	0.00	U1 All	OK
	0.758	3.16	-277.41	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	1.000	3.16	-277.41	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK

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Span	x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
	Top						Bottom				
	ft	in²	kip-ft	kip-ft			in²	kip-ft	kip-ft		
	1.166	3.16	-277.41	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
2	0.000	3.16	-277.41	-228.25	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	1.575	3.16	-277.41	-137.60	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	2.250	3.16	-277.41	-99.06	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	2.925	3.16	-277.41	-60.72	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	4.500	3.16	-277.41	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK

2.8. Longitudinal Beam Shear and Torsion Reinforcement Required

2.8.1. Section Geometrical Properties

Span	d	p_{cp}	p_h	A_{cp}	A_{oh}	A_o
	in	in	in	in²	in²	in²
1	21.50	76.00	62.00	336.000	215.250	182.963
2	21.50	76.00	62.00	336.000	215.250	182.963

2.8.2. Section Strength Properties

Span	$(A_v/s)_{min}$	ΦV_c	ΦT_{cr}	ΦS_{vt}
	in²/in	kips	kip-ft	ksi
1	0.0117	28.56	23.49	0.474
2	0.0117	28.56	23.49	0.474

2.8.3. Transverse Reinforcement Demand

Notes:
*2 - Torsion ignored (Tu < PhiTcr/4).

Span	Start	End	X_u	V_u	T_u	v_f	Comb/Patt	A_v/s	A_t/s	$A_{(v+2t)}/s$	$A_{(v+2t)}/s$	
							Required				Demand	
	ft	ft	ft	kips	kip-ft	ksi		in²/in	in²/in	in²/in	in²/in	
1	0.250	0.916	0.58	0.00	0.00	0.000	U1/All	0.0000	0.0000	0.0000	0.0000	*2
2	0.250	4.500	1.79	57.14	28.00	0.326	U1/All	$\boxed{0.0295}$	$\boxed{0.0204}$	$\boxed{0.0704}$	0.0704	

2.8.4. Required Longitudinal Reinforcement

Notes:
*2 - Torsion ignored (Tu < PhiTcr/4).

Span	Start	End	X_u	T_u	Comb/Patt	A_l	
	ft	ft	ft	kip-ft		in²	
1	0.250	0.916	0.00	0.00	U1/All	0.000	*2
2	0.250	4.500	1.79	28.00	U1/All	1.265	

2.8.5. Beam Transverse Reinforcement Details

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#4	None
2	#4	10 @ 5.3

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Page | 8
1/21/2026
2:21 PM

2.8.6. Longitudinal Torsional Reinforcement Details

	Long Bars			Short Bars		
Span	Bars	Start	Length	Bars	Start	Length
		ft	ft		ft	ft
1	---			---		
2	6-#5	0.00	4.50	---		

2.8.7. Beam Shear and Torsion Transverse Reinforcement Capacity in Terms of Required Area

Notes:
*2 - Torsion ignored (Tu < PhiTcr/4).

			Required								
Span	Start	End	X_u	V_u	T_u	v_f	Comb/Patt	A_v/s	A_t/s	$A_{(v+2t)}/s$	
	ft	ft	ft	kips	kip-ft	ksi		in²/in	in²/in	in²/in	
1	0.000	1.166	0.58	0.00	0.00	0.00	U1/All	0.0000	0.0000	0.0000	*2
2	0.000	0.250	1.79	57.14	28.00	0.33	U1/All	-----	-----	-----	
	0.250	4.250	1.79	57.14	28.00	0.33	U1/All	0.0295	0.0204	0.0704	
	4.250	4.500	4.25	0.11	0.00	0.00	U1/All	-----	-----	-----	

2.8.8. Beam Shear and Torsion Transverse Reinforcement Capacity in Terms of Provided Area

Notes:
*2 - Torsion ignored (Tu < PhiTcr/4).

			Provided			
Span	Start	End	$A_{(v+2t)}$	Sp	$A_{(v+2t)}/s$	
	ft	ft	in²	in	in²/in	
1	0.000	1.166	-----	-----	------	*2
2	0.000	0.250	-----	-----	------	
	0.250	4.250	0.400	5.33	0.0750	
	4.250	4.500	-----	-----	------	

2.8.9. Beam Torsion Longitudinal Reinforcement Capacity in Terms of Required and Provided Area

Notes:
*2 - Torsion ignored (Tu < PhiTcr/4).

			Required				Provided	
Span	Start	End	X_u	T_u	Comb/Patt	$\boxed{A_l}$	A_l	
	ft	ft	ft	kip-ft		in²	in²	
1	0.000	1.166	0.00	0.00	U1/All	0.000	-----	*2
2	0.000	0.250	1.79	28.00	U1/All	$\boxed{1.265}$	-----	
	0.250	4.250	1.79	28.00	U1/All	1.265	1.860	
	4.250	4.500	0.00	0.00	U1/All	0.000	-----	*2

2.9. Slab Shear Capacity

Span	b	d	V_{ratio}	ΦV_c	V_u	X_u
	in	in		kips	kips	ft
1	--- Not checked ---					
2	--- Not checked ---					

2.10. Material TakeOff

2.10.1. Reinforcement in the Direction of Analysis

Top Bars	60.5	lb	<=>	10.68	lb/ft	<=>	9.154 lb/ft²
Bottom Bars	0.0	lb	<=>	0.00	lb/ft	<=>	0.000 lb/ft²
Torsion Bars	28.2	lb	<=>	4.97	lb/ft	<=>	4.260 lb/ft²
Stirrups	35.6	lb	<=>	6.29	lb/ft	<=>	5.390 lb/ft²
Total Steel	124.3	lb	<=>	21.94	lb/ft	<=>	18.804 lb/ft²
Concrete	13.2	ft³	<=>	2.33	ft³/ft	<=>	2.000 ft³/ft²

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Page | **10**
1/21/2026
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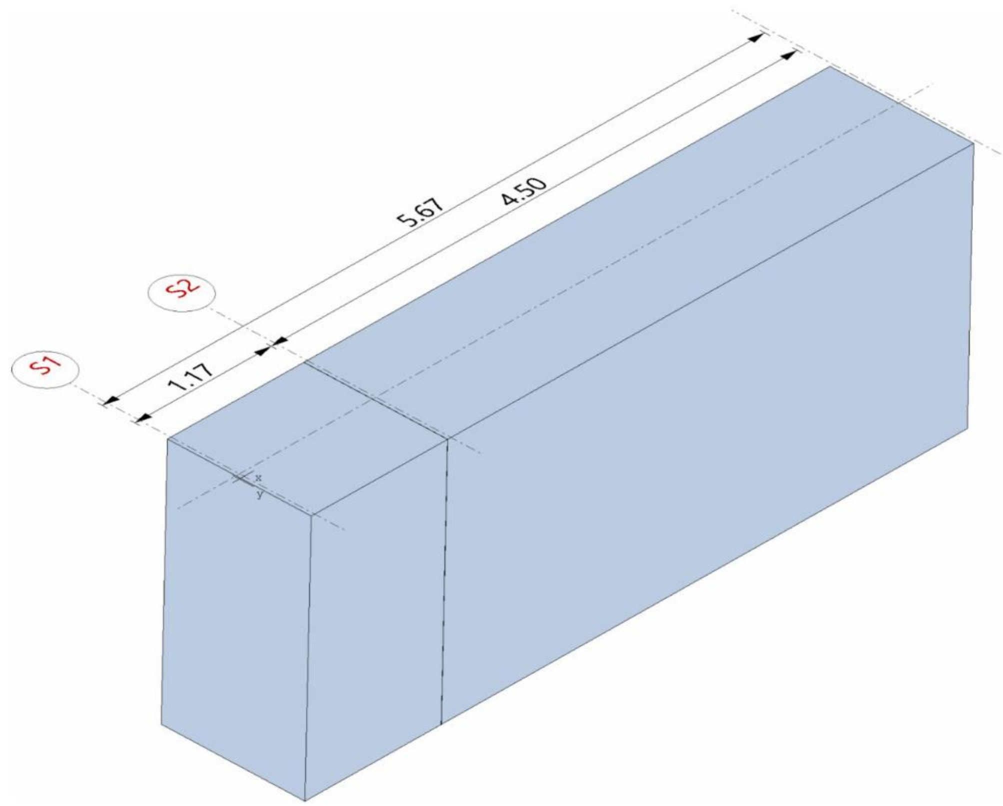
3. Screenshots

3.1. Extrude 3D view



Project:	Equilibrium Torsion
Diagram:	Model View (Extrude)
	Longitudinal beams

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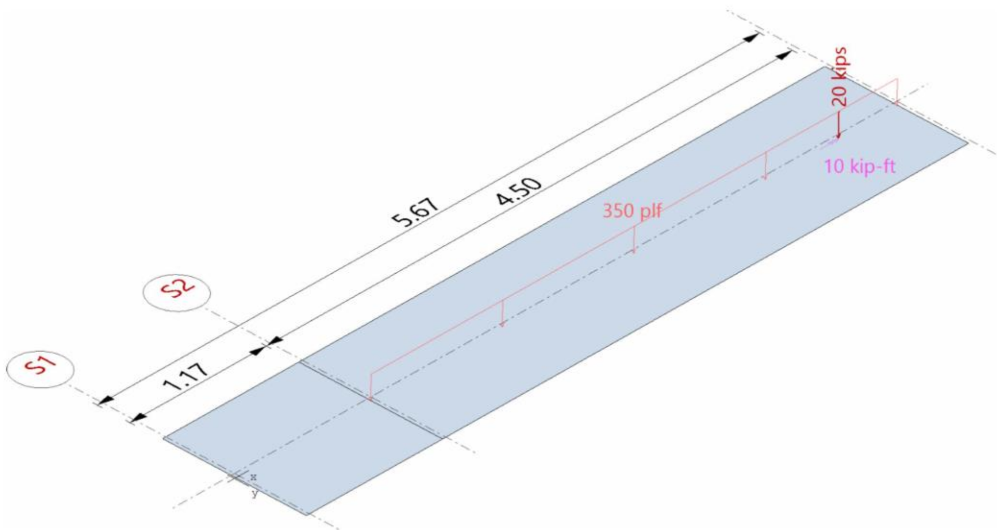
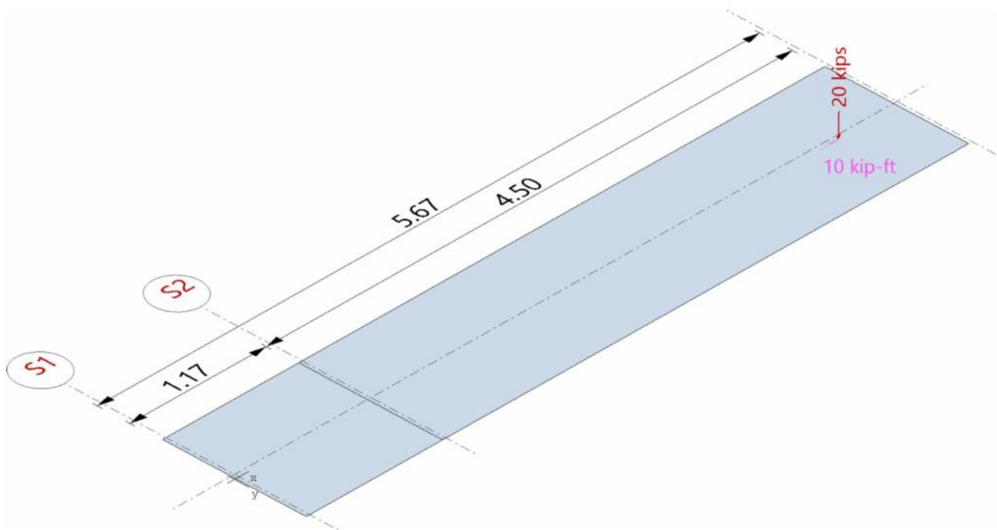
Page | 11
1/21/2026
2:21 PM

3.2. Loads - Case A - Dead



Project:	Equilibrium Torsion
Diagram:	Model View (Load Case: A - Dead)
	Longitudinal beams

3.3. Loads - Case B - Live



Project:	Equilibrium Torsion
Diagram:	Model View (Load Case: B - Live)
	Longitudinal beams

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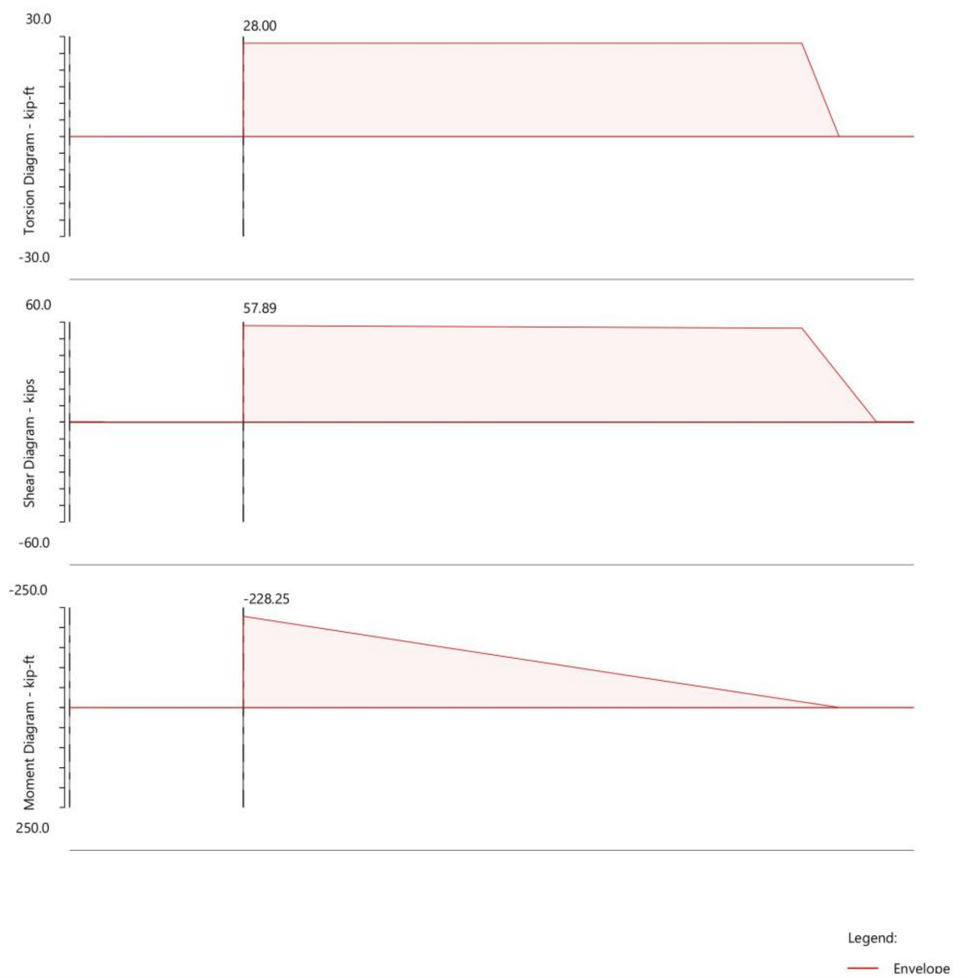
Page | 13
1/21/2026
2:21 PM

3.4. Internal Forces



Project: Equilibrium Torsion

Diagram: Internal Forces

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Page | 14
1/21/2026
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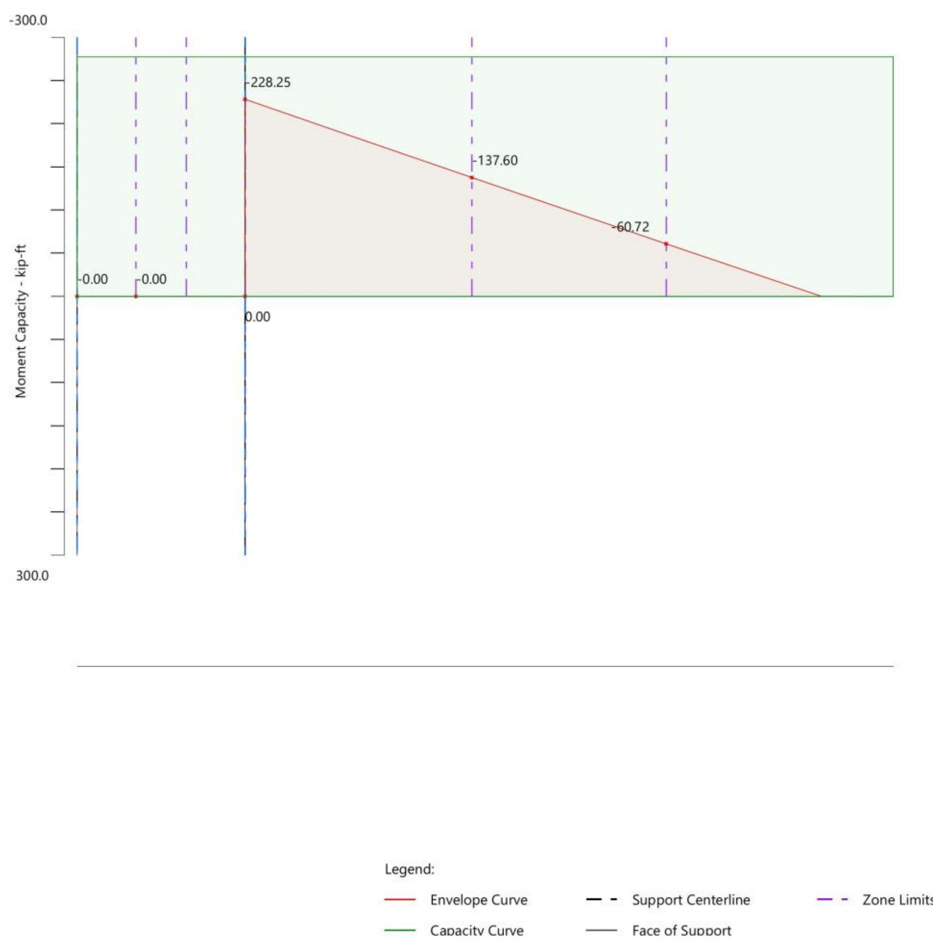
3.5. Moment Capacity



Project: Equilibrium Torsion

Diagram: Moment Capacity

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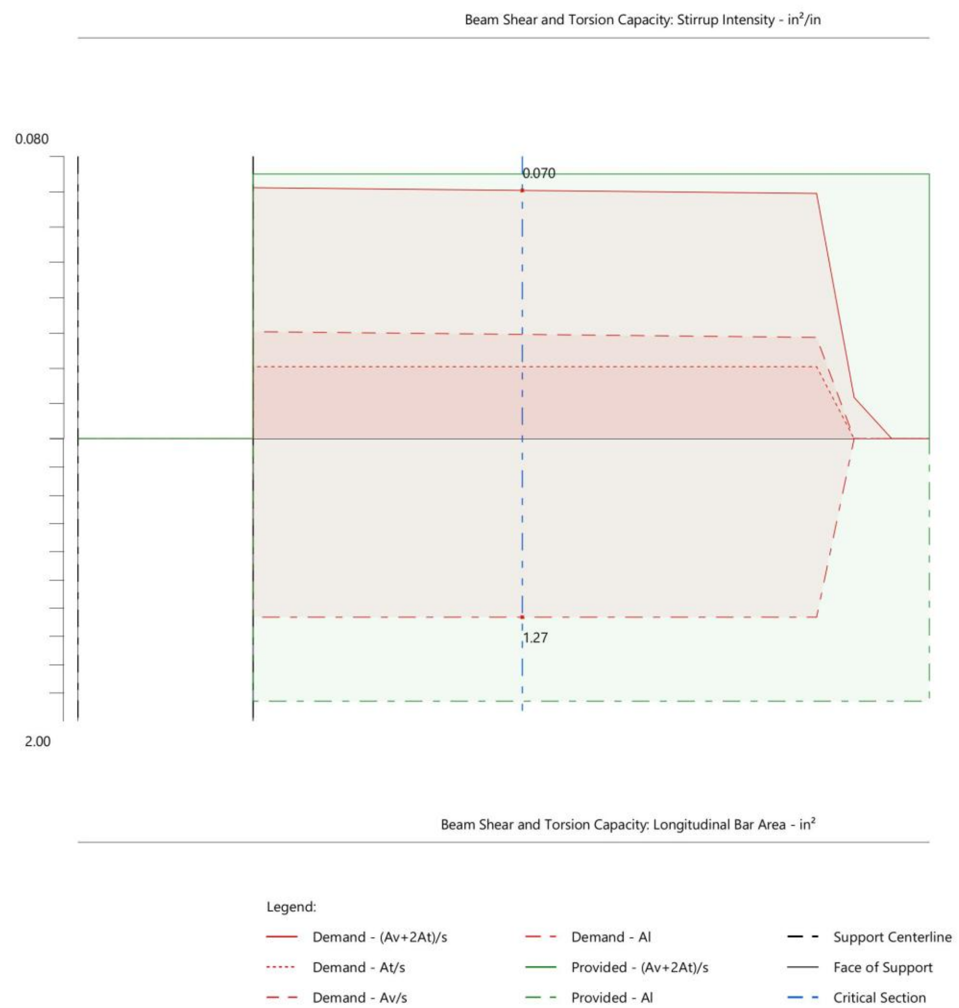
Page | 15
1/21/2026
2:21 PM

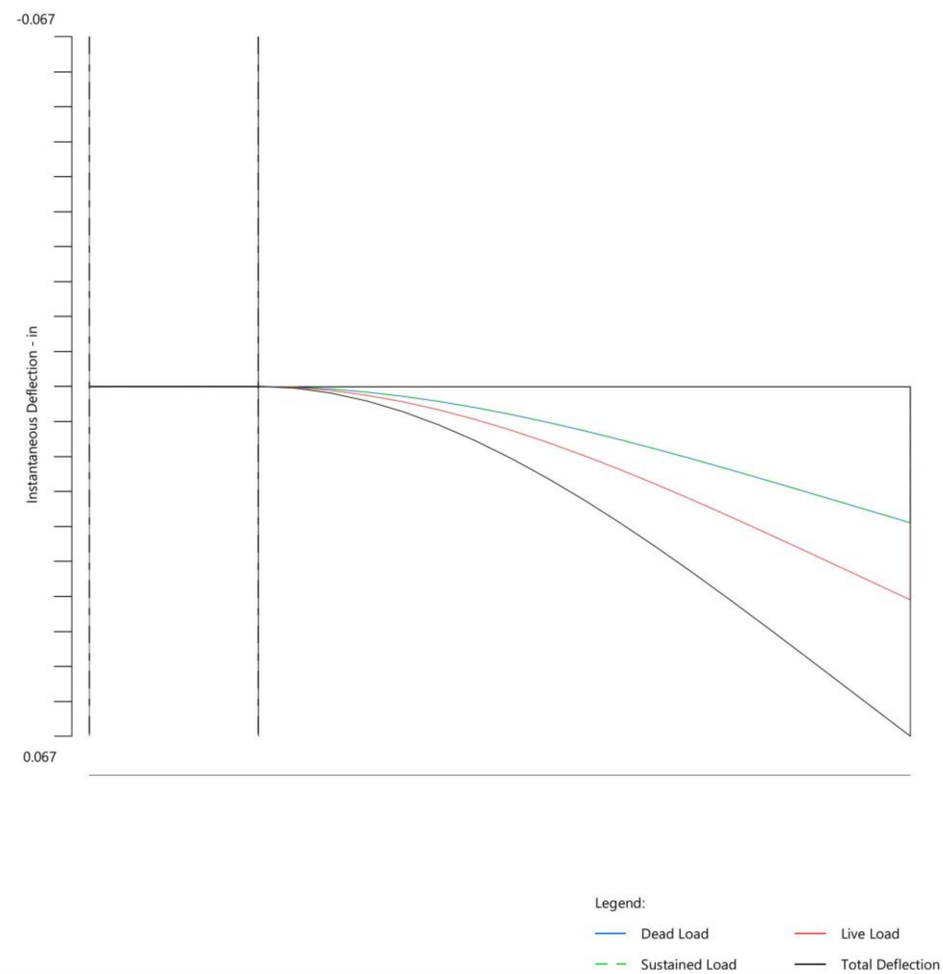
3.6. Shear Capacity



Project: Equilibrium Torsion

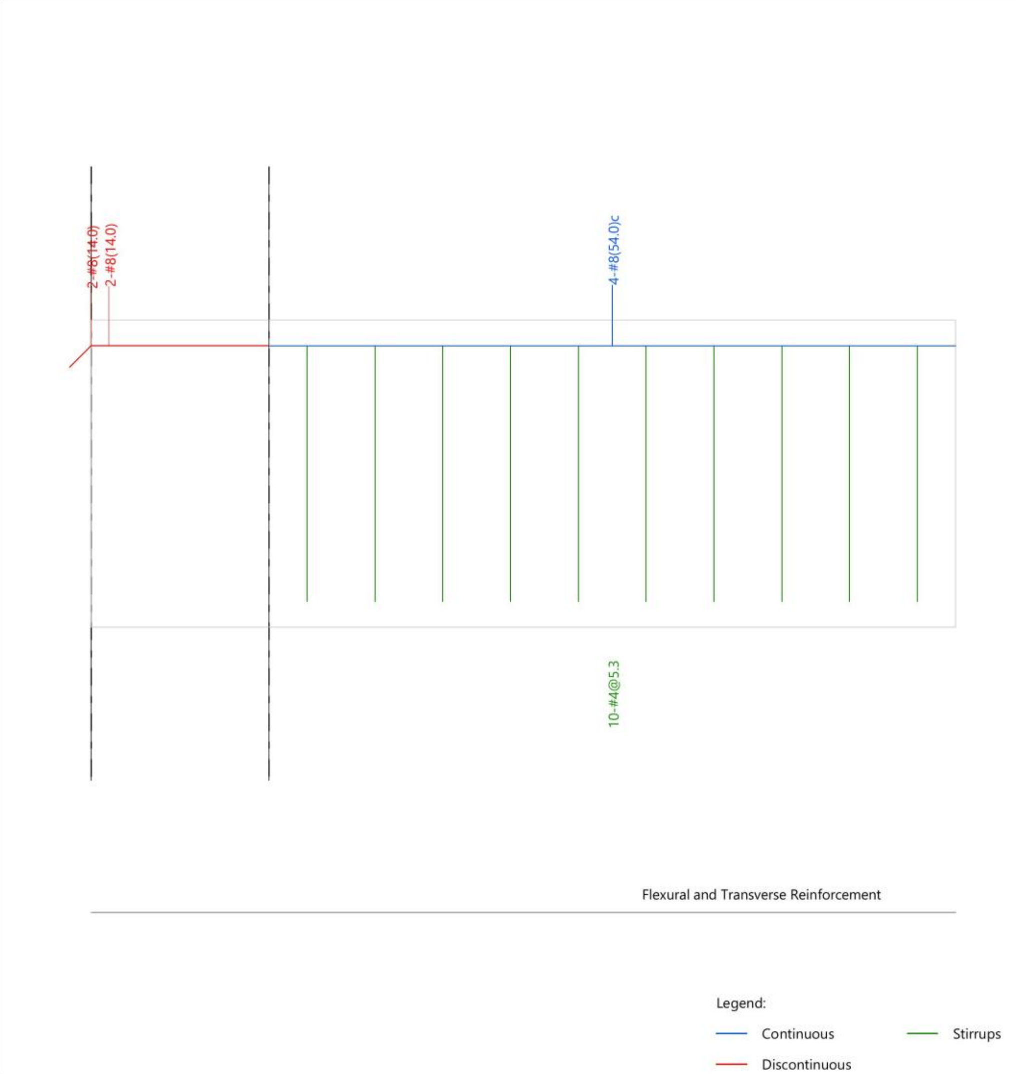
Diagram: Shear and Torsion Capacity

3.7. Deflection



Project:	Equilibrium Torsion
Diagram:	Deflection

3.8. Reinforcement



Project: Equilibrium Torsion

Diagram: Reinforcement

7. Analysis and Design Results Comparison and Conclusions

The following table show the comparison between hand results and spBeam model results.

Table 1 – Comparison of Hand Solution with spBeam Solution					
M_u (kip-ft)			T_u (kip-ft)		
Hand	**Reference**	**spBeam**	**Hand**	**Reference**	**spBeam**
228.25	228	228.25	28.00	28	28.00
V_u (kips)			A_t/s (in.^{2}/in. per leg)		
Hand	**Reference**	**spBeam**	**Hand**	**Reference**	**spBeam**
57.14	57.1	57.14	0.0204	0.0204	0.0204
A_v/s (in.^{2}/in.)			$(A_v+2A_t)/s$ (in.^{2}/in.)		
Hand	**Reference**	**spBeam**	**Hand**	**Reference**	**spBeam**
0.0295	0.0295	0.0295	0.0704	0.0703	0.0704
Required Longitudinal Reinforcement (in.^{2})			**Required Torsional Longitudinal Reinforcement (in.^{2})**		
Hand	**Reference**	**spBeam**	**Hand**	**Reference**	**spBeam**
2.550	2.56	2.550	1.265	1.27	1.265

The results of all the hand calculations and the reference used illustrated above are in precise agreement with the automated exact results obtained from the spBeam program.