One-Way Wide Module (Skip) Joist Concrete Floor System Design (ACI 318-14)
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A typical floor plan of a 5-story office building is shown below. Wide-module joists, or “skip” joists, are similar to standard one-way joists, except the pans are 53 in. or 66 in. wide. For the 53 in. pans, the pan depth varies from 16 in. to 24 in., and for 66 in. pans, the range is 14 in. to 24 in. Wide-module systems are economical for long spans with heavy loads and improved vibration resistance (see references). The gravity loads treatment is shown in this example and the lateral load effects are resisted by reinforced concrete shear walls. The design procedures shown in ACI 318-14 are illustrated in detail in this example. The hand solution is also used for a detailed comparison with the analysis and design results of the engineering software program spBeam.

Figure 1 – One-Way Wide Module Joist Concrete Floor Framing System
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Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)
Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10)

References

• Concrete Floor Systems (Guide to Estimating and Economizing), Second Edition, 2002 David A. Fanella
• spBeam Engineering Software Program Manual v5.50, STRUCTUREPOINT, 2018

Design Data

Floor-to-Floor Height = 12 ft (provided by architectural drawings)

\( w_c = 150 \text{ pcf} \)

\( f'c = 5,000 \text{ psi} \)

\( f_r = 60,000 \text{ psi} \) (For flexural reinforcement)

\( f_{rt} = 60,000 \text{ psi} \) (For shear and torsional reinforcement)

Superimposed dead load, \( SDL = 20 \text{ psf framed partitions, wood studs plaster 2 sides} \)

\( ASCE/SEI 7-10 \text{ (Table C3-1)} \)

Typical Floor Level, Live load, \( L_o = 80 \text{ psf (Office building)} \)

\( ASCE/SEI 7-10 \text{ (Table 4-1)} \)

Roof Live Load, \( L_o = 20 \text{ psf (Ordinary flat roofs)} \)

\( ASCE/SEI 7-10 \text{ (Table 4-1)} \)

Required fire resistance rating = 2 hours
1. Preliminary Member Sizing

1.1. Preliminary slab thickness and joist dimensions

In this example deflection will be calculated and checked to satisfy project deflection limits. Minimum member thickness and depths from ACI 318-14 will be used for preliminary sizing.

a) One-way Slab

Using minimum thickness for solid one-way slabs in Table 7.3.1.1 of ACI 318-14 (Table 7.3.1.1) for the solid slab spanning between the ribs.

End Span: \[ h_s = \frac{l}{24} = \frac{72}{24} = 3.00 \text{ in.} \] \( \text{ACI 318-14 (Table 7.3.1.1)} \)

Interior Span: \[ h_s = \frac{l}{28} = \frac{72}{28} = 2.57 \text{ in.} \] \( \text{ACI 318-14 (Table 7.3.1.1)} \)

The minimum slab thickness for wide-module joists for 2-hour fire rating is 4.60 in. IBC 2012 (Table 720.1(3))

Therefore, select a slab thickness of 5 in. for all spans.

b) One-way Joist

The wide-module joist systems do not meet the limitations of ACI 318-14, 9.8.1.1 through 9.8.1.4. Therefore, the structural members of this type of joist construction shall be designed with standard provisions for slabs and beams. ACI 318-14 (9.8.1.8)

Using minimum thickness for non-prestressed beams in Table 9.3.1.1. For the ribs (part of the joists) supporting the solid slab.

End Span: \[ h = \frac{l}{18.5} = \frac{384}{18.5} = 20.76 \text{ in. (governs)} \] \( \text{ACI 318-14 (Table 9.3.1.1)} \)

Interior Span: \[ h = \frac{l}{21} = \frac{384}{21} = 18.29 \text{ in.} \] \( \text{ACI 318-14 (Table 9.3.1.1)} \)

Therefore, select rib depth of 16 in. for a total joist depth of 21 in.
Figure 2 – Slab and Joist Dimensions
1.2. Preliminary column sizes

a) Interior Columns

Select a preliminary size based on the axial load demand. Determine interior column loads as follows:

The governing load combination: \( U = 1.20 \times D + 1.60 \times L + 0.50 \times L_r \)  \hspace{1cm} \textit{ACI 318-14 (Eq. 5.3.1b)}

Where:
\( D = \) Dead Load, \( L = \) Live Load, \( L_r = \) Roof Live Load

Typical Floor Level Loads

# of Floors = 4
Dead Loads, \( D \)

Self-weight of wide-module joist system (see Figure 2):

\[
\text{Joist average thickness} = \frac{2 \times A_j + A_{\text{tot}}}{\text{Total Width}} = \frac{2 \times \left( \frac{3 + 4.33}{2} \times 16 \right) + 5 \times 72}{72} = 6.63 \text{ in.} = 0.55 \text{ ft}
\]

Weight of the joist = 0.55 \times 150 \text{ pcf} = 82.88 \text{ psf}.

Superimposed dead load = 20 psf

Live Load, \( L \): Calculate the live load reduction per ASCE/SEI 7-10

\[
L = L_o \times \left( 0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \hspace{1cm} \textit{ASCE/SEI 7-10 (Eq. 4-1)}
\]

Where:
\( L \) = Reduced design live load per ft\(^2\) of area supported by the member
\( L_o \) = Unreduced design live load per ft\(^2\) of area supported by the member = 80 psf
\( K_{LL} \) = Live load element factor = 4  \hspace{1cm} \textit{ASCE/SEI 7-10 (Table 4-2)}
\( A_T \) = Tributary area = (30 \times 32) = 960 ft\(^2\)

\[
L = 80 \times \left( 0.25 + \frac{15}{\sqrt{4 \times 960}} \right) = 39.36 \text{ psf}
\]

Which satisfies 0.40 \times L_o requirement for members supporting two or more floors.  \hspace{1cm} \textit{ASCE/SEI 7-10 (4.7.2)}
Roof Level Loads

Dead Loads, $D$

Self-weight of wide-module joist system (see Figure 2):

\[
\text{Joist average thickness} = \frac{2 \times A_t + A_v}{\text{Total Width}} = \frac{2 \times \left(\frac{3 + 4.33}{2} \times 16\right) + 5 \times 72}{72} = 6.63 \text{ in.} = 0.55 \text{ ft}
\]

Weight of the joist = $0.55 \times 150 \text{ pcf} = 82.88 \text{ psf}$.

No superimposed dead load at the roof.

Roof Live Load, $L_r$: Calculate the roof live load reduction

\[
L_r = L_o \times R_1 \times R_2; \quad 12 \leq L_r \leq 20
\]

Where:

\[
L_o = 20 \text{ psf}
\]

\[
R_1 = 0.60 \text{ since } A_T = 960 \text{ ft}^2 \geq 600 \text{ ft}^2
\]

\[
R_2 = 1.00 \text{ for flat roof}
\]

\[
L_r = 20 \times 0.60 \times 1.00 = 12.00 \text{ psf}
\]

Total Factored Load on 1st story interior column (@ 1st interior support)

Total Floor Load = \[4 \times [1.20 \times (82.88 + 20) + 1.60 \times 39.36] \times 960 = 715,906.05 \text{ lb} = 715.91 \text{ kips}\]

Total Roof Load = \[1.20 \times 82.88 + 1.60 \times 12.00 \] \times 960 = 113,904.00 \text{ lb} = 113.90 \text{ kips}\]

Assume 24 in square column with 4 – #11 vertical bars with design axial strength, $\phi P_{n,max}$ of

\[
\phi P_{n,max} = 0.80 \times \phi \left[ 0.85 \times f'_c \times \left( A_y - A_n \right) + f'_y \times A_n \right]
\]

\[
\phi P_{n,max} = 0.80 \times 0.65 \times 0.85 \times 5,000 \times \left( \left( 24 \times 24 - 4 \times 1.56 \right) + 60,000 \times 4 \times 1.56 \right) = 1,453,857.60 \text{ lb}
\]

\[
\phi P_{n,max} = 1,453.86 \text{ kips}
\]

Column Self-weight = \[1.20 \times \left( \frac{24 \times 24}{144} \right) \times 0.15 \times (5 \times 12) = 43.20 \text{ kips}\]

Total Reaction @ 1st interior support = \[1.15 \times (715.91 + 113.90) + 43.20 = 997.48 \text{ kips} < 1,453.86 \text{ kips}\]

Therefore, the preliminary interior column size of 24 in. × 24 in. is adequate.
b) **Edge (Exterior) Columns**

Select a preliminary size based on the axial load demand. Therefore, the load take-down for an edge column is done as follows:

The governing load combination: \[ U = 1.20 \times D + 1.60 \times L + 0.50 \times L_r \]  

*ACI 318-14 (Eq. 5.3.1b)*

**Typical Floor Level Loads**

# of Floors = 4

Dead Loads, \( D \)

Self-weight of wide-module joist system (see Figure 2):

Weight of the joist = 82.88 psf

Superimposed dead load = 20 psf

Live Load, \( L \): Calculate the live load reduction per *ASCE/SEI 7-10*

\[
L = L_o \times \left( 0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right)
\]

*ASCE/SEI 7-10 (Eq. 4-1)*

Where:

\( L \) = Reduced design live load per ft\(^2\) of area supported by the member

\( L_o \) = Unreduced design live load per ft\(^2\) of area supported by the member = 80 psf

\( K_{LL} \) = Live load element factor = 4

\( A_T \) = Tributary area = \( \frac{30 \times 32}{2} \) = 480 ft\(^2\)

\[
L = 80 \times \left( 0.25 + \frac{15}{\sqrt{4 \times 480}} \right) = 47.39 \text{ psf}
\]

Which satisfies 0.40 \( L_o \) requirement for members supporting two or more floors. *ASCE/SEI 7-10 (4.7.2)*
Roof Level Loads

Dead Loads, $D$

Weight of the joist = 82.88 psf

No superimposed dead load at the roof

Roof Live Load, $L_r$: Calculate the roof live load reduction

$$L_r = L_o \times R_1 \times R_2; \quad 12 \leq L_r \leq 20$$  \hspace{1cm} \text{ASCE/SEI 7-10 (Eq 4-2)}

Where:

$L_o = 20$ psf

$R_1 = 1.20 - 0.001 \times A_f = 1.20 - 0.001 \times 480 = 0.72$, since $200 \text{ ft}^2 < A_f = 480 \text{ ft}^2 < 600 \text{ ft}^2$

$R_2 = 1.00$ for flat roof

$L_r = 20 \times 0.72 \times 1.00 = 14.40$ psf

Total Factored Load on 1st story edge column (@ 1st interior support)

Total Floor Load = $4 \times [1.20 \times (82.88 + 20) + 1.60 \times 47.39] \times 480 = 382,594.18 \text{ lb} = 382.59 \text{ kips}$

Total Roof Load = $[1.20 \times 82.88 + 1.60 \times 14.40] \times 480 = 58,795.20 \text{ lb} = 58.80 \text{ kips}$

Assume 20 in square column with 4 – #11 vertical bars with design axial strength, $\phi P_{n,\max}$ of

$$\phi P_{n,\max} = 0.80 \times \phi \times \left[0.85 \times f_y \times \left(A_y - A_{n}\right) + f_y \times A_{n}\right]$$  \hspace{1cm} \text{ACI 318-14 (22.4.2)}

$$\phi P_{n,\max} = 0.80 \times 0.65 \times \left[0.85 \times 5,000 \times ((20 \times 20 - 4 \times 1.56) + 60,000 \times 4 \times 1.56)\right] = 1,064,897.60 \text{ lb}$$

$\phi P_{n,\max} = 1,064.90$ kips

Column Self-weight = $1.20 \times \left(\frac{20 \times 20}{144}\right) \times 0.15 \times (5 \times 12) = 30.00$ kips

Total Reaction @ 1st interior support = $1.15 \times (382.59 + 58.80) + 30.00 = 537.60$ kips $< 1,064.90$ kips

Therefore, the preliminary edge column size of 20 in. $\times$ 20 in. is adequate.
c) **Corner Columns**

Select a preliminary size based on the axial load demand. Therefore, the load take-down for a corner column is done as follows:

The governing load combination: \( U = 1.20 \times D + 1.60 \times L + 0.50 \times L_r \) \textit{ACI 318-14 (Eq. 5.3.1b)}

**Typical Floor Level Loads**

# of Floors = 4

Dead Loads, \( D \)

Self-weight of wide-module joist system (see Figure 2):

Weight of the joist = 82.88 psf

Superimposed dead load = 20 psf

Live Load, \( L \): Calculate the live load reduction per \textit{ASCE/SEI 7-10}

\[
L = L_o \times \left( 0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \quad \textit{ASCE/SEI 7-10 (Eq. 4-1)}
\]

Where:

\( L \) = Reduced design live load per ft\(^2\) of area supported by the member

\( L_o \) = Unreduced design live load per ft\(^2\) of area supported by the member = 80 psf

\( K_{LL} \) = Live load element factor = 4 \textit{ASCE/SEI 7-10 (Table 4-2)}

\( A_T \) = Tributary area = \( \frac{30 \times 32}{4} \) = 240 ft\(^2\)

\[
L = 80 \times \left( 0.25 + \frac{15}{\sqrt{4 \times 240}} \right) = 58.73 \text{ psf}
\]

Which satisfies 0.40 \( L_o \) requirement for members supporting two or more floors. \textit{ASCE/SEI 7-10 (4.7.2)}
Roof Level Loads

Dead Loads, D

Weight of the joist = 82.88 psf

No superimposed dead load at the roof

Roof Live Load, \(L_r\): Calculate the roof live load reduction

\[
L_r = L_o \times R_1 \times R_2; \quad 12 \leq L_r \leq 20
\]

\[ASCE/SEI \ 7-10 \ (Eq \ 4-2)\]

Where:

\(L_o = 20 \text{ psf}\)

\[
R_1 = 1.20 - 0.001 \times A_T = 1.20 - 0.001 \times 240 = 0.96 \text{ since } 200 \text{ ft}^2 < A_T = 240 \text{ ft}^2 < 600 \text{ ft}^2
\]

\(R_2 = 1.00 \text{ for flat roof}\)

\(L_r = 20 \times 0.96 \times 1.00 = 19.20 \text{ psf}\)

Total Factored Load on 1st story corner column (@ exterior support)

Total Floor Load = \(4 \times [1.20 \times (82.88 + 20) + 1.60 \times 59.73] \times 240 = 208,721.02 \text{ lb} = 208.72 \text{ kips}\)

Total Roof Load = \([1.20 \times 82.88 + 1.60 \times 19.20] \times 240 = 31,240.80 \text{ lb} = 31.24 \text{ kips}\)

Assume 20 in square column with 4 – #11 vertical bars with design axial strength, \(\phi P_{n,max}\) of

\[
\phi P_{n,max} = 0.80 \times \phi \times \left[ 0.85 \times f_y \times (A_g - A_g) + f_y \times A_g \right]
\]

\[ACI \ 318-14 \ (22.4.2)\]

\[
\phi P_{n,max} = 0.80 \times 0.65 \times \left[ 0.85 \times 5,000 \times \left(20 \times 20 - 4 \times 1.56\right) + 60,000 \times 4 \times 1.56 \right] = 1,064,897.60 \text{ lb}
\]

\[
\phi P_{n,max} = 1,064.90 \text{ kips}
\]

Column Self-weight = \(1.20 \times \left( \frac{20 \times 20}{144} \right) \times 0.15 \times (5 \times 12) = 30.00 \text{ kips}\)

Total Reaction @ 1st interior support = 208.72 + 31.24 + 30.00 = 269.96 kips < 1,064.90 kips

Therefore, the preliminary edge column size of 20 in. × 20 in. is adequate.
2. Design of Structural Members

The design of the following structural members is performed and compared with results of the engineering software program spBeam:

2.1. One-Way Slab
2.2. One-Way Joist
2.3. Interior Beam
2.4. Exterior Beam
2.5. Interior Column

2.1. One-Way Slab Design

A unit strip of 1 ft is considered for the design of slab spanning between ribs. Note that *ACI 318-14* does not allow live load reduction for one-way slabs.

![Figure 3 – Partial Plan View Illustrating Slab Design Strip](image)

Slab design involves the following steps:

2.1.1. Determination of span loads
2.1.2. Determination of design moments and shears
2.1.3. Flexural Design

2.1.4. Shear Design

2.1.5. Deflections

2.1.6. Computer Program Solution

2.1.7. Summary and comparison of design results

2.1.8. Conclusions and observations

2.1.1. Determination of span loads

The following gravity load combinations are considered:

\[
U = 1.40 \times D
\]  
\[
w_u = 1.40 \times \left( \frac{5}{12} \times 0.15 \right) + 0.02 = 0.12 \text{ kips/ft per ft}
\]  
\[
U = 1.20 \times D + 1.60 \times L
\]  
\[
w_u = 1.20 \times \left( \frac{5}{12} \times 0.15 \right) + 0.02 + 1.60 \times 0.08 = 0.23 \text{ kips/ft per ft}
\]  

Span loads are governed by the second load combination.

2.1.2. Determination of design moment and shear

The factored moment and shear can be determined using the simplified method if the requirements are satisfied:

\[
ACI 318-14 \ (6.5.1)
\]

- Members are prismatic.
- Loads are uniformly distributed.
- \( L \leq 3D (0.08 \text{ kips/ft per ft} \leq 3 \times 0.0825 \text{ kips/ft per ft}) \)
- There are at least two spans.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

Thus, the approximate coefficients can be used. The factored moments and shears are determined and summarized in the following tables.

\[
ACI 318-14 \ (Table \ 6.5.2 \ and \ Table \ 6.5.3)
\]
### Table 1 – One-Way Slab Design Moment Values

<table>
<thead>
<tr>
<th>Location</th>
<th>Design Moment Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>End Spans</strong></td>
<td></td>
</tr>
<tr>
<td>Exterior Support Negative</td>
<td>( \frac{w_u \times L_u^2}{24} = \frac{0.23 \times 5.50^2}{24} = 0.29 \text{ ft-kips/ft} )</td>
</tr>
<tr>
<td>Mid-span</td>
<td>( \frac{w_u \times L_u^2}{14} = \frac{0.23 \times 5.50^2}{14} = 0.49 \text{ ft-kips/ft} )</td>
</tr>
<tr>
<td>Interior Support Negative</td>
<td>( \frac{w_u \times L_u^2}{10} = \frac{0.23 \times 5.50^2}{10} = 0.69 \text{ ft-kips/ft} )</td>
</tr>
<tr>
<td><strong>Interior Spans</strong></td>
<td></td>
</tr>
<tr>
<td>Mid-span Positive</td>
<td>( \frac{w_u \times L_u^2}{16} = \frac{0.23 \times 5.50^2}{16} = 0.43 \text{ ft-kips/ft} )</td>
</tr>
<tr>
<td>Support Negative</td>
<td>( \frac{w_u \times L_u^2}{11} = \frac{0.23 \times 5.50^2}{11} = 0.62 \text{ ft-kips/ft} )</td>
</tr>
</tbody>
</table>

### Table 2 – One-Way Slab Design Shear Values

<table>
<thead>
<tr>
<th>Location</th>
<th>Design Shear Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Span at Face of First Interior Support</td>
<td>( 1.15 \times \frac{w_u \times L_u}{2} = 1.15 \times \frac{0.23 \times 5.50}{2} = 0.72 \text{ kips/ft} )</td>
</tr>
<tr>
<td>At Face of all other Supports</td>
<td>( \frac{w_u \times L_u}{2} = \frac{0.23 \times 5.50}{2} = 0.62 \text{ kips/ft} )</td>
</tr>
</tbody>
</table>
2.1.3. Flexural Design

For the one-way slab of a wide-module joist system, a single layer of longitudinal reinforcement is provided. The first interior support negative moment governs the design as tabulated in Table 1. Therefore, it is favorable to place the single layer reinforcement closer to the top fiber of the concrete slab. The required reinforcement shall be calculated for the first interior support negative moment first. The required reinforcement for the end span positive moment shall also be calculated as the low effective depth due to the reinforcement location may govern the required reinforcement amount. Finally, the required reinforcement for design shall be checked against the minimum shrinkage and temperature reinforcement requirement per ACI 318-14 (24.4.3.2).

Calculate the required reinforcement to resist the first interior support negative moment:

\[ M_u = 0.69 \text{ ft-kips/ft} \]

Use welded wire fabric reinforcement, 6 × 6-W5.5 × W5.5 with 1.5 in. concrete cover. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, \( d \), is calculated below:

\[ d = 5.00 - 1.50 - \frac{0.26}{2} = 3.37 \text{ in.} \]

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (\( jd \)). In this example, tension-controlled section will be assumed so the reduction factor \( \phi \) is equal to 0.9, and \( jd \) will be taken equal to 0.992 \times d. The assumptions will be verified once the area of steel is finalized.

Assume \( jd = 0.992 \times d = 0.992 \times 3.37 = 3.34 \text{ in.} \)

Unit strip width, \( b = 12 \text{ in.} \)

\[ A_s = \frac{M_u}{\phi \times f_v \times jd} = \frac{0.69 \times 12,000}{0.90 \times 60,000 \times 3.34} = 0.046 \text{ in}^2/\text{ft} \]

Recalculate ‘a’ for the actual \( A_s = 0.046 \text{ in}^2/\text{ft} \):

\[ a = \frac{A_s \times f_v}{0.85 \times f'_c \times b} = \frac{0.046 \times 60,000}{0.85 \times 5,000 \times 12} = 0.054 \text{ in.} \]

\[ c = \frac{a}{\beta_t} = \frac{0.054}{0.85} = 0.063 \text{ in.} \]

\[ \epsilon_i = \left( \frac{0.003}{c} \right) \times d - 0.003 = \left( \frac{0.003}{0.063} \right) \times 3.37 - 0.003 = 0.157 > 0.005 \]
Therefore, the assumption that section is tension-controlled is valid.

\[
A_t = \frac{M_u}{\phi \times f_y \times \left( d - \frac{a}{2} \right)} = \frac{0.69 \times 12,000}{0.90 \times 60,000 \times \left( 3.37 - \frac{0.054}{2} \right)} = 0.046 \text{ ft}^2
\]

Calculate the required reinforcement to resist the positive moment:

\[M_u = 0.49 \text{ ft-kips/ft}\]

The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement:

\[d = 5.00 - 3.37 = 1.63 \text{ in.}\]

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (\(j_d\)). In this example, tension-controlled section will be assumed so the reduction factor \(\phi\) is equal to 0.9, and \(j_d\) will be taken equal to 0.975 \(\times d\). The assumptions will be verified once the area of steel is finalized.

Assume \(j_d = 0.975 \times d = 0.975 \times 1.63 = 1.59 \text{ in.}\)

Unit strip width, \(b = 12 \text{ in.}\)

\[
A_t = \frac{M_u}{\phi \times f_y \times j_d} = \frac{0.49 \times 12,000}{0.90 \times 60,000 \times 1.59} = 0.068 \text{ in}^2
\]

Recalculate ‘a’ for the actual \(A_s = 0.068 \text{ in}^2\):

\[
a = \frac{A_t \times f_y}{0.85 \times f'_c \times b} = \frac{0.070 \times 60,000}{0.85 \times 5,000 \times 12} = 0.081 \text{ in.}
\]

\[
c = \frac{a}{\beta_t} = \frac{0.081}{0.85} = 0.095 \text{ in.}
\]

\[
e_t = \left( \frac{0.003}{c} \right) \times d_t - 0.003 = \left( \frac{0.003}{0.095} \right) \times 1.63 - 0.003 = 0.049 > 0.005
\]

Therefore, the assumption that section is tension-controlled is valid.

\[
A_t = \frac{M_u}{\phi \times f_y \times \left( d - \frac{a}{2} \right)} = \frac{0.49 \times 12,000}{0.90 \times 60,000 \times \left( 1.63 - \frac{0.081}{2} \right)} = 0.068 \text{ ft}^2
\]
Check the shrinkage and temperature reinforcement requirement:

\[ A_t = 0.0018 \times b \times h = 0.0018 \times 12 \times 5 = 0.108 \text{ in}^2 \frac{\text{in}}{\text{ft}} \geq 0.0014 \text{ in}^2 \frac{\text{in}}{\text{ft}} \quad AI318-14 \text{ (Table 24.4.3.2)}

Check reinforcement spacing for crack control:

The maximum spacing of the flexural reinforcement closest to the tension face of the slab shall be:

\[ s = 15 \times \left( \frac{40,000}{f_y} \right) - 2.5 \times c_c, \text{ but not greater than } 12 \times \left( \frac{40,000}{f_y} \right) \quad AI318-14 \text{ (Table 24.3.2)}

Where:

\[ s = \text{ Maximum reinforcement spacing for crack control, in.} \]
\[ f_y = \text{ Calculated stress in reinforcement closest to the tension face at service load, ksi} \]
\[ c_c = \text{ The least distance from surface of reinforcement to the tension face, in.} \]

Use \( f_y = \frac{2}{3} \times f_{c'} = \frac{2}{3} \times 60,000 = 40,000 \text{ psi} \quad AI318-14 \text{ (24.3.2.1)}

\[ c_c = 1.50 \text{ in. for reinforcement resisting negative moment at supports (i.e. tension at the top)} \]
\[ c_c = 3.24 \text{ in. for reinforcement resisting positive moment at mid-span (i.e. tension at the bottom)} \]

Thus,

At supports

\[ s = 15 \times \left( \frac{40,000}{f_y} \right) - 2.5 \times c_c = 15 \times \left( \frac{40,000}{40,000} \right) - 2.5 \times 1.50 = 11.25 \text{ in. (governs @ support)} \]

But not greater than \( s = 12 \times \left( \frac{40,000}{f_y} \right) = 12 \times \left( \frac{40,000}{40,000} \right) = 12 \text{ in.} \)

At mid-span

\[ s = 15 \times \left( \frac{40,000}{f_y} \right) - 2.5 \times c_c = 15 \times \left( \frac{40,000}{40,000} \right) - 2.5 \times 3.24 = 6.90 \text{ in. (governs @ mid-span)} \]

But not greater than \( s = 12 \times \left( \frac{40,000}{f_y} \right) = 12 \times \left( \frac{40,000}{40,000} \right) = 12 \text{ in.} \)
Therefore, for this one-way slab, the shrinkage and temperature reinforcement requirement per *ACI 318-14 (Table 24.4.3.2)* governs the required reinforcement area ($A_s = 0.108 \text{ in}^2/\text{ft}$) and crack control requirement per *ACI 318-14 (Table 24.3.2)* governs the reinforcement spacing ($s = 6.90 \text{ in}$).

The most feasible reinforcement solution that meets both requirements mentioned above is to provide welded wire fabric reinforcement, $6 \times 6 - W5.5 \times W5.5$. Note that the welded wire reinforcement selected provides minimum shrinkage and temperature reinforcement in the slab direction parallel to the joists as well. Alternately, deformed bars can be utilized in lieu of welded wire fabric. It should be noted that two conditions specific to this design contribute to having such a stringent spacing requirement.

These are listed below:

- The 5 in. slab has a single layer reinforcement that is placed near the top surface (i.e. clear cover from the top surface to the reinforcement is 1.50 in). This result in a high $c_c$ value for the calculation of reinforcement spacing for crack control due to positive moment.

- The stress in reinforcement closest to the tension face at service load, $f_s$, is taken as $2/3 f_y$ as permitted by *ACI 318-14* without calculation. It is very likely that under the loading considered, the stress in the steel will be lower than $2/3 f_y$. The $f_s$ value is expected to be in the range of $1/3 f_y$ to $1/2 f_y$. Even if it is assumed to be $1/2 f_y$, $s$ value will be 12 in.
2.1.4. Shear Design

From Table 2 above, the shear value in end span at face of first interior support governs.

\[ V_u = 1.15 \times \frac{w_u \times l_u}{2} = 1.15 \times \frac{0.23 \times 5.50}{2} = 0.72 \text{ kips/ft} \]

The design shear at a distance, \( d \), away from the face of support,

\[ V_u = 0.72 - 0.23 \times \frac{1.63}{12} = 0.69 \text{ kips/ft} \]

Shear strength provided by concrete

\[ \phi V_c = \phi \times \left( 2 \times \frac{\sqrt{f'c}}{b_u} \times d \right) \quad \text{ACI 318-14 (Eq. 22.5.5.1)} \]

\[ \phi V_c = 0.75 \times \left( 2.00 \times 1.00 \times \sqrt{5000} \times 12 \times 1.63 \right) = 2077.20 \text{ lb/ft} = 2.08 \text{ kips/ft} \]

\[ V_u < \phi V_c = 2.08 \text{ kips/ft} \]

Therefore, the slab shear capacity is adequate.

2.1.5. Deflections

Since the preliminary slab thickness met minimum thickness requirement, the deflection calculations are not required. Unless governed by fire rating requirements; as in this example; lesser thicknesses and consequently cost savings can be achieved through deflection computations. Deflection values are calculated and provided for every model created by spBeam Program and can be used by the engineer to make additional optimization decisions.
2.1.6. Computer Program Solution

**spBeam** Program can be utilized to analysis and design beams and one-way slab systems. The one-way slab is modeled as 1-ft unit strip supported on ribs. The ribs provide some rotational stiffness at the supports. In **spBeam** solution, the rotational stiffness is assumed as 32,000 \(^*\) kip-in/rad for modeling the joist supports. Also, for one-way slab run, the rib widths assumed as 6 in. and modeled through dummy columns of 6 in. × 12 in. with zero height (i.e. column stiffness is zero, but the 6 in. dimension of the column is utilized to push the design moments 3 in. from the support centerline). In this example, user-defined bar size #2 is defined in **spBeam** to represent welded wire fabric, W5.5, with the cross-sectional area of 0.055 in\(^2\) (see Figure 4).

![Figure 4 – spBeam Reinforcement Database – User-defined Bar Set](image)

The program calculates the internal forces (shear force and bending moment), moment and shear capacities, immediate and long-term deflections, and required reinforcements. The graphical and text results are provided below for input and output of the **spBeam** program. The graphical and text results are provided here for both input and output of the **spBeam** model.

* Refer to **spBeam** manual (Chapter 2 – Special Considerations for One and Two-Way Joist Systems)
* Refer to **spBeam** manual (Chapter 4 – Defining Boundary Conditions, Rotational Stiffness)
Figure 5 – spBeam Model – Isometric View of 15 Span – 1ft Wide Unit Strip of One-Way Slab
Figure 6 – spBeam Model – Loads (Including Live Load Patterning) units in lb/ft²
Figure 7 – spBeam Model – Internal Forces (Shear Force Diagram and Bending Moment Diagram)
Figure 8 – spBeam Model – Moment Capacity Diagram
Figure 9 – spBeam Model – Shear Capacity Diagram
Figure 10 – spBeam Model – Immediate Deflection Diagram
Figure 11 – *spBeam* Model – Reinforcement Diagram
### 2.1.7. Summary and Comparison of Results

#### 1. Design Results

#### 1.1. Top Reinforcement

Notes:
* "3" - Design governed by minimum reinforcement.

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### 1.2. Bottom Reinforcement

Notes:
* 3 - Design governed by minimum reinforcement.

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<td>0.058</td>
<td>6.000</td>
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<td>1.00</td>
<td>0.43</td>
<td>3.000</td>
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<td>0.061</td>
<td>6.000</td>
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<td>6.000</td>
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<td>6.000</td>
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</tr>
<tr>
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<td>3.000</td>
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### 1.3. Slab Shear Capacity

<table>
<thead>
<tr>
<th>Span</th>
<th>$b$</th>
<th>$d$</th>
<th>$V_{\text{net}}$</th>
<th>$\Phi V_s$</th>
<th>$V_s$</th>
<th>$X_s$</th>
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<tr>
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<td>1.000</td>
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<td>0.63</td>
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<td>2.08</td>
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</table>
### Table 3 - Comparison of Hand Solution with spBeam Solution for One-Way Slab

#### Flexural Design

<table>
<thead>
<tr>
<th>Span Location</th>
<th>Design Moment (ft-kips/ft)</th>
<th>Reinforcement Required for Flexure (in²/ft)</th>
<th>Minimum Reinforcement (in²/ft) (Shrinkage &amp; Temperature Reinforcement)</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Span</td>
<td>Hand</td>
<td>spBeam</td>
<td>Hand</td>
</tr>
<tr>
<td>Interior Negative</td>
<td>0.69</td>
<td>0.63</td>
<td>0.046</td>
</tr>
<tr>
<td>Positive</td>
<td>0.49</td>
<td>0.49</td>
<td>0.068</td>
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</table>

#### Shear Design

<table>
<thead>
<tr>
<th>Span Location</th>
<th>$V_u$ (kips/ft)</th>
<th>$\phi V_u$ (kips/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Span</td>
<td>Hand</td>
<td>spBeam</td>
</tr>
<tr>
<td>Interior Negative</td>
<td>0.69</td>
<td>0.67</td>
</tr>
</tbody>
</table>

### 2.1.8. Conclusions and Observations

Minimum reinforcement requirement governed flexural design in this example. spBeam program enables the user to enter the rotational support springs as boundary conditions for joist supports and evaluate various analysis and design options beyond the limitations of the simplified method. The coefficients traditionally used to determine moments do not address various types of support and geometry.

Typically, in wide-module joist construction, one-way slab is reinforced with single layer reinforcement placed near the top in the primary direction. As seen in this example, this may cause crack control criteria to govern the reinforcement spacing and consequently, it may warrant the use of welded wire fabric reinforcement instead of deformed bar.

The maximum calculated total immediate (instantaneous) deflection (DL + LL) = 0.003 in., this value can be compared with maximum permissible calculated deflection limitation per project criteria in accordance to ACI 318-14.

`ACI 318-14 (Table 24.2.2)`

In addition to deflection results, parametric studies can be performed in spBeam to optimize design and detailing results. Note in the reinforcement diagram (Figure 11) 2-#2 for top reinforcement in the span left and right zones as well as span bottom reinforcement. One layer is suitable by inspection (Figure 8) to meet the required area of steel for top and bottom reinforcement.
2.2. One-Way Joist Design

The wide-module joists in this floor are considered as beams per *ACI 318-14 (9.8.1.8)*. Therefore, the design of the joist shall conform to the requirements of T-beams per *ACI 318-14 (9.2.4)*.

![Figure 12 – Partial Plan View Illustrating One-Way Joist to be Design](image)

Joist design involves the following steps:

2.2.1. Determination of span loads

2.2.2. Determination of design moments and shears

2.2.3. Flexural Design

2.2.4. Shear Design

2.2.5. Deflections

2.2.6. Computer Program Solution

2.2.7. Summary and comparison of design results

2.2.8. Conclusions and observations
2.2.1. Determination of span loads

The following gravity load combinations are considered:

\[ U = 1.40 \times D \quad \text{ACI 318-14 (Eq. 5.3.1a)} \]

\[ w_u = 1.40 \times \left[ \left( \frac{5}{12} \times 0.15 \right) + 0.02 \right] \times 6 + \left( \frac{6 + 8.67}{12} \times \frac{16}{12} \right) \times 0.15 = 1.40 \times 0.62 = 0.86 \text{ kips/ft} \]

\[ U = 1.20 \times D + 1.60 \times L \quad \text{ACI 318-14 (Eq. 5.3.1b)} \]

\[ w_u = 1.20 \times \left[ \left( \frac{5}{12} \times 0.15 \right) + 0.02 \right] \times 6 + \left( \frac{6 + 8.67}{12} \times \frac{16}{12} \right) \times 0.15 + 1.60 \times 0.08 \times 6 \]

\[ w_u = 1.20 \times 0.62 + 1.60 \times 0.48 = 1.51 \text{ kips/ft} \]

Span loads are governed by the second load combination.

Note that for Floor Live Load Reduction per ASCE/SEI 7-10:

\[ L = L_o \times \left( 0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right) \quad \text{ASCE/SEI 7-10 (Eq. 4-1)} \]

Where:

Live Load Element Factor, \( K_{LL} = 2 \) for interior beams \quad \text{ASCE/SEI 7-10 (Table 4-2)}

Tributary Area \( A_T = (6 \times 32) = 192 \text{ ft}^2 \)

Since \( K_{LL} \times A_T = 2 \times 192 = 384 \text{ ft}^2 < 400 \text{ ft}^2 \) live load reduction is not applicable.

2.2.2. Determination of design moment and shear

The factored moment and shear can be determined using the simplified method if the requirements are satisfied:

\[ \text{ACI 318-14 (6.5.1)} \]

- Members are prismatic.
- Loads are uniformly distributed.
- \( L \leq 3D \) (0.48 kips/ft \( \leq 3 \times 0.62 \text{ kips/ft} \))
- There are at least two spans.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent.
Thus, the approximate coefficients can be used. The factored moments and shears are determined and summarized in the following tables.

\textit{ACI 318-14 (Table 6.5.2 and Table 6.5.3)}

<table>
<thead>
<tr>
<th>Table 4 - One-Way Joist Design Moment Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbf{Location}</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>\textbf{End Spans}</td>
</tr>
<tr>
<td>Exterior Support Negative</td>
</tr>
<tr>
<td>Mid-span</td>
</tr>
<tr>
<td>Interior Support Negative</td>
</tr>
<tr>
<td>\textbf{Interior Spans}</td>
</tr>
<tr>
<td>Mid-span Positive</td>
</tr>
<tr>
<td>Support Negative</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5 - One-Way Joist Design Shear Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbf{Location}</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>End Span at Face of First Interior Support</td>
</tr>
<tr>
<td>At Face of all other Supports</td>
</tr>
</tbody>
</table>

\* When support beam is wider than the column, the clear span, $l_n$, of the joists is measured from the face of the column. For calculating negative moments, $l_n$ is taken as the average of the adjacent clear spans.

\textit{ACI 318-14 (6.5.2)}
2.2.3. Flexural Design

For the one-way joist of a wide-module joist system, the end span moment values govern the design as tabulated in Table 4.

Calculate the required reinforcement to resist the first interior support negative moment:

\[ M_u = 136.54 \text{ ft-kips} \]

Use #5 reinforcement with 1.5 in. concrete cover. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, \( d \), is calculated below:

\[
d = 21 - \left( 1.50 + \frac{3}{8} + 0.50 \times \frac{5}{8} \right) = 18.81 \text{ in.}
\]

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section \( (jd) \). In this example, tension-controlled section will be assumed so the reduction factor \( \phi \) is equal to 0.9, and \( jd \) will be taken equal to 0.909 × \( d \) since we are designing for the negative moment in a T-beam (narrow compression zone). The assumptions will be verified once the area of steel is finalized.

Assume \( jd = 0.909 \times d = 0.909 \times 18.81 = 17.11 \text{ in.} \)

Joist average width, \( b = \frac{6 + 8.67}{2} = 7.33 \text{ in.} \)

The required reinforcement at initial trial is calculated as follows:

\[
A_\text{s} = \frac{M_u}{\phi \times f_y \times jd} = \frac{136.54 \times 12,000}{0.90 \times 60,000 \times 17.11} = 1.774 \text{ in.}^2
\]

Recalculate ‘a’ for the actual \( A_s = 1.774 \text{ in.}^2 \):

\[
a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{1.774 \times 60,000}{0.85 \times 5,000 \times 7.33} = 3.414 \text{ in.}
\]

\[
c = \frac{a}{\beta_h} = \frac{3.414}{0.85} = 4.016 \text{ in.}
\]

\[
e_c = \left( \frac{0.003}{c} \right) \times d = 0.003 \times 4.016 \times 18.81 = 0.003 \times 0.011 > 0.005
\]

Therefore, the assumption that section is tension-controlled is valid.
\[ A_i = \frac{M_u}{\phi \times f_y \times \left( d - \frac{a}{2} \right)} = \frac{136.54 \times 12,000}{0.90 \times 60,000 \times \left( 18.81 - \frac{3.414}{2} \right)} = 1.774 \text{ in.}^2 \]

The minimum reinforcement shall not be less than

\[ A_{s,\text{min}} = \frac{3 \times \sqrt{f_y'}}{f_y} \times b_w \times d = \frac{3 \times \sqrt{5,000}}{60,000} \times 7.34 \times 18.81 = 0.49 \text{ in.}^2 \quad \text{ACI 318-14 (9.6.1.2(a))} \]

And not less than

\[ A_{s,\text{min}} = \frac{200}{f_y} \times b_w \times d = \frac{200}{60,000} \times 7.34 \times 18.81 = 0.46 \text{ in.}^2 \quad \text{ACI 318-14 (9.6.1.2(b))} \]

Part of the negative-moment steel shall be distributed over a width equal to the smaller of the effective flange width (72 in.) and \( \frac{l}{10} = \frac{384}{10} = 38.40 \text{ in.} \quad \text{ACI 318-14 (24.3.4)} \)

Where the effective width of the overhanging flange on each side of the beam web is the smallest of the following: \( \text{ACI 318-14 (6.3.2.1)} \)

\[ b_w + 2 \times (8 \times h) = 8.67 + 2 \times (8 \times 5) = 88.67 \text{ in.}, \text{ where } h \text{ is the slab thickness.} \]

\[ b_w + 2 \times \left( \frac{s_w}{2} \right) = 6 + 2 \times \left( \frac{5.50 \times 12}{2} \right) = 72.00 \text{ in.}, \text{ where } s_w \text{ is the clear distance to the adjacent web.} \]

\[ b_w + 2 \times \left( \frac{l}{8} \right) = 8.67 + 2 \times \left( \frac{30.17 \times 12}{8} \right) = 99.17 \text{ in.} \]

Therefore, the effective flange width is 72 in.

Provide 6-#5 bars within 38.4 in width. \( A_{s,\text{prov}} = (6 \times 0.31) = 1.86 \text{ in.}^2 > 1.774 \text{ in.}^2 \quad \text{o.k.} \)
Calculate the required reinforcement to resist the positive moment:

\[ M_u = 98.07 \text{ ft-kips} \]

In the positive moment regions, the beam acts as a T-shaped beam. The effective flange width as was calculated earlier is 72 in.

By assuming #3 bars for joist stirrups and the maximum bar size for joist bottom reinforcement as #7 and following the 1.50 in. concrete cover to reinforcement requirement of beam stirrups per \textit{ACI 318-14 (20.6.1)}, the distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, \( d \), is calculated below:

\[ d = 21 - \left( 1.50 + \frac{3}{8} \times 0.50 \times \left( \frac{7}{8} \right) \right) = 18.69 \text{ in.} \]

Since we are designing for the positive moment in a T-Beam (wide compression zone), select a moment arm, \( jd \) approximately equal to 0.994 \times d.

Assume that \( jd = 0.994 \times d = 0.994 \times 18.69 = 18.57 \text{ in.} \)

\[ A_j = \frac{M_u}{\phi \times f_y \times jd} = \frac{98.07 \times 12,000}{0.90 \times 60,000 \times 18.57} = 1.173 \text{ in.}^2 \]

Recalculate ‘a’ for the actual \( A_s = 1.173 \text{ in.}^2 \):

\[ a = \frac{A_s \times f_y}{0.85 \times f_y' \times b} = \frac{1.173 \times 60,000}{0.85 \times 5,000 \times 72} = 0.230 \text{ in.} \]

\[ c = \frac{a}{\beta_n} = \frac{0.230}{0.85} = 0.271 \text{ in.} \]

\[ \varepsilon_t = \left( \frac{0.003}{c} \right) \times d_e - 0.003 = \left( \frac{0.003}{0.27} \right) \times 18.69 - 0.003 = 0.204 > 0.005 \]

Therefore, the assumption that section is tension-controlled is valid.

\[ A_j = \frac{M_u}{\phi \times f_y \times \left( d - \frac{a}{2} \right)} = \frac{98.07 \times 12,000}{0.90 \times 60,000 \times \left( 18.69 - \frac{0.230}{2} \right)} = 1.173 \text{ in.}^2 \]

Use 2-#7 bundled bars with \( A_{s,prov} = (2 \times 0.60) = 1.20 \text{ in.}^2 > 1.173 \text{ in.}^2 \) \textit{o.k.}
Figure 13 – Cross-Sectional View at Joist Mid-Span (Section 2/2.2 in Figure 12)

Figure 14 – Cross-Sectional View at Joist Near Support Face (Section 3/2.2 in Figure 12)
### 2.2.4. Shear Design

From Table 5 above, the shear value in end span at face of first interior support governs.

\[
V_u = 1.15 \times \frac{w_u \times L_u}{2} = 1.15 \times \frac{1.51 \times 30.17}{2} = 26.17 \text{ kips}
\]

The design shear at a distance, \(d\), away from the face of support,

\[
V_u = 26.17 - 1.51 \times \frac{18.69}{12} = 23.82 \text{ kips}
\]

Shear strength provided by concrete

\[
\phi V_c = \phi \times \left(2 \times \sqrt{f'_c} \times b_w \times d\right)
\]

ACI 318-14 (Eq. 22.5.5.1)

\[
\phi V_c = 0.75 \times \left(2.00 \times 1.00 \times \sqrt{5,000} \times 7.34 \times 18.69\right) = 14,538.77 \text{ lb} = 14.54 \text{ kips}
\]

Since \(V_u > \frac{\phi V_c}{2}\), shear reinforcement is required.

Try #3, Grade 60 double-leg stirrups with a 90° hook.

The nominal shear strength required to be provided by shear reinforcement is

\[
V_s = V_u - V_c = \frac{V_u}{\phi} - V_c = \left(\frac{23.82}{0.75}\right) - 19.39 = 12.38 \text{ kips}
\]

Check whether \(V_s\) is less than \(8 \times \sqrt{f'_c} \times b_w \times d\)

If \(V_s\) is greater than \(8 \times \sqrt{f'_c} \times b_w \times d\), then the cross-section has to be revised as ACI 318-14 limits the shear capacity to be provided by stirrups to \(8 \times \sqrt{f'_c} \times b_w \times d\) ACI 318-14 (22.5.1.2)

\[
8 \times \sqrt{f'_c} \times b_w \times d = 8 \times \sqrt{5,000} \times 7.34 \times 18.69 = 77,540.09 \text{ lb} = 77.54 \text{ kips}
\]

Since \(V_s\) does not exceed \(8 \times \sqrt{f'_c} \times b_w \times d\), the cross section is adequate.

Assume #3 stirrups with two legs \((A_v = 0.22 \text{ in.}^2)\)

Calculate the required stirrup spacing as

\[
s_{\text{req/d}} = \frac{\phi \times A_v \times f_{w} \times d}{V_u - \phi \times V_c} = \frac{0.75 \times 0.22 \times 60 \times 18.69}{23.82 - 14.54} = 19.93 \text{ in.}
\]

ACI 318-14 (22.5.10.5.3)
Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per **ACI 318-14 (9.7.6.2.2)**.

Check whether $V_s$ is less than $4 \times \sqrt{f'_c \times b_w \times d}$

$$4 \times \sqrt{f'_c \times b_w \times d} = 4 \times \sqrt{5,000 \times 7.34 \times 18.69} = 38,770.05 \text{ lb} = 38.77 \text{ kips} > V_s = 12.38 \text{ kips}$$

Therefore, maximum stirrup spacing shall be the smallest of $d/2$ and 24 in. **ACI 318-14 (Table 9.7.6.2.2)**

$$s_{\text{max}} = \text{lesser of } \left[ \frac{d}{2} \right] = \text{lesser of } \left[ \frac{18.69}{2} \right] = \text{lesser of } \left[ \frac{9.34}{24 \text{ in.}} \right] = 9.34 \text{ in.}$$

This value governs over the required stirrup spacing of 19.93 in. which was based on the demand.

Joist minimum shear reinforcement requirements must be checked since wide-module joists do not satisfy **ACI 318-14 (9.8)**.

Check the maximum stirrup spacing based on minimum shear reinforcement

$$s_{\text{max}} \leq \frac{A_s \times f_y}{0.75 \times \sqrt{f'_c \times b_w}} = \frac{0.22 \times 60,000}{0.75 \times \sqrt{5,000 \times 7.34}} = 33.93 \text{ in.} \quad \text{(does not govern)} \quad \text{ACI 318-14 (10.6.2.2(a))}$$

$$s_{\text{max}} \leq \frac{A_s \times f_y}{50 \times b_w} = \frac{0.22 \times 60,000}{50 \times 7.34} = 35.99 \text{ in.} \quad \text{(does not govern)} \quad \text{ACI 318-14 (10.6.2.2(b))}$$

Therefore, $s_{\text{max}}$ value is governed by the spacing limit per **ACI 318-14 (9.7.6.2.2)**, and is equal to 9.34 in.

Use #3 @ 9 in. stirrups

$$V_s = \frac{A_s \times f_y \times d}{s} + V_c \quad \text{ACI 318-14 (22.5.1.1 and 22.5.10.5.3)}$$

$$V_s = \frac{0.22 \times 60,000 \times 18.69}{9} + 19.39 = 26.40 + 19.39 = 45.79 \text{ kips}$$

$$\phi V_s = 0.75 \times 45.79 = 34.34 \text{ kips} > V_s = 23.82 \text{ kips} \quad \text{o.k.}$$

Compute where $\frac{V_s}{\phi}$ is equal to $\frac{V_c}{2}$, and the stirrups can be stopped
\[
x = \left( \frac{V_c}{\phi} - \frac{V_c}{2} \right) \times \frac{l}{2} = \left( \frac{23.82}{0.75} - \frac{19.39}{2} \right) \times \frac{30.17 \times 12}{2} = 125.76 \text{ in.}
\]

At interior end of the exterior span, use 16-#3 @ 9 in o.c., Place 1st stirrup 2 in. from the face of supporting girder.

2.2.5. Deflections

Since the preliminary joist depth met minimum depth requirement, the deflection calculations are not required. A lesser depth maybe possible and consequently cost savings can be achieved through deflection computations. Deflection values are calculated and provided for every model created by \textit{spBeam} Program and can be used by the engineer to make additional optimization decisions.

2.2.6. Computer Program Solution

\textit{spBeam} Program can be utilized to analysis and design the one-way wide-module joist. A single wide-module joist is modeled as a five span continuous-beam.

The program calculates the internal forces (shear force and bending moment), moment and shear capacities, immediate and long-term deflection results, and required flexural reinforcement. The graphical and text results are provided here for both input and output of the \textit{spBeam} model.

The ribs are modeled as a rectangular longitudinal beam with an equivalent width of 7.33 in. and 21 in. depth to reflect the sloped sides of the forming pans.
Figure 15 – spBeam Model – One-Way Joist Section
Figure 16 – spBeam Model – Isometric View – One-Way Joist
Figure 17 – spBeam Model – Loads (Including Live Load Patterning)
Figure 19 – spBeam Model – Moment Capacity Diagram
Figure 20 – spBeam Model – Shear Capacity Diagram
Figure 21 – spBeam Model – Immediate Deflection Diagram
Figure 22 – spBeam Model – Reinforcement Diagram
2.2.7. Summary and Comparison of Results

1. Design Results

1.1. Top Reinforcement

Notes:
*5 - Number of bars governed by maximum allowable spacing.

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<th>Span Zone</th>
<th>Width (ft)</th>
<th>M&lt;sub&gt;max&lt;/sub&gt; (k-ft)</th>
<th>X&lt;sub&gt;max&lt;/sub&gt; (ft)</th>
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<th>A&lt;sub&gt;u,max&lt;/sub&gt; (in&lt;sup&gt;2&lt;/sup&gt;)</th>
<th>A&lt;sub&gt;u,req&lt;/sub&gt; (in&lt;sup&gt;2&lt;/sup&gt;)</th>
<th>S&lt;sub&gt;P,prov&lt;/sub&gt; (in)</th>
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1.2. Bottom Reinforcement

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## 1.3. Longitudinal Beam Transverse Reinforcement Demand and Capacity

### 1.3.1. Beam Transverse Reinforcement Capacity

Notes:
- *8 - Minimum transverse (stirrup) reinforcement governs.

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<th>End (ft)</th>
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<th>( V_u ) (kip)</th>
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<th>( A_{ys} ) (in²/in)</th>
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### Table 6 - Comparison of Hand Solution with spBeam Solution for One-Way Joist

#### Flexural Design

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<th>Span Location</th>
<th>Design Moment (ft-kips)</th>
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<th>Reinforcement Provided for Flexure (in.²)</th>
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#### Shear Design

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<th>( \phi V_u ) (kips/ft)</th>
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<tr>
<td>Interior Negative</td>
<td>23.82</td>
<td>22.96</td>
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</table>
2.2.8. Conclusions and Observations

In this design example, the one-way joist system is modeled as a continuous T-beam representing single one-way joist. There is a good agreement between the hand solution and computer solution. Note that the coefficients traditionally used to determine moments do not address various types of support and geometry.

The maximum calculated total immediate (instantaneous) deflection (DL + LL) = 0.316 in., this value can be compared with maximum permissible calculated deflection limitation per project criteria in accordance to ACI 318-14.

In addition to deflection results, parametric studies can be performed in spBeam to optimize design and detailing results. With a minimum spacing of 1 in. between the 2-#7, two stirrups, and 1.50 in. cover on each side, a total width of 6.50 in. is required. The rib width at the bar level is 6.385 in. which is slightly less than required. For detailing purposes, one of the following options can be used:

1. Bottom bars can be bundled. This practice is often found in joist construction.
2. Stirrups can be rotated by a small angle to preserve the minimum spacing.
3. Bottom bars can be raised sufficiently to achieve the required width taking into the account the reduction into the moment capacity.
4. Other detailing options provided by the builder/formwork supply.

Figure 23 – Joist Cross-Section
2.3. Design of Beam along Grid B (Interior Frame)

In wide-module joist construction, the supporting beam, sometimes referred to as girder, depth is typically set to match the overall joist depth. Therefore, the beam depth is set to 21 in. This depth need to satisfy the minimum depth requirement of *ACI 318-14 (Table 9.3.1.1)* so that the deflection computations can be waived.

Using the minimum depth for non-prestressed beams in *Table 9.3.1.1*.

End Span: \[ h = \frac{l}{18.5} = \frac{360}{18.5} = 19.46 \text{ in.} \] (governs) < 21 in. \textbf{ACI 318-14 (Table 9.3.1.1)}

Interior Span: \[ h = \frac{l}{21} = \frac{360}{21} = 17.14 \text{ in.} \] \textbf{ACI 318-14 (Table 9.3.1.1)}

Therefore, the preliminary beam depth satisfies the minimum depth requirement.

![Figure 24 – Partial Plan View Showing Interior Beam along grid B](image)

Beam (girder) design involves the following steps:

- 2.3.1. Determination of span loads
- 2.3.2. Determination of design moments and shears
- 2.3.3. Flexural Design
2.3.4. Shear Design

2.3.5. Deflections

2.3.6. Computer Program Solution

2.3.7. Summary and comparison of design results

2.3.8. Conclusions and observations

2.3.1. Determination of span loads

Dead Load:

Try 36 in. width for the beam (slightly larger than the column width that helps facilitate the forming, and reduces the beam longitudinal vs. column vertical bar interference)

\[
\text{Joist & Slab Weight} = \left[ \frac{5}{12} + \left( \left( \frac{6+8.67}{12} \right) \times \frac{16}{6} \right) \right] \times 0.15 \times \left( \frac{32 - \frac{36}{12}}{12} \right) = 2.40 \text{ kips/ft}
\]

\[
\text{Beam Weight} = \left( \frac{21}{12} \times \frac{36}{12} \right) \times 0.15 = 0.79 \text{ kips/ft}
\]

Superimposed Dead Load, \( SDL = 0.02 \times 32 = 0.64 \text{ kips/ft} \)

Live Load:

Check for live load reduction per \textit{ASCE/SEI 7-10}

\[
L = L_o \times \left( 0.25 + \frac{15}{\sqrt{K_{1L} \times A_T}} \right)
\]

\textit{ASCE/SEI 7-10 (Eq. 4-1)}

Where:

\[
L = \text{Reduced design live load per ft}^2 \text{ of area supported by the member}
\]

\[
L_o = \text{Unreduced design live load per ft}^2 \text{ of area supported by the member} = 80 \text{ psf}
\]

\[
K_{1L} = \text{Live load element factor} = 2 \text{ for interior beams}
\]

\[
A_T = \text{Tributary area} = (30 \times 32) = 960 \text{ ft}^2
\]

\[
L = 80 \times \left( 0.25 + \frac{15}{\sqrt{2 \times 960}} \right) = 47.39 \text{ psf}
\]

Which satisfies \( 0.50 \times L_o \) requirement for members supporting one floor. \textit{ASCE/SEI 7-10 (4.7.2)}
\[ L = \frac{47.39}{1,000} \times 32 = 1.52 \text{ kips/ft} \]

Load Combination:

The following gravity load combinations are considered:

\[ U = 1.40 \times D \quad \text{(ACI 318-14 (Eq. 5.3.1a))} \]

\[ w_u = 1.40 \times (2.40 + 0.79 + 0.64) = 1.40 \times 3.83 = 5.36 \text{ kips/ft} \]

\[ U = 1.20 \times D + 1.60 \times L \quad \text{(ACI 318-14 (Eq. 5.3.1b))} \]

\[ w_u = 1.20 \times (2.40 + 0.79 + 0.64) + 1.60 \times 1.52 = 1.20 \times 3.83 + 1.60 \times 1.52 = 7.02 \text{ kips/ft} \]

The span loads are governed by the second load combination.

2.3.2. Determination of design moment and shear

The factored moment and shear can be determined using the simplified method if the requirements are satisfied: \( ACI 318-14 \) (6.5.1)

- Members are prismatic.
- Loads are uniformly distributed.
- \( L \leq 3D \) (1.52 kips/ft \( \leq 3 \times 3.83 \) kips/ft)
- There are at least two spans.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

Thus, the approximate coefficients can be used. The factored moment and shear are determined and summarized in the following tables. \( ACI 318-14 \) (Table 6.5.2 and Table 6.5.3)
<table>
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<tr>
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<tr>
<td>End Spans</td>
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<tr>
<td>Exterior Support Negative</td>
<td>( \frac{w_u \times L_e^2}{16} = \frac{7.02 \times 28.17^2}{16} = 348.25 \text{ ft-kips} )</td>
</tr>
<tr>
<td>Mid-span</td>
<td>( \frac{w_u \times L_e^2}{14} = \frac{7.02 \times 28.17^2}{14} = 398.00 \text{ ft-kips} )</td>
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<tr>
<td>Interior Support Negative</td>
<td>( \frac{w_u \times L_e^2}{10} = \frac{7.02 \times 28.08^2}{10} = 553.90 \text{ ft-kips} )</td>
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<tr>
<td>Interior Spans</td>
<td></td>
</tr>
<tr>
<td>Mid-span Positive</td>
<td>( \frac{w_u \times L_e^2}{16} = \frac{7.02 \times 28^2}{16} = 344.14 \text{ ft-kips} )</td>
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<tr>
<td>Support Negative</td>
<td>( \frac{w_u \times L_e^2}{11} = \frac{7.02 \times 28^2}{11} = 500.56 \text{ ft-kips} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>Design Shear Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Span at Face of First Interior Support</td>
<td>( 1.15 \times \frac{w_u \times L_e}{2} = 1.15 \times \frac{7.02 \times 28.17}{2} = 113.75 \text{ kips} )</td>
</tr>
<tr>
<td>At Face of all other Supports</td>
<td>( \frac{w_u \times L_e}{2} = \frac{7.02 \times 28.17}{2} = 98.91 \text{ kips} )</td>
</tr>
</tbody>
</table>
2.3.3. Flexural Design

For this interior beam, the end span moment values govern the design as tabulated in Table 7.

Calculate the required reinforcement to resist the first interior support negative moment:

\[ M_u = 553.90 \text{ ft-kips} \]

Use #8 bars with 1.50 in. concrete cover per *ACI 318-14 (Table 20.6.1.3.1)*. To avoid interference with joist negative moment reinforcement, the clear cover to the girder top reinforcement is required to be increased by lowering the girder top reinforcement. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, \( d \), is calculated below:

\[
d = 21 - \left( 1.50 + \frac{3}{8} + \frac{5}{8} + 0.50 \times \frac{8}{8} \right) = 18.00 \text{ in.}
\]

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section \( (j'd) \). In this example, tension-controlled section will be assumed so the reduction factor \( \phi \) is equal to 0.9, and \( j'd \) will be taken equal to \( 0.919 \times d \) since we are designing for the negative moment in a rectangular beam (narrow compression zone). The assumptions will be verified once the area of steel is finalized.

Assume \( j'd = 0.919 \times d = 0.919 \times 18.00 = 16.54 \text{ in.} \)

Interior beam width, \( b = 36 \text{ in.} \)

The required reinforcement at initial trial is calculated as follows:

\[
A_t = \frac{M_u}{\phi \times f' \times j'd} = \frac{553.90 \times 12,000}{0.90 \times 60,000 \times 16.54} = 7.442 \text{ in.}^2
\]

Recalculate ‘a’ for the actual \( A_t = 7.442 \text{ in.}^2 \):

\[
a = \frac{A_t \times f'}{0.85 \times f' \times b} = \frac{7.442 \times 60,000}{0.85 \times 5,000 \times 36} = 2.918 \text{ in.}
\]

\[
c = \frac{a}{\beta_i} = \frac{2.918}{0.85} = 3.433 \text{ in.}
\]

\[
e_t = \left( \frac{0.003}{c} \right) \times d_t - 0.003 = \left( \frac{0.003}{3.433} \right) \times 18.00 - 0.003 = 0.013 > 0.005
\]

Therefore, the assumption that section is tension-controlled is valid.
\[ A_i = \frac{M_u}{\phi \times f_y \left( d - \frac{a}{2} \right)} = \frac{553.90 \times 12,000}{0.90 \times 60,000 \times \left( 18.00 - \frac{2.918}{2} \right)} = 7.442 \text{ in}^2 \]

The minimum reinforcement shall not be less than

\[ A_{s,min} = 3 \times \sqrt{\frac{f_y}{f_y}} \times b_u \times d = \frac{3 \times \sqrt{5,000}}{60,000} \times 36 \times 18 = 2.29 \text{ in}^2 \]  
**ACI 318-14 (9.6.1.2(a))**

And not less than

\[ A_{s,min} = \frac{200}{f_y} \times b_u \times d = \frac{200}{60,000} \times 36 \times 18 = 2.16 \text{ in}^2 \]  
**ACI 318-14 (9.6.1.2(b))**

Provide 10-#8 bars: \( A_s = (10 \times 0.79) = 7.90 \text{ in}^2 > 7.442 \text{ in}^2 \)  
**o.k.**

**Maximum spacing allowed:**

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

\[ s = 15 \times \left( \frac{40,000}{f_y} \right) - 2.50 \times c_e \leq 12 \times \left( \frac{40,000}{f_y} \right) \]  
**ACI 318-14 (Table 24.3.2)**

\[ c_e = 21 - \left( 18 + 0.50 \times \frac{8}{8} \right) = 2.5 \text{ in.} \]

Use \( f_y = \frac{2}{3} \times f_y = \frac{2}{3} \times 60,000 = 40,000 \text{ psi} \)  
**ACI 318-14 (24.3.2.1)**

\[ s = 15 \times \left( \frac{40,000}{40,000} \right) - 2.50 \times 2.50 = 8.75 \text{ in.} \text{ (governs)} \]

\[ s = 12 \times \left( \frac{40,000}{40,000} \right) = 12 \text{ in.} \]

Spacing provided for 10-#8 bars

\[ s_{prov} = \frac{(b_u - 2 \times d_s)}{\# \text{ of bars} - 1} = \frac{(36 - 2 \times 2.625)}{9} = 3.42 \text{ in.} < 8.75 \text{ in.} \]  
**o.k.**

Where \( d_s = 2.625 \text{ in.} \text{ for #3 stirrup as shown in the following Figure.} \)**  
**CRSI 2002 (Figure 12-9)**
Check the spacing, $s$ provided, is greater than the minimum center to center spacing, $s_{\text{min}}$ where:

$$s_{\text{min}} = d_h + \text{the largest of} \begin{cases} 
1 \text{ in.} \\
\frac{d_h}{1.33 \times \text{max. agg.}}
\end{cases}$$

Where maximum aggregate size is $\frac{3}{4}"$

$$s_{\text{min}} = 1.00 + \max \begin{cases} 
1.00 \\
1.00 \\
1.33 \times 0.75 = 1.00
\end{cases} = 1.00 + 1.00 = 2.00 \text{ in.}$$

Since the spacing provided is greater than 2.00 in. Therefore, 10-#8 bars are OK.

All the values on Table 9 are calculated based on the procedure outlined above.
2.3.4. Shear Design

From Table 8 above, the shear value in end span at face of first interior support governs.

\[ V_u = 1.15 \times \frac{w_u \times L_u}{2} = 1.15 \times \frac{7.02 \times 28.17}{2} = 113.75 \text{ kips} \]

The design shear at a distance, \( d \), away from the face of support,

\[ V_u = 113.75 - 7.02 \times \frac{18}{12} = 103.21 \text{ kips} \]

Shear strength provided by concrete

\[ \phi V_c = \phi \times \left( 2 \times \sqrt{f'_c \times b_w \times d} \right) \quad \text{ACI 318-14 (Eq. 22.5.5.1)} \]

\[ \phi V_c = 0.75 \times \left( 2.00 \times 1.00 \times \sqrt{5,000 \times 36 \times 18} \right) = 68,730.78 \text{ lb} = 68.73 \text{ kips} \]

Since \( V_u > \frac{\phi V_c}{2} \), shear reinforcement is required.

Try #3, Grade 60 four-leg stirrups \( (A_v = 0.44 \text{ in.}^2) \) with a 90° hook.

The nominal shear strength required to be provided by shear reinforcement is

\[ V_s = V_u - V_c = \frac{V_u}{\phi} - V_c = \left( \frac{103.21}{0.75} \right) = 91.64 = 45.98 \text{ kips} \]

Check whether \( V_s \) is less than \( 8 \times \sqrt{\frac{f'_c}{b_w} \times d} \)
If $V_s$ is greater than $8 \times \sqrt{f_y \cdot b_w \times d}$, then the cross-section has to be revised as **ACI 318-14** limits the shear capacity to be provided by stirrups to $8 \times \sqrt{f_y \cdot b_w \times d}$  

$$8 \times \sqrt{f_y \cdot b_w \times d} = 8 \times \sqrt{5000 \times 36 \times 18} = 366,564.16 \text{ lb} = 366.56 \text{ kips}$$

Since $V_s$ does not exceed $8 \times \sqrt{f_y \cdot b_w \times d}$, the cross section is adequate.

Calculate the required stirrup spacing as

$$s_{reqd} = \frac{\phi \times A_s \times f_y \times d}{V_u - \phi \times V_c} = \frac{0.75 \times 0.44 \times 60,000 \times 18}{103.21 - 68.73} = 10.34 \text{ in.}$$

**ACI 318-14 (22.5.10.5.3)**

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per **ACI 318-14 (9.7.6.2.2)**.

Check whether $V_s$ is less than $4 \times \sqrt{f_y \cdot b_w \times d}$

$$4 \times \sqrt{f_y \cdot b_w \times d} = 4 \times \sqrt{5000 \times 36 \times 18} = 183,282.08 \text{ lb} = 183.28 \text{ kips} > V_s = 45.98 \text{ kips}$$

Therefore, maximum stirrup spacing shall be the smallest of $d/2$ and 24 in. **ACI 318-14 (Table 9.7.6.2.2)**

$$s_{max} = \text{lesser of} \left[ \frac{d}{2} \right] = \text{lesser of} \left[ \frac{18}{2} \right] = \text{lesser of} \left[ \frac{9}{24} \right] = 9 \text{ in.}$$

This value governs over the required stirrup spacing of 10.34 in which was based on the demand. Note that since the stirrup spacing is governed by $s_{max}$, the size of the stirrup can be kept as #3. Selecting #4 stirrup size will produce capacity more than what is required and therefore, be uneconomical.

Check the maximum stirrup spacing based on minimum shear reinforcement

$$s_{max} \leq \frac{A_s \times f_y}{0.75 \times \sqrt{f_y \cdot b_w}} = \frac{0.44 \times 60,000}{0.75 \times \sqrt{5000 \times 36}} = 13.83 \text{ in.} \ (\text{does not govern})$$

**ACI 318-14 (10.6.2.2(a))**

$$s_{max} \leq \frac{A_s \times f_y}{50 \times b_w} = \frac{0.44 \times 60,000}{50 \times 36} = 14.67 \text{ in.} \ (\text{does not govern})$$

**ACI 318-14 (10.6.2.2(b))**

Therefore, $s_{max}$ value is governed by the spacing limit per **ACI 318-14 (9.7.6.2.2)**, and is equal to 9 in.

Use #3 @ 8 in. stirrups
\[ V_s = \frac{A_c \times f_{cy} \times d}{s} + V_c \]

\[ V_s = \frac{0.44 \times 60,000 \times 18}{8} + 91.64 = 59.40 + 91.64 = 151.04 \text{ kips} \]

\[ \phi V_s = 0.75 \times 151.04 = 113.28 \text{ kips} > V_s = 103.21 \text{ kips} \]

Compute where \( \frac{V_s}{\phi} \) is equal to \( \frac{V_c}{2} \), and the stirrups can be stopped

\[ x = \left( \frac{V_s}{\phi} - \frac{V_c}{2} \right) \times \frac{L}{2} = \left( \frac{103.21}{0.75} - \frac{91.64}{2} \right) \times \frac{28.17 \times 12}{2} = 112.73 \text{ in.} \]

At interior end of the exterior span, use 16-#3 @ 8 in o.c., Place 1st stirrup 2 in. from the face of the column.

### 2.3.5. Deflections

Since the preliminary beam depth met minimum depth requirement, the deflection calculations are not required. A lesser depth maybe possible and consequently cost savings can be achieved through deflection computations. Deflection values are calculated and provided for every model created by spBeam Program and can be used by the engineer to make additional optimization decisions.

### 2.3.6. Computer Program Solution

spBeam Program can be utilized to analyze and design the interior continuous beam along grid B. The beam is modeled as a three span continuous rectangular beam.

The program calculates the internal forces (shear force and bending moment), moment and shear capacities, immediate and long-term deflection results, and required flexural reinforcement. The graphical and text results are provided here for both input and output of the spBeam model.

The beam is modeled as a 36 in. by 21 in. deep rectangular longitudinal beam with column supports. The supports can be modeled as pinned, fixed, or using actual geometric properties of the beam-column joint. A value of 100 is used in this model for column stiffness share, indicating the actual column stiffness. When the percentage lies between zero and 100%, the joint stiffness contribution by the column is multiplied by that percentage. The default value is 100%.
Figure 26 – spBeam Model – Support Data
Figure 27 – spBeam Model – Isometric View – Interior Continuous Beam along Grid B
Figure 28 – **spBeam** Model – Loads (Including Live Load Patterning)
Figure 29 – spBeam Model – Internal Forces (Shear Force Diagram and Bending Moment Diagram)
Figure 30 – *spBeam* Model – Moment Capacity Diagram
Figure 31 – **spBeam** Model – Shear Capacity Diagram
Figure 32 – spBeam Model – Immediate Deflection Diagram
Figure 33 – spBeam Model – Reinforcement Diagram

File: C:\StructurePoint\spBeam\One-Way Interior Beam.slb
Project: Wide-Module Joist System
Frame: Interior Beam
Engineer: SP
Code: ACI 318-14
Date: 06/21/23
Time: 16:21:02
2.3.7. Summary and Comparison of Results

1. Design Results

1.1. Top Reinforcement

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1.2. Bottom Reinforcement

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1.3. Longitudinal Beam Transverse Reinforcement Demand and Capacity

1.3.1. Beam Transverse Reinforcement Capacity

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

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<td>2.500</td>
<td>89.23</td>
<td>U1/S2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1.250</td>
<td>13.214</td>
<td>2.500</td>
<td>89.23</td>
<td>U1/S2</td>
<td>0.0253</td>
<td>0.44</td>
<td>8.7</td>
<td>0.0506</td>
</tr>
<tr>
<td></td>
<td>13.214</td>
<td>16.786</td>
<td>13.214</td>
<td>13.96</td>
<td>U1/S2</td>
<td>0.0000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>16.786</td>
<td>28.750</td>
<td>27.500</td>
<td>89.23</td>
<td>U1/S3</td>
<td>0.0253</td>
<td>0.44</td>
<td>8.7</td>
<td>0.0506</td>
</tr>
<tr>
<td></td>
<td>28.750</td>
<td>30.000</td>
<td>27.500</td>
<td>89.23</td>
<td>U1/S3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>1.250</td>
<td>2.500</td>
<td>95.14</td>
<td>U1/S3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1.250</td>
<td>13.286</td>
<td>2.500</td>
<td>95.14</td>
<td>U1/S3</td>
<td>0.0326</td>
<td>0.44</td>
<td>8.8</td>
<td>0.0503</td>
</tr>
<tr>
<td></td>
<td>13.286</td>
<td>20.476</td>
<td>20.476</td>
<td>32.45</td>
<td>U1/Odd</td>
<td>0.0000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>20.476</td>
<td>28.917</td>
<td>27.667</td>
<td>82.97</td>
<td>U1/Odd</td>
<td>0.0176</td>
<td>0.44</td>
<td>8.8</td>
<td>0.0500</td>
</tr>
<tr>
<td></td>
<td>28.917</td>
<td>30.000</td>
<td>27.667</td>
<td>82.97</td>
<td>U1/Odd</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 10 - Comparison of Hand Solution with spBeam Solution for One-Way Interior Beam

<table>
<thead>
<tr>
<th>Span Location</th>
<th>Design Moment (ft-kips)</th>
<th>Reinforcement Required for Flexure (in.²)</th>
<th>Reinforcement Provided for Flexure (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hand</td>
<td>spBeam</td>
<td>Hand</td>
</tr>
<tr>
<td>End Span</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interior Negative</td>
<td>553.90</td>
<td>486.01</td>
<td>7.442</td>
</tr>
<tr>
<td>Positive</td>
<td>344.14</td>
<td>316.28</td>
<td>4.301</td>
</tr>
</tbody>
</table>

Shear Design

<table>
<thead>
<tr>
<th>Span Location</th>
<th>$V_u$ (kips/ft)</th>
<th>$\phi V_u$ (kips/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hand</td>
<td>spBeam</td>
</tr>
<tr>
<td>End Span</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interior Negative</td>
<td>103.21</td>
<td>95.14</td>
</tr>
</tbody>
</table>

2.3.8. Conclusions and Observations

In this design example, the interior beam is modeled as a continuous rectangular longitudinal beam. There is a good agreement between the hand solution and computer solution. Note that the coefficients traditionally used to determine moments do not address various types of support and geometry.

The maximum calculated total immediate (instantaneous) deflection (DL + LL) = 0.529 in., this value can be compared with maximum permissible calculated deflection limitation per project criteria in accordance to ACI 318-14. 

In addition to deflection results, parametric studies can be performed in spBeam to optimize design and detailing results.

The reinforcement diagram (Figure 33) shows the minimum length required (including the development length) for flexural design. The bars can be extended and detailed to provide the required support for shear stirrups.
Section A-A

Figure 34 – Interior Beam Cross-Section (Near the First Interior Support)
2.4. Design of Beam along Grid A (Exterior Frame)

In the wide-module joist construction, the supporting beam depths shall be same as the overall joist depth. Therefore, the beam depth is set to 21 in. This beam depth need to satisfy the minimum depth requirement of ACI 318-14 (Table 9.3.1.1) so that the deflection computations can be waived. The beams of the exterior frame shall be designed and detailed for the combined effects of flexure, shear, and torsion according to ACI 318.

Using the minimum depth for non-prestressed beams in Table 9.3.1.1.

End Span: \( h = \frac{l}{18.5} = \frac{360}{18.5} = 19.46 \text{ in.} \) (governs) < 21 in. \( \text{ACI 318-14 (Table 9.3.1.1)} \)

Interior Span: \( h = \frac{l}{21} = \frac{360}{21} = 17.14 \text{ in.} \) \( \text{ACI 318-14 (Table 9.3.1.1)} \)

Therefore, the preliminary beam depth satisfies the minimum depth requirement.

Beam (girder) design involves the following steps:

2.4.1. Determination of span loads
2.4.2. Determination of design moment, shear, and torsion
2.4.3. Flexural and torsion design
2.4.4. Shear and torsion design

2.4.5. Deflections

2.4.6. Computer program solution

2.4.7. Summary and comparison of design results

2.4.8. Conclusions and observations

2.4.1. Determination of span loads

**Dead Load:**

Try 24 in. width for the beam (slightly larger than the column width that helps facilitate the forming, and reduces the beam longitudinal vs. column vertical bar interference)

\[
\text{Joist & Slab Weight} = \left[ \frac{5}{12} + \left( \frac{(6.67 + 2) \times 16}{12} \right) / 6 \right] \times 0.15 \times \left( 16 - \frac{24 - 20}{2} \right) = 1.23 \text{ kips/ft}
\]

\[
\text{Beam Weight} = \left( \frac{21}{12} \times \frac{24}{12} \right) \times 0.15 = 0.53 \text{ kips/ft}
\]

Superimposed Dead Load \( SDL = 0.02 \times (16 + 10/12) = 0.34 \text{ kips/ft} \)

**Live Load:**

Check for live load reduction per *ASCE/SEI 7-10*

\[
L = L_o \times \left( 0.25 + \frac{15}{\sqrt{K_{LL} \times A_T}} \right)
\]

*ASCE/SEI 7-10 (Eq. 4-1)*

Where:

\[
L = \text{Reduced design live load per ft}^2 \text{ of area supported by the member}
\]

\[
L_o = \text{Unreduced design live load per ft}^2 \text{ of area supported by the member} = 80 \text{ psf}
\]

\[
K_{LL} = 2 \text{ (edge beams without cantilever slabs)}
\]

\[
A_T = \text{Tributary area} = (30 \times (16 - 10/12)) = 505 \text{ ft}^2
\]

\[
L = 80 \times \left( 0.25 + \frac{15}{\sqrt{2 \times 505}} \right) = 57.76 \text{ psf}
\]

Which satisfies \(0.50 \times L_o\) requirement for members supporting one floor. *ASCE/SEI 7-10 (4.7.2)*
\[
L = \frac{57.76}{1,000} \times \left( 16 \times \frac{10}{12} \right) = 0.97 \text{ kips/ft}
\]

Load Combination:

The following gravity load combinations are considered:

\[
U = 1.40 \times D \quad \text{ACI 318-14 (Eq. 5.3.1a)}
\]

\[
w_u = 1.40 \times (1.23 + 0.53 + 0.34) = 1.40 \times 2.10 = 2.93 \text{ kips/ft}
\]

\[
U = 1.20 \times D + 1.60 \times L \quad \text{ACI 318-14 (Eq. 5.3.1b)}
\]

\[
w_u = 1.20 \times (1.23 + 0.53 + 0.34) + 1.60 \times 0.97 = 1.20 \times 2.10 + 1.60 \times 0.97 = 4.06 \text{ kips/ft}
\]

The span loads are governed by the second load combination.

For factored torsional moment calculations, the beam self-weight is not included since it is applied along the beam section centerline. And the moment arm is the distance from the midspan to the centerline of the exterior beam section = 16/2 – (24/2 - 20/2)/12 = 7.83 ft.

Thus, the following load combinations are used for the calculation of the factored torsional moment:

\[
U = 1.40 \times D \quad \text{ACI 318-14 (Eq. 5.3.1a)}
\]

\[
T_u = (1.40 \times (1.23 + 0.34)) \times 7.83 = 1.40 \times 1.57 \times 7.83 = 17.17 \text{ ft-kips/ft}
\]

\[
U = 1.20 \times D + 1.60 \times L \quad \text{ACI 318-14 (Eq. 5.3.1b)}
\]

\[
T_u = (1.20 \times (1.24 + 0.34) + 1.60 \times 0.97) \times 7.83 = (1.20 \times 1.57 + 1.60 \times 0.97) \times 7.83 = 26.91 \text{ ft-kips/ft}
\]

The span factored torsional moments are governed by the second load combination.
2.4.2. Determination of design flexural moment, shear, and torsional moment

The factored moment and shear can be determined using the simplified method if the requirements are satisfied: 

- Members are prismatic.
- Loads are uniformly distributed.
- \( L \leq 3D \) (0.97 kips/ft \( \leq 3 \times 2.10 \) kips/ft)
- There are at least two spans.
- The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

Thus, the approximate coefficients can be used. The factored moments and shears are determined and summarized in the following tables.

\[ \text{ACI 318-14 (Table 6.5.2 and Table 6.5.3)} \]

<table>
<thead>
<tr>
<th>Table 11 - Exterior Beam Design Flexural Moment Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Location</strong></td>
</tr>
<tr>
<td>End Spans</td>
</tr>
<tr>
<td>Exterior Support Negative</td>
</tr>
<tr>
<td>Mid-span</td>
</tr>
<tr>
<td>Interior Support Negative</td>
</tr>
<tr>
<td>Interior Spans</td>
</tr>
<tr>
<td>Mid-span Positive</td>
</tr>
<tr>
<td>Support Negative</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 12 - Exterior Design Shear Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Location</strong></td>
</tr>
<tr>
<td>End Span at Face of First Interior Support</td>
</tr>
<tr>
<td>At Face of all other Supports</td>
</tr>
</tbody>
</table>
Any structural analysis method can be used to calculate the torsional. The following table shows the torsional moment values at the centerline of the supports:

<table>
<thead>
<tr>
<th>Location</th>
<th>Design Torsional Moment Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the centerline of all Supports</td>
<td>[ \frac{w_c \times l}{2} = \frac{26.91 \times 30}{2} = 403.59 \text{ ft-kips} ]</td>
</tr>
</tbody>
</table>
2.4.3. Flexural, Shear, and Torsion Design

For this exterior beam, the end span moment values govern the design as tabulated in Table 11.

Calculate the required reinforcement to resist the first interior support negative moment:

\[ M_u = 326.31 \text{ ft-kips} \]

Use #8 bars with 1.50 in. concrete cover per ACI 318-14 (Table 20.6.1.3.1). To avoid interference with joist negative moment reinforcement, the clear cover to the girder top reinforcement is required to be increased by lowering the girder top reinforcement. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, \( d \), is calculated below:

\[ d = 21 - \left( \frac{1.50 + 3/8 + 5/8 + 0.50 \times 8}{8} \right) = 18.00 \text{ in.} \]

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (\( jd \)). In this example, tension-controlled section will be assumed so the reduction factor \( \phi \) is equal to 0.9, and \( jd \) will be taken equal to \( 0.929 \times d \) since we are designing for the negative moment in a rectangular beam (narrow compression zone). The assumptions will be verified once the area of steel is finalized.

Assume \( jd = 0.929 \times d = 0.929 \times 18.00 = 16.73 \text{ in.} \)

Exterior beam width, \( b = 24 \text{ in.} \)

The required reinforcement at initial trial is calculated as follows:

\[ A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{326.31 \times 12,000}{0.90 \times 60,000 \times 16.73} = 4.336 \text{ in.}^2 \]

Recalculate ‘\( a \)’ for the actual \( A_s = 4.336 \text{ in.}^2 \):

\[ a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{4.336 \times 60,000}{0.85 \times 5,000 \times 24} = 2.550 \text{ in.} \]

\[ c = \frac{a}{\beta_t} = \frac{2.550}{0.85} = 3.000 \text{ in.} \]

\[ \varepsilon_t = \left( \frac{0.003}{c} \right) \times d - 0.003 = \left( \frac{0.003}{3.000} \right) \times 18.00 - 0.003 = 0.015 > 0.005 \]
Therefore, the assumption that section is tension-controlled is valid.

\[
A_s = \frac{M_u}{\phi \times f_y \times \left( d - \frac{a}{2} \right)} = \frac{326.31 \times 12,000}{0.90 \times 60,000 \times \left( 18.00 - \frac{2.550}{2} \right)} = 4.336 \text{ in.}^2
\]

The minimum reinforcement shall not be less than

\[
A_{s, \text{min}} = \frac{3 \sqrt{f_y^2}}{f_y} \times b_w \times d = \frac{3 \times \sqrt{5,000}}{60,000} \times 24 \times 18 = 1.53 \text{ in.}^2
\]

*ACI 318-14 (9.6.1.2(a))*

And not less than

\[
A_{s, \text{min}} = \frac{200}{f_y} \times b_w \times d = \frac{200}{60,000} \times 24 \times 18 = 1.44 \text{ in.}^2
\]

*ACI 318-14 (9.6.1.2(b))*

All the values on the following table are calculated based on the procedure outlined above.

<table>
<thead>
<tr>
<th>Table 14 - Reinforcing Design Summary (Flexure only) for Exterior Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Moment, $M_u$ (ft-kips)</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>Exterior Negative</td>
</tr>
<tr>
<td>Positive</td>
</tr>
<tr>
<td>Interior Negative</td>
</tr>
<tr>
<td>Positive</td>
</tr>
<tr>
<td>Exterior Negative</td>
</tr>
<tr>
<td>Positive</td>
</tr>
<tr>
<td>Effective depth, $d$ (in.)</td>
</tr>
<tr>
<td>Positive</td>
</tr>
<tr>
<td>Negative</td>
</tr>
<tr>
<td>$A_s$ req'd (in.²)</td>
</tr>
<tr>
<td>Positive</td>
</tr>
<tr>
<td>Negative</td>
</tr>
<tr>
<td>$A_s$ min (in.²)</td>
</tr>
<tr>
<td>Positive</td>
</tr>
<tr>
<td>Negative</td>
</tr>
</tbody>
</table>

* The beam top bars are to be placed below the joist top bars.

** The beam bottom bars are to be placed at the bottom-most layer. The joist bottom bars, then, shall be spliced at joist-beam intersection.

Torsion requirements for longitudinal steel have to be determined and combined with reinforcement area required for flexure.
Calculate the required reinforcement to resist torsion:

Check if torsional effects can be neglected:

If $T_u < \phi T_{th}$, it shall be permitted to neglect torsional effects. \textit{ACI 318-14 (22.7.1.1)}

Where:

$T_u = 403.59$ ft-kips

$\phi T_{th} = 12.47$ ft-kips = Threshold torsion (the calculation of $\phi T_{th}$ is shown in the next section)

Since $T_u > \phi T_{th}$, the torsional effects must be considered.

Check if the factored design torsion can be reduced:

It is permitted to reduce $T_u$ to $\phi T_{cr}$; due to redistribution of internal forces after torsional cracking; if the exterior continuous beam meet the following requirements: \textit{ACI 318-14 (22.7.3.2)}

1. The beam is statically indeterminate (continuous beam).
2. $T_u \geq \phi T_{cr}$.

To check the second condition, $\phi T_{cr}$ need to be calculated as follows:

Since the beams are cast monolithically with slab and joists, $A_{cp}$ (area enclosed by outside perimeter of concrete cross section) and $p_{cp}$ (outside perimeter of concrete cross section) for the beam can include a portion of the adjoining slab. The effective width, $b_e$, of the overhanging flange must conform to \textit{ACI 318-14 (8.4.1.8)}:

$b_e = h - h_f = 21 - 5 = 16$ in. (governs)

$b_e = 4 \times h_f = 4 \times 5 = 20$ in.

$A_{cp} = (21 \times 24) + (16 \times 5) = 584$ in.$^2$

$p_{cp} = 2 \times (21 + 24 + 16) = 122$ in.

$\frac{A_{cp}^2}{p_{cp}} = \frac{(584)^2}{122} = 2795.54$ in.$^3$

The torsional properties of the beam ignoring the overhanging flange are the following:

$A_{cp} = (21 \times 24) = 504$ in.$^2$
\( P_{cp} = 2 \times (21 + 24) = 90 \text{ in.} \)

\[
\frac{A_{cp}^2}{P_{cp}} = \frac{(504)^2}{90} = 2822.40 \text{ in.}^3 > 2795.54 \text{ in.}^3
\]

Therefore, consider the rectangular section.  

\[\phi T_u = \phi \times 4 \times \lambda \times \sqrt{T_c} \times \left( \frac{A_{cp}^2}{P_{cp}} \right) \quad ACI \ 318-14 \ (Table \ 22.7.5.1(a))\]

\[\phi T_u = 0.75 \times 4.00 \times 1.00 \times \sqrt{5,000 \times 2822.40} = 598,721.45 \text{ in-lb} = 49.89 \text{ ft-kips}\]

\[\phi T_u = \phi \times \lambda \times \sqrt{T_c} \times \left( \frac{A_{cp}^2}{P_{cp}} \right) = \frac{\phi T_u}{4} = 12.47 \text{ ft-kips}\]

Checking the second condition of  \( ACI \ 318-14 \ (22.7.3.2) \):

\[T_u = 406.59 \text{ ft-kips} > \phi T_u = 49.89 \text{ ft-kips}\]

Thus,  \( ACI \ 318-14 \) permits to reduce \( T_u \) to \( \phi T_u \).

\[T_u = \phi T_u = 49.89 \text{ ft-kips} \quad ACI \ 318-14 \ (22.7.3.2)\]

It is assumed that the torsional loading on the beam is uniformly distributed along the span.

Determine the adequacy of cross-sectional dimensions for the torsion:

For solid sections, the limit on shear and torsion is given by:

\[\sqrt{\left( \frac{V_u}{b_u \times d} \right)^2 + \left( \frac{T_u \times p_h}{1.70 \times A_{sh}^2} \right)^2} \leq \phi \times \left( \frac{V_u}{b_u \times d} + 8 \times \sqrt{T_c} \right) \quad ACI \ 318-14 \ (22.7.7.1)\]

Where:

\[A_{sh} = \text{area enclosed by centerline of outermost closed transverse torsional reinforcement.}\]

\[p_h = \text{perimeter of centerline of outermost closed transverse torsional reinforcement.}\]

Using \( d = 18.00 \text{ in.} \), the factored shear force at the critical section located at a distance \( d \) from the face of the support is:

\[V_u = 66.22 - 4.06 \times \frac{18.00}{12} = 60.13 \text{ kips}\]
Also, the nominal shear strength provided by the concrete is:

\[ V_c = 2 \times \lambda \times \sqrt{f'_c \times b_w \times d} \quad \textit{ACI 318-14 (Eq. 22.5.5.1)} \]

Using a 1.50-in. clear cover to #3 closed stirrups at bottom and 2.125 in clear cover to #3 closed stirrups at top.

\[ A_{sd} = \left[ 21 - \left( (2.125 + 1.50) + 0.375 \right) \right] \times \left[ 24 - \left( (1.50 + 1.50) + 0.375 \right) \right] = 350.63 \text{ in}^2 \]

\[ p_s = 2 \times \left\{ 21 - \left( (2.125 + 1.50) + 0.375 \right) \right\} + \left[ 24 - \left( (1.50 + 1.50) + 0.375 \right) \right] = 75.25 \text{ in.} \]

\[ \sqrt{\frac{60,125.43}{24 \times 18}} + \left( \frac{49.89 \times 12,000 \times 75.25}{1.70 \times 350.63^2} \right)^{\frac{1}{2}} = 256.60 \text{ psi} \]

\[ < 0.75 \times \left[ \frac{2 \times \sqrt{5,000 \times 24 \times 18}}{24 \times 18} + 8 \times \sqrt{5,000} \right] = 530.33 \text{ kips} \]

Therefore, the section is adequate.

Determine the transverse reinforcement required for torsion:

\[ \frac{A_t}{s} = \frac{T_u}{\phi \times 2 \times A_o \times f_{ct} \times \cot(\theta)} \quad \textit{ACI 318-14 (Eq. 22.7.6.1a)} \]

Where:

\[ A_o = 0.85 \times A_{oh} = 0.85 \times 350.63 = 298.03 \text{ in}^2 \quad \textit{ACI 318-14 (22.7.6.1.1)} \]

\[ \theta = 45^\circ \quad \textit{ACI 318-14 (22.7.6.1.2(a))} \]

Therefore,

\[ \frac{A_t}{s} = \frac{49.89 \times 12,000}{0.75 \times 2 \times 298.03 \times 60,000 \times \cot(45^\circ)} = 0.0223 \text{ in}^2/\text{in per leg} \]
Determine the transverse reinforcement required for shear:

From Table 12 above, the maximum shear value occurs at the face of the first interior support in the end span.

\[
1.15 \times \frac{w_c \times l_e}{2} = 1.15 \times \frac{4.06 \times 28.33}{2} = 66.22 \text{ kips}
\]

The design shear at a distance, \(d\), away from the face of support,

\[
V_u = 66.22 - 4.06 \times \frac{18}{12} = 60.13 \text{ kips}
\]

Shear strength provided by concrete

\[
\phi V_c = \phi \left(2 \times \sqrt{f_{y}^t \times b_x \times d}\right)
\]

\[
\phi V_c = 0.75 \times \left(2.00 \times 1.00 \times \sqrt{5,000 \times 24 \times 18}\right) = 45,820.52 \text{ lb} = 45.82 \text{ kips}
\]

Since \(V_u > \frac{\phi V_c}{2}\), shear reinforcement is required.

The nominal shear strength required to be provided by shear reinforcement is

\[
V_s = V_u - V_c = \frac{V_u}{\phi} - V_c = \left(\frac{60.13}{0.75}\right) - 61.09 = 19.07 \text{ kips}
\]

Check whether \(V_s\) is less than \(8 \times \sqrt{f_{y}^t \times b_x \times d}\)

If \(V_s\) is greater than \(8 \times \sqrt{f_{y}^t \times b_x \times d}\), then the cross-section has to be revised as ACI 318-14 limits the shear capacity to be provided by stirrups to \(8 \times \sqrt{f_{y}^t \times b_x \times d}\).

\[
8 \times \sqrt{f_{y}^t \times b_x \times d} = 8 \times \sqrt{5,000 \times 24 \times 18} = 244,376.10 \text{ lb} = 244.38 \text{ kips}
\]

Since \(V_s\) does not exceed \(8 \times \sqrt{f_{y}^t \times b_x \times d}\), the cross section is adequate.

Calculate the required transverse reinforcement for shear as

\[
\frac{A_v}{s} = \frac{V_s - \phi V_c}{\phi \times f_{y}^t \times d} = \frac{60.13 - 45.82}{0.75 \times 60.000 \times 18} = 0.0177 \text{ in}^2
\]

ACI 318-14 (22.5.10.5.3)
Calculate total required transverse reinforcement for combined shear and torsion:

\[
\frac{A_s}{s} + 2 \times \frac{A_s}{s} = 0.0177 + 2 \times 0.0223 \text{ in.}^2/\text{in. leg} = 0.0623 \text{ in.}^2/\text{in.}
\]

Minimum transverse reinforcement for shear and torsion is calculated as follows:

\[
\left( \frac{A_s + 2 \times A_t}{s} \right)_{\text{min}} = \text{greater of } \left[ 0.75 \times \sqrt{f_y} \times \left( \frac{b_w}{f_y} \right) \right] \quad \text{ACI 318-14 (9.6.4.2)}
\]

\[
\left( \frac{A_s + 2 \times A_t}{s} \right)_{\text{min}} = \text{greater of } \left[ 0.75 \times \sqrt{5,000} \times \left( \frac{24}{60,000} \right) \right] = \left[ 0.0212 \right] = 0.0212 \text{ in.}^2/\text{in} < 0.0623 \text{ in.}^2/\text{in}
\]

Then, provide \( \frac{A_s}{s} + 2 \times \frac{A_s}{s} = 0.0623 \text{ in.}^2/\text{in.} \)

Calculate the required spacing:

Maximum spacing of transverse torsion reinforcement:

\[
s_{\text{max}} = \text{lesser of } \left[ \frac{p_b}{8} \right]_{\text{12 in.}} \quad \text{ACI 318-14 (9.7.6.3.3)}
\]

\[
s_{\text{max}} = \text{lesser of } \left[ 75.25 / 8 \right]_{\text{12 in.}} = \text{lesser of } \left[ 9.41 \text{ in.} \right]_{\text{12 in.}} = 9.41 \text{ in.}
\]

Maximum spacing of transverse shear reinforcement:

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ACI 318-14 (9.7.6.2.2).

Check whether \( V_s \) is less than \( 4 \times \sqrt{f'_e} \times b_v \times d \)

\[
4 \times \sqrt{f'_e} \times b_v \times d = 4 \times \sqrt{5,000} \times 24 \times 18 = 122,188.05 \text{ lb} = 122.19 \text{ kips} > V_s = 19.07 \text{ kips}
\]

Therefore, maximum stirrup spacing shall be the smallest of \( d/2 \) and 24 in.
Using a bundle of 2-#3 closed stirrups with 2 legs (area per leg = 0.22 in.²), the required spacing, $s$, at the critical section is:

$s = 0.44 / 0.0623 = 7.06$ in, < 9.00 in.

Provide a bundle of 2-#3 closed stirrups with 2 legs spaced at 7 in. on center. #3 bars are selected for consistency with the transverse reinforcement size used for the joist and interior beam. The stirrups are bundled at outer legs to maintain $A_0$ value in calculation of $A_t/s$ for torsion. The bundle of 2-#3 bars are defined as user-defined reinforcement of size #2 in spBeam Program.

The designer may choose to utilize #4 closed stirrups with 2 legs at 6 in. on center alternatively.

In view of the shear and torsion distribution along the span length, this same reinforcement and spacing can be provided throughout the span length.

Calculate the additional required longitudinal reinforcement for torsion:

$$A_t = \frac{T_p \times \phi p_k}{f_y \times A_y \times \frac{\theta}{\cot \theta}}$$

Where:

$$A_o = 0.85 \times A_{oh} = 0.85 \times 350.63 = 298.03 \text{ in.}^2$$

$\theta = 45^\circ$

Therefore,

$$A_t = \frac{49.89 \times 12,000 \times 75.25}{0.75 \times 2.00 \times 298.03 \times 60,000 \times \cot (45^\circ)} = 1.680 \text{ in.}^2$$

The minimum total area of longitudinal torsional reinforcement:

$$A_{t,min} = \text{lesser of } \left( \frac{5 \times f_y \times A_{tp}}{f_y} - \left( \frac{A_t}{s} \right) \times p_k \times \frac{f_y}{f_y} \right)$$

$$A_{t,min} = \text{lesser of } \left[ \frac{5 \times f_y \times A_{tp}}{f_y} - \left( 25 \times b_y \right) \times p_k \times \frac{f_y}{f_y} \right]$$
\[ A_{l,\text{min}} = \text{lesser of } \left[ \frac{5 \times \sqrt{5,000 \times 504}}{60,000} \cdot \left( 0.022 \times 75.25 \times \frac{60,000}{60,000} \right) \right] \quad \left[ \frac{5 \times \sqrt{5,000 \times 504}}{60,000} \cdot \left( 25 \times 24 \times 75.25 \times \frac{60,000}{60,000} \right) \right] = \begin{bmatrix} 1.29 \\ 2.22 \end{bmatrix} = 1.29 \text{ in.}^2 \]

Since \( A_l > A_{l,\text{min}} \), use \( A_l = 1.680 \text{ in.}^2 \).

The longitudinal reinforcement is to be distributed around the perimeter of the stirrups, with a maximum spacing of 12 in. There shall be at least one longitudinal bar in each corner of the stirrups.

\textit{ACI 318-14 (9.7.5.1)}

Longitudinal bars shall have a diameter at least 0.042 times the stirrup spacing, but not less than 3/8 in.

\textit{ACI 318-14 (9.7.5.2)}

To meet the maximum spacing requirement, a bar has to be provided between corner bars at all four sides. This configuration leads to eight-bars; three at top, three at bottom, and one at each side. Therefore, the reinforcement area per bar is \( A_s = \frac{1.680}{8} = 0.21 \text{ in.}^2 \).

Then, use #5 bars for longitudinal bars which also meets minimum bar diameter requirement of 3/8 in. \( A_l \) shall be provided in addition to the required flexural reinforcement at the negative moment regions (support-top) and positive moment region (mid-span-bottom). At mid-span-top region where flexural reinforcement is not required for flexure, 3-#5 bars shall be provided. Class B lap splice is to be provided.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{Table 15 - Reinforcing Design Summary (Flexure + Torsion)} & \multicolumn{2}{c|}{\textbf{End Span}} & \multicolumn{2}{c|}{\textbf{Interior Span}} \\
\hline
\textbf{Required Longitudinal Reinforcement (in.}^2{)} & Top Reinforcing for Exterior Negative Moment & Bottom Reinforcing for Positive Moment & Top Reinforcing for Interior Negative Moment & Bottom Reinforcing for Positive Moment & Top Reinforcing for Interior Negative Moment \\
\hline
\hline
\textbf{Required Total Longitudinal Reinforcement (in.}^2{)} & \begin{tabular}{c} 1.680 \times (3/8) = 0.63 \\ \end{tabular} & \begin{tabular}{c} 1.680 \times (3/8) = 0.63 \\ \end{tabular} & \begin{tabular}{c} 1.680 \times (3/8) = 0.63 \\ \end{tabular} & \begin{tabular}{c} 1.680 \times (3/8) = 0.63 \\ \end{tabular} & \begin{tabular}{c} 1.680 \times (3/8) = 0.63 \\ \end{tabular} \\
\hline
\textbf{Reinforcement} & 5-#8 & 5-#8 & 7-#8 & 4-#8 & 6-#8 \\
\hline
\end{tabular}
\end{table}
Maximum spacing allowed:

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

\[ s = 15 \times \left( \frac{40,000}{f_y} \right) - 2.50 \times C_c \leq 12 \times \left( \frac{40,000}{f_y} \right) \]

\[ C_c = 21 - \left( 18 + 0.50 \times \left( \frac{8}{8} \right) \right) = 2.50 \text{ in.} \]

Use \( f_y = \frac{2}{3} \times f_y = \frac{2}{3} \times 60,000 = 40,000 \text{ psi} \)

\[ s = 15 \times \left( \frac{40,000}{40,000} \right) - 2.50 \times 2.50 = 8.75 \text{ in. (governs)} \]

\[ s = 12 \times \left( \frac{40,000}{40,000} \right) = 12 \text{ in.} \]

Spacing provided for 4-#8 bars

\[ s_{prov} = \frac{b_w - 2 \times d_s}{\# of bars - 1} = \left( \frac{24 - 2 \times 2.625}{3} \right) = 6.25 \text{ in.} < 8.75 \text{ in.} \text{ o.k.} \]

Where \( d_s = 2.625 \text{ in.} \) for #3 stirrup.

CRSI 2002 (Figure 12-9)

Check the spacing, \( s \) provided, is greater than the minimum center to center spacing, \( s_{min} \) where:

\[ s_{min} = d_b + \max \left\{ \frac{1}{d_b} \left( \frac{1.33 \times \text{max.agg.}}{1} \right) \right\} \]

CRSI 2002 (Figure 12-9)

Where maximum aggregate size is \( \frac{3}{4}'' \)

\[ s_{min} = 1.00 + \max \left\{ \frac{1.00}{1.00} \left( \frac{1.33 \times 0.75}{1} \right) \right\} = 1.00 + 1.00 = 2 \text{ in.} \]

Spacing provided for 7-#8 bars

\[ s_{prov} = \frac{b_w - 2 \times d_s}{\# of bars - 1} = \left( \frac{24 - 2 \times 2.625}{6} \right) = 3.13 \text{ in.} > 2.00 \text{ in.} \text{ o.k.} \]

Therefore, the reinforcement selections in the previous table meet the spacing requirements.
2.4.4. Deflections

Since the preliminary beam depth met minimum depth requirement, the deflection calculations are not required. A lesser depth maybe possible and consequently cost savings can be achieved through deflection computations. Deflection values are calculated and provided for every model created by spBeam Program and can be used by the engineer to make additional optimization decisions.

2.4.5. Computer Program Solution

spBeam Program can be utilized to analyze and design the exterior continuous beam along grid A. The beam is modeled as a three span continuous rectangular beam.

The program calculates the internal forces (shear force and bending moment), moment and shear capacities, immediate and long-term deflection results, and required flexural reinforcement. The graphical and text results are provided here for both input and output of the spBeam model.

The beam is modeled as a 24 in. by 21 in. deep rectangular longitudinal beam with column supports at 100% stiffness share.

![Figure 36 - spBeam Reinforcement (User-Defined Bar Set)](image)

The reinforcement database is selected as User-defined in order to define a bundle of 2-#3 bars as #2 with cross-sectional area of 0.22 in². (Different than #2 defined earlier for welded wire).
Torsion analysis was engaged using the torsion analysis and design check box located on the solve options tab in the input dialog box. The design for torsion is based on a thin-walled tube, space truss analogy. spBeam allows both equilibrium and compatibility torsion conditions. In the equilibrium mode, which is assumed by default, unreduced total value of the torsional design moment is used in the design. In the compatibility mode, factored torsional moments that exceed cracking moment $T_{cr}$ are reduced to the value of $T_{cr}$. However, it is user’s responsibility to determine which mode is appropriate and the program does not perform any redistribution of internal forces if compatibility torsion is selected. In this model, the following solve options were used.

![spBeam Solve Options](image)

Figure 37 – spBeam Solve Options
Figure 38 – spBeam Model – Isometric View – Exterior Continuous Beam along Grid A
**Figure 39 – spBeam Model – Loads (Including Live Load Patterning)**
Figure 40 – **spBeam** Model – Internal Forces (Shear Force Diagram and Bending Moment Diagram)
Figure 41 – spBeam Model – Moment Capacity Diagram
Figure 42 – spBeam Model – Shear Capacity Diagram
Figure 43 – spBeam Model – Immediate Deflection Diagram
Figure 44 – spBeam Model – Reinforcement Diagram
2.4.6. Summary and Comparison of Results

1. Design Results

1.1. Top Reinforcement

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<th>Width ft</th>
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<th>(X_{\text{max}}) ft</th>
<th>(A_{\text{c,req}}) in²</th>
<th>(A_{\text{c,req}}) in²</th>
<th>(S_{\text{P,req}}) in</th>
<th>Bars</th>
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1.2. Bottom Reinforcement

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<th>(A_{\text{c,req}}) in²</th>
<th>(S_{\text{P,req}}) in</th>
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<td>161.80</td>
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<td>1.580</td>
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<td>2.194</td>
<td>9.552</td>
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</table>

1.3. Longitudinal Beam Shear and Torsion Reinforcement Required

1.3.1. Transverse Reinforcement Demand

Notes:
*4 - Design torsional moment reduced to PhiTcr due to compatibility torsion.
1.3.2. Required Longitudinal Reinforcement

Notes:
*4 - Design torsional moment reduced to PhiTrcr due to compatibility torsion.
*5 - Minimum longitudinal reinforcement required.

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<th>$T_u$</th>
<th>Comb/Patt</th>
<th>$A_t$</th>
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<td>ft</td>
<td>k-ft</td>
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<td>in²</td>
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<td>24.05</td>
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<td>U/All</td>
<td>1.680 *4</td>
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<table>
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<td>$A_t/s$ (in.²/in.)</td>
</tr>
<tr>
<td>Hand</td>
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</table>

Table 16 - Comparison of Hand Solution with spBeam Solution for Exterior Beam

Table 17 - Comparison of Hand Solution with spBeam Solution (Reinforcement)
2.4.7. Conclusions and Observations

In this design example, the exterior beam is modeled as a continuous rectangular longitudinal beam. There is a good agreement between the hand solution and computer solution. Note that the coefficients traditionally used to determine moments do not address various types of support and geometry.

The maximum calculated total immediate (instantaneous) deflection \((DL + LL) = 0.368 \text{ in.}\), this value can be compared with maximum permissible calculated deflection limitation per project criteria in accordance to \textit{ACI 318-14}.

In addition to deflection results, parametric studies can be performed in spBeam to optimize design and detailing results.

The following observations can be made regarding the reinforcement diagram (Figure 44):

- The longitudinal reinforcement for the flexural design only and does not include \((A_t)\).
- \(A_t\) is shown in the text output report (Design Results) because it has to be detailed and distributed to meet the torsional spacing requirements:
  - At least one longitudinal bar must be present at each corner of the stirrups.
  - And a bar has to be provided between corner bars at all four sides to meet the maximum spacing requirement (max 12 in.).
  - This configuration leads to eight-bars in this example: three at top, three at bottom, and one at each side.
- Then, for the top bar for the interior negative section for the end span we need:
  1) The longitudinal reinforcement for the flexural design 5-%8 is required for flexure.
  2) The longitudinal reinforcement for the torsional design 3-%8 (3 top bars of the 8 bars distributed around the perimeter of the stirrups). See Table 17.
- Also, for the bottom bar for the positive section for the end span we need:
  3) The longitudinal reinforcement for the flexural design 3-%8 is required for flexure.
  4) The longitudinal reinforcement for the torsional design 3-%8 (3 bottom bars of the 8 bars distributed around the perimeter of the stirrups). See Table 17.
- Transverse reinforcement shown reflects the total size and quantity of stirrups to resist the combined effects of shear and torsion.
- Top bars minimum length required (including the development length) for flexural design is shown. The bars can be extended and detailed to provide the required support for shear stirrups.
2.5. Design of Interior, Edge, and Corner Columns

This section includes the design of interior, edge, and corner columns using spColumn software. The preliminary dimensions for these columns were calculated previously in Section 1.2.

2.5.1. Determination of factored loads

**Interior Column:**

Total Factored Load on 1st story interior column (@ 1st interior support) are reorganized based on the calculations on Section 1.2 as follows:

\[
P_D = \text{Total service dead load} = 1.15 \times \left[ \left( 4 \times (82.88 + 20) + 1 \times 82.88 \right) \times 960 \right]/1000 + 36.00 = 581.79 \text{ kips}
\]

\[
P_L = \text{Total service live load} = 1.15 \times \left[ \left( 4 \times (39.36) + 1 \times 12 \right) \times 960 \right]/1000 = 187.08 \text{ kips}
\]

\[
P_u = 1.20 \times P_D + 1.60 \times P_L = 1.20 \times 581.79 + 1.60 \times 187.08 = 997.48 \text{ kips}
\]

\[
M_{ux} = \text{unbalance flexural moment at the Interior support for the interior beam (see Figure 29)}
\]

\[
= 595.20 - 545.96 = 49.24 \text{ ft-kips}
\]

\[
M_{ux} = \text{unbalance flexural moment at the interior support for the joist (see Figure 18)}
\]

\[
= 154.53 - 141.78 = 12.75 \text{ ft-kips}
\]
The factored loads are then input into spColumn to construct the axial load – moment interaction diagram as shown in the sample input below.

**Figure 46 – spColumn Input Loads**

**Edge (Exterior) Column:**

Total Factored Load on 1st story edge column (@ 1st interior support) are reorganized based on the calculations on Section 1.2 as follows:

\[
P_D = \text{Total service dead load} = 1.15 \times \left[ \left( 4 \times (82.88 + 20) + 1 \times 82.88 \right) \times 480 \right] / 1000 + 25.00 = 297.90 \text{ kips}
\]

\[
P_L = \text{Total service live load} = 1.15 \times \left[ \left( 4 \times (47.39) + 1 \times 14.40 \right) \times 480 \right] / 1000 = 112.58 \text{ kips}
\]

\[
P_u = 1.20 \times P_D + 1.60 \times P_L = 1.20 \times 297.70 + 1.60 \times 112.58 = 537.60 \text{ kips}
\]

\[
M_u,x = \text{unbalance flexural moment at the Interior support (see Figure 40)}
\]

\[
= 333.50 - 317.49 = 16.01 \text{ ft-kips}
\]

\[
M_u,y = \text{torsional moment at the Interior support for the exterior beam (see Section 2.4.3) + unbalance flexural moment at exterior support for the joist (see Figure 18)}
\]

\[
= 2 \times T_u + 73.96 = 2 \times 49.89 + 73.96 = 173.75 \text{ ft-kips}
\]
Corner Column:

Total Factored Load on 1st story corner column (@ exterior support) are reorganized based on the calculations on Section 1.2 as follows:

\[ P_D = \text{Total service dead load} = 1.15 \times \left[ (4 \times (82.88 + 20) + 1 \times 82.88) \times 240 \right] / 1000 + 25.00 = 161.45 \text{ kips} \]

\[ P_L = \text{Total service live load} = 1.15 \times \left[ (4 \times (58.76) + 1 \times 19.20) \times 240 \right] / 1000 = 70.14 \text{ kips} \]

\[ P_s = 1.20 \times P_D + 1.60 \times P_L = 1.20 \times 161.45 + 1.60 \times 70.14 = 305.96 \text{ kips} \]

\[ M_{\alpha x} = \text{unbalance flexural moment at the Exterior support (see Figure 40)} = 249.96 \text{ ft-kips} \]

\[ M_{\alpha y} = \text{torsional moment at the Exterior support for the exterior beam (see Section 2.4.3) + unbalance flexural moment at exterior support for the joist (see Figure 18)} \]

\[ = T_x + 73.96 = 49.89 + 73.96 = 123.85 \text{ ft-kips} \]
2.5.2. Column Capacity Diagram (Axial-Moment Interaction)

The axial force and moment interaction diagram is constructed using spColumn based on geometry and load input shown below:

**Interior Column:***

![Figure 47 – spColumn Model – Biaxial Moment Interaction](image)
**Figure 48 – spColumn Model – Axial Moment Interaction**

- **General Information**
  - **Project:** Wide-Module Joint System
  - **Column:** Interior Column
  - **Engineer:** EP
  - **Code:** ACI 318-14
  - **Bar Set:** ASTM A615
  - **Units:** English
  - **Run Option:** Design
  - **Run Axes:** Biaxial
  - **Slenderness:** Not Considered
  - **Column Type:** Structural
  - **Capacity Method:** Moment capacity

- **Materials**
  - $f_y = 5$ ksi
  - $E_s = 4030.51$ ksi
  - $f_x = 60$ ksi
  - $E_s = 29000$ ksi

- **Section**
  - Type: Rectangular
  - Width: 24 in
  - Depth: 24 in
  - $A_0 = 576$ in$^2$
  - $I_x = 27648$ in$^4$
  - $I_y = 27648$ in$^4$

- **Reinforcement**
  - Pattern: All sides equal
  - Bar layout: Rectangular
  - Cover to: Transverse bars
  - Clear cover: 2 in
  - Bars: 4 @ #11

- **Confinement type:** Tied

<table>
<thead>
<tr>
<th>No.</th>
<th>$P_m$ (kip)</th>
<th>$M_{P,0}$ (k-ft)</th>
<th>$M_{P,0}$ (k-ft)</th>
<th>$\phi P_m$ (kip)</th>
<th>$\phi M_{P,0}$ (k-ft)</th>
<th>$\phi M_{P,0}$ (k-ft)</th>
<th>Capacity Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>997.5</td>
<td>49.2</td>
<td>12.8</td>
<td>997.48</td>
<td>459.56</td>
<td>119.00</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Max. Capacity Ratio: 0.11
Figure 49 – spColumn Model – Nominal and Factored Failure Surfaces
Edge (Exterior) Column:

**General Information**

<table>
<thead>
<tr>
<th>Project</th>
<th>Wide-Module Joint System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>Edge Column</td>
</tr>
<tr>
<td>Engineer</td>
<td>SP</td>
</tr>
<tr>
<td>Code</td>
<td>ACI 318-14</td>
</tr>
<tr>
<td>Bar Set</td>
<td>ASTM A615</td>
</tr>
<tr>
<td>Units</td>
<td>English</td>
</tr>
<tr>
<td>Run Option</td>
<td>Design</td>
</tr>
<tr>
<td>Run Axis</td>
<td>Biaxial</td>
</tr>
<tr>
<td>Stenderness</td>
<td>Not Considered</td>
</tr>
<tr>
<td>Column Type</td>
<td>Structural</td>
</tr>
<tr>
<td>Capacity Method</td>
<td>Moment capacity</td>
</tr>
</tbody>
</table>

**Materials**

<table>
<thead>
<tr>
<th>$f_y$</th>
<th>5 ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_y$</td>
<td>4030.51 ksi</td>
</tr>
<tr>
<td>$f_y'$</td>
<td>60 ksi</td>
</tr>
<tr>
<td>$E_y'$</td>
<td>29000 ksi</td>
</tr>
</tbody>
</table>

**Section**

<table>
<thead>
<tr>
<th>Type</th>
<th>Rectangular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>20 in</td>
</tr>
<tr>
<td>Depth</td>
<td>20 in</td>
</tr>
<tr>
<td>$A_0$</td>
<td>400 in²</td>
</tr>
<tr>
<td>$I_x$</td>
<td>13333.3 in⁴</td>
</tr>
<tr>
<td>$I_y$</td>
<td>13333.3 in⁴</td>
</tr>
</tbody>
</table>

**Reinforcement**

<table>
<thead>
<tr>
<th>Pattern</th>
<th>All sides equal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar layout</td>
<td>Rectangular</td>
</tr>
<tr>
<td>Cover to</td>
<td>Transverse bars</td>
</tr>
<tr>
<td>Clear cover</td>
<td>2 in</td>
</tr>
<tr>
<td>Bars</td>
<td>4 #11</td>
</tr>
<tr>
<td>Confinement type</td>
<td>Tied</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total steel area, $A_s$</th>
<th>6.24 in²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rho</td>
<td>1.56 %</td>
</tr>
<tr>
<td>Min. clear spacing</td>
<td>12.18 in</td>
</tr>
</tbody>
</table>

**Graph**

Figure 50 – spColumn Model – Axial Moment Interaction
Corner Column:

![Diagram of a column with dimensions and properties](image)

**General Information**
- **Project**: Wide-Flange Joist System
- **Column**: Corner Column
- **Engineer**: SP
- **Code**: ACI 318-14
- **Bar Set**: ASTM A615
- **Units**: English
- **Run Option**: Design
- **Run Axis**: Biaxial
- **Slenderness**: Not Considered
- **Column Type**: Structural
- **Capacity Method**: Moment capacity

**Materials**
- $f_y$: 5 ksi
- $E_t$: 4030.51 ksi
- $f_y$': 60 ksi
- $E_s$: 29000 ksi

**Section**
- **Type**: Rectangular
- **Width**: 20 in
- **Depth**: 20 in
- $A_t$: 400 in²
- $I_x$: 13333.3 in⁴
- $I_y$: 13333.3 in⁴

**Reinforcement**
- **Pattern**: All sides equal
- **Bar layout**: Rectangular
- **Cover to**: Transverse bars
- **Clear cover**: 2 in
- **Bars**: 4 #11
- **Confinement type**: Tied
- **Total steel area, $A_s$**: 6.24 in²
- **Rho**: 1.56 %
- **Min. clear spacing**: 12.18 in

**Table**

<table>
<thead>
<tr>
<th>No.</th>
<th>$P_{mu}$ kip</th>
<th>$M_{max}$ k-ft</th>
<th>$M_{min}$ k-ft</th>
<th>$\phi P_{mu}$ kip</th>
<th>$\phi M_{max}$ k-ft</th>
<th>$\phi M_{min}$ k-ft</th>
<th>Capacity Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>306.0</td>
<td>250.0</td>
<td>123.8</td>
<td>305.96</td>
<td>268.87</td>
<td>133.22</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Max. Capacity Ratio: 0.93

**Figure 51** – *spColumn* Model – Axial Moment Interaction