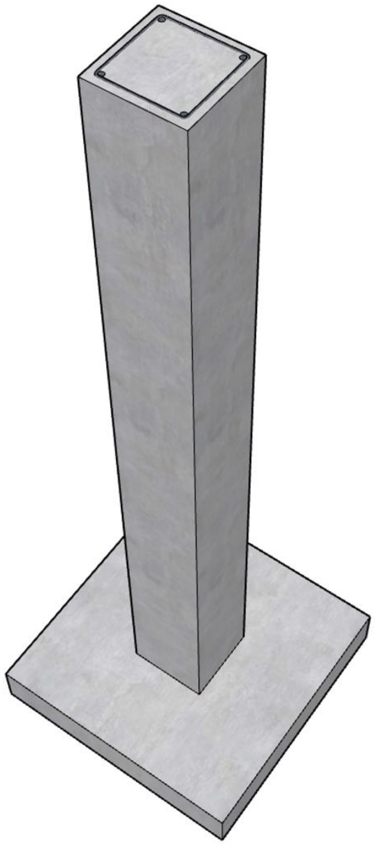
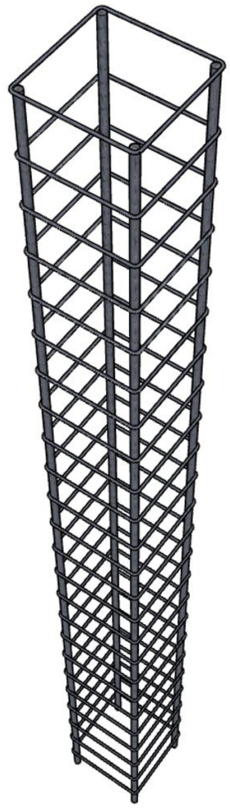
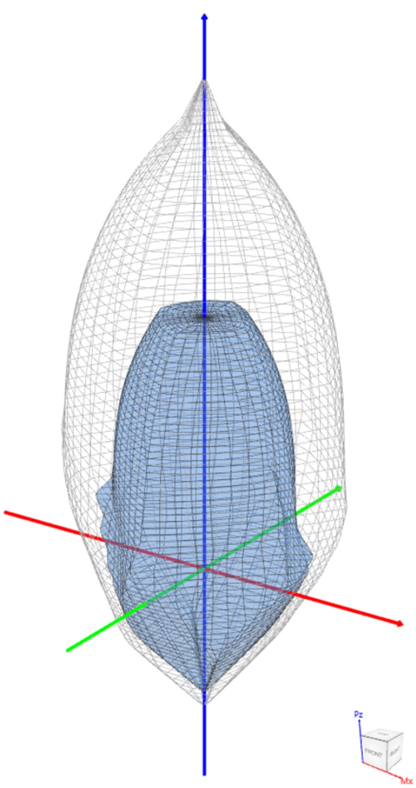
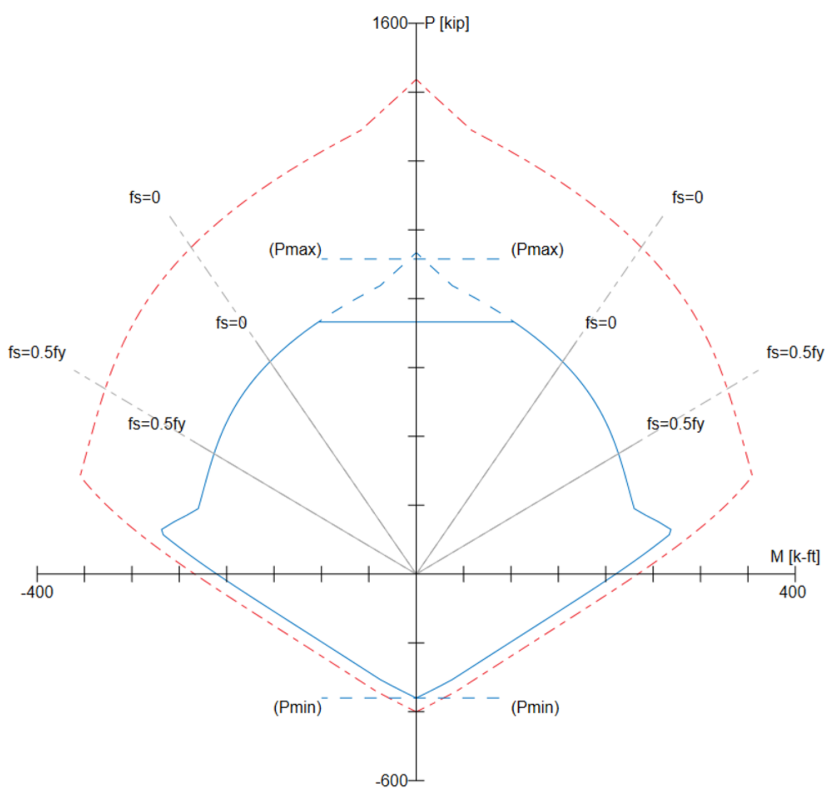
Interaction Diagram - Tied Reinforced Concrete Column with High-Strength Reinforcing Bars (ACI 318-19)









Version: February-03-2026

Interaction Diagram - Tied Reinforced Concrete Column with High-Strength Reinforcing Bars (ACI 318-19)

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using ACI 318-19. Determine seven control points on the interaction diagram and compare the calculated values with exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint. High Strength Reinforcing Bars (HSRB) with Grade 100 steel (f_y = 100 ksi) is being used to assist with congestion of reinforcement at columns/beams joints.

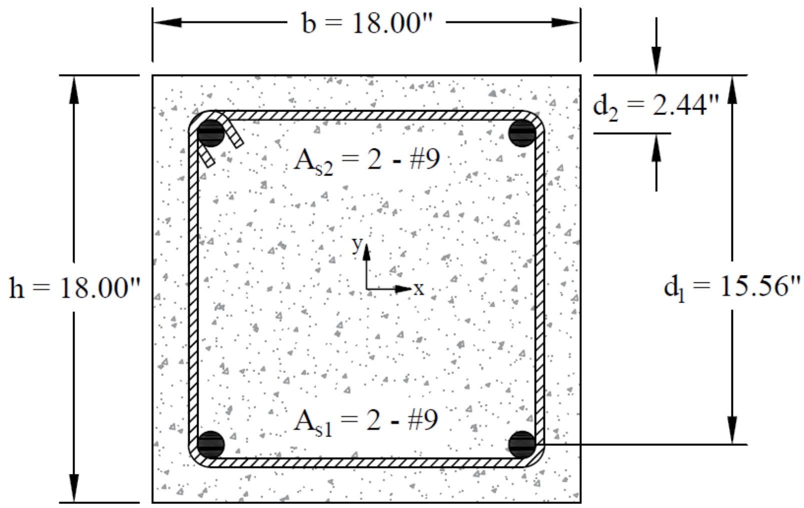


Figure 1 – Reinforced Concrete Column Cross-Section

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

- Design Guide on the ACI 318 Building Code Requirements for Structural Concrete, 1th Edition, 2020 Concrete Reinforcing Steel Institute (CRSI), Example 7.12
- "Column Design with High-Strength Reinforcing Bars per ACI 318-19" Technical Article, STRUCTUREPOINT, 2019
- "Column Design Capacity Comparison with High Strength Reinforcing Bars per ACI 318-14 and ACI 318-19" Technical Article, STRUCTUREPOINT, 2019
- "ACI 318-19 Code Revisions Impact on StructurePoint Software" Technical Article, STRUCTUREPOINT, 2019
- spColumn Engineering Software Program Manual v11.00, STRUCTUREPOINT, 2026
- "Interaction Diagram - Tied Reinforced Concrete Column Design Strength (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram - Circular Spiral Reinforced Concrete Column (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram - Barbell Concrete Shear Wall Unsymmetrical Boundary Elements (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram – Building Elevator Reinforced Concrete Core Wall Design Strength (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- Contact Support@StructurePoint.org to obtain supplementary materials (spColumn model: DE-HSRB-Fy=100-ksi-ACI-19.colx)

Design Data

f_c' = 4,000 psi

f_y = 100,000 psi

Clear Cover = 1.50 in.

Top reinforcement = 2 – #9

Bottom reinforcement = 2 – #9

Transverse reinforcement: #3 square closed ties

Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following seven control points:

Point 1: Maximum compression

Point 2: Bar stress near tension face of member equal to zero, ($f_s = 0$)

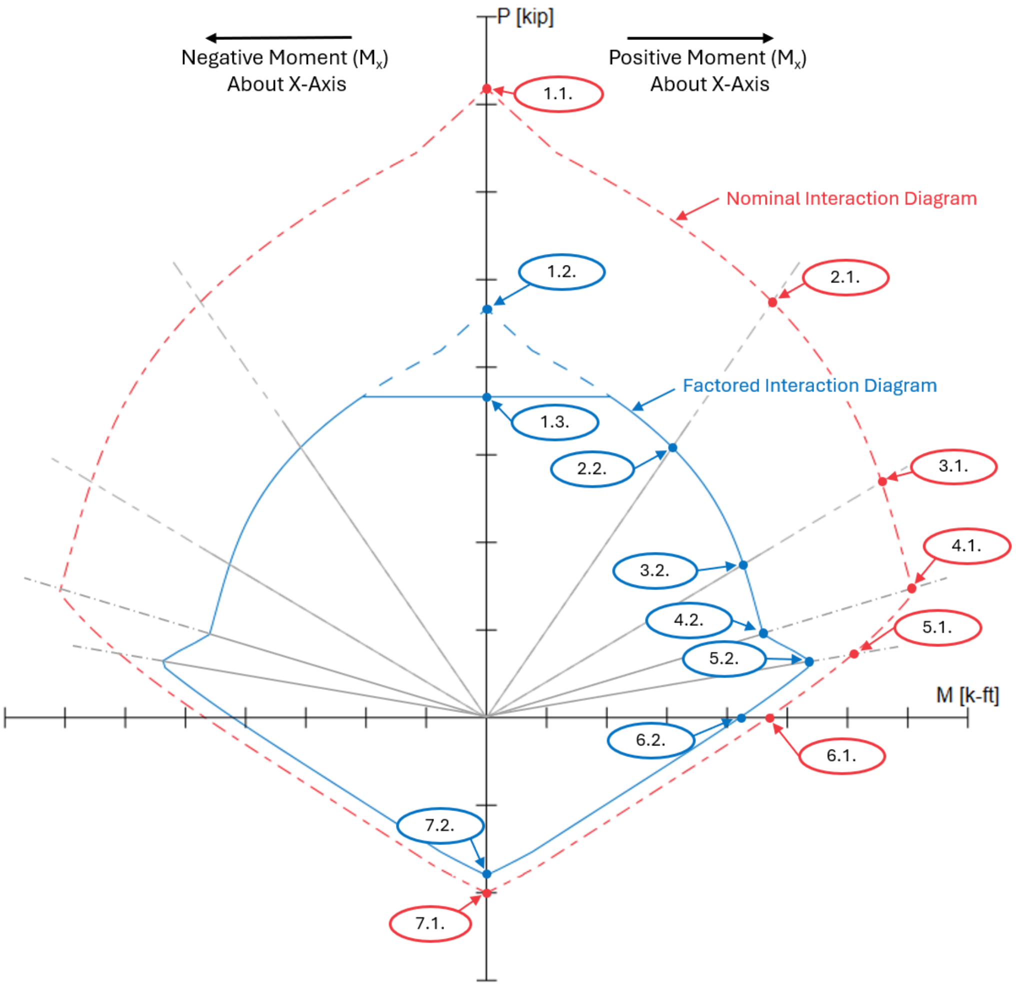
Point 3: Bar stress near tension face of member equal to 0.5 f_y ($f_s = 0.5\ f_y$)

Point 4: Bar stress near tension face of member equal to f_y ($f_s = f_y$)

Point 5: Bar strain near tension face of member equal to $\varepsilon_y + 0.003$

Point 6: Pure bending

Point 7: Maximum tension



1.1. Maximum Nominal Axial Strength in Compression (P_o)
2.1. Nominal Control Point at Zero Stress in Tension Reinforcement
3.1. Nominal Control Point at Tension Reinforcement Stress = $0.5f_y$
4.1. Nominal Control Point at Tension Reinforcement Stress = f_y
5.1. Nominal Control Point at Tension Reinforcement Stress = ε_y + 0.003 in./in.
6.1. Nominal Control Point at Pure Bending
7.1. Maximum Nominal Axial Strength in Tension

1.2. Maximum Factored Axial Strength in Compression
1.3. Allowable Factored Axial Strength in Compression ($\phi P_{n,max}$)
2.2. Factored Control Point at Zero Stress in Tension Reinforcement
3.2. Factored Control Point at Tension Reinforcement Stress = $0.5f_y$
4.2. Factored Control Point at Tension Reinforcement Stress = f_y
5.2. Factored Control Point at Tension Reinforcement Stress = ε_y + 0.003 in./in.
6.2. Factored Control Point at Pure Bending
7.2. Maximum Factored Axial Strength in Tension

Figure 2 – Tied Column Section Interaction Diagram Control Points

1. Notations

This section (based on ACI 318-19 provisions) defines notation and terminology used in this design example:

a = depth of equivalent rectangular stress block, in.

A_g = gross area of concrete section, in.^{2} For a hollow section, Ag is the area of the concrete only and does not include the area of the void(s)

A_s = area of nonprestressed longitudinal tension reinforcement, in.^{2}

A_{st} = total area of nonprestressed longitudinal reinforcement including bars or steel shapes, and excluding prestressing reinforcement, in.^{2}

c = distance from extreme compression fiber to neutral axis, in.

c_c = clear cover of reinforcement, in.

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

d_b = nominal diameter of bar, wire, or prestressing strand, in.

E_s = modulus of elasticity of reinforcement and structural steel, excluding prestressing reinforcement, psi

f_c' = specified compressive strength of concrete, psi

f_s = tensile stress in reinforcement at service loads, excluding prestressed reinforcement, psi

f_s' = compressive stress in reinforcement under factored loads, excluding prestressed reinforcement, psi

f_y = specified yield strength for nonprestressed reinforcement, psi

h = overall thickness, height, or depth of member, in.

M_n = nominal flexural strength at section, in.-lb

P_n = nominal axial compressive strength of member, lb

$P_{n,max}$ = maximum nominal axial compressive strength of a member, lb

P_{nt} = nominal axial tensile strength of member, lb

P_o = nominal axial strength at zero eccentricity, lb

β_1 = factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis

ε_{cu} = maximum usable strain at extreme concrete compression fiber

ε_t = net tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength, excluding strains due to effective prestress, creep, shrinkage, and temperature

ϕ = strength reduction factor

2. Maximum Compression

2.1. Nominal Axial Compressive Strength at Zero Eccentricity

$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$$ ***ACI 318-19 (22.4.2.2)***

$$P_o = 0.85 \times 4,000 \times \left(18 \times 18 - 4 \times 1.00\right) + 80,000 \times 4 \times 1.00 = 1,408.0 \text{ kip}$$

Note that f_y = 80 ksi is used in the previous equation since f_y is limited to 80 ksi for members subjected to pure axial compression. ***ACI 318-19 (22.4.2.1)***

2.2. Factored Axial Compressive Strength at Zero Eccentricity

Since this column is a tied column with steel strain in compression:

$$\phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_o = 0.65 \times 1,408.0 = 915.2 \text{ kip}$$

Since the section is regular (symmetrical) about the x-axis, the moment capacity associated with the maximum axial compressive strength is equal to zero.

$$M_o = \phi M_o = 0.00 \text{ kip-ft}$$

2.3. Maximum (Allowable) Factored Axial Compressive Strength

$$\phi P_{n,max} = 0.80 \times \phi P_o = 0.80 \times 915.2 = 732.2 \text{ kip}$$ ***ACI 318-19 (Table 22.4.2.1)***

3. Bar Stress Near Tension Face Equal to Zero, ($\varepsilon_s = f_s = 0$)

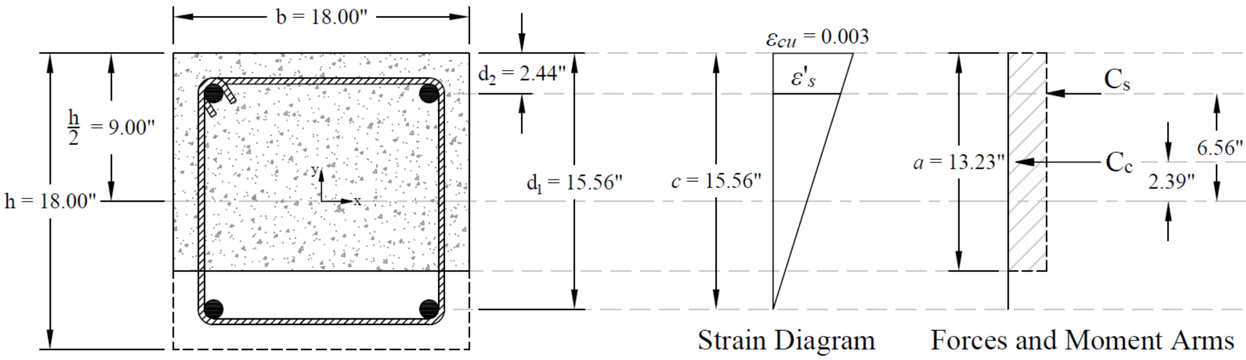


Figure 3 – Strains, Forces, and Moment Arms ($\varepsilon_t = f_s = 0$)

Strain ε_s is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. ***ACI 318-19 (10.7.5.2.1 and 2)***

3.1. *c*, *a*, and strains in the reinforcement

$$c = d_1 = 15.56 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.85 \times 15.56 = 13.23 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

a = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times \left(f_c' - 4,000\right)}{1,000} = 0.85 - \frac{0.05 \times \left(4,000 - 4,000\right)}{1,000} = 0.85$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s = 0$$

$$\therefore \phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon_s' = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (15.56 - 2.44) \times \frac{0.003}{15.56} = 0.00253 \text{ (Compression)} < \varepsilon_y = \frac{F_y}{E_s} = \frac{100}{29,000} = 0.00345$$

3.2. Forces in the Concrete and Steel

$$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 4,000 \times 13.23 \times 18 = 809.48 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = 0 \text{ psi} \rightarrow T_s = f_s \times A_{s1} = 0 \text{ kip}$$

Since $\varepsilon'_s < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f'_s = \varepsilon'_s \times E_s = 0.00253 \times 29,000,000 = 73,363.79 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\text{C}_s = (f'_s - 0.85 f'_c) \times A_{s2} = (73,363.79 - 0.85 \times 4,000) \times 2 = 139.93 \text{ kip}$$

3.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 809.48 + 139.93 - 0 = 949.4 \text{ kip}$$

$$\phi P_n = 0.65 \times 949.4 = 617.1 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 809.48 \times \left(\frac{18}{2} - \frac{13.23}{2}\right) + 139.93 \times \left(\frac{18}{2} - 2.44\right) + 0 \times \left(15.56 - \frac{18}{2}\right) = 237.5 \text{ kip-ft}$$

$$\phi M_n = 0.65 \times 237.5 = 154.37 \text{ kip-ft}$$

4. Bar Stress Near Tension Face Equal to 0.5 f_y, (f_s = 0.5 f_y)

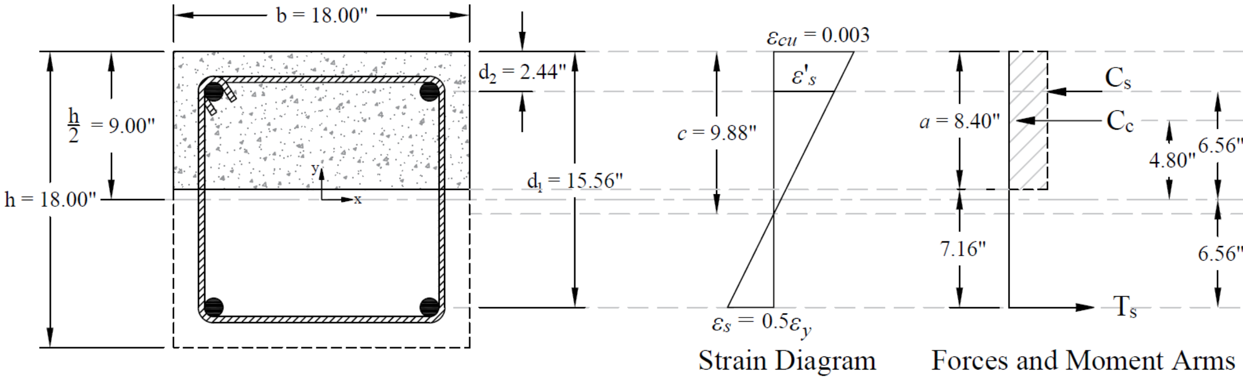


Figure 4 – Strains, Forces, and Moment Arms (f_s = 0.5 f_y)

4.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{100}{29,000} = 0.00345$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.00345}{2} = 0.00172 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$\therefore \phi = 0.65$ ***ACI 318-19 (Table 21.2.2)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{15.56}{0.00172 + 0.003} \times 0.003 = 9.88 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$a = \beta_1 \times c = 0.85 \times 9.88 = 8.40$ in. ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4,000)}{1,000} = 0.85 - \frac{0.05 \times (4,000 - 4,000)}{1,000} = 0.85$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon'_s = (c - d_2) \times \frac{0.003}{c} = (9.88 - 2.44) \times \frac{0.003}{9.88} = 0.00226 \text{ (Compression)} < \varepsilon_y$$

4.2. Forces in the Concrete and Steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 4,000 \times 8.40 \times 18 = 514.05 \text{ kip}$$ **_ACI 318-19 (22.2.2.4.1)_**

$$f_s = \varepsilon_s \times E_s = 0.00172 \times 29,000,000 = 50,000.00 \text{ psi}$$

$$T_s = f_s \times A_{s1} = 50,000 \times 2 = 100.00 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f_s' = \varepsilon_s' \times E_s = 0.00226 \times 29,000,000 = 65,526.89 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$C_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(65,526.89 - 0.85 \times 4,000\right) \times 2 = 124.25 \text{ kip}$$

4.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 514.05 + 124.25 - 100.00 = 538.3 \text{ kip}$$

$$\phi P_n = 0.65 \times 538.3 = 349.9 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 514.05 \times \left(\frac{18}{2} - \frac{8.40}{2}\right) + 124.25 \times \left(\frac{18}{2} - 2.44\right) + 100.00 \times \left(15.56 - \frac{18}{2}\right) = 328.24 \text{ kip-ft}$$

$$\phi M_n = 0.65 \times 328.24 = 213.36 \text{ kip-ft}$$

5. Bar Stress Near Tension Face Equal to f_y, ($f_s = f_y$)

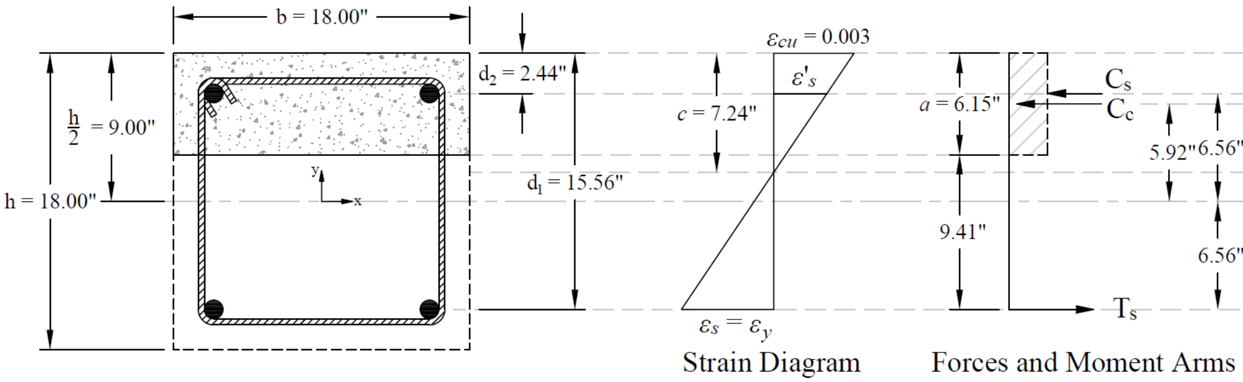


Figure 5 – Strains, Forces, and Moment Arms ($f_s = f_y$)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for ϕ for columns in which ϕ increases from 0.65 (or 0.75 for spiral columns) up to 0.90.

5.1. *c*, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{100}{29000} = 0.00345$$

$\varepsilon_s = \varepsilon_y = 0.00345 \rightarrow$ tension reinforcement has yielded

$\therefore \phi = 0.65$ **ACI 318-19 (Table 21.2.2)**

$\varepsilon_{cu} = 0.003$ **ACI 318-19 (22.2.2.1)**

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{15.56}{0.00345 + 0.003} \times 0.003 = 7.24 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis. **ACI 318-19 (22.2.2.4.2)**

$a = \beta_1 \times c = 0.85 \times 7.24 = 6.15$ in. **ACI 318-19 (22.2.2.4.1)**

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4{,}000)}{1{,}000} = 0.85 - \frac{0.05 \times (4{,}000 - 4{,}000)}{1{,}000} = 0.85$$ **ACI 318-19 (Table 22.2.2.4.3)**

$$\varepsilon'_s = (c - d_2) \times \frac{0.003}{c} = (7.24 - 2.44) \times \frac{0.003}{7.24} = 0.00199 \text{ (Compression)} < \varepsilon_y$$

5.2. Forces in the Concrete and Steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 4,000 \times 6.15 \times 18 = 376.60 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = f_y = 100,000 \text{ psi}$$

$$T_s = f_y \times A_{s1} = 100,000 \times 2 = 200.00 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f_s' = \varepsilon_s' \times E_s = 0.00199 \times 29,000,000 = 57,689.99 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$C_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(57,689.99 - 0.85 \times 4,000\right) \times 2 = 108.58 \text{ kip}$$

5.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 376.60 + 108.58 - 200.00 = 285.2 \text{ kip}$$

$$\phi P_n = 0.65 \times 285.2 = 185.4 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 376.60 \times \left(\frac{18}{2} - \frac{6.15}{2}\right) + 108.58 \times \left(\frac{18}{2} - 2.44\right) + 200.00 \times \left(15.56 - \frac{18}{2}\right) = 354.61 \text{ kip-ft}$$

$$\phi M_n = 0.65 \times 354.61 = 230.49 \text{ kip-ft}$$

6. Bar Strain Near Tension Face Equal to ε_y + 0.003, (ε_s = 0.00645 in./in.)

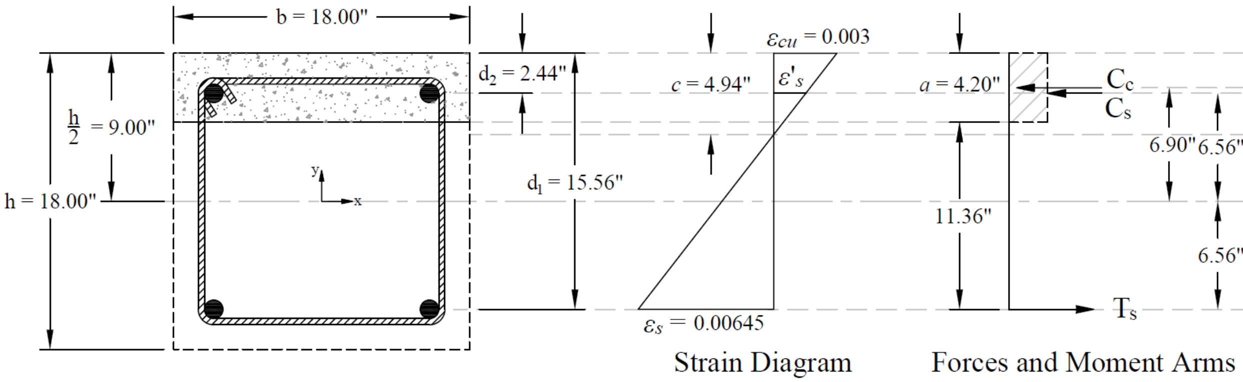


Figure 6 – Strains, Forces, and Moment Arms (ε_s = 0.00645 in./in.)

In ACI 318-19 provisions, this control point corresponds to the tension-controlled strain limit of ε_y + 0.003 (used to be 0.005 in ACI 318-14). It is the strain at the tensile limit of the transition zone for ϕ, used to define a tension-controlled section. Additional resources concerning code provision changes in ACI 318-19 can be found in "ACI 318-19 Code Revisions Impact on StructurePoint Software" technical article.

6.1. *c*, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{100}{29,000} = 0.00345$$

$$\varepsilon_s = \varepsilon_y + 0.003 = 0.00345 + 0.003 = 0.00645 > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.90$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{15.56}{0.00645 + 0.003} \times 0.003 = 4.94 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.85 \times 4.94 = 4.20 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4,000)}{1,000} = 0.85 - \frac{0.05 \times (4,000 - 4,000)}{1,000} = 0.85$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (4.94 - 2.44) \times \frac{0.003}{4.94} = 0.00152 \text{ (Compression)} < \varepsilon_y$$

6.2. Forces in the Concrete and Steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 4{,}000 \times 4.20 \times 18 = 257.03 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = f_y = 100{,}000 \text{ psi}$$

$$T_s = f_y \times A_{s1} = 100{,}000 \times 2 = 200.00 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f_s' = \varepsilon_s' \times E_s = 0.00152 \times 29{,}000{,}000 = 44{,}053.79 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$C_s = (f_s' - 0.85 f_c') \times A_{s2} = (44{,}053.79 - 0.85 \times 4{,}000) \times 2 = 81.31 \text{ kip}$$

6.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 257.03 + 81.31 - 200.00 = 138.3 \text{ kip}$$

$$\phi P_n = 0.90 \times 138.3 = 124.5 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 257.03 \times \left(\frac{18}{2} - \frac{4.20}{2}\right) + 81.31 \times \left(\frac{18}{2} - 2.44\right) + 200.00 \times \left(15.56 - \frac{18}{2}\right) = 301.60 \text{ kip-ft}$$

$$\phi M_n = 0.90 \times 301.60 = 271.44 \text{ kip-ft}$$

7. Pure Bending

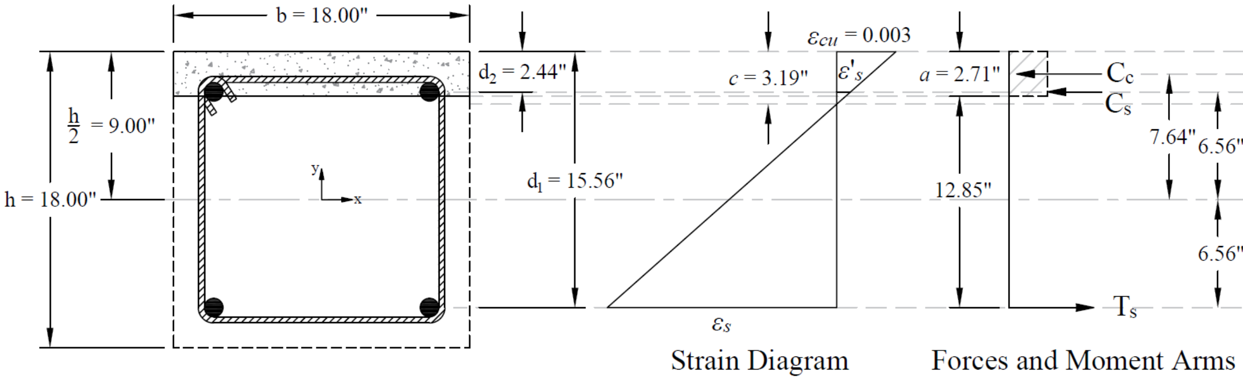


Figure 7 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the nominal axial load capacity, P_n, is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

7.1. *c*, *a*, and strains in the reinforcement

Try $c = 3.189$ in.

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$a = \beta_1 \times c = 0.85 \times 3.189 = 2.711$ in. ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4{,}000)}{1{,}000} = 0.85 - \frac{0.05 \times (4{,}000 - 4{,}000)}{1{,}000} = 0.85$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{100}{29{,}000} = 0.00345$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.003}{c}$$

$$\varepsilon_s = (15.56 - 3.189) \times \frac{0.003}{3.189} = 0.01164 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$\therefore \phi = 0.9$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c}$$

$$\varepsilon_s' = (3.189 - 2.44) \times \frac{0.003}{3.189} = 0.00071 \text{ (Compression)} < \varepsilon_y \rightarrow \text{compression reinforcement has not yielded}$$

7.2. Forces in the Concrete and Steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 4,000 \times 2.711 \times 18 = 165.88 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = f_y = 100,000 \text{ psi}$$

$$T_s = f_y \times A_{s1} = 100,000 \times 2 = 200.00 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f_s' = \varepsilon_s' \times E_s = 0.00071 \times 29,000,000 = 20,457.91 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85 f_c'$ from f_s' before computing C_s:

$$C_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(20,457.91 - 0.85 \times 4,000\right) \times 2 = 34.12 \text{ kip}$$

7.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 165.88 + 34.12 - 200.00 = 0.0 \text{ kip} \rightarrow \phi P_n = 0.0 \text{ kip}$$

The assumption that c = 3.189 in. is correct

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 165.88 \times \left(\frac{18}{2} - \frac{2.711}{2}\right) + 34.12 \times \left(\frac{18}{2} - 2.44\right) + 200.00 \times \left(15.56 - \frac{18}{2}\right) = 233.68 \text{ kip-ft}$$

$$\phi M_n = 0.90 \times 233.68 = 210.31 \text{ kip-ft}$$

8. Pure Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

8.1. P_{nt} and ϕP_{nt}

$$P_{nt} = f_y \times (A_{s1} + A_{s2}) = 100{,}000 \times (2+2) = 400.00 \text{ kip}$$ ***ACI 318-19 (22.4.3.1)***

$$\phi = 0.90$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_{nt} = 0.90 \times 400.00 = 360.0 \text{ kip}$$

8.2. M_n and ϕM_n

Since the section is symmetrical

$$M_n = \phi M_n = 0.00 \text{ kip.ft}$$

9. Column Interaction Diagram - spColumn Software

spColumn is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode was used with no loads (the program will only report control points) and no slenderness considerations using ACI 318-19.

Note that the support for 100 ksi reinforcement is currently provided in spColumn v11.00 - Beta and is only available currently to the StructurePoint advanced users group. It will be available for all users when public release is announced by StructurePoint. Contact StructurePoint to evaluate modeling high strength reinforcing steel with 100 ksi in advance of public release.

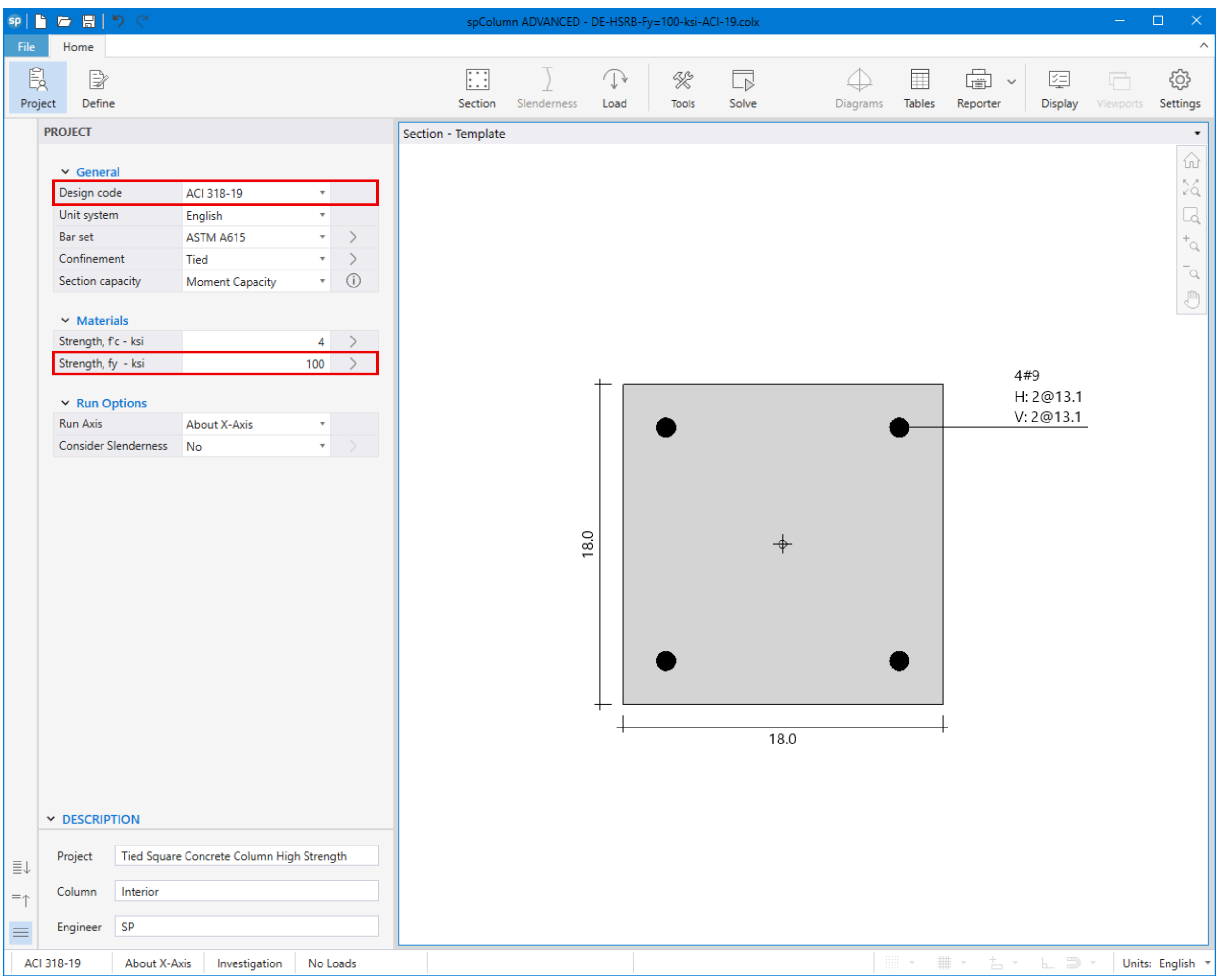


Figure 8 – spColumn Interface

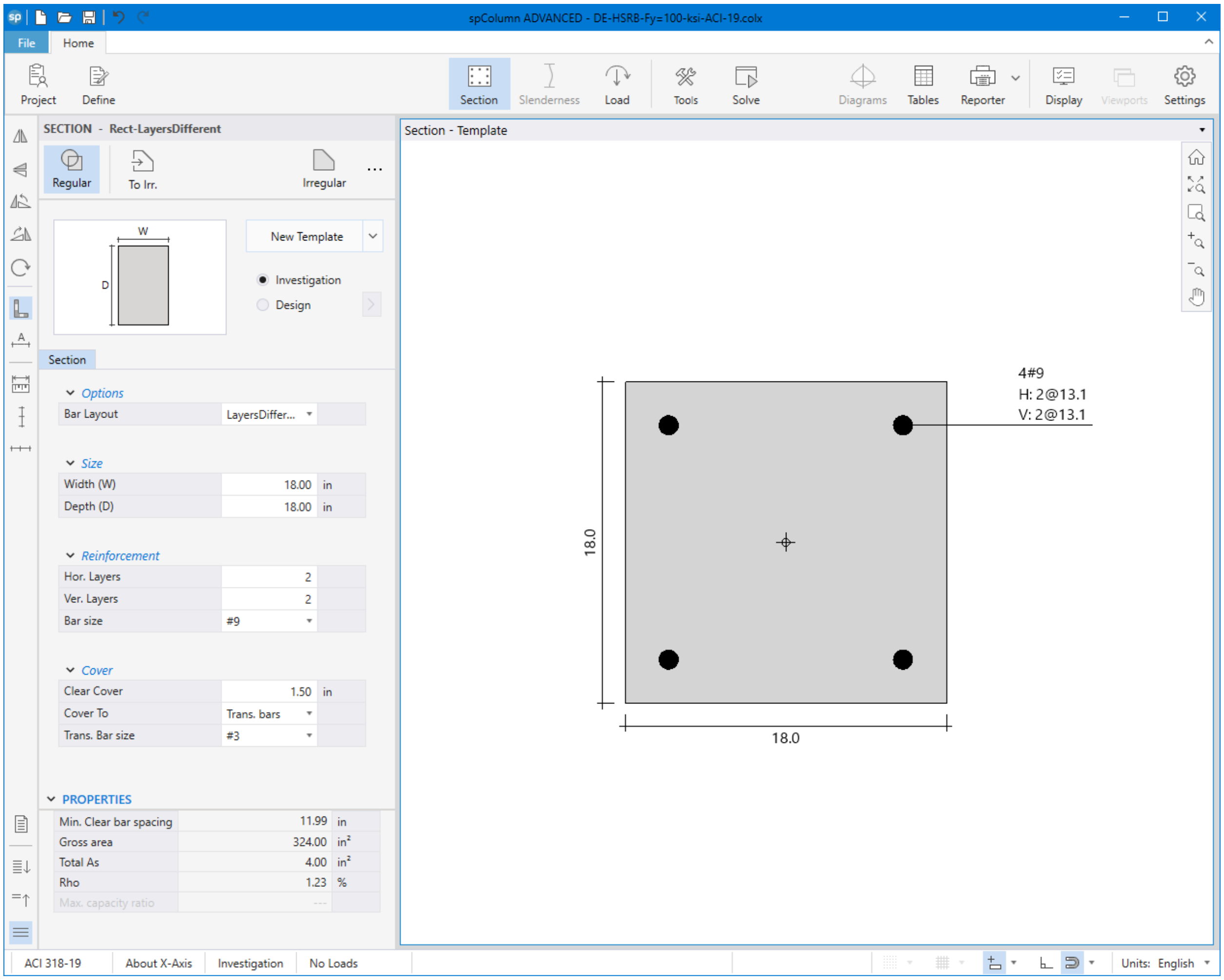


Figure 9 – spColumn Model Editor

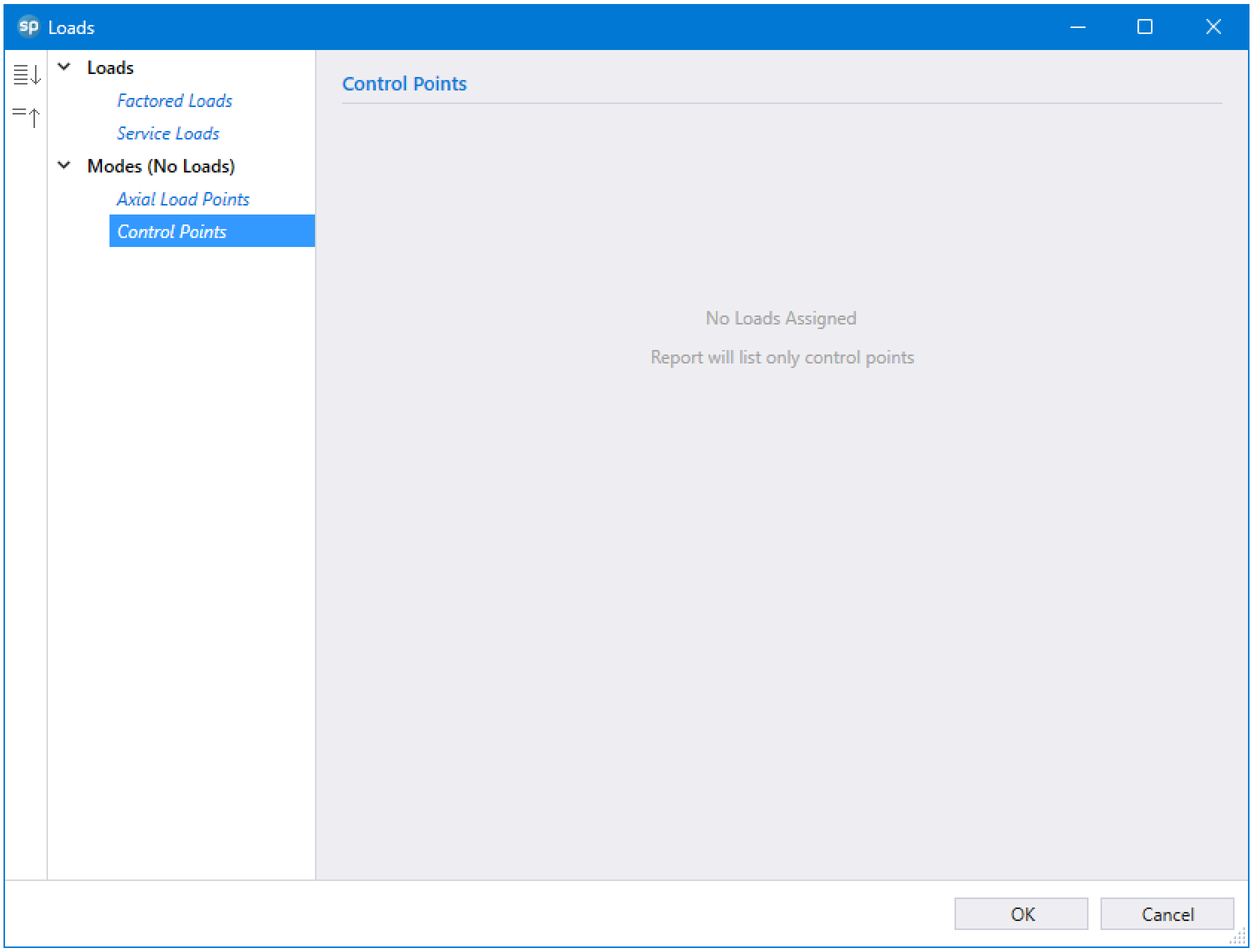


Figure 10 – Defining Loads / Modes (spColumn)

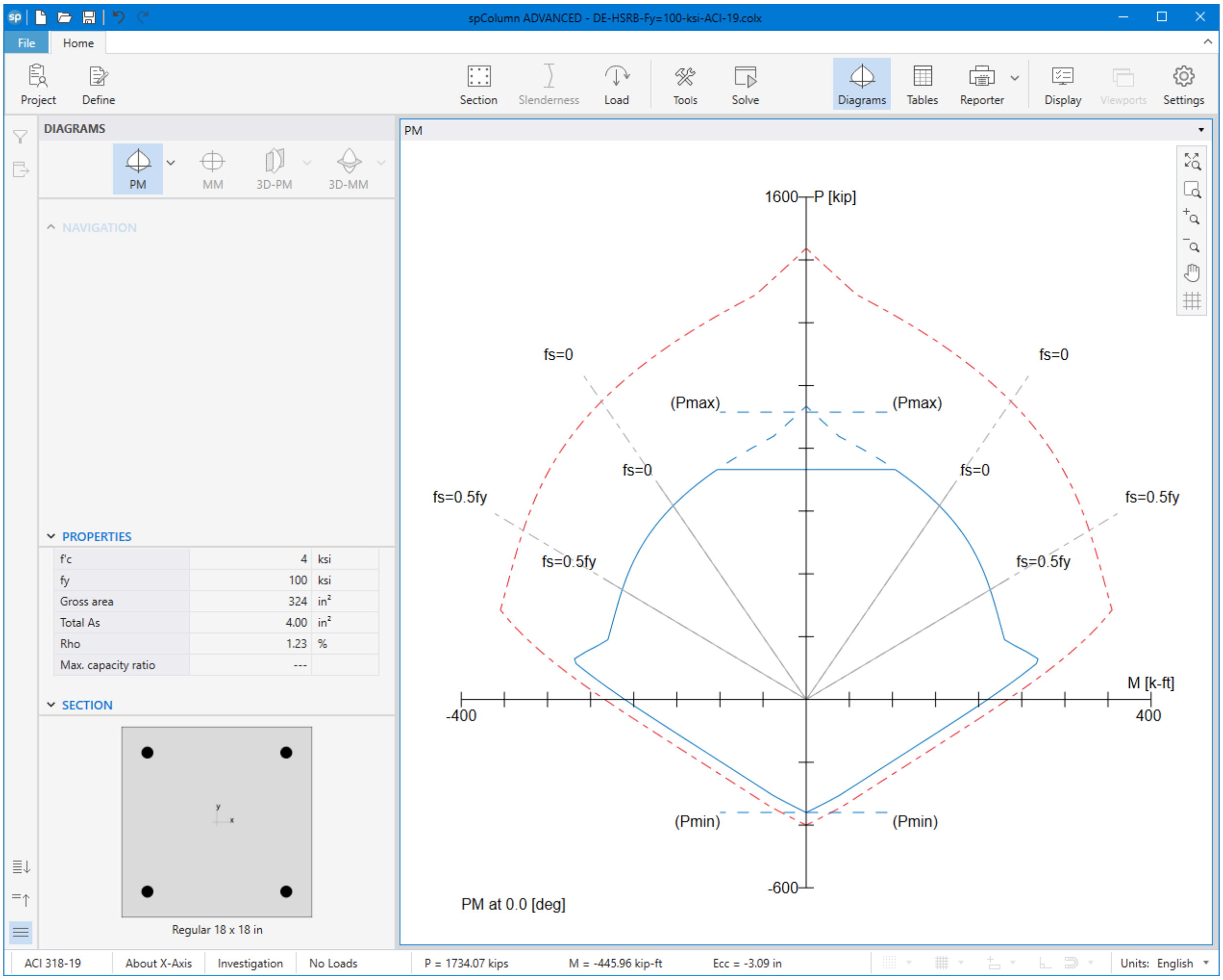
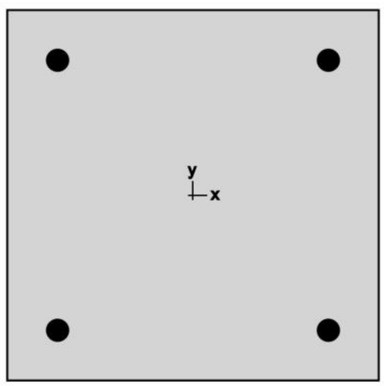


Figure 11 – Column Section Interaction Diagram about the X-Axis (spColumn)

spColumn v11.00 (TM) - Alpha 5
Computer program for the Strength Design of Reinforced Concrete Sections



Contents

List of Figures

1. General Information

File Name	F:\StructurePoint\spColumn\DE-HSRB-ACI-19.colx
Project	Tied Square Concrete Column High Strength
Column	Interior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Reinforcement Ratio	Use code defaults
Capacity Method	Moment capacity

2. Material Properties

2.1. Concrete

f'_c	4	ksi
Calculate E_c using W_c	No	
E_c	3605	ksi
f_c	3.4	ksi
ε_u	0.003	in/in
β_1	0.85	

2.2. Reinforcing Steel

f_y	100	ksi
E_s	29000	ksi
ε_{ty}	0.00344828	in/in

3. Section

3.1. Shape and Properties

Type	Regular	
A_g	324	in^2
I_x	8748	in^4
I_y	8748	in^4
r_x	5.19615	in
r_y	5.19615	in
X_o	0	in
Y_o	0	in

3.2. Section Figure

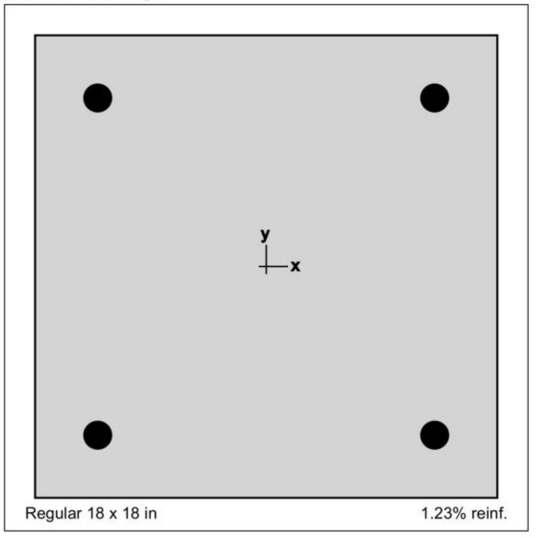


Figure 1: Column section

3.3. Solids

3.3.1. S1

Points	X in	Y in	Points	X in	Y in	Points	X in	Y in
1	-9.0	-9.0	2	9.0	-9.0	3	9.0	9.0
4	-9.0	9.0						

4. Reinforcement

4.1. Bar Set: ASTM A615

Bar	Diameter in	Area in^2	Bar	Diameter in	Area in^2	Bar	Diameter in	Area in^2
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

4.2. Confinement and Factors

Confinement type	Tied
Trans. Bar size	#3
No reduction (Nominal)	No
Capacity Reduction Factors	
Axial compression, (a)	0.8
Tension controlled ϕ, (b)	0.9
Compression controlled ϕ, (c)	0.65

4.3. Arrangement

Pattern	Rect-LayersDifferent
Clear cover	1.50 in
Cover to	Trans. bars
Total steel area, A_s	4.00 in²
Rho	1.23 %
Minimum clear spacing	11.99 in

4.4. Bars Provided

Area	X	Y	Area	X	Y	Area	X	Y
in²	in	in	in²	in	in	in²	in	in
1.00	-6.6	-6.6	1.00	6.6	-6.6	1.00	6.6	6.6
1.00	-6.6	6.6						

5. Control Points

About	Point	P	X-Moment	Y-Moment	NA Depth	d_t Depth	ε_t	ϕ
		kip	k-ft	k-ft	in	in		
X	@ Max compression	915.2	7.23	0.00	112.16	15.56	-0.00258	0.65000
X	@ Allowable comp.	732.2	103.53	0.00	18.48	15.56	-0.00047	0.65000
X	@ f_s = 0.0	617.1	154.37	0.00	15.56	15.56	0.00000	0.65000
X	@ f_s = 0.5 f_y	349.9	213.36	0.00	9.88	15.56	0.00172	0.65000
X	@ Balanced point	185.4	230.49	0.00	7.24	15.56	0.00345	0.65000
X	@ Tension control	124.5	271.44	0.00	4.94	15.56	0.00645	0.90000
X	@ Pure bending	0.0	210.32	0.00	3.19	15.56	0.01164	0.90000
X	@ Max tension	-360.0	0.00	0.00	0.00	15.56	9.99999	0.90000
-X	@ Max compression	915.2	-7.23	0.00	112.16	15.56	-0.00258	0.65000
-X	@ Allowable comp.	732.2	-103.53	0.00	18.48	15.56	-0.00047	0.65000
-X	@ f_s = 0.0	617.1	-154.37	0.00	15.56	15.56	0.00000	0.65000
-X	@ f_s = 0.5 f_y	349.9	-213.36	0.00	9.88	15.56	0.00172	0.65000
-X	@ Balanced point	185.4	-230.49	0.00	7.24	15.56	0.00345	0.65000
-X	@ Tension control	124.5	-271.44	0.00	4.94	15.56	0.00645	0.90000
-X	@ Pure bending	0.0	-210.32	0.00	3.19	15.56	0.01164	0.90000
-X	@ Max tension	-360.0	0.00	0.00	0.00	15.56	9.99999	0.90000

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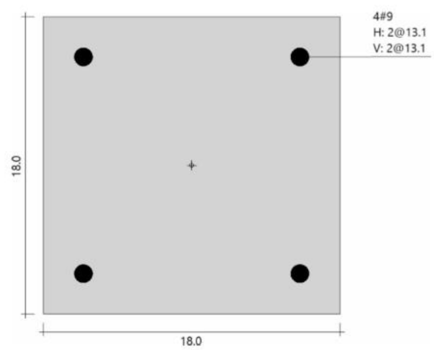
6. Diagrams

6.1. Section



General Information

Project	Tied Square Concrete C...
Column	Interior
Engineer	SP
Code	ACI 318-19
Units	English - in
Bar Set	ASTM A615
Run Axis	X - axis
Slenderness	No

Section

Type	Regular
Pattern	Rect-LayersDifferent
Gross area	324 in^2
Total As	4 in^2
Rho	1.23 %
No. of Bars	4
Confinement type	Tied
Min. Clear bar spacing	11.99 in

Materials

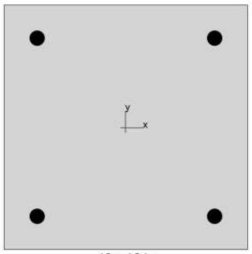
f'_c	4 ksi
f_y	100 ksi

Cover

Clear cover	1.5 in
Cover to	Trans. bars - #3

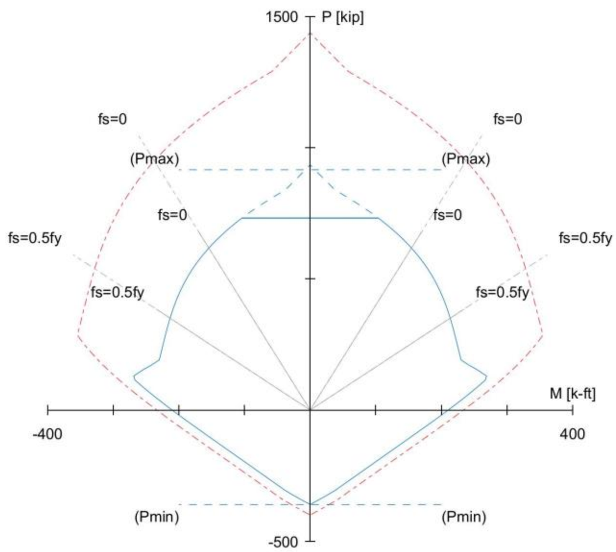
Max. capacity ratio	---

6.2. PM at θ=0 [deg]



18 x 18 in

General Information	
Project	Tied Squ... Strength
Column	Interior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Column Type	Use code defaults
Capacity Method	Moment capacity
Materials	
f'_c	4 ksi
E_c	3605 ksi
f_y	100 ksi
E_s	29000 ksi
Section	
Type	Regular
A_g	324 in²
I_x	8748 in⁴
I_y	8748 in⁴
Reinforcement	
Pattern	Rect-LayersDifferent
Clear cover	1.5 in
Cover to	Trans. bars - #3
Confinement type	Tied
Total steel area, A_s	4.00 in²
Rho	1.23 %
Min. clear spacing	11.99 in



PM at 0.0 [deg]

10. Summary and Comparison of Design Results

Table 1 – Comparison of Results						
Support	ϕP_n (kip)			ϕM_n (kip-ft)		
	Reference	Hand	spColumn	Reference	Hand	spColumn
Max compression	915.2	915.2	915.2	0.00	0.00	0.00
Allowable compression	732.2	732.2	732.2	---	---	---
f_s = 0.0	617.3	617.1	617.1	154.4	154.37	154.37
$f_s = 0.5\, f_y$	350.0	349.9	349.9	213.3	213.36	213.36
Balanced point	185.3	185.4	185.4	230.5	230.49	230.49
Tension control	---*	124.5	124.5	---*	271.44	271.44
Pure bending	0.0	0.0	0.0	210.4	210.31	210.32
Max tension	---*	360.0	360.0	---*	0.00	0.00

* Reference does not show this parameter.

In all of the hand calculations and the reference used illustrated above for this column with HSRB, the results are in precise agreement with the automated exact results obtained from the spColumn program.