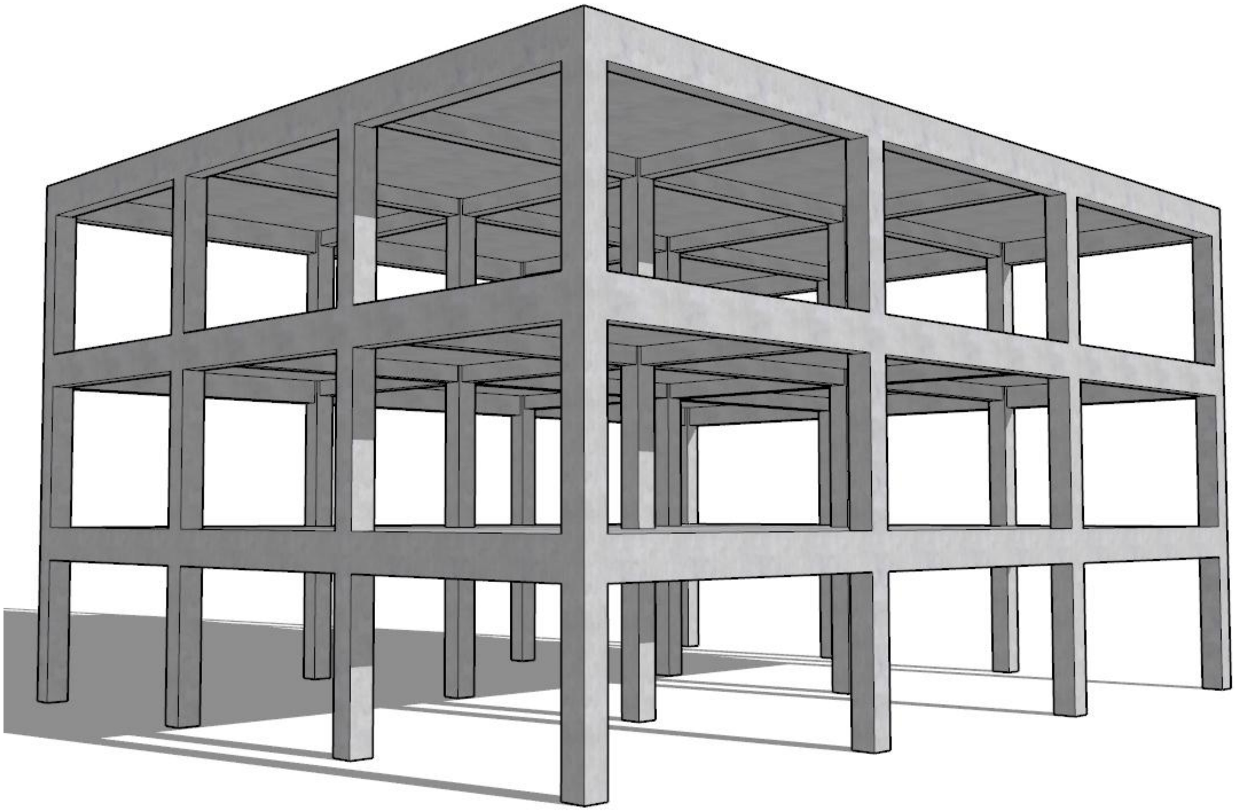
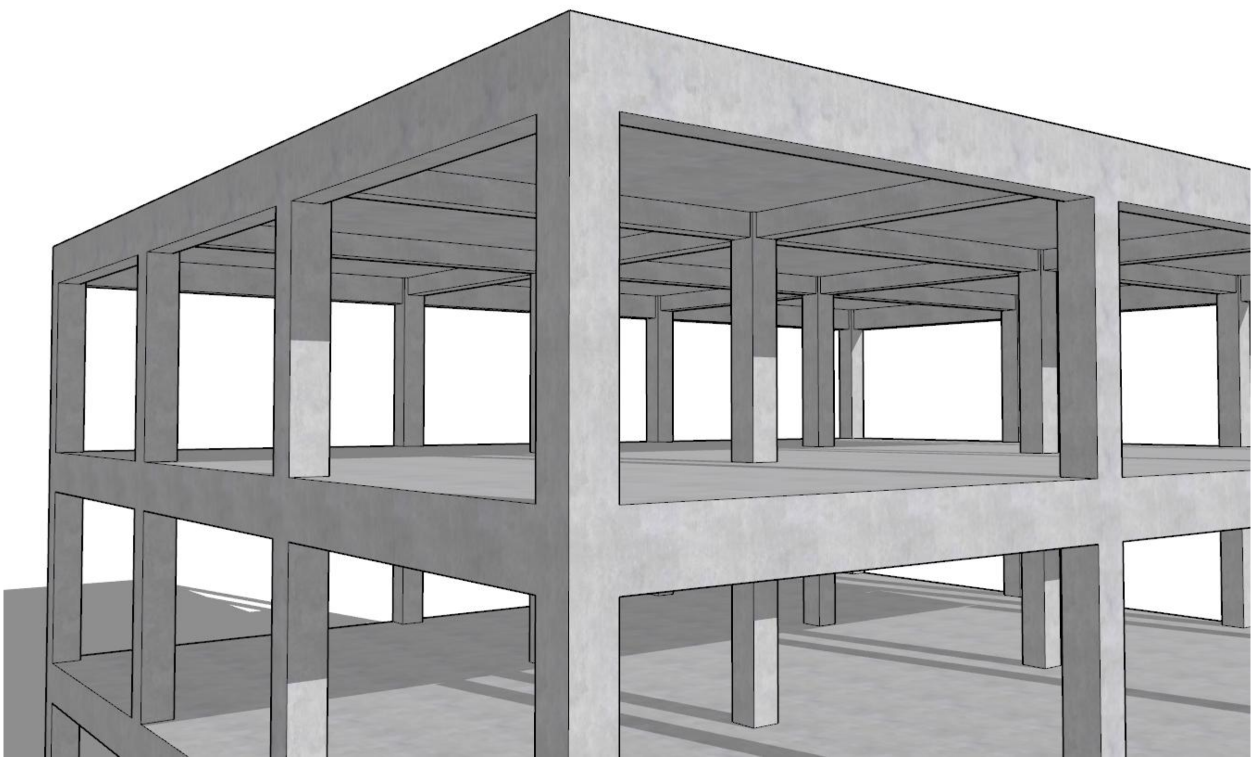
Two-Way Concrete Floor Slab with Beams System Analysis and Design (ACI 318-14)





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Design the slab system shown below for an intermediate floor where the story height = 12 ft, column cross-sectional dimensions = 18 in. × 18 in., edge beam dimensions = 14 in. × 27 in., interior beam dimensions = 14 in. × 20 in., and unfactored live load = 100 psf. The lateral loads are resisted by shear walls. Normal weight concrete with ultimate strength (f_c' = 4,000 psi) is used for all members, respectively. And reinforcement with f_y = 60,000 psi is used. Use the Equivalent Frame Method (EFM) and compare the results with spSlab engineering software program from StructurePoint.

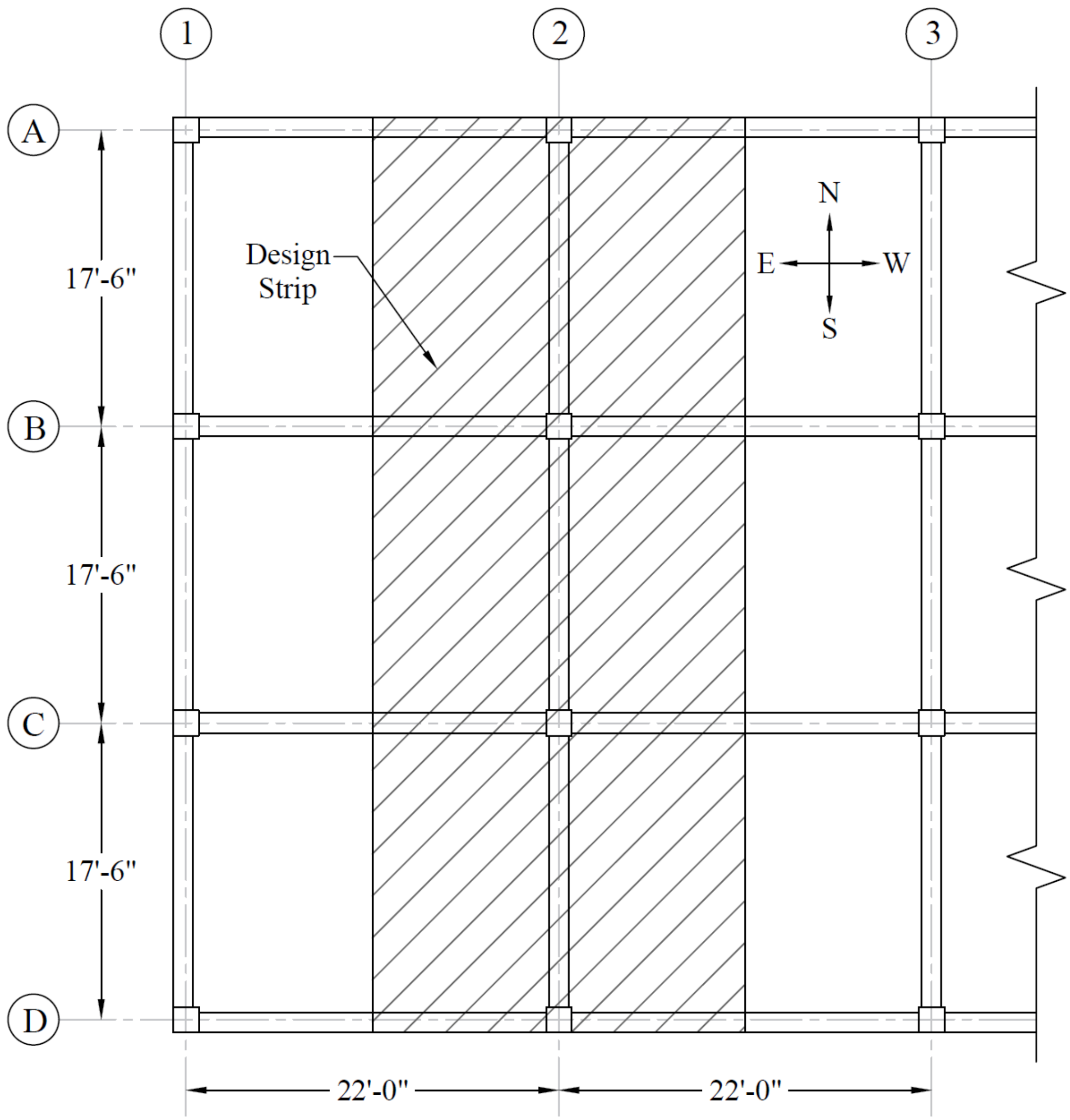


Figure 1 – Two-Way Slab with Beams Spanning between all Supports

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Minimum Design Loads for Buildings and Other Structures (ASCE/SEI 7-10)

International Code Council, 2012 International Building Code, Washington, D.C., 2012

References

- Concrete Floor Systems (Guide to Estimating and Economizing), Second Edition, 2002 David A. Fanella
- Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association, Example 20.2
- Simplified Design of Reinforced Concrete Buildings, Fourth Edition, 2011 Mahmoud E. Kamara and Lawrence C. Novak
- spSlab Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2024
- "Two-Way Flat Plate Concrete Floor System Analysis and Design (ACI 318-14)" Design Example, STRUCTUREPOINT, 2025
- "Two-Way Flat Slab (Concrete Floor with Drop Panels) System Analysis and Design (ACI 318-14)" Design Example, STRUCTUREPOINT, 2025

- Contact Support@StructurePoint.org to obtain supplementary materials (spSlab models: DE-Two-Way-Slab-with-Beams-ACI-318-14.slbx)

Design Data

Floor-to-Floor Height = 12 ft (provided by architectural drawings)

Columns = 18 × 18 in.

Interior beams = 14 × 20 in.

Edge beams = 14 × 27 in.

w_c = 150 pcf

f_c' = 4,000 psi

f_y = 60,000 psi

Live Load, L_o = 100 psf (Office building) ***ASCE/SEI 7-10 (Table 4-1)***

1. Notations

This section (based on ACI 318-14 provisions) defines notation and terminology used in this design example:

a = depth of equivalent rectangular stress block, in.

A_b = area of an individual bar or wire, in.^{2}

A_g = gross area of concrete section, in.^{2} For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s)

A_s = area of nonprestressed longitudinal tension reinforcement, in.^{2}

A_{st} = total area of nonprestressed longitudinal reinforcement including bars or steel shapes, and excluding prestressing reinforcement, in.^{2}

$A_{s,min}$ = minimum area of flexural reinforcement, in.^{2}

b = width of compression face of member, in.

b_o = perimeter of critical section for two-way shear in slabs and footings, in.

b_w = web width or diameter of circular section, in.

b_1 = dimension of the critical section b_o measured in the direction of the span for which moments are determined, in.

b_2 = dimension of the critical section b_o measured in the direction perpendicular to b_1, in.

c = distance from extreme compression fiber to neutral axis, in.

c_c = clear cover of reinforcement, in.

c_1 = dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in.

c_2 = dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction perpendicular to c_1, in.

C = cross-sectional constant to define torsional properties of slab and beam

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

d_b = nominal diameter of bar, wire, or prestressing strand, in.

E_c = modulus of elasticity of concrete, psi

E_{cb} = modulus of elasticity of beam concrete, psi

E_{cs} = modulus of elasticity of slab concrete, psi

E_s = modulus of elasticity of reinforcement and structural steel, excluding prestressing reinforcement, psi

f_c' = specified compressive strength of concrete, psi

f_r = modulus of rupture of concrete, psi

f_y = specified yield strength for nonprestressed reinforcement, psi

h = overall thickness, height, or depth of member, in.

I_b = moment of inertia of gross section of beam about centroidal axis, in.^{4}

I_{cr} = moment of inertia of cracked section transformed to concrete, in.^{4}

I_e = effective moment of inertia for calculation of deflection, in.^{4}

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in.^{4}

I_s = moment of inertia of gross section of slab about centroidal axis, in.^{4}

K_t = torsional stiffness of member; moment per unit rotation

l = span length of beam or one-way slab; clear projection of cantilever, in.

l_n = length of clear span measured face-to-face of supports, in.

l_1 = length of span in direction that moments are being determined, measured center-to-center of supports, in.

l_2 = length of span in direction perpendicular to l_1, measured center-to-center of supports, in.

M_a = maximum moment in member due to service loads at stage deflection is calculated, in.-lb

M_{cr} = cracking moment, in.-lb

M_u = factored moment at section, in.-lb

M_o = total factored static moment, in.-lb

n = number of items, such as, bars, wires, monostrand anchorage devices, anchors, or shearhead arms

P_n = nominal axial compressive strength of member, lb

P_o = nominal axial strength at zero eccentricity, lb

P_u = factored axial force; to be taken as positive for compression and negative for tension, lb

q_{Du} = factored dead load per unit area, lb/ft^2

q_{Lu} = factored live load per unit area, lb/ft^2

q_u = factored load per unit area, lb/ft^2

v_c = stress corresponding to nominal two-way shear strength provided by concrete, psi

v_u = maximum factored two-way shear stress calculated around the perimeter of a given critical section, psi

V_c = nominal shear strength provided by concrete, lb

V_n = nominal shear strength, lb

V_s = nominal shear strength provided by shear reinforcement, lb

V_u = factored shear force at section, lb

w_c = density, unit weight, of normalweight concrete or equilibrium density of lightweight concrete, lb/ft^3

y_t = distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.

α_f = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels, if any, on each side of the beam

α_{f1} = α_f in direction of l_1

α_{f2} = α_f in direction of l_2

α_s = constant used to calculate V_c in slabs and footings

β = ratio of long to short dimensions: clear spans for two-way slabs, sides of column, concentrated load or reaction area; or sides of a footing

β_t = ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports

β_1 = factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis

γ_f = factor used to determine the fraction of M_{sc} transferred by slab flexure at slab-column connections

γ_v = factor used to determine the fraction of M_{sc} transferred by eccentricity of shear at slab-column connections

ε_t = net tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength, excluding strains due to effective prestress, creep, shrinkage, and temperature

λ = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal weight concrete of the same compressive strength

λ_Δ = multiplier used for additional deflection due to long-term effects

ξ = time-dependent factor for sustained load

ρ' = ratio of A_s' to bd

ϕ = strength reduction factor

2. Preliminary Slab Thickness Sizing

Control of deflections. ***ACI 318-14 (8.3.1.2)***

In lieu of detailed calculation for deflections, ACI 318 Code gives minimum thickness for two-way slab with beams spanning between supports on all sides in ***Table 8.3.1.2***.

Beam-to-slab flexural stiffness (relative stiffness) ratio (α_f) is computed as follows:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s}$$ ***ACI 318-14 (8.10.2.7b)***

The moment of inertia for the effective beam and slab sections can be calculated as follows:

$$I_s = \frac{l_2 h^3}{12} \text{ and } I_b = \left(\frac{ba^3}{12}\right) \times f$$

Then,

$$\alpha_f = \left(\frac{b}{l_2}\right)\left(\frac{a}{h}\right)^3 \times f$$

For Edge Beams:

The effective beam and slab sections for the computation of stiffness ratio for edge beam is shown in Figure 2.

For North-South Edge Beam:

$$l_2 = \frac{22 \times 12}{2} + \frac{18}{2} = 141.00 \text{ in.}$$

$$\frac{a}{h} = \frac{27}{6} = 4.50$$

$$\frac{b}{h} = \frac{14}{6} = 2.33$$

f = 1.47 using Figure 3.

$$\alpha_f = \left(\frac{14}{141}\right)\left(\frac{27}{6}\right)^3 \times (1.47) = 13.30$$

For East-West Edge Beam:

$$l_2 = \frac{17.5 \times 12}{2} + \frac{18}{2} = 114.00 \text{ in.}$$

$$\frac{a}{h}=\frac{27}{6}=4.50$$

$$\frac{b}{h}=\frac{14}{6}=2.33$$

f = 1.47 using Figure 3.

$$\alpha_f=\left(\frac{14}{114}\right)\left(\frac{27}{6}\right)^3\times(1.47)=16.45$$

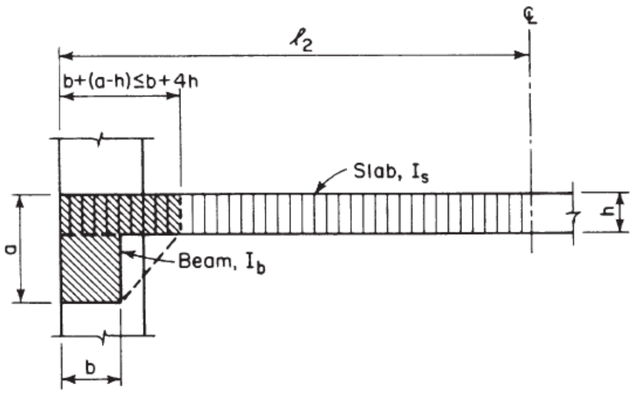


Figure 2 – Effective Beam and Slab Sections (Edge Beam)

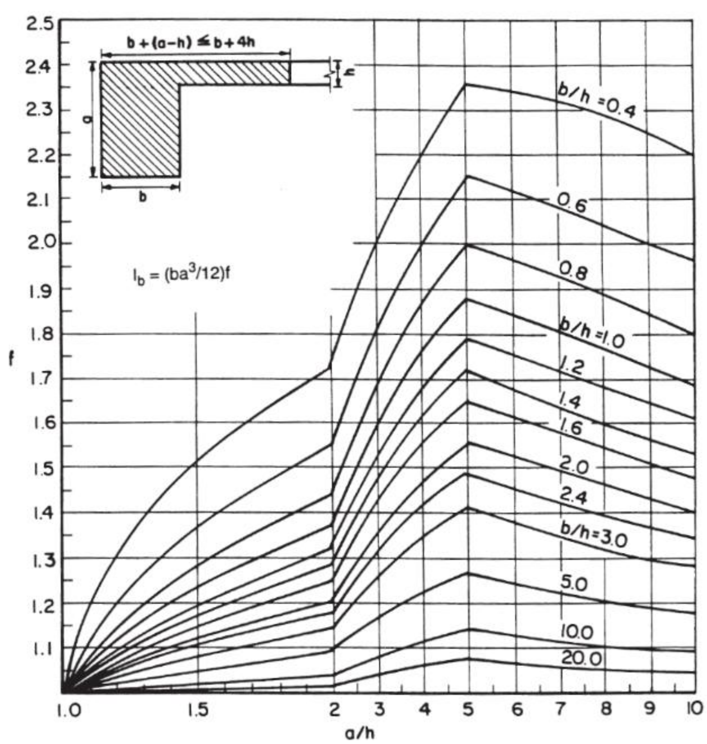


Figure 3 – Beam Stiffness (Edge Beam)

For Interior Beams:

The effective beam and slab sections for the computation of stiffness ratio for interior beam is shown in Figure 4.

For North-South Interior Beam:

$$l_2 = 22 \times 12 = 264.00 \text{ in.}$$

$$\frac{a}{h} = \frac{20}{6} = 3.33$$

$$\frac{b}{h} = \frac{14}{6} = 2.33$$

f = 1.61 using Figure 5.

$$\alpha_f = \left(\frac{14}{264}\right)\left(\frac{20}{6}\right)^3 \times (1.61) = 3.16$$

For East-West Interior Beam:

$$l_2 = 17.5 \times 12 = 210.00 \text{ in.}$$

$$\frac{a}{h} = \frac{20}{6} = 3.33$$

$$\frac{b}{h} = \frac{14}{6} = 2.33$$

f = 1.61 using Figure 5.

$$\alpha_f = \left(\frac{14}{210}\right)\left(\frac{20}{6}\right)^3 \times (1.61) = 3.98$$

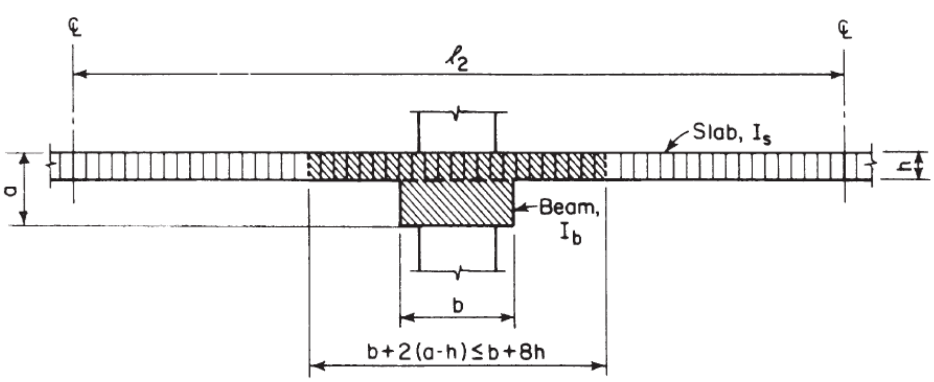


Figure 4 – Effective Beam and Slab Sections (Interior Beam)

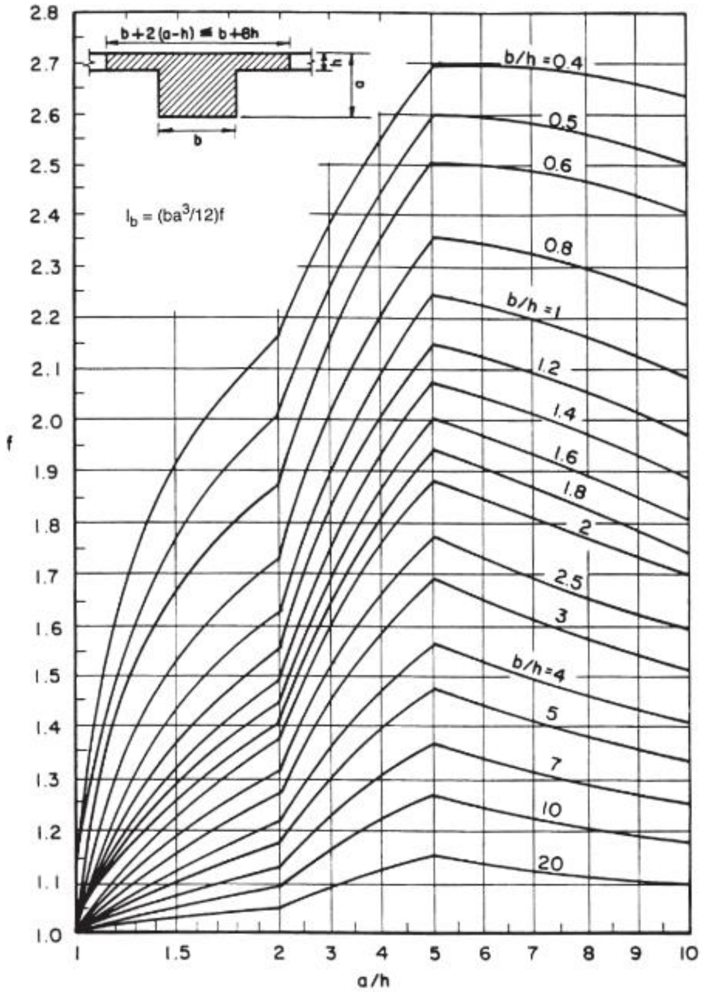


Figure 5 – Beam Stiffness (Interior Beam)

Since α_f > 2.0 for all beams, the minimum slab thickness is given by:

$$h_{\min} = \text{greater of} \begin{Bmatrix} \dfrac{l_n\left(0.8+\dfrac{f_y}{200,000}\right)}{36+9\beta} \\ 3.5 \end{Bmatrix} \qquad \textbf{\textit{\underline{ACI 318-14 (8.3.1.2)}}}$$

Where:

l_n = clear span in the long direction measured face to face of columns = 20.5 ft = 246 in.

$$\beta = \frac{\text{clear span in the long direction}}{\text{clear span in the short direction}} = \frac{22-18/12}{17.5-18/12} = 1.28$$

$$h_{\min} = \text{greater of} \begin{Bmatrix} \dfrac{246 \times \left(0.8 + \dfrac{60,000}{200,000}\right)}{36 + 9 \times 1.28} \\ 3.5 \end{Bmatrix} = \text{greater of} \begin{Bmatrix} 5.69 \\ 3.5 \end{Bmatrix} = 5.69 \text{ in.}$$

Use 6 in. slab thickness.

3. Two-Way Slab Analysis and Design – Using Equivalent Frame Method (EFM)

ACI 318 states that a slab system shall be designed by any procedure satisfying equilibrium and geometric compatibility, provided that strength and serviceability criteria are satisfied. Distinction of two-systems from one-way systems is given by ***ACI 318-14 (R8.10.2.3 & R8.3.1.2)***.

ACI 318 permits the use of Direct Design Method (DDM) and Equivalent Frame Method (EFM) for the gravity load analysis of orthogonal frames and is applicable to flat plates, flat slabs, and slabs with beams. The following sections outline the solution per EFM and spSlab software. The solution per DDM can be found in the "Two-Way Flat Plate Concrete Floor System Analysis and Design (ACI 318-14)" Design Example.

EFM is the most comprehensive and detailed procedure provided by the ACI 318 for the analysis and design of two-way slab systems where the structure is modeled by a series of equivalent frames (interior and exterior) on column lines taken longitudinally and transversely through the building.

The equivalent frame consists of three parts:

1) Horizontal slab-beam strip, including any beams spanning in the direction of the frame. Different values of moment of inertia along the axis of slab-beams should be taken into account where the gross moment of inertia at any cross section outside of joints or column capitals shall be taken, and the moment of inertia of the slab-beam at the face of the column, bracket or capital divide by the quantity $(1-c_2/l_2)^2$ shall be assumed for the calculation of the moment of inertia of slab-beams from the center of the column to the face of the column, bracket or capital. ***ACI 318-14 (8.11.3)***

2) Columns or other vertical supporting members, extending above and below the slab. Different values of moment of inertia along the axis of columns should be taken into account where the moment of inertia of columns from top and bottom of the slab-beam at a joint shall be assumed to be infinite, and the gross cross section of the concrete is permitted to be used to determine the moment of inertia of columns at any cross section outside of joints or column capitals. ***ACI 318-14 (8.11.4)***

3) Elements of the structure (Torsional members) that provide moment transfer between the horizontal and vertical members. These elements shall be assumed to have a constant cross section throughout their length consisting of the greatest of the following: (1) portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined, (2) portion of slab specified in (1) plus that part of the transverse beam above and below the slab for monolithic or fully composite construction, (3) the transverse beam includes that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness. ***ACI 318-14 (8.11.5)***

3.1. Equivalent Frame Method Limitations

In EFM, live load shall be arranged in accordance with 6.4.3 which requires slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***ACI 318-14 (8.11.1.2 & 6.4.3)***

Complete analysis must include representative interior and exterior equivalent frames in both the longitudinal and transverse directions of the floor. ***ACI 318-14 (8.11.2.1)***

Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***ACI 318-14 (8.10.2.3)***

3.2. Frame Members of Equivalent Frame

Determine moment distribution factors and fixed-end moments for the equivalent frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors k, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a) Flexural stiffness of slab-beams at both ends, K_{sb}.

$$\frac{c_{N1}}{l_1} = \frac{18}{(17.5\times12)} = 0.0857 \approx 0.1\ ,\ \frac{c_{N2}}{l_2} = \frac{18}{(22\times12)} = 0.0682$$

For $c_{F1} = c_{F2}$, stiffness factors, $k_{NF} = k_{FN} = 4.123$ ***PCA Notes on ACI 318-11 (Table A1)***

Thus, $K_{sb} = k_{NF} \times \dfrac{E_{cs}\times I_s}{l_1} = 4.123\times\dfrac{E_{cs}\times I_s}{l_1}$ ***PCA Notes on ACI 318-11 (Table A1)***

Where I_{sb} is the moment of inertia of slab-beam section shown in Figure 6 and can be computed as follows:

$$y_t = \frac{\left(14\times(20-6)\right)\times\left(\dfrac{20-6}{2}\right)+\left((22\times12)\times6\right)\times\left(20-\dfrac{6}{2}\right)}{\left(14\times(20-6)\right)+\left((22\times12)\times6\right)} = 15.90\text{ in.}$$

$$I_{sb} = \frac{14\times(20-6)^3}{12}+\left(14\times(20-6)\right)\times\left(15.90-\left(\frac{20-6}{2}\right)\right)^2 +$$
$$\frac{(22\times12)\times6^3}{12}+\left((22\times12)\times6\right)\times\left(\left(20-\frac{6}{2}\right)-15.90\right)^2 = 25{,}395.13\text{ in.}^4$$

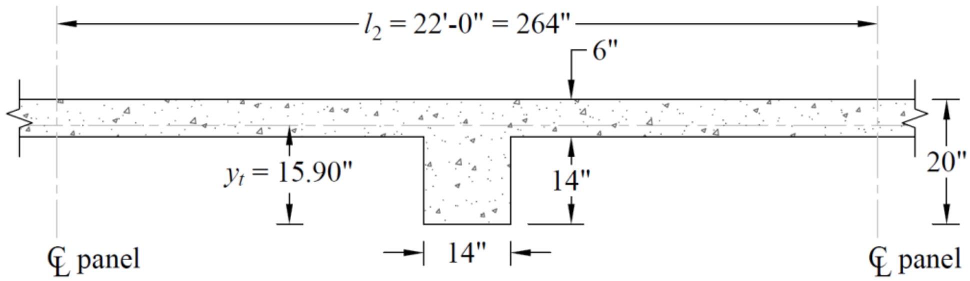


Figure 6 – Cross-Section of Slab-Beam

$$K_{sb} = 4.123 \times \frac{E_c \times 25,395.13}{17.5 \times 12} = 498.59 E_c$$

Carry-over factor $COF = 0.507$ ***PCA Notes on ACI 318-11 (Table A1)***

Fixed-end moment $FEM = 0.0843 \times w_u \times \ell_2 \times \ell_1^{\ 2}$ ***PCA Notes on ACI 318-11 (Table A1)***

b) Flexural stiffness of column members at both ends, K_c.

Referring to ***Table A7, Appendix 20A***,

For Interior Columns:

t_a = 20 – 6/2 = 17.00 in. t_b = 6/2 = 3.00 in.

H = 12 ft = 144.00 in. $H_c = H - t_a - t_b$ = 144 – 17 – 3 = 124.00 in.

$$\frac{t_a}{t_b} = \frac{17.00}{3.00} = 5.667 \qquad \frac{H}{H_c} = \frac{144.00}{124.00} = 1.161$$

Thus, $k_{c,top}$ = 6.824, $k_{c,bottom}$ = 4.984, COF_{top} = 0.513, and COF_{bottom} = 0.700 by interpolation.

$$K_c = \frac{k_c \times E_{cc} \times I_c}{l_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_{c,top} = \frac{6.824 \times 8,748.00 \times E_c}{144.00} = 414.56 E_c$$

$$K_{c,bottom} = \frac{4.984 \times 8,748.00 \times E_c}{144.00} = 302.78 E_c$$

Where, $I_c = \frac{c^4}{12} = \frac{(18)^4}{12} = 8748.00 \text{ in.}^4$

l_c = 12 ft = 144.00 in.

For Exterior Columns:

t_a = 27 – 6/2 = 24.00 in.

t_b = 6/2 = 3.00 in.

H = 12 ft = 144.00 in.

$H_c = H - t_a - t_b$ = 144 – 24 – 3 = 117.00 in.

$$\frac{t_a}{t_b} = \frac{24.00}{3.00} = 8.000$$

$$\frac{H}{H_c} = \frac{144.00}{117.00} = 1.231$$

Thus, $k_{c,top}$ = 8.589, $k_{c,bottom}$ = 5.293, COF_{top} = 0.494 and COF_{bottom} = 0.802 by interpolation.

$$K_c = \frac{k_c \times E_{cc} \times I_c}{l_c}$$

PCA Notes on ACI 318-11 (Table A7)

$$K_{c,top} = \frac{8.589 \times 8,748.00 \times E_c}{144.00} = 521.78E_c$$

$$K_{c,bottom} = \frac{5.293 \times 8,748.00 \times E_c}{144.00} = 321.55E_c$$

Where, $I_c = \frac{c^4}{12} = \frac{(18)^4}{12} = 8748.00 \text{ in.}^4$

l_c = 12 ft = 144.00 in.

c) Torsional stiffness of torsional members, K_t.

$$K_t = \frac{9 \times E_{cs} \times C}{\left[l_2 \times \left(1 - \frac{c_2}{l_2} \right)^3 \right]}$$ ***ACI 318-14 (R.8.11.5)***

For Interior Columns:

$$K_t = \frac{9 \times E_c \times 11,697.65}{264 \times (0.932)^3} = 492.88 E_c$$

Where:

$$1 - \frac{c_2}{l_2} = 1 - \frac{18}{22 \times 12} = 0.932$$

$$C = \sum \left(1 - 0.63 \times \frac{x}{y} \right) \times \left(\frac{x^3 \times y}{3} \right)$$ ***ACI 318-14 (Eq. 8.10.5.2b)***

x_1 = 14 in.	x_2 = 6 in.	x_1 = 14 in.	x_2 = 6 in.
y_1 = 14 in.	y_2 = 42 in.	y_1 = 20 in.	y_2 = 14 in.
C_1 = 4,737.97 in.^{4}	C_2 = 2,751.84 in.^{4}	C_1 = 10,225.97 in.^{4}	C_2 = 735.84 in.^{4}

$\sum C$ = 4,737.97 + 2,751.84 = 7,489.81 in.^{4}

$\sum C$ = 10,225.97 + 735.84 × 2 = 11,697.65 in.^{4}

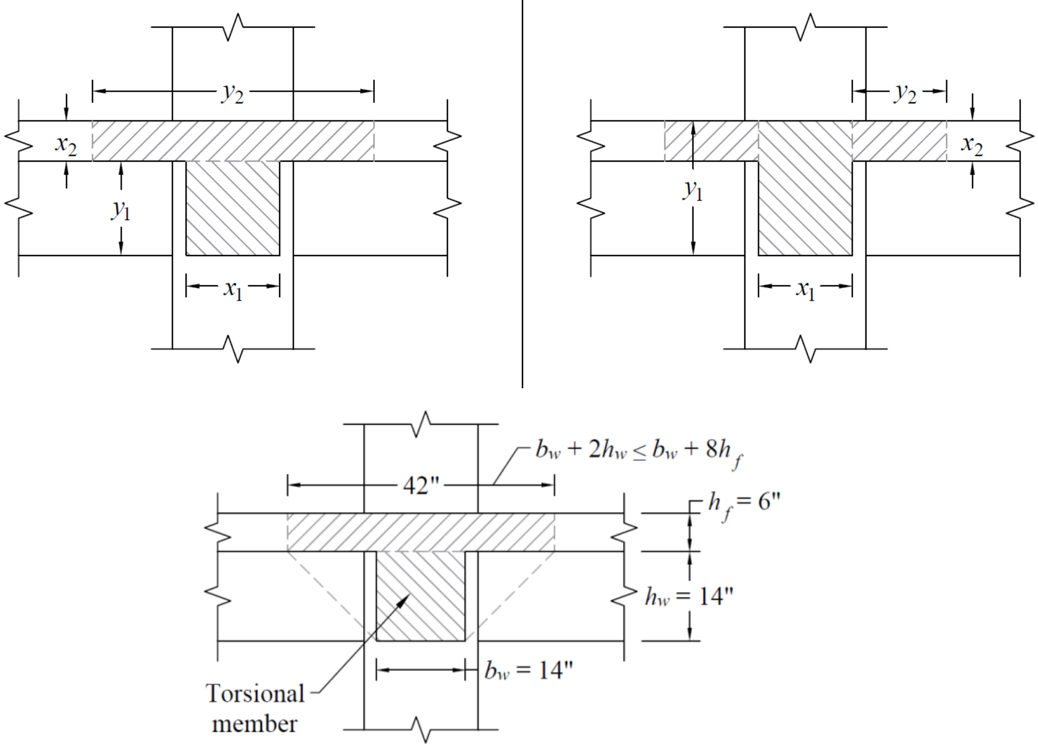


Figure 7 – Attached Torsional Member at Interior Column

For Exterior Columns:

$$K_t = \frac{9 \times E_c \times 17{,}868.48}{264 \times (0.932)^3} = 752.89 E_c$$

Where:

$$1 - \frac{c_2}{l_2} = 1 - \frac{18}{22 \times 12} = 0.932$$

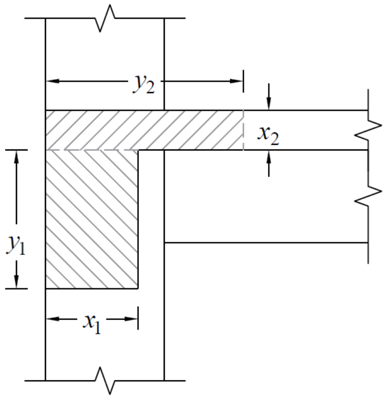
$$C = \sum \left(1 - 0.63 \times \frac{x}{y}\right) \times \left(\frac{x^3 \times y}{3}\right)$$ ***ACI 318-14 (Eq. 8.10.5.2b)***

$x_1 = 14$ in. $\quad$ $x_2 = 6$ in.

$y_1 = 21$ in. $\quad$ $y_2 = 35$ in.

$C_1 = 11{,}140.64$ in.^{4} $\quad$ $C_2 = 2{,}247.84$ in.^{4}

$\sum C = 11{,}140.64 + 2{,}247.84 = 13{,}388.48$ in.^{4}

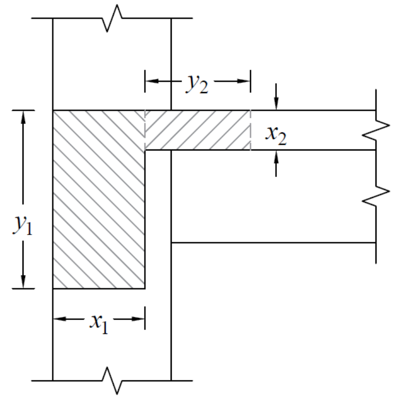


$x_1 = 14$ in. $\quad$ $x_2 = 6$ in.

$y_1 = 27$ in. $\quad$ $y_2 = 21$ in.

$C_1 = 16{,}628.64$ in.^{4} $\quad$ $C_2 = 1{,}239.84$ in.^{4}

$\sum C = 16{,}628.64 + 1{,}239.84 = 17{,}868.48$ in.^{4}



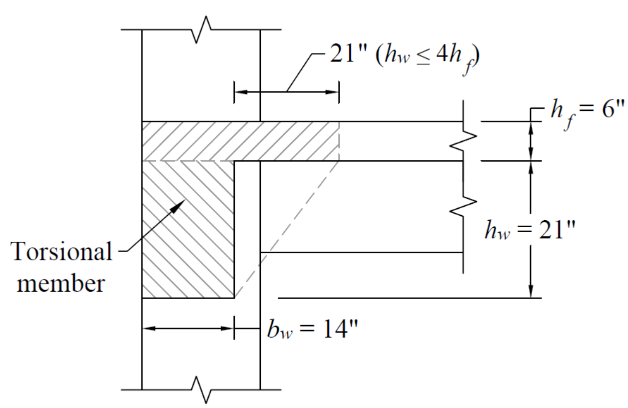


Figure 8 – Attached Torsional Member at Exterior Column

d) Increased torsional stiffness due to parallel beams, K_{ta}

For Interior Columns:

$$K_{ta} = \frac{K_t I_{sb}}{I_s} = \frac{492.88\ E_c \times 25,395.13}{4,752.00} = 2,634.01 E_c$$

Where:

$$I_{sb} = \frac{l_2 \times h^3}{12} = \frac{(22 \times 12) \times 6^3}{12} = 4,752.00\ \text{in.}^4$$

For Exterior Columns:

$$K_{ta} = \frac{K_t I_{sb}}{I_s} = \frac{752.89\ E_c \times 25,395.13}{4,752.00} = 4,023.52 E_c$$

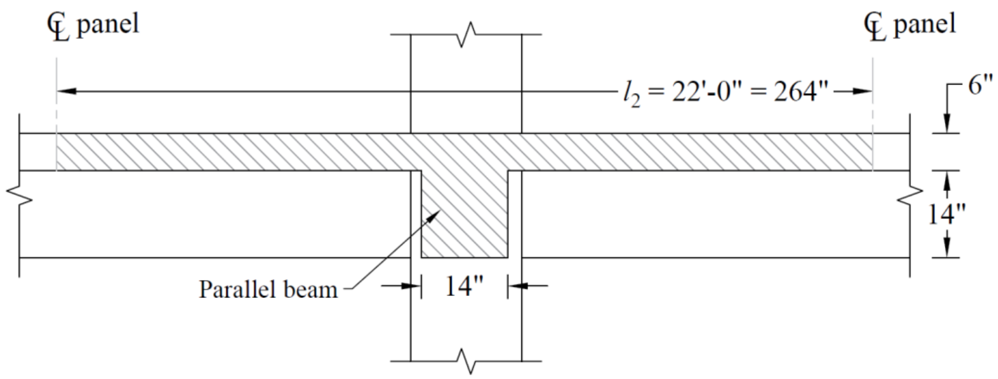


Figure 9 – Slab-Beam in the Direction of Analysis

e) Equivalent column stiffness K_{ec}.

$$K_{ec}=\frac{\sum K_c \times \sum K_{ta}}{\sum K_c + \sum K_{ta}}$$

Where $\sum K_{ta}$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

For Interior Columns:

$$K_{ec}=\frac{(302.78E_c+414.56E_c)\times(2\times 2,634.01E_c)}{(302.78E_c+414.56E_c)+(2\times 2,634.01E_c)}=631.36E_c$$

For Exterior Columns:

$$K_{ec}=\frac{(321.55E_c+521.78E_c)\times(2\times 4023.52E_c)}{(321.55E_c+521.78E_c)+(2\times 4023.52E_c)}=763.33E_c$$

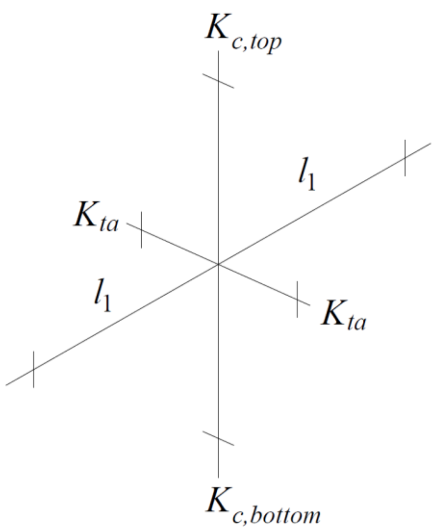


Figure 10 – Equivalent Column Stiffness

f) Slab-beam joint distribution factors, *DF*.

At exterior joint

$$DF = \frac{498.59E_c}{(498.59E_c + 763.33E_c)} = 0.395$$

At interior joint

$$DF = \frac{498.59E_c}{(498.59E_c + 498.59E_c + 631.36E_c)} = 0.306$$

COF for slab-beam = 0.507

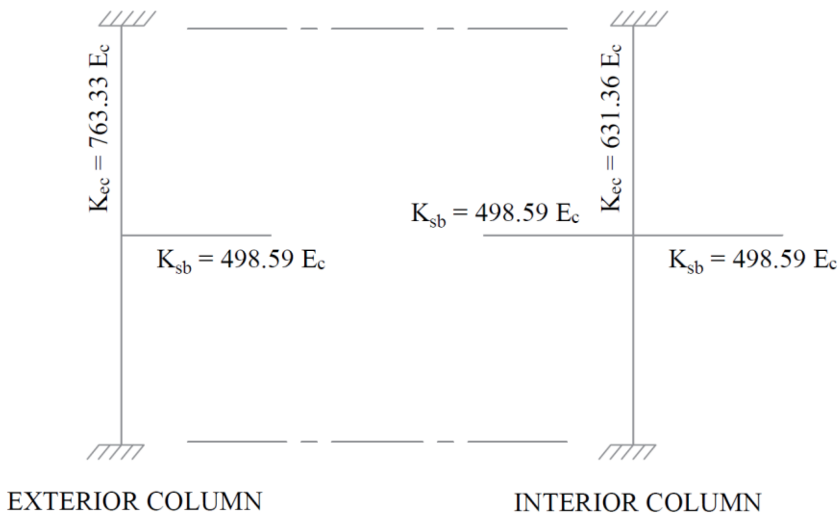


Figure 11 – Slab and Column Stiffness

3.3. Equivalent Frame Analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. With an unfactored live-to-dead load ratio:

$$\frac{L}{D} = \frac{100}{(150 \times 6/12)} = 1.33 > \frac{3}{4}$$

The frame will be analyzed for five loading conditions with pattern loading and partial live load as allowed by ***ACI 318-14 (6.4.3.3)***.

a) Factored load and Fixed-End Moments (FEM's).

Factored dead load, q_{Du} = 1.20 × (75.00 + 9.28) = 101.14 psf

Where ($\frac{14 \times (20-6)}{144} \times \frac{150}{22} = 9.28$ psf is the weight of beam stem per foot divided by l_2)

Factored live load, q_{Lu} = 1.60 × 100.00 = 160.00 psf

Total factored load, $q_u = q_{Du} + q_{Lu}$ = 261.14 psf

FEM's for slab-beam = $m_{NF} \times q_u \times l_2 \times l_1^2$ ***PCA Notes on ACI 318-11 (Table A1)***

FEM due to $q_{Du} + q_{Lu}$ = 0.0842 × (0.261 × 22) × 17.5^2 = 148.32 ft-kip

FEM due to q_{Du} + 3/4 × q_{Lu} = 0.0842 × ((0.101 + 3/4 × 0.160) × 22) × 17.5^2 = 125.60 ft-kip

FEM due to q_{Du} = 0.0842 × (0.101 × 22) × 17.5^2 = 57.44 ft-kip

b) Moment distribution.

Moment distribution for the five loading conditions is shown in Table 1. Counter-clockwise rotational moments acting on member ends are taken as positive. Maximum positive span moments are determined from the following equation:

$$M_{max}^{+} = \frac{(q_u \times l_2) \times l_1^2}{8} - \frac{M_L^- + M_R^-}{2} + \frac{(M_L^- - M_R^-)^2}{2 \times (q_u \times l_2) \times l_1^2} \text{ at distance } x_{max} = \frac{l_1}{2} + \frac{M_L^- - M_R^-}{(q_u \times l_2) \times l_1}$$

Where:

- $M_{max}{}^{+}$ = Maximum positive moment in the span
- $M_L{}^{-}$ = Negative moment in the left support
- $M_R{}^{-}$ = Negative moment in the right support
- l_1 = The span length

The reactions (shear forces) at supports are given by the following equations:

$$V_L = \frac{(q_u \times l_2) \times l_1}{2} + \frac{M_L^- - M_R^-}{l_1} \qquad V_R = \frac{(q_u \times l_2) \times l_1}{2} - \frac{M_L^- - M_R^-}{l_1}$$

Where:

- V_L = Reaction (shear force) at the left support
- V_R = Reaction (shear force) at the right support

<u>Maximum positive moment in spans 1-2 and 3-4:</u>

$$M_{max}^{+} = \frac{(0.261 \times 22) \times 17.5^2}{8} - \frac{93.15 + 167.93}{2} + \frac{(93.15 - 167.93)^2}{2 \times (0.261 \times 22) \times 17.5^2} = 90.97 \text{ ft-kips}$$

$$x_{max} = \frac{17.5}{2} + \frac{(93.15 - 167.93)}{(0.261 \times 22) \times 17.5} = 8.01 \text{ ft}$$

$$V_L = \frac{(0.261 \times 22) \times 17.5}{2} + \frac{(93.15 - 167.93)}{17.5} = 46.00 \text{ kips}$$

$$V_R = \frac{(0.261 \times 22) \times 17.5}{2} - \frac{(93.15 - 167.93)}{17.5} = 54.54 \text{ kips}$$

Where:

$M_L{}^{-}$ = 93.15 ft-kips $\qquad$ $M_R{}^{-}$ = 167.93 ft-kips

<u>Maximum positive moment in span 2-3:</u>

$$M_{max}^{+} = \frac{(0.261 \times 22) \times 17.5^2}{8} - \frac{153.86 + 153.86}{2} + \frac{(153.86 - 153.86)^2}{2 \times (0.261 \times 22) \times 17.5^2} = 66.06 \text{ ft-kips}$$

$$x_{max} = \frac{17.5}{2} + \frac{(153.86 - 153.86)}{(0.261 \times 22) \times 17.5} = 8.75 \text{ ft}$$

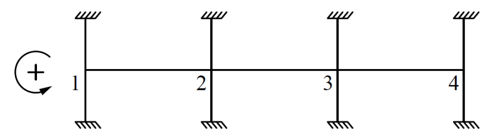
$$V_L = \frac{(0.261 \times 22) \times 17.5}{2} + \frac{(153.86 - 153.86)}{17.5} = 50.27 \text{ kips}$$

$$V_R = \frac{(0.261 \times 22) \times 17.5}{2} - \frac{(153.86 - 153.86)}{17.5} = 50.27 \text{ kips}$$

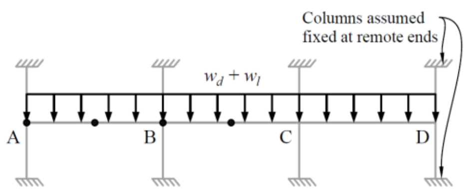
Where:

M_L^- = 153.86 ft-kips M_R^- = 153.86 ft-kips

Table 1 - Moment Distribution for Partial Frame (Transverse Direction)						
Joint	**1**	**2**		**3**		**4**
Member	**1-2**	**2-1**	**2-3**	**3-2**	**3-4**	**4-3**
DF	0.395	0.306	0.306	0.306	0.306	0.395
COF	0.507	0.507	0.507	0.507	0.507	0.507

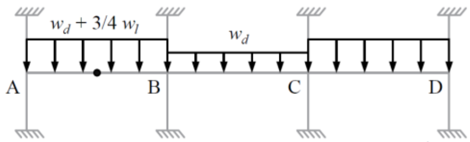


Loading (1) All spans loaded with full factored live load						
FEM	148.32	-148.32	148.32	-148.32	148.32	-148.32
Dist	-58.54	0	0	0	0	58.54
CO	0	-29.67	0	0	29.67	0
Dist	0	9.08	9.08	-9.08	-9.08	0
CO	4.60	0	-4.60	4.60	0	-4.60
Dist	-1.82	1.41	1.41	-1.41	-1.41	1.82
CO	0.71	-0.92	-0.71	0.71	0.92	-0.71
Dist	-0.28	0.50	0.50	-0.50	-0.50	0.28
CO	0.25	-0.14	-0.25	0.25	0.14	-0.25
Dist	-0.10	0.12	0.12	-0.12	-0.12	0.10
M^-_{max}	93.15	-167.93	153.86	-153.86	167.93	-93.15
M^+_{max}	90.97		66.06		90.97	



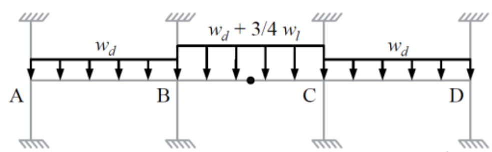
(1) Loading pattern for design moments in all spans with L ≤ 3/4 D

Loading (2) First and third spans loaded with 3/4 factored live load						
FEM	125.60	-125.60	57.44	-57.44	125.60	-125.60
Dist	-49.57	20.87	20.87	-20.87	-20.87	49.57
CO	10.58	-25.12	-10.58	10.58	25.12	-10.58
Dist	-4.17	10.93	10.93	-10.93	-10.93	4.17
CO	5.54	-2.12	-5.54	5.54	2.12	-5.54
Dist	-2.19	2.34	2.34	-2.34	-2.34	2.19
CO	1.19	-1.11	-1.19	1.19	1.11	-1.19
Dist	-0.47	0.70	0.70	-0.70	-0.70	0.47
CO	0.36	-0.24	-0.36	0.36	0.24	-0.36
Dist	-0.14	0.18	0.18	-0.18	-0.18	0.14
M^{-}_{max}	86.72	-119.15	74.81	-74.81	119.15	-86.72
M^{+}_{max}	83.66		10.36		83.66	



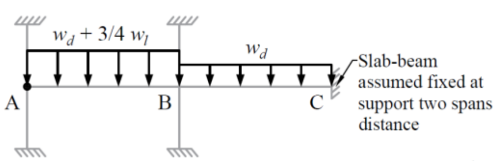
(2) Loading pattern for positive design moment in span AB*

Loading (3) Center span loaded with 3/4 factored live load						
FEM	57.44	-57.44	125.60	125.60	57.44	-57.44
Dist	-22.67	-20.87	-20.87	20.87	20.87	22.67
CO	-10.58	-11.49	10.58	-10.58	11.49	10.58
Dist	4.17	0.28	0.28	-0.28	-0.28	-4.17
CO	0.14	2.12	-0.14	0.14	-2.12	-0.14
Dist	-0.06	-0.60	-0.60	0.60	0.60	0.06
CO	-0.31	-0.03	0.31	-0.31	0.03	0.31
Dist	0.12	-0.09	-0.09	0.09	0.09	-0.12
CO	-0.04	0.06	0.04	-0.04	-0.06	0.04
Dist	0.02	-0.03	-0.03	0.03	0.03	-0.02
M^{-}_{max}	28.24	-88.10	115.07	-115.07	88.10	-28.24
M^{+}_{max}	29.64		71.17		29.64	



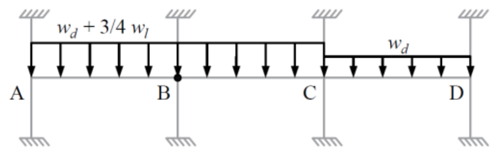
(3) Loading pattern for positive design moment in span BC*

Loading (4) First span loaded with 3/4 factored live load and beam-slab assumed fixed at support two spans away				
FEM	125.60	-125.60	57.44	-57.44
Dist	-49.57	20.87	20.87	0
CO	10.58	-25.12	0	10.58
Dist	-4.17	7.69	7.69	0
CO	3.90	-2.12	0	3.90
Dist	-1.54	0.65	0.65	0
CO	0.33	-0.78	0	0.33
Dist	-0.13	0.24	0.24	0.00
CO	0.12	-0.07	0	0.12
Dist	-0.05	0.02	0.02	0
M^{-}_{max}	85.06	-124.21	86.91	-42.52
M^{+}_{max}	82.12		21.91	

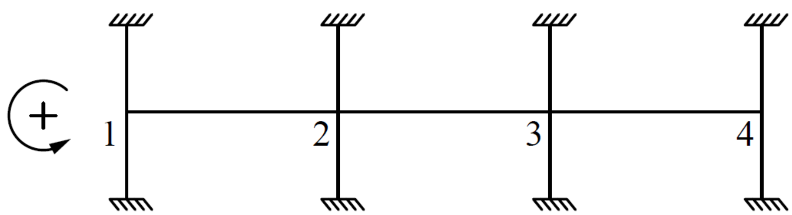


(4) Loading pattern for negative design moment at support A*

Loading (5) First and second spans loaded with 3/4 factored live load						
FEM	125.60	-125.60	125.60	-125.60	57.44	-57.44
Dist	-49.57	0	0	20.87	20.87	22.67
CO	0	-25.12	10.58	0	11.49	10.58
Dist	0	4.45	4.45	-3.52	-3.52	-4.17
CO	2.26	0	-1.78	2.26	-2.12	-1.78
Dist	-0.89	0.55	0.55	-0.04	-0.04	0.70
CO	0.28	-0.45	-0.02	0.28	0.36	-0.02
Dist	-0.11	0.15	0.15	-0.19	-0.19	0.01
CO	0.07	-0.06	-0.10	0.07	0	-0.10
Dist	-0.03	0.05	0.05	-0.02	-0.02	0.04
M^-_{max}	77.60	-146.04	139.46	-105.90	84.27	-29.52
M^+_{max}	75.99		63.93		30.48	



(5) Loading pattern for negative design moment at support B*



M^-_{max}	93.15	-167.93	153.86	-153.86	167.93	-93.15
M^+_{max}	90.97		71.17		90.97	

3.4. Design Moments

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 12. The negative design moments are taken at the faces of rectilinear supports but not at distances greater than 0.175 × l_1 from the centers of supports. ***ACI 318-14 (8.11.6.1)***

$$\frac{18 \text{ in.}}{12\times 2} = 0.75 \text{ ft} < 0.175\times 17.5 = 3.06 \text{ ft} \quad \text{(use face of support location)}$$

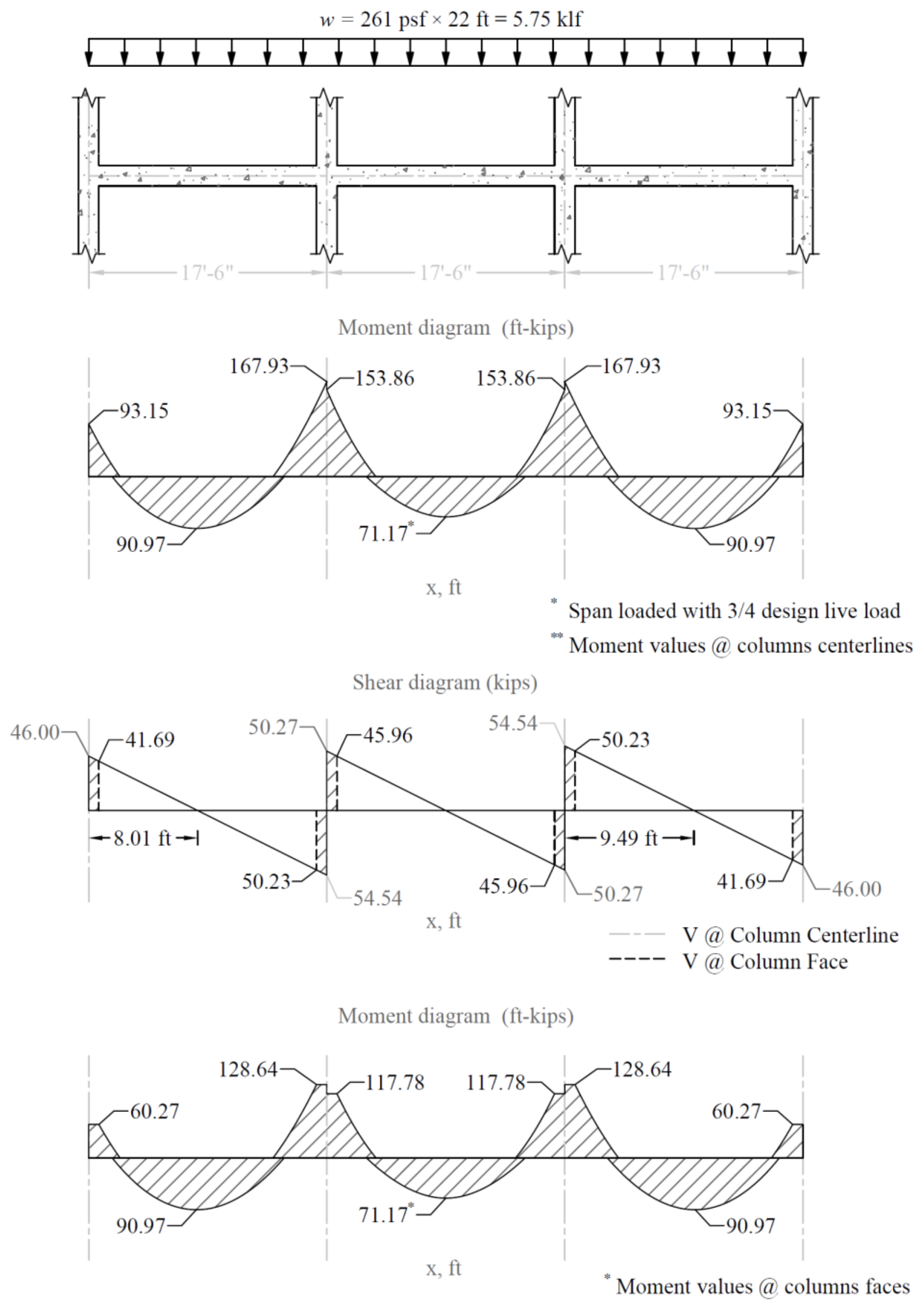


Figure 12 – Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load Except as Noted)

3.5. Distribution of Design Moments

a) Check whether the moments calculated above can take advantage of the reduction permitted by ***ACI 318-14 (8.11.6.5)***:

Slab systems within the limitations of ***ACI 318-14 (8.10.2)*** may have the resulting reduced in such proportion that the numerical sum of the positive and average negative moments not be greater than the total static moment M_o given by ***Equation 8.10.3.2*** in the ***ACI 318-14***. ***ACI 318-14 (8.11.6.5)***

Check Applicability of Direct Design Method:

1. There is a minimum of three continuous spans in each direction. ***ACI 318-14 (8.10.2.1)***
2. Successive span lengths are equal. ***ACI 318-14 (8.10.2.2)***
3. Long-to-Short ratio is 22/17.5 = 1.26 < 2.00. ***ACI 318-14 (8.10.2.3)***
4. Column are not offset. ***ACI 318-14 (8.10.2.4)***
5. Loads are gravity and uniformly distributed with service live-to-dead ratio of 1.33 < 2.00
6. Check relative stiffness for slab panel. ***ACI 318-14 (8.10.2.7)***

Interior Panel:

$$\alpha_{f1} = 3.16,\ l_2 = 22 \times 12 = 264.00 \text{ in.}$$

$$\alpha_{f2} = 3.98,\ l_2 = 17.5 \times 12 = 210.00 \text{ in.}$$

$$\frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2} = \frac{3.16 \times 264^2}{3.98 \times 210^2} = 1.25 \rightarrow 0.2 < 1.25 < 5.0$$ ***O.K.*** ***ACI 318-14 (Eq. 8.10.2.7a)***

Interior Panel:

$$\alpha_{f1} = 3.16,\ l_2 = 22 \times 12 = 264.00 \text{ in.}$$

$$\alpha_{f2} = 16.45,\ l_2 = 17.5 \times 12 = 210.00 \text{ in.}$$

$$\frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2} = \frac{3.16 \times 264^2}{16.45 \times 210^2} = 0.30 \rightarrow 0.2 < 0.30 < 5.0$$ ***O.K.*** ***ACI 318-14 (Eq. 8.10.2.7a)***

All limitation of ***ACI 318-14 (8.10.2)*** are satisfied and the provisions of ***ACI 318-14 (8.11.6.5)*** may be applied:

$$M_o = \frac{q_u \times \ell_2 \times \ell_n{}^2}{8} = \frac{0.261 \times 22 \times (17.5 - 18/12)^2}{8} = 183.84 \text{ ft-kips}$$ ***ACI 318-14 (Eq. 8.10.3.2)***

End spans: $90.97+\dfrac{60.27+128.64}{2}=185.43\text{ ft-kips}$

Interior span: $71.17+\dfrac{117.78+117.78}{2}=188.94\text{ ft-kips}$

To illustrate proper procedure, the interior span factored moments may be reduced as follows:

Permissible reduction $=\dfrac{185.43}{188.94}=0.973$

Adjusted negative design moment = 117.78 × 0.973 = 114.60 ft-kips

Adjusted positive design moment = 71.17 × 0.973 = 69.24 ft-kips

$M_o=183.84\text{ ft-kips}$

b) Distribute factored moments to column and middle strips:

The negative and positive factored moments at critical sections may be distributed to the column strip and the two half-middle strips of the slab-beam according to the Direct Design Method (DDM) in 8.10, provided that ***Eq. 8.10.2.7(a)*** is satisfied. ***ACI 318-14 (8.11.6.6)***

Since the relative stiffness of beams are between 0.2 and 5.0 (see Step 3.5), the moments can be distributed across slab-beams as specified in ***ACI 318-14 (8.10.5 and 6)*** where:

$$\frac{l_2}{l_1}=\frac{22}{17.5}=1.257$$

$$\frac{\alpha_{f1}l_2}{l_1}=3.16\times1.254=3.975$$

$$\beta_t=\frac{C}{2I_s}=\frac{17,868.48}{2\times4,752.00}=1.880$$

Where $I_s=\dfrac{22\times12\times6^3}{12}=4,752.00\text{ in.}^4$

C = 17,868.48 in.^{4} (see Figure 8)

Factored moments at critical sections are summarized in Table below.

Table 2 – Lateral Distribution of Factored Moments							
Location		**Factored Moments (ft-kips)**	**Column Strip**				**Moments in Two Half-Middle Strips** (ft-kips)**
			Percent*	**Moments (ft-kips)**	**Beam Strip Moment (ft-kips)**	**Column Strip Moment (ft-kips)**	
End Span	**Exterior Negative**	60.27	75	45.20	38.42	6.78	15.07
	Positive	90.97	67	60.95	51.81	9.14	30.02
	Interior Negative	128.64	67	86.19	73.26	12.93	42.45
Interior Span	**Negative**	117.78	67	78.91	67.07	11.84	38.87
	Positive	71.17	67	47.68	40.53	7.15	23.48

* Since $\alpha_1 l_2/l_1 > 1.0$ beams must be proportioned to resist 85 percent of column strip per ***ACI 318-14 (8.10.5.7)***

** That portion of the factored moment not resisted by the column strip is assigned to the two half-middle strips

3.6. Flexural Reinforcement Requirements

a) Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – interior negative location is provided below:

M_u = 12.93 ft-kips

Assume tension-controlled section (ϕ = 0.90)

Column strip width, $b = \dfrac{17.5 \times 12}{2} - 14 = 91.00$ in.

Use average d = 6 – 0.75 – 0.50 / 2 = 5.00 in.

$$A_s = \frac{0.85 \times f_c' \times b}{f_y}\left(d - \sqrt{d^2 - \frac{2 \times M_u}{\phi \times 0.85 \times f_c' \times b}}\right)$$

$$A_s = \frac{0.85 \times 4,000 \times 91.00}{60,000} \times \left(5.00 - \sqrt{5.00^2 - \frac{2 \times 12.93 \times 12,000}{0.90 \times 0.85 \times 4,000 \times 91.00}}\right) = 0.581 \text{ in.}^2$$

$$A_{s,min} = \max\begin{bmatrix} 0.0018 \times b \times h \\ 0.0014 \times b \times h \end{bmatrix} = \max\begin{bmatrix} 0.0018 \times 14 \times 19 \\ 0.0014 \times 14 \times 19 \end{bmatrix} = \max\begin{bmatrix} 0.983 \\ 0.764 \end{bmatrix} = 0.983 \text{ in.}^2 < 0.581 \text{ in.}^2$$

$$\therefore A_s = 0.983 \text{ in.}^2$$

Maximum spacing $s_{max} = 2h = 2 \times 6 = 12$ in. < 18 in.

Provide 8 – #4 bars with A_s = 1.60 in.^{2} and s = 91.00 / 8 = 11.38 in. $\leq s_{max}$

The flexural reinforcement calculation for the beam strip of end span – interior negative location is provided below:

M_u = 73.26 ft-kips

Assume tension-controlled section (ϕ = 0.90)

Beam strip width, b = 14.00 in.

Use average d = 20 – 0.75 – 0.50 / 2 = 19.00 in.

$$A_s = \frac{0.85 \times f_c' \times b}{f_y}\left(d - \sqrt{d^2 - \frac{2 \times M_u}{\phi \times 0.85 \times f_c' \times b}}\right)$$

$$A_s = \frac{0.85 \times 4,000 \times 14.00}{60,000} \times \left(19.00 - \sqrt{19.00^2 - \frac{2 \times 73.26 \times 12,000}{0.90 \times 0.85 \times 4,000 \times 14.00}}\right) = 0.883 \text{ in.}^2$$

$$A_{s,min} = \max\begin{bmatrix} \dfrac{3\sqrt{f_c'}}{f_y} \times b \times d \\ \dfrac{200}{f_y} \times b \times d \end{bmatrix} = \max\begin{bmatrix} \dfrac{3\sqrt{4,000}}{60,000} \times 14.00 \times 19.00 \\ \dfrac{200}{60,000} \times 14.00 \times 19.00 \end{bmatrix} = \max\begin{bmatrix} 0.841 \\ 0.887 \end{bmatrix} = 0.887 \text{ in.}^2 < 0.883 \text{ in.}^2$$

$$\therefore A_s = 0.887 \text{ in.}^2$$

Provide 5 – #4 bars with A_s = 1.00 in.^{2}

All the values in Table below are calculated based on the procedure outlined above.

Table 3 - Required Slab Reinforcement for Flexure [Equivalent Frame Method (EFM)]								
Span Location		M_u (ft-kips)	b * (in.)	d ** (in.)	A_s Req'd for flexure (in.^{2})	*Min* A_s† †† (in.^{2})	Reinforcement Provided	A_s Prov. for flexure (in.^{2})
End Span								
Beam Strip	**Exterior Negative**	38.42	14	19.00	0.456	0.608	4 – #4	0.80
	Positive	51.81	14	18.25	0.645	0.852	5 – #4	1.00
	Interior Negative	73.26	14	19.00	0.883	0.887	5 – #4	1.00
Column Strip	**Exterior Negative**	6.78	91	5.00	0.303	0.983	8 – #4	1.60
	Positive	9.14	91	5.00	0.410	0.983	8 – #4	1.60
	Interior Negative	12.93	91	5.00	0.581	0.983	8 – #4	1.60
Middle Strip	**Exterior Negative**	15.07	159	5.00	0.675	1.717	14 – #4	2.80
	Positive	30.02	159	5.00	1.355	1.717	14 – #4	2.80
	Interior Negative	42.45	159	5.00	1.928	1.717	14 – #4	2.80
Interior Span								
Beam Strip	**Positive**	40.53	14	18.25	0.502	0.670	4 – #4	0.80
Column Strip	**Positive**	7.15	91	5.00	0.320	0.983	8 – #4	1.60
Middle Strip	**Positive**	23.48	159	5.00	1.056	1.717	14 – #4	2.80

* Column strip width, b = (17.5 × 12) / 2 - 14 = 91.00 in.

* Middle strip width, b = 22 × 12 – (17.5 × 12) / 2 = 159.00 in.

* Beam strip width, b = 14.00 in.

** Use average d = 6 – 0.75 – 0.50 / 2 = 5.00 in. for Column and Middle strips

** Use average d = 20 - 1.5 - 0.50 / 2 = 18.25 in. for Beam strip Positive moment regions

** Use average d = 20 - 0.75 - 0.50 / 2 = 19 in. for Beam strip Negative moment regions

† Min. A_s = 0.0018 × b × h = 0.0108 × b for Column and Middle strips — ***ACI 318-14 (7.6.1.1)***

† Min. A_s = min (3(f_c')^0.5 / f_y × b × d, 200 / f_y × b × d) for Beam strip — ***ACI 318-14 (9.6.1.2)***

†† Min. A_s = 1.333 × A_s Req'd if A_s provided >= 1.333 × A_s Req'd for Beam strip — ***ACI 318-14 (9.6.1.3)***

s_{max} = 2 × h = 12 in. < 18 in. — ***ACI 318-14 (8.7.2.2)***

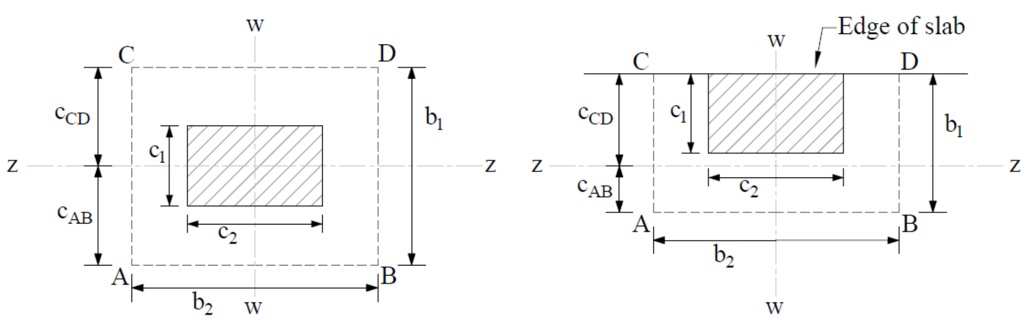
b) Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_u$ **_ACI 318-14 (8.4.2.3.1)_**

Where:

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{b_1}{b_2}}}$$ **_ACI 318-14 (8.4.2.3.2)_**

- b_1 = Dimension of the critical section b_o measured in the direction of the span for which moments are determined in ACI 318, Chapter 8 (see Figure 13).
- b_2 = Dimension of the critical section b_o measured in the direction perpendicular to b_1 in ACI 318, Chapter 8 (see Figure 13).
- b_o = Perimeter of critical section for two-way shear in slabs and footings.
- b_b = Effective slab width = c_2 + 3 × h **_ACI 318-14 (8.4.2.3.3)_**



Critical shear perimeter for interior column   Critical shear perimeter for exterior column

Figure 13 – Critical Shear Perimeters for Columns

For Exterior Column:

$$b_1 = c_1 + \frac{d}{2} = 18 + \frac{5.00}{2} = 20.50 \text{ in.}$$

$$b_2 = c_2 + d = 18 + 5.00 = 23.00 \text{ in.}$$

$$b_b = c_2 + 3 \times h = 18 + 3 \times 6.00 = 36.00 \text{ in.}$$

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{20.50}{23.00}}} = 0.614$$

$$\gamma_f M_{u,net} = 0.614 \times 93.15 = 57.17 \text{ ft-kips}$$

$$A_s = \frac{0.85 \times f_c' \times b_b}{f_y} \times \left(d - \sqrt{d^2 - \frac{2 \times \gamma_f M_{u,net}}{\phi \times 0.85 \times f_c' \times b_b}} \right)$$

$$A_s = \frac{0.85 \times 4,000 \times 36.00}{60,000} \times \left(5.00 - \sqrt{5.00^2 - \frac{2 \times 57.17 \times 12,000}{0.90 \times 0.85 \times 4,000 \times 36.00}} \right) = 2.975 \text{ in.}^2$$

$$A_{s,min} = \max \begin{bmatrix} 0.0018 \times b \times h \\ 0.0014 \times b \times h \end{bmatrix} = \max \begin{bmatrix} 0.0018 \times 14 \times 19 \\ 0.0014 \times 14 \times 19 \end{bmatrix} = \max \begin{bmatrix} 0.983 \\ 0.764 \end{bmatrix} = 0.983 \text{ in.}^2 < 2.975 \text{ in.}^2$$

$$\therefore A_{s,req'd} = 2.975 \text{ in.}^2$$

$$A_{s,provided} = \left(A_{s,provided} \right)_{(beam)} + \left(A_{s,provided} \right)_{(b_b - beam)}$$

$$A_{s,provided} = 4 \times 0.20 + 8 \times 0.20 \times \frac{36 - 14}{91} = 1.187 \text{ in.}^2 < A_{s,req'd} = 2.975 \text{ in.}^2$$

$\therefore$ Additional slab reinforcement at the exterior column is required.

$$A_{req'd,add} = 2.975 - 1.187 = 1.788 \text{ in.}^2$$

Use 10 – #4 → $A_{provided,add}$ = 10 × 0.20 = 2.00 in.^{2} < $A_{req'd,add}$ = 1.788 in.^{2}

Based on the procedure outlined above, values for all supports are given in Table below.

Table 4 - Additional Slab Reinforcement at columns for moment transfer between slab and column [Equivalent Frame Method (EFM)]									
Span Location		Effective slab width, b_b (in.)	d (in.)	γ_f	M_u^* (ft-kips)	$\gamma_f M_u$ (ft-kips)	A_s req'd within b_b (in.^{2})	A_s prov. for flexure within b_b (in.^{2})	Add'l Reinf.
End Span									
Column Strip	Exterior Negative	36.00	5.00	0.614	93.15	57.17	2.975	1.187	10-#4
	Interior Negative	36.00	5.00	0.600	44.34	26.60	1.260	1.387	-
* M_u is taken at the centerline of the support in Equivalent Frame Method solution.									

c) Determine transverse reinforcement required for beam strip shear

The transverse reinforcement calculation for the beam strip of end span – exterior location is provided below.

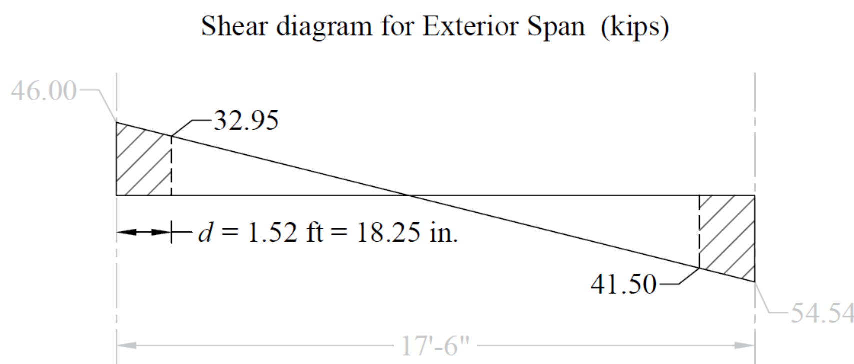


Figure 14 – Shear at Critical Sections for the End Span (at distance d from the face of the column)

$$d = h - c_{clear} - \frac{d_{stirrup}}{2} = 20 - 1.50 - \frac{0.50}{2} = 18.25 \text{ in. (using \#4 stirrups)}$$

The required shear at a distance d from the face of the supporting column V_{u_d} = 31.64 kips (Figure 14).

$$\phi_v V_c = \phi_v \times 2 \times \sqrt{f_c'} \times b \times d$$ **ACI 318-14 (22.5.5.1)**

$$\phi_v V_c = 0.75 \times 2 \times \sqrt{4,000} \times 14 \times 18.25 = 24.24 \text{ kips} < V_{u\_d} = 32.95 \text{ kips}$$ ∴Stirrups are required.

Distance from the column face beyond which minimum reinforcement is required:

$$V_s = \frac{V_{u\_d} - \phi_v V_c}{\phi_v}$$ **ACI 318-14 (22.5.10.1)**

$$V_s = \frac{32.95 - 24.24}{0.75} = 11.61 \text{ kips} < V_{s,max} = 129.27 \text{ kips}$$ ***O.K.***

$$V_{s,max} = 8 \times \sqrt{f_c'} \times b \times d = 8 \times \sqrt{4,000} \times 14 \times 18.25 = 129.27 \text{ kips}$$ **ACI 318-14 (22.5.10.1)**

$$\frac{A_{v,req'd}}{s} = \frac{V_s}{f_{yt} \times d} = \frac{11.61 \times 1{,}000}{60{,}000 \times 18.25} = 0.0106 \text{ in.}^2/\text{in.}$$ **_ACI 318-14 (22.5.10.5.3)_**

$$\frac{A_{v,min}}{s} = \max\begin{bmatrix} \dfrac{0.75\sqrt{f'_c}}{f_{yt}} b \\ \dfrac{50}{f_{yt}} b \end{bmatrix}$$ **_ACI 318-14 (9.6.3.3)_**

$$\frac{A_{v,min}}{s} = \max\begin{bmatrix} \dfrac{0.75\sqrt{4{,}000}}{60{,}000} \times 14 \\ \dfrac{50}{60{,}000} \times 14 \end{bmatrix} = \max\begin{bmatrix} 0.0111 \\ 0.0117 \end{bmatrix} = 0.0117 \text{ in.}^2/\text{in.}$$

$$\frac{A_{v,req'd}}{s} < \frac{A_{v,min}}{s} \rightarrow \therefore \text{ use } \frac{A_{s,req'd}}{s} = \frac{A_{v,min}}{s}$$

$$s_{req'd} = \frac{n \times A_{stirrup}}{\dfrac{A_{v,req'd}}{s}} = \frac{2 \times 0.20}{0.0117} = 34.29 \text{ in.}$$

$$V_s = 9.85 \text{ kips} < 4 \times \sqrt{f'_c} \times b \times d = 4 \times \sqrt{4{,}000} \times 14 \times 18.25 = 64.64 \text{ kips}$$

$$\therefore s_{max} = \text{Lesser of}\begin{bmatrix} \dfrac{d}{2} \\ 24 \end{bmatrix} = \text{Lesser of}\begin{bmatrix} \dfrac{18.25}{2} \\ 24 \end{bmatrix} = \text{Lesser of}\begin{bmatrix} 9.13 \\ 24 \end{bmatrix} = 9.13 \text{ in.}$$ **_ACI 318-14 (9.7.6.2.2)_**

Since $s_{req'd} > s_{max} \rightarrow$ use s_{max}

Select $s_{provided}$ = 8 in. #4 stirrups with first stirrup located at distance 3 in. from the column face.

The distance where the shear is zero is calculated as follows:

$$x = \frac{l}{V_{u,L} + V_{u,R}} \times V_{u,L} = \frac{17.5}{46.00 + 54.54} \times 46.00 = 8.01 \text{ ft} = 96.07 \text{ in.}$$

The distance from support beyond which minimum reinforcement is required is calculated as follows:

$$x_1 = x - \frac{x}{V_u} \times \phi_v V_c = 8.01 - \frac{8.01}{46.00} \times 24.24 = 3.79 \text{ ft} = 45.44 \text{ in.}$$

The distance at which no shear reinforcement is required is calculated as follows:

$$x_2 = x - \frac{x}{V_u} \times \frac{\phi_v V_c}{2} = 8.01 - \frac{8.01}{46.00} \times \frac{24.24}{2} = 5.90 \text{ ft} = 70.76 \text{ in.}$$

$$\#of\ \ stirrups = \frac{x_2 - 3 - \frac{c_1}{2} - \frac{s_{provided}}{2}}{s_{provided}} + 1 = \frac{70.76 - 3 - \frac{18}{2} - \frac{8}{2}}{8} + 1 = 7.84 \rightarrow \text{use 8 stirrups}$$

All the values in Table below are calculated based on the procedure outlined above.

Table 5 - Required Beam Reinforcement for Shear					
Span Location	$A_{v,min}/s$ **(in.²/in.)**	$A_{v,req'd}/s$ **(in.²/in.)**	$s_{req'd}$ **(in.)**	s_{max} **(in.)**	**Reinforcement Provided**
End Span					
Exterior	0.0117	0.0106	34.29	9.13	8 - #4 @ 8 in.*
Interior	0.0117	0.0210	19.04	9.13	10 - #4 @ 8.6 in.
Interior Span					
Interior	0.0117	0.0158	25.30	9.13	9 - #4 @ 8.6 in.
* Minimum transverse reinforcement governs					

3.7. Column Design Moments

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the actual columns above and below the slab-beam in proportion to the relative stiffness of the actual columns. Referring to Table 1, the unbalanced moment at joints 1 and 2 are:

Joint 1 = + 93.15 ft-kips (Based on Loading (1))

Joint 2= - 119.15 + 74.81 = - 44.34 ft-kips (Based on Loading (2))

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced moments to the exterior and interior columns are shown in the following Figure.

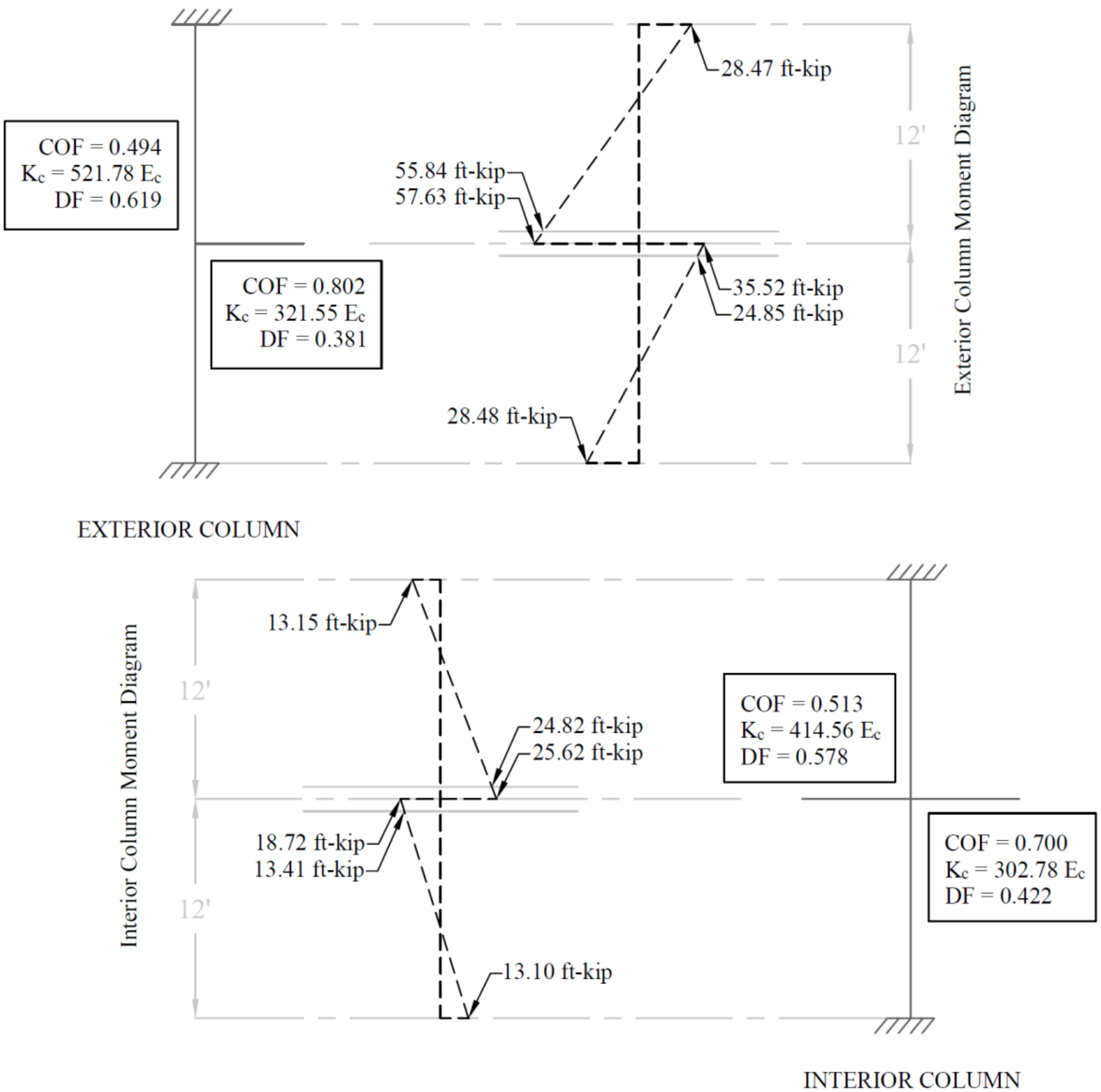


Figure 15 – Column Moments (Unbalanced Moments from Slab-Beam)

In summary:

Design moment in exterior column = 55.84 ft-kips

Design moment in interior column = 24.82 ft-kips

The moments determined above are combined with the factored axial loads (for each story) and factored moments in the transverse direction for design of column sections. A detailed analysis to obtain the moment values at the face of interior, exterior, and corner columns from the unbalanced moment values can be found in the "Two-Way Flat Plate Concrete Floor System Analysis and Design (ACI 318-14)" Design Example.

4. Design of Interior, Edge, and Corner Columns

The design of interior, edge, and corner columns is explained in the "Two-Way Flat Plate Concrete Floor System Analysis and Design (ACI 318-14)" Design Example.

5. Two-Way Slab Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with ACI 318 Chapter 22.

5.1. One-Way (Beam Action) Shear Strength

One-way shear is critical at a distance *d* from the face of the column. Figure 16 shows the V_u at the critical sections around each column. Since there is no shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$\phi V_n = \phi V_c + \phi V_s = \phi V_c$$ ***ACI 318-14 (Eq. 22.5.1.1)***

Where:

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d$$ ***ACI 318-14 (Eq. 22.5.5.1)***

$\lambda = 1$ for normal weight concrete

$$\phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{4{,}000} \times (22 \times 12 - 14) \times \frac{5}{1{,}000} = 118.59 \text{ kips}$$

Because $\phi V_c > V_u$ at all the critical section, the slab is ***o.k.*** in one-way shear.

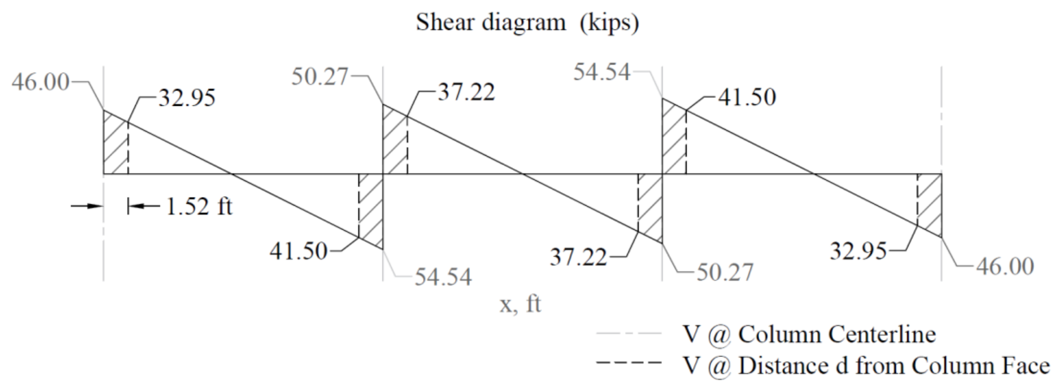


Figure 16 – One-way shear at critical sections (at distance *d* from the face of the supporting column)

5.2. Two-Way (Punching) Shear Strength

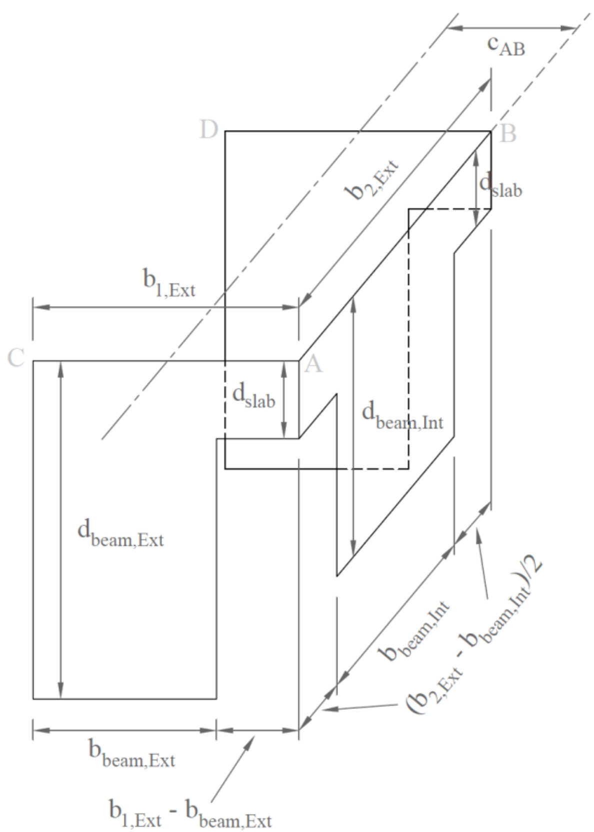
Two-way shear is critical on a rectangular section located at $d_{slab}/2$ away from the face of the column. The factored shear force V_u in the critical section is calculated as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section.

The factored unbalanced moment used for shear transfer, M_{unb}, is calculated as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

<u>**For the Exterior column:**</u>

$$V_u = 46.00 - 0.340 \times \left(\frac{20.50 \times 23.00}{144} \right) = 44.88 \text{ kips}$$

$$M_{unb} = 93.15 - 44.88 \times \left(\frac{20.50 - 9.09 - \frac{18}{2}}{12} \right) = 84.15 \text{ ft-kips}$$



<u>Figure 17 – Critical Section of Exterior support of Interior Frame</u>

For the exterior column in Figure above the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{2\times(14\times26\times(6.50+14/2)+6.5\times5\times(6.5/2))}{2\times(14\times26+6.5\times5)+14\times19+2\times4.5\times5} = 9.09 \text{ in.}$$

$$A_c = 2\times(14\times26+6.5\times5)+14\times19+2\times4.5\times5 = 1104.00 \text{ in.}^2$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\times\left(\frac{b_{beam,Ext}\times d^3_{beam,Ext}}{12}+\frac{d_{beam,Ext}\times b^3_{beam,Ext}}{12}+\left(b_{beam,Ext}\times d_{beam,Ext}\right)\times\left(\frac{b_{beam,Ext}}{2}+\left(b_1-b_{beam,Ext}\right)-c_{AB}\right)^2\right)$$
$$+2\times\left(\frac{\left(b_1-b_{beam,Ext}\right)\times d^3_{slab,Ext}}{12}+\frac{d_{slab,Ext}\times\left(b_1-b_{beam,Ext}\right)^3}{12}+\left(\left(b_1-b_{beam,Ext}\right)\times d_{slab}\right)\times\left(c_{AB}-\frac{b_1-b_{beam,Ext}}{2}\right)^2\right)$$
$$+\left(b_{beam,Int}\times d_{beam,Int}+\left(b_2-b_{beam,Int}\right)\times d_{slab}\right)\times c^2_{AB}$$

$$J_c = 2\times\left(\frac{14\times26^3}{12}+\frac{26\times14^3}{12}+(14\times26)\times\left(\frac{14}{2}+(20.50-14)-9.09\right)^2\right)$$
$$+2\times\left(\frac{(20.50-14)\times5^3}{12}+\frac{5\times(20.50-14)^3}{12}+((20.50-14)\times5)\times\left(9.09-\frac{20.50-14}{2}\right)^2\right)$$
$$+(14\times19+(23-14)\times5)\times9.09^2$$

$$J_c = 95{,}338.01 \text{ in.}^4$$

$$\gamma_v = 1-\gamma_f = 1-0.614 = 0.386$$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

Where:

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{b_1}{b_2}}}$$ ***ACI 318-14 (8.4.2.3.2)***

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{20.50}{23.00}}} = 0.614$$

$$b_1 = c_1+\frac{d_s}{2} = 18+\frac{5}{2} = 20.50 \text{ in.}$$

$$b_2 = c_2+d_s = 18+5 = 23.00 \text{ in.}$$

The length of the critical perimeter for the exterior column:

$$b_o = 2\times b_1+b_2 = 2\times20.50+23.00 = 64.00 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{44.88 \times 1,000}{1,104.00} + \frac{0.386 \times (84.15 \times 12 \times 1,000) \times 9.09}{95,338.01} = 40.65 + 37.20 = 77.86 \text{ psi}$$

$$v_c = \min \begin{Bmatrix} 4 \times \lambda \times \sqrt{f'_c} \\ \left(2 + \dfrac{4}{\beta}\right) \times \lambda \times \sqrt{f'_c} \\ \left(\dfrac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f'_c} \end{Bmatrix}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min \begin{Bmatrix} 4 \times 1 \times \sqrt{4,000} \\ \left(2 + \dfrac{4}{1}\right) \times 1 \times \sqrt{4,000} \\ \left(\dfrac{30 \times 5.00}{64.00} + 2\right) \times 1 \times \sqrt{4,000} \end{Bmatrix} = \min \begin{Bmatrix} 252.98 \\ 379.47 \\ 274.72 \end{Bmatrix} = 252.98 \text{ psi}$$

$$\phi v_c = 0.75 \times 252.98 = 189.74 \text{ psi} > v_u = 77.86 \text{ psi}$$

Because $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

For the Interior column:

$$V_u = 54.54 + 50.27 - 0.340 \times \left(\frac{23.00 \times 23.00}{144} \right) = 103.56 \text{ kips}$$

$$M_{unb} = 167.93 - 153.86 - 103.56 \times (0) = 14.07 \text{ ft-kips}$$

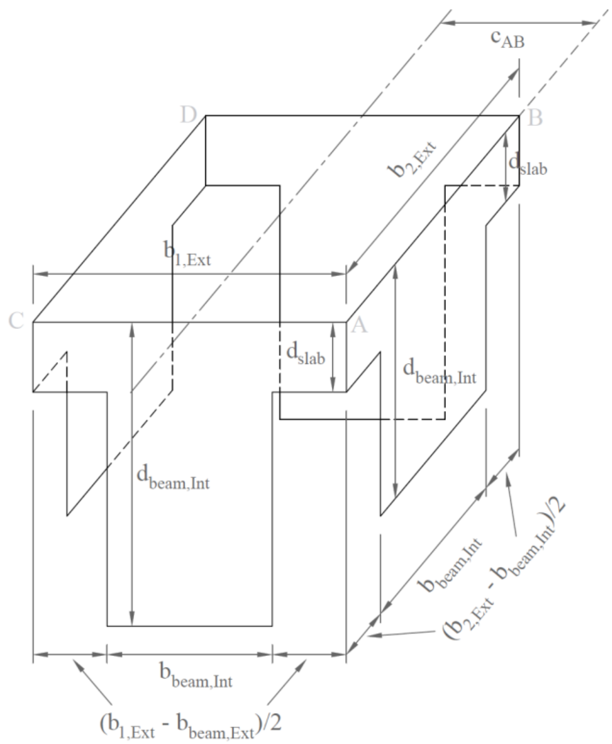


Figure 18 – Critical Section of Interior support of Interior Frame

For the interior column in Figure above the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_{1,Int}}{2} = \frac{23.00}{2} = 11.50 \text{ in.}$$

$$A_c = 4 \times (14 \times 19 + 9 \times 5) = 1244.00 \text{ in.}^2$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\times\left(\frac{b_{beam,Int}\times d_{beam,Int}^3}{12}+\frac{d_{beam,Int}\times b_{beam,Int}^3}{12}+\left(b_{beam,Int}\times d_{beam,Int}\right)\times\left(\frac{b_{beam,Int}}{2}+\left(\frac{b_1-b_{beam,Int}}{2}\right)-c_{AB}\right)^2\right)$$
$$+4\times\left(\frac{\left(\frac{b_1-b_{beam,Int}}{2}\right)\times d_{slab,Int}^3}{12}+\frac{d_{slab,Int}\times\left(\frac{b_1-b_{beam,Int}}{2}\right)^3}{12}+\left(\left(\frac{b_1-b_{beam,Int}}{2}\right)\times d_{slab,Int}\right)\times\left(c_{AB}-\frac{b_1-b_{beam,Int}}{2\times 2}\right)^2\right)$$
$$+2\times\left(b_{beam,Int}\times d_{beam,Int}+\left(b_2-b_{beam,Int}\right)\times d_{slab,Int}\right)\times c_{AB}^2$$

$$J_c = 2\times\left(\frac{14\times 19^3}{12}+\frac{19\times 14^3}{12}+\left(14\times 19\right)\times\left(\frac{14}{2}+\left(\frac{23-14}{2}\right)-11.5\right)^2\right)$$
$$+4\times\left(\frac{\left(\frac{23-14}{2}\right)\times 5^3}{12}+\frac{5\times\left(\frac{23-14}{2}\right)^3}{12}+\left(\left(\frac{23-14}{2}\right)\times 5\right)\times\left(11.5-\frac{23-14}{2\times 2}\right)^2\right)$$
$$+2\times\left(14\times 19+\left(23-14\right)\times 5\right)\times 11.5^2$$

$$J_c = 114{,}993.17 \text{ in.}^4$$

$$\gamma_v = 1-\gamma_f = 1-0.600 = 0.400$$ **_ACI 318-14 (Eq. 8.4.4.2.2)_**

Where:

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{b_1}{b_2}}}$$ **_ACI 318-14 (8.4.2.3.2)_**

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{23.00}{23.00}}} = 0.600$$

$$b_1 = c_1 + d_s = 18+5 = 23.00 \text{ in.}$$

$$b_2 = c_2 + d_s = 18+5 = 23.00 \text{ in.}$$

The length of the critical perimeter for the exterior column:

$$b_o = 2\times b_1 + 2\times b_2 = 2\times 23.00 + 2\times 23.00 = 92.00 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$ **<u>*ACI 318-14 (R.8.4.4.2.3)*</u>**

$$v_u = \frac{103.56 \times 1,000}{1,244.00} + \frac{0.400 \times (14.07 \times 12 \times 1,000) \times 11.50}{114,993.17} = 83.25 + 6.75 = 90.00 \text{ psi}$$

$$v_c = \min \begin{Bmatrix} 4 \times \lambda \times \sqrt{f_c'} \\ \left(2 + \frac{4}{\beta}\right) \times \lambda \times \sqrt{f_c'} \\ \left(\frac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f_c'} \end{Bmatrix}$$ **<u>*ACI 318-14 (Table 22.6.5.2)*</u>**

$$v_c = \min \begin{Bmatrix} 4 \times 1 \times \sqrt{4,000} \\ \left(2 + \frac{4}{1}\right) \times 1 \times \sqrt{4,000} \\ \left(\frac{30 \times 5.00}{64.00} + 2\right) \times 1 \times \sqrt{4,000} \end{Bmatrix} = \min \begin{Bmatrix} 252.98 \\ 379.47 \\ 274.72 \end{Bmatrix} = 252.98 \text{ psi}$$

$$\phi v_c = 0.75 \times 252.98 = 189.74 \text{ psi} > v_u = 90.00 \text{ psi}$$

Because $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

6. Two-Way Slab Deflection Control (Serviceability Requirements)

Since the slab thickness was selected based on the minimum slab thickness tables in ACI 318-14, the deflection calculations are not required. However, the calculations of immediate and time-dependent deflections are covered in this section for illustration and comparison with spSlab model results.

6.1. Immediate (Instantaneous) Deflections

The calculation of deflections for two-way slabs is challenging even if linear elastic behavior can be assumed. Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D + L_{Full}$) is used to obtain immediate deflections of the two-way slab in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. ***ACI 318-14 (24.2.3)***

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the slab. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr} \le I_g$$ ***ACI 318-14 (Eq. 24.2.3.5a)***

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously in this document. These moments are shown in Figure 19.

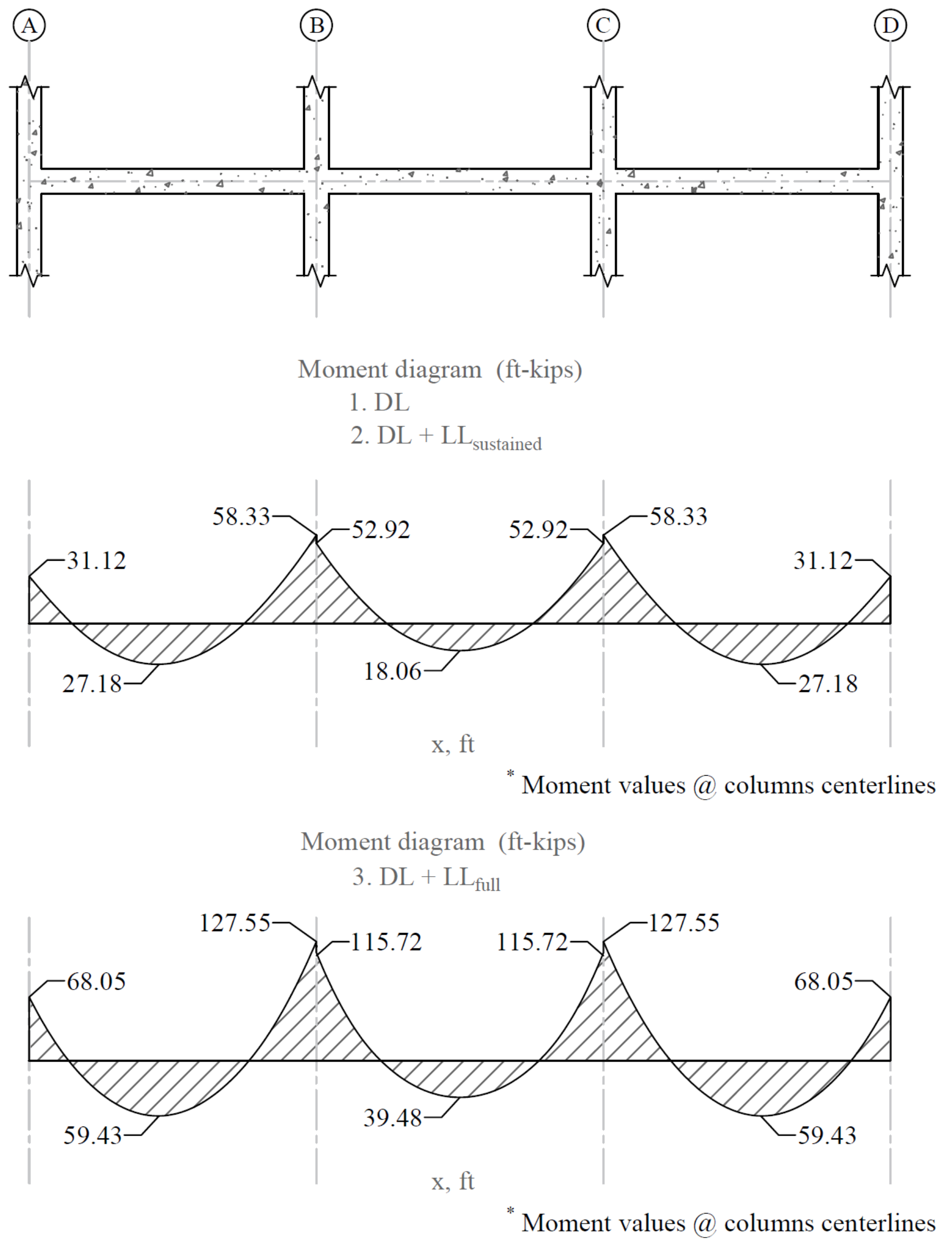


Figure 19 – Maximum Moments for the Three Service Load Levels

For positive moment (midspan) section of the exterior span:

M_{cr} = cracking moment.

$$M_{cr} = \frac{f_r \times I_g}{y_t} = \frac{474.34 \times 25,395.13}{15.90} \times \frac{1}{12 \times 1,000} = 63.14 \text{ ft-kips}$$ ***ACI 318-14 (Eq. 24.2.3.5b)***

f_r = Modulus of rapture of concrete.

$$f_r = 7.5 \times \lambda \times \sqrt{f'_c} = 7.5 \times 1.0 \times \sqrt{4,000} = 474.34 \text{ psi}$$ ***ACI 318-14 (Eq. 19.2.3.1)***

I_g = Moment of inertia of the gross uncracked concrete section

I_g = 25,395.13 in.^{4} for T-section (see Figure 20)

y_t = Distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.

y_t = 15.90 in. (see Figure 20)

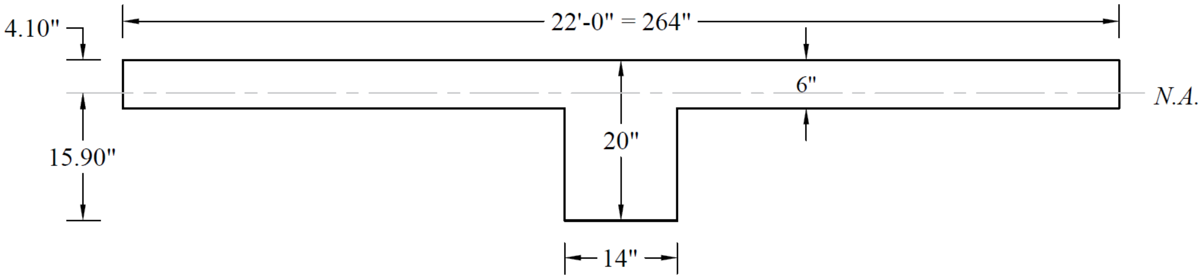


Figure 20 – I_g Calculations for Slab Section Near Support

I_{cr} = moment of inertia of the cracked section transformed to concrete. ***PCA Notes on ACI 318-11 (9.5.2.2)***

As calculated previously, the positive reinforcement for the end span frame strip is 22 #4 bars located at 1.0 in. along the slab section from the bottom of the slab and 4 #4 bars located at 1.75 in. along the beam section from the bottom of the beam. Five of the slab section bars are not continuous and will be excluded from the calculation of I_{cr}. The Figure below shows all the parameters needed to calculate the moment of inertia of the cracked section transformed to concrete at midspan.

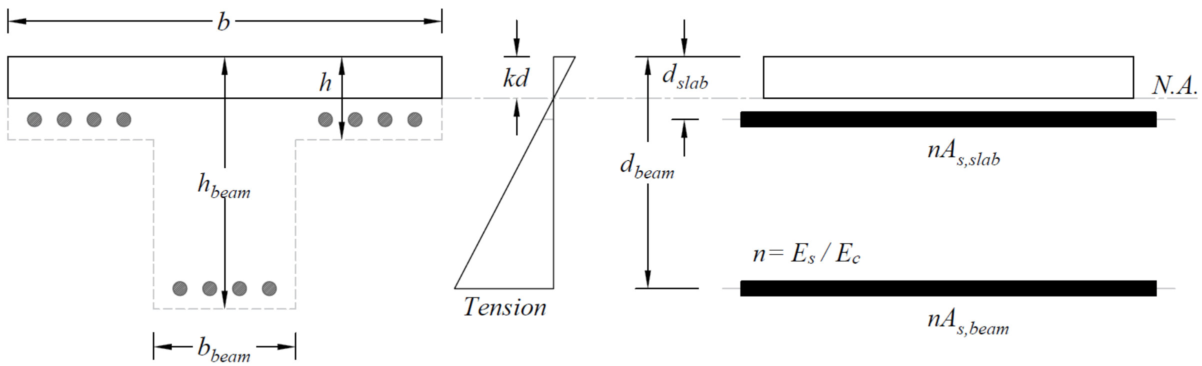


Figure 21 – Cracked Transformed Section (Positive Moment Section)

E_{cs} = Modulus of elasticity of slab concrete.

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{4,000} = 3,834 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n = \frac{E_s}{E_{cs}} = \frac{29{,}000{,}000}{3{,}834{,}000} = 7.56$$ **PCA Notes on ACI 318-11 (Table 10-2)**

$$a = \frac{b}{2} = \frac{22\times12}{2} = 132.00 \text{ in.}$$

$$b = n\times A_{s,beam} + n\times A_{s,slab} = 7.56\times(4\times0.20)+7.56\times(17\times0.20) = 31.77 \text{ in.}^2$$

$$c = -1\times\left(n\times A_{s,beam}\times d_{s,beam} + n\times A_{s,slab}\times d_{s,slab}\right)$$

$$c = -1\times\left(7.56\times(4\times0.20)\times18.25+7.56\times(17\times0.20)\times5.00\right) = -239.00 \text{ in.}^3$$

$$kd = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-31.77\pm\sqrt{31.77^2-4\times132.00\times(-239.00)}}{2\times132.00} = 1.231 \text{ in.}$$

$$I_{cr} = \frac{b\times(kd)^3}{3} + n\times A_{s,slab}\times(d_{slab}-kd)^2 + n\times A_{s,beam}\times\left(d_{beam}-kd\right)^2$$

$$I_{cr} = \frac{22\times12\times(1.231)^3}{3} + 7.56\times(17\times0.20)\times(5.00-1.231)^2 + 7.56\times(4\times0.20)\times(18.25-1.231)^2 = 2{,}282.02 \text{ in.}^4$$

<u>For negative moment section (near the interior support of the end span):</u>

The negative reinforcement for the end span frame strip near the interior support is 27 #4 bars located at 1.0 in. along the section from the top of the slab.

$$M_{cr} = \frac{f_r\times I_g}{y_t} = \frac{474.34\times9{,}333.33}{10}\times\frac{1}{12\times1{,}000} = 36.89 \text{ ft-kips}$$ **ACI 318-14 (Eq. 24.2.3.5b)**

$$f_r = 7.5\times\lambda\times\sqrt{f_c'} = 7.5\times1.0\times\sqrt{4{,}000} = 474.34 \text{ psi}$$ **ACI 318-14 (Eq. 19.2.3.1)**

$$I_g = 9{,}333.33 \text{ in.}^4$$

$$y_t = 10.00 \text{ in.}$$

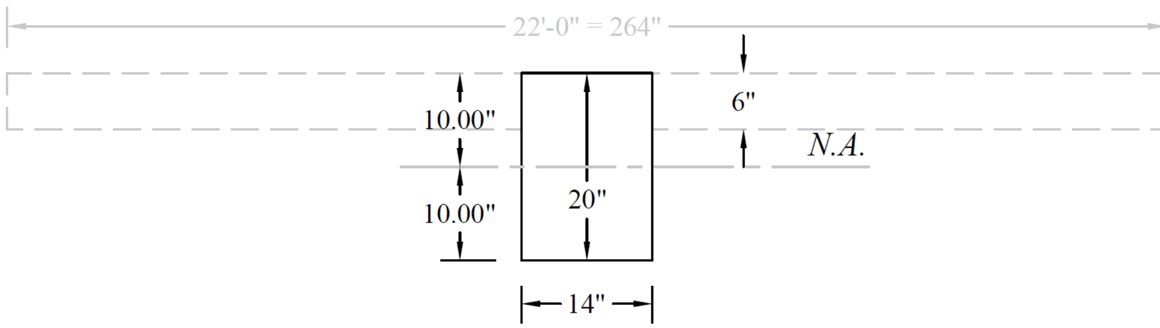


Figure 22 – I_g Calculations for Slab Section Near Support

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{4,000} = 3,834 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n = \frac{E_s}{E_{cs}} = \frac{29,000,000}{3,834,000} = 7.56$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B = \frac{b_{beam}}{n \times A_{s,total}} = \frac{14}{7.56 \times (27 \times 0.20)} = 0.34 \text{ in.}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd = \frac{\sqrt{2 \times d \times B + 1} - 1}{B} = \frac{\sqrt{2 \times 19.00 \times 0.34 + 1} - 1}{0.34} = 8.01 \text{ in.}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{b_{beam} \times (kd)^3}{3} + n \times A_{s,total} \times (d - kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{14 \times (8.01)^3}{3} + 7.56 \times (27 \times 0.20) \times (19.00 - 8.01)^2 = 7,331.24 \text{ in.}^4$$

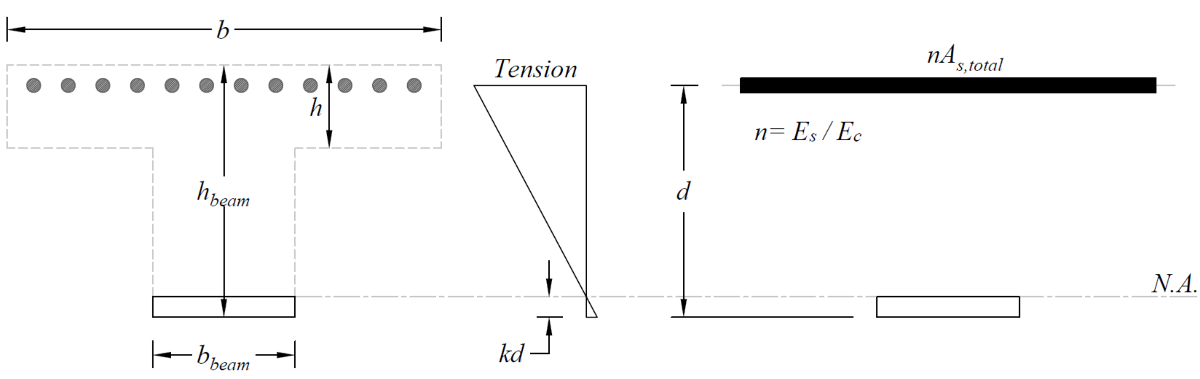


Figure 23 – Cracked Transformed Section (Interior Negative Moment Section for End Span)

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by Eq. (24.2.3.5a) unless obtained by a more comprehensive analysis.

I_e shall be permitted to be taken as the value obtained from Eq. (24.2.3.5a) at midspan for simple and continuous spans, and at the support for cantilevers. ***ACI 318-14 (24.2.3.7)***

For continuous one-way slabs and beams. I_e shall be permitted to be taken as the average of values obtained from Eq. (24.2.3.5a) for the critical positive and negative moment sections. ***ACI 318-14 (24.2.3.6)***

For the exterior span (span with one end continuous) with service load level ($D + LL_{full}$):

Since M_{cr} = 36.89 ft-kips < M_a = 127.55 ft-kips

$$I_e^- = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr}$$ ***ACI 318-14 (24.2.3.5a)***

Where I_e^- is the effective moment of inertia for the critical negative moment section (near the support).

$$I_e^- = \left(\frac{36.89}{127.55}\right)^3 \times 9,333.33 + \left[1-\left(\frac{36.89}{127.55}\right)^3\right] \times 7,331.24 = 7,379.69 \text{ in.}^4$$

$$I_e^+ = I_g = 25,395.13 \text{ in.}^4 \text{ , since } M_{cr} = 63.14 \text{ ft-kips} > M_a = 59.43 \text{ ft-kips}$$

Where I_e^+ is the effective moment of inertia for the critical positive moment section (midspan).

Since midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan section is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.85 \times I_e^+ + 0.15 \times I_e^- \text{ for end span}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(1))***

$$I_{e,avg} = 0.85 \times (25,395.13) + 0.15 \times (7,379.69) = 22,692.81 \text{ in.}^4$$

Where:

- I_e^- = The effective moment of inertia for the critical negative moment section near the support.
- I_e^+ = The effective moment of inertia for the critical positive moment section (midspan).

For the interior span (span with both ends continuous) with service load level ($D + LL_{full}$):

Since M_{cr} = 36.89 ft-kips < M_a = 115.72 ft-kips

$$I_e^- = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr}$$ ***ACI 318-14 (24.2.3.5a)***

$$I_e^- = \left(\frac{36.89}{115.72}\right)^3 \times 9,333.33 + \left[1 - \left(\frac{36.89}{115.72}\right)^3\right] \times 7,331.24 = 7,396.12 \text{ in.}^4$$

$$I_e^+ = I_g = 25,395.13 \text{ in.}^4 \text{ , since } M_{cr} = 63.14 \text{ ft-kips} > M_a = 39.48 \text{ ft-kips}$$

The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.70 \times I_e^+ + 0.15 \times \left(I_{e,l}^- + I_{e,r}^-\right) \text{ for interior span}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(2))***

$$I_{e,avg} = 0.70 \times (25,395.13) + 0.15 \times (7,396.12 + 7,396.12) = 19,995.43 \text{ in.}^4$$

Where:

- $I_{e,l}^-$ = The effective moment of inertia for the critical negative moment section near the left support.
- $I_{e,r}^-$ = The effective moment of inertia for the critical negative moment section near the right support.

The following Table provides a summary of the required parameters and calculated values needed for deflections for exterior and interior equivalent frame. It also provides a summary of the same values for column strip and middle strip to facilitate calculation of panel deflection.

Table 6 - Averaged Effective Moment of Inertia Calculations														
For Frame Strip														
Span	zone	I_g (in.^{4})	I_{cr} (in.^{4})	M_a (ft-kips)			M_{cr} (k-ft)	I_e (in.^{4})			$I_{e,avg}$ (in.^{4})		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	9,333	7,147	-30.60	-30.60	-66.91	36.89	9,333	9,333	7,513	22,762	22,762	22,693
	Midspan	25,395	2,282	27.18	27.18	59.43	63.14	25,395	25,395	25,395			
	Right	9,333	7,331	-58.33	-58.33	-127.55	36.89	7,838	7,838	7,380			
Int	Left	9,333	7,331	-52.92	-52.92	-115.72	36.89	8,009	8,009	7,396	20,179	20,179	19,995
	Mid	25,395	1,553	18.06	18.06	39.48	63.14	25,395	25,395	25,395			
	Right	9,333	7,331	-52.92	-52.92	-115.72	36.89	8,009	8,009	7,396			

Deflections in two-way slab systems shall be calculated taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. For immediate deflections in two-way slab systems, the midpanel deflection is computed as the sum of deflection at midspan of the column strip or column line in one direction (Δ_{cx} or Δ_{cy}) and deflection at midspan of the middle strip in the orthogonal direction (Δ_{mx} or Δ_{my}). Figure 24 shows the deflection computation for a rectangular panel. The average Δ for panels that have different properties in the two direction is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)

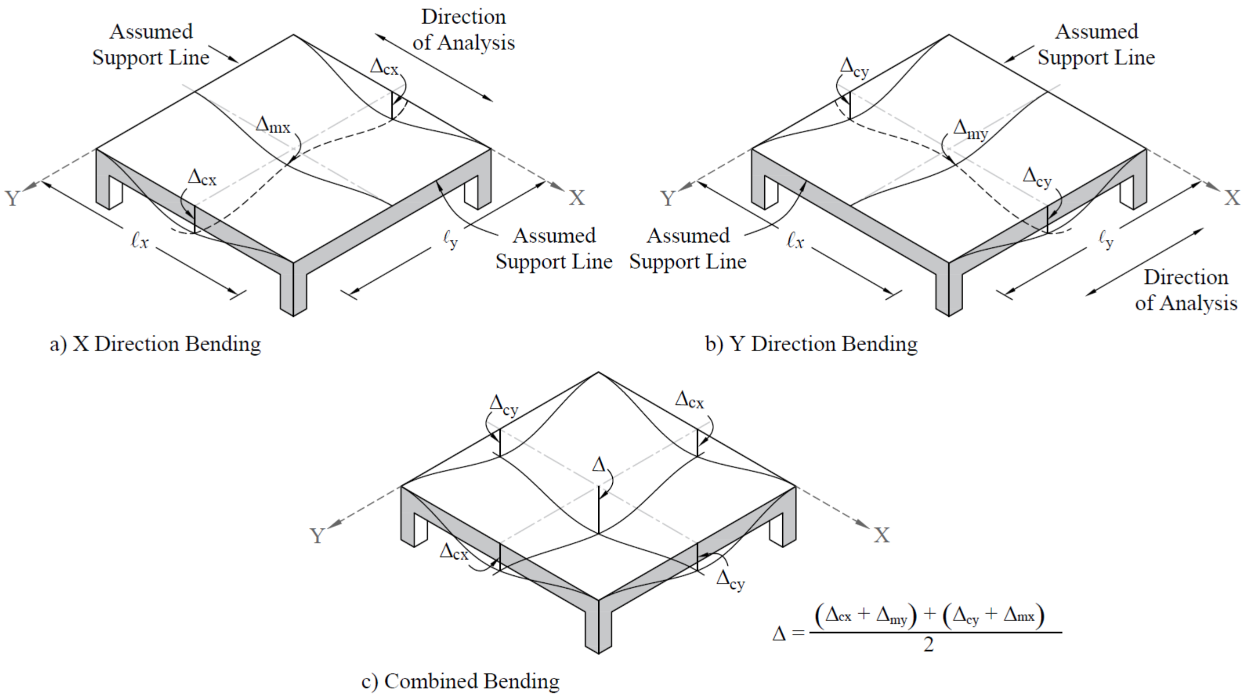


Figure 24 – Deflection Computation for a Rectangular Panel

To calculate each term of the previous equation, the following procedure should be used. Figure 25 shows the procedure of calculating the term Δ_{cx}. Same procedure can be used to find the other terms.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \left(\frac{I_{frame}}{I_c}\right)_g$$

$$LDF_c = \frac{LDF^+ + \dfrac{LDF_l^- + LDF_R^-}{2}}{2}$$

$$\Delta_{frame,fixed} = \frac{w \times l^4}{384 \times E_c \times I_{frame,average}}$$

$$\Delta\theta_{c,R} = \theta_{c,R} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{column}$$

$$\theta_{c,R} = \frac{\left(M_{net,R}\right)_{column}}{K_{ec}}$$

$$\Delta\theta_{c,L} = \theta_{c,L} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{column}$$

$$\theta_{c,L} = \frac{\left(M_{net,L}\right)_{column}}{K_{ec}}$$

Figure 25 – Δ_{cx} Calculation Procedure

For exterior span - service dead load case:

$$\Delta_{frame,fixed} = \frac{w \times l^4}{384 \times E_c \times I_{frame,averaged}}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 10)

Where:

$\Delta_{frame,fixed}$ = Deflection of column strip assuming fixed-end condition.

$$w = \text{slab weight + beam weight} = \left(150 \times \frac{6}{12} + \frac{150 \times (20-6) \times 14}{22 \times 144}\right) \times 22 = 1{,}854.17 \ \frac{\text{lb}}{\text{ft}}$$

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{4{,}000} = 3{,}834 \times 10^3 \text{ psi}$$

ACI 318-14 (19.2.2.1.a)

$I_{frame,averaged}$ = The averaged effective moment of inertia ($I_{e,avg}$) for the frame strip for service dead load case from Table 6 = 22,761.52 in.^{4}

$$\Delta_{frame,fixed} = \frac{1{,}854.17 \times (17.5 - 18/12)^4 \times 12^3}{384 \times (3{,}834 \times 10^3) \times 22{,}761.52} = 0.0063 \text{ in.}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \left(\frac{I_{frame}}{I_c}\right)_g$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 11)

Where LDF_c is the load distribution factor for the column strip. The load distribution factor for the column strip can be found from the following equation:

$$LDF_c = \frac{LDF^+ + \dfrac{LDF_l^- + LDF_R^-}{2}}{2}$$

And the load distribution factor for the middle strip can be found from the following equation:

$$LDF_m = 1 - LDF_c$$

For the end span, LDF for exterior negative region (LDF_L^-), interior negative region (LDF_R^-), and positive region (LDF_L^+) are 0.75, 0.67, and 0.67, respectively (From Table 2 of this document). Thus, the load distribution factor for the column strip for the end span is given by:

$$LDF_c = \frac{0.67 + \dfrac{0.75 + 0.67}{2}}{2} = 0.690$$

- $I_{c,g}$ = The gross moment of inertia (I_g) for the column strip (for T section) = 20,040.49 in.^{4}
- $I_{frame,g}$ = The gross moment of inertia (I_g) for the frame strip (for T section) = 25,395.13 in.^{4}

$$\Delta_{c,fixed} = 0.690 \times 0.0063 \times \frac{25,395.13}{20,040.49} = 0.0055 \text{ in.}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 12)***

Where:

$\theta_{c,L}$ = Rotation of the span left support

$(M_{net,L})_{frame}$ = 31.12 ft-kips = Net frame strip negative moment of the left support

K_{ec} = Effective column stiffness for exterior column = 763.33 × E_c = 2,927 × 10^6 in.-lb (calculated previously).

$$\theta_{c,L} = \frac{31.12 \times 12 \times 1,000}{2,927 \times 10^6} = 0.00013 \text{ rad}$$

$$\Delta\theta_{c,L} = \theta_{c,L} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 14)***

Where:

$\Delta\theta_{c,L}$ = Midspan deflection due to rotation of left support.

$(I_g / I_e)_{frame}$ = Gross to effective moment of inertia ratio for frame strip.

$$\Delta\theta_{c,L} = 0.00013 \times \frac{(17.5 - 18/12) \times 12}{8} \times \frac{25,395.13}{22,761.52} = 0.0034 \text{ in.}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}} = \frac{(58.33 - 52.92) \times 12 \times 1,000}{2,421 \times 10^6} = 0.00003 \text{ rad}$$

Where:

$\theta_{c,R}$ = rotation of the span right support.

$(M_{net,R})_{frame}$ = Net frame strip negative moment of the right support.

K_{ec} = Effective column stiffness for interior column = 631.36 × E_c = 2,421 × 10^6 in.-lb (calculated previously).

$$\Delta\theta_{c,R} = \theta_{c,R} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame} = 0.00003 \times \frac{(17.5 - 18/12) \times 12}{8} \times \frac{25,395.13}{22,761.52} = 0.00072 \text{ in.}$$

Where:

$\Delta\theta_{c,R}$ = Midspan deflection due to rotation of right support.

$$\Delta_{cx} = \Delta_{cx,\,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 9)***

$$\Delta_{cx} = 0.0055 + 0.0034 + 0.00072 = 0.010 \text{ in.}$$

Following the same procedure, Δ_{mx} can be calculated for the middle strip. This procedure is repeated for the equivalent frame in the orthogonal direction to obtain Δ_{cy}, and Δ_{my} for the end and middle spans for the other load levels ($D + LL_{sus}$ *and* $D + LL_{full}$).

Assuming square panel, $\Delta_{cx} = \Delta_{cy}$ = 0.010 in. and $\Delta_{mx} = \Delta_{my}$ = 0.021 in.

The average Δ for the corner panel is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2} = (\Delta_{cx} + \Delta_{my}) = (\Delta_{cy} + \Delta_{mx}) = 0.010 + 0.021 = 0.031 \text{ in.}$$

Table 7 - Instantaneous Deflections

Column Strip

Span	LDF	D						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx}, (in.)
Ext	0.69	0.0063	0.0055	0.00013	0.00003	0.0034	0.0007	0.010
Int	0.67	0.0071	0.0060	0.00003	0.00003	-0.0008	-0.0008	0.004

Span	LDF	$D+LL_{sus}$						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx} (in.)
Ext	0.69	0.0063	0.0055	0.00013	0.00003	0.0034	0.0007	0.010
Int	0.67	0.0071	0.0060	0.00003	0.00003	-0.0008	-0.0008	0.004

Span	LDF	$D+LL_{full}$						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx} (in.)
Ext	0.69	0.0137	0.0120	0.00028	0.00006	0.0075	0.0016	0.021
Int	0.67	0.0156	0.0132	0.00006	0.00006	-0.0018	-0.0018	0.010

Span	LDF	LL
		Δ_{cx} (in.)
Ext	0.69	0.011
Int	0.67	0.005

Middle Strip

LDF	D						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.31	0.0063	0.0172	0.00013	0.00003	0.0034	0.0007	0.021
0.33	0.0071	0.0207	0.00003	0.00003	-0.0008	-0.0008	0.019

LDF	$D+LL_{sus}$						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.31	0.0063	0.0172	0.00013	0.00003	0.0034	0.0007	0.021
0.33	0.0071	0.0207	0.00003	0.00003	-0.0008	-0.0008	0.019

LDF	$D+LL_{full}$						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.31	0.0137	0.0378	0.00028	0.00006	0.0075	0.0016	0.047
0.33	0.0156	0.0457	0.00006	0.00006	-0.0018	-0.0018	0.042

LDF	LL
	Δ_{mx} (in.)
0.31	0.025
0.33	0.023

6.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) may be estimated as follows:

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total time-dependent (long-term) deflection is calculated as:

$$\left(\Delta_{total}\right)_{lt} = \left(\Delta_{sust}\right)_{Inst} \times \left(1+\lambda_{\Delta}\right) + \left[\left(\Delta_{total}\right)_{Inst} - \left(\Delta_{sust}\right)_{Inst}\right]$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst}$ = Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_{\Delta} = \frac{\xi}{1+50 \times \rho'}$$ ***ACI 318-14 (24.2.4.1.1)***

$(\Delta_{total})_{lt}$ = Time-dependent (long-term) total deflection, in.

$(\Delta_{total})_{Inst}$ = Total immediate (instantaneous) deflection, in.

For the exterior span

ξ = 2, consider the sustained load duration to be 60 months or more. ***ACI 318-14 (Table 24.2.4.1.3)***

ρ' = 0, conservatively.

$$\lambda_{\Delta} = \frac{2}{1+50 \times 0} = 2$$

$$\Delta_{cs} = 2 \times 0.010 = 0.019 \text{ in.}$$

$$\left(\Delta_{total}\right)_{lt} = 0.010 \times (1+2) + (0.021 - 0.010) = 0.040 \text{ in.}$$

The following Table shows long-term deflections for the exterior and interior spans for the analysis in the x-direction, for column and middle strips.

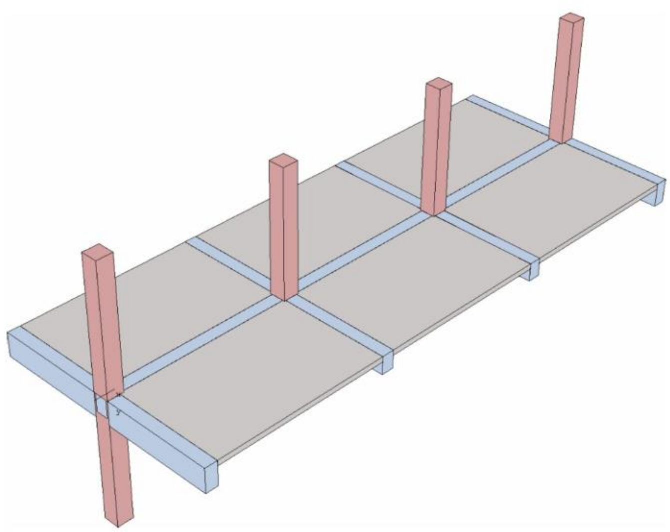
Table 8 - Long-Term Deflections					
Column Strip					
Span	**$(\Delta_{sust})_{Inst}$ (in.)**	**λ_{Δ}**	**Δ_{cs} (in.)**	**$(\Delta_{total})_{Inst}$ (in.)**	**$(\Delta_{total})_{lt}$ (in.)**
Exterior	0.010	2	0.019	0.021	0.040
Interior	0.004	2	0.009	0.010	0.018
Middle Strip					
Exterior	0.021	2	0.043	0.047	0.090
Interior	0.019	2	0.038	0.042	0.080

7. spSlab Software Program Model Solution

spSlab program utilizes the Equivalent Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the equivalent column concept ***(ACI 318-14 (R8.11.4))***.

spSlab Program models the equivalent frame as a design strip. The design strip is, then, separated by spSlab into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips.

spSlab v10.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems



Contents

1. Input Echo

1.1. General Information

File Name	F:\...\DE-Two-Way-Slab-with-Beams-ACI-318-14.slbx
Project	Slab on beams
Frame	Interior Frame
Engineer	SP
Code	ACI 318-14
Units	English
Reinforcement Database	ASTM A615
Mode	Design
Number of supports =	4 + Left Cantilever + Right Cantilever
Floor System	Two-Way

1.2. Solve Options

Live load pattern ratio = 75%
Minimum free edge distance for punching shear = 4 times slab thickness.
Circular critical section around circular supports used (if possible).
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
User-defined slab strip widths NOT selected.
User-defined distribution factors NOT selected.
One-way shear in drop panel NOT selected.
Distribution of shear to strips NOT selected.
Beam T-section design NOT selected.
Longitudinal beam contribution in negative reinforcement design over support NOT selected.
Transverse beam contribution in negative reinforcement design over support NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150	pcf
f'_c	4	ksi
E_c	3834.25	ksi
f_r	0.474342	ksi

1.3.2. Concrete: Columns

w_c	150	pcf
f'_c	4	ksi
E_c	3834.25	ksi
f_r	0.474342	ksi

1.3.3. Reinforcing Steel

f_y	60	ksi

f_{yt}	60 ksi
E_s	29000 ksi
Epoxy coated bars	No

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

1.5. Span Data

1.5.1. Slabs

Notes:
Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

Span	Loc	L1	t	wL	wR	L2L	L2R	H_{min}
		ft	in	ft	ft	ft	ft	in
1	Int	0.750	6.00	11.000	11.000	22.000	22.000	--- LC *i
2	Int	17.500	6.00	11.000	11.000	22.000	22.000	5.81 ---
3	Int	17.500	6.00	11.000	11.000	22.000	22.000	5.79 ---
4	Int	17.500	6.00	11.000	11.000	22.000	22.000	5.81 ---
5	Int	0.750	6.00	11.000	11.000	22.000	22.000	--- RC *i

1.5.2. Ribs and Longitudinal Beams

Notes:

Span	Ribs			Beams		
	b	h	Sp	b	h	Offset
	in	in	in	in	in	in
1	0.00	0.00	0.00	14.00	20.00	0.00
2	0.00	0.00	0.00	14.00	20.00	0.00
3	0.00	0.00	0.00	14.00	20.00	0.00
4	0.00	0.00	0.00	14.00	20.00	0.00
5	0.00	0.00	0.00	14.00	20.00	0.00

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	18.00	18.00	12.000	18.00	18.00	12.000	100
2	18.00	18.00	12.000	18.00	18.00	12.000	100
3	18.00	18.00	12.000	18.00	18.00	12.000	100
4	18.00	18.00	12.000	18.00	18.00	12.000	100

1.6.2. Transverse Beams

Supports	b	h	Ecc
	in	in	in
1	14.00	27.00	-2.00
2	14.00	20.00	0.00

Supports	b	h	Ecc
	in	in	in
3	14.00	20.00	0.00
4	14.00	27.00	2.00

1.6.3. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kips/in	kip-in/rad		
1	0.00	0.00	Fixed	Fixed
2	0.00	0.00	Fixed	Fixed
3	0.00	0.00	Fixed	Fixed
4	0.00	0.00	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.200	1.600

1.7.2. Area Loads

Case/Patt	Span	Wa
		psf
Dead	1	84.28
	2	84.28
	3	84.28
	4	84.28
	5	84.28
Live	1	100.00
	2	100.00
	3	100.00
	4	100.00
	5	100.00
Live/Odd	1	75.00
	3	75.00
	5	75.00
Live/Even	2	75.00
	4	75.00
Live/S1	1	75.00
	2	75.00
Live/S2	2	75.00
	3	75.00
Live/S3	3	75.00
	4	75.00
Live/S4	4	75.00
	5	75.00

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#4	#8	#4	#8
Bar spacing	in	1.00	18.00	1.00	18.00

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	in	0.75		0.75	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#4	#8	#4	#8	#4	#5
Bar spacing	in	1.00	18.00	1.00	18.00	6.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	0.75		1.51			
Layer dist.	in	1.00		1.00			
No. of legs						2	6
Side cover	in					1.50	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

2. Design Results*

*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

2.1. Strip Widths and Distribution Factors

Notes:
*Used for bottom reinforcement. **Used for top reinforcement.

Span	Strip	Width			Moment Factor		
		Left **	Right **	Bottom *	Left **	Right **	Bottom *
		ft	ft	ft			
1	Column	7.58	7.58	7.58	0.122	0.122	0.113
	Middle	13.25	13.25	13.25	0.188	0.188	0.250
	Beam	1.17	1.17	1.17	0.690	0.690	0.637
2	Column	7.58	7.58	7.58	0.113	0.101	0.101
	Middle	13.25	13.25	13.25	0.246	0.327	0.327
	Beam	1.17	1.17	1.17	0.641	0.572	0.572
3	Column	7.58	7.58	7.58	0.101	0.101	0.101
	Middle	13.25	13.25	13.25	0.327	0.327	0.327
	Beam	1.17	1.17	1.17	0.572	0.572	0.572
4	Column	7.58	7.58	7.58	0.101	0.113	0.101
	Middle	13.25	13.25	13.25	0.327	0.246	0.327
	Beam	1.17	1.17	1.17	0.572	0.641	0.572
5	Column	7.58	7.58	7.58	0.122	0.122	0.113
	Middle	13.25	13.25	13.25	0.188	0.188	0.250
	Beam	1.17	1.17	1.17	0.690	0.690	0.637

2.2. Top Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Span	Strip	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars	
			ft	kip-ft	ft	in²	in²	in²	in		
1	Column	Left	7.58	0.02	0.217	0.983	8.218	0.001	11.375	8-#4	*3 *5
		Midspan	7.58	0.06	0.402	0.983	8.218	0.003	11.375	8-#4	*3 *5
		Right	7.58	0.14	0.619	0.983	8.218	0.006	11.375	8-#4	*3 *5
	Middle	Left	13.25	0.03	0.217	1.717	14.360	0.001	11.357	14-#4	*3 *5
		Midspan	13.25	0.10	0.402	1.717	14.360	0.004	11.357	14-#4	*3 *5
		Right	13.25	0.22	0.619	1.717	14.360	0.010	11.357	14-#4	*3 *5
	Beam	Left	1.17	0.11	0.217	0.372	4.805	0.001	8.664	2-#4	*3
		Midspan	1.17	0.35	0.402	0.372	4.805	0.004	8.664	2-#4	*3
		Right	1.17	0.79	0.619	0.372	4.805	0.009	2.888	4-#4	*3
2	Column	Left	7.58	7.06	0.750	0.983	8.218	0.316	11.375	8-#4	*3 *5
		Midspan	7.58	0.00	8.750	0.000	8.218	0.000	0.000	---	
		Right	7.58	14.23	16.750	0.983	8.218	0.640	11.375	8-#4	*3 *5
	Middle	Left	13.25	15.36	0.750	1.717	14.360	0.688	11.357	14-#4	*3 *5
		Midspan	13.25	0.00	8.750	0.000	14.360	0.000	0.000	---	
		Right	13.25	46.12	16.750	1.717	14.360	2.099	11.357	14-#4	*5
	Beam	Left	1.17	40.00	0.750	0.632	4.805	0.475	2.888	4-#4	*3
		Midspan	1.17	0.00	8.750	0.000	4.805	0.000	0.000	---	
		Right	1.17	80.63	16.750	0.887	4.805	0.975	2.166	5-#4	
3	Column	Left	7.58	12.91	0.750	0.983	8.218	0.580	11.375	8-#4	*3 *5
		Midspan	7.58	0.15	11.150	0.983	8.218	0.007	11.375	8-#4	*3 *5
		Right	7.58	12.91	16.750	0.983	8.218	0.580	11.375	8-#4	*3 *5
	Middle	Left	13.25	41.84	0.750	1.717	14.360	1.900	11.357	14-#4	*5
		Midspan	13.25	0.48	11.150	1.717	14.360	0.021	11.357	14-#4	*3 *5
		Right	13.25	41.84	16.750	1.717	14.360	1.900	11.357	14-#4	*5
	Beam	Left	1.17	73.14	0.750	0.887	4.805	0.881	2.166	5-#4	*3
		Midspan	1.17	0.83	11.150	0.372	4.805	0.010	8.664	2-#4	*3
		Right	1.17	73.14	16.750	0.887	4.805	0.881	2.166	5-#4	*3
4	Column	Left	7.58	14.23	0.750	0.983	8.218	0.640	11.375	8-#4	*3 *5
		Midspan	7.58	0.00	8.750	0.000	8.218	0.000	0.000	---	
		Right	7.58	7.06	16.750	0.983	8.218	0.316	11.375	8-#4	*3 *5
	Middle	Left	13.25	46.12	0.750	1.717	14.360	2.099	11.357	14-#4	*5
		Midspan	13.25	0.00	8.750	0.000	14.360	0.000	0.000	---	
		Right	13.25	15.36	16.750	1.717	14.360	0.688	11.357	14-#4	*3 *5
	Beam	Left	1.17	80.63	0.750	0.887	4.805	0.975	2.166	5-#4	
		Midspan	1.17	0.00	8.750	0.000	4.805	0.000	0.000	---	
		Right	1.17	40.00	16.750	0.632	4.805	0.475	2.888	4-#4	*3
5	Column	Left	7.58	0.14	0.131	0.983	8.218	0.006	11.375	8-#4	*3 *5
		Midspan	7.58	0.06	0.348	0.983	8.218	0.003	11.375	8-#4	*3 *5
		Right	7.58	0.02	0.533	0.983	8.218	0.001	11.375	8-#4	*3 *5

Span	Strip	Zone	Width ft	M_{max} kip-ft	X_{max} ft	$A_{s,min}$ in²	$A_{s,max}$ in²	$A_{s,req}$ in²	Sp_{Prov} in	Bars
	Middle	Left	13.25	0.22	0.131	1.717	14.360	0.010	11.357	14-#4 *3 *5
		Midspan	13.25	0.10	0.348	1.717	14.360	0.004	11.357	14-#4 *3 *5
		Right	13.25	0.03	0.533	1.717	14.360	0.001	11.357	14-#4 *3 *5
	Beam	Left	1.17	0.79	0.131	0.372	4.805	0.009	2.888	4-#4 *3
		Midspan	1.17	0.35	0.348	0.372	4.805	0.004	8.664	2-#4 *3
		Right	1.17	0.11	0.533	0.372	4.805	0.001	8.664	2-#4 *3

2.3. Top Bar Details

NOTES:
* - Bar cut-off location does not meet ACI 318, 12.10.5.1. Revise location, unless the requirements of either 12.10.5.2 or 12.10.5.3 are manually checked and satisfied.

		Left				Continuous		Right			
Span	Strip	Bars	Length ft	Bars	Length ft	Bars	Length ft	Bars	Length ft	Bars	Length ft
1	Column	---		---		8-#4	0.75	---		---	
	Middle	---		---		14-#4	0.75	---		---	
	Beam	---		---		2-#4	0.75	1-#4	0.75	1-#4	0.75
2	Column	5-#4	3.75	3-#4	1.75	---		5-#4	6.75	3-#4	1.75
	Middle	14-#4	3.75	---		---		14-#4	6.75	---	
	Beam	4-#4	4.33	---		---		5-#4	7.33	---	
3	Column	---		---		8-#4	17.50	---		---	
	Middle	---		---		14-#4	17.50	---		---	
	Beam	2-#4 *	4.01	1-#4 *	2.59	2-#4	17.50	2-#4 *	4.01	1-#4 *	2.59
4	Column	5-#4	6.75	3-#4	1.75	---		5-#4	3.75	3-#4	1.75
	Middle	14-#4	6.75	---		---		14-#4	3.75	---	
	Beam	5-#4	7.33	---		---		4-#4	4.33	---	
5	Column	---		---		8-#4	0.75	---		---	
	Middle	---		---		14-#4	0.75	---		---	
	Beam	1-#4	0.75	1-#4	0.75	2-#4	0.75	---		---	

2.4. Top Bar Development Lengths

		Left				Continuous		Right			
Span	Strip	Bars	DevLen in	Bars	DevLen in	Bars	DevLen in	Bars	DevLen in	Bars	DevLen in
1	Column	---		---		8-#4	12.00	---		---	
	Middle	---		---		14-#4	12.00	---		---	
	Beam	---		---		2-#4	12.00	1-#4	12.00	1-#4	12.00
2	Column	5-#4	12.00	3-#4	12.00	---		5-#4	12.00	3-#4	12.00
	Middle	14-#4	12.00	---		---		14-#4	12.00	---	
	Beam	4-#4	12.00	---		---		5-#4	13.87	---	
3	Column	---		---		8-#4	12.00	---		---	
	Middle	---		---		14-#4	12.00	---		---	
	Beam	2-#4	12.54	1-#4	12.54	2-#4	12.00	2-#4	12.54	1-#4	12.54
4	Column	5-#4	12.00	3-#4	12.00	---		5-#4	12.00	3-#4	12.00

Span	Strip	Left Bars	Left DevLen (in)	Left Bars	Left DevLen (in)	Continuous Bars	Continuous DevLen (in)	Right Bars	Right DevLen (in)	Right Bars	Right DevLen (in)
	Middle	14-#4	12.00	---		---		14-#4	12.00	---	
	Beam	5-#4	13.87	---		---		4-#4	12.00	---	
5	Column	---		---		8-#4	12.00	---		---	
	Middle	---		---		14-#4	12.00	---		---	
	Beam	1-#4	12.00	1-#4	12.00	2-#4	12.00	---		---	

2.5. Bottom Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Span	Strip	Width (ft)	M_{max} (kip-ft)	X_{max} (ft)	$A_{s,min}$ (in²)	$A_{s,max}$ (in²)	$A_{s,req}$ (in²)	Sp_{Prov} (in)	Bars	
1	Column	7.58	0.00	0.309	0.000	8.218	0.000	0.000	---	
	Middle	13.25	0.00	0.309	0.000	14.360	0.000	0.000	---	
	Beam	1.17	0.00	0.309	0.000	4.613	0.000	0.000	---	
2	Column	7.58	8.50	8.000	0.983	8.218	0.381	11.375	8-#4	*3 *5
	Middle	13.25	27.55	8.000	1.717	14.360	1.242	11.357	14-#4	*3 *5
	Beam	1.17	48.16	8.000	0.797	4.613	0.599	2.888	4-#4	*3
3	Column	7.58	6.47	8.750	0.983	8.218	0.289	11.375	8-#4	*3 *5
	Middle	13.25	20.96	8.750	1.717	14.360	0.942	11.357	14-#4	*3 *5
	Beam	1.17	36.65	8.750	0.603	4.613	0.454	2.888	4-#4	*3
4	Column	7.58	8.50	9.500	0.983	8.218	0.381	11.375	8-#4	*3 *5
	Middle	13.25	27.55	9.500	1.717	14.360	1.242	11.357	14-#4	*3 *5
	Beam	1.17	48.16	9.500	0.797	4.613	0.599	2.888	4-#4	*3
5	Column	7.58	0.00	0.441	0.000	8.218	0.000	0.000	---	
	Middle	13.25	0.00	0.441	0.000	14.360	0.000	0.000	---	
	Beam	1.17	0.00	0.441	0.000	4.613	0.000	0.000	---	

2.6. Bottom Bar Details

Span	Strip	Long Bars Bars	Long Bars Start (ft)	Long Bars Length (ft)	Short Bars Bars	Short Bars Start (ft)	Short Bars Length (ft)
1	Column	---			---		
	Middle	---			---		
	Beam	---			---		
2	Column	8-#4	0.00	17.50	---		
	Middle	9-#4	0.00	17.50	5-#4	0.00	14.88
	Beam	4-#4	0.00	17.50	---		
3	Column	8-#4	0.00	17.50	---		
	Middle	9-#4	0.00	17.50	5-#4	2.63	12.25
	Beam	4-#4	0.00	17.50	---		
4	Column	8-#4	0.00	17.50	---		
	Middle	9-#4	0.00	17.50	5-#4	2.63	14.88
	Beam	4-#4	0.00	17.50	---		

Span	Strip	Long Bars Bars	Long Bars Start ft	Long Bars Length ft	Short Bars Bars	Short Bars Start ft	Short Bars Length ft
5	Column	---			---		
	Middle	---			---		
	Beam	---			---		

2.7. Bottom Bar Development Lengths

Span	Strip	Long Bars Bars	Long Bars DevLen in	Short Bars Bars	Short Bars DevLen in
1	Column	---		---	
	Middle	---		---	
	Beam	---		---	
2	Column	8-#4	12.00	---	
	Middle	9-#4	12.00	5-#4	12.00
	Beam	4-#4	12.00	---	
3	Column	8-#4	12.00	---	
	Middle	9-#4	12.00	5-#4	12.00
	Beam	4-#4	12.00	---	
4	Column	8-#4	12.00	---	
	Middle	9-#4	12.00	5-#4	12.00
	Beam	4-#4	12.00	---	
5	Column	---		---	
	Middle	---		---	
	Beam	---		---	

2.8. Flexural Capacity

Span	Strip	x ft	Top $A_{s,top}$ in²	Top ΦM_n- kip-ft	Top M_u- kip-ft	Top Comb Pat	Top Status	Bottom $A_{s,bot}$ in²	Bottom ΦM_n+ kip-ft	Bottom M_u+ kip-ft	Bottom Comb Pat	Bottom Status
1	Column	0.000	1.60	-34.88	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.217	1.60	-34.88	-0.02	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.375	1.60	-34.88	-0.05	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.402	1.60	-34.88	-0.06	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.619	1.60	-34.88	-0.14	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.750	1.60	-34.88	-0.20	U1 All	---	0.00	0.00	0.00	U1 All	---
	Middle	0.000	2.80	-61.04	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.217	2.80	-61.04	-0.03	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.375	2.80	-61.04	-0.08	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.402	2.80	-61.04	-0.10	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.619	2.80	-61.04	-0.22	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.750	2.80	-61.04	-0.30	U1 All	---	0.00	0.00	0.00	U1 All	---
	Beam	0.000	0.80	-66.58	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.217	0.80	-66.58	-0.11	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.375	0.80	-66.58	-0.31	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.402	0.80	-66.58	-0.35	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.619	0.80	-66.58	-0.79	U1 All	OK	0.00	0.00	0.00	U1 All	OK
		0.750	0.80	-66.58	-1.12	U1 All	---	0.00	0.00	0.00	U1 All	---

STRUCTUREPOINT - spSlab v10.00 (TM) Page | 12
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Span	Strip	Top						Bottom				
		x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
		ft	in²	kip-ft	kip-ft			in²	kip-ft	kip-ft		
2	Column	0.000	1.60	-34.88	-10.78	U1 All	---	1.60	34.88	0.00	U1 All	---
		0.750	1.60	-34.88	-7.06	U1 All	OK	1.60	34.88	0.00	U1 All	OK
		1.750	1.00	-22.06	-2.95	U1 Even	OK	1.60	34.88	0.00	U1 All	OK
		2.750	1.00	-22.06	0.00	U1 All	OK	1.60	34.88	0.83	U1 All	OK
		3.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	3.52	U1 All	OK
		6.350	0.00	0.00	0.00	U1 All	OK	1.60	34.88	7.81	U1 All	OK
		8.000	0.00	0.00	0.00	U1 All	OK	1.60	34.88	8.50	U1 All	OK
		8.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	8.29	U1 All	OK
		10.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	6.46	U1 Even	OK
		11.150	0.40	-8.93	0.00	U1 All	OK	1.60	34.88	5.94	U1 Even	OK
		11.750	1.00	-22.06	0.00	U1 All	OK	1.60	34.88	5.02	U1 Even	OK
		15.750	1.00	-22.06	-9.45	U1 All	OK	1.60	34.88	0.00	U1 All	OK
		16.750	1.60	-34.88	-14.23	U1 All	OK	1.60	34.88	0.00	U1 All	OK
		17.500	1.60	-34.88	-18.14	U1 All	---	1.60	34.88	0.00	U1 All	---
	Middle	0.000	2.80	-61.04	-22.97	U1 All	---	2.80	61.04	0.00	U1 All	---
		0.750	2.80	-61.04	-15.36	U1 All	OK	2.80	61.04	0.00	U1 All	OK
		2.750	2.80	-61.04	0.00	U1 All	OK	2.80	61.04	2.68	U1 All	OK
		3.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	11.41	U1 All	OK
		6.350	0.00	0.00	0.00	U1 All	OK	2.80	61.04	25.30	U1 All	OK
		8.000	0.00	0.00	0.00	U1 All	OK	2.80	61.04	27.55	U1 All	OK
		8.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	26.87	U1 All	OK
		10.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	20.94	U1 Even	OK
		11.150	1.12	-24.89	0.00	U1 All	OK	2.80	61.04	19.25	U1 Even	OK
		11.750	2.80	-61.04	0.00	U1 All	OK	2.80	61.04	16.26	U1 Even	OK
		13.875	2.80	-61.04	-8.12	U1 Odd	OK	2.80	61.04	1.02	U1 Even	OK
		14.875	2.80	-61.04	-17.70	U1 All	OK	1.80	39.69	0.00	U1 All	OK
		16.750	2.80	-61.04	-46.12	U1 All	OK	1.80	39.69	0.00	U1 All	OK
		17.500	2.80	-61.04	-59.82	U1 All	---	1.80	39.69	0.00	U1 All	---
	Beam	0.000	0.80	-66.58	-61.07	U1 All	---	0.80	63.86	0.00	U1 All	---
		0.750	0.80	-66.58	-40.00	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		3.333	0.80	-66.58	0.00	U1 All	OK	0.80	63.86	13.97	U1 All	OK
		4.333	0.00	0.00	0.00	U1 All	OK	0.80	63.86	27.31	U1 All	OK
		6.350	0.00	0.00	0.00	U1 All	OK	0.80	63.86	44.23	U1 All	OK
		8.000	0.00	0.00	0.00	U1 All	OK	0.80	63.86	48.16	U1 All	OK
		8.750	0.00	0.00	0.00	U1 All	OK	0.80	63.86	46.98	U1 All	OK
		10.167	0.00	0.00	0.00	U1 All	OK	0.80	63.86	40.07	U1 Even	OK
		11.150	0.85	-70.70	0.00	U1 All	OK	0.80	63.86	33.65	U1 Even	OK
		11.322	1.00	-82.66	0.00	U1 All	OK	0.80	63.86	32.25	U1 Even	OK
		16.750	1.00	-82.66	-80.63	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		17.000	1.00	-82.66	-87.84	U1 All	---	0.80	63.86	0.00	U1 All	---
		17.500	1.00	-82.66	-102.79	U1 All	---	0.80	63.86	0.00	U1 All	---
3	Column	0.000	1.60	-34.88	-16.55	U1 All	---	1.60	34.88	0.00	U1 All	---
		0.750	1.60	-34.88	-12.91	U1 All	OK	1.60	34.88	0.00	U1 All	OK
		6.350	1.60	-34.88	-0.15	U1 Even	OK	1.60	34.88	5.05	U1 Odd	OK
		8.750	1.60	-34.88	0.00	U1 All	OK	1.60	34.88	6.47	U1 Odd	OK
		11.150	1.60	-34.88	-0.15	U1 Even	OK	1.60	34.88	5.05	U1 Odd	OK
		16.750	1.60	-34.88	-12.91	U1 All	OK	1.60	34.88	0.00	U1 All	OK
		17.500	1.60	-34.88	-16.55	U1 All	---	1.60	34.88	0.00	U1 All	---
	Middle	0.000	2.80	-61.04	-53.64	U1 All	---	1.80	39.69	0.00	U1 All	---
		0.750	2.80	-61.04	-41.84	U1 All	OK	1.80	39.69	0.00	U1 All	OK
		2.625	2.80	-61.04	-16.97	U1 All	OK	1.80	39.69	0.00	U1 All	OK
		3.625	2.80	-61.04	-9.01	U1 S1	OK	2.80	61.04	1.12	U1 S3	OK
		6.350	2.80	-61.04	-0.48	U1 Even	OK	2.80	61.04	16.37	U1 Odd	OK

Span	Strip	Top						Bottom				
		x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
		ft	in²	kip-ft	kip-ft			in²	kip-ft	kip-ft		
		8.750	2.80	-61.04	0.00	U1 All	OK	2.80	61.04	20.96	U1 Odd	OK
		11.150	2.80	-61.04	-0.48	U1 Even	OK	2.80	61.04	16.37	U1 Odd	OK
		13.875	2.80	-61.04	-9.01	U1 S4	OK	2.80	61.04	1.12	U1 S2	OK
		14.875	2.80	-61.04	-16.97	U1 All	OK	1.80	39.69	0.00	U1 All	OK
		16.750	2.80	-61.04	-41.84	U1 All	OK	1.80	39.69	0.00	U1 All	OK
		17.500	2.80	-61.04	-53.64	U1 All	---	1.80	39.69	0.00	U1 All	---
	Beam	0.000	1.00	-82.66	-93.78	U1 All	---	0.80	63.86	0.00	U1 All	---
		0.250	1.00	-82.66	-86.70	U1 All	---	0.80	63.86	0.00	U1 All	---
		0.750	1.00	-82.66	-73.14	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		1.542	1.00	-82.66	-53.37	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		2.587	0.80	-66.58	-30.43	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		2.963	0.80	-66.58	-23.03	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		4.008	0.40	-33.75	-12.84	U1 S1	OK	0.80	63.86	6.74	U1 S3	OK
		6.350	0.40	-33.75	-0.83	U1 Even	OK	0.80	63.86	28.61	U1 Odd	OK
		8.750	0.40	-33.75	0.00	U1 All	OK	0.80	63.86	36.65	U1 Odd	OK
		11.150	0.40	-33.75	-0.83	U1 Even	OK	0.80	63.86	28.61	U1 Odd	OK
		13.492	0.40	-33.75	-12.84	U1 S4	OK	0.80	63.86	6.74	U1 S2	OK
		14.537	0.80	-66.58	-23.03	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		14.913	0.80	-66.58	-30.43	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		15.958	1.00	-82.66	-53.37	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		16.750	1.00	-82.66	-73.14	U1 All	OK	0.80	63.86	0.00	U1 All	OK
		17.250	1.00	-82.66	-86.70	U1 All	---	0.80	63.86	0.00	U1 All	---
		17.500	1.00	-82.66	-93.78	U1 All	---	0.80	63.86	0.00	U1 All	---
4	Column	0.000	1.60	-34.88	-18.14	U1 All	---	1.60	34.88	0.00	U1 All	---
		0.750	1.60	-34.88	-14.23	U1 All	OK	1.60	34.88	0.00	U1 All	OK
		1.750	1.00	-22.06	-9.45	U1 All	OK	1.60	34.88	0.00	U1 All	OK
		5.750	1.00	-22.06	0.00	U1 All	OK	1.60	34.88	5.02	U1 Even	OK
		6.350	0.40	-8.93	0.00	U1 All	OK	1.60	34.88	5.94	U1 Even	OK
		6.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	6.46	U1 Even	OK
		8.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	8.29	U1 All	OK
		9.500	0.00	0.00	0.00	U1 All	OK	1.60	34.88	8.50	U1 All	OK
		11.150	0.00	0.00	0.00	U1 All	OK	1.60	34.88	7.81	U1 All	OK
		13.750	0.00	0.00	0.00	U1 All	OK	1.60	34.88	3.52	U1 All	OK
		14.750	1.00	-22.06	0.00	U1 All	OK	1.60	34.88	0.83	U1 All	OK
		15.750	1.00	-22.06	-2.95	U1 Even	OK	1.60	34.88	0.00	U1 All	OK
		16.750	1.60	-34.88	-7.06	U1 All	OK	1.60	34.88	0.00	U1 All	OK
		17.500	1.60	-34.88	-10.78	U1 All	---	1.60	34.88	0.00	U1 All	---
	Middle	0.000	2.80	-61.04	-59.82	U1 All	---	1.80	39.69	0.00	U1 All	---
		0.750	2.80	-61.04	-46.12	U1 All	OK	1.80	39.69	0.00	U1 All	OK
		2.625	2.80	-61.04	-17.70	U1 All	OK	1.80	39.69	0.00	U1 All	OK
		3.625	2.80	-61.04	-8.12	U1 Odd	OK	2.80	61.04	1.02	U1 Even	OK
		5.750	2.80	-61.04	0.00	U1 All	OK	2.80	61.04	16.26	U1 Even	OK
		6.350	1.12	-24.89	0.00	U1 All	OK	2.80	61.04	19.25	U1 Even	OK
		6.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	20.94	U1 Even	OK
		8.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	26.87	U1 All	OK
		9.500	0.00	0.00	0.00	U1 All	OK	2.80	61.04	27.55	U1 All	OK
		11.150	0.00	0.00	0.00	U1 All	OK	2.80	61.04	25.30	U1 All	OK
		13.750	0.00	0.00	0.00	U1 All	OK	2.80	61.04	11.41	U1 All	OK
		14.750	2.80	-61.04	0.00	U1 All	OK	2.80	61.04	2.68	U1 All	OK
		16.750	2.80	-61.04	-15.36	U1 All	OK	2.80	61.04	0.00	U1 All	OK
		17.500	2.80	-61.04	-22.97	U1 All	---	2.80	61.04	0.00	U1 All	---
	Beam	0.000	1.00	-82.66	-102.79	U1 All	---	0.80	63.86	0.00	U1 All	---
		0.500	1.00	-82.66	-87.84	U1 All	---	0.80	63.86	0.00	U1 All	---

Span	Strip	Top x	Top $A_{s,top}$	Top ΦM_n-	Top M_u-	Top Comb	Top Pat	Top Status	Bottom $A_{s,bot}$	Bottom ΦM_n+	Bottom M_u+	Bottom Comb	Bottom Pat	Bottom Status
		ft	in²	kip-ft	kip-ft				in²	kip-ft	kip-ft			
		0.750	1.00	-82.66	-80.63	U1	All	OK	0.80	63.86	0.00	U1	All	OK
		6.178	1.00	-82.66	0.00	U1	All	OK	0.80	63.86	32.25	U1	Even	OK
		6.350	0.85	-70.70	0.00	U1	All	OK	0.80	63.86	33.65	U1	Even	OK
		7.333	0.00	0.00	0.00	U1	All	OK	0.80	63.86	40.07	U1	Even	OK
		8.750	0.00	0.00	0.00	U1	All	OK	0.80	63.86	46.98	U1	All	OK
		9.500	0.00	0.00	0.00	U1	All	OK	0.80	63.86	48.16	U1	All	OK
		11.150	0.00	0.00	0.00	U1	All	OK	0.80	63.86	44.23	U1	All	OK
		13.167	0.00	0.00	0.00	U1	All	OK	0.80	63.86	27.31	U1	All	OK
		14.167	0.80	-66.58	0.00	U1	All	OK	0.80	63.86	13.97	U1	All	OK
		16.750	0.80	-66.58	-40.00	U1	All	OK	0.80	63.86	0.00	U1	All	OK
		17.500	0.80	-66.58	-61.07	U1	All	---	0.80	63.86	0.00	U1	All	---
5	Column	0.000	1.60	-34.88	-0.20	U1	All	---	0.00	0.00	0.00	U1	All	---
		0.131	1.60	-34.88	-0.14	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.348	1.60	-34.88	-0.06	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.375	1.60	-34.88	-0.05	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.533	1.60	-34.88	-0.02	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.750	1.60	-34.88	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
	Middle	0.000	2.80	-61.04	-0.30	U1	All	---	0.00	0.00	0.00	U1	All	---
		0.131	2.80	-61.04	-0.22	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.348	2.80	-61.04	-0.10	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.375	2.80	-61.04	-0.08	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.533	2.80	-61.04	-0.03	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.750	2.80	-61.04	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
	Beam	0.000	0.80	-66.58	-1.12	U1	All	---	0.00	0.00	0.00	U1	All	---
		0.131	0.80	-66.58	-0.79	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.348	0.80	-66.58	-0.35	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.375	0.80	-66.58	-0.31	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.533	0.80	-66.58	-0.11	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.750	0.80	-66.58	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK

2.9. Longitudinal Beam Transverse Reinforcement Demand and Capacity

2.9.1. Section Properties

Span	d	$(A_v/s)_{min}$	ΦV_c
	in	in²/in	kips
1	18.24	0.0117	24.23
2	18.24	0.0117	24.23
3	18.24	0.0117	24.23
4	18.24	0.0117	24.23
5	18.24	0.0117	24.23

2.9.2. Beam Transverse Reinforcement Demand

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

Span	Start	End	Required X_u	Required V_u	Required Comb/Patt	Required A_v/s	Demand A_v/s	
	ft	ft	ft	kips		in²/in	in²/in	
1	0.000	0.000	0.000	0.00	U1/All	0.0000	0.0000	
2	1.000	4.122	2.270	32.32	U1/All	0.0099	0.0117	*8
	4.122	5.973	4.122	21.68	U1/All	0.0000	0.0117	*8
	5.973	7.824	5.973	11.37	U1/Even	0.0000	0.0000	

Span	Start	End	Required X_u	V_u	Comb/Patt	A_v/s	Demand A_v/s	
	ft	ft	ft	kips		in²/in	in²/in	
	7.824	9.676	9.676	10.23	U1/All	0.0000	0.0000	
	9.676	11.527	11.527	20.86	U1/All	0.0000	0.0117	*8
	11.527	13.378	13.378	31.50	U1/All	0.0089	0.0117	*8
	13.378	16.500	15.230	42.14	U1/All	0.0218	0.0218	
3	1.000	4.122	2.270	37.23	U1/All	0.0158	0.0158	
	4.122	5.973	4.122	26.59	U1/All	0.0029	0.0117	*8
	5.973	7.824	5.973	15.95	U1/All	0.0000	0.0117	*8
	7.824	9.676	9.676	6.78	U1/S3	0.0000	0.0000	
	9.676	11.527	11.527	15.95	U1/All	0.0000	0.0117	*8
	11.527	13.378	13.378	26.59	U1/All	0.0029	0.0117	*8
	13.378	16.500	15.230	37.23	U1/All	0.0158	0.0158	
4	1.000	4.122	2.270	42.14	U1/All	0.0218	0.0218	
	4.122	5.973	4.122	31.50	U1/All	0.0089	0.0117	*8
	5.973	7.824	5.973	20.86	U1/All	0.0000	0.0117	*8
	7.824	9.676	7.824	10.23	U1/All	0.0000	0.0000	
	9.676	11.527	11.527	11.37	U1/Even	0.0000	0.0000	
	11.527	13.378	13.378	21.68	U1/All	0.0000	0.0117	*8
	13.378	16.500	15.230	32.32	U1/All	0.0099	0.0117	*8
5	0.750	0.750	0.750	0.00	U1/All	0.0000	0.0000	

2.9.3. Beam Transverse Reinforcement Details

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#5	--- None ---
2	#4	8 @ 8.0 + <-- 44.4 --> + 10 @ 8.6
3	#4	10 @ 8.6 + <-- 22.2 --> + 10 @ 8.6
4	#4	10 @ 8.6 + <-- 44.4 --> + 8 @ 8.0
5	#5	--- None ---

2.9.4. Beam Transverse Reinforcement Capacity

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

Span	Start	End	Required X_u	V_u	Comb/Patt	A_v/s	Provided A_v	Sp	A_v/s	ΦV_n	
	ft	ft	ft	kips		in²/in	in²	in	in²/in	kips	
1	0.000	0.750	0.000	0.00	U1/All	-----	-----	-----	-----	-----	
2	0.000	1.000	2.270	32.32	U1/All	-----	-----	-----	-----	-----	
	1.000	5.973	2.270	32.32	U1/All	0.0099	0.40	8.0	0.0503	65.50	*8
	5.973	9.676	5.973	11.37	U1/Even	0.0000	-----	-----	-----	12.11	
	9.676	16.500	15.230	42.14	U1/All	0.0218	0.40	8.6	0.0464	62.32	
	16.500	17.500	15.230	42.14	U1/All	-----	-----	-----	-----	-----	
3	0.000	1.000	2.270	37.23	U1/All	-----	-----	-----	-----	-----	
	1.000	7.824	2.270	37.23	U1/All	0.0158	0.40	8.6	0.0464	62.32	
	7.824	9.676	9.676	6.78	U1/S3	0.0000	-----	-----	-----	12.11	
	9.676	16.500	15.230	37.23	U1/All	0.0158	0.40	8.6	0.0464	62.32	
	16.500	17.500	15.230	37.23	U1/All	-----	-----	-----	-----	-----	
4	0.000	1.000	2.270	42.14	U1/All	-----	-----	-----	-----	-----	

Span	Start	End	Required				Provided				
			X_u	V_u	Comb/Patt	A_v/s	A_v	Sp	A_v/s	ΦV_n	
	ft	ft	ft	kips		in²/in	in²	in	in²/in	kips	
	1.000	7.824	2.270	42.14	U1/All	0.0218	0.40	8.6	0.0464	62.32	
	7.824	11.527	11.527	11.37	U1/Even	0.0000	-----	-----	-----	12.11	
	11.527	16.500	15.230	32.32	U1/All	0.0099	0.40	8.0	0.0503	65.50	*8
	16.500	17.500	15.230	32.32	U1/All	-----	-----	-----	-----	-----	
5	0.000	0.750	0.750	0.00	U1/All	-----	-----	-----	-----	-----	

2.10. Slab Shear Capacity

Span	b	d	V_{ratio}	ΦV_c	V_u	X_u
	in	in		kips	kips	ft
1	250.00	5.00	0.000	118.59	0.00	0.00
2	250.00	5.00	0.000	118.59	0.00	16.33
3	250.00	5.00	0.000	118.59	0.00	16.33
4	250.00	5.00	0.000	118.59	0.00	1.17
5	250.00	5.00	0.000	118.59	0.00	0.00

2.11. Flexural Transfer of Negative Unbalanced Moment at Supports

Support	Width	Width-c	d	M_{unb}	Comb	Patt	γ_f	$A_{s,req}$	$A_{s,prov}$	Add Bars
	in	in	in	kip-ft				in²	in²	
1	36.00	36.00	5.00	93.20	U1	All	0.687	3.420	1.187	12-#4
2	36.00	36.00	5.00	46.72	U1	Even	0.600	1.333	1.387	---
3	36.00	36.00	5.00	46.72	U1	Even	0.600	1.333	1.387	---
4	36.00	36.00	5.00	93.20	U1	All	0.687	3.420	1.187	12-#4

2.12. Punching Shear Around Columns

2.12.1. Critical Section Properties

Support	Type	b_1	b_2	b_0	d_{avg}	CG	$c_{(left)}$	$c_{(right)}$	A_c	J_c
		in	in	in	in	in	in	in	in²	in⁴
1	Rect	20.50	44.00	64.00	17.25	2.41	11.41	9.09	1104.00	95338.01
2	Rect	23.00	23.00	92.00	13.52	0.00	11.50	11.50	1244.00	114993.17
3	Rect	23.00	23.00	92.00	13.52	0.00	11.50	11.50	1244.00	114993.17
4	Rect	20.50	44.00	64.00	17.25	-2.41	9.09	11.41	1104.00	95338.01

2.12.2. Punching Shear Results

Support	V_u	v_u	M_{unb}	Comb	Patt	γ_v	v_u	ΦV_c
	kips	psi	kip-ft				psi	psi
1	48.47	43.9	83.48	U1	All	0.313	73.8	189.7
2	104.49	84.0	-16.77	U1	All	0.400	92.0	189.7
3	104.49	84.0	16.77	U1	All	0.400	92.0	189.7
4	48.47	43.9	-83.48	U1	All	0.313	73.8	189.7

2.13. Material TakeOff

2.13.1. Reinforcement in the Direction of Analysis

Top Bars	673.5	lb	<=>	12.47	lb/ft	<=>	0.567	lb/ft²
Bottom Bars	876.8	lb	<=>	16.24	lb/ft	<=>	0.738	lb/ft²
Stirrups	183.9	lb	<=>	3.41	lb/ft	<=>	0.155	lb/ft²
Total Steel	1734.2	lb	<=>	32.11	lb/ft	<=>	1.460	lb/ft²
Concrete	817.2	ft³	<=>	15.13	ft³/ft	<=>	0.688	ft³/ft²

3. Deflection Results: Summary

3.1. Section Properties

3.1.1. Frame Section Properties

Notes:
M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Span	Zone	M+ve I_g (in⁴)	M+ve I_{cr} (in⁴)	M+ve M_{cr} (kip-ft)	M-ve I_g (in⁴)	M-ve I_{cr} (in⁴)	M-ve M_{cr} (kip-ft)
1	Left	25395	0	63.14	9333	6766	-36.89
	Midspan	25395	0	63.14	9333	7147	-36.89
	Right	433026	0	1267.92	433026	23081	-1267.92
2	Left	25395	1552	63.14	9333	7147	-36.89
	Midspan	25395	2280	63.14	9333	0	-36.89
	Right	25395	1552	63.14	9333	7331	-36.89
3	Left	25395	1552	63.14	9333	7331	-36.89
	Midspan	25395	1552	63.14	9333	6766	-36.89
	Right	25395	1552	63.14	9333	7331	-36.89
4	Left	25395	1552	63.14	9333	7331	-36.89
	Midspan	25395	2280	63.14	9333	0	-36.89
	Right	25395	1552	63.14	9333	7147	-36.89
5	Left	433026	0	1267.92	433026	23081	-1267.92
	Midspan	25395	0	63.14	9333	7147	-36.89
	Right	25395	0	63.14	9333	6766	-36.89

3.1.2. Frame Effective Section Properties

Span	Zone	Weight	Dead M_{max} (kip-ft)	Dead I_e (in⁴)	Sustained M_{max} (kip-ft)	Sustained I_e (in⁴)	Dead+Live M_{max} (kip-ft)	Dead+Live I_e (in⁴)
1	Right	1.000	-0.52	433026	-0.52	433026	-1.14	433026
	Span Avg	----	----	433026	----	433026	----	433026
2	Middle	0.850	27.18	25395	27.18	25395	59.43	25395
	Right	0.150	-58.33	7838	-58.33	7838	-127.55	7380
	Span Avg	----	----	22762	----	22762	----	22693
3	Left	0.150	-52.92	8010	-52.92	8010	-115.72	7396
	Middle	0.700	18.06	25395	18.06	25395	39.48	25395
	Right	0.150	-52.92	8010	-52.92	8010	-115.72	7396
	Span Avg	----	----	20179	----	20179	----	19995
4	Left	0.150	-58.33	7838	-58.33	7838	-127.55	7380
	Middle	0.850	27.18	25395	27.18	25395	59.43	25395
	Span Avg	----	----	22762	----	22762	----	22693
5	Left	1.000	-0.52	433026	-0.52	433026	-1.14	433026
	Span Avg	----	----	433026	----	433026	----	433026

Load Level: Dead, Sustained, Dead+Live.

3.1.3. Strip Section Properties at Midspan

Notes:
Load distribution factor, LDF, averages moment distribution factors listed in Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

Span	Column Strip I_g (in⁴)	Column Strip LDF	Column Strip Ratio	Middle Strip I_g (in⁴)	Middle Strip LDF	Middle Strip Ratio
1	20040.5	0.781	0.990	2862	0.219	1.943
2	20040.5	0.693	0.878	2862	0.307	2.723

Span	Column Strip			Middle Strip		
	I_g in⁴	LDF	Ratio	I_g in⁴	LDF	Ratio
3	20040.5	0.673	0.853	2862	0.327	2.903
4	20040.5	0.693	0.878	2862	0.307	2.723
5	20040.5	0.781	0.990	2862	0.219	1.943

3.2. Instantaneous Deflections

3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.011	---	0.013	0.013	0.011	0.024
		Loc	ft	8.000	---	8.000	8.000	8.000	8.000
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.006	---	0.007	0.007	0.006	0.013
		Loc	ft	8.750	---	8.750	8.750	8.750	8.750
	Up	Def	in	0.000	---	0.000	0.000	0.000	-0.001
		Loc	ft	1.000	---	1.000	1.000	1.000	1.000
4	Down	Def	in	0.011	---	0.013	0.013	0.011	0.024
		Loc	ft	9.500	---	9.500	9.500	9.500	9.500
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	ft	0.750	---	0.750	0.750	0.750	0.750

3.2.2. Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.010	---	0.012	0.012	0.010	0.023
		Loc	ft	8.000	---	8.000	8.000	8.000	8.000
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.005	---	0.006	0.006	0.005	0.011
		Loc	ft	8.750	---	8.750	8.750	8.750	8.750
	Up	Def	in	0.000	---	0.000	0.000	0.000	-0.001
		Loc	ft	1.000	---	1.000	1.000	1.000	1.000
4	Down	Def	in	0.010	---	0.012	0.012	0.010	0.023
		Loc	ft	9.500	---	9.500	9.500	9.500	9.500
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	ft	0.750	---	0.750	0.750	0.750	0.750

3.2.3. Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.022	---	0.026	0.026	0.022	0.049
		Loc	ft	8.500	---	8.500	8.500	8.500	8.500
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.020	---	0.024	0.024	0.020	0.044
		Loc	ft	8.750	---	8.750	8.750	8.750	8.750
	Up	Def	in	0.000	---	0.000	0.000	0.000	0.000
		Loc	ft	0.750	---	0.750	0.750	0.750	0.750
4	Down	Def	in	0.022	---	0.026	0.026	0.022	0.049
		Loc	ft	9.000	---	9.000	9.000	9.000	9.000
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.001	---	-0.001	-0.001	-0.001	-0.003
		Loc	ft	0.750	---	0.750	0.750	0.750	0.750

3.3. Long-term Deflections

3.3.1. Long-term Column Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span	Zone	M_{+ve} $A_{s,top}$ in²	M_{+ve} b in	M_{+ve} d in	M_{+ve} Rho' %	M_{+ve} Lambda	M_{-ve} $A_{s,bot}$ in²	M_{-ve} b in	M_{-ve} d in	M_{-ve} Rho' %	M_{-ve} Lambda
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

3.3.2. Long-term Middle Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span	Zone	M_{+ve} $A_{s,top}$ in²	M_{+ve} b in	M_{+ve} d in	M_{+ve} Rho' %	M_{+ve} Lambda	M_{-ve} $A_{s,bot}$ in²	M_{-ve} b in	M_{-ve} d in	M_{-ve} Rho' %	M_{-ve} Lambda
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

3.3.3. Extreme Long-term Column Strip Deflections and Corresponding Locations

Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.002	-0.004	-0.004	-0.005
		Loc	ft	0.000	0.000	0.000	0.000
2	Down	Def	in	0.021	0.033	0.033	0.043
		Loc	ft	8.000	8.000	8.000	8.000
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
3	Down	Def	in	0.010	0.015	0.015	0.020
		Loc	ft	8.750	8.750	8.750	8.750
	Up	Def	in	-0.001	-0.001	-0.001	-0.001
		Loc	ft	1.000	1.000	1.000	1.000
4	Down	Def	in	0.021	0.033	0.033	0.043
		Loc	ft	9.500	9.500	9.500	9.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
5	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.002	-0.004	-0.004	-0.005
		Loc	ft	0.750	0.750	0.750	0.750

3.3.4. Extreme Long-term Middle Strip Deflections and Corresponding Locations

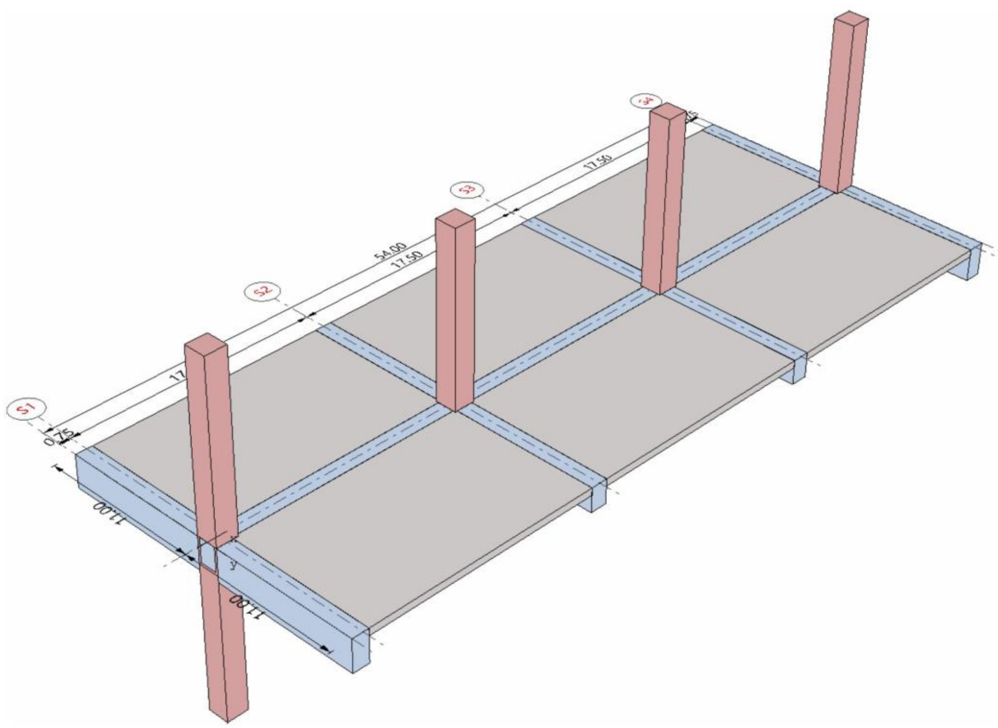
Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.002	-0.004	-0.004	-0.005
		Loc	ft	0.000	0.000	0.000	0.000
2	Down	Def	in	0.044	0.071	0.071	0.093
		Loc	ft	8.500	8.500	8.500	8.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
3	Down	Def	in	0.040	0.064	0.064	0.084
		Loc	ft	8.750	8.750	8.750	8.750
	Up	Def	in	0.000	-0.001	-0.001	-0.001
		Loc	ft	0.750	0.750	0.750	0.750
4	Down	Def	in	0.044	0.071	0.071	0.093
		Loc	ft	9.000	9.000	9.000	9.000
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
5	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.002	-0.004	-0.004	-0.005

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
		Loc	ft	0.750	0.750	0.750	0.750

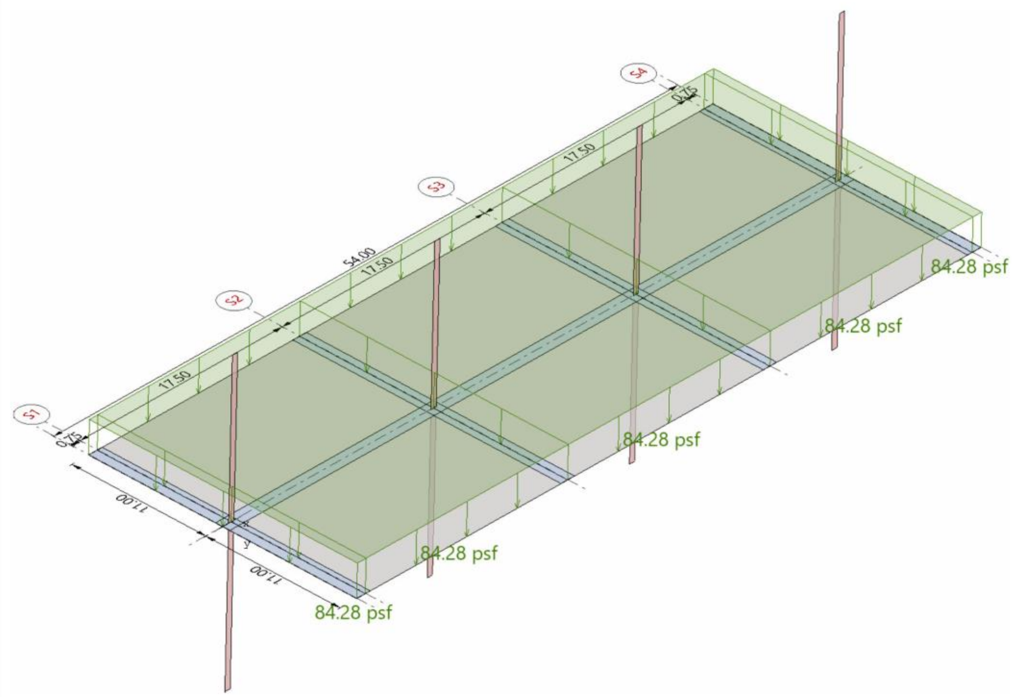
4. Screenshots

4.1. Extrude 3D view



Project:	Slab on beams
Diagram:	Model View (Extrude)
	Slabs; Longitudinal beams; Transverse Beams

4.2. Loads - Case A - Dead



Project: Slab on beams

Diagram: Model View (Load Case: A - Dead)

Slabs; Columns; Longitudinal beams; Transverse Beams

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Page | 24
5/8/2025
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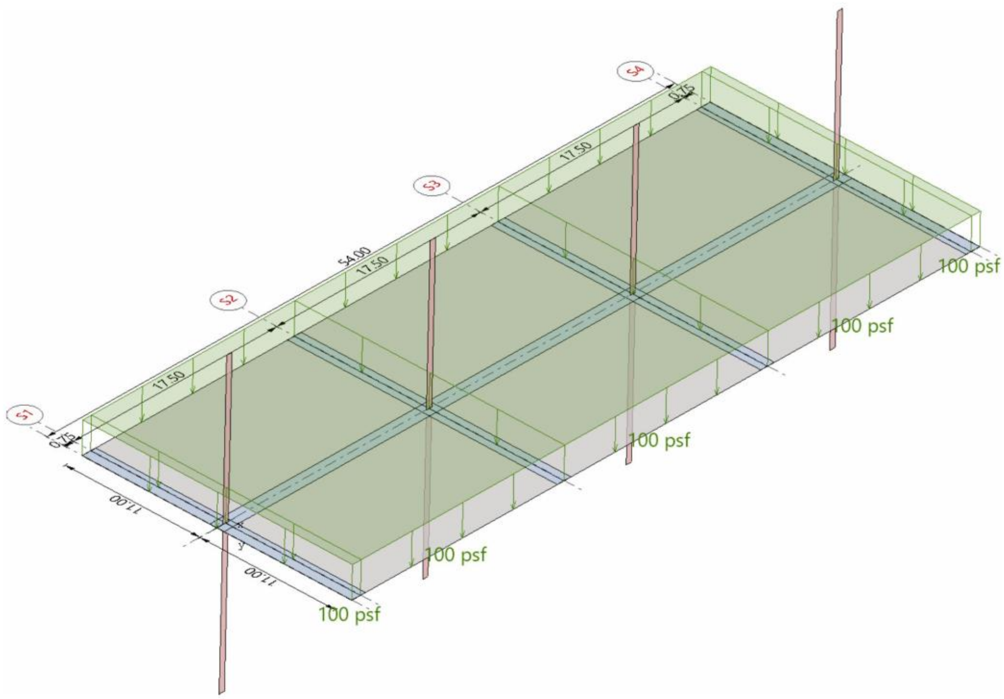
4.3. Loads - Case B - Live

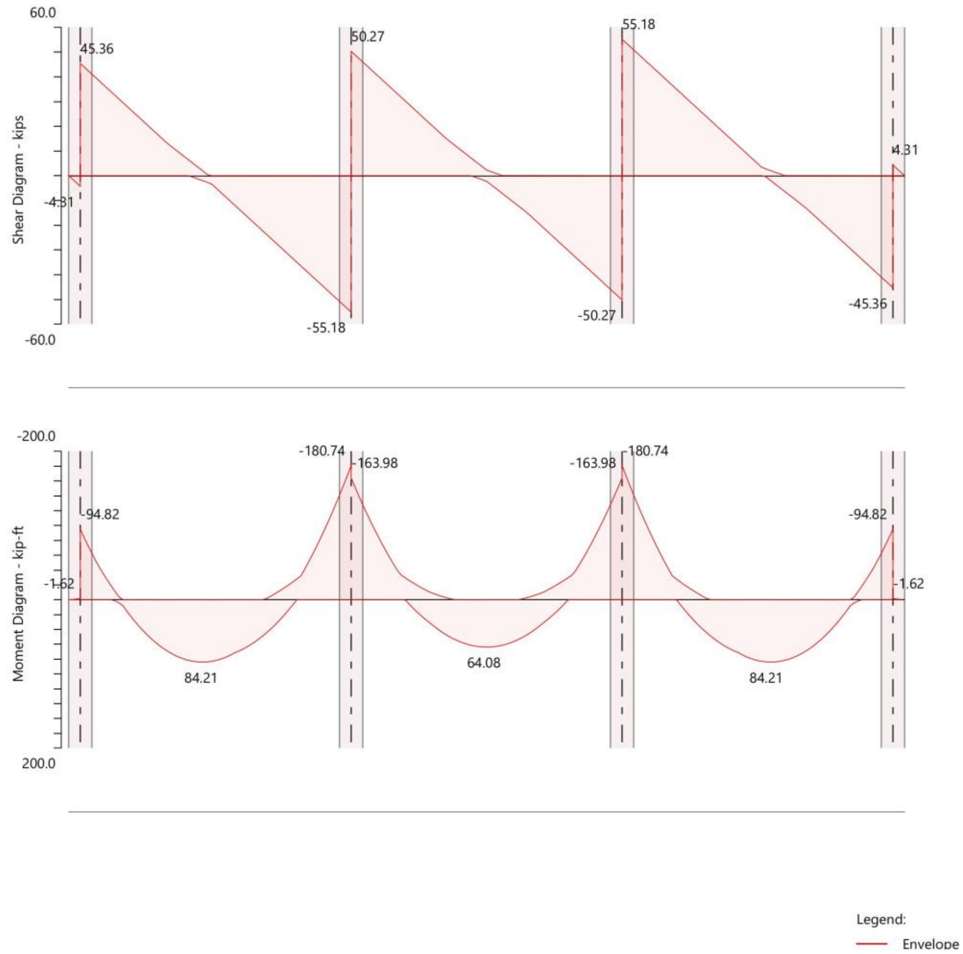


Project: Slab on beams

Diagram: Model View (Load Case: B - Live)

Slabs; Columns; Longitudinal beams; Transverse Beams

4.4. Internal Forces



Project: Slab on beams

Diagram: Internal Forces

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Page | 26
5/8/2025
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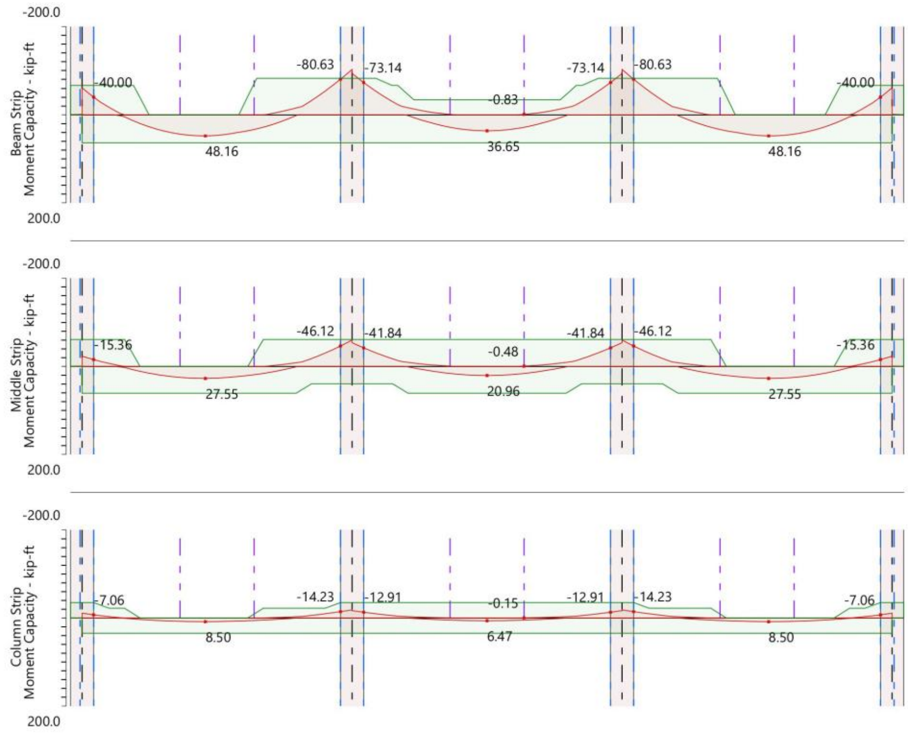
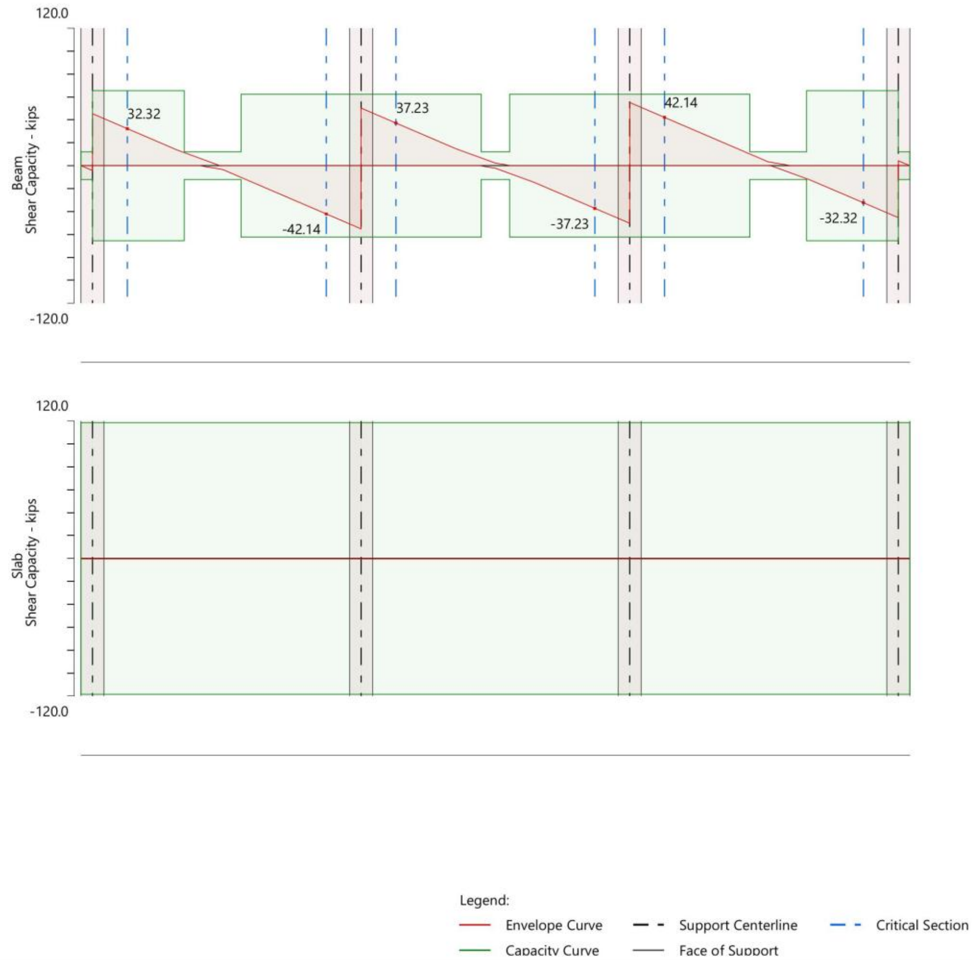
4.5. Moment Capacity



Project: Slab on beams

Diagram: Moment Capacity

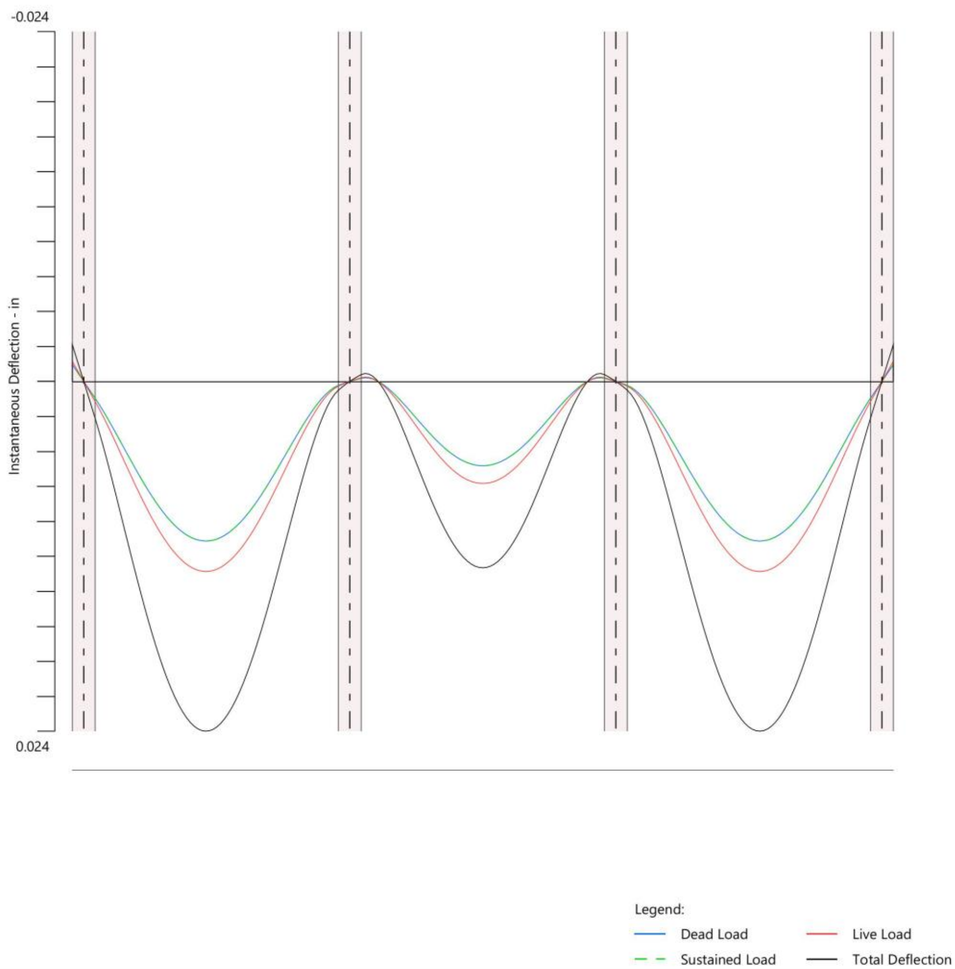
4.6. Shear Capacity



Project: Slab on beams

Diagram: Shear Capacity

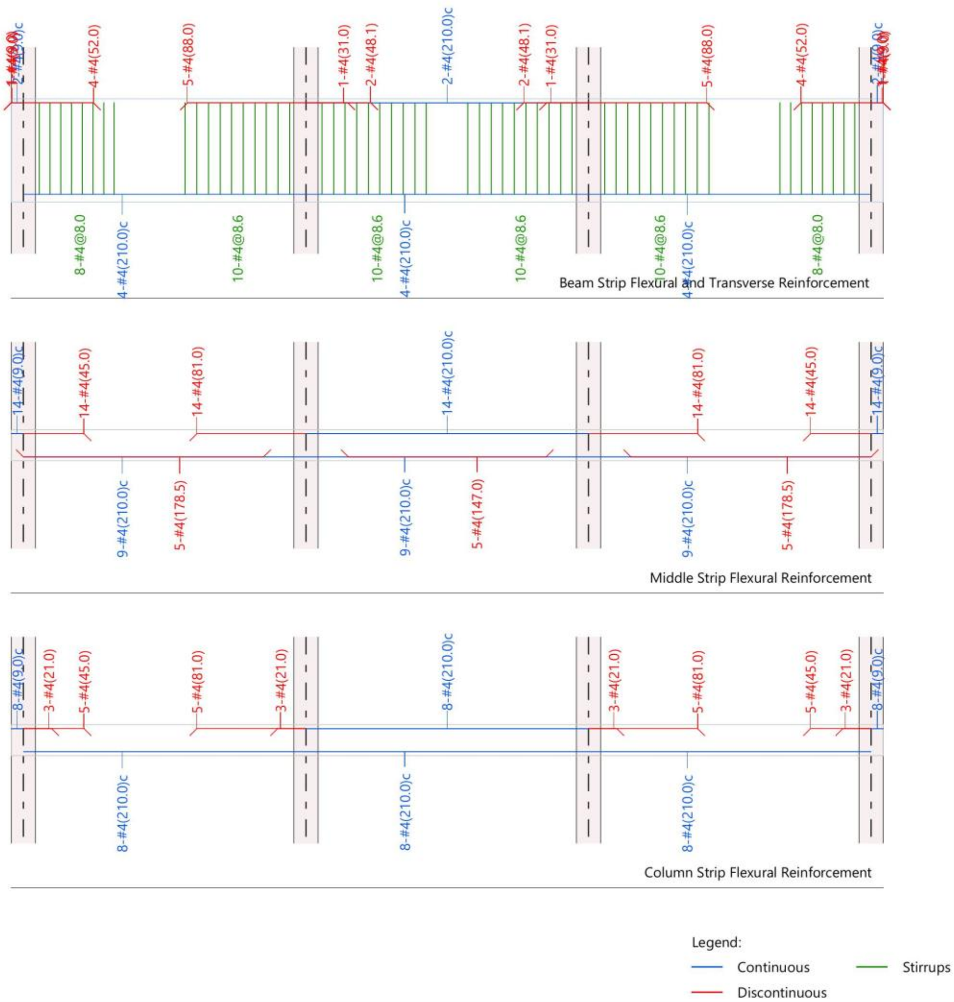
4.7. Deflection



Project: Slab on beams

Diagram: Deflection

4.8. Reinforcement



Project: Slab on beams

Diagram: Reinforcement

8. Summary and Comparison of Design Results

Table 9 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution (ft-kips)			
		Hand (EFM)	**spSlab**
Exterior Span			
Beam Strip	**Exterior Negative***	38.42	40.00
	Positive	51.81	48.16
	Interior Negative*	73.26	80.63
Column Strip	**Exterior Negative***	6.78	7.06
	Positive	9.14	8.50
	Interior Negative*	12.93	14.23
Middle Strip	**Exterior Negative***	15.07	15.36
	Positive	30.02	27.55
	Interior Negative*	42.45	46.12
Interior Span			
Beam Strip	**Interior Negative***	67.07	73.14
	Positive	40.53	36.65
Column Strip	**Interior Negative***	11.84	12.91
	Positive	7.15	6.47
Middle Strip	**Interior Negative***	38.87	41.84
	Positive	23.48	20.96
* Negative moments are taken at the faces of supports			

Table 10 - Comparison of Reinforcement Results							
Span Location		Reinforcement Provided for Flexure		Additional Reinforcement Provided for Unbalanced Moment Transfer		Total Reinforcement Provided	
		Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior Span							
Beam Strip	Exterior Negative	4 - #4	4 - #4	n/a	n/a	4 - #4	4 - #4
	Positive	5 - #4	4 - #4	n/a	n/a	5 - #4	4 - #4
	Interior Negative	5 - #4	5 - #4	---	---	5 - #4	5 - #4
Column Strip	Exterior Negative	8 - #4	8 - #4	10 - #4	12 - #4	18 - #4	20 - #4
	Positive	8 - #4	8 - #4	n/a	n/a	8 - #4	8 - #4
	Interior Negative	8 - #4	8 - #4	---	---	8 - #4	8 - #4
Middle Strip	Exterior Negative	14 - #4	14 - #4	n/a	n/a	14 - #4	14 - #4
	Positive	14 - #4	14 - #4	n/a	n/a	14 - #4	14 - #4
	Interior Negative	14 - #4	14 - #4	n/a	n/a	14 - #4	14 - #4
Interior Span							
Beam Strip	Positive	4 - #4	4 - #4	n/a	n/a	4 - #4	4 - #4
Column Strip	Positive	8 - #4	8 - #4	n/a	n/a	8 - #4	8 - #4
Middle Strip	Positive	14 - #4	14 - #4	n/a	n/a	14 - #4	14 - #4

Table 11 - Comparison of Beam Shear Reinforcement Results		
Span Location	Reinforcement Provided	
	Hand	spSlab
End Span		
Exterior	8 - #4 @ 8 in.	8 - #4 @ 8 in.
Interior	10 - #4 @ 8.6 in.	10 - #4 @ 8.6 in.
Interior Span		
Interior	9 - #4 @ 8.6 in.	10 - #4 @ 8.6 in.

Table 12 - Comparison of Two-Way (Punching) Shear Check Results (around Columns Faces)										
Support	**b_1 (in.)**		**b_2 (in.)**		**b_o (in.)**		**V_u (kips)**		**c_{AB} (in.)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	20.50	20.50	23.00	44.00	64.00	64.00	44.88	48.47	9.09	9.09
Interior	23.00	23.00	23.00	23.00	92.00	92.00	103.56	104.49	11.50	11.50
Support	**J_c (in.^{4})**		**γ_v**		**M_{unb} (ft-kips)**		**v_u (psi)**		**ϕv_c (psi)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	95,338	95,338	0.386	0.313	84.15	83.48	77.86	73.80	189.7	189.7
Interior	114,993	114,993	0.400	0.400	14.07	16.77	90.00	92.00	189.7	189.7

Table 13 - Comparison of Immediate Deflection Results (in.)								
Column Strip								
Span	**D**		**D + LL_{sus}**		**D + LL_{full}**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	0.010	0.010	0.010	0.010	0.021	0.023	0.011	0.012
Interior	0.004	0.005	0.004	0.005	0.010	0.011	0.005	0.006
Middle Strip								
Span	**D**		**D + LL_{sus}**		**D + LL_{full}**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	0.021	0.022	0.021	0.022	0.047	0.049	0.025	0.026
Interior	0.019	0.020	0.019	0.020	0.042	0.044	0.023	0.024

Table 14 - Comparison of Time-Dependent Deflection Results						
Column Strip						
Span	λ_Δ		Δ_{cs} (in.)		Δ_{total} (in.)	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	0.019	0.021	0.040	0.043
Interior	2.0	2.0	0.009	0.010	0.018	0.020
Middle Strip						
Span	λ_Δ		Δ_{cs} (in.)		Δ_{total} (in.)	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	0.043	0.044	0.090	0.093
Interior	2.0	2.0	0.038	0.040	0.080	0.084

In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the spSlab model.

9. Conclusions & Observations

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in ***ACI 318-14 Chapter 8 (8.2.1)***.

Direct Design Method (DDM) is an approximate method and is applicable to two-way slab concrete floor systems that meet the stringent requirements of ***ACI 318-14 (8.10.2)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Equivalent Frame Method (EFM) does not have the limitations of Direct Design Method. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StucturePoint's spSlab software program solution utilizes the Equivalent Frame Method to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable ACI 318-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
8.10.2.1	Minimum of three continuous spans in each direction	☑		
8.10.2.2	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
8.10.2.3	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
8.10.2.4	Column offset shall not exceed 10% of the span in direction of offset from either axis between centerlines of successive columns	☑		
8.10.2.5	All loads shall be due to gravity only	☑		
8.10.2.5	All loads shall be uniformly distributed over an entire panel (q_u)	☑		
8.10.2.6	Unfactored live load shall not exceed two times the unfactored dead load	☑		
8.10.2.7	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑		
8.7.4.2	Structural integrity steel detailing	☑	☑	☑
8.5.4	Openings in slab systems	☑	☑	☑
8.2.2	Concentrated loads	Not permitted	☑	☑
8.11.1.2	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
R8.10.4.5*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)		Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb})