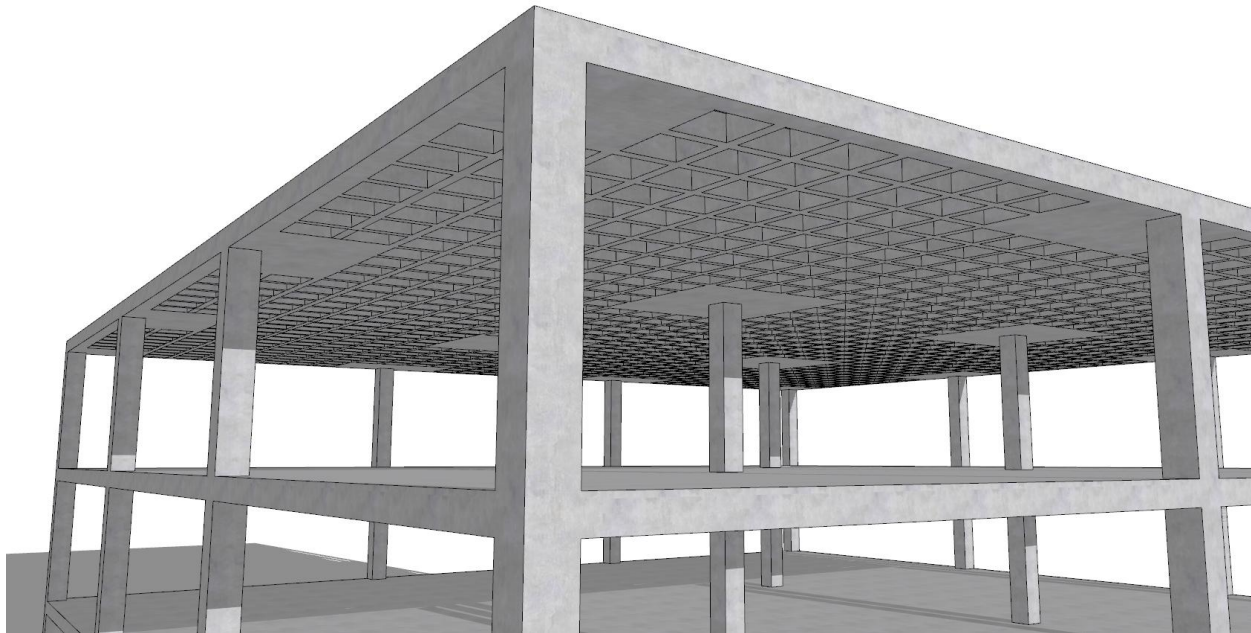
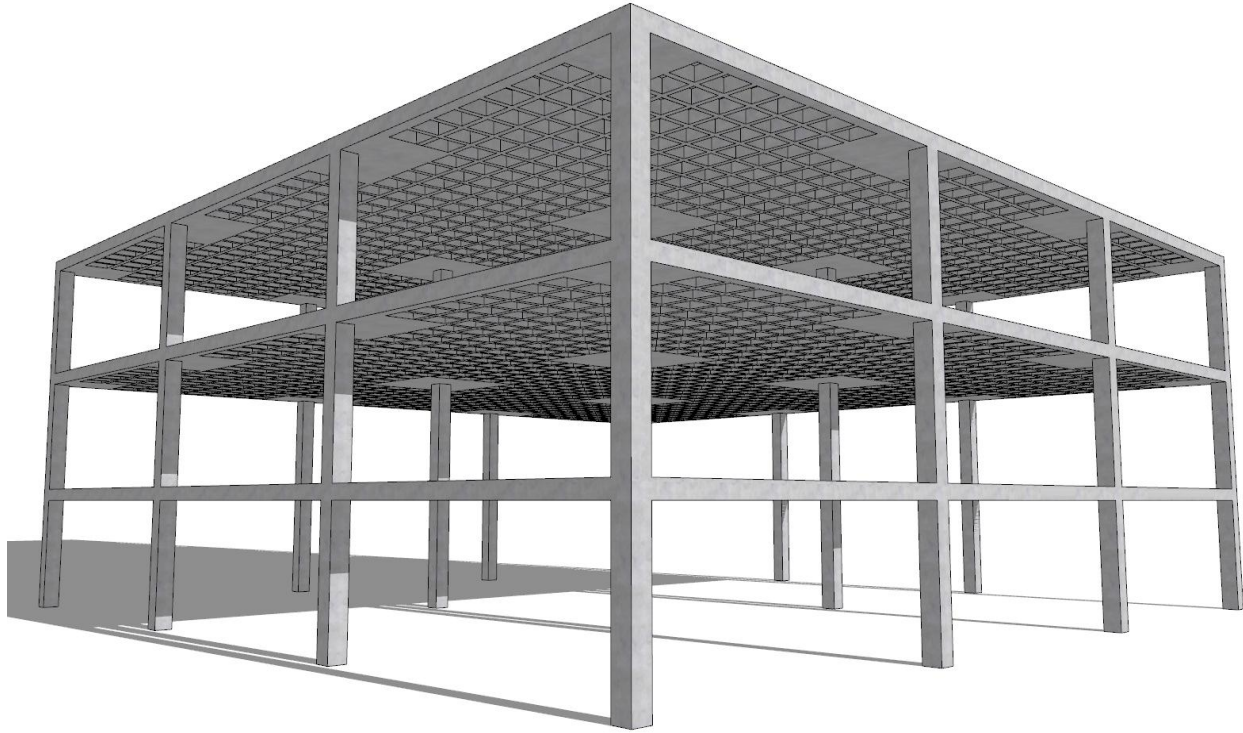


**Two-Way Joist Concrete Slab Floor (Waffle Slab) System Analysis and Design (ACI 318-14)**



## Two-Way Joist Concrete Slab Floor (Waffle Slab) System Analysis and Design (ACI 318-14)

Design the concrete floor slab system shown below for an intermediate floor with partition weight of 50 psf, and unfactored live load of 100 psf. The lateral loads are independently resisted by shear walls. A flat plate system will be considered first to illustrate the impact longer spans and heavier applied loads. A waffle slab system will be investigated since it is economical for longer spans with heavy loads. The dome voids reduce the dead load and electrical fixtures can be fixed in the voids. Waffle system provides an attractive ceiling that can be left exposed when possible producing savings in architectural finishes. The Equivalent Frame Method (EFM) shown in ACI 318 is used in this example. The hand solution from EFM is also used for a detailed comparison with the model results of [spSlab](#) engineering software program from [StructurePoint](#).

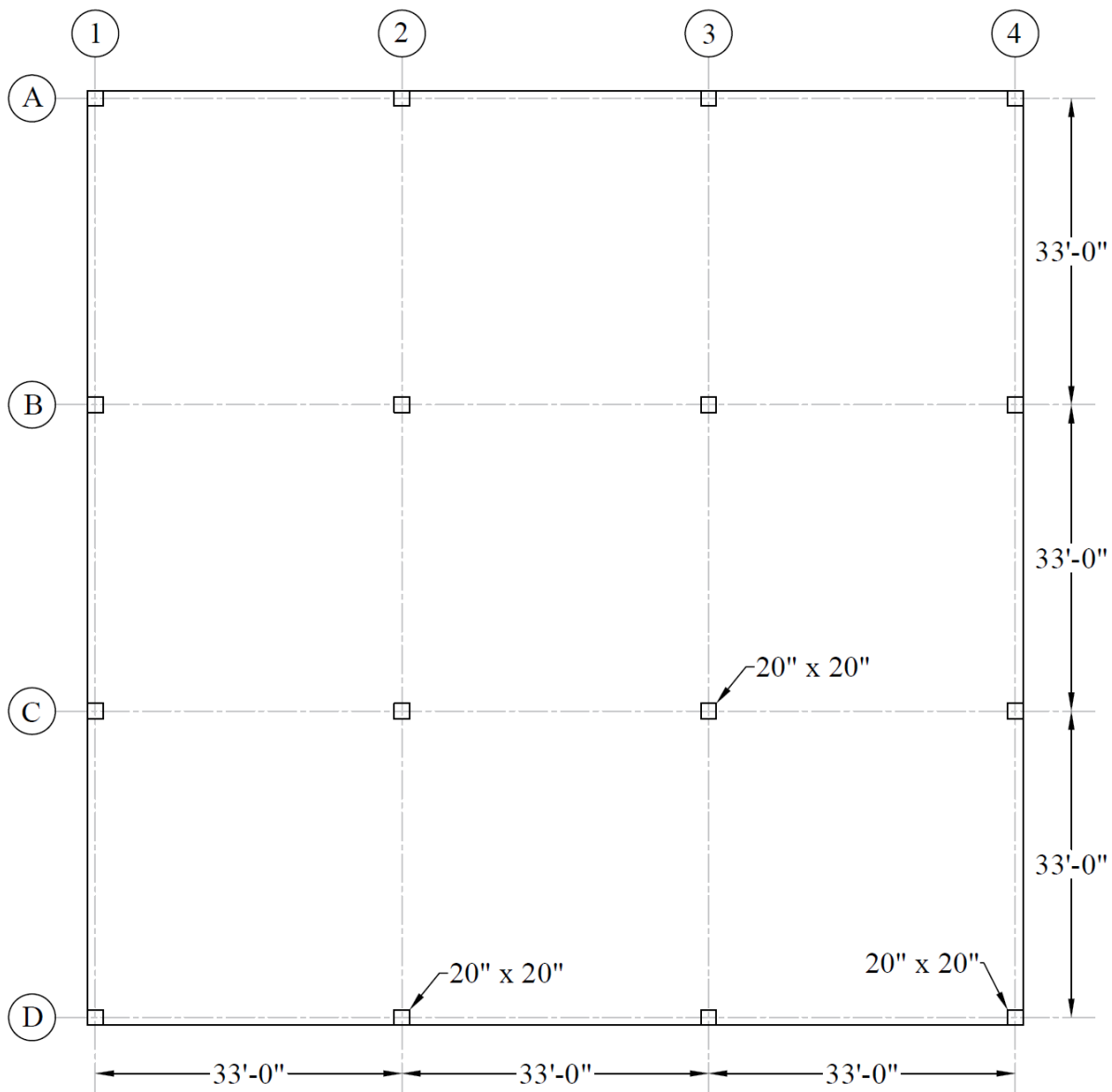


Figure 1 – Two-Way Flat Concrete Floor System

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## Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

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- “[Two-Way Flat Plate Concrete Floor System Analysis and Design \(ACI 318-14\)](#)” Design Example, [STRUCTUREPOINT](#), 2025
- “[Two-Way Flat Slab \(Concrete Floor with Drop Panels\) System Analysis and Design \(ACI 318-14\)](#)” Design Example, [STRUCTUREPOINT](#), 2025
- “[Two-Way Concrete Floor Slab with Beams System Analysis and Design \(ACI 318-14\)](#)” Design Example, [STRUCTUREPOINT](#), 2025
- Contact [Support@StructurePoint.org](mailto:Support@StructurePoint.org) to obtain supplementary materials ([spSlab](#) models: DE-Two-Way-Joist-Waffle-System-ACI-318-14.slbx)

## Design Data

Story Height = 13 ft (provided by architectural drawings)

Superimposed Dead Load,  $SDL = 50$  psf for Frame walls, hollow concrete masonry unit wythe, 12 in. thick, 125 pcf unit density, with no grout **ASCE/SEI 7-10 (Table C3-1)**

Live Load,  $LL = 100$  psf for Recreational uses – Gymnasiums **ASCE/SEI 7-10 (Table 4-1)**

$f_c' = 5,000$  psi (for slab)

$f_c' = 6,000$  psi (for columns)

$f_y = 60,000$  psi

## 1. Notations

This section (based on ACI 318-14 provisions) defines notation and terminology used in this design example:

$a$  = depth of equivalent rectangular stress block, in.

$A_b$  = area of an individual bar or wire, in.<sup>2</sup>

$A_g$  = gross area of concrete section, in.<sup>2</sup> For a hollow section,  $A_g$  is the area of the concrete only and does not include the area of the void(s)

$A_s$  = area of nonprestressed longitudinal tension reinforcement, in.<sup>2</sup>

$A_{s,min}$  = minimum area of flexural reinforcement, in.<sup>2</sup>

$b$  = width of compression face of member, in.

$b_o$  = perimeter of critical section for two-way shear in slabs and footings, in.

$b_v$  = width of cross section at contact surface being investigated for horizontal shear, in.

$b_w$  = web width or diameter of circular section, in.

$b_1$  = dimension of the critical section  $b_o$  measured in the direction of the span for which moments are determined, in.

$b_2$  = dimension of the critical section  $b_o$  measured in the direction perpendicular to  $b_1$ , in.

$c$  = distance from extreme compression fiber to neutral axis, in.

$c_c$  = clear cover of reinforcement, in.

$c_1$  = dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in.

$c_2$  = dimension of rectangular or equivalent rectangular column, capital, or bracket measured in the direction perpendicular to  $c_1$ , in.

$C$  = cross-sectional constant to define torsional properties of slab and beam

$d$  = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.

$d_b$  = nominal diameter of bar, wire, or prestressing strand, in.

$E_c$  = modulus of elasticity of concrete, psi

$E_{cs}$  = modulus of elasticity of slab concrete, psi

$E_s$  = modulus of elasticity of reinforcement and structural steel, excluding prestressing reinforcement, psi

$f'_c$  = specified compressive strength of concrete, psi

- $f_r$  = modulus of rupture of concrete, psi
- $f_y$  = specified yield strength for nonprestressed reinforcement, psi
- $h$  = overall thickness, height, or depth of member, in.
- $h_w$  = height of entire wall from base to top, or clear height of wall segment or wall pier considered, in.
- $I_b$  = moment of inertia of gross section of beam about centroidal axis, in.<sup>4</sup>
- $I_{cr}$  = moment of inertia of cracked section transformed to concrete, in.<sup>4</sup>
- $I_e$  = effective moment of inertia for calculation of deflection, in.<sup>4</sup>
- $I_g$  = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement, in.<sup>4</sup>
- $I_s$  = moment of inertia of gross section of slab about centroidal axis, in.<sup>4</sup>
- $K_t$  = torsional stiffness of member; moment per unit rotation
- $l$  = span length of beam or one-way slab; clear projection of cantilever, in.
- $l_n$  = length of clear span measured face-to-face of supports, in.
- $l_1$  = length of span in direction that moments are being determined, measured center-to-center of supports, in.
- $l_2$  = length of span in direction perpendicular to  $l_1$ , measured center-to-center of supports, in.
- $M_a$  = maximum moment in member due to service loads at stage deflection is calculated, in.-lb
- $M_{cr}$  = cracking moment, in.-lb
- $M_{sc}$  = factored slab moment that is resisted by the column at a joint, in.-lb
- $M_u$  = factored moment at section, in.-lb
- $M_o$  = total factored static moment, in.-lb
- $n$  = number of items, such as, bars, wires, monostrand anchorage devices, anchors, or shearhead arms
- $P_n$  = nominal axial compressive strength of member, lb
- $P_o$  = nominal axial strength at zero eccentricity, lb
- $P_u$  = factored axial force; to be taken as positive for compression and negative for tension, lb
- $q_{Du}$  = factored dead load per unit area, lb/ft<sup>2</sup>
- $q_{Lu}$  = factored live load per unit area, lb/ft<sup>2</sup>
- $q_u$  = factored load per unit area, lb/ft<sup>2</sup>
- $v_c$  = stress corresponding to nominal two-way shear strength provided by concrete, psi
- $v_u$  = maximum factored two-way shear stress calculated around the perimeter of a given critical section, psi

- $V_c$  = nominal shear strength provided by concrete, lb
- $V_n$  = nominal shear strength, lb
- $V_s$  = nominal shear strength provided by shear reinforcement, lb
- $V_u$  = factored shear force at section, lb
- $w_c$  = density, unit weight, of normalweight concrete or equilibrium density of lightweight concrete, lb/ft<sup>3</sup>
- $y_t$  = distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.
- $\beta$  = ratio of long to short dimensions: clear spans for two-way slabs, sides of column, concentrated load or reaction area; or sides of a footing
- $\beta_t$  = ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports
- $\beta_1$  = factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis
- $\gamma_f$  = factor used to determine the fraction of  $M_{sc}$  transferred by slab flexure at slab-column connections
- $\gamma_v$  = factor used to determine the fraction of  $M_{sc}$  transferred by eccentricity of shear at slab-column connections
- $\varepsilon_t$  = net tensile strain in extreme layer of longitudinal tension reinforcement at nominal strength, excluding strains due to effective prestress, creep, shrinkage, and temperature
- $\lambda$  = modification factor to reflect the reduced mechanical properties of lightweight concrete relative to normal weight concrete of the same compressive strength
- $\lambda_\Delta$  = multiplier used for additional deflection due to long-term effects
- $\zeta$  = time-dependent factor for sustained load
- $\rho'$  = ratio of  $A_s'$  to  $bd$
- $\phi$  = strength reduction factor

## 2. Preliminary Member Sizing

### 2.1. Preliminary Flat Plate (without Joists)

#### 2.1.1. Slab Minimum Thickness – Deflection

ACI 318-14 (8.3.1.1)

In lieu of detailed calculation for deflections, ACI 318 Code gives minimum slab thickness for two-way construction without interior beams in **Table 8.3.1.1**.

For flat plate slab system, the minimum slab thickness per **ACI 318-14** are:

Exterior Panels:  $h_s = \frac{l_n}{30} = \frac{376}{30} = 12.53 \text{ in.}$

ACI 318-14 (Table 8.3.1.1)

But not less than 5 in.

ACI 318-14 (8.3.1.1(a))

Interior Panels:  $h_s = \frac{l_n}{33} = \frac{376}{33} = 11.39 \text{ in.}$

ACI 318-14 (Table 8.3.1.1)

But not less than 5 in.

ACI 318-14 (8.3.1.1(a))

Where  $l_n$  = length of clear span in the long direction =  $33 \times 12 - 20 = 376 \text{ in.}$

Use 13 in. slab for all panels (self-weight =  $150 \text{ pcf} \times 13 \text{ in.} / 12 = 162.50 \text{ psf}$ )



### 2.1.2. Slab Shear Strength – One Way Shear

Evaluate the average effective depth (Figure 2):

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 13 - 0.75 - 0.75 - \frac{0.75}{2} = 11.13 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 13 - 0.75 - \frac{0.75}{2} = 11.88 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{11.13 + 11.88}{2} = 11.50 \text{ in.}$$

Where:

$c_{clear} = 3/4 \text{ in.}$  for # 6 steel bar

ACI 318-14 (Table 20.6.1.3.1)

$d_b = 0.75 \text{ in.}$  for # 6 steel bar

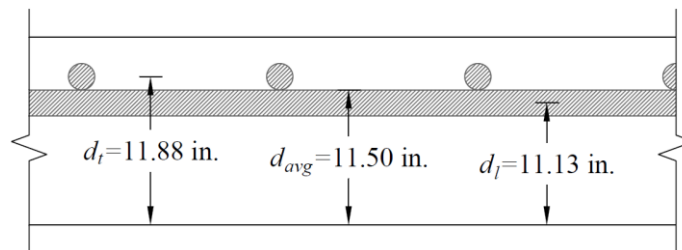


Figure 2 – Average Effective Depth for Flat Plate

Factored dead load,  $q_{Du} = 1.20 \times (162.50 + 50.00) = 255.00 \text{ psf}$

Factored live load,  $q_{Lu} = 1.60 \times 100.00 = 160.00 \text{ psf}$

ACI 318-14 (5.3.1)

Total factored load,  $q_u = 255.00 + 160.00 = 415.00 \text{ psf}$

Check the adequacy of slab thickness for beam action (one-way shear)

ACI 318-14 (22.5)

At an interior column:

Consider a 12-in. wide strip. The critical section for one-way shear is located at a distance  $d$ , from the face of support (see [Figure 3](#)):

Tributary area for one-way shear is:

$$A_{Tributary} = \left[ \frac{33}{2} - \frac{20}{2 \times 12} - \frac{11.50}{12} \right] \times \frac{12}{12} = 14.71 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.415 \times 14.71 = 6.10 \text{ kips}$$

$$V_c = 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d$$

ACI 318-14 (Eq. 22.5.5.1)

Where  $\lambda = 1$  for normal weight concrete, more information can be found in “[Concrete Type Classification Based on Unit Density](#)” technical article.

$$\phi V_c = 0.75 \times 2.0 \times 1.0 \times \sqrt{5,000} \times 12 \times \frac{11.50}{1,000} = 14.64 \text{ kips} > V_u = 6.10 \text{ kips}$$

Slab thickness of 13 in. is adequate for one-way shear.

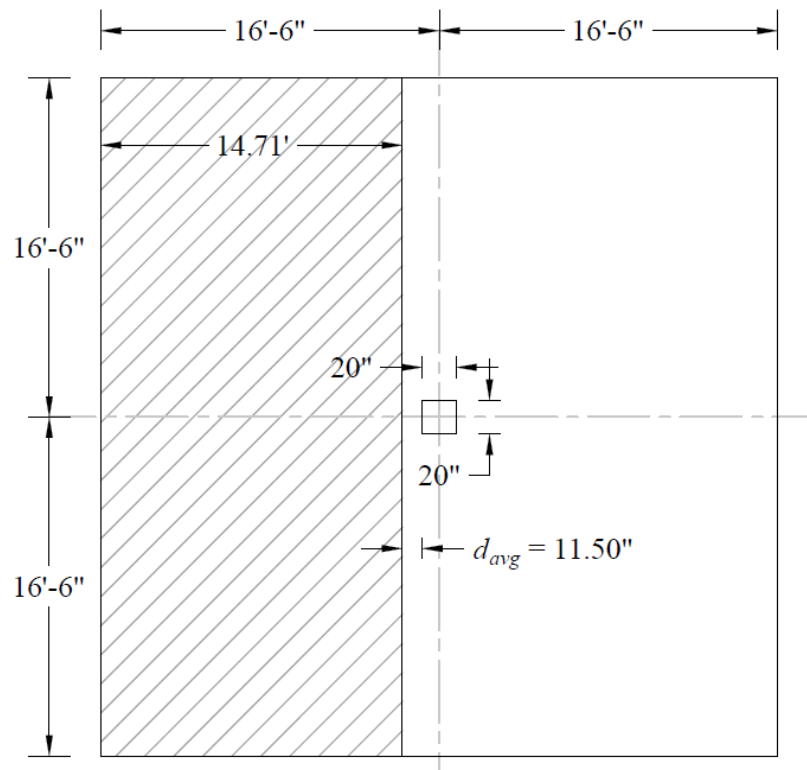


Figure 3 – Critical Section for One-Way Shear

### 2.1.3. Slab Shear Strength – Two-Way Shear

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column ([Figure 4](#)):

Tributary area for two-way shear is:

$$A_{Tributary} = (33 \times 33) - \left( \frac{20 + 11.50}{12} \right)^2 = 1,082.11 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.415 \times 1,082.11 = 449.08 \text{ kips}$$

$$V_c = 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d \quad (\text{For square interior column})$$

ACI 318-14 (Table 22.6.5.2(a))

$$V_c = 4 \times 1 \times \sqrt{5,000} \times (4 \times (20 + 11.50)) \times \frac{11.50}{1,000} = 409.84 \text{ kips}$$

$$\phi V_c = 0.75 \times 409.84 = 307.38 \text{ kips} < V_u = 449.08 \text{ kips}$$

Slab thickness of 13 in. is not adequate for two-way shear. This is expected as the self-weight and applied loads are very challenging for a flat plate system.

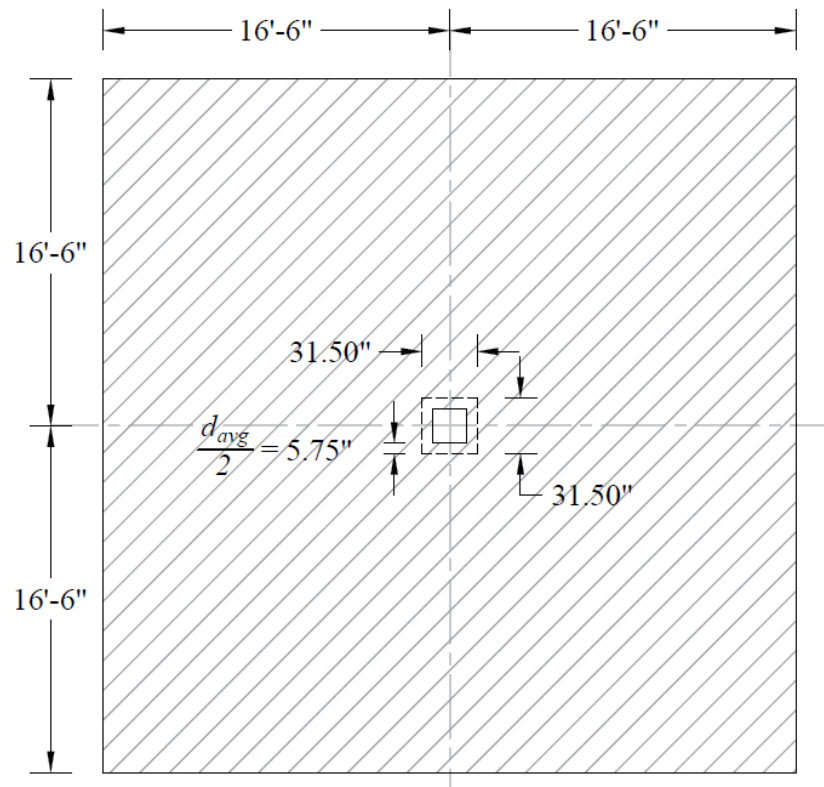


Figure 4 – Critical Section for Two-Way Shear

In this case, four options can be considered: 1) to increase the slab thickness further, 2) use headed shear reinforcement in the slab, 3) apply drop panels at columns, or 4) use two-way joist slab system. In this example, the latter option will be used to achieve better understanding for the design of two-way joist slab often called two-way ribbed slab or waffle slab.

Check the applicable joist dimensional limitations as follows:

- 1) Width of ribs shall be at least 4 in. at any location along the depth. ACI 318-14 (9.8.1.2)

Use ribs with 6 in. width.

- 2) Overall depth of ribs shall not exceed 3.5 times the minimum width. ACI 318-14 (9.8.1.3)

$3.5 \times 6 \text{ in.} = 21 \text{ in.} \rightarrow$  Use ribs with 14 in. depth.

- 3) Clear spacing between ribs shall not exceed 30 in. ACI 318-14 (9.8.1.4)

Use 30 in. clear spacing.

- 4) Slab thickness (with removable forms) shall be at least the greater of: ACI 318-14 (8.8.3.1)

a)  $1/12$  clear distance between ribs  $= 1/12 \times 30 = 2.50 \text{ in.}$

b) 2 in.

Use a slab thickness of 3.00 in.  $> 2.50 \text{ in.}$

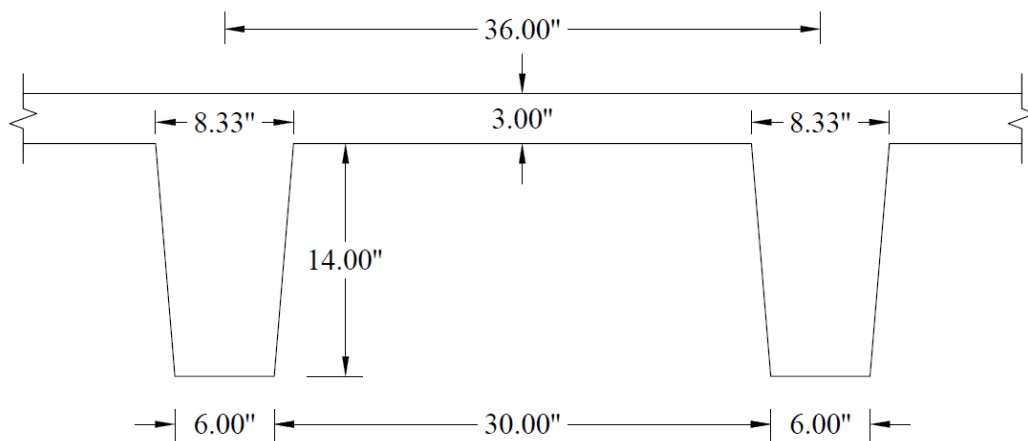


Figure 5 – Joists Dimensions

In waffle slabs a drop panel is automatically invoked to guarantee adequate two-way (punching) shear resistance at column supports. This is evident from the flat plate check conducted using 13 in. indicating insufficient punching shear capacity above. Check the drop panel dimensional limitations as follows:

- 1) The drop panel shall project below the slab at least one-fourth of the adjacent slab thickness.

**ACI 318-14 (8.2.4(a))**

Since the slab thickness ( $h_{MI}$  – calculated in [page 12](#) this document) is 12 in., the thickness of the drop panel should be at least:

$$h_{dp,min} = 0.25 \times h_{MI} = 0.25 \times 12 = 3.00 \text{ in.}$$

Drop panel depth are also controlled by the rib depth (both at the same level). For nominal lumber size (2x),

$$h_{dp} = h_{rib} = 14.00 \text{ in.} > h_{dp,min} = 3.00 \text{ in.}$$

The total thickness including the actual slab and the drop panel thickness ( $h$ ) =  $h_s + h_{dp} = 3.00 + 14.00 = 17.00 \text{ in.}$

- 2) The drop panel shall extend in each direction from the centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.

**ACI 318-14 (8.2.4(b))**

$$l_{1,dp,min} = \frac{1}{6} \times l_1 + \frac{1}{6} \times l_1 = \frac{1}{6} \times 33 + \frac{1}{6} \times 33 = 11.00 \text{ ft}$$

$$l_{2,dp,min} = \frac{1}{6} \times l_2 + \frac{1}{6} \times l_2 = \frac{1}{6} \times 33 + \frac{1}{6} \times 33 = 11.00 \text{ ft}$$

$$\text{Use } l_{1,dp} = l_{2,dp} = 12.00 \text{ ft} > l_{1,dp,min} = l_{2,dp,min} = 11.00 \text{ ft}$$

Based on the previous discussion, [Figure 6](#) shows the dimensions of the selected two-way joist system.

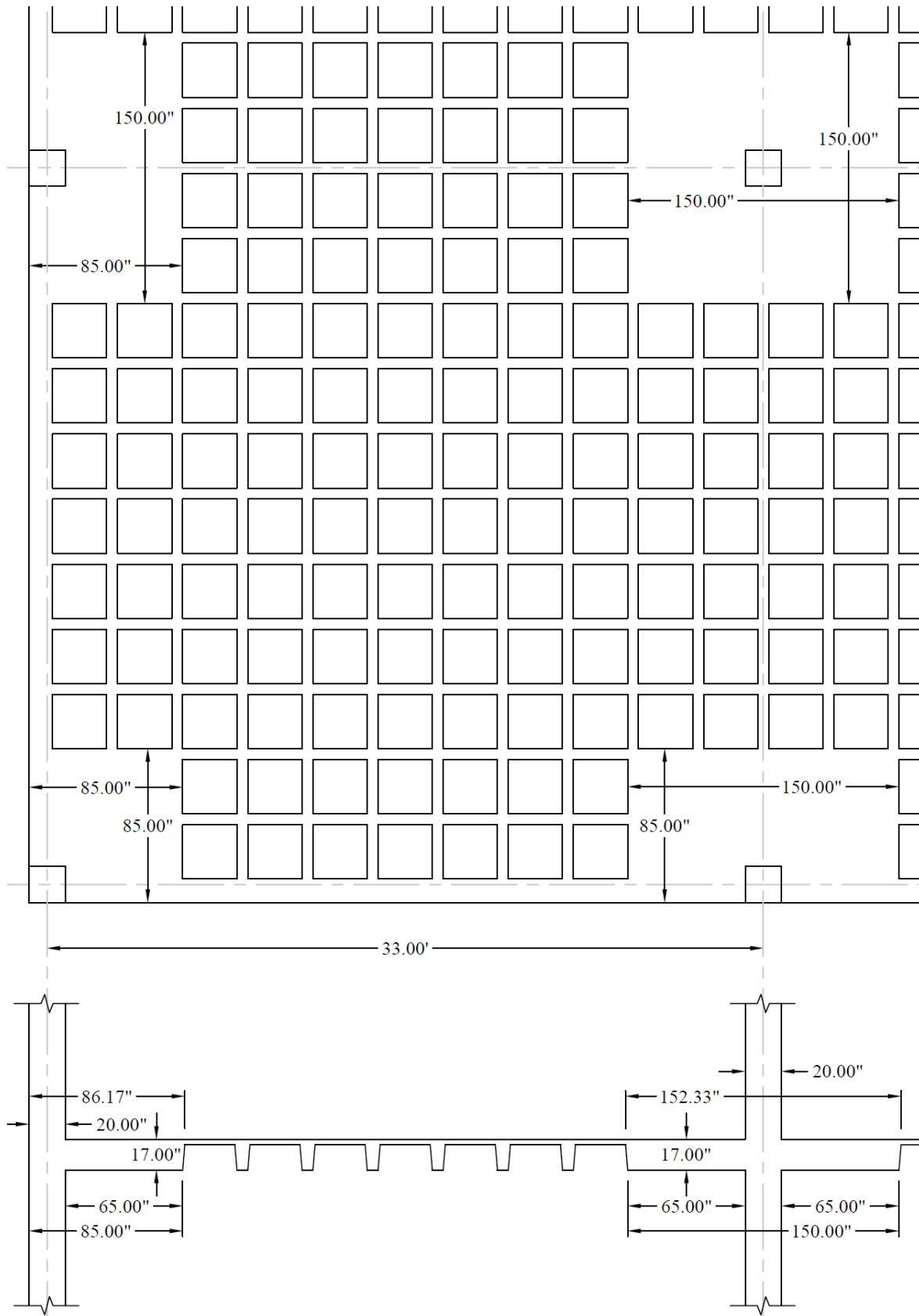


Figure 6 – Two-Way Joist (Waffle) Slab

## 2.2. Preliminary Two-Way Joist Slab (Waffle Slab)

For slabs with changes in thickness and subjected to bending in two directions, it is necessary to check shear at multiple sections as defined in the ACI 318-14. The critical sections shall be located with respect to:

1) Edges or corners of columns. ACI 318-14 (22.6.4.1(a))

2) Changes in slab thickness, such as edges of drop panels. ACI 318-14 (22.6.4.1(b))

### 2.2.1. Slab Minimum Thickness – Deflection

ACI 318-14 (8.3.1.1)

In lieu of detailed calculation for deflections, ACI 318 Code gives minimum slab thickness for two-way construction without interior beams in **Table 8.3.1.1**.

For this slab system, the minimum slab thicknesses per **ACI 318-14** are:

Exterior Panels:  $h_s = \frac{l_n}{33} = \frac{376}{33} = 11.39$  in. ACI 318-14 (Table 8.3.1.1)

But not less than 4 in. ACI 318-14 (8.3.1.1(b))

Interior Panels:  $h_s = \frac{l_n}{36} = \frac{376}{36} = 10.44$  in. ACI 318-14 (Table 8.3.1.1)

But not less than 4 in. ACI 318-14 (8.3.1.1(b))

Where  $l_n$  = length of clear span in the long direction =  $33 \times 12 - 20 = 376$  in.

For the purposes of analysis and design, the ribbed slab will be replaced with a solid slab of equivalent moment of inertia, weight, punching shear capacity, and one-way shear capacity.

The equivalent thickness based on moment of inertia is used to find slab stiffness considering the ribs in the direction of the analysis only. The ribs spanning in the transverse direction are not considered in the stiffness computations. This thickness,  $h_{MI}$ , is given by:

$$h_{MI} = \left( \frac{12 \times I_{rib}}{b_{rib}} \right)^{1/3} = \left( \frac{12 \times 5134.87}{36} \right)^{1/3} = 12.00 \text{ in.} \quad \text{spSlab Software Manual (Eq. 2-11)}$$

Where:

$I_{rib}$  = Moment of inertia of one joist section between centerlines of ribs (see [Figure 7](#)).

$b_{rib}$  = The center-to-center distance of two ribs (clear rib spacing plus rib width) (see [Figure 7](#)).

Since  $h_{MI} = 12.00$  in.  $> h_{min} = 11.39$  in., the deflection calculation can be neglected. However, the deflection calculation will be included in this example for comparison with the [spSlab](#) software results.

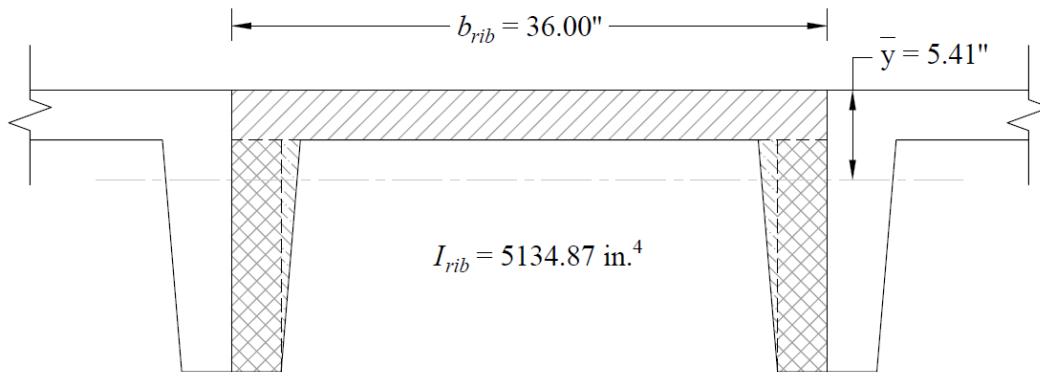
The drop panel depth for two-way joist (waffle) slab is set equal to the rib depth. The equivalent drop depth based on moment of inertia,  $d_{MI}$ , is given by:

$$d_{MI} = h_{MI} + h_{rib} = 12.00 + 5.00 = 17.00 \text{ in.}$$

*spSlab Software Manual (Eq. 2-12)*

Where:

$$h_{rib} = 3.00 + 14.00 - 12.00 = 5.00 \text{ in.}$$



**Figure 7 – Equivalent Thickness Based on Moment of Inertia**

Find system self-weight using the equivalent thickness based on the weight of individual components (see the [Figure 8](#)). This thickness,  $h_w$ , is given by:

$$h_w = \frac{V_{mod}}{A_{mod}} = \frac{66.037}{99.000} = 8.005 \text{ in.}$$

*spSlab Software Manual (Eq. 2-10)*

Where:

$V_{mod}$  = The Volume of one joist module (the transverse joists are included – 11 joists in the frame strip).

$$V_{mod} = V_{Longitudinal \text{ Joist}} + V_{Transverse \text{ Joists}} - V_{Intersection \text{ between Joists}}$$

$$V_{Longitudinal \text{ Joist}} = \left( \frac{6 + 8.33}{2} \times 14.00 + 3.00 \times 36.00 \right) \times (33.00 \times 12.00) = 47.74 \text{ ft}^3$$

$$V_{Transverse \text{ Joists}} = 11 \times \left( \frac{6 + 8.33}{2} \times 14.00 \right) \times 36.00 = 22.99 \text{ ft}^3$$

$$V_{Intersection \text{ between Joists}} = 11 \times \left( \frac{6^2 + 8.33^2}{2} \times 14.00 \right) = 4.70 \text{ ft}^3$$



$$V_{mod} = 47.74 + 22.99 - 4.70 = 66.04 \text{ ft}^3$$

$$A_{mod} = \text{The plan area of one joist module} = 33 \times 36/12 = 99.00 \text{ ft}^2.$$

$$\text{Self-weight for slab section without drop panel} = 150 \text{ pcf} \times 8.00 \text{ in.} / 12 = 100.057 \text{ psf}$$

$$\text{Self-weight for slab section with drop panel} = 150 \text{ pcf} \times (14.00 + 3.00 - 8.00) \text{ in.} / 12 = 112.443 \text{ psf}$$

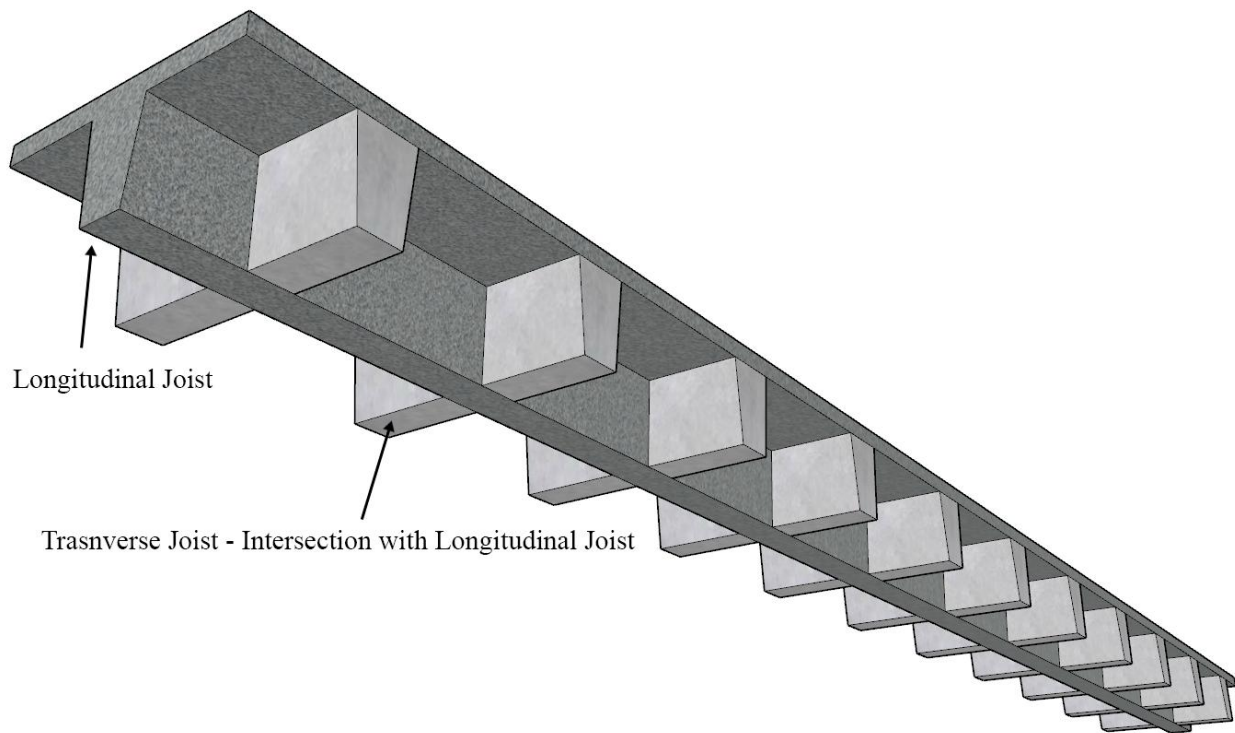


Figure 8 – Equivalent Thickness Based on the Weight of Individual Components

### 2.2.2. Slab Shear Strength – One Way Shear

For critical section at distance  $d$  from the edge of the column (slab section with drop panel):

Evaluate the average effective depth:

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 17.00 - 0.75 - 0.75 - \frac{0.75}{2} = 15.13 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 17.00 - 0.75 - \frac{0.75}{2} = 15.88 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{15.13 + 15.88}{2} = 15.50 \text{ in.}$$

Where:

$c_{clear} = 3/4 \text{ in.}$  for # 6 steel bar

ACI 318-14 (Table 20.6.1.3.1)

$d_b = 0.75 \text{ in.}$  for # 6 steel bar

$h_s = 17.00 \text{ in.}$  = The drop depth ( $d_{MI}$ )

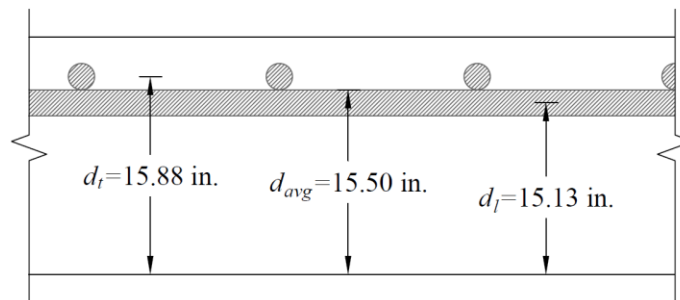


Figure 9 – Average Effective Depth for Slab Section with Drop Panel

Factored dead load,  $q_{Du} = 1.20 \times (150 \times 17.00 / 12 + 50.00) = 315.00 \text{ psf}$

Factored live load,  $q_{Lu} = 1.60 \times 100.00 = 160.00 \text{ psf}$

ACI 318-14 (5.3.1)

Total factored load,  $q_u = 315.00 + 160.00 = 475.00 \text{ psf}$

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior column

**ACI 318-14 (22.5)**

Consider a 12-in. wide strip. The critical section for one-way shear is located at a distance  $d$ , from the edge of the column (see [Figure 10](#))

Tributary area for one-way shear is:

$$A_{Tributary} = \left[ \frac{33}{2} - \frac{20}{2 \times 12} - \frac{15.50}{12} \right] \times \frac{12}{12} = 14.38 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.475 \times 14.38 = 6.83 \text{ kips}$$

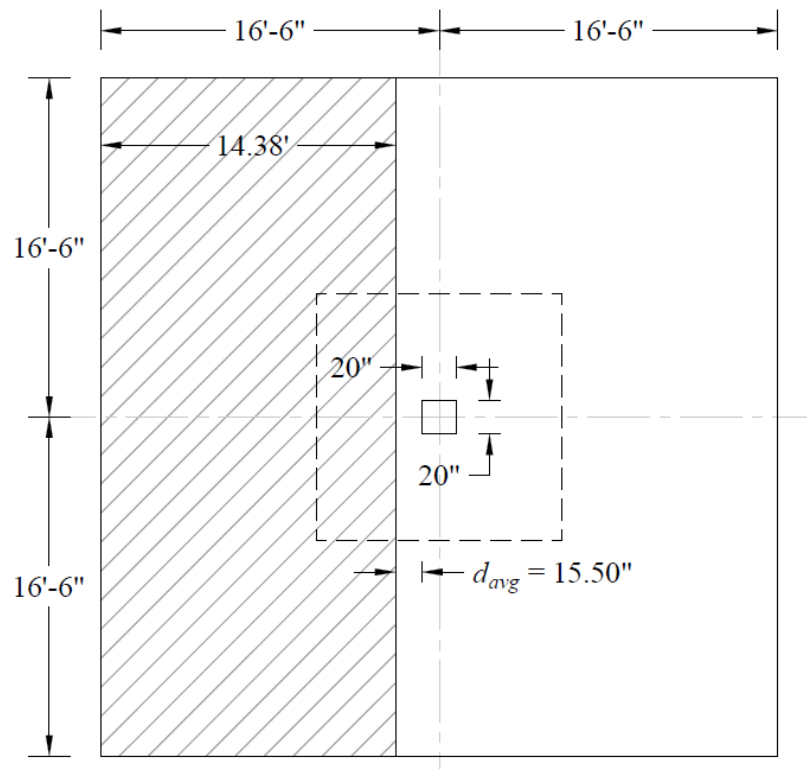
$$V_c = 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d$$

**ACI 318-14 (Eq. 22.5.5.1)**

Where  $\lambda = 1$  for normal weight concrete

$$\phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{5,000} \times 12 \times \frac{15.50}{1,000} = 19.73 \text{ kips} > V_u = 6.83 \text{ kips}$$

Slab thickness is adequate for one-way shear for the first critical section (from the edge of the column).



**Figure 10 – Critical Section at Distance  $d$  from the Edge of the Column for One-Way Shear**

For critical section at the edge of the drop panel (slab section without drop panel):

Evaluate the average effective depth:

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 12 - 0.75 - 0.75 - \frac{0.75}{2} = 10.13 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 12 - 0.75 - \frac{0.75}{2} = 10.88 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{10.13 + 10.88}{2} = 10.50 \text{ in.}$$

Where:

$c_{clear} = 3/4 \text{ in.}$  for # 6 steel bar

ACI 318-14 (Table 20.6.1.3.1)

$d_b = 0.75 \text{ in.}$  for # 6 steel bar

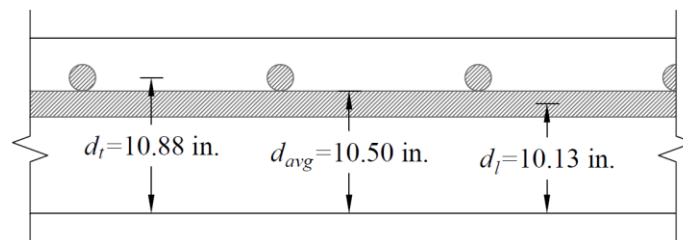


Figure 11 – Average Effective Depth for Slab Section without Drop Panel

Factored dead load,  $q_{Du} = 1.20 \times (100.057 + 50.00) = 180.07 \text{ psf}$

Factored live load,  $q_{Lu} = 1.60 \times 100.00 = 160.00 \text{ psf}$

ACI 318-14 (5.3.1)

Total factored load,  $q_u = 180.07 + 160.00 = 340.07 \text{ psf}$

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior drop panel

ACI 318-14 (22.5)

Consider a 12-in. wide strip. The critical section for one-way shear is located at the face of the solid head (see [Figure 12](#))

Tributary area for one-way shear is:

$$A_{Tributary} = \left[ \frac{33}{2} - \frac{12}{2} \right] \times \frac{12}{12} = 10.50 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.340 \times 10.50 = 3.57 \text{ kips}$$

$$V_c = 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d$$

ACI 318-14 (Eq. 22.5.5.1)

Where  $\lambda = 1$  for normal weight concrete

$$\phi V_c = 0.75 \times 2.0 \times 1.0 \times \sqrt{5,000} \times 12 \times \frac{10.50}{1,000} = 13.36 \text{ kips} > V_u = 3.57 \text{ kips}$$

Slab thickness of 12 in. is adequate for one-way shear for the second critical section (at the edge of the drop panel).

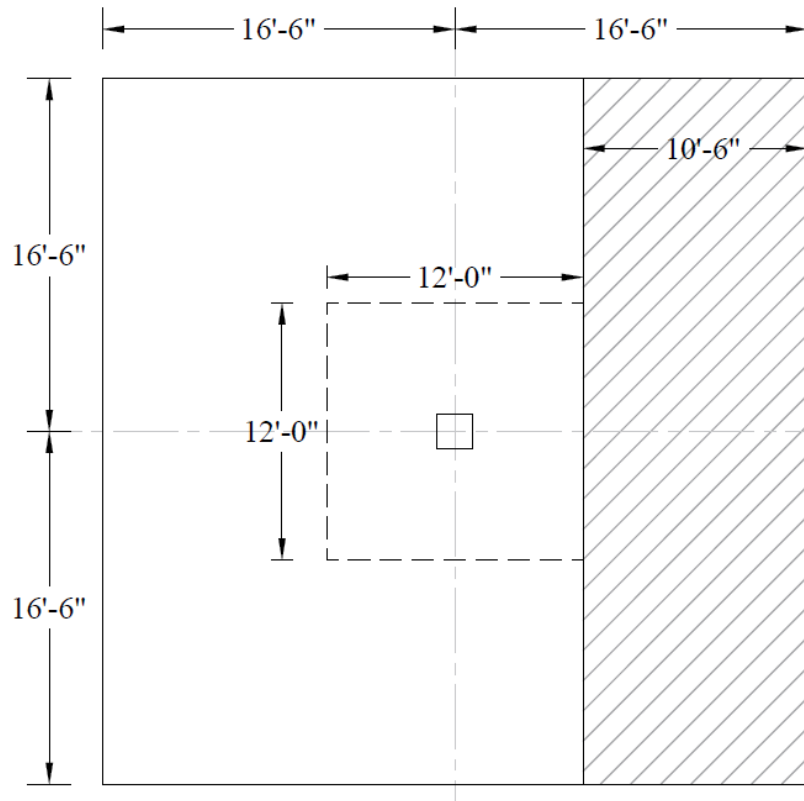


Figure 12 – Critical Section at the Face of the Drop Panel for One-Way Shear

### 2.2.3. Slab Shear Strength – Two-Way Shear

For critical section at distance  $d/2$  from the edge of the column (slab section with drop panel):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column ([Figure 13](#)):

Tributary area of two-way shear for the slab without the drop panel is:

$$A_{Tributary\_1} = (33 \times 33) - (12 \times 12)^2 = 945.00 \text{ ft}^2$$

Tributary area of two-way shear for the slab with the drop panel is:

$$A_{Tributary\_2} = (12 \times 12) - \left( \frac{20 + 15.50}{12} \right)^2 = 135.25 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.340 \times 945.00 + 0.475 \times 135.25 = 385.61 \text{ kips}$$

$$V_c = 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d \quad (\text{For square interior column})$$

ACI 318-14 (Table 22.6.5.2(a))

$$V_c = 4 \times 1.0 \times \sqrt{5,000} \times (4 \times (20 + 15.50)) \times \frac{15.50}{1,000} = 622.54 \text{ kips}$$

$$\phi V_c = 0.75 \times 622.54 = 466.90 \text{ kips} > V_u = 385.61 \text{ kips}$$

Slab thickness is adequate for two-way shear for the first critical section (from the edge of the column).

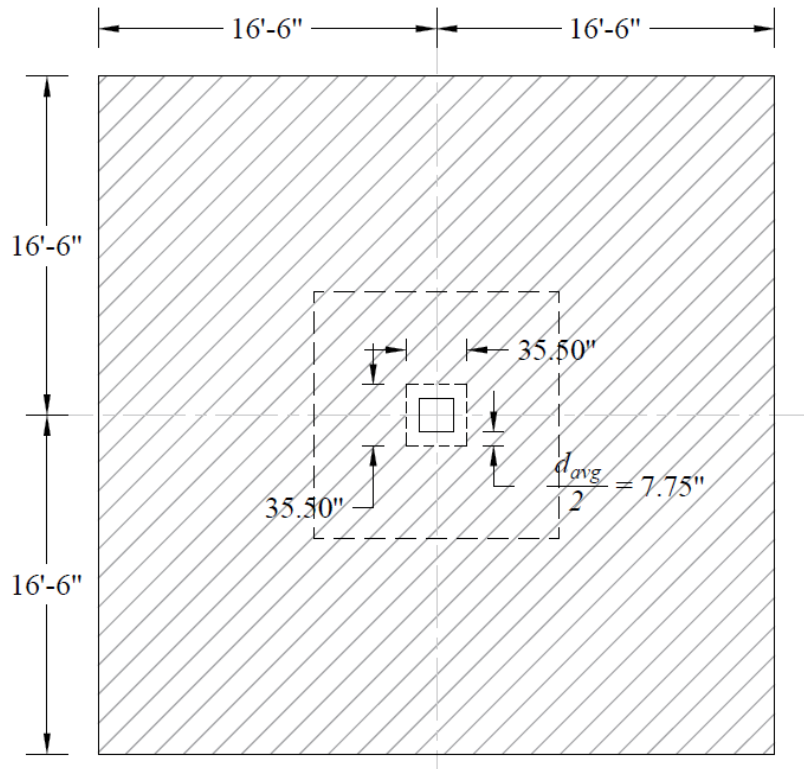


Figure 13 – Critical Section at  $d/2$  from the Edge of the Column for Two-Way Shear

For critical section at the edge of the drop panel (slab section without drop panel):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior drop panel ([Figure 14](#)):

Tributary area for two-way shear is:

$$A_{Tributary} = (33 \times 33) - \left(12 + \frac{10.50}{12}\right)^2 = 923.23 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.340 \times 923.23 = 313.96 \text{ kips}$$

$$V_c = 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d \quad (\text{For square interior column})$$

ACI 318-14 (Table 22.6.5.2(a))

$$V_c = 4 \times 1.0 \times \sqrt{5,000} \times (4 \times (144 + 10.50)) \times \frac{10.50}{1,000} = 1,835.37 \text{ kips}$$

$$\phi V_c = 0.75 \times 1,835.37 = 1,376.52 \text{ kips} > V_u = 313.96 \text{ kips}$$

Slab thickness of 12 in. is adequate for two-way shear for the second critical section (from the edge of the drop panel).

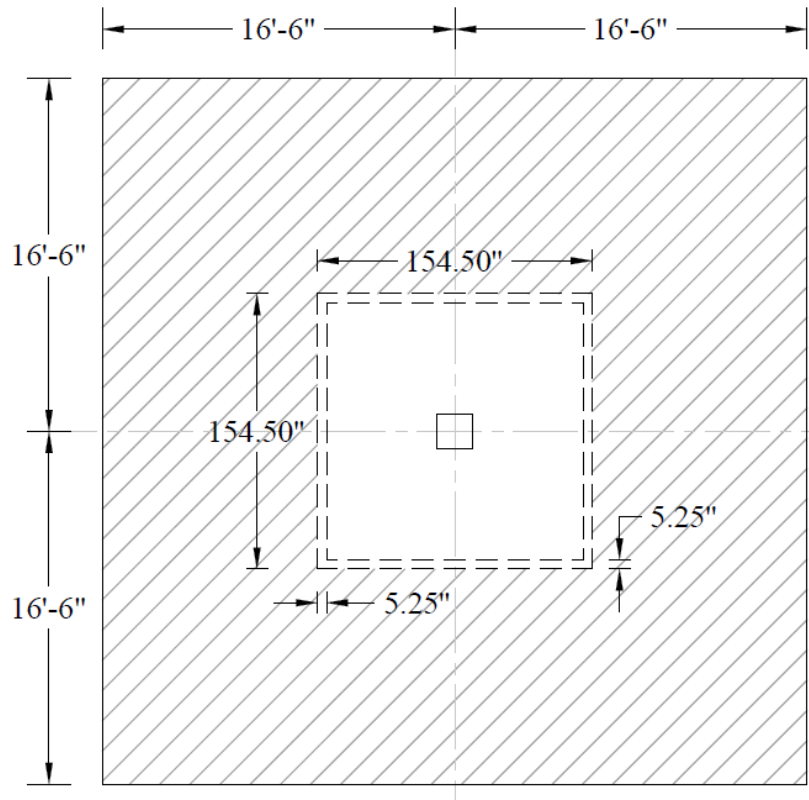


Figure 14 – Critical Section at  $d/2$  from the Edge of the Drop Panel for Two-Way Shear

#### 2.2.4. Column Dimensions - Axial Load

Check the adequacy of column dimensions for axial load:

For live load, superimposed dead load, and self-weight of the slab around an interior column:

$$q_u = 340.07 \text{ psf (see [on page 17](#))}$$

$$A_{Tributary} = 33 \times 33 = 1,089.00 \text{ ft}^2$$

For self-weight of additional slab thickness due to the presence of the drop panel around an interior column:

$$q_u = 475.00 - 340.07 = 134.93 \text{ psf (see [on page 15](#) and [on page 17](#))}$$

$$A_{Tributary} = 12 \times 12 = 144.00 \text{ ft}^2$$

Assuming four story building

$$P_u = n \times q_u \times A_{Tributary} = 4 \times (0.340 \times 1,089.00 + 0.135 \times 144.00) = 1,559.06 \text{ kips}$$

Assume 20 in. square column with 12 – No. 11 vertical bars with design axial strength,  $\phi P_{n,max}$  of

$$\phi P_{n,max} = 0.80 \times \phi \times (0.85 \times f'_c \times (A_g - A_{st}) + f_y \times A_{st}) \quad \text{ACI 318-14 (22.4.2)}$$

$$\phi P_{n,max} = 0.80 \times 0.65 \times (0.85 \times 6,000 \times (20 \times 20 - 12 \times 1.56) + 60,000 \times 12 \times 1.56) = 1,595.22 \text{ kips}$$

$$\phi P_{n,max} = 1,595.22 \text{ kips} > P_u = 1,559.06 \text{ kips}$$

Column dimensions of 20 in.  $\times$  20 in. are adequate for axial load.



### 3. Flexural Analysis and Design

ACI 318 states that a slab system shall be designed by any procedure satisfying equilibrium and geometric compatibility, provided that strength and serviceability criteria are satisfied. Distinction of two-systems from one-way systems is given by ACI 318-14 (R8.10.2.3 & R8.3.1.2).

ACI 318 permits the use of Direct Design Method (DDM) and Equivalent Frame Method (EFM) for the gravity load analysis of orthogonal frames and is applicable to flat plates, flat slabs, and slabs with beams. The following sections outline the solution per EFM and [spSlab](#) software. For the solution per DDM, check the “[Two-Way Flat Plate Concrete Floor System Analysis and Design \(ACI 318-14\)](#)” Design Example.

#### 3.1. Equivalent Frame Method (EFM)

EFM is the most comprehensive and detailed procedure provided by the ACI 318 for the analysis and design of two-way slab systems where the structure is modeled by a series of equivalent frames (interior and exterior) on column lines taken longitudinally and transversely through the building.

The equivalent frame consists of three parts (for a detailed discussion of this method, refer to “[Two-Way Flat Plate Concrete Floor System Analysis and Design \(ACI 318-14\)](#)”):

- 1) Horizontal slab-beam strip.
- 2) Columns or other vertical supporting members.
- 3) Elements of the structure (Torsional members) that provide moment transfer between the horizontal and vertical members.

##### 3.1.1. Limitations for Use of Equivalent Frame Method

In EFM, live load shall be arranged in accordance with 6.4.3 which requires slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ACI 318-14 (8.11.1.2 & 6.4.3)

Complete analysis must include representative interior and exterior equivalent frames in both the longitudinal and transverse directions of the floor. ACI 318-14 (8.11.2.1)

Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ACI 318-14 (8.10.2.3)

### 3.1.2. Frame Members of Equivalent Frame

Determine moment distribution factors and fixed-end moments for the equivalent frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors  $k$ , carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at Appendix 20A of PCA Notes on ACI 318-11. These calculations are shown below.

a) Flexural stiffness of slab-beams at both ends,  $K_{sb}$ .

$$\frac{c_{N1}}{l_1} = \frac{20}{(33 \times 12)} = 0.051, \quad \frac{c_{N2}}{l_2} = \frac{20}{(33 \times 12)} = 0.051$$

Slab thickness =  $h = h_{MI} = 12.00$  in. and drop thickness =  $d_{MI} - h_{MI} = 17.00 - 12.00 = 5.00$  in.

$$\frac{\text{drop thickness}}{\text{slab thickness}} = \frac{5.00}{12.00} = 0.4167$$

For  $c_{F1} = c_{F2}$ , stiffness factors,  $k_{NF} = k_{FN} = 5.541$

PCA Notes on ACI 318-11 (Table A2 & A3)

$$\text{Thus, } K_{sb} = k_{NF} \times \frac{E_{cs} \times I_s}{l_1} = 5.541 \times \frac{E_{cs} \times I_s}{l_1}$$

PCA Notes on ACI 318-11 (Table A2 & A3)

$$K_{sb} = 5.541 \times \frac{4,287 \times 10^3 \times 57,024.00}{396.00} = 3,420,614,448 \text{ in.-lb}$$

$$\text{Where, } I_s = \frac{l_2 \times h^3}{12} = \frac{396 \times (12)^3}{12} = 57,024.00 \text{ in.}^4$$

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$

ACI 318-14 (19.2.2.1.a)

Carry-over factor  $COF = 0.576$

PCA Notes on ACI 318-11 (Table A2 & A3)

$$\text{Fixed-end moment } FEM = \sum_{i=1}^n m_{NFi} \times w_i \times l_i^2$$

PCA Notes on ACI 318-11 (Table A2 & A3)

Uniform load fixed end moment coefficient,  $m_{NF1} = 0.0913$

Fixed end moment coefficient for (b-a) = 0.2 when a = 0,  $m_{NF2} = 0.0162$

Fixed end moment coefficient for (b-a) = 0.2 when a = 0.8,  $m_{NF3} = 0.0020$

b) Flexural stiffness of column members at both ends,  $K_c$ .

Referring to Table A7, Appendix 20A,

For the Bottom Column (Below):

$$t_a = 3.00 / 2 + 14.00 = 15.50 \text{ in.}$$

$$t_b = 3.00 / 2 = 1.50 \text{ in.}$$

$$H = 13 \text{ ft} = 156.00 \text{ in.}$$

$$H_c = H - t_a - t_b = 156.00 - 15.50 - 1.50 = 139.00 \text{ in.}$$

$$\frac{t_a}{t_b} = \frac{15.50}{1.50} = 10.333$$

$$\frac{H}{H_c} = \frac{156.00}{139.00} = 1.122$$

Thus,  $k_{AB} = 6.178$  and  $C_{AB} = 0.500$  by interpolation.

$$K_{c, \text{bottom}} = \frac{6.178 \times E_{cc} \times I_c}{l_c}$$

PCA Notes on ACI 318-11 (Table A7)

$$K_{c, \text{bottom}} = 6.178 \times \frac{4,696 \times 10^3 \times 13,333.33}{156} = 2,479,648,547 \text{ in.-lb}$$

$$\text{Where, } I_c = \frac{c^4}{12} = \frac{(20)^4}{12} = 13,333.33 \text{ in.}^4$$

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{6,000} = 4,696 \times 10^3 \text{ psi}$$

ACI 318-14 (19.2.2.1.a)

$$l_c = 13 \text{ ft} = 156 \text{ in.}$$

For the Top Column (Above):

$$\frac{t_b}{t_a} = \frac{1.50}{15.50} = 0.097$$

$$\frac{H}{H_c} = \frac{156.00}{139.00} = 1.122$$

Thus,  $k_{BA} = 4.624$  and  $C_{BA} = 0.667$  by interpolation.

$$K_c = \frac{4.624 \times E_{cc} \times I_c}{l_c}$$

PCA Notes on ACI 318-11 (Table A7)

$$K_{c, \text{top}} = 4.624 \times \frac{4,696 \times 10^3 \times 13,333.33}{156} = 1,855,923,419 \text{ in.-lb}$$

c) Torsional stiffness of torsional members,  $K_t$ .

$$K_t = \frac{9 \times E_{cs} \times C}{l_2 \times \left(1 - \frac{c_2}{l_2}\right)^3} \quad \text{ACI 318-14 (R.8.11.5)}$$

$$K_t = \frac{9 \times 4,287 \times 10^3 \times 15,213.92}{33 \times 12 \times \left(1 - \frac{20}{33 \times 12}\right)^3} = 1,731,665,695 \text{ in.-lb}$$

$$\text{Where } C = \sum \left(1 - 0.63 \times \frac{x}{y}\right) \times \left(\frac{x^3 \times y}{3}\right) \quad \text{ACI 318-14 (Eq. 8.10.5.2b)}$$

$$C = \left(1 - 0.63 \times \frac{17.00}{20.00}\right) \times \left(\frac{17.00^3 \times 20.00}{3}\right) = 15,213.92 \text{ in.}^4$$

$$c_2 = 20 \text{ in.}, l_2 = 33 \text{ ft} = 396 \text{ in.}$$

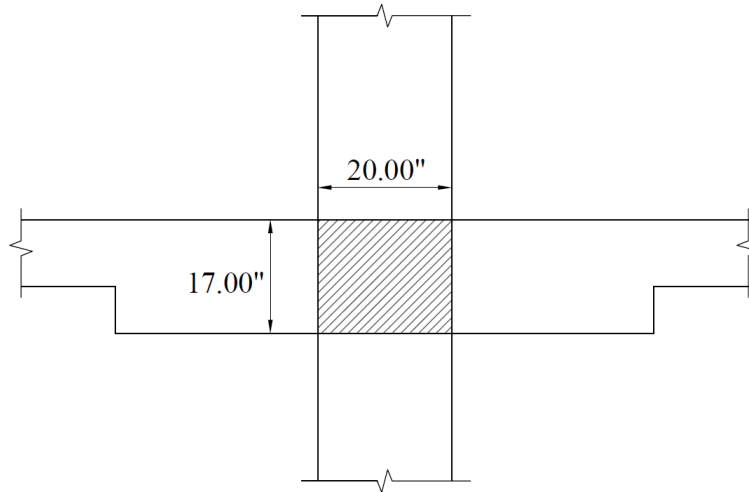


Figure 15 – Torsional Member

d) Equivalent column stiffness,  $K_{ec}$ .

$$K_{ec} = \frac{\sum K_c \times \sum K_t}{\sum K_c + \sum K_t}$$

$$K_{ec} = \frac{(2,479.65 + 1,855.92) \times (2 \times 1,731.67)}{[(2,479.65 + 1,855.92) + (2 \times 1,731.67)]} \times 10^6 = 1,925,337,678 \text{ in.-lb}$$

Where  $\sum K_t$  is for two torsional members one on each side of the column, and  $\sum K_c$  is for the upper and lower columns at the slab-beam joint of an intermediate floor.

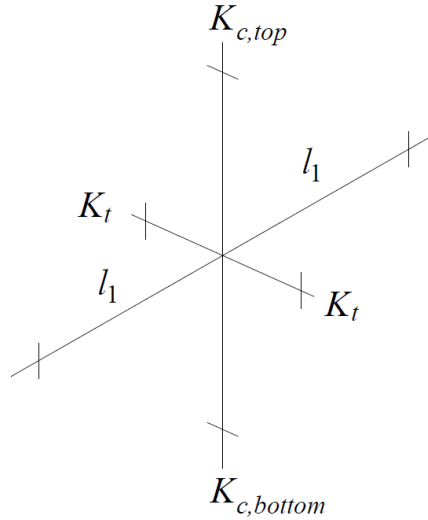


Figure 16 – Column and Edge of Slab

e) Slab-beam joint distribution factors,  $DF$ .

At exterior joint

$$DF = \frac{3,420.61}{(3,420.61 + 1,925.34)} = 0.640$$

At interior joint

$$DF = \frac{3,420.61}{(3,420.61 + 3,420.61 + 1,925.34)} = 0.390$$

COF for slab-beam = 0.576

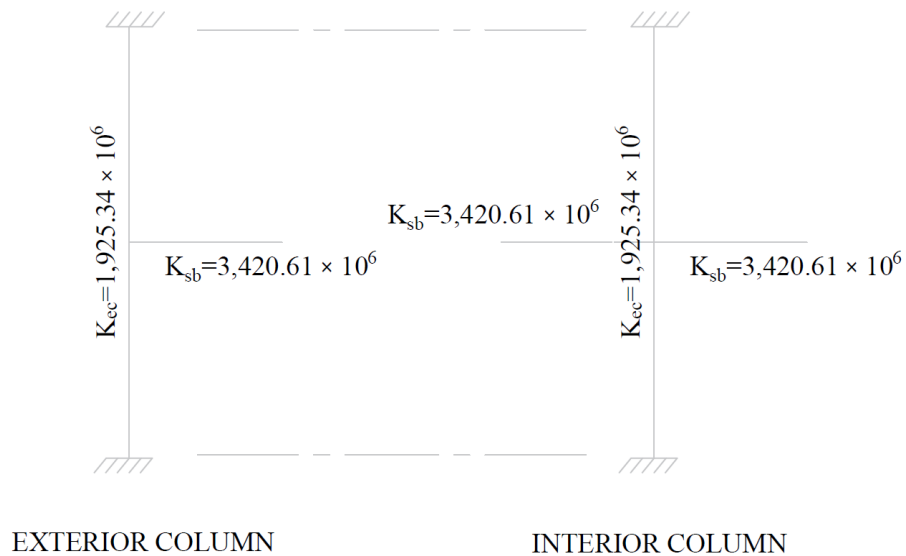


Figure 17 – Slab and Column Stiffness

### 3.1.3. Equivalent Frame Analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. ACI 318-14 (6.4.3.2)

$$\frac{L}{D} = \frac{100}{(100 + 50)} = 0.67 < \frac{3}{4}$$

a) Factored load and Fixed-End Moments (FEM's).

For slab:

$$\text{Factored dead load, } q_{Du} = 1.20 \times (100.00 + 50.00) = 180.00 \text{ psf}$$

$$\text{Factored live load, } q_{Lu} = 1.60 \times 100.00 = 160.00 \text{ psf} \quad \text{ACI 318-14 (5.3.1)}$$

$$\text{Total factored load, } q_u = q_{Du} + q_{Lu} = 340.00 \text{ psf}$$

For drop panels:

$$\text{Factored dead load, } q_{Du} = 1.20 \times (150.00 \times 9.00 / 12) = 135.00 \text{ psf}$$

$$\text{Factored live load, } q_{Lu} = 1.60 \times 0.00 = 0.00 \text{ psf} \quad \text{ACI 318-14 (5.3.1)}$$

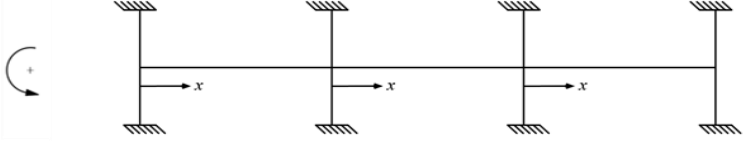
$$\text{Total factored load, } q_u = q_{Du} + q_{Lu} = 135.00 \text{ psf}$$

$$\text{Fixed-end moment } FEM = \sum_{i=1}^n m_{NFi} \times w_i \times l_i^2 \quad \text{PCA Notes on ACI 318-11 (Table A2 \& A3)}$$

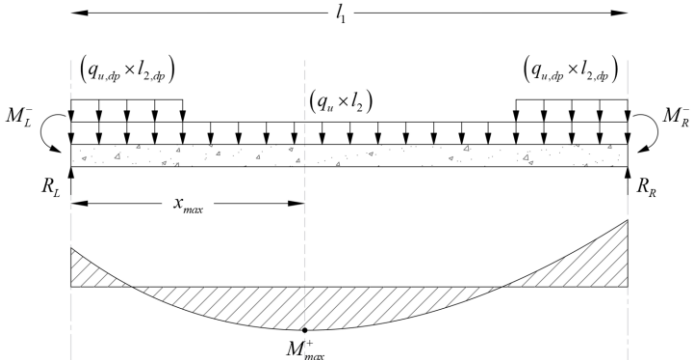
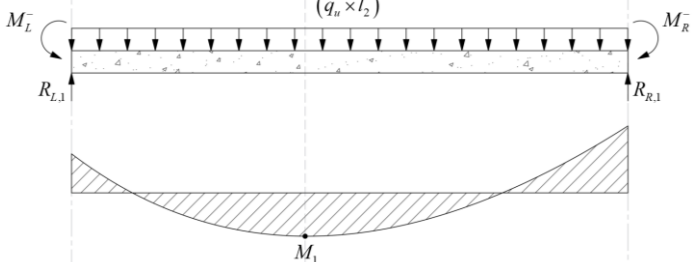
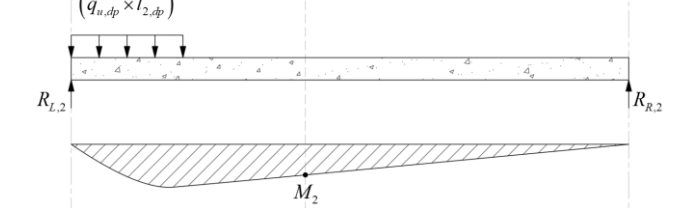
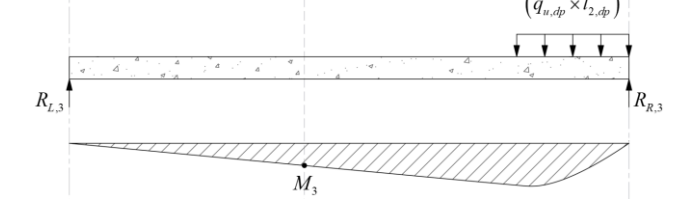
$$FEM = 0.0913 \times 0.340 \times 33 \times 33^2 + 0.0162 \times 0.135 \times 12 \times 33^2 + 0.0020 \times 0.135 \times 12 \times 33^2$$

$$FEM = 1,147.66 \text{ ft-kips}$$

- b) Moment distribution. Computations are shown in the [Table below](#). Counterclockwise rotational moments acting on the member ends are taken as positive.

Table 1 - Moment Distribution for Equivalent Frame						
						
Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.640	0.390	0.390	0.390	0.390	0.640
COF	0.576	0.576	0.576	0.576	0.576	0.576
FEM	1,147.66	-1,147.66	1,147.66	-1,147.66	1,147.66	-1,147.66
Dist	-734.33	0	0	0	0	734.33
CO	0	-422.89	0	0	422.89	0
Dist	0	165.01	165.01	-165.01	-165.01	0
CO	95.03	0	-95.03	95.03	0	-95.03
Dist	-60.80	37.08	37.08	-37.08	-37.08	60.80
CO	21.35	-35.01	-21.35	21.35	35.01	-21.35
Dist	-13.66	21.99	21.99	-21.99	-21.99	13.66
CO	12.67	-7.87	-12.67	12.67	7.87	-12.67
Dist	-8.10	8.01	8.01	-8.01	-8.01	8.10
CO	4.61	-4.67	-4.61	4.61	4.67	-4.61
Dist	-2.95	3.62	3.62	-3.62	-3.62	2.95
CO	2.09	-1.70	-2.09	2.09	1.70	-2.09
Dist	-1.33	1.48	1.48	-1.48	-1.48	1.33
CO	0.85	-0.77	-0.85	0.85	0.77	-0.85
Dist	-0.54	0.63	0.63	-0.63	-0.63	0.54
CO	0.36	-0.31	-0.36	0.36	0.31	-0.36
Dist	-0.23	0.26	0.26	-0.26	-0.26	0.23
CO	0.15	-0.13	-0.15	0.15	0.13	-0.15
Dist	-0.10	0.11	0.11	-0.11	-0.11	0.10
CO	0.06	-0.06	-0.06	0.06	0.06	-0.06
Dist	-0.04	0.05	0.05	-0.05	-0.05	0.04
CO	0.03	-0.02	-0.03	0.03	0.02	-0.03
Dist	-0.02	0.02	0.02	-0.02	-0.02	0.02
M <sub>max</sub>	462.75	-1,382.84	1,248.72	-1,248.72	1,382.84	-462.75
V	166.97	-222.73	194.85	-194.85	222.73	-166.97
x <sub>max</sub>	14.02		16.50		18.98	
M <sup>+</sup> <sub>max</sub>	668.33		307.76		668.33	

Maximum positive span moments are determined from the following equations:

$M_{max}^+ = M_1 + M_2 + M_3$ $R_L = R_{L,1} + R_{L,2} + R_{L,3}$ $R_R = R_{R,1} + R_{R,2} + R_{R,3}$ $x_{max} = \frac{l_1}{2} + \frac{M_L^- - M_R^-}{(q_u \times l_2) \times l_1}$	
$M_1 = \frac{(q_u \times l_2) \times l_1^2}{8} - \frac{M_L^- + M_R^-}{2} + \frac{(M_L^- - M_R^-)^2}{2 \times (q_u \times l_2) \times l_1^2}$ $R_{L,1} = \frac{(q_u \times l_2) \times l_1}{2} + \frac{M_L^- - M_R^-}{l_1}$ $R_{R,1} = \frac{(q_u \times l_2) \times l_1}{2} - \frac{M_L^- - M_R^-}{l_1}$	
$M_2 = R_{R,2} \times (l_1 - x_{max})$ $R_{L,2} = \frac{(q_{u,dp} \times l_{2,dp}) \times \frac{l_{1,dp}}{2}}{2 \times l_1} \times \left( 2 \times \frac{9 \times l_{1,dp}}{11} + \frac{l_{1,dp}}{2} \right)$ $R_{R,2} = \frac{(q_{u,dp} \times l_{2,dp}) \times \frac{l_{1,dp}}{2}}{2 \times l_1} \times \left( \frac{l_{1,dp}}{2} \right)$	
$M_3 = R_{L,3} \times x_{max}$ $R_{L,3} = \frac{(q_{u,dp} \times l_{2,dp}) \times \frac{l_{1,dp}}{2}}{2 \times l_1} \times \left( \frac{l_{1,dp}}{2} \right)$ $R_{R,3} = \frac{(q_{u,dp} \times l_{2,dp}) \times \frac{l_{1,dp}}{2}}{2 \times l_1} \times \left( 2 \times \frac{9 \times l_{1,dp}}{11} + \frac{l_{1,dp}}{2} \right)$	



Maximum positive moment in spans 1-2 and 3-4:

$$M_{max}^+ = M_1 + M_2 + M_3 = 639.17 + 16.78 + 12.38 = 668.33 \text{ ft-kips}$$

$$V_L = R_L = R_{L,1} + R_{L,2} + R_{L,3} = 157.25 + 8.84 + 0.88 = 166.97 \text{ kips}$$

$$V_R = R_R = R_{R,1} + R_{R,2} + R_{R,3} = 213.01 + 0.88 + 8.84 = 222.73 \text{ kips}$$

$$x_{max} = \frac{33}{2} + \frac{(462.75 - 1,382.84)}{(0.340 \times 33) \times 33} = 14.02 \text{ ft}$$

Where:

$$M_L^- = 462.75 \text{ ft-kips}$$

$$M_R^- = 1,382.84 \text{ ft-kips}$$

$$M_1 = \frac{(0.340 \times 33) \times 33^2}{8} - \frac{462.75 + 1,382.84}{2} + \frac{(462.75 - 1,382.84)^2}{2 \times (0.340 \times 33) \times 33^2} = 639.17 \text{ ft-kips}$$

$$M_2 = \frac{(0.135 \times 12) \times \left(\frac{12}{2}\right)}{2 \times 33} \times \left(\frac{12}{2}\right) \times (33 - 14.02) = 16.78 \text{ ft-kips}$$

$$M_3 = \frac{(0.135 \times 12) \times \left(\frac{12}{2}\right)}{2 \times 33} \times \left(\frac{12}{2}\right) \times 14.02 = 12.38 \text{ ft-kips}$$

And:

$$R_{L,1} = \frac{(0.340 \times 33) \times 33}{2} + \frac{(462.75 - 1,382.84)}{33} = 157.25 \text{ kips}$$

$$R_{L,2} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times \left(2 \times \frac{9 \times 33}{11} + 6\right) = 8.84 \text{ kips}$$

$$R_{L,3} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times (6) = 0.88 \text{ kips}$$

$$R_{R,1} = \frac{(0.340 \times 33) \times 33}{2} - \frac{(462.75 - 1,382.84)}{33} = 213.01 \text{ kips}$$

$$R_{R,2} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times (6) = 0.88 \text{ kips}$$

$$R_{R,3} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times \left(2 \times \frac{9 \times 33}{11} + 6\right) = 8.84 \text{ kips}$$

Maximum positive moment in span 2-3:

$$M_{max}^+ = M_1 + M_2 + M_3 = 278.60 + 14.58 + 14.58 = 307.76 \text{ ft-kips}$$

$$V_L = R_L = R_{L,1} + R_{L,2} + R_{L,3} = 185.13 + 8.84 + 0.88 = 194.85 \text{ kips}$$

$$V_R = R_R = R_{R,1} + R_{R,2} + R_{R,3} = 185.13 + 0.88 + 8.84 = 194.85 \text{ kips}$$

$$x_{max} = \frac{33}{2} + \frac{(1,248.72 - 1,248.72)}{(0.340 \times 33) \times 33} = 16.50 \text{ ft}$$

Where:

$$M_L^- = 1,248.72 \text{ ft-kips}$$

$$M_R^- = 1,248.72 \text{ ft-kips}$$

$$M_1 = \frac{(0.340 \times 33) \times 33^2}{8} - \frac{1,248.72 + 1,248.72}{2} + \frac{(1,248.72 - 1,248.72)^2}{2 \times (0.340 \times 33) \times 33^2} = 278.60 \text{ ft-kips}$$

$$M_2 = \frac{(0.135 \times 12) \times \left(\frac{12}{2}\right)}{2 \times 33} \times \left(\frac{12}{2}\right) \times (33 - 16.50) = 14.58 \text{ ft-kips}$$

$$M_3 = \frac{(0.135 \times 12) \times \left(\frac{12}{2}\right)}{2 \times 33} \times \left(\frac{12}{2}\right) \times 16.50 = 14.58 \text{ ft-kips}$$

and:

$$R_{L,1} = \frac{(0.340 \times 33) \times 33}{2} + \frac{(1,248.72 - 1,248.72)}{33} = 185.13 \text{ kips}$$

$$R_{L,2} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times \left(2 \times \frac{9 \times 33}{11} + 6\right) = 8.84 \text{ kips}$$

$$R_{L,3} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times (6) = 0.88 \text{ kips}$$

$$R_{R,1} = \frac{(0.340 \times 33) \times 33}{2} - \frac{(1,248.72 - 1,248.72)}{33} = 185.13 \text{ kips}$$

$$R_{R,2} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times (6) = 0.88 \text{ kips}$$

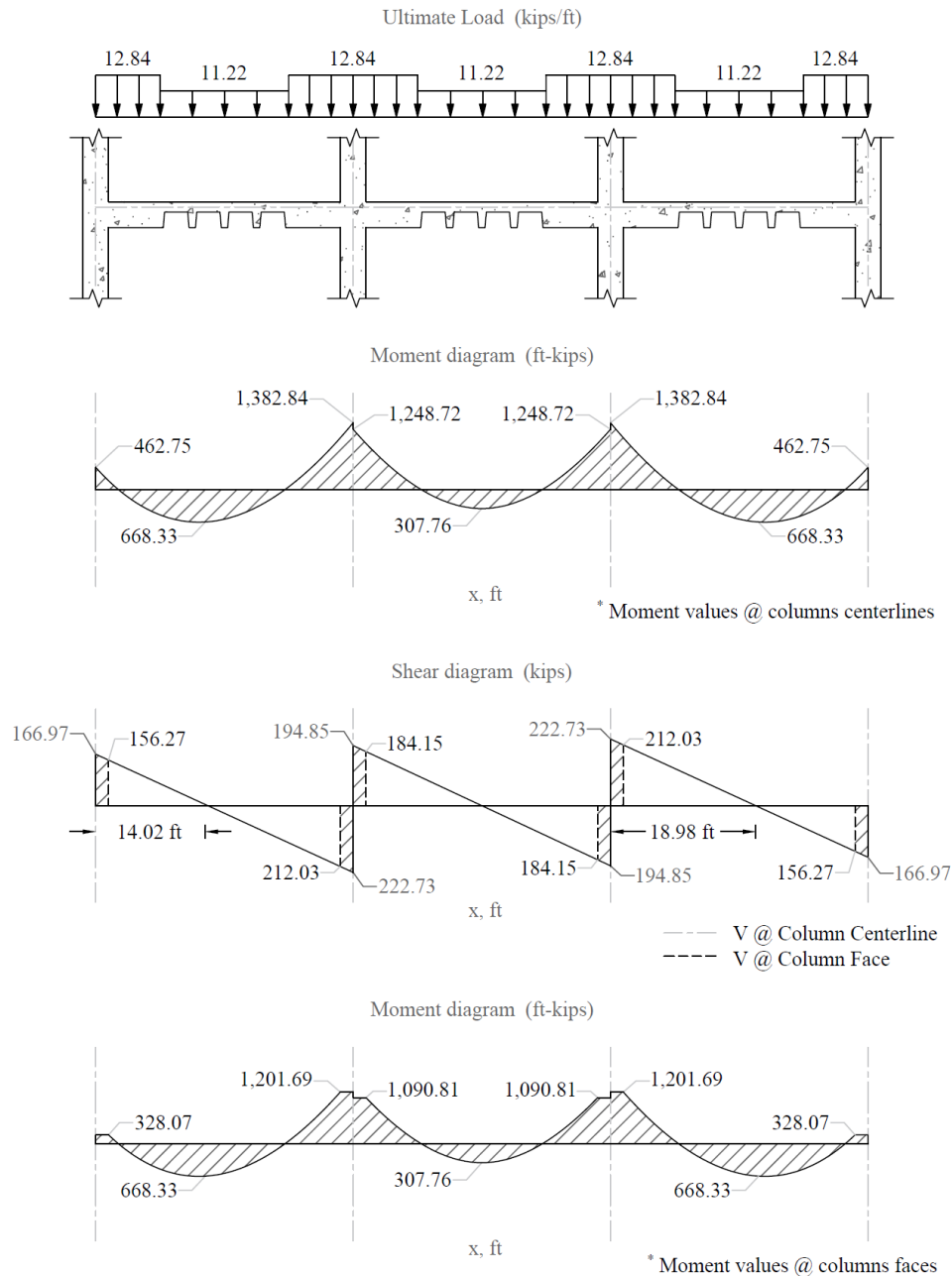
$$R_{R,3} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times \left(2 \times \frac{9 \times 33}{11} + 6\right) = 8.84 \text{ kips}$$

### 3.1.4. Factored Moments Used for Design

Positive and negative factored moments for the slab system in the direction of analysis are plotted in [Figure 18](#). The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than  $0.175 \times l_1$  from the centers of supports.

**ACI 318-14 (8.11.6.1)**

$$\frac{20 \text{ in.}}{12 \times 2} = 0.83 \text{ ft} < 0.175 \times 33 = 5.78 \text{ ft (use face of support location)}$$



**Figure 18 – Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)**

### 3.1.5. Factored Moments in Slab-Beam Strip

- a) Check whether the moments calculated above can take advantage of the reduction permitted by ACI 318-14 (8.11.6.5):

If the slab system analyzed using EFM within the limitations of ACI 318-14 (8.10.2), it is permitted by the ACI code to reduce the calculated moments obtained from EFM in such proportion that the absolute sum of the positive and average negative design moments need not exceed the total static moment  $M_o$  given by Equation 8.10.3.2 in the ACI 318-14.

#### Check Applicability of Direct Design Method:

1. There is a minimum of three continuous spans in each direction. ACI 318-14 (8.10.2.1)
2. Successive span lengths are equal. ACI 318-14 (8.10.2.2)
3. Long-to-Short ratio is  $33/33 = 1.00 < 2.00$ . ACI 318-14 (8.10.2.3)
4. Column are not offset. ACI 318-14 (8.10.2.4)
5. Loads are gravity and uniformly distributed with service live-to-dead ratio of  $0.67 < 2.00$   
(Note: The self-weight of the drop panels is not uniformly distributed entirely along the span. However, the variation in load magnitude is small). ACI 318-14 (8.10.2.5 and 6)
6. Check relative stiffness for slab panel. ACI 318-14 (8.10.2.7)

Slab system is without beams and this requirement is not applicable.

All limitation of ACI 318-14 (8.10.2) are satisfied and the provisions of ACI 318-14 (8.11.6.5) may be applied:

$$M_o = \frac{q_u \times l_2 \times l_n^2}{8} = \frac{0.340 \times 33 \times (33 - 20/12)^2}{8} = 1,376.94 \text{ ft-kips} \quad \text{ACI 318-14 (Eq. 8.10.3.2)}$$

$$\text{End spans: } 668.33 + \frac{462.75 + 1,382.84}{2} = 1,591.13 \text{ ft-kips} > M_o$$

$$\text{Interior span: } 307.76 + \frac{1,248.72 + 1,248.72}{2} = 1,556.48 \text{ ft-kips} > M_o$$

To illustrate proper procedure, the interior span factored moments may be reduced as follows:

$$\text{Permissible reduction} = \frac{1,376.94}{1,556.48} = 0.885$$

$$\text{Adjusted negative design moment} = 1,248.72 \times 0.885 = 1,104.68 \text{ ft-kips}$$

$$\text{Adjusted positive design moment} = 307.76 \times 0.885 = 272.26 \text{ ft-kips}$$

$$M_o = 272.26 + \frac{1,104.68 + 1,104.68}{2} = 1,376.94 \text{ ft-kips}$$

ACI 318 allows the reduction of the moment values based on the previous procedure. Since the drop panels may cause gravity loads not to be uniform (Check limitation #5 and [Figure 18](#)), the moment values obtained from EFM will be used for comparison reasons.

b) Distribute factored moments to column and middle strips:

After the negative and positive moments have been determined for the slab-beam strip, the ACI code permits the distribution of the moments at critical sections to the column strips, beams (if any), and middle strips in accordance with the DDM.

**ACI 318-14 (8.11.6.6)**

Distribution of factored moments at critical sections is summarized in [Table below](#).

Table 2 - Distribution of Factored Moments						
Location		Slab-beam Strip	Column Strip		Middle Strip	
		Moment (ft-kips)	Percent	Moment (ft-kips)	Percent	Moment (ft-kips)
End Span	Exterior Negative	328.07	100	328.07	0	0.00
	Positive	668.33	60	401.00	40	267.33
	Interior Negative	1,201.69	75	901.27	25	300.42
Interior Span	Negative	1,090.81	75	818.10	25	272.70
	Positive	307.76	60	184.66	40	123.10

### 3.1.6. Flexural Reinforcement Requirements

- a) Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – interior negative location is provided below:

$$M_u = 901.27 \text{ ft-kips}$$

Use  $d = 15.88$  in. (slab with drop panel where  $h = 17$  in.)

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section ( $jd$ ). In this example, tension-controlled section will be assumed so the reduction factor  $\phi$  is equal to 0.90, and  $jd$  will be taken equal to  $0.971 \times d$ . The assumptions will be verified once the area of steel is finalized.

Assume  $jd = 0.971 \times d = 15.41$  in.

$$\text{Column strip width, } b = \frac{33 \times 12}{2} = 198.00 \text{ in.}$$

$$\text{Middle strip width, } b = 33 \times 12 - 198.00 = 198.00 \text{ in.}$$

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{901.27 \times 12,000}{0.90 \times 60,000 \times 15.41} = 12.995 \text{ in.}^2$$

Recalculate 'a' for the actual  $A_s = 12.995 \text{ in.}^2$ :

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{12.995 \times 60,000}{0.85 \times 5,000 \times 198.00} = 0.927 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.927}{0.85} = 1.090 \text{ in.}$$

$$\epsilon_t = \left( \frac{0.003}{c} \right) \times d_t - 0.003 = \left( \frac{0.003}{1.090} \right) \times 15.88 - 0.003 = 0.0407 \geq 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left( d - \frac{a}{2} \right)} = \frac{901.27 \times 12,000}{0.90 \times 60,000 \times \left( 15.88 - \frac{0.927}{2} \right)} = 12.995 \text{ in.}^2$$

Two values of thickness must be considered. The slab thickness in the column strip is 17.00 in. with the drop panel and 8.00 in. for the equivalent slab without the drop panel based on the system weight.

The weighted slab thickness:

$$h_w = \frac{17.00 \times (12) + 8 \times \left(\frac{33}{2} - 12\right)}{(12) + \left(\frac{33}{2} - 12\right)} = 14.55 \text{ in.}$$

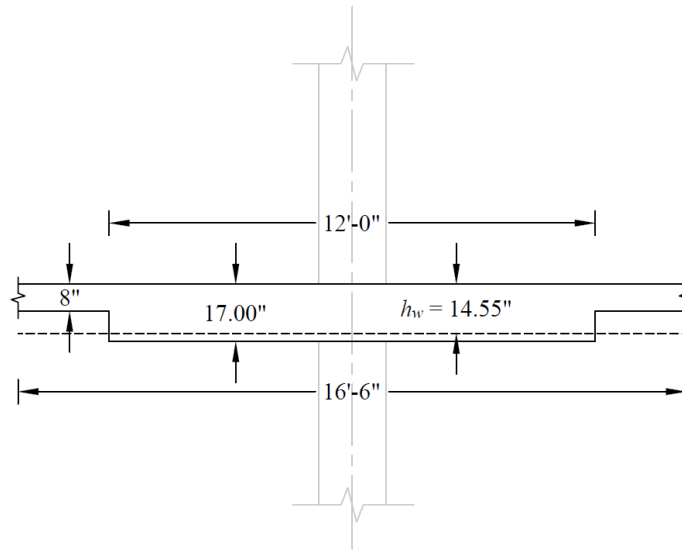


Figure 19 – The Weighted Slab Thickness

$$A_{s,min} = 0.0018 \times b \times h_w$$

ACI 318-14 (24.4.3.2)

$$A_{s,min} = 0.0018 \times 198 \times 14.55 = 5.184 \text{ in.}^2 < 12.995 \text{ in.}^2$$

$$s_{max} = \text{lesser of } \left[ \frac{5h}{18 \text{ in.}} \right] = \text{lesser of } \left[ \frac{5 \times 3 = 15 \text{ in.}}{18 \text{ in.}} \right] = 15.00 \text{ in.}$$

ACI 318-14 (24.4.3.3)

Provide 30 – #6 bars with  $A_s = 13.20 \text{ in.}^2$  and  $s = \frac{198}{30} = 6.60 \text{ in.} \leq s_{max} = 15.00 \text{ in.}$

The flexural reinforcement calculation for the column strip of interior span – positive location is provided below:

$$M_u = 184.66 \text{ ft-kips}$$

Use  $d = 15.88$  in. (slab with rib where  $h = 17$  in.)

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section ( $jd$ ). In this example, tension-controlled section will be assumed so the reduction factor  $\phi$  is equal to 0.90, and  $jd$  will be taken equal to  $0.994 \times d$ . The assumptions will be verified once the area of steel is finalized.

Assume  $jd = 0.994 \times d = 15.78$  in.

$$\text{Column strip width, } b = \frac{33 \times 12}{2} = 198.00 \text{ in.}$$

Middle strip width,  $b = 33 \times 12 - 198.00 = 198.00$  in.

$$A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{184.66 \times 12,000}{0.90 \times 60,000 \times 15.78} = 2.600 \text{ in.}^2$$

Recalculate 'a' for the actual  $A_s = 2.600 \text{ in.}^2$ :

$$a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{2.600 \times 60,000}{0.85 \times 5,000 \times 198.00} = 0.185 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.185}{0.85} = 0.218 \text{ in.}$$

$$\varepsilon_t = \left( \frac{0.003}{c} \right) \times d_t - 0.003 = \left( \frac{0.003}{0.218} \right) \times 15.88 - 0.003 = 0.2154 \geq 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi \times f_y \times \left( d - \frac{a}{2} \right)} = \frac{184.66 \times 12,000}{0.90 \times 60,000 \times \left( 15.88 - \frac{0.185}{2} \right)} = 2.600 \text{ in.}^2$$

$$A_{s,\min} = 0.0018 \times b \times h_{eq}$$

ACI 318-14 (24.4.3.2)



$$A_{s,min} = 0.0018 \times 198 \times 8.00 = 2.851 \text{ in.}^2 > 2.600 \text{ in.}^2$$

$$\therefore \text{ use } A_s = A_{s,min} = 2.851 \text{ in.}^2$$

Since column strip has 5 ribs  $\rightarrow$  provide 10 – #6 bars (2 bars / rib):

$$A_{s,provided} = 10 \times 0.44 = 4.40 \text{ in.}^2 > A_{s,required} = 2.851 \text{ in.}^2$$

Based on the procedure outlined above, values for all span locations are given in [Table below](#).

Table 3 - Required Slab Reinforcement for Flexure [Equivalent Frame Method (EFM)]								
Span Location		$M_u$ (ft-kips)	$b$ (in.)	$d$ (in.)	$A_{s,req}$ (in. <sup>2</sup> )	$A_{s,min}$ (in. <sup>2</sup> )	Reinforcement Provided	$A_{s,provided}$ (in. <sup>2</sup> )
End Span								
Column Strip	Exterior Negative	328.07	198	15.88	4.641	5.184	14 – #6 * **	6.16
	Positive	401.00	198	15.81	5.709	2.851	10 – #7 (2 bars / rib)	6.00
	Interior Negative	901.27	198	15.88	12.995	5.184	30 – #6	13.20
Middle Strip	Exterior Negative	0.00	198	15.88	0	5.184	14 – #6 * **	6.16
	Positive	267.33	198	15.88	3.774	2.851	12 – #6 (2 bars / rib)	5.28
	Interior Negative	300.42	198	15.88	4.246	5.184	14 – #6 * **	6.16
Interior Span								
Column Strip	Positive	184.66	198	15.88	2.600	2.851	10 – #6 * (2 bars / rib)	4.40
Middle Strip	Positive	123.10	198	15.88	1.730	2.851	12 – #6 * (2 bars / rib)	5.28
* Design governed by minimum reinforcement.								
** Number of bars governed by maximum allowable spacing.								

- b) Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

The factored slab moment resisted by the column ( $\gamma_f \times M_{sc}$ ) shall be assumed to be transferred by flexure.

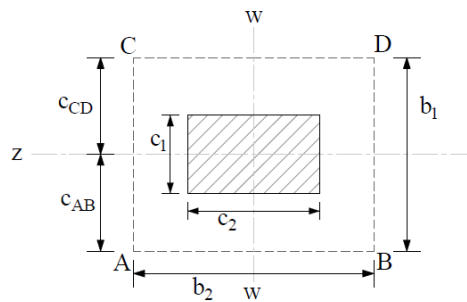
Concentration of reinforcement over the column by closer spacing or additional reinforcement shall be used to resist this moment. The fraction of slab moment not calculated to be resisted by flexure shall be assumed to be resisted by eccentricity of shear. ACI 318-14 (8.4.2.3)

Portion of the unbalanced moment transferred by flexure is  $\gamma_f \times M_{sc}$  ACI 318-14 (8.4.2.3.1)

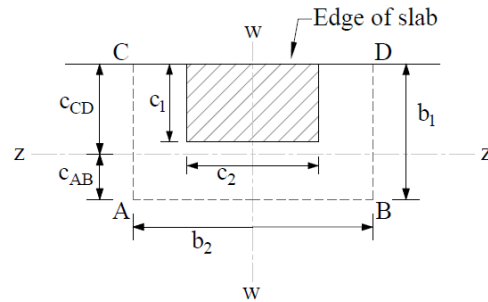
Where:

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{b_1}{b_2}}} \quad \text{ACI 318-14 (8.4.2.3.2)}$$

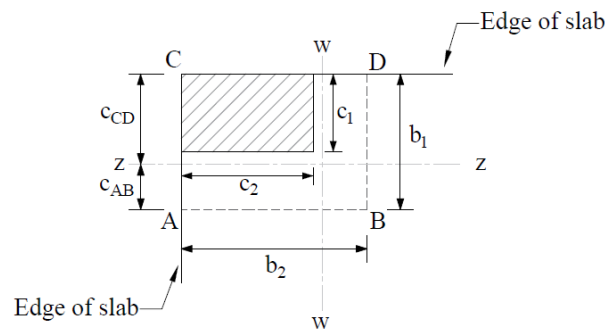
- $b_1$  = Dimension of the critical section  $b_o$  measured in the direction of the span for which moments are determined in ACI 318, Chapter 8 (see [Figure 20](#)).
- $b_2$  = Dimension of the critical section  $b_o$  measured in the direction perpendicular to  $b_1$  in ACI 318, Chapter 8 (see [Figure 20](#)).
- $b_b$  = Effective slab width =  $c_2 + 3 \times h$  ACI 318-14 (8.4.2.3.3)



Critical shear perimeter for interior column



Critical shear perimeter for exterior column



Critical shear perimeter for corner column

Figure 20 – Critical Shear Perimeters for Columns

For exterior support:

$$d = h - c_{clear} - \frac{d_b}{2} = 17.00 - 0.75 - \frac{0.75}{2} = 15.88 \text{ in.}$$

$$M_{sc} = 462.75 \text{ ft-kips}$$

$$A_{s(prov)} = 6.16 \text{ in.}^2$$

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{15.88}{2} = 27.94 \text{ in.}$$

$$b_2 = c_2 + d = 20 + 15.88 = 35.88 \text{ in.}$$

$$b_b = c_2 + 3 \times h = 20 + 3 \times 17.00 = 71.00 \text{ in.}$$

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{27.94}{35.88}}} = 0.630$$

$$A_s = \frac{0.85 \times f'_c \times b_b}{f_y} \times \left( d - \sqrt{d^2 - \frac{2 \times \gamma_f \times M_{sc}}{\phi \times 0.85 \times f'_c \times b_b}} \right)$$

$$A_s = \frac{0.85 \times 5,000 \times 71.00}{60,000} \times \left( 15.88 - \sqrt{15.88^2 - \frac{2 \times 0.630 \times 462.75}{0.90 \times 0.85 \times 5,000 \times 71.00}} \right) = 4.188 \text{ in.}^2$$

However, the area of steel provided to resist the flexural moment within the effective slab width,  $b_b$ :

$$A_{s, provided \text{ within } b_b} = A_{s, provided} \times \frac{b_b}{b} = 4.188 \times \frac{71.00}{198} = 2.209 \text{ in.}^2$$

Then, the required additional reinforcement at exterior column for moment transfer between slab and column:

$$A_{s, additional} = A_s - A_{s, provided \text{ within } b_b} = 4.188 - 2.209 = 1.979 \text{ in.}^2$$

Provide 5 – #6 additional bars with  $A_s = 2.20 \text{ in.}^2$

Based on the procedure outlined above, values for all supports are given in [Table below](#).

Table 4 - Additional Slab Reinforcement required for moment transfer between slab and column (EFM)									
Span Location		$M_{sc}^*$ (ft-kips)	$\gamma_f$	$\gamma_f M_{sc}$ (ft-kips)	$b_b$ (in.)	$d$ (in.)	$A_s$ req'd within $b_b$ (in. <sup>2</sup> )	$A_s$ prov. For flexure within $b_b$ (in. <sup>2</sup> )	Add'l Reinf.
End Span									
Column Strip	Exterior Negative	462.75	0.630	291.35	71.00	15.88	4.188	2.209	5-#6
	Interior Negative	134.12	0.600	80.47	71.00	15.88	2.029	4.733	-
* $M_{sc}$ is taken at the centerline of the support in Equivalent Frame Method solution.									

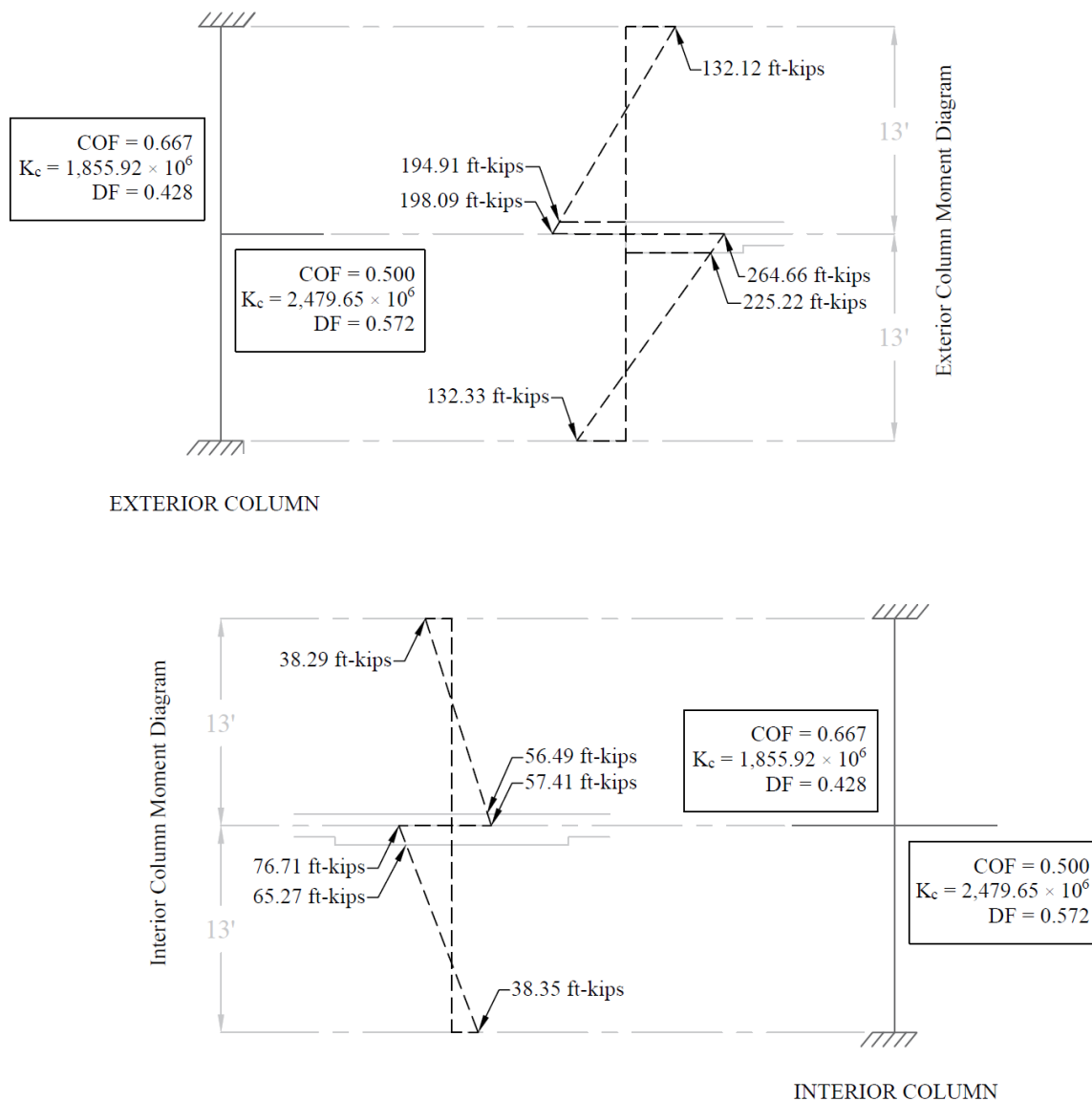
### 3.1.7. Factored Moments in Columns

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the support columns above and below the slab-beam in proportion to the relative stiffness of the support columns. Referring to [Figure 18](#), the unbalanced moment at the exterior and interior joints are:

Exterior Joint = + 462.75 ft-kips

Joint 2 = - 1,382.84 + 1,248.72 = -134.12 ft-kips

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced slab moments ( $M_{sc}$ ) to the exterior and interior columns are shown in the [Figure 21](#).



**Figure 21 – Column Moments (Unbalanced Moments from Slab-Beam)**

In summary:

For Top column (Above):

$$M_{col, Exterior} = 194.91 \text{ ft-kips}$$

$$M_{col, Interior} = 56.49 \text{ ft-kips}$$

For Bottom column (Below):

$$M_{col, Exterior} = 225.22 \text{ ft-kips}$$

$$M_{col, Interior} = 65.27 \text{ ft-kips}$$

The moments determined above are combined with the factored axial loads (for each story) and factored moments in the transverse direction for design of column sections. The moment values at the face of interior, exterior, and corner columns from the unbalanced moment values are shown in the [following Table](#).

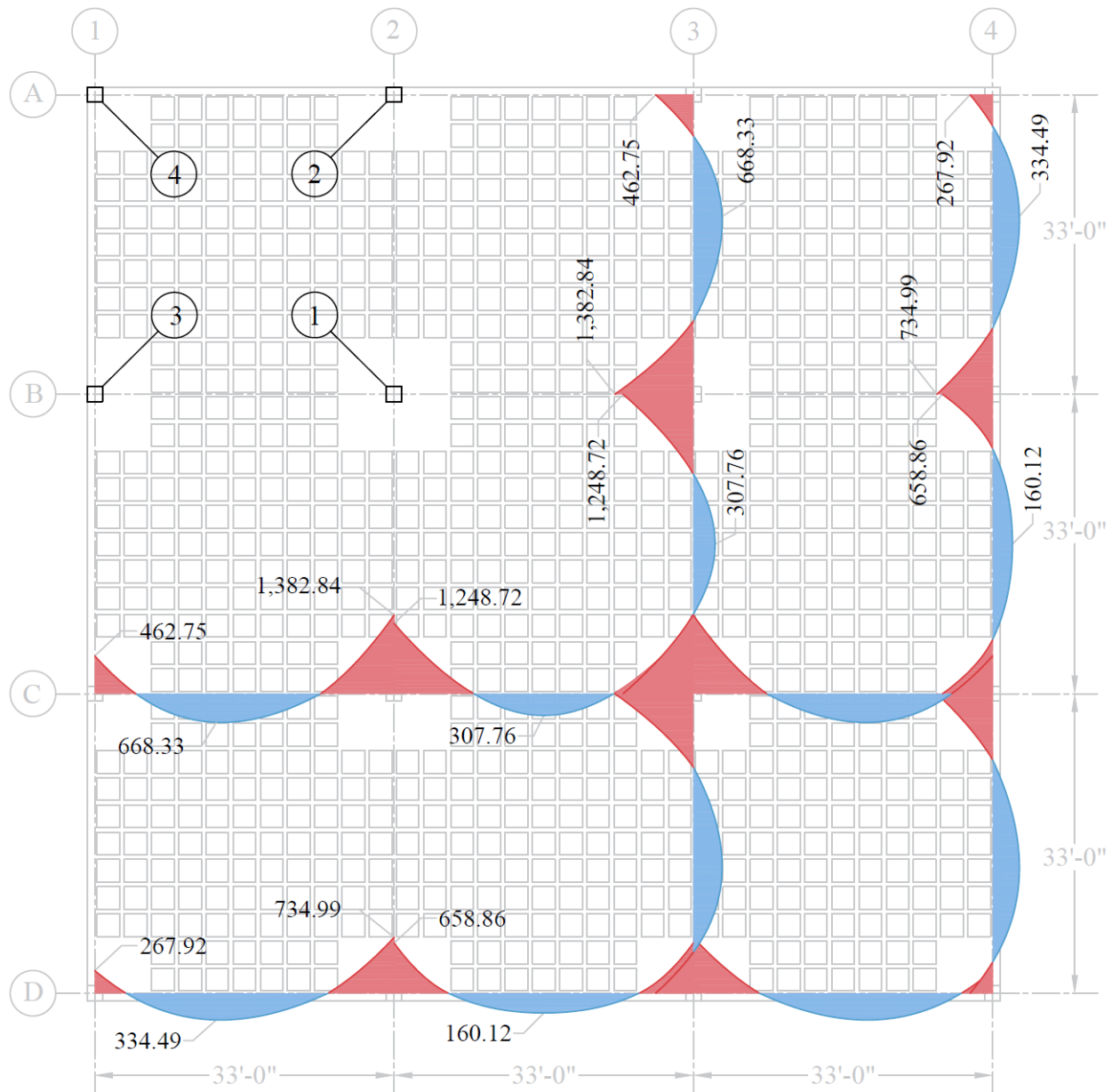


Figure 22 – Moment Diagrams (kips-ft)

Table 5 - Factored Moments in Columns			
$M_u$ (kips-ft)	Column Location		
	Interior	Exterior	Corner
$M_{ux}$	65.27	225.22	225.22
$M_{uy}$	65.27	65.27	225.22

#### 4. Design of Columns by spColumn

This section includes the design of interior, edge, and corner columns using [spColumn](#) software. The preliminary dimensions for these columns were calculated previously in section one. The reduction of live load per [ASCE 7-10](#) will be ignored in this example. However, the detailed procedure to calculate the reduced live loads is explained in the “[One-Way Wide Module \(Skip Joist\) Concrete Floor System Design \(ACI 318-14\)](#)” Design Example.

##### 4.1. Determination of Factored Loads

Assume 4 story building

###### Interior Column:

Tributary area for interior column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = (33 \times 33) = 1,089.00 \text{ ft}^2$$

Tributary area for interior column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = (12 \times 12) = 144.00 \text{ ft}^2$$

- $P_u = 4 \times q_u \times A_{Tributary} = 4 \times (0.340 \times 1,089.00 + 0.135 \times 144.00) = 1,558.80 \text{ kips}$
- $M_{u,x} = 65.27 \text{ ft-kips}$  ([see the previous Table](#))
- $M_{u,y} = 65.27 \text{ ft-kips}$  ([see the previous Table](#))

###### Edge (Exterior) Column:

Tributary area for exterior column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left( \frac{33}{2} + \frac{20/2}{12} \right) \times 33 = 572.00 \text{ ft}^2$$

Tributary area for exterior column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = \left( \frac{12}{2} + \frac{20/2}{12} \right) \times 12 = 82.00 \text{ ft}^2$$

- $P_u = 4 \times q_u \times A_{Tributary} = 4 \times (0.340 \times 572.00 + 0.135 \times 82.00) = 822.20 \text{ kips}$
- $M_{u,x} = 225.22 \text{ ft-kips}$  ([see the previous Table](#))
- $M_{u,y} = 65.27 \text{ ft-kips}$  ([see the previous Table](#))

Corner Column:

Tributary area for corner column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left( \frac{33}{2} + \frac{20/2}{12} \right) \times \left( \frac{33}{2} + \frac{20/2}{12} \right) = 300.44 \text{ ft}^2$$

Tributary area for corner column for self-weight of additional slab thickness due to the presence of the drop panel is

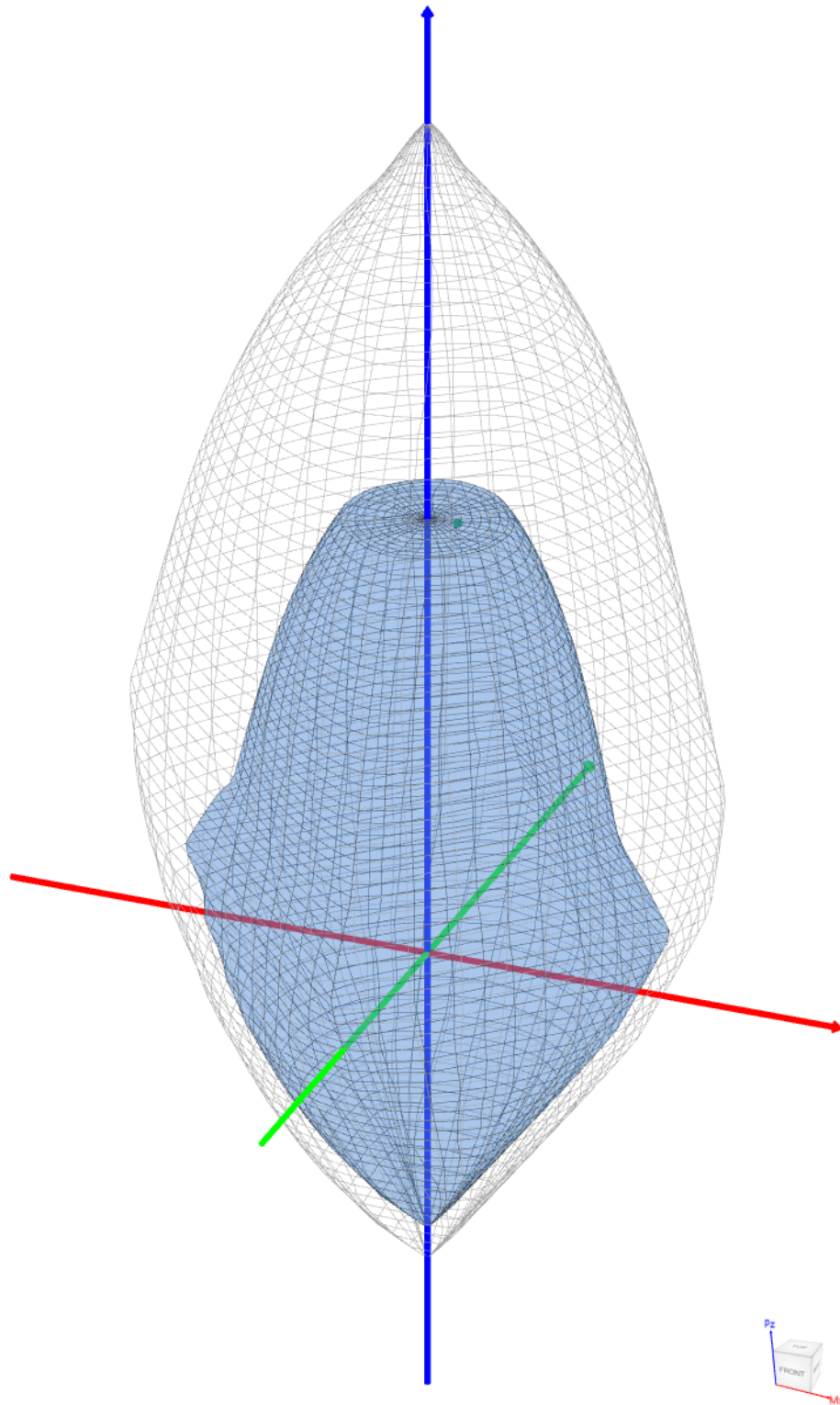
$$A_{Tributary} = \left( \frac{12}{2} + \frac{20/2}{12} \right) \times \left( \frac{12}{2} + \frac{20/2}{12} \right) = 46.69 \text{ ft}^2$$

- $P_u = 4 \times q_u \times A_{Tributary} = 4 \times (0.340 \times 300.44 + 0.135 \times 46.69) = 433.82 \text{ kips}$
- $M_{u,x} = 225.22 \text{ ft-kips}$  ([see the previous Table](#))
- $M_{u,y} = 225.22 \text{ ft-kips}$  ([see the previous Table](#))

The factored loads are then input into [spColumn](#) to construct the axial load – moment interaction diagram.

## 4.2. Moment Interaction Diagram

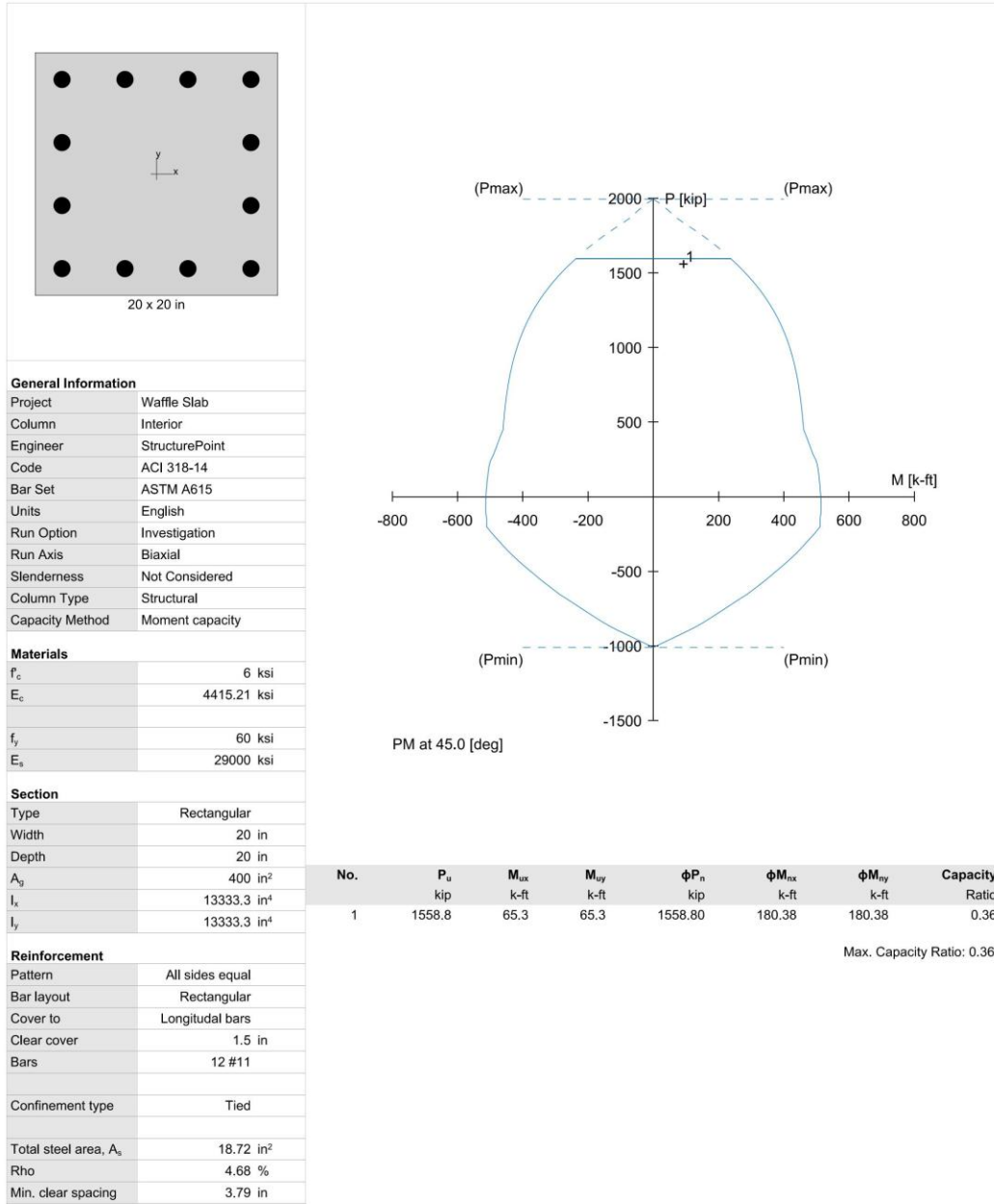
Interior Column:





STRUCTUREPOINT - spColumn v10.10a (TM)  
Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-2B1B4-2B1B4  
F:\StructurePoint\spColumn\Interior Column.colx

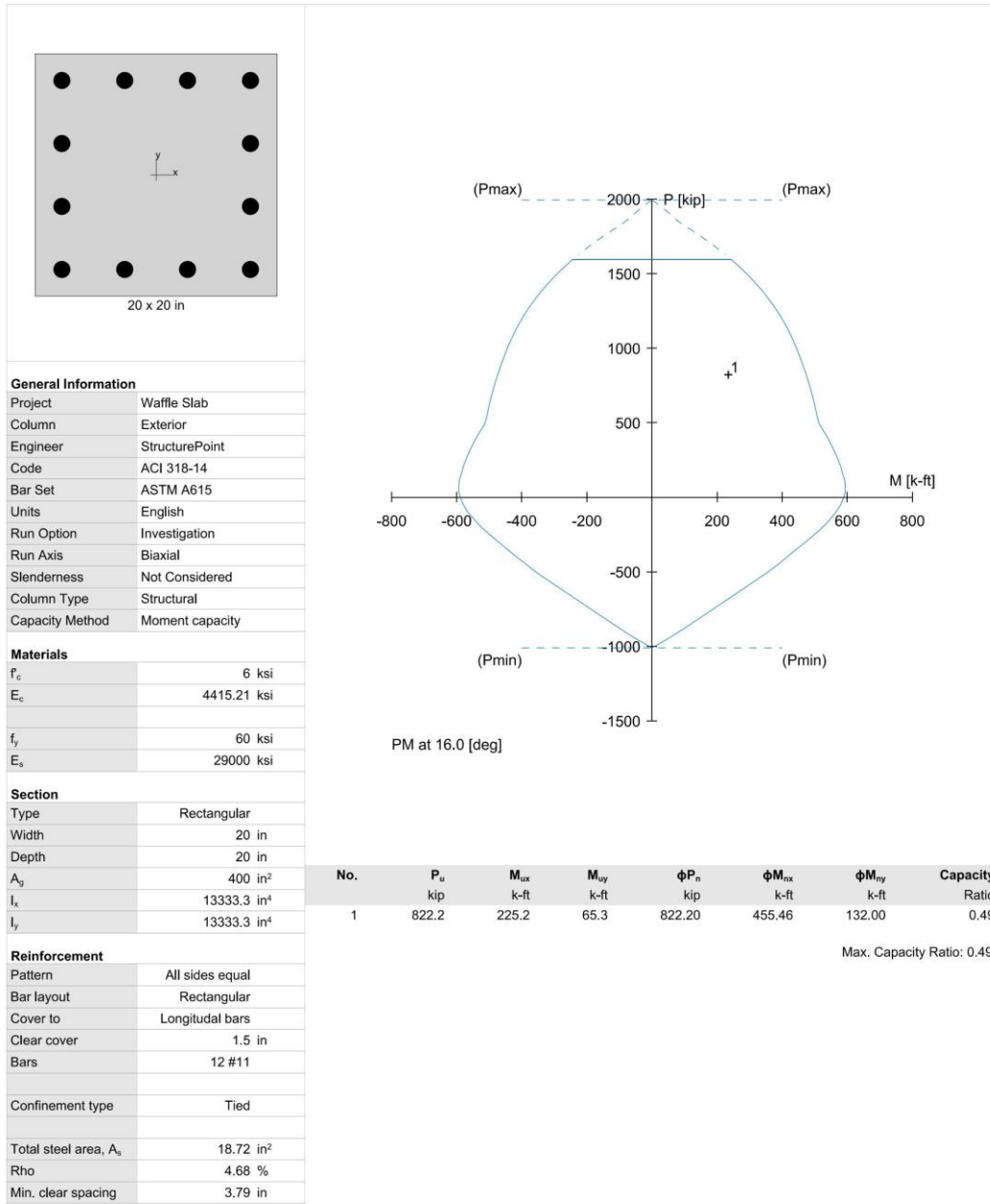
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Edge Column:

STRUCTUREPOINT - spColumn v10.10a (TM)  
Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-2B1B4-2B1B4  
F:\StructurePoint\spColumn\Exterior Column.colx

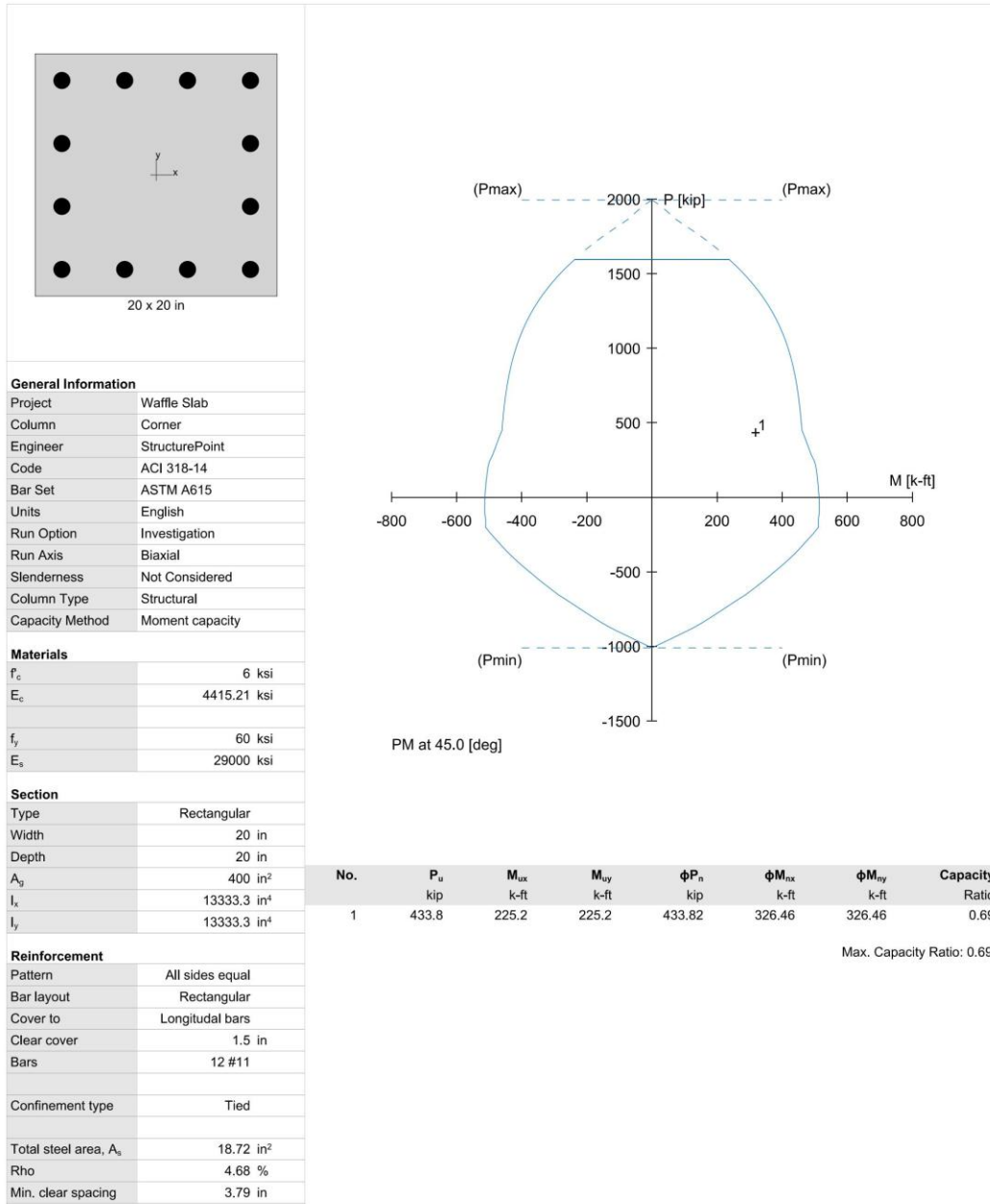
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Corner Column:

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## 5. Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with ACI 318 Chapter 22.

### 5.1. One-Way (Beam Action) Shear Strength

ACI 318-14 (22.5)

One-way shear is critical at a distance  $d$  from the face of the column as shown in [Figure 3](#). [Figure 24](#) and [Figure 26](#) show the factored shear forces ( $V_u$ ) at the critical sections around each column and each drop panel, respectively. In members without shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$\phi V_n = \phi V_c + \phi V_s = \phi V_c, (\phi V_s = 0) \quad \text{ACI 318-14 (Eq. 22.5.1.1)}$$

Where:

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d \quad \text{ACI 318-14 (Eq. 22.5.5.1)}$$

One-way shear capacity is calculated assuming the shear cross-section area consisting of the drop panel (if any), the ribs, and the slab portion above them, decreased by concrete cover. For such section the equivalent shear width for single rib is calculated from the formula:

$$b_v = b + \frac{d}{12} \quad \text{spSlab Software Manual (Eq. 2-13)}$$

Where:

$b$  = rib width, in.

$d$  = distance from extreme compression fiber to tension reinforcement centroid.

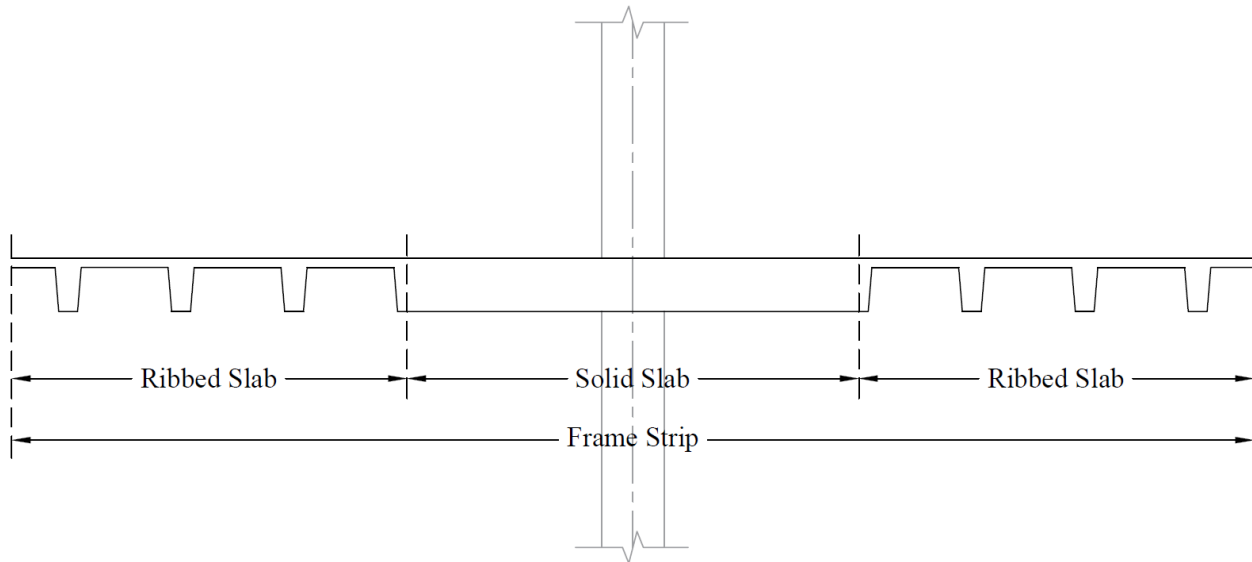
#### 5.1.1. At Distance $d$ from the Supporting Column

$$d = h - c_{clear} - \frac{d_b}{2} = 17.00 - 0.75 - \frac{0.75}{2} = 15.88 \text{ in. for middle span with \#6 reinforcement.}$$

$$b_v = 6.00 + \frac{15.88}{12} = 7.32 \text{ in.}$$

Where  $\lambda = 1$  for normal weight concrete

$$b = l_{2,drop} + n_{ribs} \times b_v = 12 \times 12 + 7 \times 7.32 = 195.26 \text{ in. (see the following Figure)}$$



**Figure 23 – Frame Strip Cross Section (at Distance  $d$  from the Face of the Supporting Column)**

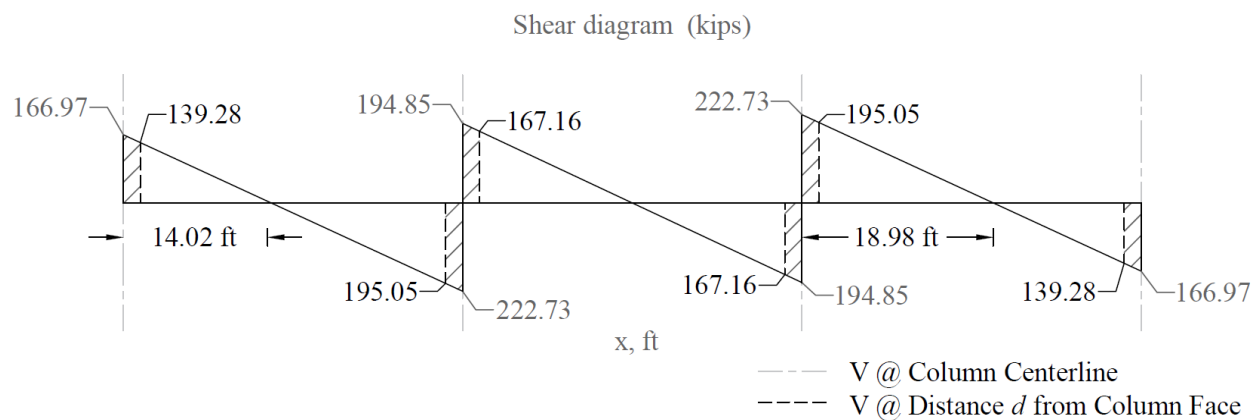
The one-way shear capacity for the ribbed slab portions shown in [Figure 23](#) is permitted to be increased by 10%. **ACI 318-14 (9.8.1.5)**

$$\phi V_c = (\phi V_c)_{\text{Solid Slab}} + 1.10 \times (\phi V_c)_{\text{Ribbed Slab}}$$

$$\phi V_c = 0.75 \times 2.0 \times 1.0 \times \frac{\sqrt{5,000}}{1,000} \times (12 \times 12) \times 15.88 + 1.10 \times 0.75 \times 2.0 \times 1.0 \times \frac{\sqrt{5,000}}{1,000} \times (7 \times 7.32) \times 15.88$$

$$\phi V_c = 337.41 \text{ kips}$$

Because  $\phi V_c \geq V_u$  at all the critical sections, the slab has adequate one-way shear strength.



**Figure 24 – One-way Shear at Critical Sections (at Distance  $d$  from the Face of the Supporting Column)**

### 5.1.2. At the Face of the Drop Panel

$$d = h - c_{clear} - \frac{d_b}{2} = 17.00 - 0.75 - \frac{0.75}{2} = 15.88 \text{ in. for middle span with \#6 reinforcement.}$$

$$b_v = 6.00 + \frac{15.88}{12} = 7.32 \text{ in.}$$

Where  $\lambda = 1$  for normal weight concrete

$$b = n_{ribs} \times b_v = 11 \times 7.32 = 80.55 \text{ in. (see the following Figure)}$$

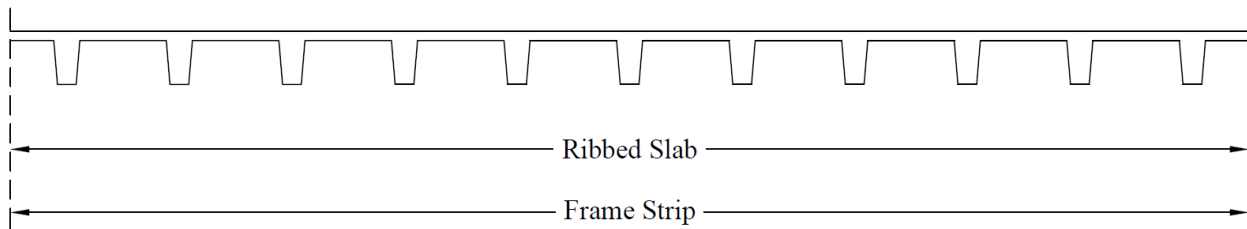


Figure 25 – Frame Strip Cross Section (at Distance  $d$  from the Face of the Drop Panel)

The one-way shear capacity for the ribbed slab portions shown in Figure 25 is permitted to be increased by 10%. **ACI 318-14 (9.8.1.5)**

$$\phi V_c = 1.10 \times (\phi V_c)_{\text{Ribbed Slab}} = 1.10 \times 0.75 \times 2.0 \times 1.0 \times \frac{\sqrt{5,000}}{1,000} \times (11 \times 7.32) \times 15.88 = 149.20 \text{ kips}$$

Because  $\phi V_c \geq V_u$  at all the critical sections, the slab has adequate one-way shear strength.

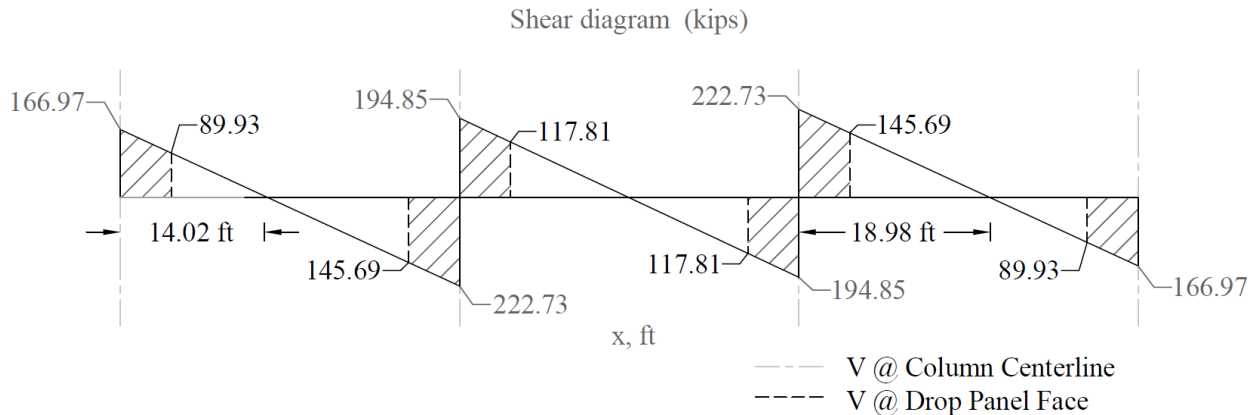


Figure 26 – One-Way Shear at Critical Sections (at the Face of the Drop Panel)

## 5.2. Two-Way (Punching) Shear Strength

ACI 318-14 (22.6)

### 5.2.1. Around the Columns Faces

Two-way shear is critical on a rectangular section located at  $d/2$  away from the face of the column as shown in [Figure 20](#).

#### a) Exterior column:

The factored shear force ( $V_u$ ) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section ( $d/2$  away from column face).

$$V_u = V - q_u \times (b_1 \times b_2) = 166.97 - 0.475 \times \left( \frac{27.94 \times 35.88}{144} \right) = 163.66 \text{ kips}$$

The factored unbalanced moment used for shear transfer,  $M_{unb}$ , is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$M_{unb} = M - V_u \times \left( b_1 - c_{AB} - \frac{c_1}{2} \right) = 462.75 - 163.66 \times \left( \frac{27.94 - 8.51 - \frac{20}{2}}{12} \right) = 334.13 \text{ ft-kips}$$

For the exterior column in [Figure 20](#) the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{b_1^2}{2 \times b_1 + b_2} = \frac{27.94^2}{2 \times 27.94 + 35.88} = 8.51 \text{ in.}$$

Where:

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{15.88}{2} = 27.94 \text{ in.} \quad b_2 = c_2 + d = 20 + 15.88 = 35.88 \text{ in.}$$

The polar moment  $J_c$  of the shear perimeter is:

$$J_c = 2 \times \left( \frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \left( \frac{b_1}{2} - c_{AB} \right)^2 \right) + b_2 \times d \times c_{AB}^2$$

$$J_c = 2 \times \left( \frac{27.94 \times 15.88^3}{12} + \frac{15.88 \times 27.94^3}{12} + (27.94 \times 15.88) \times \left( \frac{27.94}{2} - 8.51 \right)^2 \right) + 35.88 \times 15.88 \times 8.51^2$$

$$J_c = 143,997.36 \text{ in.}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.630 = 0.370$$

ACI 318-14 (Eq. 8.4.4.2.2)

Where:

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{b_1}{b_2}}}$$

ACI 318-14 (8.4.2.3.2)

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{27.94}{35.88}}} = 0.630$$

The length of the critical perimeter for the exterior column:

$$b_o = 2 \times b_1 + b_2 = 2 \times 27.94 + 35.88 = 91.75 \text{ in.}$$

The two-way shear stress ( $v_u$ ) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$

ACI 318-14 (R.8.4.4.2.3)

$$v_u = \frac{163.66 \times 1,000}{91.75 \times 15.88} + \frac{0.370 \times (334.13 \times 12 \times 1,000) \times 8.51}{143,997.36} = 112.36 + 87.74 = 200.10 \text{ psi}$$

$$v_c = \min \left\{ \begin{array}{l} 4 \times \lambda \times \sqrt{f'_c} \\ \left( 2 + \frac{4}{\beta} \right) \times \lambda \times \sqrt{f'_c} \\ \left( \frac{\alpha_s \times d}{b_o} + 2 \right) \times \lambda \times \sqrt{f'_c} \end{array} \right\}$$

ACI 318-14 (Table 22.6.5.2)

$$v_c = \min \left\{ \begin{array}{l} 4 \times 1 \times \sqrt{5,000} \\ \left( 2 + \frac{4}{1} \right) \times 1 \times \sqrt{5,000} \\ \left( \frac{30 \times 15.88}{91.75} + 2 \right) \times 1 \times \sqrt{5,000} \end{array} \right\} = \min \left\{ \begin{array}{l} 282.84 \\ 424.26 \\ 508.46 \end{array} \right\} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75 \times 282.84 = 212.13 \text{ psi}$$

Because  $\phi v_c > v_u$  at the critical section, the slab has adequate two-way shear strength at this joint.



**b) Interior column:**

$$V_u = V - q_u \times (b_1 \times b_2) = (222.73 + 194.85) - 0.475 \times \left( \frac{35.88 \times 35.88}{144} \right) = 413.34 \text{ kips}$$

$$M_{unb} = M - V_u \times \left( b_1 - c_{AB} - \frac{c_1}{2} \right) = (1,382.84 - 1,248.72) - 413.34 \times (0) = 134.12 \text{ ft-kips}$$

For the interior column in [Figure 20](#), the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{35.88}{2} = 17.94 \text{ in.}$$

Where:

$$b_1 = c_1 + d = 20 + 15.88 = 35.88 \text{ in.} \quad b_2 = c_2 + d = 20 + 15.88 = 35.88 \text{ in.}$$

The polar moment  $J_c$  of the shear perimeter is:

$$J_c = 2 \times \left( \frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left( \frac{b_1}{2} - c_{AB} \right)^2 \right) + 2 \times b_2 \times d \times c_{AB}^2$$

$$J_c = 2 \times \left( \frac{35.88 \times 15.88^3}{12} + \frac{15.88 \times 35.88^3}{12} + (35.88 \times 15.88) \times (0)^2 \right) + 2 \times 35.88 \times 15.88 \times 17.94^2$$

$$J_c = 512,571.48 \text{ in.}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.600 = 0.400$$

ACI 318-14 (Eq. 8.4.4.2.2)

Where:

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{b_1}{b_2}}}$$

ACI 318-14 (8.4.2.3.2)

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{35.88}{35.88}}} = 0.600$$

The length of the critical perimeter for the interior column:

$$b_o = 2 \times (b_1 + b_2) = 2 \times (35.88 + 35.88) = 143.50 \text{ in.}$$

The two-way shear stress ( $v_u$ ) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c} \quad \text{ACI 318-14 (R.8.4.4.2.3)}$$

$$v_u = \frac{413.34 \times 1,000}{143.50 \times 15.88} + \frac{0.400 \times (134.12 \times 12 \times 1,000) \times 17.94}{512,571.48} = 181.44 + 22.53 = 203.97 \text{ psi}$$

$$v_c = \min \left\{ \begin{array}{l} 4 \times \lambda \times \sqrt{f'_c} \\ \left( 2 + \frac{4}{\beta} \right) \times \lambda \times \sqrt{f'_c} \\ \left( \frac{\alpha_s \times d}{b_o} + 2 \right) \times \lambda \times \sqrt{f'_c} \end{array} \right\} \quad \text{ACI 318-14 (Table 22.6.5.2)}$$

$$v_c = \min \left\{ \begin{array}{l} 4 \times 1 \times \sqrt{5,000} \\ \left( 2 + \frac{4}{1} \right) \times 1 \times \sqrt{5,000} \\ \left( \frac{40 \times 15.88}{143.50} + 2 \right) \times 1 \times \sqrt{5,000} \end{array} \right\} = \min \left\{ \begin{array}{l} 282.84 \\ 424.26 \\ 454.32 \end{array} \right\} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75 \times 282.84 = 212.13 \text{ psi}$$

Since  $\phi v_c > v_u$  at the critical section, the slab has adequate two-way shear strength at this joint.

c) **Corner column:**

In this example, interior equivalent frame strip was selected where it only has exterior and interior supports (no corner supports are included in this strip). However, the two-way shear strength of corner supports usually governs. Thus, the two-way shear strength for the corner column in this example will be checked for educational purposes. Same procedure is used to find the reaction and factored unbalanced moment used for shear transfer at the centroid of the critical section for the corner support for the exterior equivalent frame strip.

$$V_u = V - q_u \times (b_1 \times b_2) = 94.32 - 0.475 \times \left( \frac{27.94 \times 27.94}{144} \right) = 91.75 \text{ kips}$$

$$M_{unb} = M - V_u \times \left( b_1 - c_{AB} - \frac{c_1}{2} \right) = 265.55 - 91.75 \times \left( \frac{27.94 - 6.98 - \frac{20}{2}}{12} \right) = 181.81 \text{ ft-kips}$$

For the corner column in [Figure 20](#), the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{b_1^2}{2 \times (b_1 + b_2)} = \frac{27.94^2}{2 \times (27.94 + 27.94)} = 6.98 \text{ in.}$$

Where:

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{15.88}{2} = 27.94 \text{ in.} \quad b_2 = c_2 + \frac{d}{2} = 20 + \frac{15.88}{2} = 27.94 \text{ in.}$$

The polar moment  $J_c$  of the shear perimeter is:

$$J_c = \left( \frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left( \frac{b_1}{2} - c_{AB} \right)^2 \right) + b_2 \times d \times c_{AB}^2$$

$$J_c = \left( \frac{27.94 \times 15.88^3}{12} + \frac{15.88 \times 27.94^3}{12} + (27.94 \times 15.88) \times \left( \frac{27.94}{2} - 6.98 \right)^2 \right) + 27.94 \times 15.88 \times 6.98^2$$

$$J_c = 81,430.82 \text{ in.}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.600 = 0.400$$

**ACI 318-14 (Eq. 8.4.4.2.2)**

Where:

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{b_1}{b_2}}} \quad \text{ACI 318-14 (8.4.2.3.2)}$$

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{27.94}{27.94}}} = 0.600$$

The length of the critical perimeter for the corner column:

$$b_o = b_1 + b_2 = 27.94 + 27.94 = 55.88 \text{ in.}$$

The two-way shear stress ( $v_u$ ) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c} \quad \text{ACI 318-14 (R.8.4.4.2.3)}$$

$$v_u = \frac{91.75 \times 1,000}{55.88 \times 15.88} + \frac{0.400 \times (181.81 \times 12 \times 1,000) \times 6.98}{81,430.82} = 103.43 + 74.85 = 178.28 \text{ psi}$$

$$v_c = \min \left\{ \begin{array}{l} 4 \times \lambda \times \sqrt{f'_c} \\ \left( 2 + \frac{4}{\beta} \right) \times \lambda \times \sqrt{f'_c} \\ \left( \frac{\alpha_s \times d}{b_o} + 2 \right) \times \lambda \times \sqrt{f'_c} \end{array} \right\} \quad \text{ACI 318-14 (Table 22.6.5.2)}$$

$$v_c = \min \left\{ \begin{array}{l} 4 \times 1 \times \sqrt{5,000} \\ \left( 2 + \frac{4}{1} \right) \times 1 \times \sqrt{5,000} \\ \left( \frac{20 \times 15.88}{55.88} + 2 \right) \times 1 \times \sqrt{5,000} \end{array} \right\} = \min \left\{ \begin{array}{l} 282.84 \\ 424.26 \\ 543.22 \end{array} \right\} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75 \times 282.84 = 212.13 \text{ psi}$$

Since  $\phi v_c > v_u$  at the critical section, the slab has adequate two-way shear strength at this joint.

### 5.2.2. Around Drop Panels

Two-way shear is critical on a rectangular section located at  $d/2$  away from the face of the drop panel.

The factored shear force ( $V_u$ ) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section ( $d/2$  away from column face).

**Note:** For simplicity, it is conservative to deduct only the self-weight of the slab and joists in the critical section from the shear reaction in punching shear calculations. This approach is also adopted in the [spSlab](#) program for the punching shear check around the drop panels.

#### a) Exterior drop panel:

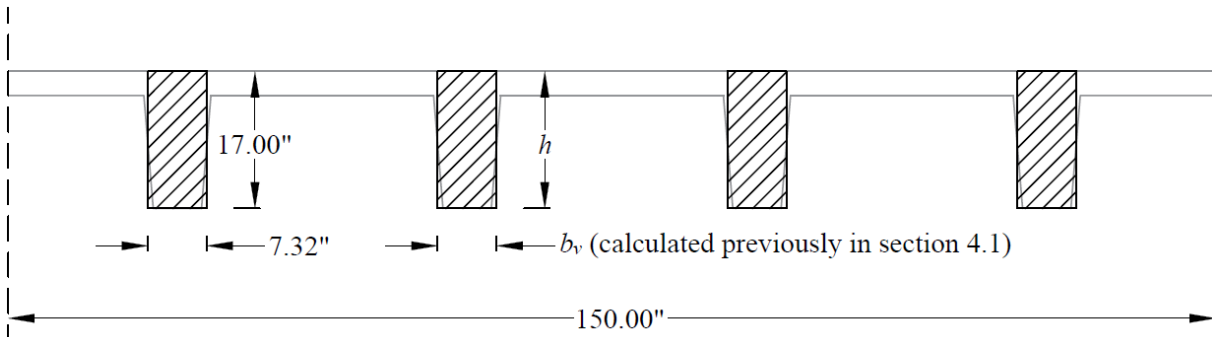
$$V_u = V - q_u \times (b_1 \times b_2) = 166.97 - 0.340 \times \left( \frac{89.94 \times 159.88}{144} \right) = 133.02 \text{ kips}$$

$d$  value is used in the calculation of  $v_u$  is given by (see the [following Figure](#)).

$$d = \frac{(\text{\# of ribs within the drop panel width}) \times h \times b_v}{\text{the drop panel width}}$$

*spSlab Software Manual (Eq. 2-14)*

$$d = \frac{4 \times 17 \times 7.32}{150} = 3.32 \text{ in.}$$



**Figure 27 – Equivalent Thickness based on Shear Area Calculation)**

The length of the critical perimeter for the exterior drop panel:

$$b_o = 2 \times 89.94 + 159.88 = 339.75 \text{ in.}$$

Where:

$$b_1 = \frac{c_1}{2} + \frac{d}{2} + \frac{l_{1,dp}}{2} = \frac{20}{2} + \frac{15.88}{2} + \frac{12 \times 12}{2} = 89.94 \text{ in.} \quad b_2 = d + l_{2,dp} = 15.88 + 12 \times 12 = 159.88 \text{ in.}$$

$$c_{AB} = \frac{b_1^2}{2 \times b_1 + b_2} = \frac{89.94^2}{2 \times 89.94 + 159.88} = 23.81 \text{ in.}$$

The polar moment  $J_c$  of the shear perimeter is:

$$J_c = 2 \times \left( \frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \left( \frac{b_1}{2} - c_{AB} \right)^2 \right) + b_2 \times d \times c_{AB}^2$$

$$J_c = 2 \times \left( \frac{89.94 \times 3.32^3}{12} + \frac{3.32 \times 89.94^3}{12} + (89.94 \times 3.32) \times \left( \frac{89.94}{2} - 23.81 \right)^2 \right) + 159.88 \times 3.32 \times 23.81^2$$

$$J_c = 971,273.30 \text{ in.}^4$$

The two-way shear stress ( $v_u$ ) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} \quad \text{ACI 318-14 (R.8.4.4.2.3)}$$

$$v_u = \frac{133.02 \times 1,000}{339.75 \times 3.32} = 117.94 \text{ psi}$$

The two-way shear capacity for the ribbed slab is permitted to be increased by 10%.

ACI 318-14 (9.8.1.5)

$$v_c = \min \left\{ \begin{array}{l} 1.10 \times 4 \times \lambda \times \sqrt{f'_c} \\ 1.10 \times \left( 2 + \frac{4}{\beta} \right) \times \lambda \times \sqrt{f'_c} \\ 1.10 \times \left( \frac{\alpha_s \times d}{b_o} + 2 \right) \times \lambda \times \sqrt{f'_c} \end{array} \right\} \quad \text{ACI 318-14 (Table 22.6.5.2)}$$

$$v_c = \min \left\{ \begin{array}{l} 1.10 \times 4 \times 1 \times \sqrt{5,000} \\ 1.10 \times \left( 2 + \frac{4}{1} \right) \times 1 \times \sqrt{5,000} \\ 1.10 \times \left( \frac{30 \times 3.32}{339.75} + 2 \right) \times 1 \times \sqrt{5,000} \end{array} \right\} = \min \left\{ \begin{array}{l} 311.13 \\ 466.69 \\ 178.36 \end{array} \right\} = 178.36 \text{ psi}$$

$$\phi v_c = 0.75 \times 178.36 = 133.77 \text{ psi}$$

In waffle slab design where the drop panels create a large critical shear perimeter, the factor  $(b_o/d)$  has limited contribution and is traditionally neglected for simplicity and conservatism. This approach is adopted in this calculation and in the [spSlab](#) program ([spSlab software manual](#), [Eq. 2-46](#)).

The two-way shear capacity for the ribbed slab is permitted to be increased by 10%.

**ACI 318-14 (9.8.1.5)**

$$v_c = 1.10 \times 2 \times \lambda \times \sqrt{f'_c}$$

**spSlab Software Manual (Eq. 2-46)**

$$v_c = 1.10 \times 2 \times 1 \times \sqrt{5,000} = 155.56 \text{ psi}$$

$$\phi v_c = 0.75 \times 155.56 = 116.67 \text{ psi}$$

Since  $\phi v_c < v_u$  at the critical section, the slab does not have adequate two-way shear strength around this drop panel.

**b) Interior drop panel:**

$$V_u = V - q_u \times (b_1 \times b_2) = 222.73 + 194.85 - 0.340 \times \left( \frac{159.88 \times 159.88}{144} \right) = 357.23 \text{ kips}$$

The length of the critical perimeter for the interior drop panel:

$$b_o = 2 \times (159.88 + 159.88) = 639.50 \text{ in.}$$

Where:

$$b_1 = d + l_{1,dp} = 15.88 + 12 \times 12 = 159.88 \text{ in.} \quad b_2 = d + l_{2,dp} = 15.88 + 12 \times 12 = 159.88 \text{ in.}$$

$$c_{AB} = \frac{b_1}{2} = \frac{159.88}{2} = 79.94 \text{ in.}$$

The polar moment  $J_c$  of the shear perimeter is:

$$J_c = 2 \times \left( \frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left( \frac{b_1}{2} - c_{AB} \right)^2 \right) + 2 \times b_2 \times d \times c_{AB}^2$$

$$J_c = 2 \times \left( \frac{159.88 \times 3.32^3}{12} + \frac{3.32 \times 159.88^3}{12} + (159.88 \times 3.32) \times (0)^2 \right) + 2 \times 159.88 \times 3.32 \times 79.94^2$$

$$J_c = 9,044,800.03 \text{ in.}^4$$

The two-way shear stress ( $v_u$ ) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} \quad \text{ACI 318-14 (R.8.4.4.2.3)}$$

$$v_u = \frac{357.23 \times 1,000}{639.50 \times 3.32} = 168.27 \text{ psi}$$

The two-way shear capacity for the ribbed slab is permitted to be increased by 10%.

ACI 318-14 (9.8.1.5)



$$v_c = \min \left\{ \begin{array}{l} 1.10 \times 4 \times \lambda \times \sqrt{f'_c} \\ 1.10 \times \left( 2 + \frac{4}{\beta} \right) \times \lambda \times \sqrt{f'_c} \\ 1.10 \times \left( \frac{\alpha_s \times d}{b_o} + 2 \right) \times \lambda \times \sqrt{f'_c} \end{array} \right\} \quad \text{ACI 318-14 (Table 22.6.5.2)}$$

$$v_c = \min \left\{ \begin{array}{l} 1.10 \times 4 \times 1 \times \sqrt{5,000} \\ 1.10 \times \left( 2 + \frac{4}{1} \right) \times 1 \times \sqrt{5,000} \\ 1.10 \times \left( \frac{40 \times 3.32}{639.50} + 2 \right) \times 1 \times \sqrt{5,000} \end{array} \right\} = \min \left\{ \begin{array}{l} 311.13 \\ 466.69 \\ 171.71 \end{array} \right\} = 171.71 \text{ psi}$$

$$\phi v_c = 0.75 \times 171.71 = 128.79 \text{ psi}$$

$$v_c = 1.10 \times 2 \times \lambda \times \sqrt{f'_c} \quad \text{spSlab Software Manual (Eq. 2-46)}$$

$$v_c = 1.10 \times 2 \times 1 \times \sqrt{5,000} = 155.56 \text{ psi}$$

$$\phi v_c = 0.75 \times 155.56 = 116.67 \text{ psi}$$

Since  $\phi v_c < v_u$  at the critical section, the slab does not have adequate two-way shear strength around this drop panel.

c) **Corner drop panel:**

$$V_u = V - q_u \times (b_1 \times b_2) = 94.32 - 0.340 \times \left( \frac{89.94 \times 89.94}{144} \right) = 75.22 \text{ kips}$$

The length of the critical perimeter for the corner drop panel:

$$b_o = 89.94 + 89.94 = 179.88 \text{ in.}$$

Where:

$$b_1 = \frac{l_{1,dp}}{2} + \frac{d}{2} + \frac{c_1}{2} = \frac{12 \times 12}{2} + \frac{15.88}{2} + \frac{20}{2} = 89.94 \text{ in.} \quad b_2 = \frac{l_{2,dp}}{2} + \frac{d}{2} + \frac{c_2}{2} = \frac{12 \times 12}{2} + \frac{15.88}{2} + \frac{20}{2} = 89.94 \text{ in.}$$

$$c_{AB} = \frac{b_1^2}{2 \times (b_1 + b_2)} = \frac{89.94^2}{2 \times (89.94 + 89.94)} = 22.48 \text{ in.}$$

The polar moment  $J_c$  of the shear perimeter is:

$$J_c = \left( \frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left( \frac{b_1}{2} - c_{AB} \right)^2 \right) + b_2 \times d \times c_{AB}^2$$

$$J_c = \left( \frac{89.94 \times 3.32^3}{12} + \frac{3.32 \times 89.94^3}{12} + (89.94 \times 3.32) \times \left( \frac{89.94}{2} - 22.48 \right)^2 \right) + 89.94 \times 3.32 \times 22.48^2$$

$$J_c = 503,407.36 \text{ in.}^4$$

The two-way shear stress ( $v_u$ ) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} \quad \text{ACI 318-14 (R.8.4.4.2.3)}$$

$$v_u = \frac{75.22 \times 1,000}{179.88 \times 3.32} = 125.97 \text{ psi}$$

The two-way shear capacity for the ribbed slab is permitted to be increased by 10%.

ACI 318-14 (9.8.1.5)

$$v_c = \min \left\{ \begin{array}{l} 1.10 \times 4 \times \lambda \times \sqrt{f'_c} \\ 1.10 \times \left( 2 + \frac{4}{\beta} \right) \times \lambda \times \sqrt{f'_c} \\ 1.10 \times \left( \frac{\alpha_s \times d}{b_o} + 2 \right) \times \lambda \times \sqrt{f'_c} \end{array} \right\} \quad \text{ACI 318-14 (Table 22.6.5.2)}$$

$$v_c = \min \left\{ \begin{array}{l} 1.10 \times 4 \times 1 \times \sqrt{5,000} \\ 1.10 \times \left( 2 + \frac{4}{1} \right) \times 1 \times \sqrt{5,000} \\ 1.10 \times \left( \frac{20 \times 3.32}{179.88} + 2 \right) \times 1 \times \sqrt{5,000} \end{array} \right\} = \min \left\{ \begin{array}{l} 311.13 \\ 466.69 \\ 184.27 \end{array} \right\} = 184.27 \text{ psi}$$

$$\phi v_c = 0.75 \times 184.27 = 138.21 \text{ psi}$$

$$v_c = 1.10 \times 2 \times \lambda \times \sqrt{f'_c} \quad \text{spSlab Software Manual (Eq. 2-46)}$$

$$v_c = 1.10 \times 2 \times 1 \times \sqrt{5,000} = 155.56 \text{ psi}$$

$$\phi v_c = 0.75 \times 155.56 = 116.67 \text{ psi}$$

Since  $\phi v_c < v_u$  at the critical section, the slab does not have adequate two-way shear strength around this drop panel.

To mitigate the deficiency in two-way shear capacity an evaluation of possible options is required:

1. Increase the thickness of the slab system
2. Increasing the dimensions of the drop panels (length and/or width)
3. Increasing the concrete strength
4. Reduction of the applied loads
5. Reduction of the panel spans
6. Using less conservative punching shear allowable (gain of 5-10%)
7. Refine the deduction of drop panel weight from the shear reaction (gain of 2-5%)

This example will be continued without the required modification discussed above to continue the illustration of the analysis and design procedure.

## 6. Serviceability Requirements (Deflection Check)

Since the slab thickness was selected to meet the minimum slab thickness tables in ACI 318-14, the deflection calculations of immediate and time-dependent deflections are not required. They are shown below for illustration purposes and comparison with [spSlab](#) software results.

### 6.1. Immediate (Instantaneous) Deflections

The calculation of deflections for two-way slabs is challenging even if linear elastic behavior can be assumed. Elastic analysis for three service load levels ( $D$ ,  $D + L_{sustained}$ ,  $D + L_{Full}$ ) is used to obtain immediate deflections of the two-way slab in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. ACI 318-14 (24.2.3)

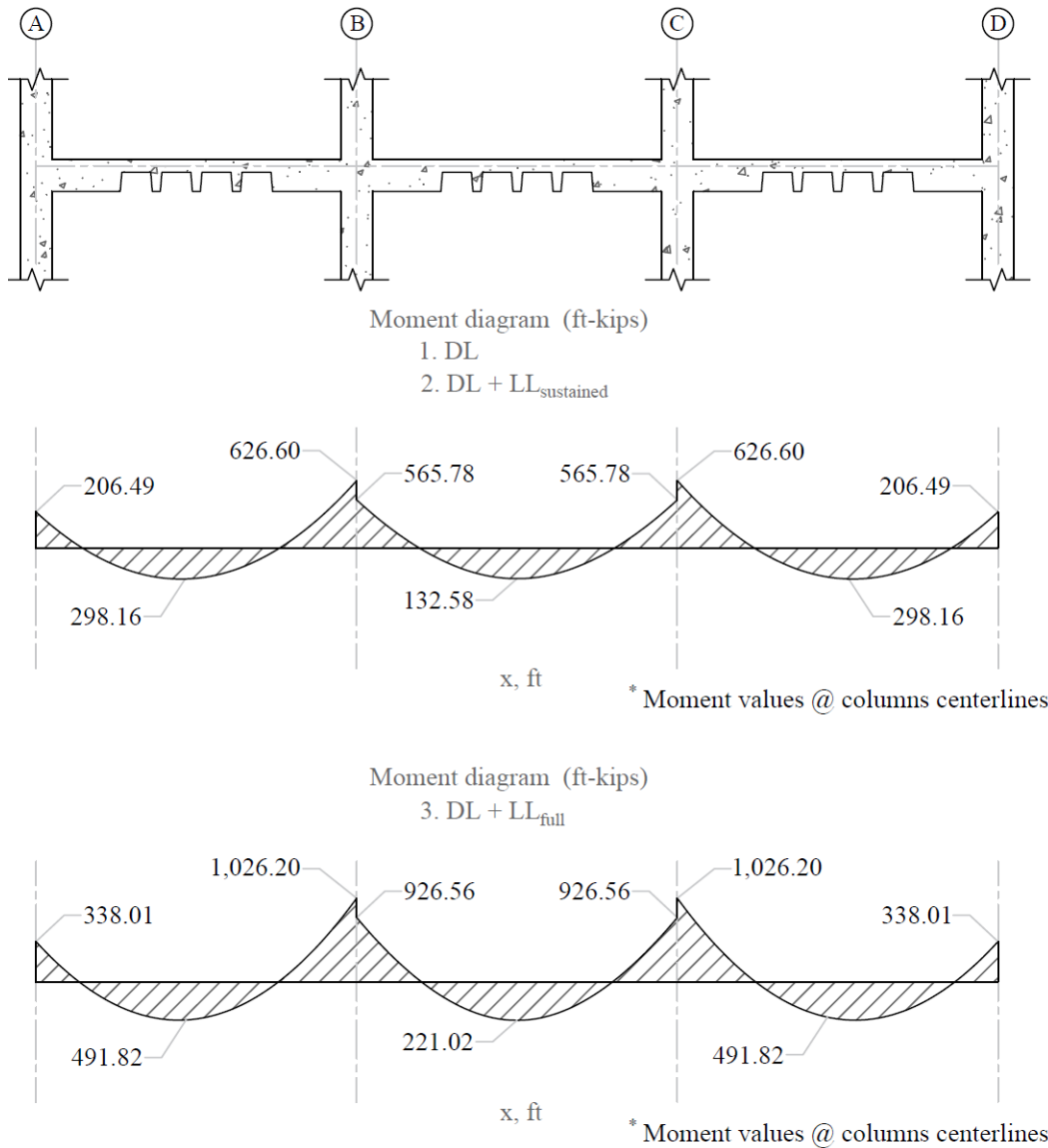
The effective moment of inertia ( $I_e$ ) is used to account for the cracking effect on the flexural stiffness of the slab.  $I_e$  for uncracked section ( $M_{cr} > M_a$ ) is equal to  $I_g$ . When the section is cracked ( $M_{cr} < M_a$ ), then the following equation should be used:

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^3 \times I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \times I_{cr} \leq I_g \quad \text{ACI 318-14 (Eq. 24.2.3.5a)}$$

Where:

$M_a$  = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously in this document. These moments are shown in [Figure 28](#).



**Figure 28 – Maximum Moments for the Three Service Load Levels**

For positive moment (midspan) section:

$M_{cr}$  = cracking moment.

$$M_{cr} = \frac{f_r \times I_g}{y_t} = \frac{530.33 \times 60,254.73}{11.41} \times \frac{1}{12 \times 1,000} = 233.46 \text{ ft-kips}$$

**ACI 318-14 (Eq. 24.2.3.5b)**

$f_r$  = Modulus of rupture of concrete.

$$f_r = 7.5 \times \lambda \times \sqrt{f'_c} = 7.5 \times 1.0 \times \sqrt{5,000} = 530.33 \text{ psi}$$

**ACI 318-14 (Eq. 19.2.3.1)**

$I_g$  = Moment of inertia of the gross uncracked concrete section. See the [following Figure](#).

$$I_g = I_{g/rib} \times \# \text{ of ribs} = 5,477.70 \times 11 = 60,254.73 \text{ in.}^2$$

$y_t$  = Distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.

$$y_t = h_{rib} - y_{bar} = 17.00 - 5.59 = 11.41 \text{ in.}$$

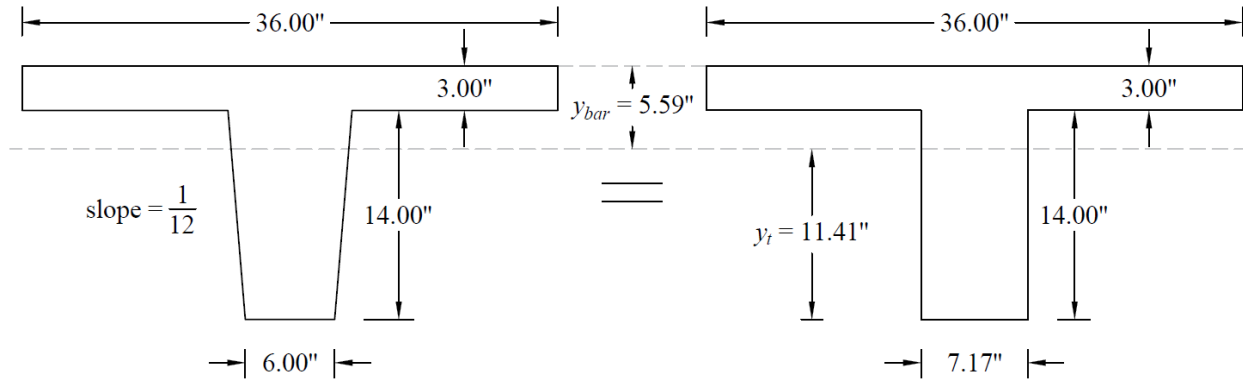


Figure 29 – Equivalent Gross Section for One Rib – Positive Moment Section

$I_{cr}$  = moment of inertia of the cracked section transformed to concrete.

**PCA Notes on ACI 318-11 (9.5.2.2)**

As calculated previously, the positive reinforcement for the middle span frame strip is 22 #6 bars located at 1.125 in. along the section from the bottom of the slab. The [Figure below](#) shows all the parameters needed to calculate the moment of inertia of the cracked section transformed to concrete at midspan.

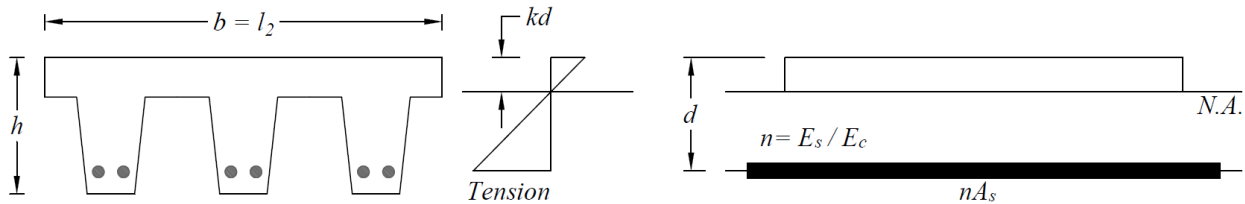


Figure 30 – Cracked Transformed Section – Positive Moment Section

$E_{cs}$  = Modulus of elasticity of slab concrete.

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$

**ACI 318-14 (19.2.2.1.a)**

$$n = \frac{E_s}{E_{cs}} = \frac{29,000,000}{4,287,000} = 6.76$$

**PCA Notes on ACI 318-11 (Table 10-2)**

$$B = \frac{b}{n \times A_s} = \frac{33 \times 12}{6.76 \times (22 \times 0.44)} = 6.05 \text{ in.}^{-1}$$

**PCA Notes on ACI 318-11 (Table 10-2)**

$$kd = \frac{\sqrt{2 \times d \times B + 1} - 1}{B} = \frac{\sqrt{2 \times 15.88 \times 6.05 + 1} - 1}{6.05} = 2.13 \text{ in.}$$

PCA Notes on ACI 318-11 (Table 10-2)

$$I_{cr} = \frac{b \times (kd)^3}{3} + n \times A_s \times (d - kd)^2$$

PCA Notes on ACI 318-11 (Table 10-2)

$$I_{cr} = \frac{33 \times 12 \times (2.13)^3}{3} + 6.76 \times (22 \times 0.44) \times (15.88 - 2.13)^2 = 13,646.72 \text{ in.}^4$$

For negative moment section (near the interior support of the end span):

The negative reinforcement for the end span frame strip near the interior support is 45 #6 bars located at 1.125 in. along the section from the top of the slab.

$$M_{cr} = \frac{f_r \times I_g}{y_t} = \frac{530.33 \times 103,622.30}{9.65} \times \frac{1}{12 \times 1,000} = 474.40 \text{ ft-kips}$$

ACI 318-14 (Eq. 24.2.3.5b)

$$f_r = 7.5 \times \lambda \times \sqrt{f'_c} = 7.5 \times 1.0 \times \sqrt{5,000} = 530.33 \text{ psi}$$

ACI 318-14 (Eq. 19.2.3.1)

$$I_g = 103,622.30 \text{ in.}^2 \text{ (See the following Figure)}$$

Note: A lower value of  $I_g$  (60,254.73 in.<sup>4</sup>) excluding the drop panel is conservatively adopted in calculating waffle slab deflection by the [spSlab](#) software.

$$y_t = 9.65 \text{ in.}$$

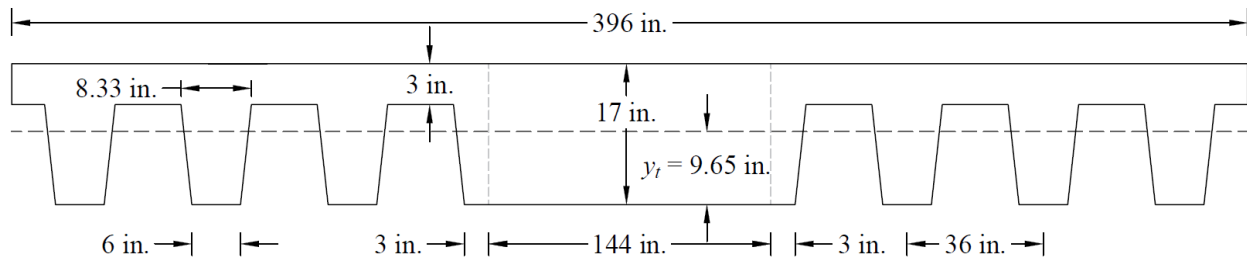


Figure 31 – Gross Section – Negative Moment Section

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$

ACI 318-14 (19.2.2.1.a)

$$n = \frac{E_s}{E_{cs}} = \frac{29,000,000}{4,287,000} = 6.76$$

PCA Notes on ACI 318-11 (Table 10-2)

$$B = \frac{b_{total}}{n \times A_s} = \frac{194.17}{6.76 \times (45 \times 0.44)} = 1.45 \text{ in.}^{-1}$$

PCA Notes on ACI 318-11 (Table 10-2)

Where  $b_{total} = 144.00 + 7 \times 7.17 = 194.17$  in. (See [Figure 31](#) and [Figure 32](#))

$$kd = \frac{\sqrt{2 \times d \times B + 1} - 1}{B} = \frac{\sqrt{2 \times 15.88 \times 1.45 + 1} - 1}{1.45} = 4.04 \text{ in.}$$

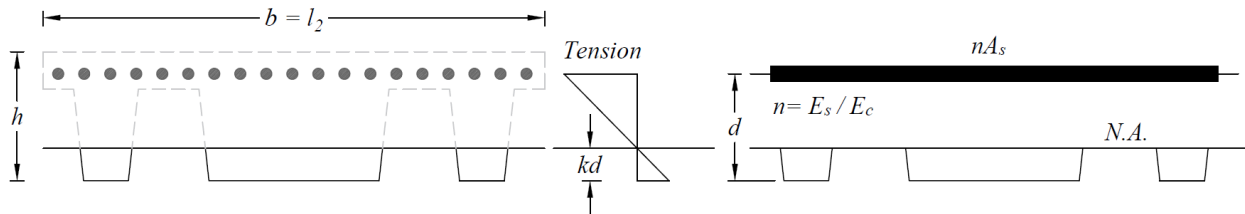
**PCA Notes on ACI 318-11 (Table 10-2)**

$$I_{cr} = \frac{b_{total} \times (kd)^3}{3} + n \times A_s \times (d - kd)^2$$

**PCA Notes on ACI 318-11 (Table 10-2)**

$$I_{cr} = \frac{194.17 \times (4.04)^3}{3} + 6.76 \times (45 \times 0.44) \times (15.88 - 4.04)^2 = 23,028.31 \text{ in.}^4$$

Note: A lower value of  $I_{cr}$  (18,722.37 in.<sup>4</sup>) excluding the drop panel is conservatively adopted in calculating waffle slab deflection by the [spSlab](#) software.



**Figure 32 – Cracked Transformed Section – Negative Moment Section**

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia,  $I_e$ , was developed to provide a transition between the upper and lower bounds of  $I_g$  and  $I_{cr}$  as a function of the ratio  $M_{cr}/M_a$ . For conventionally reinforced (nonprestressed) members, the effective moment of inertia,  $I_e$ , shall be calculated by Eq. (24.2.3.5a) unless obtained by a more comprehensive analysis.

$I_e$  shall be permitted to be taken as the value obtained from Eq. (24.2.3.5a) at midspan for simple and continuous spans, and at the support for cantilevers. **ACI 318-14 (24.2.3.7)**

For continuous one-way slabs and beams,  $I_e$  shall be permitted to be taken as the average of values obtained from Eq. (24.2.3.5a) for the critical positive and negative moment sections. **ACI 318-14 (24.2.3.6)**

For the middle span (span with two ends continuous) with service load level ( $D + LL_{full}$ ):

Since  $M_{cr} = 474.40$  ft-kips  $< M_a = 926.56$  ft-kips

$$I_e^- = \left( \frac{M_{cr}}{M_a} \right)^3 \times I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \times I_{cr}$$

**ACI 318-14 (24.2.3.5a)**

Where  $I_e^-$  is the effective moment of inertia for the critical negative moment section (near the support).



$$I_e^- = \left(\frac{474.40}{926.56}\right)^3 \times 103,622.30 + \left[1 - \left(\frac{474.40}{926.56}\right)^3\right] \times 23,028.31 = 33,845.29 \text{ in.}^4$$

$$I_e^+ = I_g = 60,254.73 \text{ in.}^4, \text{ since } M_{cr} = 233.46 \text{ ft-kips} > M_a = 221.02 \text{ ft-kips}$$

Where  $I_e^+$  is the effective moment of inertia for the critical positive moment section (midspan).

Since midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan section is heavily represented in calculation of  $I_e$  and this is considered satisfactory in approximate deflection calculations. Both the midspan stiffness ( $I_e^+$ ) and averaged span stiffness ( $I_{e,avg}$ ) can be used in the calculation of immediate (instantaneous) deflection.

The averaged effective moment of inertia ( $I_{e,avg}$ ) is given by:

$$I_{e,avg} = 0.70 \times I_e^+ + 0.15 \times (I_{e,l}^- + I_{e,r}^-) \text{ for interior span} \quad \text{PCA Notes on ACI 318-11 (9.5.2.4(2))}$$

$$I_{e,avg} = 0.85 \times I_e^+ + 0.15 \times I_e^- \text{ for end span} \quad \text{PCA Notes on ACI 318-11 (9.5.2.4(1))}$$

However, these expressions lead to improved results only for continuous prismatic members. The drop panels in this example result in non-prismatic members and the following expressions are recommended according to ACI 318-89:

$$I_{e,avg} = 0.50 \times I_e^+ + 0.25 \times (I_{e,l}^- + I_{e,r}^-) \text{ for interior span} \quad \text{ACI 435R-95 (2.14)}$$

For the middle span (span with two ends continuous) with service load level ( $D + LL_{full}$ ):

$$I_{e,avg} = 0.50 \times 60,254.73 + 0.25 \times (33,845.29 + 33,845.29) = 47,050.01 \text{ in.}^4$$

$$I_{e,avg} = 0.50 \times I_e^+ + 0.50 \times I_e^- \text{ for end span} \quad \text{ACI 435R-95 (2.14)}$$

For the end span (span with one end continuous) with service load level ( $D + LL_{full}$ ):

$$I_{e,avg} = 0.50 \times 20,378.52 + 0.50 \times 30,990.47 = 25,684.49 \text{ in.}^4$$

Where:

- $I_{e,l}^-$  = The effective moment of inertia for the critical negative moment section near the left support.
- $I_{e,r}^-$  = The effective moment of inertia for the critical negative moment section near the right support.
- $I_e^+$  = The effective moment of inertia for the critical positive moment section (midspan).

Note: The prismatic member equations excluding the effect of the drop panel are conservatively adopted in calculating waffle slab deflection by [spSlab](#).

The [following Table](#) provides a summary of the required parameters and calculated values needed for deflections for exterior and interior spans.

**Table 6 - Averaged Effective Moment of Inertia Calculations**

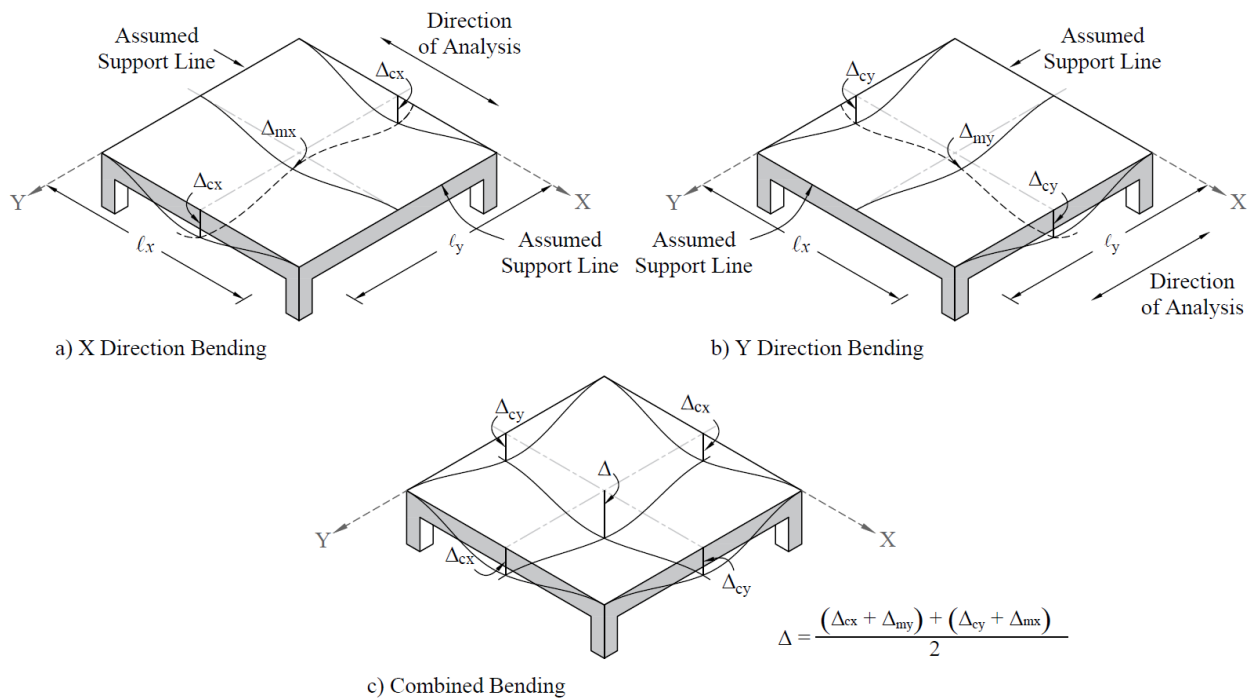
**For Frame Strip**

Span	zone	$I_g$ (in. <sup>4</sup> )	$I_{cr}$ (in. <sup>4</sup> )	$M_a$ (ft-kips)			$M_{cr}$ (k-ft)	$I_e$ (in. <sup>4</sup> )			$I_{e,avg}$ (in. <sup>4</sup> )		
				D	D + LL <sub>Sus</sub>	D + L <sub>full</sub>		D	D + LL <sub>Sus</sub>	D + L <sub>full</sub>	D	D + LL <sub>Sus</sub>	D + L <sub>full</sub>
Ext	Left	103,622	15,504	206.49	206.49	338.01	474.40	103,622	103,622	103,622	47,520	47,520	25,684
	Midspan	60,255	15,603	298.16	298.16	491.82	233.46	37,037	37,037	20,379			
	Right	103,622	23,028	626.60	626.60	1,026.20	474.40	58,003	58,003	30,990			
Int	Left	103,622	23,028	565.78	565.78	926.56	474.40	70,538	70,538	33,845	65,396	65,396	47,050
	Midspan	60,255	13,647	132.58	132.58	221.02	233.46	60,255	60,255	60,255			
	Right	103,622	23,028	565.78	565.78	926.56	474.40	70,538	70,538	33,845			

Deflections in two-way slab systems shall be calculated taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. For immediate deflections in two-way slab systems, the midpanel deflection is computed as the sum of deflection at midspan of the column strip or column line in one direction ( $\Delta_{cx}$  or  $\Delta_{cy}$ ) and deflection at midspan of the middle strip in the orthogonal direction ( $\Delta_{mx}$  or  $\Delta_{my}$ ). [Figure 33](#) shows the deflection computation for a rectangular panel. The average  $\Delta$  for panels that have different properties in the two direction is calculated as follows:

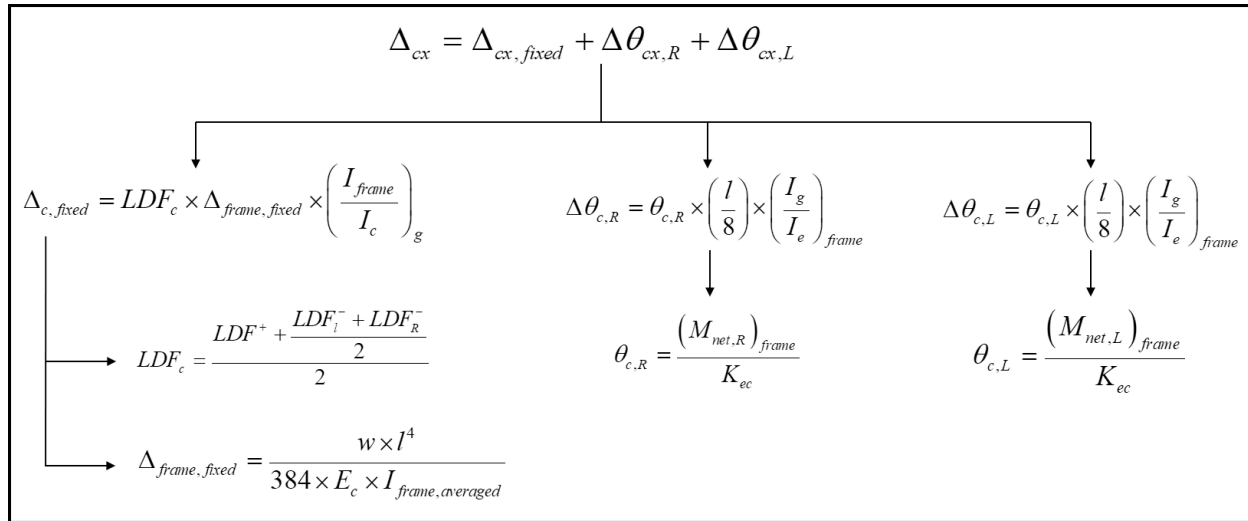
$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2}$$

***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)***



**Figure 33 – Deflection Computation for a Rectangular Panel**

To calculate each term of the previous equation, the following procedure should be used. [Figure 34](#) shows the procedure of calculating the term  $\Delta_{cx}$ . Same procedure can be used to find the other terms.



**Figure 34 –  $\Delta_{cx}$  Calculation Procedure**

For end span - service dead load case:

$$\Delta_{frame, fixed} = \frac{w \times l^4}{384 \times E_c \times I_{frame, averaged}}$$

**PCA Notes on ACI 318-11 (9.5.3.4 Eq. 10)**

Where:

$\Delta_{frame, fixed}$  = Deflection of frame strip assuming fixed-end condition.

$$w_{SDL+slab} = \left( 50.00 + 150 \times \frac{8}{12} \right) \times 33 = 4,950.00 \frac{\text{lb}}{\text{ft}}$$

$$w_{drop\ panel} = \left( 150 \times \frac{17}{12} \right) \times 12 = 2,550.00 \frac{\text{lb}}{\text{ft}}$$

$$w = \frac{4,950.00 \times (33 - 12) + (4,950.00 + 2,550.00) \times (12 - 20/12)}{(33 - 20/12)} = 5,790.96 \frac{\text{lb}}{\text{ft}}$$

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$

**ACI 318-14 (19.2.2.1.a)**

$I_{frame, averaged}$  = The averaged effective moment of inertia ( $I_{e, avg}$ ) for the frame strip for service dead load case from [Table 6](#) = 47,520.23 in.<sup>4</sup>

$$\Delta_{frame, fixed} = \frac{5,790.96 \times (33 - 20/12)^4 \times 12^3}{384 \times (4,287 \times 10^3) \times 47,520.23} = 0.1233 \text{ in.}$$

$$\Delta_{c, fixed} = LDF_c \times \Delta_{frame, fixed} \times \left( \frac{I_{frame}}{I_c} \right)_g$$

**PCA Notes on ACI 318-11 (9.5.3.4 Eq. 11)**

$LDF_c$  is the load distribution factor for the column strip. The load distribution factor for the column strip can be found from the following equation:

$$LDF_c = \frac{LDF^+ + \frac{LDF_l^- + LDF_r^-}{2}}{2}$$

**spSlab Software Manual (Eq. 2-114)**

And the load distribution factor for the middle strip can be found from the following equation:

$$LDF_m = 1 - LDF_c$$

**spSlab Software Manual (Eq. 2-115)**

Taking for example the end span where highest deflections are expected, the LDF for exterior negative region ( $LDF_l^-$ ), interior negative region ( $LDF_r^-$ ), and positive region ( $LDF^+$ ) are 1.00, 0.75, and 0.60, respectively (From [Table 2](#) of this document). Thus, the load distribution factor for the column strip for the end span is given by:

$$LDF_c = \frac{0.6 + \frac{1.0 + 0.75}{2}}{2} = 0.738$$

$I_{c,g}$  = The gross moment of inertia ( $I_g$ ) for the column strip for service dead load = 28,289.32 in.<sup>4</sup>

$$\Delta_{c, fixed} = 0.738 \times 0.1233 \times \frac{60,254.73}{28,289.32} = 0.1937 \text{ in.}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$

**PCA Notes on ACI 318-11 (9.5.3.4 Eq. 12)**

Where:

$\theta_{c,L}$  = Rotation of the span left support

$(M_{net,L})_{frame}$  = 206.49 ft-kips = Net frame strip negative moment of the left support

$K_{ec}$  = Effective column stiffness =  $1,925.34 \times 10^6$  in.-lb ([calculated above](#)).

$$\theta_{c,L} = \frac{206.49 \times 12 \times 1,000}{1,925.34 \times 10^6} = 0.00129 \text{ rad}$$

$$\Delta\theta_{c,L} = \theta_{c,L} \times \left( \frac{l}{8} \right) \times \left( \frac{I_g}{I_e} \right)_{frame}$$

**PCA Notes on ACI 318-11 (9.5.3.4 Eq. 14)**

Where:

$\Delta\theta_{c,L}$  = Midspan deflection due to rotation of left support.

$(I_g / I_e)_{frame}$  = Gross-to-effective moment of inertia ratio for frame strip.

$$\Delta\theta_{c,L} = 0.00129 \times \frac{33 \times 12 - 20}{8} \times \frac{60,254.73}{47,520.23} = 0.0767 \text{ in.}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}} = \frac{(626.60 - 565.78) \times 12 \times 1,000}{1,925.34 \times 10^6} = 0.00038 \text{ rad}$$

Where:

$\theta_{c,R}$  = rotation of the end span right support.

$(M_{net,R})_{frame}$  = Net frame strip negative moment of the right support.

$$\Delta\theta_{c,R} = \theta_{c,R} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame} = 0.00038 \times \frac{33 \times 12 - 20}{8} \times \frac{60,254.73}{47,520.23} = 0.0226 \text{ in.}$$

Where:

$\Delta\theta_{c,R}$  = Midspan deflection due to rotation of right support.

$$\Delta_{cx} = \Delta_{cx, fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

**PCA Notes on ACI 318-11 (9.5.3.4 Eq. 9)**

$$\Delta_{cx} = 0.1937 + 0.0767 + 0.0226 = 0.2930 \text{ in.}$$

Following the same procedure,  $\Delta_{mx}$  can be calculated for the middle strip. This procedure is repeated for the equivalent frame in the orthogonal direction to obtain  $\Delta_{cy}$  and  $\Delta_{my}$  for the end and middle spans for the other load levels ( $D + LL_{sus}$  and  $D + LL_{full}$ ).

Since this example has square panels,  $\Delta_{cx} = \Delta_{cy} = 0.2930 \text{ in.}$ , and  $\Delta_{mx} = \Delta_{my} = 0.1682 \text{ in.}$

The average  $\Delta$  for the corner panel is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2} = (\Delta_{cx} + \Delta_{my}) = (\Delta_{cy} + \Delta_{mx}) = 0.2930 + 0.1682 = 0.4612 \text{ in.}$$

**Table 7 - Immediate (Instantaneous) Deflections in the x-direction**

Column Strip									Middle Strip							
Span	LDF	D							LDF	D						
		$\Delta_{\text{frame-fixed}}$ (in.)	$\Delta_{\text{c-fixed}}$ (in.)	$\theta_{\text{c1}}$ (rad)	$\theta_{\text{c2}}$ (rad)	$\Delta\theta_{\text{c1}}$ (in.)	$\Delta\theta_{\text{c2}}$ (in.)	$\Delta_{\text{cx}}$ (in.)		$\Delta_{\text{frame-fixed}}$ (in.)	$\Delta_{\text{m-fixed}}$ (in.)	$\theta_{\text{m1}}$ (rad)	$\theta_{\text{m2}}$ (rad)	$\Delta\theta_{\text{m1}}$ (in.)	$\Delta\theta_{\text{m2}}$ (in.)	$\Delta_{\text{mx}}$ (in.)
Ext	0.738	0.1233	0.1937	0.00129	0.00038	0.0767	0.0226	0.2930	0.263	0.1233	0.0689	0.00129	0.00038	0.0767	0.0226	0.1682
Int	0.675	0.0896	0.1288	-0.00038	-0.00038	-0.0164	-0.0164	0.0960	0.325	0.0896	0.0620	-0.00038	-0.00038	-0.0164	-0.0164	0.0292

Span	LDF	D+LL <sub>sus</sub>							LDF	D+LL <sub>sus</sub>						
		$\Delta_{\text{frame-fixed}}$ (in.)	$\Delta_{\text{c-fixed}}$ (in.)	$\theta_{\text{c1}}$ (rad)	$\theta_{\text{c2}}$ (rad)	$\Delta\theta_{\text{c1}}$ (in.)	$\Delta\theta_{\text{c2}}$ (in.)	$\Delta_{\text{cx}}$ (in.)		$\Delta_{\text{frame-fixed}}$ (in.)	$\Delta_{\text{m-fixed}}$ (in.)	$\theta_{\text{m1}}$ (rad)	$\theta_{\text{m2}}$ (rad)	$\Delta\theta_{\text{m1}}$ (in.)	$\Delta\theta_{\text{m2}}$ (in.)	$\Delta_{\text{mx}}$ (in.)
Ext	0.738	0.1233	0.1937	0.00129	0.00038	0.0767	0.0226	0.2930	0.263	0.1233	0.0689	0.00129	0.00038	0.0767	0.0226	0.1682
Int	0.675	0.0896	0.1288	-0.00038	-0.00038	-0.0164	-0.0164	0.0960	0.325	0.0896	0.0620	-0.00038	-0.00038	-0.0164	-0.0164	0.0292

Span	LDF	D+LL <sub>full</sub>							LDF	D+LL <sub>full</sub>						
		$\Delta_{\text{frame-fixed}}$ (in.)	$\Delta_{\text{c-fixed}}$ (in.)	$\theta_{\text{c1}}$ (rad)	$\theta_{\text{c2}}$ (rad)	$\Delta\theta_{\text{c1}}$ (in.)	$\Delta\theta_{\text{c2}}$ (in.)	$\Delta_{\text{cx}}$ (in.)		$\Delta_{\text{frame-fixed}}$ (in.)	$\Delta_{\text{m-fixed}}$ (in.)	$\theta_{\text{m1}}$ (rad)	$\theta_{\text{m2}}$ (rad)	$\Delta\theta_{\text{m1}}$ (in.)	$\Delta\theta_{\text{m2}}$ (in.)	$\Delta_{\text{mx}}$ (in.)
Ext	0.738	0.3581	0.5625	0.00211	0.00062	0.2323	0.0685	0.8633	0.263	0.3581	0.2002	0.00211	0.00062	0.2323	0.0685	0.5010
Int	0.675	0.1955	0.2811	-0.00062	-0.00062	-0.0374	-0.0374	0.2063	0.325	0.1955	0.1353	-0.00062	-0.00062	-0.0374	-0.0374	0.0606

Span	LDF	LL
		$\Delta_{\text{cx}}$ (in.)
Ext	0.738	0.5703
Int	0.675	0.1103

LDF	LL
	$\Delta_{\text{mx}}$ (in.)
0.263	0.3328
0.325	0.0314



## 6.2. Time-Dependent (Long-Term) Deflections ( $\Delta_{lt}$ )

The additional time-dependent (long-term) deflection resulting from creep and shrinkage ( $\Delta_{cs}$ ) may be estimated as follows:

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst} \quad \text{PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)}$$

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1 + \lambda_{\Delta}) + [(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}] \quad \text{CSA A23.3-04 (N9.8.2.5)}$$

Where:

$(\Delta_{sust})_{Inst}$  = Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_{\Delta} = \frac{\xi}{1 + 50 \times \rho'} \quad \text{ACI 318-14 (24.2.4.1.1)}$$

$(\Delta_{total})_{lt}$  = Time-dependent (long-term) total deflection, in.

$(\Delta_{total})_{Inst}$  = Total immediate (instantaneous) deflection, in.

For the exterior span

$\xi = 2$ , consider the sustained load duration to be 60 months or more. ACI 318-14 (Table 24.2.4.1.3)

$\rho' = 0$ , conservatively.

$$\lambda_{\Delta} = \frac{2}{1 + 50 \times 0} = 2$$

$$\Delta_{cs} = 2 \times 0.2930 = 0.5859 \text{ in.}$$

$$(\Delta_{total})_{lt} = 0.2930 \times (1 + 2) + (0.8633 - 0.2930) = 1.4492 \text{ in.}$$

The [following Table](#) shows long-term deflections for the exterior and interior spans for the analysis in the x-direction, for column and middle strips.

Table 8 - Long-Term Deflections					
Column Strip					
Span	$(\Delta_{\text{sust}})_{\text{Inst}}$ (in.)	$\lambda_{\Delta}$	$\Delta_{\text{cs}}$ (in.)	$(\Delta_{\text{total}})_{\text{Inst}}$ (in.)	$(\Delta_{\text{total}})_{\text{lt}}$ (in.)
Exterior	0.2930	2	0.5859	0.8633	1.4492
Interior	0.0960	2	0.1920	0.2063	0.3983
Middle Strip					
Exterior	0.1682	2	0.3365	0.5010	0.8374
Interior	0.0292	2	0.0584	0.0606	0.1189

## 7. spSlab Software Program Model Solution

[spSlab](#) program utilizes the Equivalent Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems with drop panels. [spSlab](#) uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept (ACI 318-14 (R8.11.4)).

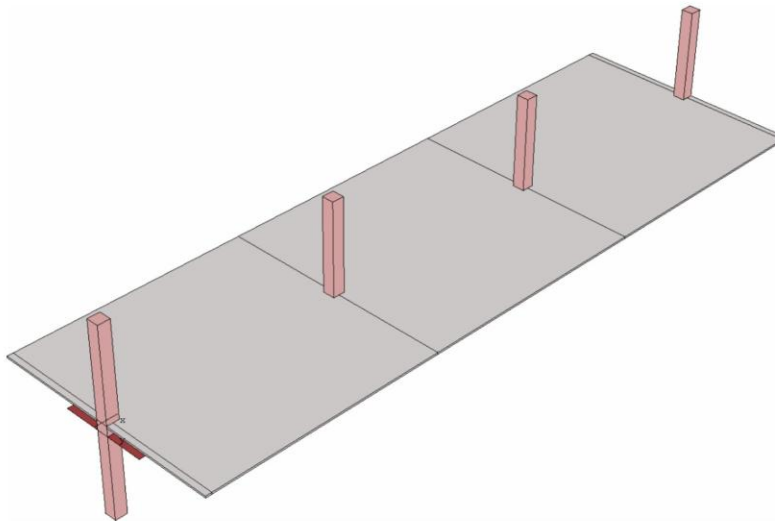
[spSlab](#) Program models the equivalent frame as a design strip. The design strip is, then, separated by [spSlab](#) into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips.



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spSlab v10.00 (TM)  
A Computer Program for Analysis, Design, and Investigation of  
Reinforced Concrete Beams, One-way and Two-way Slab Systems  
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## 1. Input Echo

### 1.1. General Information

File Name	...DE-Two-Way-Joist-Waffle-System-ACI-318-14....
Project	Two-Way Joist (Waffle) System
Frame	Interior Frame
Engineer	SP
Code	ACI 318-14
Units	English
Reinforcement Database	ASTM A615
Mode	Design
Number of supports =	4 + Left Cantilever + Right Cantilever
Floor System	Two-Way

### 1.2. Solve Options

Live load pattern ratio = 0%
Minimum free edge distance for punching shear = 4 times slab thickness.
Circular critical section around circular supports used (if possible).
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
User-defined slab strip widths NOT selected.
User-defined distribution factors NOT selected.
One-way shear in drop panel selected.
Distribution of shear to strips NOT selected.
Beam T-section design NOT selected.
Longitudinal beam contribution in negative reinforcement design over support NOT selected.
Transverse beam contribution in negative reinforcement design over support NOT selected.

### 1.3. Material Properties

#### 1.3.1. Concrete: Slabs / Beams

$w_c$	150 pcf
$f'_c$	5 ksi
$E_c$	4286.83 ksi
$f_r$	0.53033 ksi

#### 1.3.2. Concrete: Columns

$w_c$	150 pcf
$f'_c$	6 ksi
$E_c$	4695.98 ksi
$f_r$	0.580948 ksi

#### 1.3.3. Reinforcing Steel

$f_y$	60 ksi
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$f_{yt}$	60 ksi
$E_s$	29000 ksi
Epoxy coated bars	No

#### 1.4. Reinforcement Database

Size	Db in	Ab in <sup>2</sup>	Wb lb/ft	Size	Db in	Ab in <sup>2</sup>	Wb lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

#### 1.5. Span Data

##### 1.5.1. Slabs

Notes:

Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:

\*i - cantilever end span (LC, RC) support condition

Span	Loc	L1 ft	t in	wL ft	wR ft	L2L ft	L2R ft	H <sub>min</sub> in
1	Int	0.833	3.00	16.500	16.500	33.000	33.000	--- LC *i
2	Int	33.000	3.00	16.500	16.500	33.000	33.000	2.50 ---
3	Int	33.000	3.00	16.500	16.500	33.000	33.000	2.50 ---
4	Int	33.000	3.00	16.500	16.500	33.000	33.000	2.50 ---
5	Int	0.833	3.00	16.500	16.500	33.000	33.000	--- RC *i

##### 1.5.2. Ribs and Longitudinal Beams

Notes:

Span	Ribs			Beams		
	b in	h in	Sp in	b in	h in	Offset in
1	6.00	14.00	30.00	0.00	0.00	0.00
2	6.00	14.00	30.00	0.00	0.00	0.00
3	6.00	14.00	30.00	0.00	0.00	0.00
4	6.00	14.00	30.00	0.00	0.00	0.00
5	6.00	14.00	30.00	0.00	0.00	0.00

#### 1.6. Support Data

##### 1.6.1. Columns

Support	c1a in	c2a in	Ha ft	c1b in	c2b in	Hb ft	Red %
1	20.00	20.00	13.000	20.00	20.00	13.000	100
2	20.00	20.00	13.000	20.00	20.00	13.000	100
3	20.00	20.00	13.000	20.00	20.00	13.000	100
4	20.00	20.00	13.000	20.00	20.00	13.000	100



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### 1.6.2. Drop Panels

Notes:

Support	h in	Li ft	Lr ft	WI ft	Wr ft
1	0.00	0.833	6.000	6.000	6.000
2	0.00	6.000	6.000	6.000	6.000
3	0.00	6.000	6.000	6.000	6.000
4	0.00	6.000	0.833	6.000	6.000

### 1.6.3. Boundary Conditions

Support	Spring		Far End	
	K <sub>x</sub> kips/in	K <sub>y</sub> kip-in/rad	Above	Below
1	0.00	0.00	Fixed	Fixed
2	0.00	0.00	Fixed	Fixed
3	0.00	0.00	Fixed	Fixed
4	0.00	0.00	Fixed	Fixed

## 1.7. Load Data

### 1.7.1. Load Cases and Combinations

Case	SELF	Dead	Live
Type	DEAD	DEAD	LIVE
U1	1.200	1.200	1.600

### 1.7.2. Area Loads

Case/Patt	Span	Wa psf
SELF	1	100.06
	2	100.06
	3	100.06
	4	100.06
	5	100.06
Dead	1	50.00
	2	50.00
	3	50.00
	4	50.00
	5	50.00
Live	1	100.00
	2	100.00
	3	100.00
	4	100.00
	5	100.00

### 1.7.3. Line Loads

Case/Patt	Span	Wa plf	La ft	Wb plf	Lb ft
SELF	1	1349.32	0.000	1349.32	0.833
	2	1349.32	0.000	1349.32	6.000
	2	1349.32	27.000	1349.32	33.000
	3	1349.32	0.000	1349.32	6.000
	3	1349.32	27.000	1349.32	33.000
	4	1349.32	0.000	1349.32	6.000
	4	1349.32	27.000	1349.32	33.000

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Case/Patt	Span	Wa plf	La ft	Wb plf	Lb ft
	5	1349.32	0.000	1349.32	0.833

## 1.8. Reinforcement Criteria

### 1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#6	#8	#6	#8
Bar spacing	in	1.00	18.00	1.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	in	0.75		0.75	

There is NOT more than 12 in of concrete below top bars.

### 1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#5	#8	#5	#8	#3	#5
Bar spacing	in	1.00	18.00	1.00	18.00	6.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	1.50		1.50			
Layer dist.	in	1.00		1.00			
No. of legs						2	6
Side cover	in					1.50	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

## 2. Design Results\*

\*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

### 2.1. Strip Widths and Distribution Factors

Notes:

\*Used for bottom reinforcement. \*\*Used for top reinforcement.

Span	Strip	Width			Moment Factor		
		Left **	Right **	Bottom *	Left **	Right **	Bottom *
		ft	ft	ft			
1	Column	16.50	16.50	16.50	1.000	1.000	0.600
	Middle	16.50	16.50	16.50	0.000	0.000	0.400
2	Column	16.50	16.50	16.50	1.000	0.750	0.600
	Middle	16.50	16.50	16.50	0.000	0.250	0.400
3	Column	16.50	16.50	16.50	0.750	0.750	0.600
	Middle	16.50	16.50	16.50	0.250	0.250	0.400
4	Column	16.50	16.50	16.50	0.750	1.000	0.600
	Middle	16.50	16.50	16.50	0.250	0.000	0.400
5	Column	16.50	16.50	16.50	1.000	1.000	0.600
	Middle	16.50	16.50	16.50	0.000	0.000	0.400

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## 2.2. Top Reinforcement

Notes:

\*3 - Design governed by minimum reinforcement.

\*5 - Number of bars governed by maximum allowable spacing.

Span Strip	Zone	Width ft	M <sub>max</sub> kip-ft	X <sub>max</sub> ft	A <sub>s,min</sub> in <sup>2</sup>	A <sub>s,max</sub> in <sup>2</sup>	A <sub>s,req</sub> in <sup>2</sup>	Sp <sub>Prov</sub> in	Bars
1 Column	Left	16.50	0.41	0.241	2.853	10.120	0.006	14.143	14-#6 *3 *5
	Midspan	16.50	1.32	0.447	2.853	10.120	0.018	14.143	14-#6 *3 *5
	Right	16.50	3.09	0.688	5.184	51.614	0.043	14.143	14-#6 *3 *5
	Middle	Left	16.50	0.00	0.000	2.853	12.144	14.143	14-#6 *3 *5
		Midspan	16.50	0.00	0.344	2.853	12.144	14.143	14-#6 *3 *5
		Right	16.50	0.00	0.688	2.853	12.144	14.143	14-#6 *3 *5
	2 Column	Left	16.50	323.83	0.833	5.184	51.614	4.594	14.143 14-#6 *3 *5
		Midspan	16.50	0.00	16.500	0.000	10.120	0.000	---
		Right	16.50	907.33	32.167	5.184	51.614	13.208	6.387 31-#6
2 Column	Middle	Left	16.50	1.39	2.063	2.853	12.144	0.019	14.143 14-#6 *3 *5
		Midspan	16.50	0.00	16.500	0.000	12.144	0.000	---
		Right	16.50	302.44	32.167	2.853	12.144	4.482	14.143 14-#6 *5
	3 Column	Left	16.50	823.67	0.833	5.184	51.614	11.945	6.387 31-#6
		Midspan	16.50	0.00	16.500	0.000	10.120	0.000	---
		Right	16.50	823.68	32.167	5.184	51.614	11.945	6.387 31-#6
	Middle	Left	16.50	274.56	0.833	2.853	12.144	4.045	14.143 14-#6 *5
		Midspan	16.50	0.00	16.500	0.000	12.144	0.000	---
		Right	16.50	274.56	32.167	2.853	12.144	4.046	14.143 14-#6 *5
3 Column	4 Column	Left	16.50	907.32	0.833	5.184	51.614	13.208	6.387 31-#6
		Midspan	16.50	0.00	16.500	0.000	10.120	0.000	---
		Right	16.50	323.84	32.167	5.184	51.614	4.595	14.143 14-#6 *3 *5
	Middle	Left	16.50	302.44	0.833	2.853	12.144	4.482	14.143 14-#6 *5
		Midspan	16.50	0.00	16.500	0.000	12.144	0.000	---
		Right	16.50	1.39	30.937	2.853	12.144	0.019	14.143 14-#6 *3 *5
	5 Column	Left	16.50	3.10	0.146	5.184	51.614	0.043	14.143 14-#6 *3 *5
		Midspan	16.50	1.32	0.386	2.853	10.120	0.018	14.143 14-#6 *3 *5
		Right	16.50	0.41	0.593	2.853	10.120	0.006	14.143 14-#6 *3 *5
4 Column	Middle	Left	16.50	0.00	0.146	2.853	12.144	0.000	14.143 14-#6 *3 *5
		Midspan	16.50	0.00	0.490	2.853	12.144	0.000	14.143 14-#6 *3 *5
		Right	16.50	0.00	0.833	2.853	12.144	0.000	14.143 14-#6 *3 *5

## 2.3. Top Bar Details

NOTES:

\* - Bar cut-off location does not meet ACI 318, 12.10.5.1. Revise location, unless the requirements of either 12.10.5.2 or 12.10.5.3 are manually checked and satisfied.

Span Strip	Bars	Left		Continuous		Right			
		Length ft	Bars	Length ft	Bars	Length ft	Bars	Length ft	Bars
1 Column	---		---		14-#6	0.83	---		---
Middle	---		---		14-#6	0.83	---		---

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Span Strip	Bars	Left		Length ft	Continuous		Length ft	Right		Length ft	
		Bars	Length ft		Bars	Length ft		Bars	Length ft		
2 Column	12-#6	11.17	2-#6	7.10	---	---	---	16-#6	11.17	15-#6 *	7.10
Middle	14-#6	7.73	---	---	---	---	---	14-#6	10.21	---	---
3 Column	16-#6	11.21	15-#6 *	7.10	---	---	---	16-#6	11.21	15-#6 *	7.10
Middle	14-#6	11.21	---	---	---	---	---	14-#6	11.21	---	---
4 Column	16-#6	11.17	15-#6 *	7.10	---	---	---	12-#6	11.17	2-#6	7.10
Middle	14-#6	10.21	---	---	---	---	---	14-#6	7.73	---	---
5 Column	---	---	---	---	14-#6	0.83	---	---	---	---	---
Middle	---	---	---	---	14-#6	0.83	---	---	---	---	---

#### 2.4. Top Bar Development Lengths

Span Strip	Left				Continuous		Right			
	Bars	DevLen in	Bars	DevLen in	Bars	DevLen in	Bars	DevLen in	Bars	DevLen in
1 Column	---		---		14-#6	12.00	---		---	
Middle	---		---		14-#6	12.00	---		---	
2 Column	12-#6	18.99	2-#6	18.99	---		16-#6	24.65	15-#6	24.65
Middle	14-#6	12.00	---		---		14-#6	18.52	---	
3 Column	16-#6	22.29	15-#6	22.29	---		16-#6	22.29	15-#6	22.29
Middle	14-#6	16.72	---		---		14-#6	16.72	---	
4 Column	16-#6	24.65	15-#6	24.65	---		12-#6	18.99	2-#6	18.99
Middle	14-#6	18.52	---		---		14-#6	12.00	---	
5 Column	---		---		14-#6	12.00	---		---	
Middle	---		---		14-#6	12.00	---		---	

#### 2.5. Bottom Reinforcement

Notes:

\*3 - Design governed by minimum reinforcement.

Span Strip	Width ft	M <sub>max</sub> kip-ft	X <sub>max</sub> ft	A <sub>s,min</sub> in <sup>2</sup>	A <sub>s,max</sub> in <sup>2</sup>	A <sub>s,req</sub> in <sup>2</sup>	Sp <sub>Prov</sub> in	Bars
1 Column	16.50	0.00	0.344	0.000	66.794	0.000	0.000	---
Middle	16.50	0.00	0.344	0.000	66.794	0.000	0.000	---
2 Column	16.50	400.59	14.000	2.853	66.531	5.703	3.823	10-#7
Middle	16.50	267.06	14.000	2.853	66.794	3.770	3.938	12-#6
3 Column	16.50	180.35	16.500	2.853	66.794	2.539	3.938	10-#6 *3
Middle	16.50	120.24	16.500	2.853	66.794	1.690	3.938	12-#6 *3
4 Column	16.50	400.59	19.000	2.853	66.531	5.703	3.823	10-#7
Middle	16.50	267.06	19.000	2.853	66.794	3.770	3.938	12-#6
5 Column	16.50	0.00	0.833	0.000	66.794	0.000	0.000	---
Middle	16.50	0.00	0.833	0.000	66.794	0.000	0.000	---

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## 2.6. Bottom Bar Details

Span	Strip	Long Bars			Short Bars			Waffle		
		Bars	Start	Length	Bars	Start	Length	Ribs	Bars/Rib	A <sub>s</sub> /Rib
			ft	ft		ft	ft			in <sup>2</sup>
1	Column	---			---			---		
	Middle	---			---			---		
2	Column	10-#7	0.00	33.00	---			5	2-#7	1.200
	Middle	12-#6	0.00	33.00	---			6	2-#6	0.880
3	Column	10-#6	0.00	33.00	---			5	2-#6	0.880
	Middle	12-#6	0.00	33.00	---			6	2-#6	0.880
4	Column	10-#7	0.00	33.00	---			5	2-#7	1.200
	Middle	12-#6	0.00	33.00	---			6	2-#6	0.880
5	Column	---			---			---		
	Middle	---			---			---		

## 2.7. Bottom Bar Development Lengths

Span	Strip	Long Bars		Short Bars	
		Bars	DevLen	Bars	DevLen
			in		in
1	Column	---		---	
	Middle	---		---	
2	Column	10-#7	39.00	---	
	Middle	12-#6	18.18	---	
3	Column	10-#6	14.69	---	
	Middle	12-#6	12.00	---	
4	Column	10-#7	39.00	---	
	Middle	12-#6	18.18	---	
5	Column	---		---	
	Middle	---		---	

## 2.8. Flexural Capacity

Span	Strip	Top						Bottom					
		x	A <sub>s,top</sub>	ΦM <sub>n</sub> -	M <sub>u</sub> -	Comb Pat	Status	A <sub>s,bot</sub>	ΦM <sub>n</sub> +	M <sub>u</sub> +	Comb Pat	Status	
		ft	in <sup>2</sup>	kip-ft	kip-ft			in <sup>2</sup>	kip-ft	kip-ft			
1	Column	0.000	6.16	-399.88	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.241	6.16	-399.88	-0.41	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.417	6.16	-399.88	-1.11	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.447	6.16	-432.18	-1.32	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.688	6.16	-432.18	-3.09	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.833	6.16	-432.18	-4.46	U1 All	---	0.00	0.00	0.00	U1 All	---	
	Middle	0.000	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.241	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.417	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.447	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.688	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK	
		0.833	6.16	-406.57	0.00	U1 All	---	0.00	0.00	0.00	U1 All	---	

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Span Strip	Top						Bottom				
	x ft	A <sub>s,top</sub> in²	ΦM <sub>n</sub> - kip-ft	M <sub>u</sub> - kip-ft	Comb Pat	Status	A <sub>s,bot</sub> in²	ΦM <sub>n</sub> + kip-ft	M <sub>u</sub> + kip-ft	Comb Pat	Status
2 Column	0.000	6.16	-432.18	-461.26	U1 All	---	6.00	421.16	0.00	U1 All	---
	0.833	6.16	-432.18	-323.83	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	5.518	6.16	-432.18	0.00	U1 All	OK	6.00	421.16	159.36	U1 All	OK
	6.000	5.89	-413.70	0.00	U1 All	OK	6.00	421.16	186.15	U1 All	OK
	6.000	5.89	-384.13	0.00	U1 All	OK	6.00	421.16	186.19	U1 All	OK
	7.100	5.28	-347.67	0.00	U1 All	OK	6.00	421.16	241.16	U1 All	OK
	9.591	5.28	-347.67	0.00	U1 All	OK	6.00	421.16	335.68	U1 All	OK
	11.173	0.00	0.00	0.00	U1 All	OK	6.00	421.16	374.01	U1 All	OK
	11.800	0.00	0.00	0.00	U1 All	OK	6.00	421.16	384.55	U1 All	OK
	14.000	0.00	0.00	0.00	U1 All	OK	6.00	421.16	400.59	U1 All	OK
	16.500	0.00	0.00	0.00	U1 All	OK	6.00	421.16	379.22	U1 All	OK
	21.200	0.00	0.00	0.00	U1 All	OK	6.00	421.16	225.08	U1 All	OK
	21.827	0.00	0.00	0.00	U1 All	OK	6.00	421.16	193.28	U1 All	OK
	23.881	7.04	-450.44	0.00	U1 All	OK	6.00	421.16	70.55	U1 All	OK
	25.900	7.04	-450.44	-103.71	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	27.000	10.57	-616.27	-224.26	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	27.000	10.58	-732.25	-224.33	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	27.954	13.64	-935.78	-336.00	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	32.167	13.64	-935.78	-907.33	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	32.375	13.64	-935.78	-938.65	U1 All	---	6.00	421.16	0.00	U1 All	---
	33.000	13.64	-935.78	-1034.21	U1 All	---	6.00	421.16	0.00	U1 All	---
Middle	0.000	6.16	-406.57	3.05	U1 All	---	5.28	372.72	0.00	U1 All	---
	0.833	6.16	-406.57	0.00	U1 All	OK	5.28	372.72	0.00	U1 All	OK
	2.063	6.16	-406.57	-1.39	U1 All	OK	5.28	372.72	0.00	U1 All	OK
	6.727	6.16	-406.57	0.00	U1 All	OK	5.28	372.72	148.95	U1 All	OK
	7.727	0.00	0.00	0.00	U1 All	OK	5.28	372.72	179.27	U1 All	OK
	11.800	0.00	0.00	0.00	U1 All	OK	5.28	372.72	256.36	U1 All	OK
	14.000	0.00	0.00	0.00	U1 All	OK	5.28	372.72	267.06	U1 All	OK
	16.500	0.00	0.00	0.00	U1 All	OK	5.28	372.72	252.81	U1 All	OK
	21.200	0.00	0.00	0.00	U1 All	OK	5.28	372.72	150.05	U1 All	OK
	22.792	0.00	0.00	0.00	U1 All	OK	5.28	372.72	92.79	U1 All	OK
	24.335	6.16	-406.57	0.00	U1 All	OK	5.28	372.72	26.38	U1 All	OK
	32.167	6.16	-406.57	-302.44	U1 All	OK	5.28	372.72	0.00	U1 All	OK
	33.000	6.16	-406.57	-357.07	U1 All	---	5.28	372.72	0.00	U1 All	---
3 Column	0.000	13.64	-935.78	-942.14	U1 All	---	4.40	311.22	0.00	U1 All	---
	0.833	13.64	-935.78	-823.67	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	5.242	13.64	-935.78	-308.27	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	6.000	10.95	-757.30	-238.55	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	6.000	10.95	-617.63	-238.50	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	7.100	7.04	-450.44	-146.47	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	9.351	7.04	-450.44	0.00	U1 All	OK	4.40	311.22	8.22	U1 All	OK
	11.208	0.00	0.00	0.00	U1 All	OK	4.40	311.22	86.05	U1 All	OK
	11.800	0.00	0.00	0.00	U1 All	OK	4.40	311.22	105.95	U1 All	OK
	16.500	0.00	0.00	0.00	U1 All	OK	4.40	311.22	180.35	U1 All	OK
	21.200	0.00	0.00	0.00	U1 All	OK	4.40	311.22	105.95	U1 All	OK
	21.792	0.00	0.00	0.00	U1 All	OK	4.40	311.22	86.05	U1 All	OK
	23.649	7.04	-450.44	0.00	U1 All	OK	4.40	311.22	8.22	U1 All	OK
	25.900	7.04	-450.44	-146.47	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	27.000	10.95	-617.63	-238.50	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	27.000	10.95	-757.29	-238.55	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	27.758	13.64	-935.78	-308.27	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	32.167	13.64	-935.78	-823.68	U1 All	OK	4.40	311.22	0.00	U1 All	OK
	33.000	13.64	-935.78	-942.14	U1 All	---	4.40	311.22	0.00	U1 All	---



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Span Strip	Top						Bottom				
	x ft	A <sub>s,top</sub> in²	ΦM <sub>n</sub> - kip-ft	M <sub>u</sub> - kip-ft	Comb Pat	Status	A <sub>s,bot</sub> in²	ΦM <sub>n</sub> + kip-ft	M <sub>u</sub> + kip-ft	Comb Pat	Status
Middle	0.000	6.16	-406.57	-314.05	U1 All	---	5.28	372.72	0.00	U1 All	---
	0.833	6.16	-406.57	-274.56	U1 All	OK	5.28	372.72	0.00	U1 All	OK
	9.815	6.16	-406.57	0.00	U1 All	OK	5.28	372.72	19.91	U1 All	OK
	11.208	0.00	0.00	0.00	U1 All	OK	5.28	372.72	57.37	U1 All	OK
	11.800	0.00	0.00	0.00	U1 All	OK	5.28	372.72	70.63	U1 All	OK
	16.500	0.00	0.00	0.00	U1 All	OK	5.28	372.72	120.24	U1 All	OK
	21.200	0.00	0.00	0.00	U1 All	OK	5.28	372.72	70.63	U1 All	OK
	21.792	0.00	0.00	0.00	U1 All	OK	5.28	372.72	57.37	U1 All	OK
	23.185	6.16	-406.57	0.00	U1 All	OK	5.28	372.72	19.91	U1 All	OK
	32.167	6.16	-406.57	-274.56	U1 All	OK	5.28	372.72	0.00	U1 All	OK
	33.000	6.16	-406.57	-314.05	U1 All	---	5.28	372.72	0.00	U1 All	---
4 Column	0.000	13.64	-935.78	-1034.21	U1 All	---	6.00	421.16	0.00	U1 All	---
	0.625	13.64	-935.78	-938.64	U1 All	---	6.00	421.16	0.00	U1 All	---
	0.833	13.64	-935.78	-907.32	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	5.046	13.64	-935.78	-335.99	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	6.000	10.58	-732.26	-224.33	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	6.000	10.57	-616.27	-224.26	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	7.100	7.04	-450.44	-103.71	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	9.119	7.04	-450.44	0.00	U1 All	OK	6.00	421.16	70.55	U1 All	OK
	11.173	0.00	0.00	0.00	U1 All	OK	6.00	421.16	193.28	U1 All	OK
	11.800	0.00	0.00	0.00	U1 All	OK	6.00	421.16	225.08	U1 All	OK
	16.500	0.00	0.00	0.00	U1 All	OK	6.00	421.16	379.22	U1 All	OK
	19.000	0.00	0.00	0.00	U1 All	OK	6.00	421.16	400.59	U1 All	OK
	21.200	0.00	0.00	0.00	U1 All	OK	6.00	421.16	384.55	U1 All	OK
	21.827	0.00	0.00	0.00	U1 All	OK	6.00	421.16	374.01	U1 All	OK
	23.409	5.28	-347.67	0.00	U1 All	OK	6.00	421.16	335.67	U1 All	OK
	25.900	5.28	-347.67	0.00	U1 All	OK	6.00	421.16	241.15	U1 All	OK
	27.000	5.89	-384.13	0.00	U1 All	OK	6.00	421.16	186.19	U1 All	OK
	27.000	5.89	-413.70	0.00	U1 All	OK	6.00	421.16	186.15	U1 All	OK
	27.482	6.16	-432.18	0.00	U1 All	OK	6.00	421.16	159.35	U1 All	OK
	32.167	6.16	-432.18	-323.84	U1 All	OK	6.00	421.16	0.00	U1 All	OK
	33.000	6.16	-432.18	-461.26	U1 All	---	6.00	421.16	0.00	U1 All	---
	0.000	6.16	-406.57	-357.07	U1 All	---	5.28	372.72	0.00	U1 All	---
	0.833	6.16	-406.57	-302.44	U1 All	OK	5.28	372.72	0.00	U1 All	OK
	8.665	6.16	-406.57	0.00	U1 All	OK	5.28	372.72	26.38	U1 All	OK
	10.208	0.00	0.00	0.00	U1 All	OK	5.28	372.72	92.79	U1 All	OK
	11.800	0.00	0.00	0.00	U1 All	OK	5.28	372.72	150.05	U1 All	OK
	16.500	0.00	0.00	0.00	U1 All	OK	5.28	372.72	252.81	U1 All	OK
	19.000	0.00	0.00	0.00	U1 All	OK	5.28	372.72	267.06	U1 All	OK
	21.200	0.00	0.00	0.00	U1 All	OK	5.28	372.72	256.36	U1 All	OK
	25.273	0.00	0.00	0.00	U1 All	OK	5.28	372.72	179.27	U1 All	OK
	26.273	6.16	-406.57	0.00	U1 All	OK	5.28	372.72	148.95	U1 All	OK
	30.937	6.16	-406.57	-1.39	U1 All	OK	5.28	372.72	0.00	U1 All	OK
	32.167	6.16	-406.57	0.00	U1 All	OK	5.28	372.72	0.00	U1 All	OK
	33.000	6.16	-406.57	3.05	U1 All	---	5.28	372.72	0.00	U1 All	---
5 Column	0.000	6.16	-432.18	-4.46	U1 All	---	0.00	0.00	0.00	U1 All	---
	0.146	6.16	-432.18	-3.10	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.386	6.16	-399.88	-1.32	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.417	6.16	-399.88	-1.12	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.593	6.16	-399.88	-0.41	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.833	6.16	-399.88	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
Middle	0.000	6.16	-406.57	0.00	U1 All	---	0.00	0.00	0.00	U1 All	---

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Span Strip	Top						Bottom				
	x ft	A <sub>s,top</sub> in <sup>2</sup>	ΦM <sub>n-</sub> kip-ft	M <sub>u-</sub> kip-ft	Comb Pat	Status	A <sub>s,bot</sub> in <sup>2</sup>	ΦM <sub>n+</sub> kip-ft	M <sub>u+</sub> kip-ft	Comb Pat	Status
	0.146	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.386	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.417	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.593	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK
	0.833	6.16	-406.57	0.00	U1 All	OK	0.00	0.00	0.00	U1 All	OK

## 2.9. Slab Shear Capacity

Span	b in	d in	V <sub>ratio</sub>	ΦV <sub>c</sub> kips	V <sub>u</sub> kips	X <sub>u</sub> ft
1	80.55	15.87	1.000	149.20	0.00	0.00
	195.26	15.88	1.000	337.41	0.00	0.00
2	195.22	15.81	1.000	336.01	138.98	2.15
	80.49	15.81	1.000	148.50	146.11	27.00
	195.22	15.81	1.000	336.01	195.53	30.85
3	195.26	15.88	1.000	337.41	167.19	2.16
	80.55	15.87	1.000	149.20	117.83	27.00
	195.26	15.88	1.000	337.41	167.19	30.84
4	195.22	15.81	1.000	336.01	195.53	2.15
	80.49	15.81	1.000	148.50	146.11	6.00
	195.22	15.81	1.000	336.01	138.99	30.85
5	195.26	15.88	1.000	337.41	0.00	0.83
	80.55	15.87	1.000	149.20	0.00	0.83

## 2.10. Flexural Transfer of Negative Unbalanced Moment at Supports

Support	Width in	Width-c in	d in	M <sub>unb</sub> kip-ft	Comb Patt	γ <sub>f</sub>	A <sub>s,req</sub> in <sup>2</sup>	A <sub>s,prov</sub> in <sup>2</sup>	Add Bars
1	71.00	71.00	15.88	453.75	U1 All	0.630	4.105	2.209	5-#6
2	71.00	71.00	15.88	135.09	U1 All	0.600	1.143	4.891	---
3	71.00	71.00	15.88	135.09	U1 All	0.600	1.143	4.891	---
4	71.00	71.00	15.88	453.75	U1 All	0.630	4.105	2.209	5-#6

## 2.11. Punching Shear Around Columns

### 2.11.1. Critical Section Properties

Support	Type	b <sub>1</sub> in	b <sub>2</sub> in	b <sub>0</sub> in	d <sub>avg</sub> in	CG in	C <sub>(left)</sub> in	C <sub>(right)</sub> in	A <sub>c</sub> in <sup>2</sup>	J <sub>c</sub> in <sup>4</sup>
1	Rect	27.94	35.88	91.75	15.88	9.43	19.43	8.51	1456.53	143997.36
2	Rect	35.88	35.88	143.50	15.88	0.00	17.94	17.94	2278.06	512571.48
3	Rect	35.88	35.88	143.50	15.88	0.00	17.94	17.94	2278.06	512571.48
4	Rect	27.94	35.88	91.75	15.88	-9.43	8.51	19.43	1456.53	143997.36

### 2.11.2. Punching Shear Results

Support	V <sub>u</sub> kips	V <sub>u</sub> psi	M <sub>unb</sub> kip-ft	Comb	Patt	γ <sub>v</sub>	V <sub>u</sub> psi	ΦV <sub>c</sub> psi
1	174.86	120.1	316.33	U1	All	0.370	203.1	212.1
2	414.86	182.1	-135.09	U1	All	0.400	204.8	212.1
3	414.86	182.1	135.09	U1	All	0.400	204.8	212.1
4	174.87	120.1	-316.33	U1	All	0.370	203.1	212.1



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## 2.12. Punching Shear Around Drops

### 2.12.1. Critical Section Properties

Support	Type	b <sub>1</sub> in	b <sub>2</sub> in	b <sub>0</sub> in	d <sub>avg</sub> in	CG in	C <sub>(left)</sub> in	C <sub>(right)</sub> in	A <sub>c</sub> in <sup>2</sup>	J <sub>c</sub> in <sup>4</sup>
1	Rect	89.94	159.88	339.75	3.32	56.13	66.13	23.81	1126.99	970513.92
2	Rect	159.88	159.88	639.50	3.32	0.00	79.94	79.94	2121.30	9037734.82
3	Rect	159.88	159.88	639.50	3.32	0.00	79.94	79.94	2121.30	9037734.82
4	Rect	89.94	159.88	339.75	3.32	-56.13	23.81	66.13	1126.99	970513.92

### 2.12.2. Punching Shear Results

Support	V <sub>u</sub> kips	Comb	Pat	v <sub>u</sub> psi	ΦV <sub>c</sub> psi
1	143.27	U1	All	127.1	116.7 *EXCEEDED
2	357.54	U1	All	168.5	116.7 *EXCEEDED
3	357.54	U1	All	168.5	116.7 *EXCEEDED
4	143.28	U1	All	127.1	116.7 *EXCEEDED

## 2.13. Material TakeOff

### 2.13.1. Reinforcement in the Direction of Analysis

Top Bars	3456.8 lb <=> 34.34 lb/ft <=> 1.041 lb/ft <sup>2</sup>
Bottom Bars	3629.1 lb <=> 36.05 lb/ft <=> 1.092 lb/ft <sup>2</sup>
Stirrups	0.0 lb <=> 0.00 lb/ft <=> 0.000 lb/ft <sup>2</sup>
Total Steel	7085.9 lb <=> 70.39 lb/ft <=> 2.133 lb/ft <sup>2</sup>
Concrete	2215.9 ft <sup>3</sup> <=> 22.01 ft <sup>3</sup> /ft <=> 0.667 ft <sup>3</sup> /ft <sup>2</sup>

## 3. Deflection Results: Summary

### 3.1. Section Properties

#### 3.1.1. Frame Section Properties

Notes:

M+ve values are for positive moments (tension at bottom face).

M-ve values are for negative moments (tension at top face).

Span Zone	M <sub>+ve</sub>			M <sub>-ve</sub>		
	I <sub>g</sub> in <sup>4</sup>	I <sub>cr</sub> in <sup>4</sup>	M <sub>cr</sub> kip-ft	I <sub>g</sub> in <sup>4</sup>	I <sub>cr</sub> in <sup>4</sup>	M <sub>cr</sub> kip-ft
1 Left	60255	0	233.46	60255	13128	-476.06
Midspan	60255	0	233.46	60255	13128	-476.06
Right	60255	0	233.46	60255	13128	-476.06
2 Left	60255	12200	233.46	60255	13128	-476.06
Midspan	60255	15599	233.46	60255	0	-476.06
Right	60255	12200	233.46	60255	18722	-476.06
3 Left	60255	10861	233.46	60255	18722	-476.06
Midspan	60255	13647	233.46	60255	0	-476.06
Right	60255	10861	233.46	60255	18722	-476.06
4 Left	60255	12200	233.46	60255	18722	-476.06
Midspan	60255	15599	233.46	60255	0	-476.06
Right	60255	12200	233.46	60255	13128	-476.06
5 Left	60255	0	233.46	60255	13128	-476.06
Midspan	60255	0	233.46	60255	13128	-476.06
Right	60255	0	233.46	60255	13128	-476.06

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### 3.1.2. Frame Effective Section Properties

Span Zone	Weight	Load Level					
		Dead		Sustained		Dead+Live	
		M <sub>max</sub> kip-ft	I <sub>e</sub> in <sup>4</sup>	M <sub>max</sub> kip-ft	I <sub>e</sub> in <sup>4</sup>	M <sub>max</sub> kip-ft	I <sub>e</sub> in <sup>4</sup>
1 Right	1.000	-2.19	60255	-2.19	60255	-3.33	60255
Span Avg	----	----	60255	----	60255	----	60255
2 Middle	0.850	298.16	37035	298.16	37035	491.82	20375
Right	0.150	-626.60	36936	-626.60	36936	-1026.20	22869
Span Avg	----	----	37020	----	37020	----	20749
3 Left	0.150	-565.78	43465	-565.78	43465	-926.56	24356
Middle	0.700	132.58	60255	132.58	60255	221.01	60255
Right	0.150	-565.78	43465	-565.78	43465	-926.56	24356
Span Avg	----	----	55218	----	55218	----	49485
4 Left	0.150	-626.60	36936	-626.60	36936	-1026.20	22869
Middle	0.850	298.16	37035	298.16	37035	491.82	20375
Span Avg	----	----	37020	----	37020	----	20749
5 Left	1.000	-2.19	60255	-2.19	60255	-3.34	60255
Span Avg	----	----	60255	----	60255	----	60255

### 3.1.3. Strip Section Properties at Midspan

Notes:

Load distribution factor, LDL, averages moment distribution factors listed in Design Results.  
Ratio refers to proportion of strip to frame deflections under fix-end conditions.

Span	Column Strip			Middle Strip		
	I <sub>g</sub> in <sup>4</sup>	LDF	Ratio	I <sub>g</sub> in <sup>4</sup>	LDF	Ratio
1	28289.3	0.800	1.704	28289.3	0.200	0.426
2	28289.3	0.738	1.571	28289.3	0.262	0.559
3	28289.3	0.675	1.438	28289.3	0.325	0.692
4	28289.3	0.738	1.571	28289.3	0.262	0.559
5	28289.3	0.800	1.704	28289.3	0.200	0.426

## 3.2. Instantaneous Deflections

### 3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

Span	Direction	Value	Units	Live				Total	
				Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.253	---	0.374	0.374	0.253	0.627
		Loc	ft	15.000	---	15.500	15.500	15.000	15.250
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.070	---	0.070	0.070	0.070	0.140
		Loc	ft	16.500	---	16.500	16.500	16.500	16.500
	Up	Def	in	-0.004	---	-0.001	-0.001	-0.004	-0.005
		Loc	ft	1.571	---	1.325	1.325	1.571	1.325
4	Down	Def	in	0.253	---	0.374	0.374	0.253	0.627
		Loc	ft	18.000	---	17.500	17.500	18.000	17.750
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---

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Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

### 3.2.2. Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
2	Down	Def	in	0.000	---	0.000	0.000	0.000	0.000
		Loc	ft	0.337	---	0.531	0.531	0.337	0.867
	Up	Def	in	15.500	---	15.750	15.750	15.500	15.750
3	Down	Def	in	---	---	---	---	---	---
		Loc	ft	0.116	---	0.107	0.107	0.116	0.222
	Up	Def	in	16.500	---	16.500	16.500	16.500	16.500
4	Down	Def	in	-0.003	---	-0.001	-0.001	-0.003	-0.004
		Loc	ft	1.325	---	1.079	1.079	1.325	1.079
	Up	Def	in	0.337	---	0.531	0.531	0.337	0.867
5	Down	Def	in	17.500	---	17.250	17.250	17.500	17.250
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	---	---	---	---	---	---
	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

### 3.2.3. Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
2	Down	Def	in	0.000	---	0.000	0.000	0.000	0.000
		Loc	ft	0.189	---	0.254	0.254	0.189	0.443
	Up	Def	in	14.500	---	14.750	14.750	14.500	14.750
3	Down	Def	in	---	---	---	---	---	---
		Loc	ft	0.039	---	0.043	0.043	0.039	0.082
	Up	Def	in	16.500	---	16.500	16.500	16.500	16.500
4	Down	Def	in	-0.005	---	-0.002	-0.002	-0.005	-0.006
		Loc	ft	2.310	---	1.571	1.571	2.310	1.817
	Up	Def	in	0.189	---	0.254	0.254	0.189	0.443
5	Down	Def	in	18.500	---	18.250	18.250	18.500	18.250
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	---	---	---	---	---	---
	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

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### 3.3. Long-term Deflections

#### 3.3.1. Long-term Column Strip Deflection Factors

Notes:

Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.  
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span Zone	M <sub>ve</sub>					M <sub>ve</sub>				
	A <sub>s,top</sub> in <sup>2</sup>	b in	d in	Rho' %	Lambda	A <sub>s,bot</sub> in <sup>2</sup>	b in	d in	Rho' %	Lambda
1 Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2 Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3 Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4 Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5 Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

#### 3.3.2. Long-term Middle Strip Deflection Factors

Notes:

Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.  
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span Zone	M <sub>ve</sub>					M <sub>ve</sub>				
	A <sub>s,top</sub> in <sup>2</sup>	b in	d in	Rho' %	Lambda	A <sub>s,bot</sub> in <sup>2</sup>	b in	d in	Rho' %	Lambda
1 Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2 Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3 Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4 Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5 Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

#### 3.3.3. Extreme Long-term Column Strip Deflections and Corresponding Locations

Notes:

Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.

Incremental deflections after partitions are installed can be estimated by deflections due to:

- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.

Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.033	-0.050	-0.050	-0.067
		Loc	ft	0.000	0.000	0.000	0.000
	Down	Def	in	0.673	1.204	1.204	1.540
		Loc	ft	15.500	15.750	15.750	15.500
2	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Down	Def	in	0.231	0.338	0.338	0.454
		Loc	ft	16.500	16.500	16.500	16.500
	Up	Def	in	-0.005	-0.006	-0.006	-0.009
		Loc	ft	1.325	1.079	1.079	1.325
3	Down	Def	in	0.673	1.204	1.204	1.540
		Loc	ft	17.500	17.250	17.250	17.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
5	Down	Def	in	---	---	---	---

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Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
		Loc	ft	---	---	---	---
	Up	Def	in	-0.033	-0.050	-0.050	-0.067
		Loc	ft	0.833	0.833	0.833	0.833

### 3.3.4. Extreme Long-term Middle Strip Deflections and Corresponding Locations

Notes:

Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.

Incremental deflections after partitions are installed can be estimated by deflections due to:

- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,

- creep and shrinkage plus live load (cs+l), if live load applied after partitions.

Total deflections consist of dead, live, and creep and shrinkage deflections.

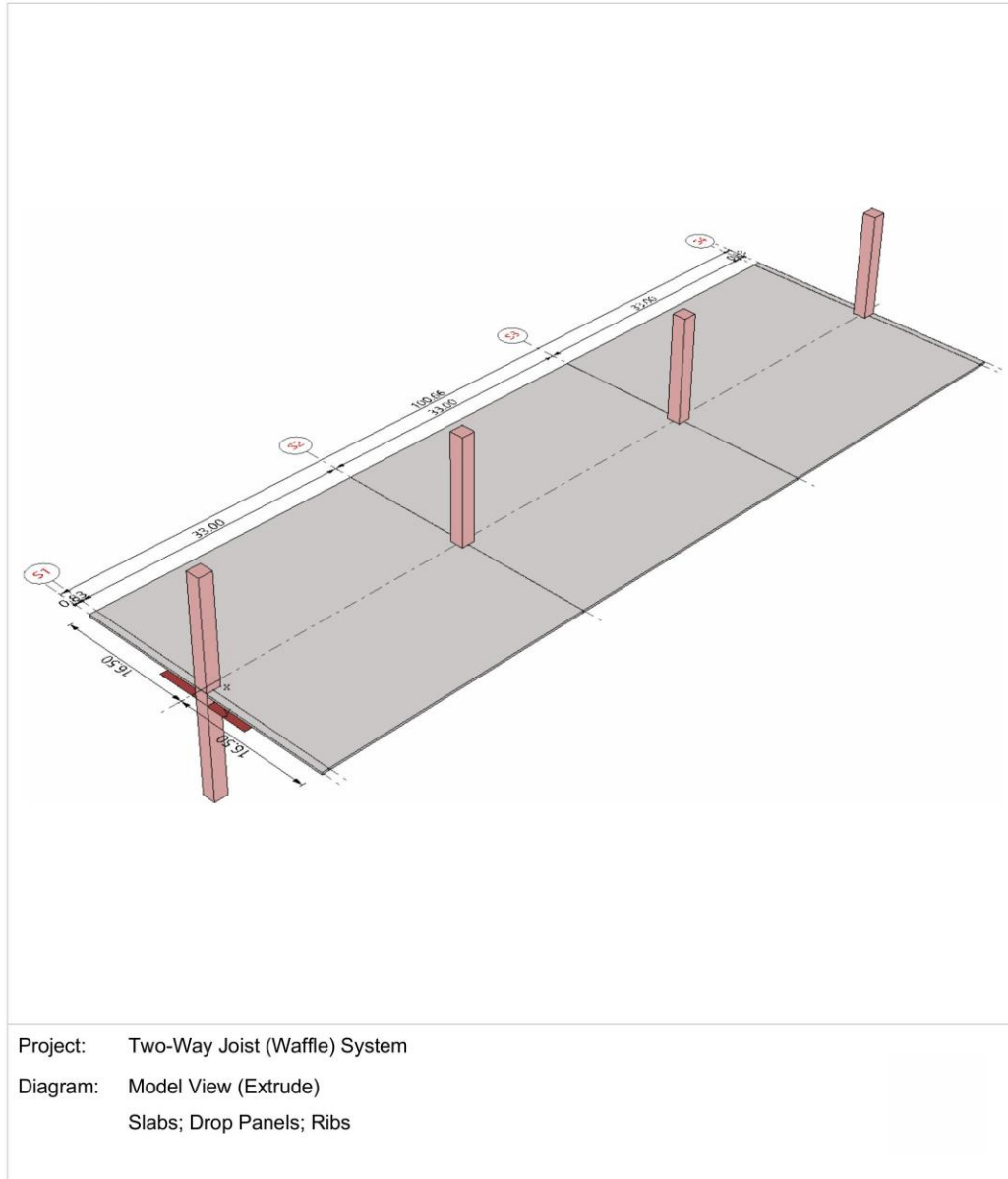
Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.033	-0.050	-0.050	-0.067
		Loc	ft	0.000	0.000	0.000	0.000
2	Down	Def	in	0.378	0.632	0.632	0.821
		Loc	ft	14.500	14.500	14.500	14.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
3	Down	Def	in	0.078	0.121	0.121	0.160
		Loc	ft	16.500	16.500	16.500	16.500
	Up	Def	in	-0.009	-0.011	-0.011	-0.015
		Loc	ft	2.310	2.063	2.063	2.063
4	Down	Def	in	0.378	0.632	0.632	0.821
		Loc	ft	18.500	18.500	18.500	18.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
5	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.033	-0.050	-0.050	-0.067
		Loc	ft	0.833	0.833	0.833	0.833

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#### 4. Screenshots

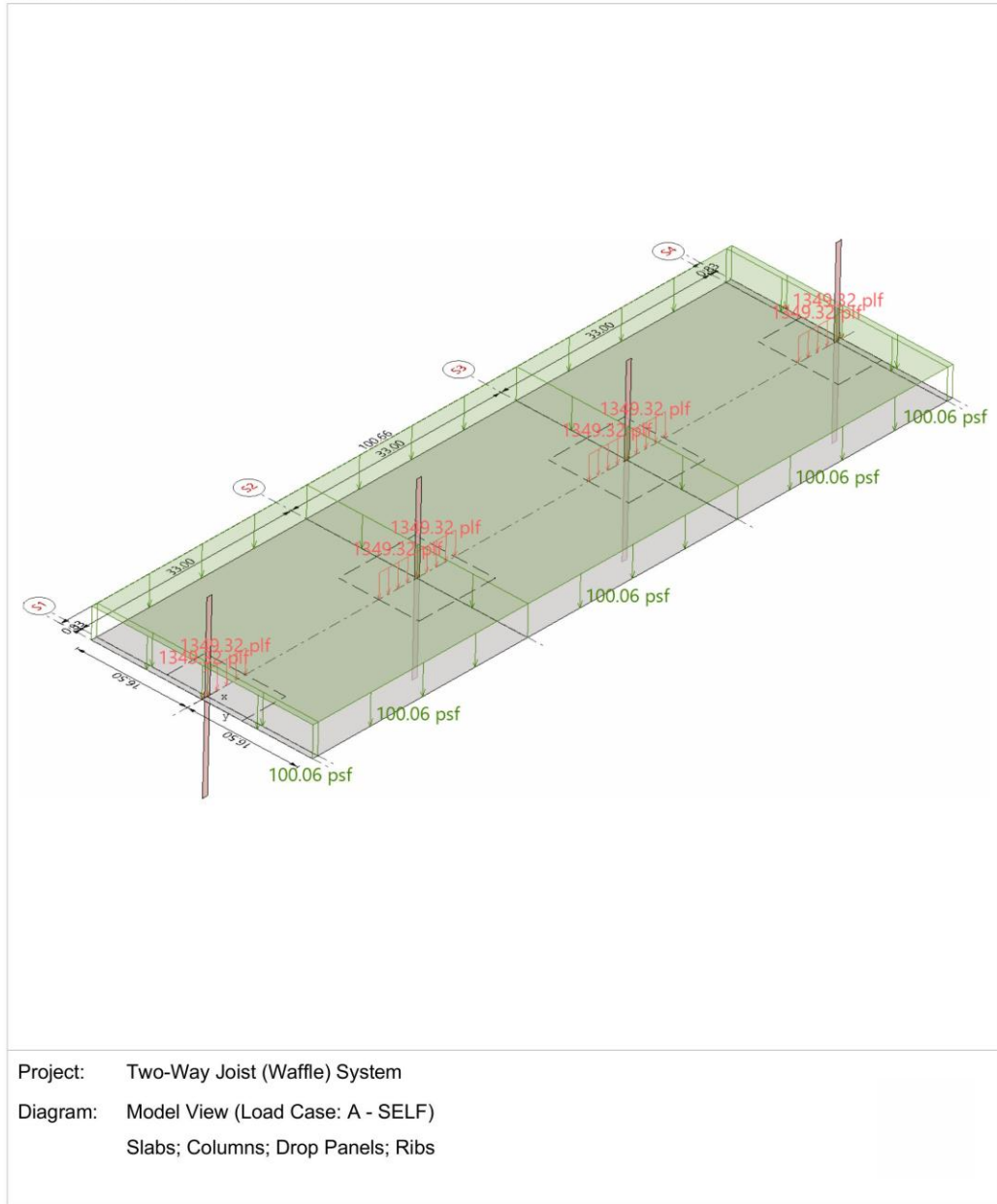
#### 4.1. Extrude 3D view



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#### 4.2. Loads - Case A - SELF

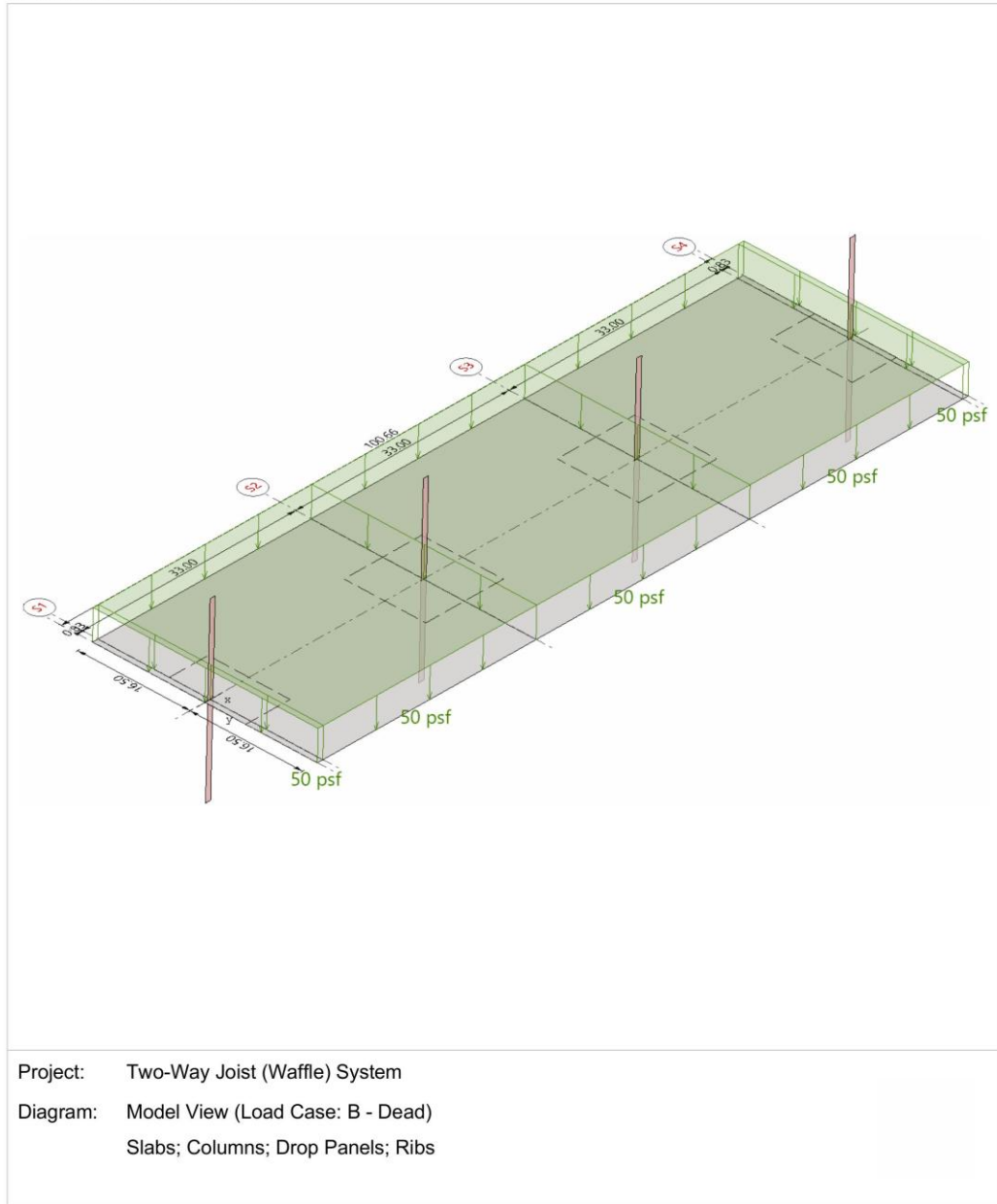




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#### 4.3. Loads - Case B - Dead

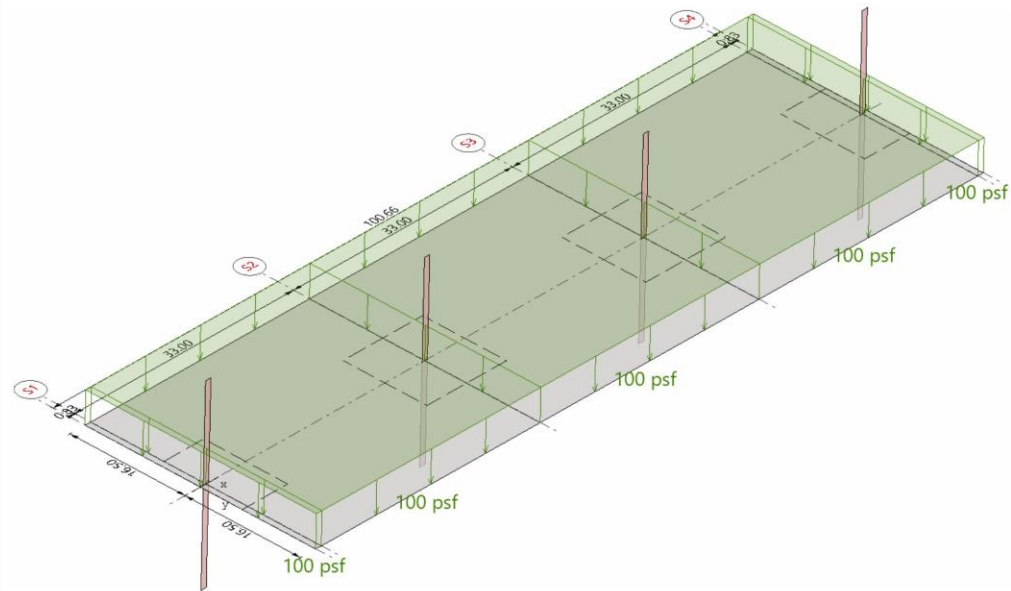




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#### 4.4. Loads - Case C - Live

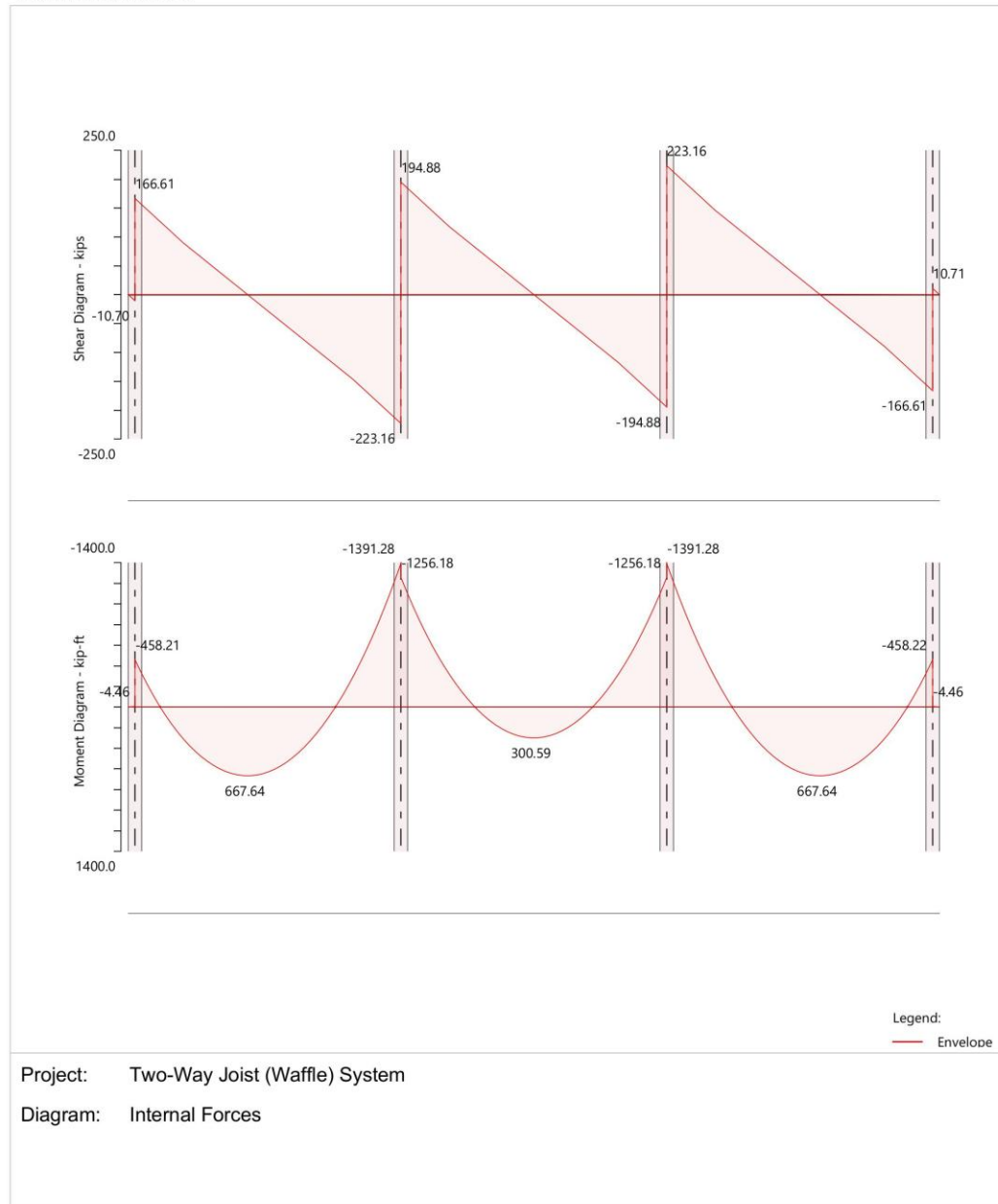


Project: Two-Way Joist (Waffle) System  
Diagram: Model View (Load Case: C - Live)  
Slabs; Columns; Drop Panels; Ribs

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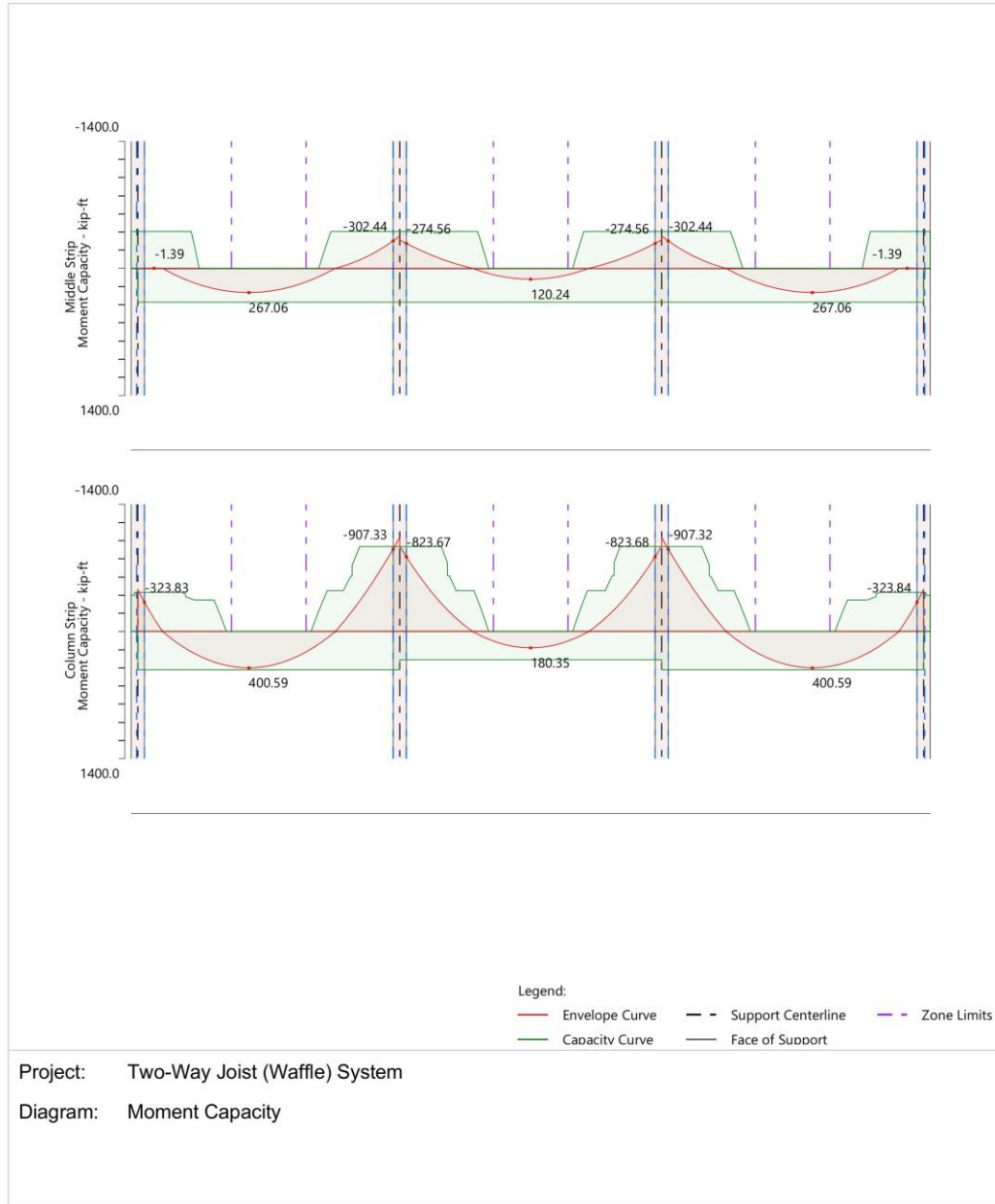
#### 4.5. Internal Forces



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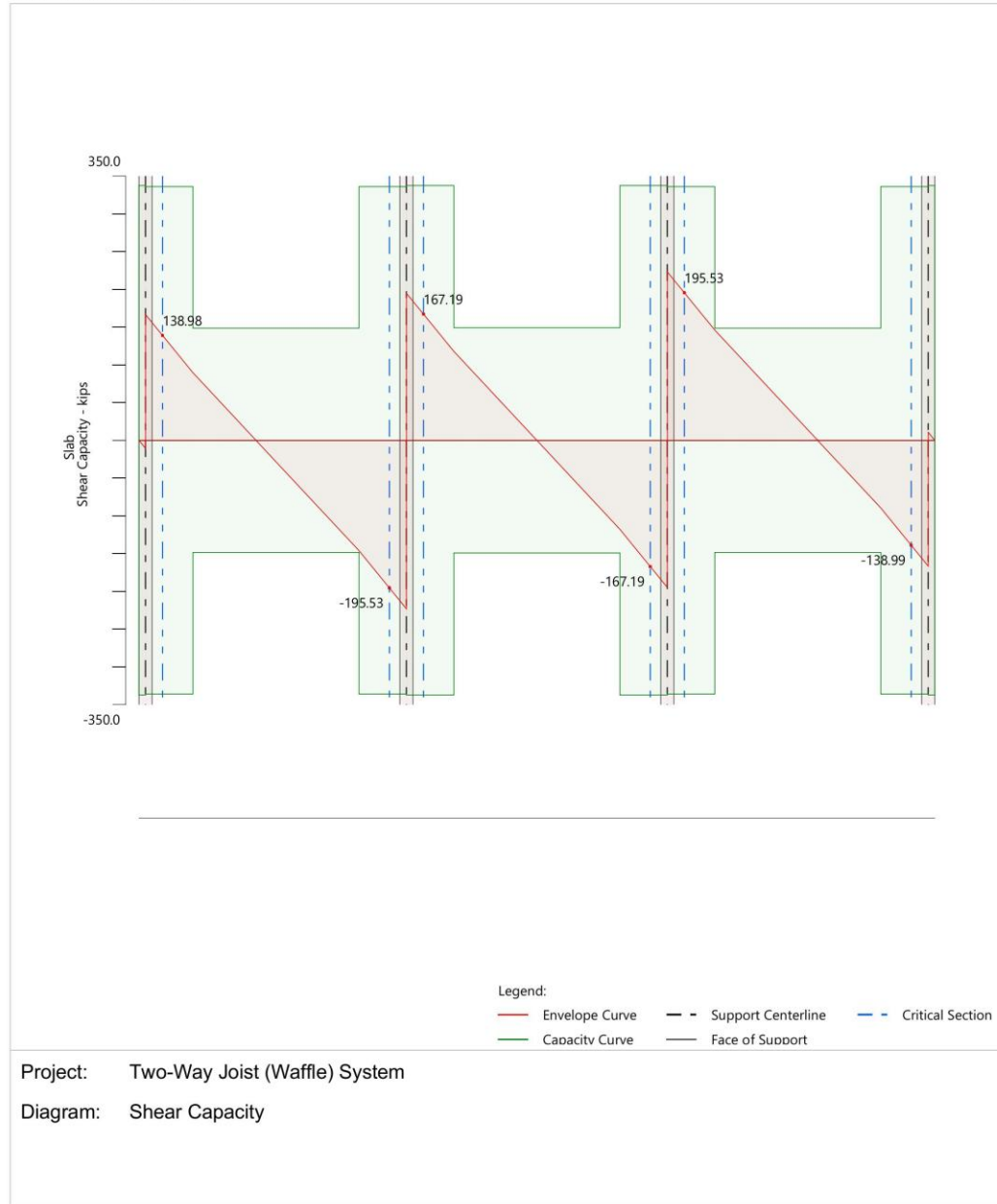
#### 4.6. Moment Capacity



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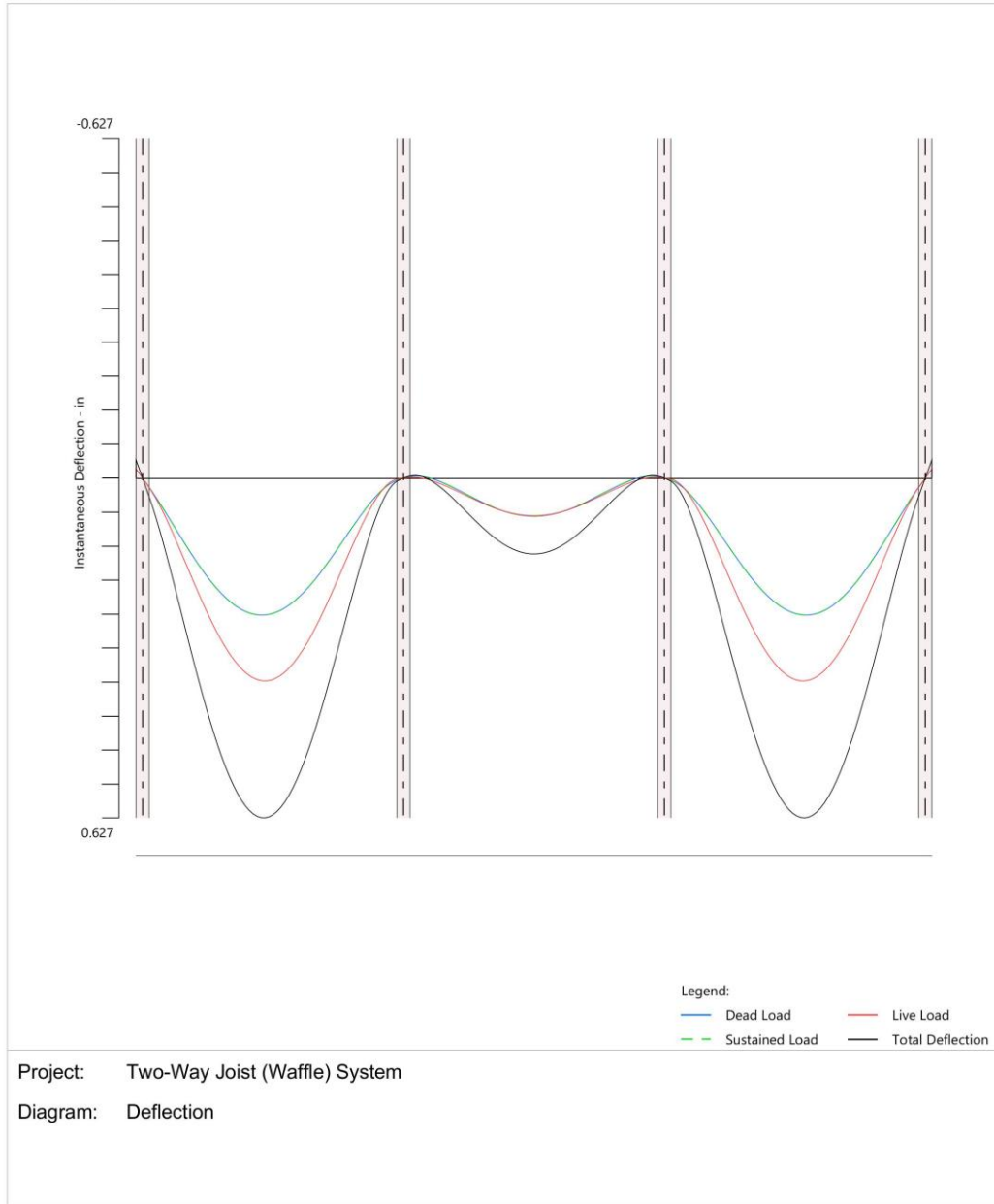
#### 4.7. Shear Capacity



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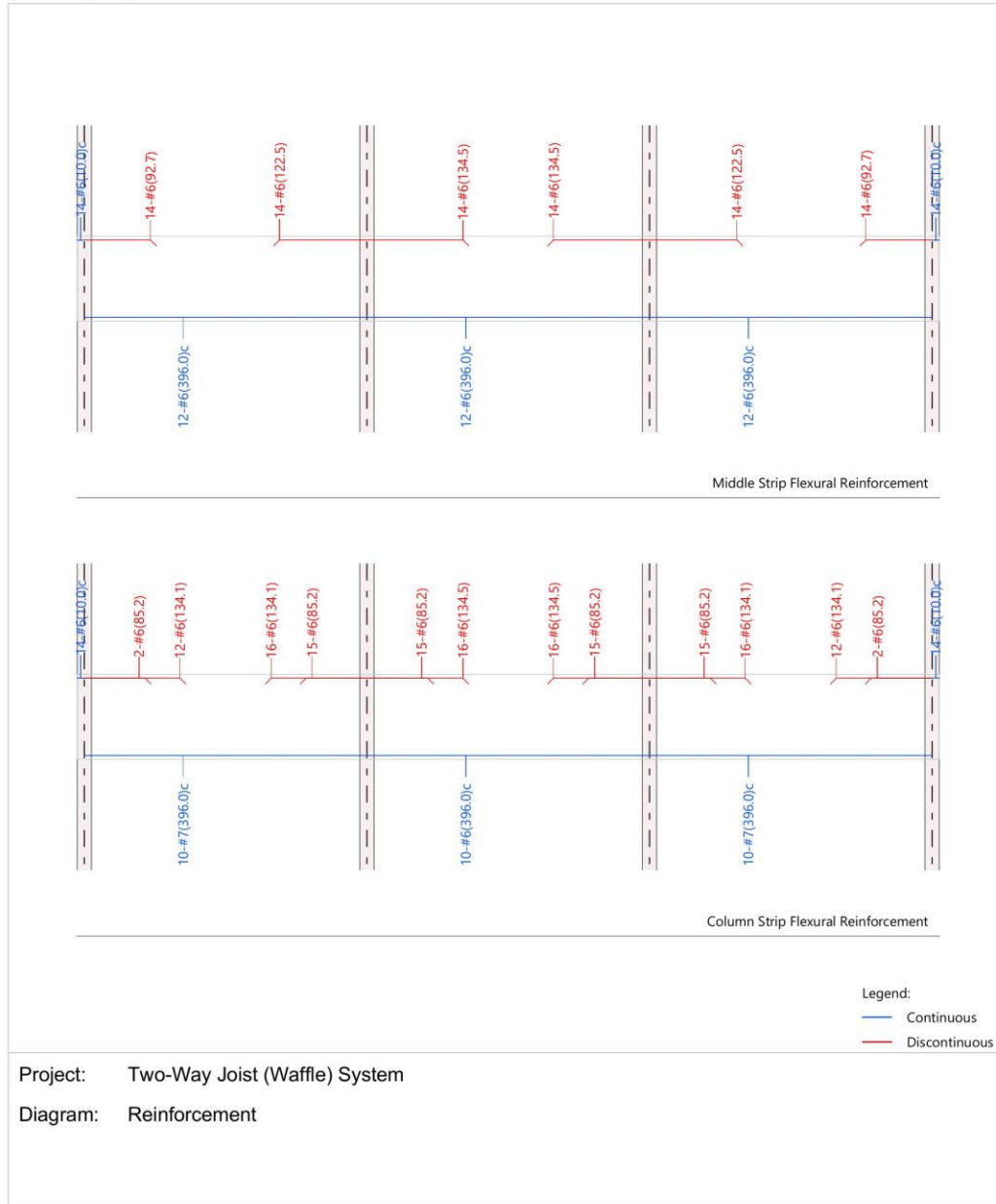
#### 4.8. Deflection



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#### 4.9. Reinforcement



## 8. Summary and Comparison of Design Results

Table 9 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution (ft-kips)				
		Hassoun (DDM)**	Hand (EFM)	spSlab
Exterior Span				
Column Strip	Exterior Negative*	370.00	328.07	323.83
	Positive	444.00	401.00	400.59
	Interior Negative*	748.00	901.27	907.33
Middle Strip	Exterior Negative*	---	0.00	0.00
	Positive	---	267.33	267.06
	Interior Negative*	---	300.42	302.44
Interior Span				
Column Strip	Interior Negative*	---	818.10	823.67
	Positive	---	184.66	180.35
Middle Strip	Interior Negative*	249.00	272.70	274.56
	Positive	296.00	123.10	120.24
<p>* Negative moments are taken at the faces of supports</p> <p>** Direct Design Method does not distinguish between interior and exterior spans nor explicitly address the effect of column contribution at joints</p>				

**Table 10 - Comparison of Reinforcement Results**

Span Location		Reinforcement Provided for Flexure			Additional Reinforcement Provided for Unbalanced Moment Transfer			Total Reinforcement Provided		
		Hassoun	Hand	spSlab	Hassoun	Hand	spSlab	Hassoun	Hand	spSlab
<b>Exterior Span</b>										
<b>Column Strip</b>	<b>Exterior Negative</b>	14 – #6	14 – #6	14 – #6	---	5 – #6	5 – #6	14 – #6	19 – #6	19 – #6
	<b>Positive</b>	10 – #8 2 bars / rib	10 – #7 2 bars / rib	10 – #7 2 bars / rib	---	n/a	n/a	10 – #8 2 bars / rib	10 – #7 2 bars / rib	10 – #7 2 bars / rib
	<b>Interior Negative</b>	28 – #6	30 – #6	31 – #6	---	---	---	28 – #6	30 – #6	31 – #6
<b>Middle Strip</b>	<b>Exterior Negative</b>	10 – #6	14 – #6	14 – #6	---	n/a	n/a	10 – #6*	14 – #6	14 – #6
	<b>Positive</b>	12 – #7 2 bars / rib	12 – #6 2 bars / rib	12 – #6 2 bars / rib	---	n/a	n/a	12 – #7 2 bars / rib	12 – #6 2 bars / rib	12 – #6 2 bars / rib
	<b>Interior Negative</b>	10 – #6	14 – #6	14 – #6	---	n/a	n/a	10 – #6*	14 – #6	14 – #6
<b>Interior Span</b>										
<b>Column Strip</b>	<b>Positive</b>	10 – #7 2 bars / rib	10 – #6 2 bars / rib	10 – #6 2 bars / rib	---	n/a	n/a	10 – #7 2 bars / rib	10 – #6 2 bars / rib	10 – #6 2 bars / rib
<b>Middle Strip</b>	<b>Positive</b>	10 – #6 2 bars / rib	12 – #6 2 bars / rib	12 – #6 2 bars / rib	---	n/a	n/a	10 – #6 2 bars / rib	12 – #6 2 bars / rib	12 – #6 2 bars / rib
* Max spacing requirement exceeded (not checked)										

**Table 11 - Comparison of One-Way (Beam Action) Shear Check Results**

Span	$V_u @ d$ (kips)		$V_u @$ drop panel (kips)		$\phi V_c @ d$ (kips)		$\phi V_c @$ drop panel (kips)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
<b>Exterior</b>	195.05	195.53	145.69	146.11	336.01	336.01	148.50	148.50
<b>Interior</b>	167.16	167.19	117.81	117.83	337.41	337.41	149.20	149.20
* One-way shear check is not provided in the reference (Hassoun and Al-Manaseer)								



**Table 12 - Comparison of Two-Way (Punching) Shear Check Results (around Columns Faces)**

Support	$b_1$ (in.)		$b_2$ (in.)		$b_o$ (in.)		$V_u$ (kips)		$c_{AB}$ (in.)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	27.94	27.94	35.88	35.88	91.75	91.75	163.66	174.86	8.51	8.51
Interior	35.88	35.88	35.88	35.88	143.50	143.50	413.34	414.86	17.94	17.94
Corner	27.94	27.94	27.94	27.94	55.88	55.87	91.75	92.43	6.98	6.98
Support	$J_c$ (in. <sup>4</sup> )		$\gamma_v$		$M_{unb}$ (ft-kips)		$v_u$ (psi)		$\phi v_c$ (psi)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	143,997	143,997	0.370	0.370	334.13	316.33	200.1	203.1	212.1	212.1
Interior	512,571	512,571	0.400	0.400	134.12	135.09	204.0	204.8	212.1	212.1
Corner	81,431	81,431	0.400	0.400	181.81	181.19	178.3	178.8	212.1	212.1

**Table 13 - Comparison of Two-Way (Punching) Shear Check Results (around Drop Panels)**

Support	$b_1$ (in.)		$b_2$ (in.)		$b_o$ (in.)		$V_u$ (kips)		$c_{AB}$ (in.)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	89.94	89.94	159.88	159.88	339.75	339.75	133.02	143.27	23.81	23.81
Interior	159.88	159.88	159.88	159.88	639.50	639.50	357.23	357.54	79.94	79.94
Corner	89.94	89.94	89.94	89.94	179.88	179.87	75.22	75.17	22.48	22.48
Support	$J_c$ (in. <sup>4</sup> )		$\gamma_v$		$M_{unb}$ (ft-kips)		$v_u$ (psi)		$\phi v_c$ (psi)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	971,273	970,514	N.A.	N.A.	N.A.	N.A.	117.9	127.1	116.7	116.7
Interior	9,044,800	9,037,735	N.A.	N.A.	N.A.	N.A.	168.3	168.5	116.7	116.7
Corner	503,407	503,013	N.A.	N.A.	N.A.	N.A.	126.0	126.0	116.7	116.7

General notes:

1. Red values are exceeding permissible shear capacity
2. Hand Calculation fail to capture analysis details possible in [spSlab](#) like accounting for the exact value of the moments and shears at supports and including the loads for the small slab section extending beyond the supporting column centerline.
3. Shear stresses from [spSlab](#) are higher than hand calculations since it considers the load effects beyond the column centerline known in the model as right/left cantilevers. This small increase is often neglected in simplified hand calculations like the one used here.

**Table 14 - Comparison of Immediate Deflection Results (in.)**

Column Strip								
Span	D		D+LL <sub>sus</sub>		D+LL <sub>full</sub>		LL	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	0.293	0.337	0.293	0.337	0.863	0.867	0.570	0.531
Interior	0.096	0.116	0.096	0.116	0.206	0.222	0.110	0.107
Middle Strip								
Span	D		D+LL <sub>sus</sub>		D+LL <sub>full</sub>		LL	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	0.168	0.189	0.168	0.189	0.501	0.443	0.333	0.254
Interior	0.029	0.039	0.029	0.039	0.061	0.082	0.031	0.043

**Table 15 - Comparison of Time-Dependent Deflection Results**

Column Strip							
Span	$\lambda_{\Delta}$		$\Delta_{cs}$ (in.)		$\Delta_{total}$ (in.)		
	Hand	spSlab	Hand	spSlab	Hand	spSlab	
Exterior	2.0	2.0	0.586	0.673	1.449	1.540	
Interior	2.0	2.0	0.192	0.231	0.398	0.454	
Middle Strip							
Span	$\lambda_{\Delta}$		$\Delta_{cs}$ (in.)		$\Delta_{total}$ (in.)		
	Hand	spSlab	Hand	spSlab	Hand	spSlab	
Exterior	2.0	2.0	0.337	0.378	0.837	0.821	
Interior	2.0	2.0	0.058	0.078	0.119	0.160	

In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the [spSlab](#) model. The deflection results from [spSlab](#) are, however, more conservative than hand calculations for two main reasons explained previously: 1) Values of  $I_g$  and  $I_{cr}$  at the negative section exclude the stiffening effect of the drop panel and 2) The  $I_{e,avg}$  used by [spSlab](#) considers equations for prismatic members.

## 9. Conclusions & Observations

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in **ACI 318-14 Chapter 8 (8.2.1)**.

Direct Design Method (DDM) is an approximate method and is applicable to two-way slab concrete floor systems that meet the stringent requirements of **ACI 318-14 (8.10.2)**. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Equivalent Frame Method (EFM) does not have the limitations of Direct Design Method. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StructurePoint's [spSlab](#) software program solution utilizes the Equivalent Frame Method to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as [spMats](#). Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable ACI 318- 14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand// <a href="#">spSlab</a> )	FEM ( <a href="#">spMats</a> )
8.10.2.1	Minimum of three continuous spans in each direction	<input checked="" type="checkbox"/>		
8.10.2.2	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	<input checked="" type="checkbox"/>		
8.10.2.3	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	
8.10.2.4	Column offset shall not exceed 10% of the span in direction of offset from either axis between centerlines of successive columns	<input checked="" type="checkbox"/>		
8.10.2.5	All loads shall be due to gravity only	<input checked="" type="checkbox"/>		
8.10.2.5	All loads shall be uniformly distributed over an entire panel ( $q_u$ )	<input checked="" type="checkbox"/>		
8.10.2.6	Unfactored live load shall not exceed two times the unfactored dead load	<input checked="" type="checkbox"/>		
8.10.2.7	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	<input checked="" type="checkbox"/>		
8.7.4.2	Structural integrity steel detailing	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
8.5.4	Openings in slab systems	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
8.2.2	Concentrated loads	Not permitted	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
8.11.1.2	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
R8.10.4.5*	Reinforcement for unbalanced slab moment transfer to column ( $M_{sc}$ )	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)		Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with <a href="#">spSlab</a> output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. <a href="#">spSlab</a> )	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. <a href="#">spMats</a> )
* The unbalanced slab moment transferred to the column $M_{sc}$ ( $M_{unb}$ ) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain $M_{sc}$ ( $M_{unb}$ ). In EFM where a frame analysis is used, moments at the column center line are used to obtain $M_{sc}$ ( $M_{unb}$ ).				