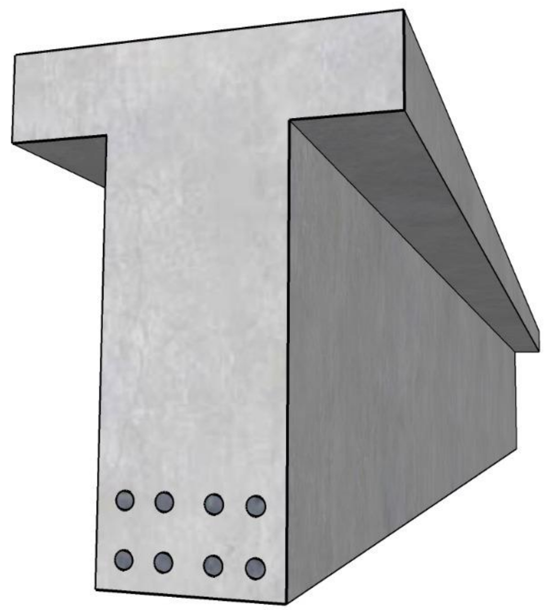
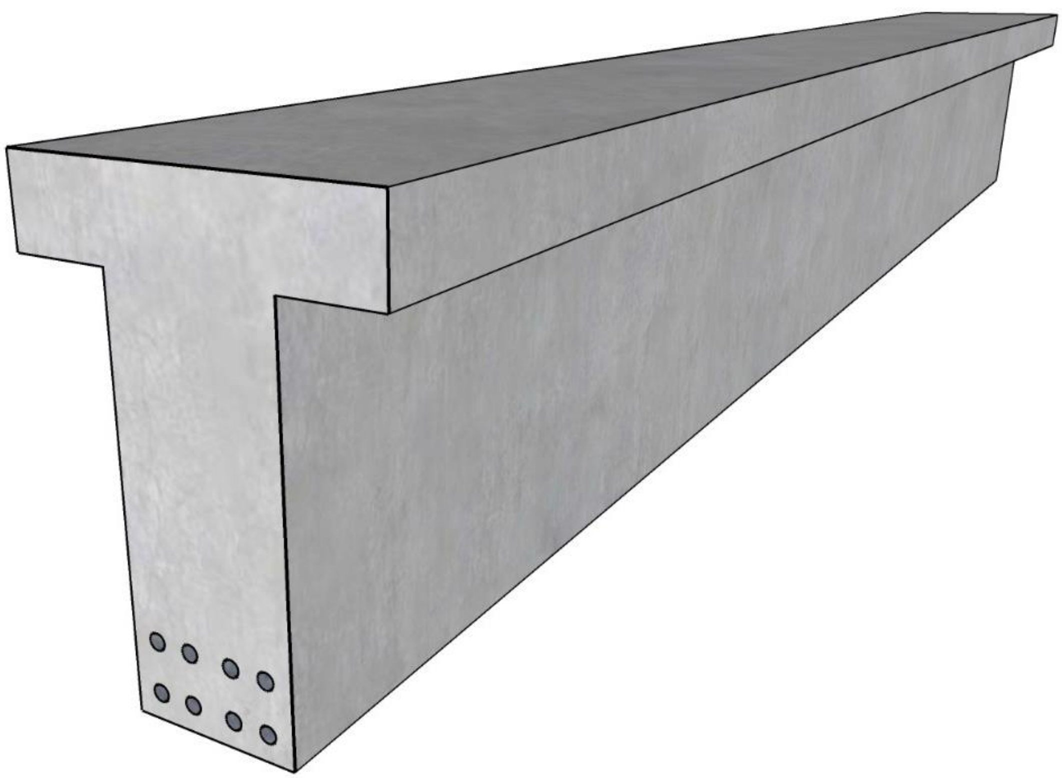
Flexural Design of Reinforced Concrete T-Beams (ACI 318-14)





Flexural Design of Reinforced Concrete T-Beams (ACI 318-14)

This example aims to determine the required amount of tension reinforcing steel in the flanged concrete T-Beam section shown in Figure 1. It is designed in accordance with the ACI 318-14 code to carry a combination of applied dead and live load moments. The T-Beam is reinforced with #4 stirrups. Concrete compressive strength and reinforcement yield strength is as shown in design data below. A comparison of the design results in the Reference and the hand calculations is provided against values obtained by spBeam engineering software program from StructurePoint.

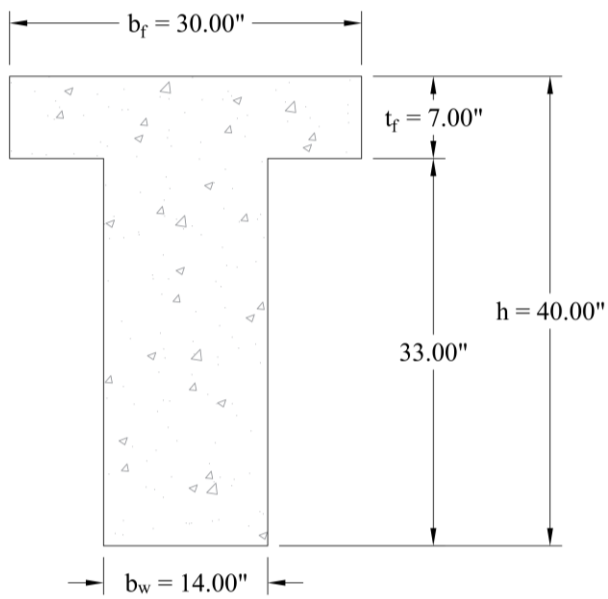


Figure 1 – Flanged Reinforced Concrete Beam Cross-Section

Contents

Code

- Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

[1] Reinforced Concrete Design, 8th Edition, 2017, Chu-Kia Wang, Charles G. Salmon, Jose A. Pincheira, Gustavo J. Parra-Montesinos, Oxford University Press, Example 4.5.1.

[2] spBeam Engineering Software Program Manual v5.50, StructurePoint LLC., 2018.

[3] "Flexural Strength of Flanged Reinforced Concrete Beam (T-Beam Section) - Case One" Case Study, STRUCTUREPOINT, 2021.

[4] "Flexural Strength of Flanged Reinforced Concrete Beam (T-Beam Section) - Case Two" Case Study, STRUCTUREPOINT, 2021.

[5] "Flexural Strength of Flanged Reinforced Concrete Beam (T-Beam Section) - Case Three" Case Study, STRUCTUREPOINT, 2021.

[6] "Flexural Strength of Flanged Reinforced Concrete Beam (T-Beam Section) - Case Four" Case Study, STRUCTUREPOINT, 2021.

[7] "Flexural Strength of Flanged Reinforced Concrete Beam (T-Beam Section) - Case Five" Case Study, STRUCTUREPOINT, 2021.

Design Data

f_c'= 4,000 psi normal weight concrete

f_y = 60,000 psi

Dead load moment, M_D = 560 ft-kips

Live load moment, M_L = 700 ft-kips

T-Beam overall height, h = 40 in. (Including flange thickness)

Flange thickness, t_f = 7 in.

Flange width, b_f = 30 in.

Clear cover = 1.5 in.

Solution

1. Design Factored Moment

$$M_u = 1.2 \times M_D + 1.6 \times M_L = 1.2 \times 560 \text{ ft-kips} + 1.6 \times 700 \text{ ft-kips} = 1792 \text{ ft-kips}$$ ***ACI 318-14 (Table 5.3.1, Eq. b)***

Required nominal moment M_n is,

$$M_{n,req} = \frac{M_u}{\phi}$$

Assume ϕ = 0.9

$$M_{n,req} = \frac{1792 \text{ ft-kips}}{0.9} = 1991.1 \text{ ft-kips}$$

Since the effective depth of the beam *"d"* is required to proceed with the subsequent procedures, many iterations have been carried out to find the required tension reinforcement and it is estimated to be two layers of reinforcement of #11 rebars.

2. Equivalent Compressive Block Depth

Determine whether the equivalent compressive block depth *"a"* will be greater than the flange thickness, t_f = 7 in.

For $a = t_f$,

$$C = 0.85 \times f_c' \times b_f \times t_f = 0.85 \times 4 \text{ ksi} \times 30 \text{ in.} \times 7 \text{ in.} = 714 \text{ kips}$$

As discussed earlier, assume that the beam is reinforced with two layers of #11 reinforcement and clear spacing between layers = 1 in., the effective depth is:

$$d = h - \text{ clear cover} - \text{stirrup diameter} - \text{bar diameters} - \frac{\text{clear spacing between layers}}{2}$$

$$d = 40 \text{ in.} - 1.5 \text{ in.} - 0.5 \text{ in.} - 1.41 \text{ in.} - \frac{1 \text{ in.}}{2} = 36 \text{ in.}$$

$$M_n = C \times \left(d - \frac{t_f}{2}\right) = 714 \text{ kips} \times \left(36 \text{ in.} - \frac{7 \text{ in.}}{2}\right) \times \frac{1 \text{ ft}}{12 \text{ in.}} = 1933.8 \text{ ft-kips}$$

Since the required nominal moment from Section 1 ($M_{n,req}$ = 1991.1 ft-kips) exceeds M_n = 1933.8 ft-kips, the actual "*a*" must exceed "t_f".

3. Required Tensile Reinforcement

For $a > t_f$

$$M_{n,req} = 0.85 \times f_c^{'} \times A_1 \times \left(d - \frac{a}{2}\right) + 0.85 \times f_c^{'} \times A_2 \times \left(d - \frac{t_f}{2}\right)$$

$$1991.1 \text{ ft-kips} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 0.85 \times 4 \text{ ksi} \times (14 \text{ in.} \times a) \times \left(36 \text{ in.} - \frac{a}{2}\right) + 0.85 \times 4 \text{ ksi} \times (16 \text{ in.} \times 7 \text{ in.}) \times \left(36 \text{ in.} - \frac{7 \text{ in.}}{2}\right)$$

$$a^2 - 72 \times a - 483.9 = 0$$

By solving the parabolic equation above,

$$a = 7.5 \text{ in.} > t_f$$

$$C_1 = 0.85 \times f_c^{'} \times b_w \times a = 0.85 \times 4 \text{ ksi} \times 14 \text{ in.} \times 7.5 \text{ in.} = 357 \text{ kips} = T_1$$

$$C_2 = 0.85 \times f_c^{'} \times \left(b_f - b_w\right) \times t_f = 0.85 \times 4 \text{ ksi} \times 16 \text{ in.} \times 7 \text{ in.} = 380.8 \text{ kips} = T_2$$

$$A_{s1} = \frac{T_1}{f_y} = \frac{357 \text{ kips}}{60 \text{ ksi}} = 5.95 \text{ in.}^2$$

$$A_{s2} = \frac{T_2}{f_y} = \frac{380.8 \text{ kips}}{60 \text{ ksi}} = 6.347 \text{ in.}^2$$

$$A_{s,req} = A_{s1} + A_{s2} = 5.95 \text{ in.}^2 + 6.347 \text{ in.}^2 = 12.29 \text{ in.}^2$$

ACI 318-14 requirement for the minimum required area of tensile reinforcement: ***ACI 318-14 (9.6.1.2)***

$$A_{s,\min} = \max \text{ of } \begin{bmatrix} \dfrac{3 \times \sqrt{f_c^{'}}}{f_y} \times b_w \times d \\ \dfrac{200}{f_y} \times b_w \times d \end{bmatrix} = \max \text{ of } \begin{bmatrix} \dfrac{3 \times \sqrt{4000 \text{ psi}}}{60{,}000 \text{ psi}} \times 14 \text{ in.} \times 36 \text{ in.} \\ \dfrac{200}{60{,}000 \text{ psi}} \times 14 \text{ in.} \times 36 \text{ in.} \end{bmatrix} = \max \text{ of } \begin{bmatrix} 1.59 \text{ in.}^2 \\ 1.68 \text{ in.}^2 \end{bmatrix}$$

$$A_{s,\min} = 1.68 \text{ in.}^2 < A_{s,req} = 12.29 \text{ in.}^2$$

Therefore, reinforcing the beam with 8-#11 reinforcement bars of an area A_s =12.48 in.^{2} arranged in two layers is sufficient.

Longitudinal reinforcement spacing requirements are checked below:

$$w_{bend} = \left(1 - \frac{\sqrt{2}}{2}\right) \times \left(r - \frac{d_{b,longitudinal}}{2}\right)$$ ***spBeam Manual (Eq. 2-96)***

Where "*r*" is the inside radius of bend for stirrup = 4 × stirrup radius = 4 × 0.5 in. / 2 = 1 in.

$$w_{bend} = \left(1 - \frac{\sqrt{2}}{2}\right) \times \left(1 \text{ in.} - \frac{1.41 \text{ in.}}{2}\right) = 0.086$$

$$d_s = \text{Side Cover } + d_{b,stirrup} + w_{bend} + \frac{d_{b,longitudinal}}{2}$$ ***spBeam Manual (Figure 2.21)***

For the purpose of conducting a valid comparison with the reference example a value of 1 in. is used for the side cover.

$$d_s = 1 \text{ in. } + 0.5 \text{ in.} + 0.086 \text{ in.} + \frac{1.41 \text{ in.}}{2} = 2.291 \text{ in.}$$

$$s_{provided} = \left(\frac{b - 2 \times d_s}{\# of \;\; bars - 1}\right) = \left(\frac{14 \text{ in.} - 2 \times 2.291 \text{ in.}}{4 - 1}\right) = 3.139 \text{ in.}$$

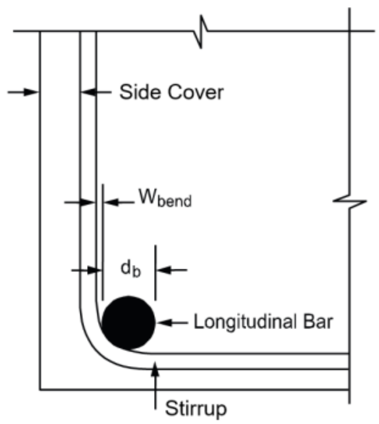


Figure 2 – Width Due to Stirrup Bend (spBeam Manual – Figure 2.21)

The maximum allowed spacing (s_{max}):

$$s_{\max} = 15\left(\frac{40,000}{f_s}\right) - 2.5c_c \le 12\left(\frac{40,000}{f_s}\right)$$ ***ACI 318-14 (Table 24.3.2)***

c_c = the least distance from surface of reinforcement to the tension face which equals to 2.0 in.

Use f_s = 2/3 f_y = 40,000 psi ***ACI 318-14 (24.3.2.1)***

$$s_{\max} = \min \left\{ \begin{array}{c} 15\left(\frac{40,000}{40,000}\right) - 2.5 \times 2.0 \\ 12\left(\frac{40,000}{40,000}\right) \end{array} \right\} = \min \left\{ \begin{array}{c} 10 \text{ in.} \\ 12 \text{ in.} \end{array} \right\} = 10 \text{ in.}$$

$s_{provided}$ = 3.139 in. < s_{max} = 10 in.

Thus, s_{max} requirement is satisfied.

The minimum allowed spacing (s_{min}): **ACI 318-14 (25.2.1)**

$$s_{\min} = d_b + \max\begin{Bmatrix} 1 \text{ in.} \\ d_b \end{Bmatrix} = 1.41 \text{ in.} + \max\begin{Bmatrix} 1 \text{ in.} \\ 1.41 \text{ in.} \end{Bmatrix} = 2.82 \text{ in.}$$

Minimum spacing requirement based on the aggregate size in **ACI 318-14 (25.2.1)** is omitted in this check for s_{min} since it is not given by the reference book.

$s_{provided}$ = 3.139 in. > s_{min} = 2.82 in.

Thus, s_{min} requirement is satisfied.

To check the tensile strain in extreme tension bars to assure the use of ϕ = 0.9, the distance d_t to the extreme tension steel is calculated as:

$$d_t = d + 0.5 \text{ in.} + \frac{1.41 \text{ in.}}{2} = 36 \text{ in.} + 1.21 \text{ in.} = 37.21 \text{ in.}$$

Since f_c' = 4,000 psi:

$\beta_1 = 0.85$ **ACI 318-14 (Table 22.2.2.4.3)**

$$c = \frac{a}{\beta_1} = \frac{7.5 \text{ in.}}{0.85} = 8.82 \text{ in.}$$

$$\varepsilon_t = 0.003 \times \frac{d_t - c}{c} = 0.003 \times \frac{37.21 \text{ in.} - 8.82 \text{ in.}}{8.82 \text{ in.}} = 0.00966 > 0.005$$

Therefore, ϕ = 0.90 (function of the extreme-tension layer of bars strain) as assumed **ACI 318-14 (21.2.1)**

Thus, using 8-#11 bars in two layers is sufficient. Beam cross-section is shown in Figure 3.

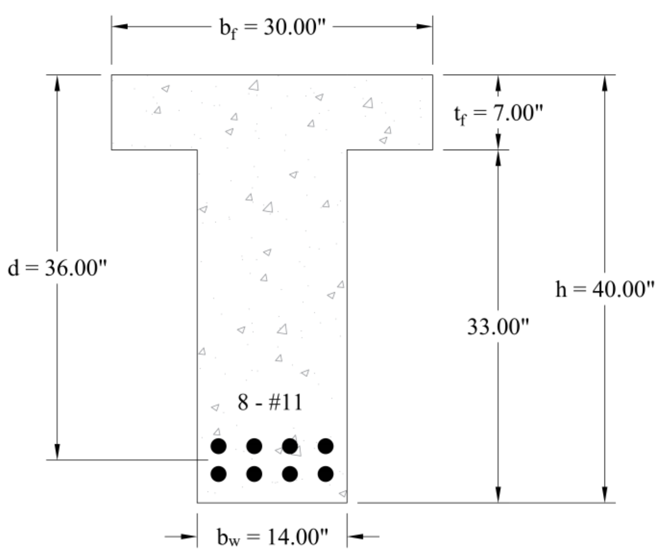


Figure 3 –Beam Cross-Section with Tension Reinforcement

4. Design of T-Section Reinforced Concrete Beam (ACI 318-14) – spBeam Software

spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per American (ACI 318) and Canadian (CSA A23.3) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

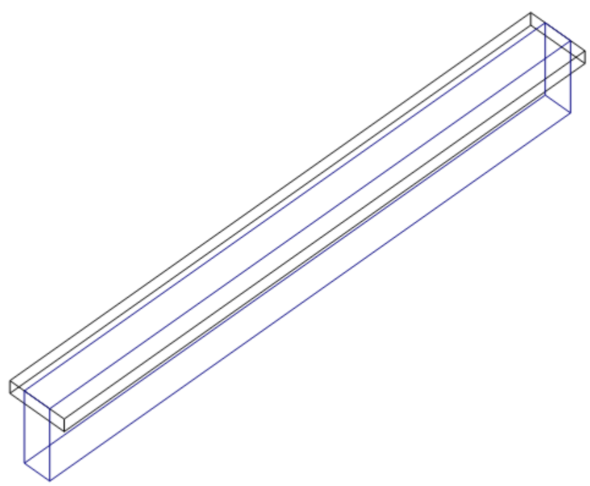
spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or flanges (T-Beam) sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, spBeam provide users with options and flexibility to:

1. Design the beam as a rectangular cross-section or a flanges (T-Beam) section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.
4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia
6. Design the beam as singly or doubly reinforced section.

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the beam covered in this design example.

spBeam v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams and One-way Slab Systems



Contents

1. Input Echo

1.1. General Information

File Name	C:\ACI 318 Exa...\Design of T Section - 4.5.1.slb
Project	Design of T section
Frame	Example 4.5.1
Engineer	SP
Code	ACI 318-14
Reinforcement Database	User Defined
Mode	Design
Number of supports =	2
Floor System	One-Way/Beam

1.2. Solve Options

Live load pattern ratio = 0%
Deflections are based on gross section properties.
Long-term deflections are NOT calculated.
Compression reinforcement calculations selected.
Default incremental rebar design selected.
Moment redistribution NOT selected.
Effective flange width calculations selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150 lb/ft^3
f'_c	4 ksi
E_c	3834.3 ksi
f_r	0.47434 ksi

1.3.2. Concrete: Columns

w_c	150 lb/ft^3
f'_c	4 ksi
E_c	3834.3 ksi
f_r	0.47434 ksi

1.3.3. Reinforcing Steel

f_y	60 ksi
f_{yt}	60 ksi
E_s	29000 ksi
Epoxy coated bars	No

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in^2	lb/ft		in	in^2	lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

1.5. Span Data

1.5.1. Slabs

Span	Loc	L1	t	wL	wR	bE_{ff}	H_{min}
		ft	in	ft	ft	in	in
1	Int	24.000	7.00	1.250	1.250	30.00	0.00

1.5.2. Ribs and Longitudinal Beams

Span	Ribs			Beams		Span
	b	h	Sp	b	h	H_{min}
	in	in	in	in	in	in
1	0.00	0.00	0.00	14.00	40.00	18.00

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	0.00	0.00	0.000	0.00	0.00	0.000	100
2	0.00	0.00	0.000	0.00	0.00	0.000	100

1.6.2. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kip/in	kip-in/rad		
1	0	0	Pinned	Pinned
2	0	0	Pinned	Pinned

1.7. Load Data

*Problem is not solved. Selfweight and pattern loads are not included in load listing.

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.200	1.600

1.7.2. Support Loads

Case	Support	F_z	M_y
		kip	k-ft
Dead	1	0.00	-560.00
Live	1	0.00	-700.00

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#3	#4	#3	#4
Bar spacing	in	1.00	18.00	1.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	in	3.00		3.00	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#3	#3	#11	#11	#4	#4
Bar spacing	in	1.00	18.00	1.00	18.00	6.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	2.00		2.00			
Layer dist.	in	1.00		1.00			
No. of legs						2	6
Side cover	in					1.00	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

2. Design Results

2.1. Top Reinforcement

Span	Zone	Width	M_{max}	X_{max}	A'_s	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
		ft	k-ft	ft	in²	in²	in²	in²	in	
1	Left	2.40	0.00	0.000	0.000	0.000	9.562	0.000	0.000	---
	Midspan	2.40	0.00	12.000	0.000	0.000	9.562	0.000	0.000	---
	Right	2.40	0.00	24.000	0.000	0.000	9.562	0.000	0.000	---

2.2. Top Bar Details

Span	Left				Continuous		Right			
	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
		ft		ft		ft		ft		ft
1	---		---		---		---		---	

2.3. Top Bar Development Lengths

Span	Left				Continuous		Right			
	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
		in		in		in		in		in
1	---		---		---		---		---	

2.4. Bottom Reinforcement

Span	Width	M_{max}	X_{max}	A'_s	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
	ft	k-ft	ft	in²	in²	in²	in²	in	
1	1.17	1792.00	0.000	0.000	1.684	15.473	12.260	3.139	8-#11 2L

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2.5. Bottom Bar Details

Span	Long Bars			Short Bars		
	Bars	Start	Length	Bars	Start	Length
		ft	ft		ft	ft
1	8-#11	0.00	24.00	---		

2.6. Bottom Bar Development Lengths

Span	Long Bars		Short Bars	
	Bars	DevLen	Bars	DevLen
		in		in
1	8-#11	88.54	---	

2.7. Flexural Capacity

Span	Top						Bottom				
	x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
	ft	in²	k-ft	k-ft			in²	k-ft	k-ft		
1	0.000	0.00	0.00	0.00	U1 All	OK	12.48	1820.17	1792.00	U1 All	OK
	8.400	0.00	0.00	0.00	U1 All	OK	12.48	1820.17	1164.80	U1 All	OK
	12.000	0.00	0.00	0.00	U1 All	OK	12.48	1820.17	896.00	U1 All	OK
	15.600	0.00	0.00	0.00	U1 All	OK	12.48	1820.17	627.20	U1 All	OK
	24.000	0.00	0.00	0.00	U1 All	OK	12.48	1820.17	0.00	U1 All	OK

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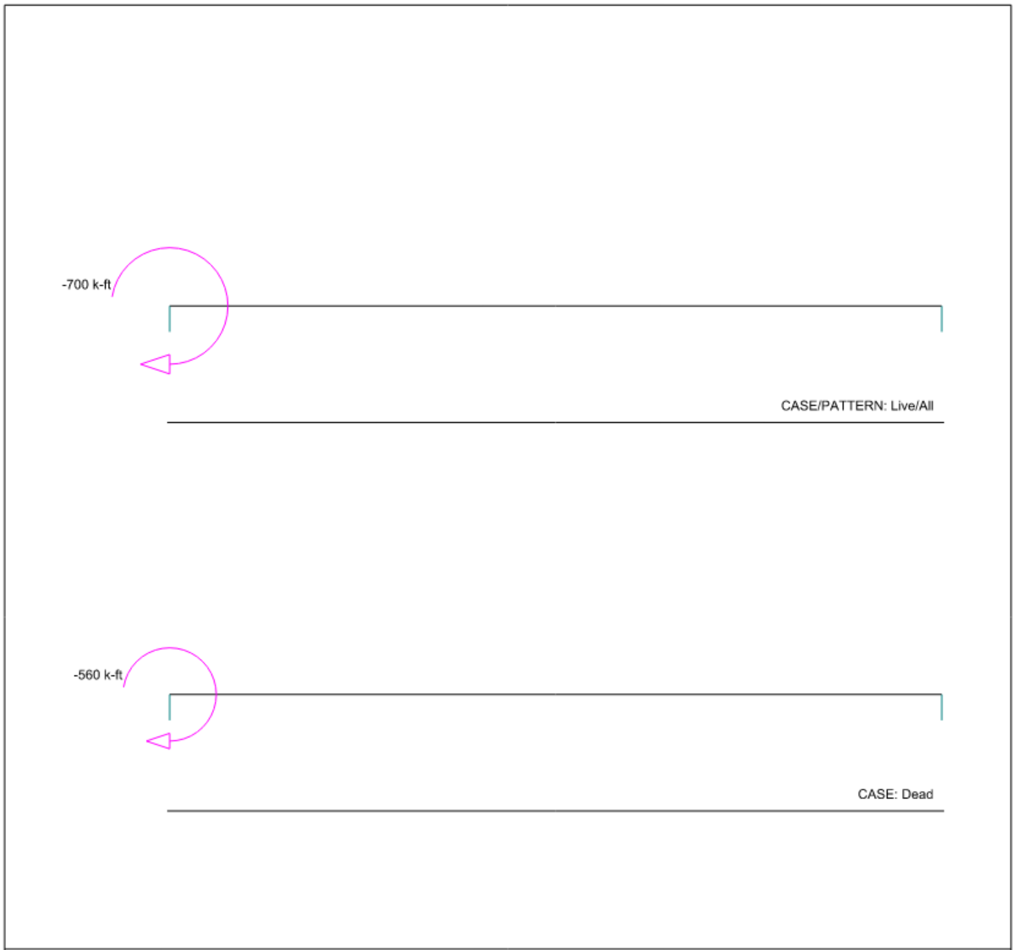
3. Diagrams

3.1. Loads



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Project: Design of T section
Frame: Example 4.5.1
Engineer: SP
Code: ACI 318-14
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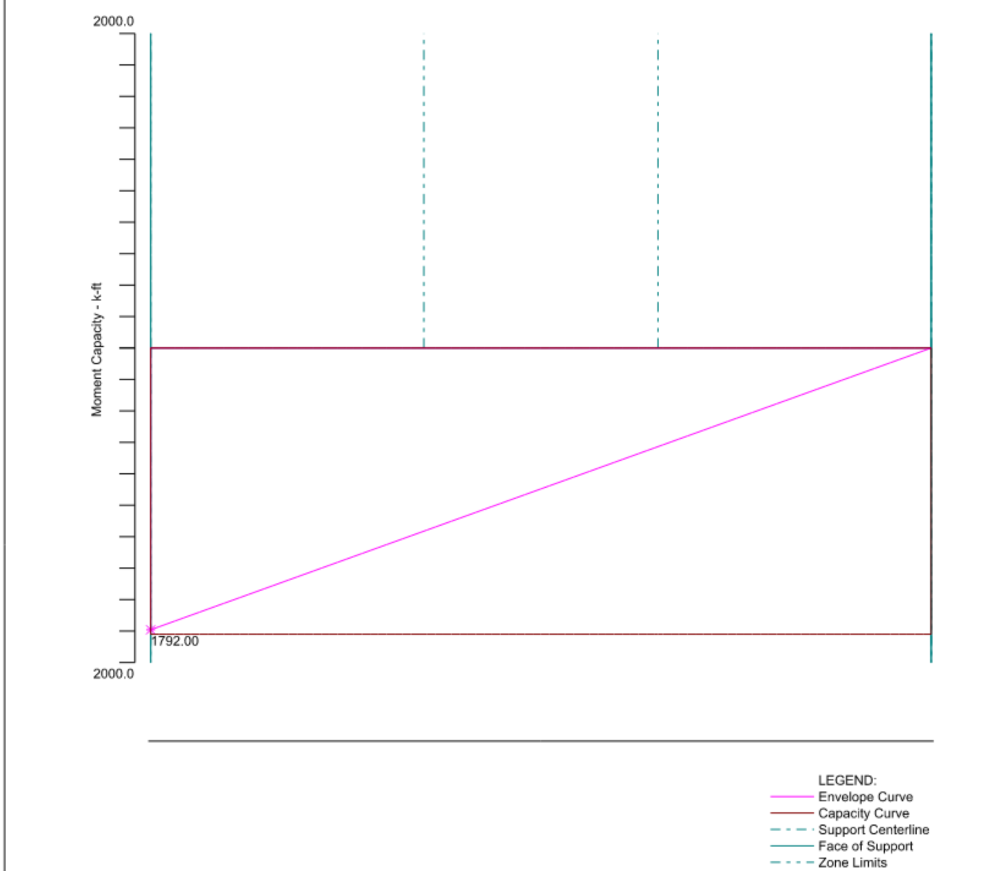
3.2. Moment Capacity



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Project: Design of T section

Frame: Example 4.5.1

Engineer: SP

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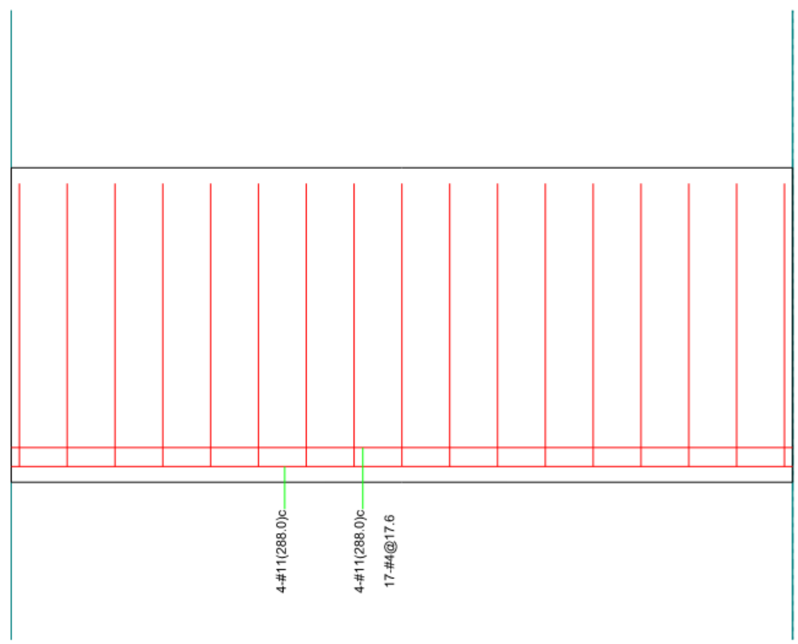
3.3. Reinforcement



Flexural and Transverse Reinforcement

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Project: Design of T section

Frame: Example 4.5.1

Engineer: SP

Code: ACI 318-14

Date: 11/04/21

Time: 16:18:26

5. Comparison of Design Results

Table 1 - Comparison of Results							
Method	**b_f,** in.	**M_u,** kip-ft	**Reinforcement**	**$A_{s,provided}$,** in.^{2}	**$S_{provided}$,** in.	**$A_{s,req}$,** in.^{2}	**$A_{s,min}$,** in.^{2}
Reference	30	1792	8 - #11	12.48	---	12.30	1.68
Hand	30	1792	8 - #11	12.48	3.139	12.29	1.68
spBeam	30	1792	8 - #11	12.48	3.139	12.26	1.68

In the hand calculations and the reference used illustrated above, the results are in excellent agreement with the automated exact results obtained from the spBeam program.

The required amount of tension reinforcement in the section can be reasonably estimated using the following equation that is based on practical experience.

$$A_{s\_est} = \frac{M_n}{0.8 \times f_y \times h} = \frac{1991.1 \text{ ft-kips} \times \dfrac{12 \text{ in.}}{1 \text{ ft}}}{0.8 \times 60 \text{ ksi} \times 40 \text{ in.}} = 12.44 \text{ in.}^2$$

The design process of reinforced concrete beams (rectangular or flanged) for flexure is a time-and-budget-consuming process in any design project as beam members are mainly and repeatedly compose conventional building frames. Therefore, using spBeam by industry professionals for the repetitive analysis and design of important structural members is beneficial in term of time and budgets while achieving accuracy and proficiency.