Flexural Stiffness for Critical Buckling Load of Concrete Columns (CSA A23.3)

A primary concern in calculating the critical axial load P_c is the choice of the flexural stiffness, *EI*, that reasonably approximates the variations due to cracking, creep, and concrete nonlinearity. *EI* is used in the process of determining the moment magnification at column ends and along the column length in sway and nonsway frames. Per CSA A23.3-94, 04, and 14:

$$P_c = \frac{\pi^2 EI}{\left(kl_u\right)^2}$$

CSA A23.3-04 & 14 (10.15.3.1)

CSA A23.3-94 (10.15.3)

$$M_c = \frac{C_m M_2}{1 - \dfrac{P_f}{\phi_m P_c}} \geq M_2 \text{ (For nonsway frames)}$$

CSA A23.3-04 & 14 (10.15.3.1)

CSA A23.3-94 (10.15.3)

Where $\phi_m = 0.75$

$$\delta_s = \frac{1}{1 - \dfrac{\Sigma P_f}{\phi_m \times \Sigma P_c}} \quad \text{(For sway frames)}$$

CSA A23.3-94, 04 & 14 (10.16.3.2)

Where $\phi_m = 0.75$

CSA A23.3 provides two options to calculate *EI* as follows:

$$EI = \left\{ \begin{array}{ll} \text{(a)} & \dfrac{0.2E_c I_g + E_s I_{se}}{1+\beta_d} \\ \text{(b)} & \dfrac{0.4E_c I_g}{1+\beta_d} \end{array} \right\}$$

CSA A23.3-04 & 14 (10.15.3.1)

$$EI = \left\{ \begin{array}{ll} \text{(a)} & \dfrac{0.2E_c I_g + E_s I_{se}}{1+\beta_d} \\ \text{(b)} & 0.25E_c I_g \end{array} \right\}$$

CSA A23.3-94 (10.15.3)

spColumn Program utilizes equation (a) for the calculation of the flexural stiffness of compression members. Equation (b) is also permitted by the CSA A23.3.

Comparison of *EI* values for Individual Columns

CSA A23.3 states that both equations (a) and (b) give approximate lower bound expressions for the flexural stiffness of individual compression members. Also, the effect of sustained loads must be considered for both of them. Since both equations are lower bounds, it follows logically that it is appropriate to select the larger value. If the reinforcing steel is not yet chosen, I_{se} cannot be computed and equation (b) is the only option to compute an initial value for *EI*. However, equation (a) gives more accurate values by considering the reinforcing steel influence on flexural stiffness.

A different value for the magnitude of the magnified moment is possible for each option of *EI.* So which equation will lead to the optimum column design based on the CSA A23.3-14 provision? To answer this question equation (a) is set equal to equation (b) as follows:

$$\frac{0.2E_cI_g + E_sI_{se}}{1+\beta_d} = \frac{0.4E_cI_g}{1+\beta_d}$$

This will lead to the relative stiffness non-dimensional factor α_r to be used to illustrate the comparison:

$$\alpha_r = \frac{E_sI_{se}}{E_cI_g} = 0.2$$

When α_r is greater than 0.2, *EI* obtained from equation (a) will be greater than *EI* obtained from equation (b) resulting in a lower value for the magnification factor and a lower magnified moment.

Conclusions

The designer may utilize any of the options for *EI* provided in CSA A23.3, Clause 10.15.3 given the available input data at a given design stage and the desired level of accuracy. spColumn Program utilizes equation (a) which is more accurate than equation (b). The designer may choose between equations (a) and (b) to optimize the required moment magnification and finalize column size and reinforcement.

References

[1] Design of Concrete Structures (CSA A23.3-94) and Explanatory Notes on CSA Standard A23.3-94

[2] Design of Concrete Structures (CSA A23.3-04) and Explanatory Notes on CSA Standard A23.3-04

[3] Design of Concrete Structures (CSA A23.3-14) and Explanatory Notes on CSA Standard A23.3-14

[4] Reinforced Concrete Mechanics and Design, First Canadian Edition, 2000, James MacGregor and Michael Bartlett, Prentice Hall.