Interaction Diagram - Circular Spiral Reinforced Concrete Column (ACI 318-19)
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Develop an interaction diagram for the circular concrete column shown in the figure below about the x-axis. Determine seven control points on the interaction diagram and compare the calculated values with the Reference and exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.

![Circular Reinforced Concrete Column Cross-Section](image)

**Figure 1 – Circular Reinforced Concrete Column Cross-Section**
Contents

1. Maximum Compression ........................................................................................................................................... 4
   1.1. Nominal axial compressive strength at zero eccentricity .................................................................................. 4
   1.2. Factored axial compressive strength at zero eccentricity .................................................................................. 4
   1.3. Maximum (allowable) factored axial compressive strength ............................................................................. 4
2. Bar Stress Near Tension Face Equal to Zero, ($\varepsilon_s = f_s = 0$) ............................................................................. 5
3. Bar Stress Near Tension Face Equal to 0.5 $f_y$, ($f_s = 0.5 f_y$) .................................................................................. 8
4. Bar Stress Near Tension Face Equal to $f_y$, ($f_s = f_y$) ............................................................................................ 11
5. Bar Strain Near Tension Face Equal to $\varepsilon_y + 0.003 \text{ in./in.}$, ($\varepsilon_s = 0.00507 \text{ in./in.}$) ...................... 14
6. Pure Bending ......................................................................................................................................................... 17
7. Maximum Tension .................................................................................................................................................. 20
8. Column Interaction Diagram - spColumn Software .............................................................................................. 21
9. Summary and Comparison of Design Results ..................................................................................................... 31
10. Conclusions & Observations .............................................................................................................................. 32
Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

- spColumn Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2021
- “Interaction Diagram - Tied Reinforced Concrete Column Design Strength (ACI 318-19)” Design Example, STRUCTUREPOINT, 2022
- “Interaction Diagram - Tied Reinforced Concrete Column with High-Strength Reinforcing Bars (ACI 318-19)” Design Example, STRUCTUREPOINT, 2022
- “Interaction Diagram - Barbell Concrete Shear Wall Unsymmetrical Boundary Elements (ACI 318-19)” Design Example, STRUCTUREPOINT, 2022
- “Interaction Diagram – Building Elevator Reinforced Concrete Core Wall Design Strength (ACI 318-19)” Design Example, STRUCTUREPOINT, 2022

Design Data

\[ f_c' = 5000 \text{ psi} \]
\[ f_y = 60000 \text{ psi} \]
Clear Cover = 1.5 in.
Column Diameter = 20 in.

Stirrups, longitudinal reinforcement and reinforcement locations are shown in Figure 1 and Table 1.

<table>
<thead>
<tr>
<th>Layer, ( i )</th>
<th>( d_i ) in</th>
<th>( n_i )</th>
<th>( A_{si}, \text{in}^2 )</th>
<th>( nA_{si}, \text{in}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.64</td>
<td>1</td>
<td>1.27</td>
<td>1.27</td>
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<td>2</td>
<td>4.79</td>
<td>2</td>
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<td>2</td>
<td>1.27</td>
<td>2.54</td>
</tr>
<tr>
<td>4</td>
<td>15.21</td>
<td>2</td>
<td>1.27</td>
<td>2.54</td>
</tr>
<tr>
<td>5</td>
<td>17.37</td>
<td>1</td>
<td>1.27</td>
<td>1.27</td>
</tr>
</tbody>
</table>

\[ A_{st} = \sum n_i A_{si} \]

\[ A_{st} = 10.16 \]
Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following seven control points:

Point 1: Maximum compression
Point 2: Bar stress near tension face equal to zero, \((f_s = 0)\)
Point 3: Bar stress near tension face equal to 0.5 \(f_y\) \((f_s = 0.5 f_y)\)
Point 4: Bar stress near tension face equal to \(f_y\) \((f_s = f_y)\)
Point 5: Bar strain near tension face equal to \(\varepsilon_y + 0.003\)
Point 6: Pure bending
Point 7: Maximum tension

Several terms are used to facilitate the following calculations:

\[ A_{comp} = \text{concrete area in compression, in}^2. \]
\[ \bar{y} = \text{geometric centroid location along the y-axis, in.} \]
\[ P_o = \text{nominal axial compressive strength, kip} \]
\[ \phi P_o = \text{factored axial compressive strength, kip} \]
\[ \phi P_{n,max} = \text{maximum (allowable) factored axial compressive strength, kip} \]
\[ c = \text{distance from the fiber of maximum compressive strain to the neutral axis, in.} \]
\[ a = \text{depth of equivalent rectangular stress block, in.} \]
\[ C_c = \text{compression force in equivalent rectangular stress block, kip} \]
\[ \varepsilon_{s,1} = \text{strain value in reinforcement layer, in./in.} \]
\[ C_t = \text{compression force in reinforcement, kip} \]
\[ T_s = \text{tension force in reinforcement, kip} \]
Figure 2 – Circular Column Section Interaction Diagram Control Points

1.1. Maximum Nominal Axial Strength in Compression (\(P_n\))
2.1. Nominal Control Point at Zero Stress in Tension Reinforcement
3.1. Nominal Control Point at Tension Reinforcement Stress = 0.5\(f_y\)
4.1. Nominal Control Point at Tension Reinforcement Stress = \(f_y\)
5.1. Nominal Control Point at Tension Reinforcement Stress = \(\varepsilon_y + 0.003\) in./in.
6.1. Nominal Control Point at Pure Bending
7.1. Maximum Nominal Axial Strength in Tension

1.2. Maximum Factored Axial Strength in Compression
1.3. Allowable Factored Axial Strength in Compression (\(\Phi P_{n,allow}\))
2.2. Factored Control Point at Zero Stress in Tension Reinforcement
3.2. Factored Control Point at Tension Reinforcement Stress = 0.5\(f_y\)
4.2. Factored Control Point at Tension Reinforcement Stress = \(f_y\)
5.2. Factored Control Point at Tension Reinforcement Stress = \(\varepsilon_y + 0.003\) in./in.
6.2. Factored Control Point at Pure Bending
7.2. Maximum Factored Axial Strength in Tension
1. Maximum Compression

1.1. Nominal axial compressive strength at zero eccentricity

\[ P_o = 0.85f'(A_y - A_n) + f_nA_n \]

\[ P_o = 0.85 \times 5000 \times \left( \frac{\pi}{4} \times 20^2 - 8 \times 1.27 \right) + 60000 \times 8 \times 1.27 = 1902 \text{ kips} \]

ACI 318-19 (22.4.2.2)

1.2. Factored axial compressive strength at zero eccentricity

Since this column is a spiral column with steel strain in compression:

\[ \phi = 0.75 \]

\[ \phi P_o = 0.75 \times 1902 = 1426.2 \text{ kips} \]

ACI 318-19 (Table 21.2.2)

Since the section is regular (symmetrical) about the x-axis, the moment capacity associated with the maximum axial compressive strength is equal to zero.

\[ M_o = \phi M_o = 0.00 \text{ kip-ft} \]

1.3. Maximum (allowable) factored axial compressive strength

\[ \phi P_{o,\text{max}} = 0.85 \times \phi P_o = 0.85 \times 1426.2 = 1212.3 \text{ kips} \]

ACI 318-19 (Table 22.4.2.1)
2. Bar Stress Near Tension Face Equal to Zero, \( (\varepsilon_s = f_s = 0) \)

Strain \( \varepsilon_s \) is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices.

**ACI 318-19 (10.7.5.2.1 and 2)**

For Concrete:

\[
c = d_s = 17.37 \text{ in.}
\]

\[
\varepsilon_{s5} = 0 < \varepsilon_y = \frac{F_y}{E_y} = \frac{60}{29000} = 0.00207
\]

\[
\therefore \phi = 0.75
\]

\[
\varepsilon_{cs} = 0.003
\]

**ACI 318-19 (Table 21.2.2)**

Where \( c \) is the distance from the fiber of maximum compressive strain to the neutral axis.

**ACI 318-19 (22.2.2.1)**

\[
a = \beta_i \times c = 0.80 \times 17.37 = 13.89 \text{ in.}
\]

**ACI 318-19 (22.2.4.2)**

Where:

\[
a = \text{Depth of equivalent rectangular stress block}
\]

\[
\beta_i = 0.85 - \frac{0.05 \times (f' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80
\]

**ACI 318-19 (Table 22.2.4.3)**

\[
C_c = 0.85 \times f' \times A_{\text{comp}} = \frac{0.85 \times 5000 \times 232.9}{232.9} = 989.9 \text{ kip} \quad \text{(Compression)}
\]

**ACI 318-19 (22.2.4.1)**

Where:
\[
\theta = \cos^{-1}\left(\frac{D-a}{2D}\right) = \cos^{-1}\left(\frac{2D-a}{2} - \frac{13.89}{20}\right) = 112.9^\circ
\]

\[
A_{\text{comp}} = D^2 \times \frac{\theta - \sin(\theta) \times \cos(\theta)}{4}
\]

\[
A_{\text{comp}} = 20^2 \times \frac{(112.9^\circ \times \frac{\pi}{180^\circ}) - \sin(112.9^\circ) \times \cos(112.9^\circ)}{4} = 232.9 \text{ in.}^2
\]

\[
\overline{y} = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{\text{comp}}} = \frac{20^3 \times \sin^3(112.9^\circ)}{12 \times 232.9} = 2.24 \text{ in.}
\]

Figure 4 – Cracked Column Section Properties \((\epsilon_t = f_s = 0)\)

For Reinforcement:

\[
\epsilon_{s4} = \left(\frac{c-d_s}{c}\right) \times \frac{e_{cu}}{c} = (17.37 - 15.21) \times \frac{0.003}{17.37} = 0.00037 \text{ (Compression)} < \epsilon_y = \frac{F_y}{E_s} = \frac{60,000}{29,000} = 0.00207
\]

\[
f_{s5} = 0 \text{ psi} \rightarrow F_{s5} = f_{s5} \times A_{s5} = 0 \text{ kip}
\]

Since \(\epsilon_{s4} < \epsilon_y\) \(\rightarrow\) reinforcement has not yielded

\[
\therefore f_{s4} = \epsilon_{s4} \times E_s = 0.00037 \times 29000000 = 10808 \text{ psi}
\]

The area of the reinforcement in this layer is not included in the area \((ab)\) used to compute \(C_c\). As a result, it is NOT necessary to subtract 0.85\(f_{s4}^\prime\) from \(f_{s4}\) before computing \(F_{s4}\).
\[ F_{s4} = f_{s4} \times A_{s4} = 10808 \times (2 \times 1.27) = 27.45 \text{ kip (Compression)} \]

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

<table>
<thead>
<tr>
<th>Layer</th>
<th>d, in.</th>
<th>( \varepsilon ), in./in.</th>
<th>( f_s ), psi</th>
<th>( F_s ), kip</th>
<th>( C_c ), kip</th>
<th>Moment arm (r), in.</th>
<th>Moment, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>989.86</td>
<td>2.24</td>
<td>184.55</td>
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<td>1</td>
<td>2.64</td>
<td>0.00254</td>
<td>60000</td>
<td>70.80*</td>
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<td>7.37</td>
<td>43.46</td>
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<td>4.79</td>
<td>0.00217</td>
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<td>5.21</td>
<td>61.45</td>
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<td>3</td>
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<tr>
<td>4</td>
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<td>27.45</td>
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<td>-11.91</td>
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<tr>
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<td>17.37</td>
<td>0.00000</td>
<td>0</td>
<td>0.00</td>
<td>---</td>
<td>-7.37</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Axial Force and Bending Moment

\[ P_n, \text{kip} \quad 1312.64 \quad M_n, \text{kip-ft} \quad 277.54 \]

\[ \phi P_n, \text{kip} \quad 984.48 \quad \phi M_n, \text{kip-ft} \quad 208.16 \]

* The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, \(0.85f'_c\) is subtracted from \(f_s\) in the computation of \(F_s\).

Where:

\[ P_n = C_c + \sum F_s \quad (+) = \text{Compression} \quad (-) = \text{Tension} \]

\[ \phi P_n = \phi \times P_n = 0.75 \times P_n \]

\[ M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + \sum_{i=1}^{n-1} \left( F_s \times \left( \frac{h}{2} - d_i \right) \right) \quad (+) = \text{Counter Clockwise} \quad (-) = \text{Clockwise} \]

\[ \phi M_n = \phi \times M_n = 0.75 \times M_n \]
3. Bar Stress Near Tension Face Equal to 0.5 $f_t$, ($f_i = 0.5 f_t$)

![Strain Diagram](image)

**Figure 5 – Strains, Forces, and Moment Arms ($f_i = 0.5 f_t$)**

For Concrete:

$$\varepsilon_y = \frac{f_i}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_{cs} = \frac{\varepsilon_y}{2} = \frac{0.00207}{2} = -0.00103 \text{ (Tension)} < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$$\varepsilon_{cs} < \varepsilon_y$$

$$\therefore \phi = 0.75 \quad \text{ACI 318-19 (Table 21.2.2)}$$

$$\varepsilon_{cu} = 0.003 \quad \text{ACI 318-19 (22.2.2.1)}$$

$$c = \frac{d_i}{\varepsilon_{cs} + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{17.37 \times 0.003}{0.00103 + 0.003} = 12.91 \text{ in.}$$

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

$$a = \beta_i \times c = 0.80 \times 12.91 = 10.33 \text{ in.} \quad \text{ACI 318-19 (22.2.4.2)}$$

Where:

$a = \text{Depth of equivalent rectangular stress block}$

$$\beta_i = 0.85 - \frac{0.05 \times (f_i' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80 \quad \text{ACI 318-19 (Table 22.2.4.3)}$$

$$C_c = 0.85 \times f_i' \times A_{comp} = 0.85 \times 5000 \times 163.7 = 695.63 \text{ kip (Compression)} \quad \text{ACI 318-19 (22.2.4.1)}$$

Where:
\[ \theta = \cos^{-1}\left(\frac{D - a}{2 \frac{D}{2}}\right) = \cos^{-1}\left(\frac{20 - 10.33}{20}\right) = 91.9^\circ \]

\[ A_{\text{comp}} = D^2 \times \frac{\theta - \sin(\theta) \times \cos(\theta)}{4} \]

\[ A_{\text{comp}} = 20^2 \times \frac{91.9^\circ \times \frac{\pi}{180^\circ} - \sin(91.9^\circ) \times \cos(91.9^\circ)}{4} = 163.7 \text{ in.}^2 \]

\[ \overline{y} = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{\text{comp}}} = \frac{20^3 \times \sin^3(91.9^\circ)}{12 \times 163.7} = 4.07 \text{ in.} \]

**Figure 6 – Cracked Column Section Properties \( f_s = 0.5 f_y \)**

For Reinforcement:

\[ \varepsilon_{s5} = \frac{\varepsilon_y}{2} = -\frac{0.00207}{2} = -0.00103 \text{ (Tension)} < \varepsilon_y \rightarrow \text{reinforcement has not yielded} \]

\[ f_{s5} = \varepsilon_{s5} \times E_s = -0.00053 \times 29000000 = -30000 \text{ psi} \]

The area of the reinforcement in this layer is not included in the area \( ab \) used to compute \( C_c \). As a result, it is NOT necessary to subtract 0.85\( f'_s \) from \( f_{s5} \) before computing \( F_{s5} \):

\[ F_{s5} = f_{s5} \times A_{s5} = -30000 \times (1 \times 1.27) = -38.1 \text{ kip (Tension)} \]
\( \epsilon_{s4} = \left( c - d_s \right) \times \frac{E_{su}}{E} = (12.91 - 15.21) \times \frac{0.003}{12.91} = -0.00053 \) (Tension) \( \times \epsilon_y \) → reinforcement has not yielded

\[ \therefore f_{s4} = \epsilon_{s4} \times E_y = -0.00053 \times 29000000 = -15465 \text{ psi} \]

The area of the reinforcement in this layer is not included in the area \((ab)\) used to compute \(C_c\). As a result, it is NOT necessary to subtract \(0.85f_c'\) from \(f_{s4}\) before computing \(F_{s4}\):

\[ F_{s4} = f_{s4} \times A_{s4} = -15465 \times (2 \times 1.27) = -39.28 \text{ kip} \text{ (Tension)} \]

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

<table>
<thead>
<tr>
<th>Layer</th>
<th>(d, \text{ in.})</th>
<th>(\epsilon, \text{ in./in.})</th>
<th>(f_{s}, \text{ psi})</th>
<th>(F_{s}, \text{ kip})</th>
<th>(C_c, \text{ kip})</th>
<th>Moment arm ((r), in.)</th>
<th>Moment, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>---</td>
<td>0.00300</td>
<td>---</td>
<td>---</td>
<td>695.63</td>
<td>4.07</td>
<td>235.73</td>
</tr>
<tr>
<td>1</td>
<td>2.64</td>
<td>0.00239</td>
<td>60000</td>
<td>70.80*</td>
<td>---</td>
<td>7.37</td>
<td>43.46</td>
</tr>
<tr>
<td>2</td>
<td>4.79</td>
<td>0.00189</td>
<td>54712</td>
<td>128.17*</td>
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<td>5.21</td>
<td>55.62</td>
</tr>
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<td>3</td>
<td>10.00</td>
<td>0.00068</td>
<td>19623</td>
<td>39.05*</td>
<td>---</td>
<td>0.00</td>
<td>0.00</td>
</tr>
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<td>0.00053</td>
<td>-15465</td>
<td>-39.28</td>
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<td>-5.21</td>
<td>17.05</td>
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<tr>
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<td>-0.00103</td>
<td>-30000</td>
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<td>-7.37</td>
<td>23.38</td>
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<tr>
<td>Axial Force and Bending Moment</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P_n, \text{ kip})</td>
<td>856.27</td>
<td>(M_n, \text{ kip-ft})</td>
<td>375.24</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(\phi P_n, \text{ kip})</td>
<td>642.20</td>
<td>(\phi M_n, \text{ kip-ft})</td>
<td>281.43</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, \(0.85f_c'\) is subtracted from \(f_i\) in the computation of \(F_i\).

Where:

\[ P_n = C_c + \sum F_s \]  

\( (+) = \text{Compression} \quad (-) = \text{Tension} \)

\[ \phi P_n = \phi \times P_n = 0.75 \times P_n \]

\[ M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + \sum_{i=1}^{n} \left( F_s \times \left( \frac{h}{2} - d_s \right) \right) \]  

\( (+) = \text{Counter Clockwise} \quad (-) = \text{Clockwise} \)

\[ \phi M_n = \phi \times M_n = 0.75 \times M_n \]

---

10
4. Bar Stress Near Tension Face Equal to \( f_t = f_y \)

![Figure 7 – Strains, Forces, and Moment Arms (f_t = f_y)](image)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for \( \phi \) for columns in which \( \phi \) increases from 0.75 for spiral columns (or 0.65 for tied columns) up to 0.90.

For Concrete:

\[
\varepsilon_y = \frac{f_y}{E} = \frac{60}{29,000} = 0.00207
\]

\[
\varepsilon_{s5} = -\varepsilon_y = -0.00207 \quad \text{(Tension)} = \varepsilon_y \rightarrow \text{tension reinforcement has yielded}
\]

\[
\therefore \phi = 0.75
\]

\[
\varepsilon_{cu} = 0.003
\]

\[
c = \frac{d_5}{\varepsilon_{s5} + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{17.37}{0.00207 + 0.003} \times 0.003 = 10.28 \text{ in.}
\]

Where \( c \) is the distance from the fiber of maximum compressive strain to the neutral axis.

\[
a = \beta_i \times c = 0.80 \times 10.28 = 8.22 \text{ in.}
\]

Where:

\( a = \text{Depth of equivalent rectangular stress block} \)

\[
\beta_i = 0.85 - \frac{0.05 \times (f'_y - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80
\]

ACI 318-19 (Table 21.2.2)

ACI 318-19 (22.2.2.1)

ACI 318-19 (22.2.4.2)

ACI 318-19 (22.2.4.1)

ACI 318-19 (Table 22.2.4.3)
\[ C_c = 0.85 \times f'_c \times A_{comp} = 0.85 \times 5000 \times 121.7 = 517.24 \text{ kip (Compression)} \]

Where:
\[
\theta = \cos^{-1}\left(\frac{D - a}{D/2}\right) = \cos^{-1}\left(\frac{20}{2} - \frac{8.22}{2}\right) = 79.8°
\]
\[
A_{comp} = D^2 \times \frac{\theta - \sin(\theta) \times \cos(\theta)}{4}
\]
\[
A_{comp} = 20^2 \times \frac{\left(79.8° \times \frac{\pi}{180°}\right) - \sin(79.8°) \times \cos(79.8°)}{4} = 121.7 \text{ in}^2
\]
\[
y = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{comp}} = \frac{20^3 \times \sin^3(79.8°)}{12 \times 121.7} = 5.22 \text{ in.}
\]

Figure 8 – Cracked Column Section Properties \((f_s = f_y)\)

For Reinforcement:
\[
\varepsilon_{s5} = -\varepsilon_y = -0.00207 \text{ (Tension)} = \varepsilon_y \rightarrow \text{reinforcement has yielded}
\]
\[
\therefore f_{s5} = f_y = -60000 = -60000 \text{ psi}
\]

The area of the reinforcement in this layer is not included in the area \((ab)\) used to compute \(C_c\). As a result, it is NOT necessary to subtract \(0.85f'_c\) from \(f_{s5}\) before computing \(F_{s5}\):

\[
F_{s5} = f_{s5} \times A_{s5} = -60000 \times (1 \times 1.27) = -76.2 \text{ kip (Tension)}
\]
\[ \varepsilon_{st} = (e - d_s) \times \frac{E_{cu}}{E_s} = (10.28 - 15.21) \times \frac{0.003}{10.28} = -0.00144 \] (Tension) \< \varepsilon_s \rightarrow \text{reinforcement has not yielded}

\[ : \quad f_{st} = \varepsilon_{st} \times E_s = -0.00144 \times 29000000 = -41739 \text{ psi} \]

The area of the reinforcement in this layer is not included in the area \((ab)\) used to compute \(C_c\). As a result, it is NOT necessary to subtract \(0.85f'_c\) from \(f_{st}\) before computing \(F_{st}\):

\[ F_{st} = f_{st} \times A_{st} = -41739 \times (2 \times 1.27) = -106.02 \text{ kip (Tension)} \]

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

<table>
<thead>
<tr>
<th>Layer</th>
<th>(d, \text{ in.})</th>
<th>(\varepsilon, \text{ in./in.})</th>
<th>(f_s, \text{ psi})</th>
<th>(F_s, \text{ kip})</th>
<th>(C_c, \text{ kip})</th>
<th>Moment arm (r), in.</th>
<th>Moment, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>---</td>
<td>0.00300</td>
<td>---</td>
<td>---</td>
<td>517.24</td>
<td>5.22</td>
<td>225.00</td>
</tr>
<tr>
<td>1</td>
<td>2.64</td>
<td>0.00223</td>
<td>60000</td>
<td>70.8*</td>
<td>---</td>
<td>7.37</td>
<td>43.46</td>
</tr>
<tr>
<td>2</td>
<td>4.79</td>
<td>0.00160</td>
<td>46433</td>
<td>107.14*</td>
<td>---</td>
<td>5.21</td>
<td>46.50</td>
</tr>
<tr>
<td>3</td>
<td>10.00</td>
<td>0.00008</td>
<td>2347</td>
<td>5.96*</td>
<td>---</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>15.21</td>
<td>-0.00144</td>
<td>-41739</td>
<td>-106.02</td>
<td>---</td>
<td>-5.21</td>
<td>46.01</td>
</tr>
<tr>
<td>5</td>
<td>17.37</td>
<td>-0.00207</td>
<td>-60000</td>
<td>-76.20</td>
<td>---</td>
<td>-7.37</td>
<td>46.77</td>
</tr>
<tr>
<td>Axial Force and Bending Moment</td>
<td>(P_n, \text{ kip})</td>
<td>(\phi P_n, \text{ kip})</td>
<td>518.93</td>
<td>389.20</td>
<td>(M_n, \text{ kip-ft})</td>
<td>(\phi M_n, \text{ kip-ft})</td>
<td>407.73</td>
</tr>
</tbody>
</table>

* The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, \(0.85f'_c\) is subtracted from \(f_{st}\) in the computation of \(F_{st}\).

Where:

\[ P_n = C_c + \sum_{i=1}^{n} F_s \]  
\( (+) = \text{Compression} \quad (-) = \text{Tension} \)

\[ \phi P_n = \phi \times P_n = 0.75 \times P_n \]

\[ M_n = C_c \times \left( \frac{h - a}{2} \right) + \sum_{i=1}^{n} \left( F_s \times \left( \frac{h}{2} - d_i \right) \right) \]  
\( (+) = \text{Counter Clockwise} \quad (-) = \text{Clockwise} \)

\[ \phi M_n = \phi \times M_n = 0.75 \times M_n \]
5. **Bar Strain Near Tension Face Equal to** \( \varepsilon_y + 0.003 \text{ in./in.} \), \( (\varepsilon_s = 0.00507 \text{ in./in.}) \)

**Figure 9 – Strains, Forces, and Moment Arms** \( (\varepsilon_s = 0.00507 \text{ in./in.}) \)

In ACI 318-19 provisions, this control point corresponds to the tension-controlled strain limit of \( \varepsilon_y + 0.003 \) (used to be 0.005 in ACI 318-14). It is the strain at the tensile limit of the transition zone for \( \phi \), used to define a tension-controlled section. Additional resources concerning code provision changes in ACI 318-19 can be found in “ACI 318-19 Code Revisions Impact on StructurePoint Software” technical article.

For Concrete:

\[
\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207
\]

\[
\varepsilon_{s5} = \varepsilon_y + 0.003 = 0.00207 + 0.003 = -0.00507 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}
\]

\[\therefore \phi = 0.90\]

\[
\varepsilon_{cu} = 0.003
\]

\[
c = \frac{d_s}{\varepsilon_{s5} + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{17.37}{0.00507 + 0.003} \times 0.003 = 6.46 \text{ in.}
\]

Where \( c \) is the distance from the fiber of maximum compressive strain to the neutral axis.

\[\text{ACI 318-19 (Table 21.2.2)}\]

\[a = \beta \times c = 0.80 \times 6.46 = 5.16 \text{ in.}\]

Where:

\[a = \text{Depth of equivalent rectangular stress block}\]

\[\beta = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80 \]

\[\text{ACI 318-19 (Table 22.2.4.3)}\]

\[C_c = 0.85 \times f'_c \times A_{comp} = 0.85 \times 5000 \times 64.29 = 273.24 \text{ kip (Compression)} \]

\[\text{ACI 318-19 (22.2.4.1)}\]
Where:

\[ \theta = \cos^{-1}\left( \frac{D - a}{\frac{D}{2}} \right) = \cos^{-1}\left( \frac{\frac{20}{2} - 5.16}{\frac{20}{2}} \right) = 61.1^\circ \]

\[ A_{\text{comp}} = D^2 \times \frac{\theta - \sin(\theta) \times \cos(\theta)}{4} \]

\[ A_{\text{comp}} = 20^2 \times \frac{\left(61.1^\circ \times \frac{\pi}{180^\circ}\right) - \sin\left(61.1^\circ\right) \times \cos\left(61.1^\circ\right)}{4} = 64.29 \text{ in.}^2 \]

\[ y = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{\text{comp}}} = \frac{20^3 \times \sin^3\left(61.1^\circ\right)}{12 \times 64.29} = 6.95 \text{ in.} \]

Figure 10 – Cracked Column Section Properties (\(\varepsilon_s = 0.00507 \text{ in./in.}\))

For Reinforcement:

\(\varepsilon_{s5} = -0.00507 \text{ (Tension)} > \varepsilon_s \rightarrow \text{reinforcement has yielded} \)

\[ \therefore f_{s5} = f_s = -60000 \text{ psi} \]

The area of the reinforcement in this layer is not included in the area \((ab)\) used to compute \(C_c\). As a result, it is NOT necessary to subtract \(0.85f'_s\) from \(f_{s5}\) before computing \(F_{s5}\):

\[ F_{s5} = f_{s5} \times A_{s5} = -60000 \times (1 \times 1.27) = -76.2 \text{ kip (Tension)} \]
\[ \varepsilon_{s4} = (c - d_s) \times \frac{E_{cu}}{E_s} = (6.46 - 15.21) \times \frac{0.003}{6.46} = -0.00407 \text{ (Tension)} \]  
\[ \Rightarrow \varepsilon_s \rightarrow \text{reinforcement has yielded} \]

\[ f_{s4} = f_s = -60000 \text{ psi} \]

The area of the reinforcement in this layer is not included in the area \((ab)\) used to compute \(C_c\). As a result, it is NOT necessary to subtract \(0.85f'_c\) from \(f_{s4}\) before computing \(F_{s4}\):

\[ F_{s4} = f_{s4} \times A_{s4} = -60000 \times (2 \times 1.27) = -152.4 \text{ kip (Tension)} \]

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

<table>
<thead>
<tr>
<th>Layer</th>
<th>(d, \text{ in.})</th>
<th>(\varepsilon, \text{ in./in.})</th>
<th>(f_s, \text{ psi})</th>
<th>(F_s, \text{ kip})</th>
<th>(C_c, \text{ kip})</th>
<th>Moment arm ((r)), \text{ in.}</th>
<th>Moment, \text{ kip-ft}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>---</td>
<td>0.00300</td>
<td>---</td>
<td>---</td>
<td>273.24</td>
<td>6.95</td>
<td>158.36</td>
</tr>
<tr>
<td>1</td>
<td>2.64</td>
<td>0.00178</td>
<td>51492</td>
<td>60.00(^*)</td>
<td>---</td>
<td>7.37</td>
<td>36.82</td>
</tr>
<tr>
<td>2</td>
<td>4.79</td>
<td>0.00077</td>
<td>22423</td>
<td>46.16(^*)</td>
<td>---</td>
<td>5.21</td>
<td>20.03</td>
</tr>
<tr>
<td>3</td>
<td>10.00</td>
<td>-0.00165</td>
<td>-47754</td>
<td>-121.29</td>
<td>---</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>15.21</td>
<td>-0.00407</td>
<td>-60000</td>
<td>-152.4</td>
<td>---</td>
<td>-5.21</td>
<td>66.14</td>
</tr>
<tr>
<td>5</td>
<td>17.37</td>
<td>-0.00507</td>
<td>-60000</td>
<td>-76.20</td>
<td>---</td>
<td>-7.37</td>
<td>46.77</td>
</tr>
</tbody>
</table>

Axial Force and Bending Moment

\[ P_n = C_c + \sum F_i \]

\[ \phi P_n = \phi \times P_n = 0.90 \times P_n \]

\[ M_n = C_r \times \left( \frac{h - a}{2} \right) + \sum_{i=1}^{n-1} \left( F_i \times \left( \frac{h}{2} - d_i \right) \right) \]

\[ \phi M_n = \phi \times M_n = 0.90 \times M_n \]

\(^*\) The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, \(0.85f'_c\) is subtracted from \(f_s\) in the computation of \(F_s\).
6. Pure Bending

![Strain Diagram Forces and Moment Arms](image)

Figure 11 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the nominal axial load capacity, \( P_n \), is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

Try \( c = 6.256 \) in.

Where \( c \) is the distance from the fiber of maximum compressive strain to the neutral axis.

\[
\varepsilon_c = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207
\]

\[
\varepsilon_{c5} = (c - d_s) \times \frac{\varepsilon_{cu}}{c} = \left( 6.256 - 17.36 \right) \times \frac{0.003}{6.256} = -0.00533 \text{ (Tension) > } \varepsilon_c \text{ → reinforcement has yielded}
\]

\( \varepsilon_{c5} > 0.005 \)

\( \therefore \phi = 0.9 \)

\[
a = \beta_i \times c = 0.80 \times 6.256 = 5.00 \text{ in.}
\]

\[
\varepsilon_{cu} = 0.003
\]

Where:

\( a \) = Depth of equivalent rectangular stress block

\[
\beta_i = 0.85 - \frac{0.05 \times (f'_y - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80
\]

\[
C_c = 0.85 \times f'_y \times A_{comp} = 0.85 \times 5000 \times 61.5 = 261.38 \text{ kip (Compression)}
\]

Where:
\[ \theta = \cos^{-1}\left(\frac{D - a}{D} - \frac{20}{2}ight) = \cos^{-1}\left(\frac{20 - 5.00}{20} \right) = 60.0^\circ \]

\[ A_{comp} = D^2 \times \frac{\theta - \sin(\theta) \times \cos(\theta)}{4} \]

\[ A_{comp} = 20^2 \times \frac{60.0^\circ \times \frac{\pi}{180^\circ} - \sin(60.0^\circ) \times \cos(60.0^\circ)}{4} = 61.5 \text{ in.}^2 \]

\[ y = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{comp}} = \frac{20^3 \times \sin^3(60.0^\circ)}{12 \times 61.5} = 7.05 \text{ in.} \]

**Figure 12 – Cracked Column Section Properties (Pure Moment)**

\[ \varepsilon_{s5} = -0.00533 \text{ (Tension)} > \varepsilon_y \rightarrow \text{reinforcement has yielded} \]

\[ \therefore f_{s5} = f_y = -60000 \text{ psi} \]

The area of the reinforcement in this layer is not included in the area \((ab)\) used to compute \(C_c\). As a result, it is NOT necessary to subtract 0.85\(f'_c\) from \(f_{s5}\) before computing \(F_{s5}\):

\[ F_{s5} = f_{s5} \times A_{s5} = -60000 \times (1 \times 1.27) = -76.2 \text{ kip (Tension)} \]

\[ \varepsilon_{s4} = \left(c - d_s\right) \times \frac{\varepsilon_{sw}}{c} = \left(6.256 - 15.21\right) \times \frac{0.003}{6.256} = -0.00429 \text{ (Tension)} > \varepsilon_y \rightarrow \text{reinforcement has yielded} \]

\[ \therefore f_{s4} = f_y = -60000 \text{ psi} \]
The area of the reinforcement in this layer is not included in the area \((ab)\) used to compute \(C_c\). As a result, it is NOT necessary to subtract \(0.85f'_c\) from \(f_s\) before computing \(F_s\):

\[
F_{s4} = f_{s4} \times A_{s4} = -60000 \times (2 \times 1.27) = -152.4 \text{ kip (Tension)}
\]

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

<table>
<thead>
<tr>
<th>Layer</th>
<th>(d), in.</th>
<th>(\epsilon), in./in.</th>
<th>(f_s), psi</th>
<th>(F_s), kip</th>
<th>(C_c), kip</th>
<th>Moment arm ((r)), in.</th>
<th>Moment, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>---</td>
<td>0.00300</td>
<td>---</td>
<td>---</td>
<td>261.38</td>
<td>7.05</td>
<td>153.51</td>
</tr>
<tr>
<td>1</td>
<td>2.64</td>
<td>0.00174</td>
<td>50356</td>
<td>58.55*</td>
<td>---</td>
<td>7.37</td>
<td>35.94</td>
</tr>
<tr>
<td>2</td>
<td>4.79</td>
<td>0.00070</td>
<td>20357</td>
<td>40.91*</td>
<td>---</td>
<td>5.21</td>
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<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>15.21</td>
<td>-0.00429</td>
<td>-60000</td>
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<td>---</td>
<td>-5.21</td>
<td>66.14</td>
</tr>
<tr>
<td>5</td>
<td>17.37</td>
<td>-0.00533</td>
<td>-60000</td>
<td>-76.20</td>
<td>---</td>
<td>-7.37</td>
<td>46.77</td>
</tr>
<tr>
<td>Axial Force and Bending Moment</td>
<td>(P_n), kip</td>
<td>0.00</td>
<td>(M_n), kip-ft</td>
<td>320.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\phi P_n), kip</td>
<td>0.00</td>
<td>(\phi M_n), kip-ft</td>
<td>288.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, \(0.85f'_c\) is subtracted from \(f_c\) in the computation of \(F_c\).

Where:

\[
P_n = C_c + \sum F_i
\]

\[
\phi P_n = \phi \times P_n = 0.90 \times P_n
\]

Since \(P_n = \phi P_n = 0\) kip, the assumption that \(c = 3.25\) in. is correct.

\[
M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + \sum F_i \times \left(\frac{h}{2} - d_i\right)
\]

\[
\phi M_n = \phi \times M_n = 0.90 \times M_n
\]
7. Maximum Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

\[ P_{nt} = f_y \times \sum_{i=1}^{n} n_i A_{ni} = 60,000 \times (8 \times 1.27) = -609.60 \text{ kip (Tension)} \quad \text{ACI 318-19 (22.4.3.1)} \]

\[ \phi = 0.90 \quad \text{ACI 318-19 (Table 21.2.2)} \]

\[ \phi P_{nt} = 0.90 \times 609.60 = -548.64 \text{ kip (Tension)} \]

Since the section is symmetrical

\[ M_n = \phi M_n = 0 \text{ kip.ft} \]
**Column Interaction Diagram - spColumn Software**

*spColumn* is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode was used with no loads (the program will only report control points) and no slenderness considerations using ACI 318-19.

**Figure 13 – spColumn Interface**
Figure 14 – spColumn Model Editor
Figure 15 – Defining Loads / Modes (spColumn)
Figure 16 – Column P-M Interaction Diagram about the X-Axis (spColumn)
Contents
1. General Information ................................................................. 3
2. Material Properties ...................................................................... 3
  2.1. Concrete ................................................................. 3
  2.2. Steel ................................................................. 3
3. Section ........................................................................ 3
  3.1. Shape and Properties ......................................................... 3
  3.2. Section Figure ................................................................... 4
4. Reinforcement ........................................................................ 4
  4.1. Bar Set, ASTM A615 ....................................................... 4
  4.2. Confinement and Factors .................................................. 4
  4.3. Arrangement ..................................................................... 4
5. Control Points ........................................................................ 5
6. Diagrams ........................................................................ 6
  6.1. PM at θ=0 [deg] ............................................................ 6

List of Figures
Figure 1: Column section ................................................................. 4
1. General Information

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2. Material Properties

2.1. Concrete

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<td>$f_y$</td>
<td>4.25 ksi</td>
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<tr>
<td>$e_y$</td>
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<tr>
<td>$k_y$</td>
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</table>

2.2. Steel

<table>
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</tr>
</thead>
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<td>$f_y$</td>
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</tr>
<tr>
<td>$E_s$</td>
<td>29000 ksi</td>
</tr>
<tr>
<td>$e_s$</td>
<td>0.00206867 in/lin</td>
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</table>

3. Section

3.1. Shape and Properties

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<th>Circular</th>
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</thead>
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<tr>
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<td>314.159 in²</td>
</tr>
<tr>
<td>$I_s$</td>
<td>7853.98 in⁴</td>
</tr>
<tr>
<td>$t_s$</td>
<td>5 in</td>
</tr>
<tr>
<td>$f_s$</td>
<td>5 in</td>
</tr>
<tr>
<td>$X_s$</td>
<td>0 in</td>
</tr>
<tr>
<td>$Y_s$</td>
<td>0 in</td>
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</table>
3.2. Section Figure

Figure 1: Column section

4. Reinforcement

4.1. Bar Set: ASTM A615

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<tr>
<th>Bar</th>
<th>Diameter (in)</th>
<th>Area (in²)</th>
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<th>Area (in²)</th>
<th>Bar</th>
<th>Diameter (in)</th>
<th>Area (in²)</th>
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<tr>
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<td>0.88</td>
<td>0.60</td>
<td>#8</td>
<td>1.00</td>
<td>0.79</td>
</tr>
<tr>
<td>#9</td>
<td>1.13</td>
<td>1.00</td>
<td>#10</td>
<td>1.27</td>
<td>1.27</td>
<td>#11</td>
<td>1.41</td>
<td>1.56</td>
</tr>
<tr>
<td>#14</td>
<td>1.69</td>
<td>2.25</td>
<td>#18</td>
<td>2.28</td>
<td>4.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2. Confinement and Factors

- Confinement type: Spiral
- For #10 bars or less: #3 ties
- For larger bars: #4 ties

4.3. Arrangement

<table>
<thead>
<tr>
<th>Pattern</th>
<th>All sides equal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar layout</td>
<td>Circular</td>
</tr>
<tr>
<td>Cover to</td>
<td>Longitudinal bars</td>
</tr>
<tr>
<td>Clear cover</td>
<td>2 in</td>
</tr>
<tr>
<td>Bars</td>
<td>8 #10</td>
</tr>
</tbody>
</table>
Total steel area, $A_s$ 10.16 in$^2$
Rho 3.23 %
Minimum clear spacing 4.37 in

### 5. Control Points

<table>
<thead>
<tr>
<th>About</th>
<th>Point</th>
<th>P</th>
<th>X-Moment</th>
<th>Y-Moment</th>
<th>NA Depth</th>
<th>$d_L$ Depth</th>
<th>$e_L$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X @ Max compression</td>
<td>1426.2</td>
<td>0.00</td>
<td>0.00</td>
<td>55.95</td>
<td>17.36</td>
<td>-0.000207</td>
<td>0.75000</td>
</tr>
<tr>
<td></td>
<td>X @ Allowable comp.</td>
<td>1212.3</td>
<td>110.60</td>
<td>0.00</td>
<td>21.50</td>
<td>17.36</td>
<td>-0.000598</td>
<td>0.75000</td>
</tr>
<tr>
<td></td>
<td>X @ $f_s = 0.0$</td>
<td>984.5</td>
<td>-206.16</td>
<td>0.00</td>
<td>17.36</td>
<td>17.36</td>
<td>0.000000</td>
<td>0.75000</td>
</tr>
<tr>
<td></td>
<td>X @ Balanced point</td>
<td>642.2</td>
<td>281.43</td>
<td>0.00</td>
<td>12.91</td>
<td>17.36</td>
<td>0.001030</td>
<td>0.75000</td>
</tr>
<tr>
<td></td>
<td>X @ Tension control</td>
<td>389.2</td>
<td>305.80</td>
<td>0.00</td>
<td>10.28</td>
<td>17.36</td>
<td>0.000207</td>
<td>0.75000</td>
</tr>
<tr>
<td></td>
<td>X @ Pure bending</td>
<td>26.6</td>
<td>295.31</td>
<td>0.00</td>
<td>6.46</td>
<td>17.36</td>
<td>0.000597</td>
<td>0.90000</td>
</tr>
<tr>
<td></td>
<td>X @ Max tension</td>
<td>-546.6</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>17.36</td>
<td>0.009969</td>
<td>0.90000</td>
</tr>
<tr>
<td>-X @ Max compression</td>
<td></td>
<td>1426.2</td>
<td>0.00</td>
<td>0.00</td>
<td>55.95</td>
<td>17.36</td>
<td>-0.000207</td>
<td>0.75000</td>
</tr>
<tr>
<td>-X @ Allowable comp.</td>
<td></td>
<td>1212.3</td>
<td>-110.60</td>
<td>0.00</td>
<td>21.50</td>
<td>17.36</td>
<td>-0.000598</td>
<td>0.75000</td>
</tr>
<tr>
<td>-X @ $f_s = 0.0$</td>
<td></td>
<td>984.5</td>
<td>-206.16</td>
<td>0.00</td>
<td>17.36</td>
<td>17.36</td>
<td>0.000000</td>
<td>0.75000</td>
</tr>
<tr>
<td>-X @ Balanced point</td>
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<td>12.91</td>
<td>17.36</td>
<td>0.001030</td>
<td>0.75000</td>
</tr>
<tr>
<td>-X @ Tension control</td>
<td></td>
<td>389.2</td>
<td>-305.80</td>
<td>0.00</td>
<td>10.28</td>
<td>17.36</td>
<td>0.000207</td>
<td>0.75000</td>
</tr>
<tr>
<td>-X @ Pure bending</td>
<td></td>
<td>26.6</td>
<td>-295.31</td>
<td>0.00</td>
<td>6.46</td>
<td>17.36</td>
<td>0.000597</td>
<td>0.90000</td>
</tr>
<tr>
<td>-X @ Max tension</td>
<td></td>
<td>-546.6</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>17.36</td>
<td>0.999969</td>
<td>0.90000</td>
</tr>
</tbody>
</table>
6. Diagrams
6.1. PM at θ=0 [deg]

**General Information**
- Project: Pinheiro Ex 10.18.1
- Column: Interior
- Engineer: SP
- Code: ACI 318-19
- Bar Set: ASTM A615
- Units: English
- Run Option: Investigation
- Run Axes: X - axle
- Stiffness: Not Considered
- Column Type: Structural
- Capacity Method: Moment capacity

**Materials**
- $f_y$: 5 ksi
- $E$: 4903.51 ksi
- $f_y'$: 60 ksi
- $E'$: 29000 ksi

**Section**
- Type: Circular
- Diameter: 20 in
- $A_o$: 314.159 in$^2$
- $I_o$: 7853.99 in$^4$
- $I_o'$: 7853.99 in$^4$

**Reinforcement**
- Pattern: All sides equal
- Bar layout: Circular
- Cover to: Longitudinal bars
- Clear cover: 2 in
- Bars: 8 #10
- Confinement type: Spiral
- Total steel area, $A_s$: 19.15 in$^2$
- $f_y'$: 3.23 ksi
- Min. clear spacing: 4.37 in
9. Summary and Comparison of Design Results

Table 7 - Comparison of Results (Balanced Point)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reference</th>
<th>Hand</th>
<th>spColumn</th>
</tr>
</thead>
<tbody>
<tr>
<td>c, in.</td>
<td>10.28</td>
<td>10.28</td>
<td>10.28</td>
</tr>
<tr>
<td>d₃, in.</td>
<td>17.36</td>
<td>17.36</td>
<td>17.36</td>
</tr>
<tr>
<td>εₛ₃, in./in.</td>
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<td>0.00207</td>
<td>0.00207</td>
</tr>
<tr>
<td>Pₚ, kip</td>
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<td>519</td>
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</tr>
<tr>
<td>Mₚ, kip-ft</td>
<td>408</td>
<td>408</td>
<td>408</td>
</tr>
</tbody>
</table>

Table 8 - Comparison of Results

<table>
<thead>
<tr>
<th>Control Point</th>
<th>ΦPₚ, kip</th>
<th>ΦMₚ, kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hand</td>
<td>spColumn</td>
</tr>
<tr>
<td>Max compression</td>
<td>1426.2</td>
<td>1426.2</td>
</tr>
<tr>
<td>Allowable compression</td>
<td>1212.3</td>
<td>1212.3</td>
</tr>
<tr>
<td>fₛ = 0.0</td>
<td>984.5</td>
<td>984.5</td>
</tr>
<tr>
<td>fₛ = 0.5 f_y</td>
<td>642.2</td>
<td>642.2</td>
</tr>
<tr>
<td>Balanced point</td>
<td>389.2</td>
<td>389.2</td>
</tr>
<tr>
<td>Tension control</td>
<td>26.6</td>
<td>26.6</td>
</tr>
<tr>
<td>Pure bending</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Max tension</td>
<td>-548.6</td>
<td>-548.6</td>
</tr>
</tbody>
</table>

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.
10. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to $M_x$ and $M_y$ due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint’s spColumn program can also evaluate column sections in biaxial mode to produce the results shown in the following Figures for the column section in this example.
Figure 17 – Nominal & Design Interaction Diagram in Two Directions (Biaxial) (spColumn)
Figure 18 – Circular Column Interaction Diagram and 3D failure Surface Viewer (spColumn)
Figure 19 – Circular Column 3D Failure Surface with a Horizontal Plane Cut at P = 500 kip (spColumn)
Figure 20 – Circular Column 3D Failure Surface with a Vertical Plane Cut at 45° (spColumn)