Interaction Diagram - Tied Reinforced Concrete Column (CSA A23.3-19)
Interaction Diagram - Tied Reinforced Concrete Column (CSA A23.3-19)

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using CSA A23.3-19 provisions. Determine six control points on the interaction diagram and compare the calculated values with exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.

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Code

Design of Concrete Structures (CSA A23.3-19)

References

- spColumn Engineering Software Program Manual v10.10, STRUCTUREPOINT, 2023

Design Data

\[ f'_c = 35 \text{ MPa} \]
\[ f_y = 400 \text{ MPa} \]

Cover = 55 mm to bar center

Column 400 mm \( \times \) 400 mm

Top reinforcement = 4 No. 30

Bottom reinforcement = 4 No. 30

Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following six control points:

Point 1: Pure compression

Point 2: Bar stress near tension face of member equal to zero, \( f_s = 0 \)

Point 3: Bar stress near tension face of member equal to 0.5 \( f_y \), \( f_s = -0.5 f_y \)

Point 4: Bar stress near tension face of member equal to \( f_y \), \( f_s = - f_y \)

Point 5: Pure bending

Point 6: Pure tension
Figure 2 – Control Points
1. Pure Compression

1.1. Nominal axial resistance at zero eccentricity

\[ P_o = \alpha_i f'_c (A_y - A_y) + f_y A_y \]

\[ P_o = 0.798 \times 35 \times (400 \times 400 - 8 \times 700) + 400 \times 8 \times 700 = 6,549.7 \text{ kN} \]

Where \( \alpha_i = 0.85 - 0.0015 f'_c \geq 0.67 \)  
\( \alpha_i = 0.85 - 0.0015 \times 35 = 0.798 \geq 0.67 \)  

CSA A23.3-19 (Equation 10.1)

1.2. Factored axial load resistance at zero eccentricity

Since this column is a tied column with steel strain in compression:

\[ P_{ro} = \alpha \phi_c f'_c (A_y - A_y) + \phi_s f_y A_y \]  

\[ P_{ro} = 0.798 \times 0.65 \times 35 \times (400 \times 400 - 8 \times 700) + 0.85 \times 400 \times 8 \times 700 = 4,705.3 \text{ kN} \]

Where:
\( \phi_c = 0.65 \)  
\( \phi_s = 0.85 \)  

CSA A23.3-19 (Equation 10.11)

CSA A23.3-19 (8.4.2)

CSA A23.3-19 (8.4.3(a))

1.3. Maximum factored axial load resistance

\[ P_{r,max} = (0.2 + 0.002h) P_{ro} \leq 0.80 P_{ro} \]  

\[ P_{r,max} = (0.2 + 0.002 \times 400) \times 4,705.3 = 4,705.3 \text{ kN} \leq 0.80 \times 4,705.3 = 3,764.2 \text{ kN} \]

\[ P_{r,max} = 3,764.2 \text{ kN} \]

CSA A23.3-19 (Equation 10.9)
2. Bar Stress Near Tension Face of Member Equal to Zero, ($\varepsilon_s = f_s = 0$)

![Strain Diagram and Forces and Moment Arms](image)

Figure 3 – Strains, Forces, and Moment Arms ($\varepsilon_s = f_s = 0$)

Strain $\varepsilon_s$ is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. **CSA A23.3-19 (12.15 and 16)**

2.1. $c$, $a$, and strains in the reinforcement

\[ c = d_t = 345 \text{ mm} \]

Where $c$ is depth of the neutral axis measured from the compression edge of the column section. **CSA A23.3-19 (3.2)**

\[ a = \beta_i \times c = 0.883 \times 345 = 304 \text{ mm} \]

Where:

\[ \beta_i = 0.97 - 0.0025 \times f_s' = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \]

**CSA A23.3-19 (Equation 10.2)**

\[ \varepsilon_s = 0 \]

**CSA A23.3-19 (8.4.2)**

\[ \phi_e = 0.65 \]

**CSA A23.3-19 (8.4.3(a))**

\[ \phi_s = 0.85 \]

**CSA A23.3-19 (10.1.3)**

\[ \varepsilon_{cu} = 0.0035 \]

\[ \varepsilon' = (c - d_t) \times \frac{\varepsilon_{cu}}{c} = (345 - 55) \times \frac{0.0035}{345} = 0.00294 \text{ (Compression)} > \varepsilon_s = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.00200 \]
2.2. Forces in the concrete and steel

\[ C_{rc} = \alpha_t \times \phi_c \times f' \times a \times b = 0.798 \times 0.65 \times 35 \times 304 \times 400 = 2,209.6 \text{ kN} \]

\[ f_s = 0 \text{ kN} \rightarrow T_{ns} = \phi_s \times f \times A_{t1} = 0 \text{ kN} \]

Since \( \varepsilon'_c > \varepsilon_y \rightarrow \) compression reinforcement has yielded

\[ \therefore f'_c = f_y = 400 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \( C_{rc}. \) As a result, it is necessary to subtract \( \alpha_1 \phi f_c ' \) from \( \phi f_s ' \) before computing \( C_{rs}: \)

\[ C_{rs} = (\phi_x \times f'_x \times \alpha_1 \times \phi \times f'_c) \times A_{t2} = (0.85 \times 400 - 0.798 \times 0.65 \times 35) \times 2,800 = 901.2 \text{ kN} \]

2.3. \( P_r \) and \( M_r \)

\[ P_r = C_{rc} + C_{ns} - T_{ns} = 2,209.6 + 901.2 - 0 = 3,110.8 \text{ kN} \]

\[ M_r = C_{rc} \times \left( \frac{h - a}{2} \right) + C_{ns} \times \left( \frac{h - d}{2} \right) + T_{ns} \times \left( d_1 - \frac{h}{2} \right) \]

\[ M_r = 2,209.6 \times \left( \frac{400}{2} - \frac{304}{2} \right) + 901.2 \times \left( \frac{400}{2} - 55 \right) + 0 \times \left( 345 - \frac{400}{2} \right) = 236.22 \text{ kN-m} \]
3. Bar Stress Near Tension Face of Member Equal to 0.5 $f_y$, ($f_s = -0.5 f_y$)

![Figure 4 – Strains, Forces, and Moment Arms ($f_s = -0.5 f_y$)](image)

3.1. $c$, $a$, and strains in the reinforcement

\[
\varepsilon_y = \frac{F_y}{E_y} = \frac{400}{200,000} = 0.00200
\]

\[
\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.00200}{2} = 0.00100 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}
\]

$\phi_c = 0.65$  \hspace{1cm} \text{CSA A23.3-19 (8.4.2)}

$\phi_s = 0.85$  \hspace{1cm} \text{CSA A23.3-19 (8.4.3(a))}

$\varepsilon_{cu} = 0.0035$  \hspace{1cm} \text{CSA A23.3-19 (10.1.3)}

\[
c = \frac{d_t}{\varepsilon_s + \varepsilon_{cu}} = \frac{345}{0.00100 + 0.0035} \times 0.0035 = 268 \text{ mm}
\]

Where $c$ is depth of the neutral axis measured from the compression edge of the column section.  \hspace{1cm} \text{CSA A23.3-19 (3.2)}

\[
a = \beta_t \times c = 0.883 \times 268 = 237 \text{ mm}
\]

Where:

$a = \text{Depth of equivalent rectangular stress block}$  \hspace{1cm} \text{CSA A23.3-19 (3.2)}

$\beta_t = 0.97 - 0.0025 \times f'_{se} = 0.97 - 0.0025 \times 35 = 0.883 > 0.67$  \hspace{1cm} \text{CSA A23.3-19 (Equation 10.2)}

\[
\varepsilon' = (c - d_t) \times \frac{\varepsilon_{cu}}{c} = (268 - 55) \times \frac{0.0035}{268} = 0.00278 \text{ (Compression) > } \varepsilon_y
\]
3.2. Forces in the concrete and steel

\[ C_{rc} = \alpha_t \times f_c' \times a \times b = 0.798 \times 0.65 \times 35 \times 237 \times 400 = 1,718.5 \text{ kN} \]

\[ f_s = \varepsilon_s \times E_s = 0.00100 \times 200,000 = 200 \text{ MPa} \]

\[ T_{rs} = \phi_r \times f_s \times A_s = 0.85 \times 200 \times 2,800 = 476.0 \text{ kN} \]

Since \( \varepsilon_s > \varepsilon_y \), compression reinforcement has yielded

\[ f_s' = f_y = 400 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_{rc}\). As a result, it is necessary to subtract \(\alpha_t \phi_r f_c'\) from \(\phi_s f_s'\) before computing \(C_{rs}\):

\[ C_{rs} = (\phi_s \times f_s' - \alpha_t \times \phi_r \times f_c') \times A_s = (0.85 \times 400 - 0.798 \times 0.65 \times 35) \times 2,800 = 901.2 \text{ kN} \]

3.3. \(P_r\) and \(M_r\)

\[ P_r = C_{rc} + C_{rs} - T_{rs} = 1,718.5 + 901.2 - 476.0 = 2,143.7 \text{ kN} \]

\[ M_r = C_{rc} \times \left( \frac{h - a}{2} \right) + C_{rs} \times \left( \frac{h - d_2}{2} \right) + T_{rs} \times \left( \frac{d_1 - h}{2} \right) \]

\[ M_r = 1,718.5 \times \left( \frac{400}{2} - \frac{237}{2} \right) + 901.2 \times \left( \frac{400}{2} - 55 \right) + 476.0 \times \left( 345 - \frac{400}{2} \right) = 339.92 \text{ kN-m} \]
4. Bar Stress Near Tension Face of Member Equal to $f$, ($f = -f_y$)

![Figure 5 – Strains, Forces, and Moment Arms ($f = -f_y$)](image)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement.

4.1. $c$, $a$, and strains in the reinforcement

\[
\varepsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.00200
\]

\[
\varepsilon_s = \varepsilon_y = 0.00200 \rightarrow \text{tension reinforcement has yielded}
\]

\[
\phi_c = 0.65 \quad \text{CSA A23.3-19 (8.4.2)}
\]

\[
\phi_s = 0.85 \quad \text{CSA A23.3-19 (8.4.3(a))}
\]

\[
\varepsilon_{cu} = 0.0035 \quad \text{CSA A23.3-19 (10.1.3)}
\]

\[
c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{345}{0.00200 + 0.0035} \times 0.0035 = 220 \text{ mm}
\]

Where $c$ is depth of the neutral axis measured from the compression edge of the column section.

\[
a = \beta_a \times c = 0.883 \times 220 = 194 \text{ mm} \quad \text{CSA A23.3-19 (10.1.7)}
\]

Where:

\[
a = \text{Depth of equivalent rectangular stress block} \quad \text{CSA A23.3-19 (3.2)}
\]

\[
\beta_a = 0.97 - 0.0025 \times f' = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \quad \text{CSA A23.3-19 (Equation 10.2)}
\]

\[
\varepsilon' = (c - d_y) \times \varepsilon_{cu} = (220 - 55) \times \frac{0.0035}{220} = 0.00262 \text{ (Compression) > } \varepsilon_y
\]
4.2. Forces in the concrete and steel

\[ C_{rc} = \alpha_i \times f_c \times f'_s \times a \times b = 0.798 \times 0.65 \times 35 \times 194 \times 400 = 1,406.1 \text{ kN} \]

CSA A23.3-19 (10.1.7)

\[ f_s = f'_s = 400 \text{ MPa} \]

\[ T_{rs} = \phi_s \times f_s \times A_s = 0.85 \times 400 \times 2,800 = 952.0 \text{ kN} \]

Since \( \varepsilon'_c > \varepsilon_y \), compression reinforcement has yielded

\[ \therefore f'_s = f_s = 400 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_{rc}\). As a result, it is necessary to subtract \(\alpha_i \phi f'_c\) from \(\phi f_s\) before computing \(C_{rs}\):

\[ C_{rs} = (\phi_s \times f'_s - \alpha_i \times \phi f'_c) \times A_s = (0.85 \times 400 - 0.798 \times 0.65 \times 35) \times 2,800 = 901.2 \text{ kN} \]

4.3. \(P_r\) and \(M_r\)

\[ P_r = C_{rc} + C_{rs} - T_{rs} = 1,406.1 + 901.2 - 952.0 = 1,355.3 \text{ kN} \]

\[ M_r = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left( \frac{h}{2} - d_a \right) + T_{rs} \times \left( d_i - \frac{h}{2} \right) \]

\[ M_r = 1,406.1 \times \left( \frac{400}{2} - \frac{194}{2} \right) + 901.2 \times \left( \frac{400}{2} - 55 \right) + 952.0 \times \left( 345 - \frac{400}{2} \right) = 413.72 \text{ kN-m} \]
5. Pure Bending

![Figure 6 - Strains, Forces, and Moment Arms (Pure Moment)](image)

This corresponds to the case where the factored axial load resistance, $P_r$, is equal to zero. Iterative procedure is used to determine the factored moment resistance as follows:

5.1. $c$, $a$, and strains in the reinforcement

Try $c = 78.56$ mm

Where $c$ is depth of the neutral axis measured from the compression edge of the column section.

\[ a = \beta_1 \times c = 0.883 \times 78.56 = 69 \text{ mm} \]

Where:

\[ \beta_1 = 0.97 - 0.0025 \times f'_y = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \]

\[ \varepsilon_{cu} = 0.0035 \]

\[ \varepsilon_y = \frac{F_y}{E_y} = \frac{400}{200,000} = 0.00200 \]

\[ \varepsilon = (d_1 - c) \times \frac{\varepsilon_{cu}}{c} = (345 - 78.56) \times \frac{0.0035}{78.56} = 0.01187 \text{ (Tension) > } \varepsilon_y \rightarrow \text{tension reinforcement has yielded} \]

\[ \phi_1 = 0.65 \]

\[ \phi_s = 0.85 \]

\[ \varepsilon' = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (78.56 - 55) \times \frac{0.0035}{78.56} = 0.00105 \text{ (Compression) < } \varepsilon_y \]
5.2. Forces in the concrete and steel

\[ C_{rc} = \alpha_{c} \times f_{c}' \times a \times b = 0.798 \times 0.65 \times 35 \times 69 \times 400 = 503.1 \text{ kN} \quad \text{CSA A23.3-19 (10.1.7)} \]

\[ f_{c}' = f_{c} = 400 \text{ MPa} \]

\[ T_{ss} = \phi_{s} \times f_{s} \times A_{s} = 0.85 \times 400 \times 2,800 = 952.0 \text{ kN} \]

Since \( e'_{c} < e_{c} \rightarrow \) compression reinforcement has not yielded

\[ \therefore f_{s}' = e'_{c} \times E_{s} = 0.00105 \times 200,000 = 210 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_{rc}\). As a result, it is necessary to subtract \(\alpha_{c} \phi_{c} f_{c}'\) from \(\phi_{s} f_{s}'\) before computing \(C_{rs}\):

\[ C_{rs} = (\phi_{s} \times f_{s}' - \alpha_{c} \times \phi_{c} \times f_{c}') \times A_{s} = (0.85 \times 210 - 0.798 \times 0.65 \times 35) \times 2,800 = 448.8 \text{ kN} \]

5.3. \(P_{r}\) and \(M_{r}\)

\[ P_{r} = C_{rc} + C_{rs} - T_{ss} = 503.1 + 448.8 - 952.0 = 0 \text{ kN} \]

The assumption that \(c = 78.56 \text{ mm}\) is correct

\[ M_{r} = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left( \frac{h}{2} - d_{2} \right) + T_{ss} \times \left( d_{1} - \frac{h}{2} \right) \]

\[ M_{r} = 503.1 \times \left( \frac{400}{2} - \frac{69}{2} \right) + 448.8 \times \left( \frac{400}{2} - 55 \right) + 952.0 \times \left( 345 - \frac{400}{2} \right) = 286.31 \text{ kN-m} \]
6. Pure Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

6.1. Strength under pure axial tension \( (P_{rt}) \)

\[
P_{rt} = \phi \times f_y \times (A_{s1} + A_{s2}) = 0.85 \times 400 \times (2,800 + 2,800) = 1,904.0 \text{ kN}
\]

6.2. Corresponding Moment \( (M_{rt}) \)

Since the section is symmetrical

\[
M_{rt} = 0.00 \text{ kN-m}
\]
7. **Column Interaction Diagram - *spColumn* Software**

*spColumn* is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode was used with no loads (the program will only report control points) and no slenderness considerations using CSA A23.3-19.

![Figure 7 – *spColumn* Interface](image-url)
Figure 8 – spColumn Model Editor
Figure 9 – Defining Loads / Modes (spColumn)
Figure 10 – Column P-M Interaction Diagram about the X-Axis (spColumn)
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2. Material Properties

2.1. Concrete

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3. Section

3.1. Shape and Properties

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3.2. Section Figure

![Figure 1: Column section](image)

Rectangular 400 x 400 mm 3.50% reinf.

4. Reinforcement

4.1. Bar Set: CSA G30.18

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<td>#25</td>
<td>25.20</td>
<td>500.00</td>
<td>#30</td>
<td>29.90</td>
<td>700.00</td>
<td>#35</td>
<td>35.70</td>
<td>1000.00</td>
</tr>
<tr>
<td>#45</td>
<td>43.70</td>
<td>1500.00</td>
<td>#55</td>
<td>56.40</td>
<td>2500.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2. Confinement and Factors

- Confinement type: Tied
- For #35 bars or less: #10 ties
- For larger bars: #15 ties

Material Resistance Factors

- Axial compression (a): 0.8
- Steel (f_s): 0.85
- Concrete (f_c): 0.65

Minimum dimension, h: 400 mm

4.3. Arrangement

- Pattern: Sides different
- Bar layout: Rectangular
- Cover to: Longitudinal bars
- Clear cover: ___
4.4. Bars Provided

<table>
<thead>
<tr>
<th></th>
<th>Bars</th>
<th>Clear cover</th>
</tr>
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<tbody>
<tr>
<td>Top</td>
<td>4</td>
<td>#30</td>
</tr>
<tr>
<td>Bottom</td>
<td>4</td>
<td>#30</td>
</tr>
<tr>
<td>Left</td>
<td>0</td>
<td>#10</td>
</tr>
<tr>
<td>Right</td>
<td>0</td>
<td>#10</td>
</tr>
</tbody>
</table>

5. Control Points

<table>
<thead>
<tr>
<th>About</th>
<th>Point</th>
<th>P</th>
<th>X-Moment</th>
<th>Y-Moment</th>
<th>NA Depth</th>
<th>d, Depth</th>
<th>εs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>kN</td>
<td>kN.mm</td>
<td>kN.mm</td>
<td>mm</td>
<td>mm</td>
<td></td>
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<tr>
<td>X @ Max comp.</td>
<td>4705.3</td>
<td>0.00</td>
<td>0.00</td>
<td>805</td>
<td>345</td>
<td>-0.00200</td>
<td></td>
</tr>
<tr>
<td>X @ Allow. comp.</td>
<td>3764.2</td>
<td>146.13</td>
<td>0.00</td>
<td>412</td>
<td>345</td>
<td>-0.00557</td>
<td></td>
</tr>
<tr>
<td>X @ f_e = 0.0</td>
<td>3119.8</td>
<td>236.22</td>
<td>0.00</td>
<td>345</td>
<td>345</td>
<td>0.00000</td>
<td></td>
</tr>
<tr>
<td>X @ f_e = 0.5 f_c</td>
<td>2143.7</td>
<td>339.92</td>
<td>0.00</td>
<td>268</td>
<td>345</td>
<td>0.00100</td>
<td></td>
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<tr>
<td>X @ Balanced pt.</td>
<td>1555.3</td>
<td>413.72</td>
<td>0.00</td>
<td>220</td>
<td>345</td>
<td>0.00200</td>
<td></td>
</tr>
<tr>
<td>X @ Pure bending</td>
<td>9.0</td>
<td>286.31</td>
<td>0.00</td>
<td>79</td>
<td>345</td>
<td>0.01187</td>
<td></td>
</tr>
<tr>
<td>X @ Max tension</td>
<td>-1904.0</td>
<td>0.00</td>
<td>0.00</td>
<td>345</td>
<td>9.99999</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- X @ Max comp. | 4705.3  | 0.00  | 0.00    | 805      | 345      | -0.00200 |
- X @ Allow. comp.| 3764.2   | -146.13 | 0.00    | 412      | 345      | -0.00557 |
- X @ f_e = 0.0  | 3119.8   | -236.22 | 0.00    | 345      | 345      | 0.00000 |
- X @ f_e = 0.5 f_c | 2143.7   | -339.92 | 0.00    | 268      | 345      | 0.00100 |
- X @ Balanced pt.| 1555.3   | -413.72 | 0.00    | 220      | 345      | 0.00200 |
- X @ Pure bending | 9.0       | -286.31 | 0.00    | 79       | 345      | 0.01187 |
- X @ Max tension | -1904.0  | 0.00   | 0.00    | 345      | 9.99999  |
6. Diagrams

6.1. PM at θ=0 [deg]

<table>
<thead>
<tr>
<th>General Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project</td>
</tr>
<tr>
<td>Column</td>
</tr>
<tr>
<td>Engineer</td>
</tr>
<tr>
<td>Code</td>
</tr>
<tr>
<td>Bar Set</td>
</tr>
<tr>
<td>Units</td>
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<td>Run Option</td>
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<tr>
<td>Run Axis</td>
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<tr>
<td>Stiffness</td>
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<tr>
<td>Column Type</td>
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<tr>
<td>Capacity Method</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_y$</td>
</tr>
<tr>
<td>$f_y'$</td>
</tr>
<tr>
<td>$f_y$</td>
</tr>
<tr>
<td>$E_y$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Width</td>
</tr>
<tr>
<td>Depth</td>
</tr>
<tr>
<td>$A_y$</td>
</tr>
<tr>
<td>$I_y$</td>
</tr>
<tr>
<td>$I_y'$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern</td>
</tr>
<tr>
<td>Bar layout</td>
</tr>
<tr>
<td>Cover to</td>
</tr>
<tr>
<td>Clear cover</td>
</tr>
<tr>
<td>Bars</td>
</tr>
<tr>
<td>Confinement type</td>
</tr>
<tr>
<td>Total steel area, $A_s$</td>
</tr>
<tr>
<td>Reinforcement ratio, $Rho$</td>
</tr>
<tr>
<td>Min. clear spacing</td>
</tr>
</tbody>
</table>

![Diagram of PM at 0.0 [deg]](image-url)
8. Summary and Comparison of Design Results

<table>
<thead>
<tr>
<th>Support</th>
<th>$P_n$, kN</th>
<th>$M_n$, kN-m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hand</td>
<td>spColumn</td>
</tr>
<tr>
<td>Max compression</td>
<td>4705.3</td>
<td>4705.3</td>
</tr>
<tr>
<td>Allowable compression</td>
<td>3764.2</td>
<td>3764.2</td>
</tr>
<tr>
<td>$f_t = 0.0$</td>
<td>3110.8</td>
<td>3110.8</td>
</tr>
<tr>
<td>$f_t = 0.5 f_c$</td>
<td>2143.7</td>
<td>2143.7</td>
</tr>
<tr>
<td>Balanced point</td>
<td>1355.3</td>
<td>1355.3</td>
</tr>
<tr>
<td>Pure bending</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Max tension</td>
<td>1904.0</td>
<td>1904.0</td>
</tr>
</tbody>
</table>

In all of the hand calculations illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.
9. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where \( f_s = f_y \)).

![Figure 11 - Strains, Forces, and Moment Arms (f_s = -f_y, Moments About x- and y-axis)](image-url)
When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure. Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

![Interaction Diagram About the X-Axis](image1)

![Interaction Diagram About the Y-Axis](image2)

**Figure 12** – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)
In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to $M_x$ and $M_y$ due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint’s spColumn program can also evaluate column sections in biaxial mode to produce the results shown in the following Figure for the column section in this example.

Figure 13 – Nominal & Design 3D Failure Surfaces (Biaxial) (spColumn)
Figure 14 – Tied Column Interaction Diagram and 3D failure Surface Viewer (spColumn)
Figure 15 – Tied Column 3D Failure Surface with a Horizontal Plane Cut at $P = 1,550.0$ kN (spColumn)
Figure 16 – Tied Column 3D Failure Surface with a Vertical Plane Cut at 45° (spColumn)