Interaction Diagram - Tied Reinforced Concrete Column (CSA A23.3-19)


PM at 0.0 [deg]

## Interaction Diagram - Tied Reinforced Concrete Column (CSA A23.3-19)

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x -axis using CSA A23.3-19 provisions. Determine six control points on the interaction diagram and compare the calculated values with exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.


Figure 1 - Reinforced Concrete Column Cross-Section

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## Code

Design of Concrete Structures (CSA A23.3-19)

## References

- Reinforced Concrete Mechanics and Design, $1^{\text {st }}$ Canadian Edition, 2000, James MacGregor and Fred Michael Bartlett, Prentice Hall Canada Inc., Example 11-1
- spColumn Engineering Software Program Manual v10.10, STRUCTUREPOINT, 2023


## Design Data

$f_{c}{ }^{\prime}=35 \mathrm{MPa}$
$f_{y}=400 \mathrm{MPa}$
Cover $=55 \mathrm{~mm}$ to bar center

Column $400 \mathrm{~mm} \times 400 \mathrm{~mm}$
Top reinforcement $=4$ No. 30
Bottom reinforcement $=4$ No. 30

## Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following six control points:

Point 1: Pure compression
Point 2: Bar stress near tension face of member equal to zero, $\left(f_{s}=0\right)$
Point 3: Bar stress near tension face of member equal to $0.5 f_{y}\left(f_{s}=-0.5 f_{y}\right)$
Point 4: Bar stress near tension face of member equal to $f_{y}\left(f_{s}=-f_{y}\right)$
Point 5: Pure bending
Point 6: Pure tension


Factored moment resistance, Mr (kN.m)
Figure 2 - Control Points

## 1. Pure Compression

1.1. Nominal axial resistance at zero eccentricity
$P_{o}=\alpha_{1} f_{c}^{\prime}\left(A_{g}-A_{s t}\right)+f_{y} A_{s t}$
$P_{o}=0.798 \times 35 \times(400 \times 400-8 \times 700)+400 \times 8 \times 700=6,549.7 \mathrm{kN}$
Where $\alpha_{1}=0.85-0.0015 f_{c}^{\prime} \geq 0.67$
CSA A23.3-19 (Equation 10.1)
$\alpha_{1}=0.85-0.0015 \times 35=0.798 \geq 0.67$
1.2. Factored axial load resistance at zero eccentricity

Since this column is a tied column with steel strain in compression:
$P_{r o}=\alpha_{1} \phi_{c} f_{c}^{\prime}\left(A_{g}-A_{s t}\right)+\phi_{s} f_{y} A_{s t}$
CSA A23.3-19 (Equation 10.11)
$P_{r o}=0.798 \times 0.65 \times 35 \times(400 \times 400-8 \times 700)+0.85 \times 400 \times 8 \times 700=4,705.3 \mathrm{kN}$
Where:
$\phi_{c}=0.65$
CSA A23.3-19 (8.4.2)
$\phi_{s}=0.85$
CSA A23.3-19 (8.4.3(a))
1.3. Maximum factored axial load resistance
$P_{r, \text { max }}=(0.2+0.002 h) P_{r o} \leq 0.80 P_{r o}$
CSA A23.3-19 (Equation 10.9)
$P_{r, \text { max }}=(0.2+0.002 \times 400) \times 4,705.3=4,705.3 \mathrm{kN} \leq 0.80 P_{r o}=0.80 \times 4,705.3=3,764.2 \mathrm{kN}$
$P_{r, \text { max }}=3,764.2 \mathrm{kN}$

## 2. Bar Stress Near Tension Face of Member Equal to Zero, $\left(\varepsilon_{s}=f_{s}=0\right)$



Figure 3 - Strains, Forces, and Moment Arms $\left(\varepsilon_{t}=f_{s}=0\right)$

Strain $\varepsilon_{\mathrm{s}}$ is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices.

CSA A23.3-19 (12.15 and 16)
2.1. $\mathrm{c}, a$, and strains in the reinforcement
$c=d_{1}=345 \mathrm{~mm}$

Where $c$ is depth of the neutral axis measured from the compression edge of the column section.
CSA A23.3-19 (3.2)
$a=\beta_{1} \times c=0.883 \times 345=304 \mathrm{~mm}$
CSA A23.3-19 (10.1.7)

Where:
$a=$ Depth of equivalent rectangular stress block
CSA A23.3-19 (3.2)
$\beta_{1}=0.97-0.0025 \times f_{c}^{\prime}=0.97-0.0025 \times 35=0.883>0.67$
CSA A23.3-19 (Equation 10.2)
$\varepsilon_{s}=0$
$\phi_{c}=0.65$
CSA A23.3-19 (8.4.2)

CSA A23.3-19 (8.4.3(a))

CSA A23.3-19 (10.1.3)
$\varepsilon_{s}^{\prime}=\left(c-d_{2}\right) \times \frac{\varepsilon_{c u}}{c}=(345-55) \times \frac{0.0035}{345}=0.00294($ Compression $)>\varepsilon_{y}=\frac{F_{y}}{E_{s}}=\frac{400}{200,000}=0.00200$
2.2. Forces in the concrete and steel

$$
C_{r c}=\alpha_{1} \times \phi_{c} \times f_{c}^{\prime} \times a \times b=0.798 \times 0.65 \times 35 \times 304 \times 400=2,209.6 \mathrm{kN}
$$

$$
f_{s}=0 \mathrm{kN} \rightarrow \mathrm{~T}_{r s}=\phi_{s} \times f_{s} \times A_{s 1}=0 \mathrm{kN}
$$

Since $\varepsilon_{s}^{\prime}>\varepsilon_{y} \rightarrow$ compression reinforcement has yielded

$$
\therefore f_{s}^{\prime}=f_{y}=400 \mathrm{MPa}
$$

The area of the reinforcement in this layer has been included in the area ( $a b$ ) used to compute $C_{r c}$. As a result, it is necessary to subtract $\alpha_{l} \phi_{c} f_{c}$ ' from $\phi_{s} f_{s}$ ' before computing $C_{r s}$ :

$$
\mathrm{C}_{r s}=\left(\phi_{s} \times f_{s}^{\prime}-\alpha_{1} \times \phi_{c} \times f_{c}^{\prime}\right) \times A_{s 2}=(0.85 \times 400-0.798 \times 0.65 \times 35) \times 2,800=901.2 \mathrm{kN}
$$

2.3. $\underline{P}_{r}$ and $M_{r}$

$$
\begin{aligned}
& P_{r}=C_{r c}+C_{r s}-T_{r s}=2,209.6+901.2-0=3,110.8 \mathrm{kN} \\
& M_{r}=C_{r c} \times\left(\frac{h}{2}-\frac{a}{2}\right)+C_{r s} \times\left(\frac{h}{2}-d_{2}\right)+T_{r s} \times\left(d_{1}-\frac{h}{2}\right) \\
& M_{r}=2,209.6 \times\left(\frac{400}{2}-\frac{304}{2}\right)+901.2 \times\left(\frac{400}{2}-55\right)+0 \times\left(345-\frac{400}{2}\right)=236.22 \mathrm{kN}-\mathrm{m}
\end{aligned}
$$

3. Bar Stress Near Tension Face of Member Equal to $0.5 f_{y},\left(f_{s}=-0.5 f_{y}\right)$


Figure 4 - Strains, Forces, and Moment Arms $\left(f_{s}=-0.5 f_{y}\right)$
3.1. $\mathrm{c}, a$, and strains in the reinforcement
$\varepsilon_{y}=\frac{F_{y}}{E_{s}}=\frac{400}{200,000}=0.00200$
$\varepsilon_{s}=\frac{\varepsilon_{y}}{2}=\frac{0.00200}{2}=0.00100<\varepsilon_{y} \rightarrow$ tension reinforcement has not yielded
$\phi_{c}=0.65$
CSA A23.3-19 (8.4.2)
$\phi_{s}=0.85$
CSA A23.3-19 (8.4.3(a))
$\varepsilon_{c u}=0.0035$
CSA A23.3-19 (10.1.3)
$c=\frac{d_{1}}{\varepsilon_{s}+\varepsilon_{c u}} \times \varepsilon_{c u}=\frac{345}{0.00100+0.0035} \times 0.0035=268 \mathrm{~mm}$
Where $c$ is depth of the neutral axis measured from the compression edge of the column section.
CSA A23.3-19 (3.2)
$a=\beta_{1} \times c=0.883 \times 268=237 \mathrm{~mm}$
CSA A23.3-19 (10.1.7)

Where:
$a=$ Depth of equivalent rectangular stress block
CSA A23.3-19 (3.2)
$\beta_{1}=0.97-0.0025 \times f_{c}^{\prime}=0.97-0.0025 \times 35=0.883>0.67$
CSA A23.3-19 (Equation 10.2)
$\varepsilon_{s}^{\prime}=\left(c-d_{2}\right) \times \frac{\varepsilon_{c u}}{c}=(268-55) \times \frac{0.0035}{268}=0.00278($ Compression $)>\varepsilon_{y}$
3.2. Forces in the concrete and steel

$$
C_{r c}=\alpha_{1} \times \phi_{c} \times f_{c}^{\prime} \times a \times b=0.798 \times 0.65 \times 35 \times 237 \times 400=1,718.5 \mathrm{kN}
$$

$$
f_{s}=\varepsilon_{s} \times E_{s}=0.00100 \times 200,000=200 \mathrm{MPa}
$$

$$
\mathrm{T}_{r s}=\phi_{s} \times f_{s} \times A_{s 1}=0.85 \times 200 \times 2,800=476.0 \mathrm{kN}
$$

Since $\varepsilon_{s}^{\prime}>\varepsilon_{y} \rightarrow$ compression reinforcement has yielded
$\therefore f_{s}^{\prime}=f_{y}=400 \mathrm{MPa}$

The area of the reinforcement in this layer has been included in the area ( $a b$ ) used to compute $C_{r c}$. As a result, it is necessary to subtract $\alpha_{l} \phi_{c} f_{c}$ ' from $\phi_{s} f_{s}$ ' before computing $C_{r s}$ :

$$
\mathrm{C}_{r s}=\left(\phi_{s} \times f_{s}^{\prime}-\alpha_{1} \times \phi_{c} \times f_{c}^{\prime}\right) \times A_{s 2}=(0.85 \times 400-0.798 \times 0.65 \times 35) \times 2,800=901.2 \mathrm{kN}
$$

3.3. $\underline{P}_{r}$ and $M_{r}$

$$
\begin{aligned}
& P_{r}=C_{r c}+C_{r s}-T_{r s}=1,718.5+901.2-476.0=2,143.7 \mathrm{kN} \\
& M_{r}=C_{r c} \times\left(\frac{h}{2}-\frac{a}{2}\right)+C_{r s} \times\left(\frac{h}{2}-d_{2}\right)+T_{r s} \times\left(d_{1}-\frac{h}{2}\right) \\
& M_{r}=1,718.5 \times\left(\frac{400}{2}-\frac{237}{2}\right)+901.2 \times\left(\frac{400}{2}-55\right)+476.0 \times\left(345-\frac{400}{2}\right)=339.92 \mathrm{kN}-\mathrm{m}
\end{aligned}
$$

## 4. Bar Stress Near Tension Face of Member Equal to $f_{y},\left(f_{s}=-f_{y}\right)$



Figure 5 - Strains, Forces, and Moment Arms $\left(f_{s}=-f_{v}\right)$
This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement.
4.1. $\mathrm{c}, a$, and strains in the reinforcement
$\varepsilon_{y}=\frac{F_{y}}{E_{s}}=\frac{400}{200,000}=0.00200$
$\varepsilon_{s}=\varepsilon_{y}=0.00200 \rightarrow$ tension reinforcement has yielded
$\phi_{c}=0.65$
CSA A23.3-19 (8.4.2)
$\phi_{s}=0.85$
CSA A23.3-19 (8.4.3(a))
$\varepsilon_{c u}=0.0035$
CSA A23.3-19 (10.1.3)
$c=\frac{d_{1}}{\varepsilon_{s}+\varepsilon_{c u}} \times \varepsilon_{c u}=\frac{345}{0.00200+0.0035} \times 0.0035=220 \mathrm{~mm}$
Where $c$ is depth of the neutral axis measured from the compression edge of the column section.
CSA A23.3-19 (3.2)
$a=\beta_{1} \times c=0.883 \times 220=194 \mathrm{~mm}$
CSA A23.3-19 (10.1.7)

Where:
$a=$ Depth of equivalent rectangular stress block
CSA A23.3-19 (3.2)
$\beta_{1}=0.97-0.0025 \times f_{c}^{\prime}=0.97-0.0025 \times 35=0.883>0.67$
CSA A23.3-19 (Equation 10.2)
$\varepsilon_{s}^{\prime}=\left(c-d_{2}\right) \times \frac{\varepsilon_{c u}}{c}=(220-55) \times \frac{0.0035}{220}=0.00262($ Compression $)>\varepsilon_{y}$
4.2. Forces in the concrete and steel

$$
C_{r c}=\alpha_{1} \times \phi_{c} \times f_{c}^{\prime} \times a \times b=0.798 \times 0.65 \times 35 \times 194 \times 400=1,406.1 \mathrm{kN}
$$

$$
f_{s}=f_{y}=400 \mathrm{MPa}
$$

$$
\mathrm{T}_{r s}=\phi_{s} \times f_{s} \times A_{s 1}=0.85 \times 400 \times 2,800=952.0 \mathrm{kN}
$$

Since $\varepsilon_{s}^{\prime}>\varepsilon_{y} \rightarrow$ compression reinforcement has yielded

$$
\therefore f_{s}^{\prime}=f_{y}=400 \mathrm{MPa}
$$

The area of the reinforcement in this layer has been included in the area ( $a b$ ) used to compute $C_{r c}$. As a result, it is necessary to subtract $\alpha_{l} \phi_{c} f_{c}$ ' from $\phi_{s} f_{s}$ ' before computing $C_{r s}$ :

$$
\mathrm{C}_{r s}=\left(\phi_{s} \times f_{s}^{\prime}-\alpha_{1} \times \phi_{c} \times f_{c}^{\prime}\right) \times A_{s 2}=(0.85 \times 400-0.798 \times 0.65 \times 35) \times 2,800=901.2 \mathrm{kN}
$$

4.3. $\underline{P}_{\underline{r}}$ and $M_{\underline{r}}$

$$
\begin{aligned}
& P_{r}=C_{r c}+C_{r s}-T_{r s}=1,406.1+901.2-952.0=1,355.3 \mathrm{kN} \\
& M_{r}=C_{r c} \times\left(\frac{h}{2}-\frac{a}{2}\right)+C_{r s} \times\left(\frac{h}{2}-d_{2}\right)+T_{r s} \times\left(d_{1}-\frac{h}{2}\right) \\
& M_{r}=1,406.1 \times\left(\frac{400}{2}-\frac{194}{2}\right)+901.2 \times\left(\frac{400}{2}-55\right)+952.0 \times\left(345-\frac{400}{2}\right)=413.72 \mathrm{kN}-\mathrm{m}
\end{aligned}
$$

## 5. Pure Bending



Figure 6 - Strains, Forces, and Moment Arms (Pure Moment)
This corresponds to the case where the factored axial load resistance, $P_{r}$, is equal to zero. Iterative procedure is used to determine the factored moment resistance as follows:
5.1. $\mathrm{c}, a$, and strains in the reinforcement

Try $c=78.56 \mathrm{~mm}$

Where $c$ is depth of the neutral axis measured from the compression edge of the column section.
CSA A23.3-19 (3.2)
$a=\beta_{1} \times c=0.883 \times 78.56=69 \mathrm{~mm}$
CSA A23.3-19 (10.1.7)

Where:
$\beta_{1}=0.97-0.0025 \times f_{c}^{\prime}=0.97-0.0025 \times 35=0.883>0.67$
CSA A23.3-19 (Equation 10.2)
$\varepsilon_{c u}=0.0035$
CSA A23.3-19 (10.1.3)
$\varepsilon_{y}=\frac{F_{y}}{E_{s}}=\frac{400}{200,000}=0.00200$
$\varepsilon_{s}=\left(d_{1}-c\right) \times \frac{\varepsilon_{c u}}{c}=(345-78.56) \times \frac{0.0035}{78.56}=0.01187$ (Tension) $>\varepsilon_{y} \rightarrow$ tension reinforcement has yielded
$\phi_{c}=0.65$
CSA A23.3-19 (8.4.2)
$\phi_{s}=0.85$
CSA A23.3-19 (8.4.3(a))
$\varepsilon_{s}^{\prime}=\left(c-d_{2}\right) \times \frac{\varepsilon_{c u}}{c}=(78.56-55) \times \frac{0.0035}{78.56}=0.00105($ Compression $)<\varepsilon_{y}$
5.2. Forces in the concrete and steel

$$
C_{r c}=\alpha_{1} \times \phi_{c} \times f_{c}^{\prime} \times a \times b=0.798 \times 0.65 \times 35 \times 69 \times 400=503.1 \mathrm{kN}
$$

$f_{s}=f_{y}=400 \mathrm{MPa}$
$\mathrm{T}_{r s}=\phi_{s} \times f_{s} \times A_{s 1}=0.85 \times 400 \times 2,800=952.0 \mathrm{kN}$
Since $\varepsilon_{s}^{\prime}<\varepsilon_{y} \rightarrow$ compression reinforcement has not yielded

$$
\therefore f_{s}^{\prime}=\varepsilon_{s}^{\prime} \times E_{s}=0.00105 \times 200,000=210 \mathrm{MPa}
$$

The area of the reinforcement in this layer has been included in the area ( $a b$ ) used to compute $C_{r c}$. As a result, it is necessary to subtract $\alpha_{l} \phi_{c} f_{c}$ ' from $\phi_{s} f_{s}$ ' before computing $C_{r s}$ :

$$
\mathrm{C}_{r s}=\left(\phi_{s} \times f_{s}^{\prime}-\alpha_{1} \times \phi_{c} \times f_{c}^{\prime}\right) \times A_{s 2}=(0.85 \times 210-0.798 \times 0.65 \times 35) \times 2,800=448.8 \mathrm{kN}
$$

5.3. $\underline{P}_{r}$ and $M_{r}$

$$
P_{r}=C_{r c}+C_{r s}-T_{r s}=503.1+448.8-952.0 \approx 0 \mathrm{kN}
$$

The assumption that $\mathrm{c}=78.56 \mathrm{~mm}$ is correct

$$
\begin{aligned}
& M_{r}=C_{r c} \times\left(\frac{h}{2}-\frac{a}{2}\right)+C_{r s} \times\left(\frac{h}{2}-d_{2}\right)+T_{r s} \times\left(d_{1}-\frac{h}{2}\right) \\
& M_{r}=503.1 \times\left(\frac{400}{2}-\frac{69}{2}\right)+448.8 \times\left(\frac{400}{2}-55\right)+952.0 \times\left(345-\frac{400}{2}\right)=286.31 \mathrm{kN}-\mathrm{m}
\end{aligned}
$$

## 6. Pure Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.
6.1. Strength under pure axial tension $\left(P_{r t}\right)$
$P_{r t}=\phi_{s} \times f_{y} \times\left(A_{s 1}+A_{s 2}\right)=0.85 \times 400 \times(2,800+2,800)=1,904.0 \mathrm{kN}$

### 6.2. Corresponding Moment $\left(M_{r t}\right)$

Since the section is symmetrical

$$
M_{r t}=0.00 \mathrm{kN}-\mathrm{m}
$$

## 7. Column Interaction Diagram - spColumn Software

spColumn is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode was used with no loads (the program will only report control points) and no slenderness considerations using CSA A23.3-19.


Figure 7 - spColumn Interface


Figure 8 - spColumn Model Editor

## Structure Point



Figure 9 - Defining Loads / Modes (spColumn)


Figure 10 - Column P-M Interaction Diagram about the X-Axis (spColumn)

## sficolumn.



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E:IStructurePointlspColumnlTied Square Column - CSA.colx 4:53 PM

## 1. General Information

| File Name | E:IStructurePoin...ITied Square <br> Column-CSA.colx |
| :--- | :--- |
| Project | Column Interaction Diagram CSA |
| Column | Interior |
| Engineer | SP |
| Code | CSA A23.3-19 |
| Bar Set | CSA G30.18 |
| Units | Metric |
| Run Option | Investigation |
| Run Axis | X-axis |
| Slenderness | Not Considered |
| Column Type | Structural |
| Capacity Method | Moment capacity |

## 2. Material Properties

### 2.1. Concrete

| Type | Standard |
| :--- | :--- |
| $\mathbf{f}_{c}$ | 35 MPa |
| $\mathrm{E}_{\mathrm{c}}$ | 28164.7 MPa |
| $\mathrm{f}_{\mathrm{c}}$ | 27.9125 MPa |
| $\varepsilon_{u}$ | $0.0035 \mathrm{~mm} / \mathrm{mm}$ |
| $\beta_{1}$ | 0.8825 |

2.2. Steel

| Type | Standard |
| :--- | :---: |
| $\mathrm{f}_{\mathrm{y}}$ | 400 MPa |
| $\mathrm{E}_{\mathrm{s}}$ | 200000 MPa |
| $\varepsilon_{\mathrm{ty}}$ | $0.002 \mathrm{~mm} / \mathrm{mm}$ |

## 3. Section

3.1. Shape and Properties

| Type | Rectangular |
| :--- | ---: |
| Width | 400 mm |
| Depth | 400 mm |
| $\mathrm{~A}_{\mathrm{g}}$ | $160000 \mathrm{~mm}^{2}$ |
| $\mathrm{I}_{\mathrm{x}}$ | $2.13333 \mathrm{e}+009 \mathrm{~mm}^{4}$ |
| $\mathrm{I}_{\mathrm{y}}$ | $2.13333 \mathrm{e}+009 \mathrm{~mm}^{4}$ |
| $\mathrm{r}_{\mathrm{x}}$ | 115.47 mm |
| $\mathrm{r}_{\mathrm{y}}$ | 115.47 mm |
| $\mathrm{X}_{0}$ | 0 mm |
| $\mathrm{Y}_{0}$ | 0 mm |

### 3.2. Section Figure



Figure 1: Column section

## 4. Reinforcement

4.1. Bar Set: CSA G30.18

| Bar | Diameter <br> mm | Area <br> mm | Bar | Diameter <br> mm | Area <br> $\mathrm{mm}^{2}$ | Bar | Diameter <br> mm | Area <br> $\mathrm{mm}^{2}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\# 10$ | 11.30 | 100.00 | $\# 15$ | 16.00 | 200.00 | $\# 20$ | 19.50 | 300.00 |
| $\# 25$ | 25.20 | 500.00 | $\# 30$ | 29.90 | 700.00 | $\# 35$ | 35.70 | 1000.00 |
| $\# 45$ | 43.70 | 1500.00 | $\# 55$ | 56.40 | 2500.00 |  |  |  |

### 4.2. Confinement and Factors

| Confinement type | Tied |
| :--- | :---: |
| For \#55 bars or less | \#10 ties |
| For larger bars | \#15 ties |
|  |  |
| Material Resistance Factors |  |
| Axial compression, (a) | 0.8 |
| Steel $\left(\phi_{\mathrm{s}}\right)$ | 0.85 |
| Concrete $\left(\phi_{\mathrm{c}}\right)$ | 0.65 |
| Minimum dimension, h |  |

### 4.3. Arrangement

| Pattern | Sides different |
| :--- | ---: |
| Bar layout | Rectangular |
| Cover to | Longitudal bars |
| Clear cover | --- |


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| Bars | --- |
| :--- | :---: |
|  |  |
| Total steel area, $\mathrm{A}_{\mathrm{s}}$ | $5600 \mathrm{~mm}^{2}$ |
| Rho | $3.50 \%$ |
| Minimum clear spacing | 67 mm |

### 4.4. Bars Provided

|  |  | Bars | Clear cover |
| :--- | :--- | :--- | ---: |
|  |  |  | mm |
| Top | 4 | $\# 30$ | 40.05 |
| Bottom | 4 | $\# 30$ | 40.05 |
| Left | 0 | $\# 10$ | 40.05 |
| Right | 0 | $\# 10$ | 40.05 |

## 5. Control Points

| About Point | P | X-Moment | Y-Moment | NA Depth | $\mathrm{d}_{\text {t }}$ Depth | $\varepsilon_{\mathrm{t}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | kN | kNm | kNm | mm | mm |  |
| X @ Max compression | 4705.3 | 0.00 | 0.00 | 805 | 345 | -0.00200 |
| X @ Allowable comp. | 3764.2 | 146.13 | 0.00 | 412 | 345 | -0.00057 |
| $X @ f_{s}=0.0$ | 3110.8 | 236.22 | 0.00 | 345 | 345 | 0.00000 |
| X @ $\mathrm{f}_{\mathrm{s}}=0.5 \mathrm{f}_{\mathrm{y}}$ | 2143.7 | 339.92 | 0.00 | 268 | 345 | 0.00100 |
| X @ Balanced point | 1355.3 | 413.72 | 0.00 | 220 | 345 | 0.00200 |
| X @ Pure bending | 0.0 | 286.31 | 0.00 | 79 | 345 | 0.01187 |
| X @ Max tension | -1904.0 | 0.00 | 0.00 | 0 | 345 | 9.99999 |
| -X @ Max compression | 4705.3 | 0.00 | 0.00 | 805 | 345 | -0.00200 |
| -X @ Allowable comp. | 3764.2 | -146.13 | 0.00 | 412 | 345 | -0.00057 |
| -X @ $\mathrm{f}_{\mathrm{s}}=0.0$ | 3110.8 | -236.22 | 0.00 | 345 | 345 | 0.00000 |
| -X@ $\mathrm{f}_{\mathrm{s}}=0.5 \mathrm{f}_{\mathrm{y}}$ | 2143.7 | -339.92 | 0.00 | 268 | 345 | 0.00100 |
| -X @ Balanced point | 1355.3 | -413.72 | 0.00 | 220 | 345 | 0.00200 |
| -X @ Pure bending | 0.0 | -286.31 | 0.00 | 79 | 345 | 0.01187 |
| -X@ Max tension | -1904.0 | 0.00 | 0.00 | 0 | 345 | 9.99999 |


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| :--- | ---: |
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| E:IStructurePointlspColumnlTied Square Column - CSA.colx | $4: 53$ PM |

E:IStructurePointlspColumnlTied Square Column - CSA.colx
6. Diagrams 6.1. $P M$ at $\theta=0$ [deg]

8. Summary and Comparison of Design Results

| Table 1 - Comparison of Results |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Support |  | $\boldsymbol{P}_{\mathbf{r}}, \mathbf{k N}$ |  | $\boldsymbol{M}_{\boldsymbol{r}}, \mathbf{k N} \mathbf{- m}$ |  |
|  | Hand | spColumn | Hand | spColumn |  |
| Max compression | 4705.3 | 4705.3 | 0.00 | 0.00 |  |
| Allowable compression | 3764.2 | 3764.2 | --- | --- |  |
| $f_{s}=0.0$ | 3110.8 | 3110.8 | 236.22 | 236.22 |  |
| $f_{s}=0.5 f_{y}$ | 2143.7 | 2143.7 | 339.92 | 339.92 |  |
| Balanced point | 1355.3 | 1355.3 | 413.72 | 413.72 |  |
| Pure bending | 0.0 | 0.0 | 286.31 | 286.31 |  |
| Max tension | 1904.0 | 1904.0 | 0.00 | 0.00 |  |

In all of the hand calculations illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.

## 9. Conclusions \& Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where $f_{s}=$ $f_{y}$ ).


Strain Diagram
Forces and Moment Arms

$\underline{\text { Figure } 11 \text { - Strains, Forces, and Moment Arms }\left(f_{\underline{s}}=-f_{2} \text { Moments About x- and y-axis) }\right.}$

When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure. Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.


Figure 12 - Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to $\mathrm{M}_{\mathrm{x}}$ and $\mathrm{M}_{\mathrm{y}}$ due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint's spColumn program can also evaluate column sections in biaxial mode to produce the results shown in the following Figure for the column section in this example.


Figure 13 - Nominal \& Design 3D Failure Surfaces (Biaxial) (spColumn)


Figure 14 - Tied Column Interaction Diagram and 3D failure Surface Viewer (spColumn)

$\underline{\text { Figure } 15 \text { - Tied Column 3D Failure Surface with a Horizontal Plane Cut at } \mathrm{P}=1,550.0 \mathrm{kN} \text { (spColumn) }}$


Figure 16 - Tied Column 3D Failure Surface with a Vertical Plane Cut at $45^{\circ}$ (spColumn)


[^0]:    Structure Point
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