Interaction Diagram - Tied Reinforced Concrete Column Design Strength (ACI 318-19)
Interaction Diagram - Tied Reinforced Concrete Column Design Strength (ACI 318-19)

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using ACI 318-19 provisions. Determine seven control points on the interaction diagram and compare the calculated values with the Reference and exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.

![Reinforced Concrete Column Cross-Section](image)

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Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

- spColumn Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2021
- “Interaction Diagram - Circular Spiral Reinforced Concrete Column (ACI 318-19)” Design Example, STRUCTUREPOINT, 2022
- “Interaction Diagram - Tied Reinforced Concrete Column with High-Strength Reinforcing Bars (ACI 318-19)” Design Example, STRUCTUREPOINT, 2022
- “Interaction Diagram - Barbell Concrete Shear Wall Unsymmetrical Boundary Elements (ACI 318-19)” Design Example, STRUCTUREPOINT, 2022
- “Interaction Diagram – Building Elevator Reinforced Concrete Core Wall Design Strength (ACI 318-19)” Design Example, STRUCTUREPOINT, 2022

Design Data

\( f_c' = 5000 \text{ psi} \)
\( f_y = 60,000 \text{ psi} \)
Cover = 2.5 in. to the center of the reinforcement
Column 16 in. x 16 in.
Top reinforcement = 4 #9
Bottom reinforcement = 4 #9
Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following seven control points:

Point 1: Maximum compression
Point 2: Bar stress near tension face equal to zero, \( f_s = 0 \)
Point 3: Bar stress near tension face equal to 0.5 \( f_y \) \( (f_s = 0.5 f_y) \)
Point 4: Bar stress near tension face equal to \( f_y \) \( (f_s = f_y) \)
Point 5: Bar strain near tension face equal to \( \varepsilon_y + 0.003 \)
Point 6: Pure bending
Point 7: Maximum tension

Several terms are used to facilitate the following calculations:

\[
\begin{align*}
  P_o & = \text{nominal axial compressive strength, kip} \\
  \phi P_o & = \text{factored axial compressive strength, kip} \\
  \phi P_{o,\text{max}} & = \text{maximum (allowable) factored axial compressive strength, kip} \\
  c & = \text{distance from the fiber of maximum compressive strain to the neutral axis, in.} \\
  a & = \text{depth of equivalent rectangular stress block, in.} \\
  C_c & = \text{compression force in equivalent rectangular stress block, kip} \\
  \varepsilon_s & = \text{strain value in reinforcement, in./in.} \\
  C_s & = \text{compression force in reinforcement, kip} \\
  T_s & = \text{tension force in reinforcement, kip}
\end{align*}
\]
Figure 2 – Tied Column Section Interaction Diagram Control Points

1. Maximum Nominal Axial Strength in Compression ($P_a$)
2. Nominal Control Point at Zero Stress in Tension Reinforcement
3. Nominal Control Point at Tension Reinforcement Stress = 0.5$f_y$
4. Nominal Control Point at Tension Reinforcement Stress = $f_y$
5. Nominal Control Point at Tension Reinforcement Stress = $\epsilon_y + 0.003 \text{ in./in.}$
6. Nominal Control Point at Pure Bending
7. Maximum Nominal Axial Strength in Tension

1. Maximum Factored Axial Strength in Compression
2. Allowable Factored Axial Strength in Compression ($\phi P_{u,m}$)
3. Factored Control Point at Zero Stress in Tension Reinforcement
4. Factored Control Point at Tension Reinforcement Stress = 0.5$f_y$
5. Factored Control Point at Tension Reinforcement Stress = $f_y$
6. Factored Control Point at Tension Reinforcement Stress = $\epsilon_y + 0.003 \text{ in./in.}$
7. Factored Control Point at Pure Bending
8. Maximum Factored Axial Strength in Tension
1. Maximum Compression

1.1. Nominal axial compressive strength at zero eccentricity

\[ P_o = 0.85 f'_c (A_s - A_e) + f_y A_e \]

\[ P_o = 0.85 \times 5000 \times (16 \times 16 - 8 \times 1.00) + 60000 \times 8 \times 1.00 = 1,534.0 \text{ kips} \]

1.2. Factored axial compressive strength at zero eccentricity

Since this column is a tied column with steel strain in compression:

\[ \phi = 0.65 \]

\[ \phi P_o = 0.65 \times 1,534.0 = 997.1 \text{ kips} \]

Since the section is regular (symmetrical) about the x-axis, the moment capacity associated with the maximum axial compressive strength is equal to zero.

\[ M_o = \phi M_o = 0.00 \text{ kip.ft} \]

1.3. Maximum (allowable) factored axial compressive strength

\[ \phi P_{n,max} = 0.80 \times \phi P_o = 0.80 \times 997 = 797.7 \text{ kips} \]
2. Bar Stress Near Tension Face Equal to Zero, \( \varepsilon_s = f_s = 0 \)

![Strain Diagram](image)

**Figure 3 – Strains, Forces, and Moment Arms \( \varepsilon_t = f_s = 0 \)**

Strain \( \varepsilon_s \) is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices.

*ACI 318-19 (10.7.5.2.1 and 2)*

2.1. \( c, a, \) and strains in the reinforcement

\[
c = d_i = 13.5 \text{ in.}
\]

Where \( c \) is the distance from the fiber of maximum compressive strain to the neutral axis.

*ACI 318-19 (22.2.2.4.2)*

\[
a = \beta_i \times c = 0.80 \times 13.5 = 10.80 \text{ in.}
\]

Where:

\( a = \) Depth of equivalent rectangular stress block

\[
\beta_i = 0.85 - \frac{0.05 \times (f_s' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80
\]

*ACI 318-19 (Table 22.2.2.4.3)*

\[
\varepsilon_s = 0
\]

\[
\therefore \phi = 0.65
\]

*ACI 318-19 (Table 21.2.2)*

\[
\varepsilon_{cu} = 0.003
\]

*ACI 318-19 (22.2.2.1)*

\[
\varepsilon_c' = (c - d_i) \times \frac{\varepsilon_{cu}}{c} = (13.50 - 2.5) \times \frac{0.003}{13.50} = 0.00244 \text{ (Compression)} > \varepsilon_y = F_y \frac{c}{E_s} = \frac{60}{29,000} = 0.00207
\]
2.2. Forces in the concrete and steel

\[ C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 10.80 \times 16 = 734.4 \text{ kip} \]  

\[ f_s = 0 \text{ psi} \rightarrow T_s = f_s \times A_{s1} = 0 \text{ kip} \]

Since \( \varepsilon'_{c} > \varepsilon_y \) compression reinforcement has yielded

\[ \therefore f'_c = f_y = 60,000 \text{ psi} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, it is necessary to subtract 0.85\(f'_c\) from \(f'_s\) before computing \(C_s\):

\[ C_s = (f'_s - 0.85 f'_c) \times A_{s2} = (60,000 - 0.85 \times 5,000) \times 4 = 223 \text{ kip} \]

2.3. \(\phi P_n\) and \(\phi M_a\)

\[ P_n = C_c + C_s - T_s = 734.4 + 223 - 0 = 957.4 \text{ kip} \]

\[ \phi P_n = 0.65 \times 957 = 622.3 \text{ kip} \]

\[ M_a = C_c \times \left( \frac{h - a}{2} \right) + C_s \times \left( \frac{h - d_2}{2} \right) + T_s \times \left( \frac{d_1 - h}{2} \right) \]

\[ M_a = 734.4 \times \left( \frac{16}{2} - \frac{10.80}{2} \right) + 223 \times \left( \frac{16}{2} - 2.5 \right) + 0 \times \left( 13.50 - \frac{16}{2} \right) = 261.33 \text{ kip.ft} \]

\[ \phi M_a = 0.65 \times 261 = 169.86 \text{ kip.ft} \]
3. Bar Stress Near Tension Face Equal to 0.5 $f_r$, ($f_i = 0.5 f_r$)

![Figure 4 – Strains, Forces, and Moment Arms ($f_i = 0.5 f_r$)](image)

3.1. $\varepsilon_c, \varepsilon_a$, and strains in the reinforcement

$$\varepsilon_c = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_a = \frac{\varepsilon_c}{2} = \frac{0.00207}{2} = 0.00103 < \varepsilon_c \rightarrow \text{tension reinforcement has not yielded}$$

$$\therefore \phi = 0.65$$

$$\varepsilon_{cu} = 0.003$$

$$c = \frac{d_i}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00103 + 0.003} \times 0.003 = 10.04 \text{ in.}$$

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (Table 22.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 10.04 = 8.03 \text{ in.}$$

ACI 318-19 (22.2.4.1)

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$

ACI 318-19 (Table 22.2.4.3)

$$\varepsilon'_c = (c - d_i) \times \frac{0.003}{c} = (10.04 - 2.5) \times \frac{0.003}{10.04} = 0.00225 \text{ (Compression)} > \varepsilon_c$$

3.2. Forces in the concrete and steel

$$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 8.03 \times 16 = 546.1 \text{ kip}$$

ACI 318-19 (22.2.4.1)

$$f_s = \varepsilon_i \times E_s = 0.00103 \times 29,000,000 = 30,000 \text{ psi}$$
\[ T_s = f_s \times A_s = 30,000 \times 4 = 120 \text{ kip} \]

Since \( \varepsilon_s' > \varepsilon_y \rightarrow \) compression reinforcement has yielded

\[ \therefore f'_s = f_y = 60,000 \text{ psi} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, it is necessary to subtract \(0.85f'_s\) from \(f'_s\) before computing \(C_s\):

\[ C_s = \left(f'_s - 0.85f'_s\right) \times A_s = (60,000 - 0.85 \times 5,000) \times 4 = 223 \text{ kip} \]

3.3. \( \phi P_n \) and \( \phi M_n \)

\[ P_n = C_s + C_s - T_s = 546.1 + 223 - 120 = 649.1 \text{ kip} \]

\[ \phi P_n = 0.65 \times 649 = 421.9 \text{ kip} \]

\[ M_n = C_s \times \left(\frac{h - a}{2}\right) + C_s \times \left(\frac{h - d_s}{2}\right) + T_s \times \left(d_1 - \frac{h}{2}\right) \]

\[ M_n = 546.1 \times \left(\frac{16}{2} - \frac{8.03}{2}\right) + 223 \times \left(\frac{16}{2} - 2.5\right) + 120 \times \left(13.50 - \frac{16}{2}\right) = 338.54 \text{ kip.ft} \]

\[ \phi M_n = 0.65 \times 339 = 220.05 \text{ kip.ft} \]
4. Bar Stress Near Tension Face Equal to $f_s$, ($f_s = f_y$)

![Figure 5 – Strains, Forces, and Moment Arms ($f_s = f_y$)](image)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for $\phi$ for columns in which $\phi$ increases from 0.65 (or 0.75 for spiral columns) up to 0.90.

4.1. $c$, $a$, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \varepsilon_y = 0.00207 \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.65 \quad \text{ACI 318-19 (Table 21.2.2)}$$

$$\varepsilon_{cu} = 0.003 \quad \text{ACI 318-19 (22.2.2.1)}$$

$$c = \frac{d_i}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00207 + 0.003} \times 0.003 = 7.99 \text{ in.}$$

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

$$a = \beta_1 \times c = 0.80 \times 7.99 = 6.39 \text{ in.} \quad \text{ACI 318-19 (22.2.2.4.2)}$$

Where:

$$\beta_1 = 0.85 - 0.05 \times \frac{(f'_y - 4000)}{1000} = 0.85 - 0.05 \times \frac{(5000 - 4000)}{1000} = 0.80 \quad \text{ACI 318-19 (Table 22.2.2.4.3)}$$

$$\varepsilon'_s = \left(\frac{c - d_i}{c}\right) \times \frac{0.003}{\varepsilon_y} = \frac{(7.99 - 2.5)}{7.99} \times \frac{0.003}{0.00207} = 0.00206 \text{ (Compression) < } \varepsilon_y$$
4.2. Forces in the concrete and steel

\[ C_c = 0.85 \times f' \times a \times b = 0.85 \times 5,000 \times 6.39 \times 16 = 434.6 \text{ kip} \]

\[ f_s = f_y = 60,000 \text{ psi} \]

\[ T_s = f_s \times A_s = 60,000 \times 4 = 240 \text{ kip} \]

Since \( \varepsilon' < \varepsilon_y \), compression reinforcement has not yielded

\[ \therefore f' = \varepsilon' \times E = 0.00206 \times 29,000,000 = 59,778 \text{ psi} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \( C_c \). As a result, it is necessary to subtract \( 0.85f' \) from \( f' \) before computing \( C_s \):

\[ C_s = (f' - 0.85f') \times A_{s2} = (59,778 - 0.85 \times 5,000) \times 4 = 222.1 \text{ kip} \]

4.3. \( \phi P_n \) and \( \phi M_n \)

\[ P_n = C_c + C_i - T_s = 434.6 + 222.1 - 240 = 416.8 \text{ kip} \]

\[ \phi P_n = 0.65 \times 417 = 270.9 \text{ kip} \]

\[ M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_i \times \left( \frac{h}{2} - d_2 \right) + T_i \times \left( d_i - \frac{h}{2} \right) \]

\[ M_n = 434.6 \times \left( \frac{16}{2} - \frac{6.39}{2} \right) + 222.1 \times \left( \frac{16}{2} - 2.5 \right) + 240 \times \left( 13.50 - \frac{16}{2} \right) = 385.81 \text{ kip.ft} \]

\[ \phi M_n = 0.65 \times 386 = 250.77 \text{ kip.ft} \]
5. Bar Strain Near Tension Face Equal to $\varepsilon_y + 0.003$, $(\varepsilon_s = 0.00507 \text{ in./in.})$

![Strain Diagram](image)

Figure 6 – Strains, Forces, and Moment Arms $(\varepsilon_s = 0.00507 \text{ in./in.})$

In ACI 318-19 provisions, this control point corresponds to the tension-controlled strain limit of $\varepsilon_y + 0.003$ (used to be 0.005 in ACI 318-14). It is the strain at the tensile limit of the transition zone for $\phi$, used to define a tension-controlled section. Additional resources concerning code provision changes in ACI 318-19 can be found in “ACI 318-19 Code Revisions Impact on StructurePoint Software” technical article.

5.1. $c$, $a$, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = \varepsilon_y + 0.003 = 0.00207 + 0.003 = 0.00507 > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$\therefore \phi = 0.9$

$\varepsilon_{cu} = 0.003$

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00507 + 0.003} \times 0.003 = 5.02 \text{ in.}$$

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

$\alpha = \beta_1 \times c = 0.80 \times 5.02 = 4.02 \text{ in.}$

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_s - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$

$\varepsilon'_s = (c - d_1) \times \frac{0.003}{c} = (5.02 - 2.5) \times \frac{0.003}{5.02} = 0.00151 \text{ (Compression)} < \varepsilon_y$
5.2. Forces in the concrete and steel

\[ C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 4.02 \times 16 = 273.1 \text{ kip} \]  

\[ f_s = f_s = 60,000 \text{ psi} \]

\[ T_s = f_s \times A_s = 60,000 \times 4 = 240 \text{ kip} \]

Since \( \varepsilon_s' < \varepsilon_y \rightarrow \) compression reinforcement has not yielded

\[ \therefore f_s' = \varepsilon_s' \times E_s = 0.00151 \times 29,000,000 = 43,667 \text{ psi} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \( C_c \). As a result, it is necessary to subtract \( 0.85 f'_c \) from \( f_s' \) before computing \( C_s \):

\[ C_s = (f_s' - 0.85 f'_c) \times A_s = (43,667 - 0.85 \times 5,000) \times 4 = 157.7 \text{ kip} \]

5.3. \( \phi P_n \) and \( \phi M_n \)

\[ P_n = C_s + C_t - T_s = 273.1 + 157.7 - 240 = 190.7 \text{ kip} \]

\[ \phi P_n = 0.90 \times 191 = 171.6 \text{ kip} \]

\[ M_n = C_c \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_s \times \left( \frac{h}{2} - d_2 \right) + T_s \times \left( d_1 - \frac{h}{2} \right) \]

\[ M_n = 273.1 \times \left( \frac{16}{2} - \frac{4.02}{2} \right) + 157.7 \times \left( \frac{16}{2} - 2.5 \right) + 240 \times \left( 13.50 - \frac{16}{2} \right) = 318.61 \text{ kip.ft} \]

\[ \phi M_n = 0.90 \times 320 = 286.75 \text{ kip.ft} \]
6. Pure Bending

Figure 7 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the nominal axial load capacity, \( P_n \), is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

6.1. \( c \), \( a \), and strains in the reinforcement

Try \( c = 3.25 \) in.

Where \( c \) is the distance from the fiber of maximum compressive strain to the neutral axis.

\[
\alpha = \beta_i \times c = 0.80 \times 3.25 = 2.60 \text{ in.}
\]

Where:

\[
\beta_i = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80 \quad \text{(ACI 318-19 (Table 22.2.4.3))}
\]

\[
\varepsilon_{cu} = 0.003
\]

\[
\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207
\]

\[
\varepsilon_c = (d_i - c) \times \frac{0.003}{c} = (13.50 - 3.25) \times \frac{0.003}{3.25} = 0.00946 \quad \text{(Tension) > \varepsilon_y} \quad \text{tension reinforcement has yielded}
\]

\[\therefore \phi = 0.9 \quad \text{(ACI 318-19 (Table 21.2.2))}\]

\[
\varepsilon'_c = (c - d_i) \times \frac{0.003}{c} = (3.25 - 2.5) \times \frac{0.003}{3.25} = 0.00069 \quad \text{(Compression) < \varepsilon_y}
\]

6.2. Forces in the concrete and steel

\[
C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5,000 \times 2.6 \times 16 = 176.8 \text{ kip} \quad \text{(ACI 318-19 (22.2.4.1))}
\]

\[
f_s = f_y = 60,000 \text{ psi}
\]
\[ T_s = f_s \times A_s = 60,000 \times 4 = 240 \text{ kip} \]

Since \( \varepsilon' < \varepsilon_y \) → compression reinforcement has not yielded

\[ \therefore f'_s = \varepsilon'_s \times E_s = 0.00069 \times 29,000,000 = 20,077 \text{ psi} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \( C_c \). As a result, it is necessary to subtract \( 0.85f'_c \) from \( f'_s \) before computing \( C_s \):

\[ C_s = \left( f'_s - 0.85f'_c \right) \times A_s = \left( 20,077 - 0.85 \times 5,000 \right) \times 4 = 63.3 \text{ kip} \]

6.3. \( \phi P_n \) and \( \phi M_n \)

\[ P_n = C_c + C_s - T_s = 176.8 + 63.3 - 240 = 0 \text{ kip} \rightarrow \phi P_n = 0 \text{ kip} \]

The assumption that \( c = 3.25 \text{ in.} \) is correct

\[ M_n = C_c \times \left( \frac{h - a}{2} \right) + C_s \times \left( \frac{h - d_2}{2} \right) + T_s \times \left( d_1 - \frac{h}{2} \right) \]

\[ M_n = 176.8 \times \left( \frac{16 - 2.60}{2} \right) + 63.3 \times \left( \frac{16 - 2.5}{2} \right) + 240 \times \left( 13.50 - \frac{16}{2} \right) = 237.73 \text{ kip.ft} \]

\[ \phi M_n = 0.90 \times 238 = 213.96 \text{ kip.ft} \]
7. Maximum Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

7.1. $P_{nt}$ and $\phi P_{nt}$

$$P_{nt} = f_y \times (A_{y1} + A_{y2}) = 60,000 \times (4 + 4) = 480.0 \text{ kip}$$

$\phi = 0.9$

$$\phi P_{nt} = 0.90 \times 480.0 = 432.0 \text{ kip}$$

ACI 318-19 (22.4.3.1)

ACI 318-19 (Table 21.2.2)

7.2. $M_n$ and $\phi M_n$

Since the section is symmetrical

$$M_n = \phi M_n = 0.00 \text{ kip.ft}$$
8. Column Interaction Diagram - spColumn Software

spColumn is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode was used with no loads (the program will only report control points) and no slenderness considerations using ACI 318-19.

Figure 8 – spColumn Interface
Figure 9 – spColumn Model Editor
Figure 10 – Defining Loads / Modes (spColumn)
Figure 11 – Column P-M Interaction Diagram about the X-Axis (spColumn)
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1. General Information

<table>
<thead>
<tr>
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<td>SP</td>
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2. Material Properties

2.1. Concrete

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<tr>
<td>E_c</td>
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<td>f_t</td>
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<tr>
<td>e_c</td>
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<td>β_c</td>
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2.2. Steel

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<tr>
<td>E_s</td>
<td>29000 ksi</td>
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<tr>
<td>f_y</td>
<td>0.00206897 in/in</td>
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3. Section

3.1. Shape and Properties

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<td>Width</td>
<td>16 in</td>
</tr>
<tr>
<td>Depth</td>
<td>16 in</td>
</tr>
<tr>
<td>A_y</td>
<td>256 in²</td>
</tr>
<tr>
<td>I_y</td>
<td>5461.33 in⁴</td>
</tr>
<tr>
<td>I_x</td>
<td>5461.33 in⁴</td>
</tr>
<tr>
<td>t_y</td>
<td>4.6188 in</td>
</tr>
<tr>
<td>t_x</td>
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<tr>
<td>X_c</td>
<td>0 in</td>
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<tr>
<td>Y_c</td>
<td>0 in</td>
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</table>
3.2. Section Figure

Rectangular 16 x 16 in  3.13% rein.

Figure 1: Column section

4. Reinforcement
4.1. Bar Set: ASTM A615

<table>
<thead>
<tr>
<th>Bar</th>
<th>Diameter</th>
<th>Area</th>
<th>Bar</th>
<th>Diameter</th>
<th>Area</th>
<th>Bar</th>
<th>Diameter</th>
<th>Area</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>in</td>
<td></td>
<td></td>
<td>in</td>
<td></td>
<td></td>
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<td></td>
<td>in²</td>
<td></td>
<td></td>
<td>in²</td>
<td></td>
<td></td>
<td>in²</td>
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<tr>
<td>#3</td>
<td>0.38</td>
<td>0.11</td>
<td>#4</td>
<td>0.50</td>
<td>0.20</td>
<td>#5</td>
<td>0.63</td>
<td>0.31</td>
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<tr>
<td>#6</td>
<td>0.75</td>
<td>0.44</td>
<td>#7</td>
<td>0.88</td>
<td>0.60</td>
<td>#8</td>
<td>1.00</td>
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</tr>
<tr>
<td>#9</td>
<td>1.13</td>
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<td>#10</td>
<td>1.27</td>
<td>1.27</td>
<td>#11</td>
<td>1.41</td>
<td>1.56</td>
</tr>
<tr>
<td>#14</td>
<td>1.69</td>
<td>2.25</td>
<td>#18</td>
<td>2.28</td>
<td>4.00</td>
<td></td>
<td></td>
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4.2. Confinement and Factors

Confinement type: Tied
For #10 bars or less: #3 ties
For larger bars: #4 ties

Capacity Reduction Factors
Axial compression, (a): 0.8
Tension controlled, (b): 0.9
Compression controlled, (c): 0.65

4.3. Arrangement
Pattern: Sides different
Bar layout: Rectangular
Cover to: Longitudinal bars
Clear cover: —
Bars: —
### 4.4. Bars Provided

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<thead>
<tr>
<th></th>
<th>Bars</th>
<th>Clear cover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>4</td>
<td>#9</td>
</tr>
<tr>
<td>Bottom</td>
<td>4</td>
<td>#9</td>
</tr>
<tr>
<td>Left</td>
<td>0</td>
<td>#9</td>
</tr>
<tr>
<td>Right</td>
<td>0</td>
<td>#9</td>
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### 5. Control Points

<table>
<thead>
<tr>
<th>About Point</th>
<th>P (kip)</th>
<th>X-Moment (k-ft)</th>
<th>Y-Moment (k-ft)</th>
<th>NA Depth (in)</th>
<th>d, Depth (in)</th>
<th>ε, (°)</th>
<th>φ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X @ Max comp.</td>
<td>997.1</td>
<td>0.00</td>
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<td>43.50</td>
<td>13.50</td>
<td>-0.00207</td>
<td>0.65000</td>
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<tr>
<td>X @ Allowable comp.</td>
<td>797.7</td>
<td>102.64</td>
<td>0.00</td>
<td>17.35</td>
<td>13.50</td>
<td>-0.00067</td>
<td>0.65000</td>
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<td>X @ f ≤ 0.1</td>
<td>622.3</td>
<td>169.89</td>
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<td>X @ f = 0.5 f</td>
<td>421.9</td>
<td>226.05</td>
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<td>10.04</td>
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<td>X @ Balanced point</td>
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<td>X @ Tension control</td>
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<td>286.75</td>
<td>0.00</td>
<td>5.02</td>
<td>13.50</td>
<td>0.00507</td>
<td>0.90000</td>
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<tr>
<td>X @ Pure bending</td>
<td>0.0</td>
<td>213.91</td>
<td>0.00</td>
<td>3.25</td>
<td>13.50</td>
<td>0.00946</td>
<td>0.90000</td>
</tr>
<tr>
<td>X @ Max tension</td>
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<td>0.00</td>
<td>0.00</td>
<td>13.50</td>
<td>9.598999</td>
<td>0.90000</td>
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</tr>
<tr>
<td>X @ Allowable comp.</td>
<td>797.7</td>
<td>-102.64</td>
<td>0.00</td>
<td>17.35</td>
<td>13.50</td>
<td>-0.00067</td>
<td>0.65000</td>
</tr>
<tr>
<td>X @ f = 0.0</td>
<td>622.3</td>
<td>-169.88</td>
<td>0.00</td>
<td>13.50</td>
<td>13.50</td>
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<tr>
<td>X @ f = 0.5 f</td>
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<td>-226.05</td>
<td>0.00</td>
<td>10.04</td>
<td>13.50</td>
<td>0.00103</td>
<td>0.65000</td>
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<tr>
<td>X @ Balanced point</td>
<td>270.9</td>
<td>-256.77</td>
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<td>171.6</td>
<td>-286.75</td>
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<td>X @ Pure bending</td>
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<td>-213.91</td>
<td>0.00</td>
<td>3.25</td>
<td>13.50</td>
<td>0.00946</td>
<td>0.90000</td>
</tr>
<tr>
<td>X @ Max tension</td>
<td>-432.0</td>
<td>0.00</td>
<td>0.00</td>
<td>13.50</td>
<td>9.598999</td>
<td>0.90000</td>
<td></td>
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6. Diagrams
6.1. PM at θ=0 [deg]

General Information
- Project: Tied Square, etc. Column
- Column: Interior
- Engineer: SP
- Code: ACI 318-19
- Bar Set: ASTM A615
- Units: English
- Run Option: Investigation
- Run Area: X - axis
- Stiffness: Not Considered
- Column Type: Structural
- Capacity Method: Moment capacity

Materials
- $f_y$: 5 ksi
- $E$: 430.51 ksi
- $f_y$: 60 ksi
- $E$: 29000 ksi

Section
- Type: Rectangular
- Width: 16 in
- Depth: 16 in
- $A_y$: 256 in$^2$
- $I_y$: 5401.33 in$^4$
- $I_y$: 5401.33 in$^4$

Reinforcement
- Pattern: Sides different
- Bar layout: Rectangular
- Cover to: Longitudinal bars
- Clear cover: ---
- Bars: ---
- Confinement type: Tied

Total steel area, $A_s$: 8.02 in$^2$
- Rho: 3.13 %
- Min. clear spacing: 2.54 in
9. Summary and Comparison of Design Results

<table>
<thead>
<tr>
<th>Support</th>
<th>$\phi P_n$, kip</th>
<th>$\phi M_n$, kip.ft</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>Pure bending</td>
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</tr>
<tr>
<td>Max tension</td>
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<td>432.0</td>
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</tbody>
</table>

* Reference rounded the strain value from 0.00507 to 0.0051.

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.
10. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where $f_s = f_y$).

![Diagram](image1)

*Figure 12 – Strains, Forces, and Moment Arms ($f_s = f_y$, Moments About x- and y-axis)*
When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure. Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

![Interaction Diagram About the X-Axis](image1.png)

![Interaction Diagram About the Y-Axis](image2.png)

Figure 13 – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)
In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to $M_x$ and $M_y$ due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint’s spColumn program can also investigate column and wall sections in biaxial mode to produce the results shown in the following Figure for the column section in this example. In biaxial run mode, $M_x$ and $M_y$ diagrams at each axial force level can be viewed in 2D and 3D views.

Figure 14 – Nominal & Design 3D Failure Surfaces (Biaxial) (spColumn)
Figure 15 – Tied Column Interaction Diagram and 3D failure Surface Viewer (spColumn)
Figure 16 – Tied Column 3D Failure Surface with a Horizontal Plane Cut at P = 350 kip (spColumn)
Figure 17 – Tied Column 3D Failure Surface with a Vertical Plane Cut at 45° (spColumn)