Interaction Diagram - Tied Reinforced Concrete Column (Using CSA A23.3-14)
Interaction Diagram - Tied Reinforced Concrete Column

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using CSA A23.3-14 provisions. Determine six control points on the interaction diagram and compare the calculated values in the Reference and with exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.

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Code
Design of Concrete Structures (CSA A23.3-14)

Reference

Design Data
\[ f' = 35 \text{ MPa} \]
\[ f_y = 400 \text{ MPa} \]
Cover = 55 mm to the center of the reinforcement
Column 400 mm x 400 mm
Top reinforcement = 4 No. 30
Bottom reinforcement = 4 No. 30

Solution
Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following six control points:

Point 1: Pure compression
Point 2: Bar stress near tension face of member equal to zero, \( f_s = 0 \)
Point 3: Bar stress near tension face of member equal to 0.5 \( f_y \) \( (f_s = -0.5 f_y) \)
Point 4: Bar stress near tension face of member equal to \( f_y \) \( (f_s = -f_y) \)
Point 5: Pure bending
Point 6: Pure tension
Figure 2 – Control Points

- Pure Compression
- Pure Bending
- Balanced Failure $f_s = -f_y$
- $P_{t,\text{max}}$
- $P_r, M_r$
- $f_s = 0$
- $f_s = -0.5 f_y$
1. Pure Compression

1.1. Nominal axial resistance at zero eccentricity

\[ P_o = \alpha_1 f'_c (A_x - A_y) + f_y A_y \]

\[ P_o = 0.798 \times 35 \times (400 \times 400 - 8 \times 700) + 400 \times 8 \times 700 = 6550 \text{ kN} \]

Where \( \alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67 \)

\[ \alpha_1 = 0.85 - 0.0015 \times 35 = 0.798 \geq 0.67 \]

**CSA A23.3-14 (Equation 10.1)**

1.2. Factored axial load resistance at zero eccentricity

Since this column is a tied column with steel strain in compression:

\[ P_{ro} = \alpha \phi_s f'_c (A_x - A_y) + \phi_y f_y A_y \]

\[ P_{ro} = 0.798 \times 0.65 \times 35 \times (400 \times 400 - 8 \times 700) + 0.85 \times 400 \times 8 \times 700 = 4705 \text{ kN} \]

Where:

\[ \phi_s = 0.65 \]

**CSA A23.3-14 (Equation 10.11)**

\[ \phi_y = 0.85 \]

**CSA A23.3-14 (8.4.2)**

1.3. Maximum factored axial load resistance

\[ P_{o,\text{max}} = (0.2 + 0.002h) P_{ro} \leq 0.80 P_{ro} \]

\[ P_{o,\text{max}} = (0.2 + 0.002 \times 400) \times 4705 = 4705 \text{ kN} \leq 0.80 \times 4705 = 3764 \text{ kN} \]

\[ P_{o,\text{max}} = 3764 \text{ kN} \]

**CSA A23.3-14 (Equation 10.9)**
2. Bar Stress Near Tension Face of Member Equal to Zero, \( (\varepsilon_s = f_s = 0) \)

Strain \( \varepsilon_s \) is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices.

CSA A23.3-14 (12.15 and 16)

2.1. \( c, a, \) and strains in the reinforcement

\[ c = d_1 = 345 \text{ mm} \]

Where \( c \) is depth of the neutral axis measured from the compression edge of the column section.

CSA A23.3-14 (3.2)

\[ a = \beta_1 \times c = 0.883 \times 345 = 304 \text{ mm} \]

Where:

\( a = \) Depth of equivalent rectangular stress block

CSA A23.3-14 (10.1.7)

\[ \beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \]

CSA A23.3-14 (Equation 10.2)

\[ \varepsilon_c = 0 \]

CSA A23.3-14 (8.4.2)

\[ \phi_c = 0.65 \]

CSA A23.3-14 (8.4.3(a))

\[ \phi_f = 0.85 \]

CSA A23.3-14 (10.1.3)

\[ \varepsilon_{uc} = 0.0035 \]

CSA A23.3-14 (10.1.3)

\[ \varepsilon_s = (c - d_2) \times \frac{\varepsilon_{uc}}{c} = (345 - 55) \times \frac{0.0035}{345} = 0.00294 \text{ (Compression)} > \varepsilon_s = \frac{F_s}{E_s} = \frac{400}{200,000} = 0.002 \]

2.2. Forces in the concrete and steel

\[ C_{rc} = \alpha_t \times \phi_f \times f'_c \times a \times b = 0.798 \times 0.65 \times 35 \times 304 \times 400 = 2210 \text{ kN} \]

CSA A23.3-14 (10.1.7)

\[ f'_s = 0 \text{ kN} \rightarrow T_{rs} = \phi_c \times f'_s \times A_{sl} = 0 \text{ kN} \]
Since $e'_c > e_s \rightarrow$ compression reinforcement has yielded

$$f'_s = f_s = 400 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, it is necessary to subtract \(\alpha \phi f'_c\) from \(\phi f_s\) before computing \(C_{rs}\):

$$C_{rs} = \left(\phi_c f'_c - \alpha_c \phi_c f'_c\right) \times A_2 = \left(0.85 \times 400 - 0.798 \times 0.65 \times 35\right) \times 2800 = 901 \text{ kN}$$

2.3. \(P_r\) and \(M_r\)

$$P_r = C_{rs} + C_{sc} - T_n = 2210 + 901 - 0 = 3111 \text{ kN}$$

$$M_r = C_{rc} \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_{rs} \times \left(\frac{h}{2} - d_2\right) + T_n \times \left(d_1 - \frac{h}{2}\right)$$

$$M_r = 2210 \times \left(\frac{400}{2} - \frac{304}{2}\right) + 901 \times \left(\frac{400}{2} - 55\right) + 0 \times \left(345 - \frac{400}{2}\right) = 236 \text{ kN.m}$$
3. Bar Stress Near Tension Face of Member Equal to 0.5 $f_y$, ($f_t = -0.5 f_y$)

![Figure 4 - Strains, Forces, and Moment Arms ($f_t = -0.5 f_y$)]

3.1. $c$, $a$, and strains in the reinforcement

$$\varepsilon_y = \frac{F_y}{E_y} = \frac{400}{200,000} = 0.002$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.002}{2} = 0.001 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$$\phi_c = 0.65 \quad \text{CSA A23.3-14 (8.4.2)}$$

$$\phi_s = 0.85 \quad \text{CSA A23.3-14 (8.4.3(a))}$$

$$\varepsilon_{cu} = 0.0035 \quad \text{CSA A23.3-14 (10.1.3)}$$

$$c = \frac{d_l}{\varepsilon_s + \varepsilon_{cu}} = \frac{345}{0.001 + 0.0035} \times 0.0035 = 268 \text{ mm}$$

Where $c$ is depth of the neutral axis measured from the compression edge of the column section.

$$a = \beta_l \times c = 0.883 \times 268 = 237 \text{ mm} \quad \text{CSA A23.3-14 (10.1.7)}$$

Where:

$a = \text{Depth of equivalent rectangular stress block} \quad \text{CSA A23.3-14 (3.2)}$

$$\beta_l = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \quad \text{CSA A23.3-14 (Equation 10.2)}$$

$$\varepsilon'_s = (c - d_l) \times \frac{\varepsilon_{cu}}{c} = (268 - 55) \times \frac{0.0035}{268} = 0.00278 \text{ (Compression)} > \varepsilon_s$$

3.2. Forces in the concrete and steel

$$C_{rc} = \alpha_t \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.65 \times 237 \times 400 = 1719 \text{ kN} \quad \text{CSA A23.3-14 (10.1.7)}$$

$$f'_c = \varepsilon_s \times E_s = 0.001 \times 200,000 = 200 \text{ MPa}$$
\[ T_n = \phi_f \times f_s \times A_{1} = 0.85 \times 200 \times 2800 = 476 \text{ kN} \]

Since \( \varepsilon_n > \varepsilon_y \rightarrow \) compression reinforcement has yielded

\[ \therefore f'_s = f_y = 400 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, it is necessary to subtract \(\alpha_f \phi_f f'_s\)' from \(\phi f'_s\)' before computing \(C_{rs}\):

\[ C_{rs} = (\phi_f f'_s - \alpha_f \phi_f f'_c) \times A_{2} = (0.85 \times 400 - 0.798 \times 0.65 \times 35) \times 2800 = 901 \text{ kN} \]

3.3. \(P_r\) and \(M_r\)

\[ P_r = C_{rc} + C_{ns} - T_n = 1719 + 901 - 476 = 2144 \text{ kN} \]

\[ M_r = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{ns} \times \left( \frac{h}{2} - d_2 \right) + T_n \times \left( d_1 - \frac{h}{2} \right) \]

\[ M_r = 1719 \times \left( \frac{400}{2} - \frac{237}{2} \right) + 901 \times \left( \frac{400}{2} - 55 \right) + 476 \times \left( 345 - \frac{400}{2} \right) = 340 \text{ kN.m} \]
4. Bar Stress Near Tension Face of Member Equal to \( f_y \), \( f_s = -f_y \)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement.

4.1. \( c, a, \) and strains in the reinforcement

\[
\varepsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002
\]

\( \varepsilon_c = \varepsilon_y = 0.002 \rightarrow \) tension reinforcement has yielded

\( \phi_c = 0.65 \quad \text{CSA A23.3-14 (8.4.2)} \)

\( \phi_s = 0.85 \quad \text{CSA A23.3-14 (8.4.3(a))} \)

\( \varepsilon_{ca} = 0.0035 \quad \text{CSA A23.3-14 (10.1.3)} \)

\[
c = \frac{d_i}{\varepsilon_i + \varepsilon_{ca}} \times \varepsilon_{cu} = \frac{345}{0.002 + 0.0035} \times 0.0035 = 220 \text{ mm}
\]

Where \( c \) is depth of the neutral axis measured from the compression edge of the column section.

\( a = \beta_i \times c = 0.883 \times 220 = 194 \text{ mm} \quad \text{CSA A23.3-14 (10.1.7)} \)

Where:

\( a = \) Depth of equivalent rectangular stress block

\( \beta_i = 0.97 - 0.0025 \times f_y = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \quad \text{CSA A23.3-14 (3.2)} \)

\[
\varepsilon_s = \frac{(c - d_2) \times \varepsilon_{ca}}{c} = \frac{(220 - 55) \times 0.0035}{220} = 0.00262 \text{ (Compression) > } \varepsilon_y
\]
4.2. Forces in the concrete and steel

\[ C_c = \alpha_c \times f_c \times a \times b = 0.798 \times 0.65 \times 35 \times 194 \times 400 = 1406 \, \text{kN} \]  \hspace{1cm} \text{CSA A23.3-14 (10.1.7)}

\[ f_c = f_y = 400 \, \text{MPa} \]

\[ T_n = \phi_c \times f_c \times A_1 = 0.85 \times 400 \times 2800 = 952 \, \text{kN} \]

Since \( \varepsilon_c > \varepsilon_y \), compression reinforcement has yielded

\[ \therefore f_y' = f_y = 400 \, \text{MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \( C_c \). As a result, it is necessary to subtract \( \alpha_c \phi_c f_c \) from \( \phi_c f_c' \) before computing \( C_r \):

\[ C_r = \left( \phi_c f_y' - \alpha_c \phi_c f_c \right) \times A_2 = \left( 0.85 \times 400 - 0.798 \times 0.65 \times 35 \right) \times 2800 = 901 \, \text{kN} \]

4.3. \( P_r \) and \( M_r \)

\[ P_r = C_r + C_r - T_n = 1406 + 901 - 952 = 1355 \, \text{kN} \]

\[ M_r = C_r \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_r \times \left( \frac{h}{2} - d_2 \right) + T_n \times \left( d_1 - \frac{h}{2} \right) \]

\[ M_r = 1406 \times \left( \frac{400}{2} - \frac{194}{2} \right) + 901 \times \left( \frac{400}{2} - 55 \right) + 952 \times \left( 345 - \frac{400}{2} \right) = 414 \, \text{kN.m} \]
5. Pure Bending

Figure 6 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the factored axial load resistance, $P_r$, is equal to zero. Iterative procedure is used to determine the factored moment resistance as follows:

5.1. $c$, $a$, and strains in the reinforcement

Try $c = 78.55$ mm

Where $c$ is depth of the neutral axis measured from the compression edge of the column section.

\[
\alpha = \beta_y \times c = 0.883 \times 78.55 = 69 \text{ mm}
\]

Where:

\[
\beta_y = 0.97 - 0.0025 \times f_y = 0.97 - 0.0025 \times 35 = 0.883 > 0.67
\]

\[
\varepsilon_{cu} = 0.0035
\]

\[
\varepsilon_y = \frac{F_y}{E_y} = \frac{400}{200,000} = 0.002
\]

\[
\varepsilon_s = (d_1 - c) \times \frac{E_{cu}}{c} = (345 - 78.55) \times \frac{0.0035}{78.55} = 0.01187 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}
\]

\[
\phi_e = 0.65
\]

\[
\phi_s = 0.85
\]

\[
\varepsilon_s = (c - d_2) \times \frac{E_{cu}}{c} = (78.55 - 55) \times \frac{0.0035}{78.55} = 0.00105 \text{ (Compression)} < \varepsilon_y
\]

5.2. Forces in the concrete and steel

\[
C_{rc} = \alpha \times \phi_e \times f'_c \times a \times b = 0.798 \times 0.65 \times 35 \times 69 \times 400 = 503 \text{ kN}
\]

\[
f_s = f'_y = 400 \text{ MPa}
\]

CSA A23.3-14 (3.2)

CSA A23.3-14 (10.1.7)

CSA A23.3-14 (Equation 10.2)

CSA A23.3-14 (10.1.3)

CSA A23.3-14 (8.4.2)

CSA A23.3-14 (8.4.3(a))
\[ T_n = \phi_i \times f_s \times A_{s1} = 0.85 \times 400 \times 2800 = 952 \text{ kN} \]

Since \( \varepsilon' < \varepsilon_y \) → compression reinforcement has not yielded

\[ f_s' = \varepsilon' \times E_s = 0.00105 \times 200,000 = 210 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, it is necessary to subtract \(a_1\phi f_s'\) from \(\phi f_s'\) before computing \(C_{rs}\):

\[ C_n = (\phi_i \times f_s' - a_1 \times \phi \times f_s') \times A_{s2} = (0.85 \times 210 - 0.798 \times 0.65 \times 35) \times 2800 = 449 \text{ kN} \]

5.3. \(P_r\) and \(M_r\)

\[ P_r = C_{rc} + C_{rs} - T_n = 503 + 449 - 952 \approx 0 \text{ kN} \]

The assumption that \(c = 78.55 \text{ mm}\) is correct

\[ M_r = C_{rc} \times \left(\frac{h - a}{2}\right) + C_{rs} \times \left(\frac{h - d_2}{2}\right) + T_n \times \left(\frac{d_1 - h}{2}\right) \]

\[ M_r = 503 \times \left(\frac{400 - 69}{2}\right) + 449 \times \left(\frac{400 - 55}{2}\right) + 952 \times \left(\frac{345 - 400}{2}\right) = 286 \text{ kN.m} \]
6. Pure Tension
The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

6.1. Strength under pure axial tension \( P_{rt} \)

\[
P_{rt} = \phi \times f_y \times (A_{s1} + A_{s2}) = 0.85 \times 400 \times (2800 + 2800) = 1904 \text{ kN}
\]

6.2. Corresponding Moment \( M_{rt} \)

Since the section is symmetrical

\[ M_{rt} = 0 \text{ kN.m} \]
7. Column Interaction Diagram - spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with control points using the CSA A23.3-14. In lieu of using program shortcuts, spSection (Figure 9) was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

Figure 7 – Generating spColumn Model
Figure 8 – spColumn Model Editor (spSection)
Figure 9 – Column Section Interaction Diagram about the X-Axis (spColumn)
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1. General Information

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2. Material Properties

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3. Section

3.1. Shape and Properties

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3.2. Section Figure

![Section Figure Diagram]

Rectangular 400 x 400 mm 3.50% reinf.

Figure 1: Column section

4. Reinforcement

4.1. Bar Set: CSA G30.18

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4.2. Confinement and Factors

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<tr>
<td>For larger bars</td>
<td>#15 ties</td>
</tr>
</tbody>
</table>

Material Resistance Factors

| Axial compression, (a) | 0.8 |
| Steel (Φ_s) | 0.85 |
| Concrete (Φ_c) | 0.65 |

Minimum dimension, h: 400 mm

4.3. Arrangement

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Sides different</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar layout</td>
<td>---</td>
</tr>
<tr>
<td>Cover to</td>
<td>Longitudinal bars</td>
</tr>
<tr>
<td>Clear cover</td>
<td>---</td>
</tr>
</tbody>
</table>
### 4.4. Bars Provided

<table>
<thead>
<tr>
<th></th>
<th>Bars</th>
<th>Cover</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>mm</td>
</tr>
<tr>
<td>Top</td>
<td>4</td>
<td>#30</td>
</tr>
<tr>
<td>Bottom</td>
<td>4</td>
<td>#30</td>
</tr>
<tr>
<td>Left</td>
<td>0</td>
<td>#10</td>
</tr>
<tr>
<td>Right</td>
<td>0</td>
<td>#10</td>
</tr>
</tbody>
</table>

### 5. Control Points

<table>
<thead>
<tr>
<th>About Point</th>
<th>P</th>
<th>X-Moment</th>
<th>Y-Moment</th>
<th>NA Depth</th>
<th>d1 Depth</th>
<th>f1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kN</td>
<td>kNm</td>
<td>kNm</td>
<td>mm</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>X @ Max compression</td>
<td>4705.3</td>
<td>0.00</td>
<td>0.00</td>
<td>805</td>
<td>345</td>
<td>-0.00200</td>
</tr>
<tr>
<td>X @ Allowable comp.</td>
<td>3764.2</td>
<td>146.16</td>
<td>0.00</td>
<td>412</td>
<td>345</td>
<td>-0.00057</td>
</tr>
<tr>
<td>X @ 0.0</td>
<td>3111.1</td>
<td>236.23</td>
<td>0.00</td>
<td>345</td>
<td>345</td>
<td>0.00000</td>
</tr>
<tr>
<td>X @ 0.5 fy</td>
<td>2144.0</td>
<td>339.98</td>
<td>0.00</td>
<td>268</td>
<td>345</td>
<td>0.00100</td>
</tr>
<tr>
<td>X @ Balanced point</td>
<td>1355.5</td>
<td>413.81</td>
<td>0.00</td>
<td>220</td>
<td>345</td>
<td>0.00200</td>
</tr>
<tr>
<td>X @ Pure bending</td>
<td>0.0</td>
<td>286.30</td>
<td>0.00</td>
<td>79</td>
<td>345</td>
<td>0.01188</td>
</tr>
<tr>
<td>X @ Max tension</td>
<td>-1904.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
<td>345</td>
<td>0.99999</td>
</tr>
<tr>
<td>-X @ Max compression</td>
<td>4705.3</td>
<td>0.00</td>
<td>0.00</td>
<td>805</td>
<td>345</td>
<td>-0.00200</td>
</tr>
<tr>
<td>-X @ Allowable comp.</td>
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<td>-146.16</td>
<td>0.00</td>
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<td>-0.00057</td>
</tr>
<tr>
<td>-X @ 0.0</td>
<td>3111.1</td>
<td>-236.23</td>
<td>0.00</td>
<td>345</td>
<td>345</td>
<td>0.00000</td>
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<tr>
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<td>-339.98</td>
<td>0.00</td>
<td>268</td>
<td>345</td>
<td>0.00100</td>
</tr>
<tr>
<td>-X @ Balanced point</td>
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<td>-413.81</td>
<td>0.00</td>
<td>220</td>
<td>345</td>
<td>0.00200</td>
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<tr>
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<td>-286.30</td>
<td>0.00</td>
<td>79</td>
<td>345</td>
<td>0.01188</td>
</tr>
<tr>
<td>-X @ Max tension</td>
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<td>0.00</td>
<td>0.00</td>
<td>0</td>
<td>345</td>
<td>0.99999</td>
</tr>
</tbody>
</table>
8. Summary and Comparison of Design Results

<table>
<thead>
<tr>
<th>Support</th>
<th>$P_e$, kN</th>
<th>$M_e$, kN.m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hand</td>
<td>Reference</td>
</tr>
<tr>
<td>Max compression</td>
<td>4705</td>
<td>4490</td>
</tr>
<tr>
<td>Allowable compression</td>
<td>3764</td>
<td>3592</td>
</tr>
<tr>
<td>$f_s = 0.0$</td>
<td>3111</td>
<td>2945</td>
</tr>
<tr>
<td>$f_s = 0.5$ $f_y$</td>
<td>2144</td>
<td>2015</td>
</tr>
<tr>
<td>Balanced point</td>
<td>1355</td>
<td>1253</td>
</tr>
<tr>
<td>Pure bending</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Max tension</td>
<td>1904</td>
<td>1904</td>
</tr>
</tbody>
</table>

** The reference used CSA A23.3-94 where the resistance factor for concrete ($\phi_c$) is 0.60. The hand calculation and spColumn used CSA A23.3-14 where the resistance factor for concrete ($\phi_c$) is 0.65. (Check Column Interaction Diagram Using CSA A23.3-94 Example)
9. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where $f_s = f_y$).

![Figure 10 – Strains, Forces, and Moment Arms ($f_s = -f_y$, Moments About x- and y-axis)](image)
When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure.

Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to $M_x$ and $M_y$ due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint’s spColumn program can also evaluate column sections in biaxial mode to produce the results shown in the following Figure for the column section in this example.
Figure 12 – Nominal & Design Interaction Diagram in Two Directions (Biaxial) (spColumn)