Interaction Diagram - Tied Reinforced Concrete Column (Using CSA A23.3-94)
Interaction Diagram - Tied Reinforced Concrete Column

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using CSA A23.3-14 provisions. Determine six control points on the interaction diagram and compare the calculated values in the Reference and with exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.

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Code

Design of Concrete Structures (CSA A23.3-94)

Reference


Design Data

\( f' = 35 \text{ MPa} \)
\( f_y = 400 \text{ MPa} \)

Cover = 55 mm to the center of the reinforcement

Column 400 mm x 400 mm

Top reinforcement = 4 No. 30

Bottom reinforcement = 4 No. 30

Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following six control points:

Point 1: Pure compression

Point 2: Bar stress near tension face of member equal to zero, \((f_s = 0)\)

Point 3: Bar stress near tension face of member equal to 0.5 \(f_y\) \((f_s = -0.5 f_y)\)

Point 4: Bar stress near tension face of member equal to \(f_y\) \((f_s = -f_y)\)

Point 5: Pure bending

Point 6: Pure tension
Figure 2 – Control Points
1. Pure Compression

1.1. Nominal axial resistance at zero eccentricity

\[ P_o = \alpha_i f'c (A_x - A_i) + f_s A_i \]

\[ P_o = 0.798 \times 35 \times (400 \times 400 - 8 \times 700) + 400 \times 8 \times 700 = 6550 \text{ kN} \]

Where \( \alpha_i = 0.85 - 0.0015 f'_c \geq 0.67 \)  
\( \alpha_i = 0.85 - 0.0015 \times 35 = 0.798 \geq 0.67 \)  

**CSA A23.3-94 (Equation 10-1)**

1.2. Factored axial load resistance at zero eccentricity

Since this column is a tied column with steel strain in compression:

\[ P_{ro} = \alpha_i \phi_s f'c (A_x - A_i) + \phi_s f_s A_i \]

\[ P_{ro} = 0.798 \times 0.6 \times 35 \times (400 \times 400 - 8 \times 700) + 0.85 \times 400 \times 8 \times 700 = 4490 \text{ kN} \]

Where:
\( \phi_s = 0.6 \)  
\( \phi_i = 0.85 \)

**CSA A23.3-94 (Equation 10-10)**

1.3. Maximum factored axial load resistance

\[ P_{r,\text{max}} = 0.80 P_{ro} \]

\[ P_{r,\text{max}} = (0.2 + 0.002 \times 400) \times 4490 = 4490 \text{ kN} \leq 0.80 P_{ro} = 0.80 \times 4490 = 3592 \text{ kN} \]

\[ P_{r,\text{max}} = 3592 \text{ kN} \]

**CSA A23.3-94 (Equation 10-9)**
2. Bar Stress Near Tension Face of Member Equal to Zero, (εs = f_s = 0)

Strain ε_s is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices.  

CSA A23.3-94 (12.15 and 16)

### 2.1. c, a, and strains in the reinforcement

\[ c = d_t = 345 \text{ mm} \]

Where \( c \) is distance from extreme compression fiber to neutral axis.

CSA A23.3-94 (10.0)

\[ a = \beta f \times c = 0.883 \times 345 = 304 \text{ mm} \]

Where:

- \( a \) = Depth of equivalent rectangular stress block  
  - CSA A23.3-94 (10.0)
- \( \beta f \) = 0.97 − 0.0025 × \( f_s \) = 0.97 − 0.0025 × 35 = 0.883 > 0.67  
  - CSA A23.3-94 (Equation 10-2)
- \( \varepsilon_s = 0 \)

- \( \phi_s = 0.6 \)  
  - CSA A23.3-94 (8.4.2)
- \( \phi_f = 0.85 \)  
  - CSA A23.3-94 (8.4.3)

- \( \varepsilon_{cu} = 0.0035 \)  
  - CSA A23.3-94 (10.1.3)

\[ \varepsilon_s = (c - d_t) \times \frac{\varepsilon_{cu}}{c} = (345 - 55) \times \frac{0.0035}{345} = 0.00294 \text{ (Compression)} \times \varepsilon_s = \frac{F_s}{E_s} = \frac{400}{200,000} = 0.002 \]

### 2.2. Forces in the concrete and steel

\[ C_{rc} = \alpha_1 \times \phi_s \times f_s \times a \times b = 0.798 \times 0.6 \times 35 \times 304 \times 400 = 2040 \text{ kN} \]

CSA A23.3-94 (10.1.7)
\[ f_s = 0 \text{ kN} \rightarrow T_{rs} = \phi_s \times f_s \times A_i = 0 \text{ kN} \]

Since \( \varepsilon_s > \varepsilon_y \rightarrow \) compression reinforcement has yielded

\[ \therefore f_s = f_y = 400 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, it is necessary to subtract \(\alpha_i f_s\) from \(\phi_s f_s\) before computing \(C_{rs}\):

\[ \left( \phi_s \times f_s - \alpha_i \times \phi_s \times f_s \right) \times A_{s2} = \left(0.85 \times 400 - 0.798 \times 0.6 \times 35\right) \times 2800 = 905 \text{ kN} \]

2.3. \(P_r\) and \(M_r\)

\[ P_r = C_{rc} + C_{rs} - T_{rs} = 2040 + 905 - 0 = 2945 \text{ kN} \]

\[ M_r = C_{rc} \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_{rs} \times \left(\frac{h}{2} - d_2\right) + T_{rs} \times \left(d_1 - \frac{h}{2}\right) \]

\[ M_r = 2040 \times \left(\frac{400}{2} - \frac{304}{2}\right) + 905 \times \left(\frac{400}{2} - 55\right) + 0 \times \left(345 - \frac{400}{2}\right) = 229 \text{ kN.m} \]
3. Bar Stress Near Tension Face of Member Equal to 0.5 \( f_y \), \( f_s = -0.5 f_y \)

![Figure 4 – Strains, Forces, and Moment Arms (\( f_s = -0.5 f_y \))](image)

3.1. \( c \), \( a \), and strains in the reinforcement

\[
\varepsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002
\]

\[
\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.002}{2} = 0.001 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}
\]

\[\phi_s = 0.6 \quad \text{CSA A23.3-94 (8.4.2)}\]

\[\phi_c = 0.85 \quad \text{CSA A23.3-94 (8.4.3)}\]

\[\varepsilon_{cu} = 0.0035 \quad \text{CSA A23.3-94 (10.1.3)}\]

\[
c = \frac{d_i \times \varepsilon_y}{\varepsilon_y + \varepsilon_{cu}} = \frac{345}{0.001 + 0.0035} \times 0.0035 = 268 \text{ mm}
\]

Where \( c \) is depth of the neutral axis measured from the compression edge of the column section.

\[\text{CSA A23.3-94 (10.0)}\]

\[a = \beta_s \times c = 0.883 \times 268 = 237 \text{ mm} \quad \text{CSA A23.3-94 (10.1.7)}\]

Where:

\[a = \text{Depth of equivalent rectangular stress block} \quad \text{CSA A23.3-94 (10.0)}\]

\[\beta_s = 0.97 - 0.0025 \times f'_s = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \quad \text{CSA A23.3-94 (Equation 10-2)}\]

\[\varepsilon'_c = \frac{(c - d_i) \times \varepsilon_{cu}}{c} = \frac{(268 - 55) \times 0.0035}{268} = 0.00278 \text{ (Compression)} > \varepsilon_y\]

3.2. Forces in the concrete and steel

\[
C_{cs} = \alpha_c \times \phi' \times f'_s \times a \times b = 0.798 \times 0.6 \times 35 \times 237 \times 400 = 1586 \text{ kN} \quad \text{CSA A23.3-94 (10.1.7)}
\]
\[ f_s = \varepsilon_s \times E_s = 0.001 \times 200,000 = 200 \text{ MPa} \]

\[ T_{rs} = \phi_s \times f_s \times A_{s1} = 0.85 \times 200 \times 2800 = 476 \text{ kN} \]

Since \( \varepsilon_s > \varepsilon_y \rightarrow \) compression reinforcement has yielded

\[ f_s' = f_s = 400 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \(C_c\). As a result, it is necessary to subtract \(\alpha_s f_s'\) from \(\phi_s f_s\) before computing \(C_{rs}\):

\[ C_{rs} = (\phi_s \times f_s' - \alpha_s \times \phi_s \times f_s) \times A_{s2} = (0.85 \times 400 - 0.798 \times 0.6 \times 35) \times 2800 = 905 \text{ kN} \]

3.3. \(P_r\) and \(M_r\)

\[ P_r = C_{rc} + C_{rs} - T_{rs} = 1586 + 905 - 476 = 2015 \text{ kN} \]

\[ M_r = C_{rc} \times \left( \frac{h - a}{2} \right) + C_{rs} \times \left( \frac{h - d_2}{2} \right) + T_{rs} \times \left( d_1 - \frac{h}{2} \right) \]

\[ M_r = 1586 \times \left( \frac{400}{2} - \frac{237}{2} \right) + 905 \times \left( \frac{400}{2} - 55 \right) + 476 \times \left( 345 - \frac{400}{2} \right) = 330 \text{ kN.m} \]
4. Bar Stress Near Tension Face of Member Equal to \( f_y \), \( f_s = -f_y \)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement.

4.1. \( c \), \( a \), and strains in the reinforcement

\[
\varepsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002
\]

\[
\varepsilon_s = \varepsilon_y = 0.002 \rightarrow \text{tension reinforcement has yielded}
\]

\[
\phi_c = 0.6
\]

\[
\phi_s = 0.85
\]

\[
\varepsilon_{cu} = 0.0035
\]

\[
c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} = \frac{345}{0.002 + 0.0035} \times 0.0035 = 220 \text{ mm}
\]

Where \( c \) is depth of the neutral axis measured from the compression edge of the column section.

\[
a = \beta_i \times c = 0.883 \times 220 = 194 \text{ mm}
\]

Where:

\[
\beta_i = 0.97 - 0.0025 \times f'_{cu} = 0.97 - 0.0025 \times 35 = 0.883 > 0.67
\]

\[
\varepsilon'_s = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (220 - 55) \times \frac{0.0035}{220} = 0.00262 \quad \text{(Compression)} > \varepsilon_y
\]
4.2. Forces in the concrete and steel

\[ C_{rc} = \alpha \times f_{c} \times a \times b = 0.798 \times 0.6 \times 35 \times 194 \times 400 = 1298 \text{ kN} \quad \text{CSA A23.3-94 (10.1.7)} \]

\[ f_s = f_y = 400 \text{ MPa} \]

\[ T_{rs} = \phi_s \times f_s \times A_{s1} = 0.85 \times 400 \times 2800 = 952 \text{ kN} \]

Since \( \varepsilon_r > \varepsilon_y \), compression reinforcement has yielded

\[ \therefore f_s = f_y = 400 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \( C_c \). As a result, it is necessary to subtract \( \alpha \phi f_c' \) from \( \phi f_s' \) before computing \( C_{rs} \):

\[ C_{rs} = (\phi_s \times f_s' - \alpha \times f_c \times f_c') \times A_{s2} = (0.85 \times 400 - 0.798 \times 0.6 \times 35) \times 2800 = 905 \text{ kN} \]

4.3. \( P_r \) and \( M_r \)

\[ P_r = C_{rc} + C_{rs} - T_{rs} = 1406 + 901 - 952 = 1355 \text{ kN} \]

\[ M_r = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left( \frac{h}{2} - d_2 \right) + T_{rs} \times \left( d_1 - \frac{h}{2} \right) \]

\[ M_r = 1298 \times \left( \frac{400}{2} - \frac{194}{2} \right) + 905 \times \left( \frac{400}{2} - 55 \right) + 952 \times \left( 345 - \frac{400}{2} \right) = 403 \text{ kN.m} \]
5. Pure Bending

This corresponds to the case where the factored axial load resistance, $P_r$, is equal to zero. Iterative procedure is used to determine the factored moment resistance as follows:

5.1. $c$, $a$, and strains in the reinforcement

Try $c = 78.55$ mm

Where $c$ is depth of the neutral axis measured from the compression edge of the column section.

\[ a = \beta c = 0.883 \times 78.55 = 69 \text{ mm} \]

Where:

\[ \beta = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \]

\[ \varepsilon_{cu} = 0.0035 \]

\[ \varepsilon_y = \frac{F}{E_s} = \frac{400}{200,000} = 0.002 \]

\[ \varepsilon_s = (d_i - c) \times \frac{\varepsilon_{cu}}{c} = (345 - 78.55) \times \frac{0.0035}{78.55} = 0.01187 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded} \]

\[ \phi_c = 0.6 \]

\[ \phi_s = 0.85 \]

\[ \varepsilon'_s = (c - d_j) \times \frac{\varepsilon_{cu}}{c} = (78.55 - 55) \times \frac{0.0035}{78.55} = 0.00105 \text{ (Compression)} < \varepsilon_y \]

5.2. Forces in the concrete and steel

\[ C_{rc} = \alpha_c \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 69 \times 400 = 474 \text{ kN} \]

\[ f_s = f'_s = 400 \text{ MPa} \]
\[ T_{rs} = \phi_f \times f_s \times A_{s1} = 0.85 \times 400 \times 2800 = 952 \text{ kN} \]

Since \( \varepsilon_s < \varepsilon_y \rightarrow \) compression reinforcement has not yielded

\[ \therefore f_s = \varepsilon_s \times E_s = 0.00105 \times 200,000 = 210 \text{ MPa} \]

The area of the reinforcement in this layer has been included in the area \((ab)\) used to compute \( C_c \). As a result, it is necessary to subtract \( \alpha \phi f_s \) from \( \phi f_s \) before computing \( C_{rs} \):

\[ C_{rs} = (\phi \times f_s - \alpha \times \phi \times f_s') \times A_{s2} = (0.85 \times 210 - 0.798 \times 0.6 \times 35) \times 2800 = 477 \text{ kN} \]

5.3. \( P_r \) and \( M_r \)

\[ P_r = C_{rc} + C_{rs} - T_{rc} = 474 + 477 - 952 \approx 0 \text{ kN} \]

The assumption that \( c = 78.55 \text{ mm} \) is correct

\[ M_r = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left( \frac{h}{2} - d_s \right) + T_{rc} \times \left( d_i - \frac{h}{2} \right) \]

\[ M_r = 474 \times \left( \frac{400}{2} - \frac{69}{2} \right) + 477 \times \left( \frac{400}{2} - 55 \right) + 952 \times \left( 345 - \frac{400}{2} \right) = 285 \text{ kN.m} \]
6. **Pure Tension**

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

6.1. **Strength under pure axial tension** \( (P_{rt}) \)

\[
P_{rt} = \phi \times f_y \times (A_{s1} + A_{s2}) = 0.85 \times 400 \times (2800 + 2800) = 1904 \text{ kN}
\]

6.2. **Corresponding Moment** \( (M_{rt}) \)

Since the section is symmetrical

\[
M_{rt} = 0 \text{ kN.m}
\]
spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with control points using the CSA A23.3-94. In lieu of using program shortcuts, spSection (Figure 9) was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

Figure 7 – Generating spColumn Model
Figure 8 – spColumn Model Editor (spSection)
Figure 9 – Column Section Interaction Diagram about the X-Axis (spColumn)
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# 1. General Information

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# 2. Material Properties

## 2.1. Concrete

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## 2.2. Steel

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# 3. Section

## 3.1. Shape and Properties

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3.2. Section Figure

![Section Figure](image)

Figure 1: Column section

4. Reinforcement

4.1. Bar Set: CSA G30.18

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<th>Diameter (mm)</th>
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4.2. Confinement and Factors

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Material Resistance Factors

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<td>Concrete (Φ_c)</td>
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Minimum dimension, h: 400 mm

4.3. Arrangement

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<td>Cover to</td>
<td>Longitudinal bars</td>
</tr>
<tr>
<td>Clear cover</td>
<td>---</td>
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</table>
4.4. Bars Provided

<table>
<thead>
<tr>
<th></th>
<th>Bars</th>
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<tbody>
<tr>
<td>Top</td>
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<td>#30</td>
</tr>
<tr>
<td>Bottom</td>
<td>4</td>
<td>#30</td>
</tr>
<tr>
<td>Left</td>
<td>0</td>
<td>#10</td>
</tr>
<tr>
<td>Right</td>
<td>0</td>
<td>#10</td>
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</tbody>
</table>

5. Control Points

<table>
<thead>
<tr>
<th>About Point</th>
<th>P (kN)</th>
<th>X-Moment (kNm)</th>
<th>Y-Moment (kNm)</th>
<th>NA Depth (mm)</th>
<th>d (Depth) (mm)</th>
<th>ε₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>X @ Max compression</td>
<td>4705.3</td>
<td>0.00</td>
<td>0.00</td>
<td>805</td>
<td>345</td>
<td>-0.00200</td>
</tr>
<tr>
<td>X @ Allowable comp.</td>
<td>3764.2</td>
<td>146.16</td>
<td>0.00</td>
<td>412</td>
<td>345</td>
<td>-0.00057</td>
</tr>
<tr>
<td>X @ fₚ = 0.0</td>
<td>3111.1</td>
<td>236.23</td>
<td>0.00</td>
<td>345</td>
<td>345</td>
<td>0.00000</td>
</tr>
<tr>
<td>X @ fₚ = 0.5 fₚ</td>
<td>2144.0</td>
<td>339.98</td>
<td>0.00</td>
<td>268</td>
<td>345</td>
<td>0.00100</td>
</tr>
<tr>
<td>X @ Balanced point</td>
<td>1355.5</td>
<td>413.81</td>
<td>0.00</td>
<td>220</td>
<td>345</td>
<td>0.00200</td>
</tr>
<tr>
<td>X @ Pure bending</td>
<td>0.0</td>
<td>286.39</td>
<td>0.00</td>
<td>79</td>
<td>345</td>
<td>0.01188</td>
</tr>
<tr>
<td>X @ Max tension</td>
<td>-1904.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
<td>345</td>
<td>9.99999</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>About Point</th>
<th>P (kN)</th>
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<th>Y-Moment (kNm)</th>
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<th>d (Depth) (mm)</th>
<th>ε₀</th>
</tr>
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<td>412</td>
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<td>-0.00057</td>
</tr>
<tr>
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<td>345</td>
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<td>345</td>
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<td>0.01188</td>
</tr>
<tr>
<td>X @ Max tension</td>
<td>-1904.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
<td>345</td>
<td>9.99999</td>
</tr>
</tbody>
</table>
8. Summary and Comparison of Design Results

<table>
<thead>
<tr>
<th>Support</th>
<th>$P$, kN</th>
<th>$M$, kN.m</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>Reference*</td>
</tr>
<tr>
<td>Max compression</td>
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<td>4490</td>
</tr>
<tr>
<td>Allowable compression</td>
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<tr>
<td>$f_s = 0.0$</td>
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<tr>
<td>$f_s = 0.5 f_y$</td>
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<td>2015</td>
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<tr>
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<td>Pure bending</td>
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<td>0</td>
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<tr>
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<td>1904</td>
<td>1904</td>
</tr>
</tbody>
</table>


In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.
9. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where $f_s = f_y$).

![Strains, Forces, and Moment Arms](Figure 10 – Strains, Forces, and Moment Arms ($f_s = -f_y$, Moments About x- and y-axis))
When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure.

![Interaction Diagram About the x-axis](image1.png) ![Interaction Diagram About the y-axis](image2.png)

**Figure 11 – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)**

Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to \( M_x \) and \( M_y \) due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint’s [spColumn](#) program can also evaluate column sections in biaxial mode to produce the results shown in the following Figure for the column section in this example.
Figure 12 – Nominal & Design Interaction Diagram in Two Directions (Biaxial) (spColumn)