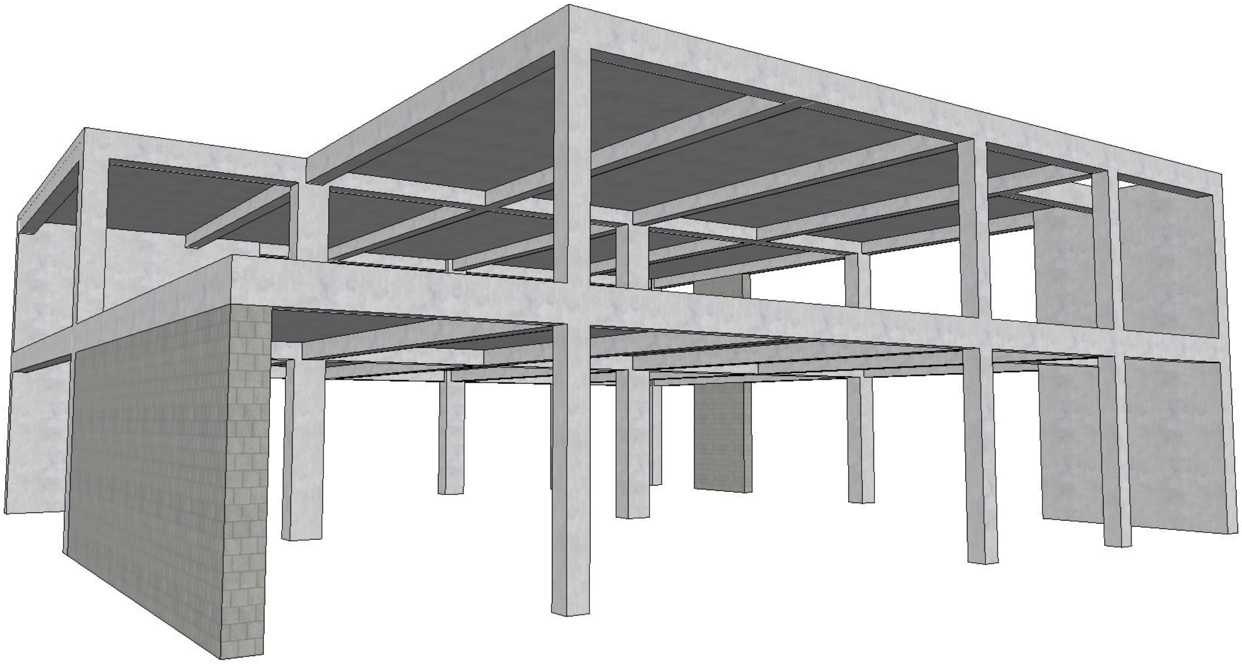
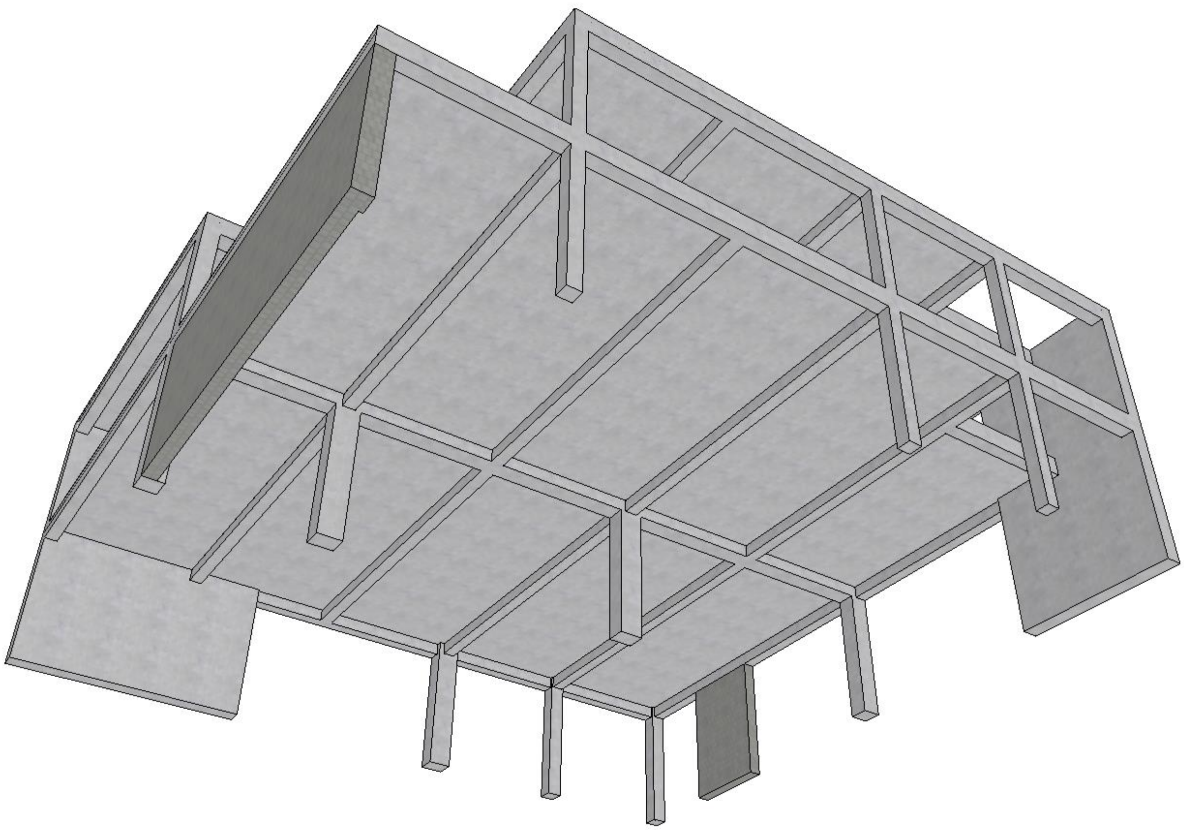
Reinforced Concrete Continuous Beam Analysis and Design (ACI 318-14)





Reinforced Concrete Continuous Beam Analysis and Design (ACI 318-14)

A structural reinforced concrete continuous beams at an intermediate building floor provides gravity load resistance for the applied dead and live loads.

The continuous beam along grid 3 is selected to demonstrate the analysis and design of continuous T-beams (structural analysis, flexural design, shear design, deflection checks) and the results of hand calculations are then compared with numerical analysis results obtained from the spBeam engineering software program.

Additionally, the boundary conditions for each T-beam (Grids 1 thru 6 and A thru C) are selected to demonstrate and explore in detail the actual interaction between the beam and the supporting members. Similar evaluation is performed using computer software to reflect recommended modeling procedures in spBeam to obtain the most accurate results.

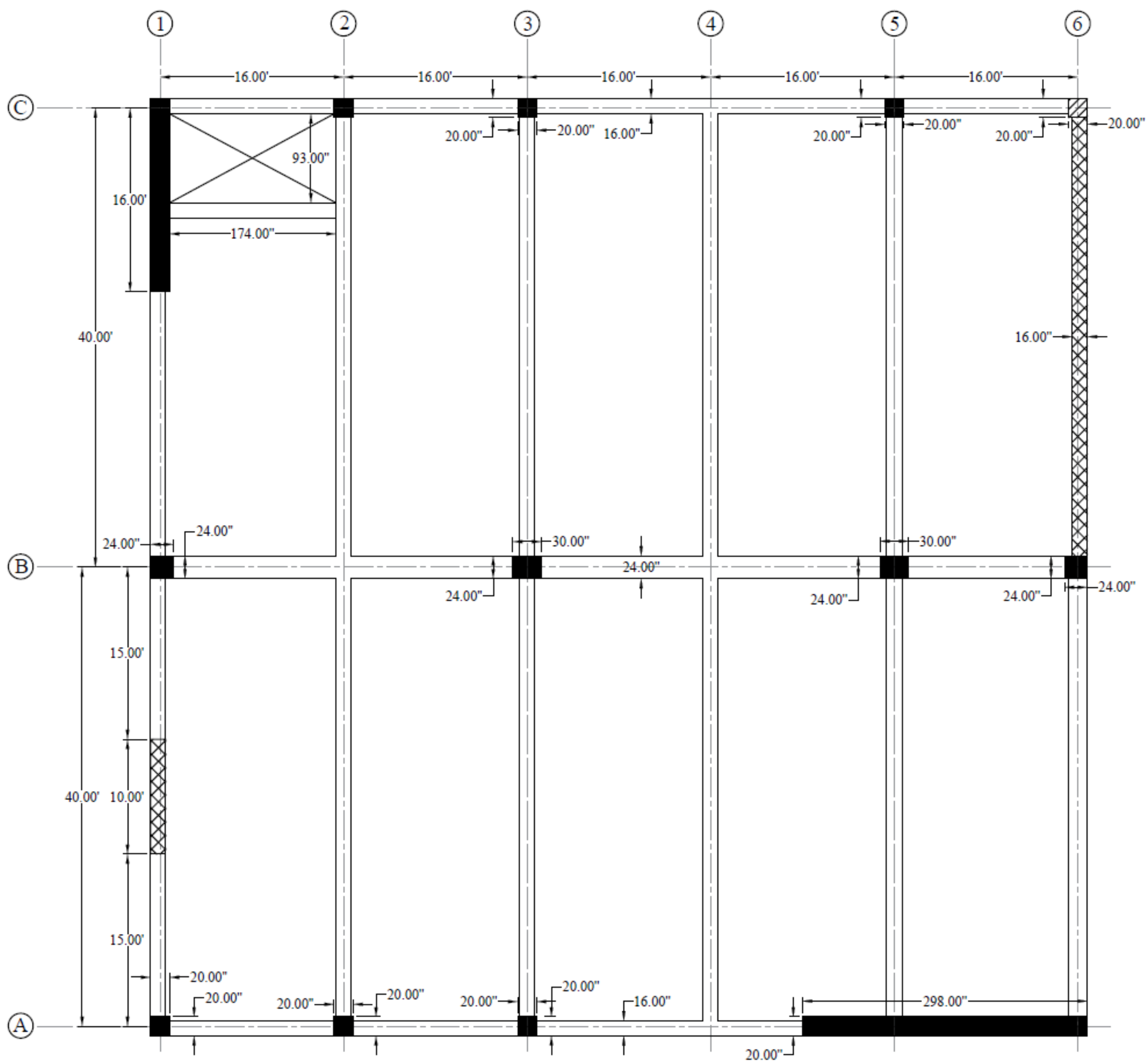


Figure 1 – Reinforced Concrete Continuous Beams at intermediate building floor

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association

Reinforced Concrete Mechanics and Design, 7th Edition, 2016, James Wight, Pearson

Reinforced Concrete Structures, 2nd Edition, 2016, David A. Fanella, McGraw-Hill Education

spBeam Engineering Software Program Manual v5.00, STRUCTUREPOINT, 2015

Design Data

f_c' = 4,000 psi normal weight concrete (w_c = 150 pcf)

f_y = 60,000 psi

Superimposed dead load, SDL = 20 psf framed partitions, wood studs plaster 2 sides

ASCE/SEI 7-10 (Table C3-1)

Typical Floor Level, Live load, L_o = 50 psf (Office building) ***ASCE/SEI 7-10 (Table 4-1)***

Solution

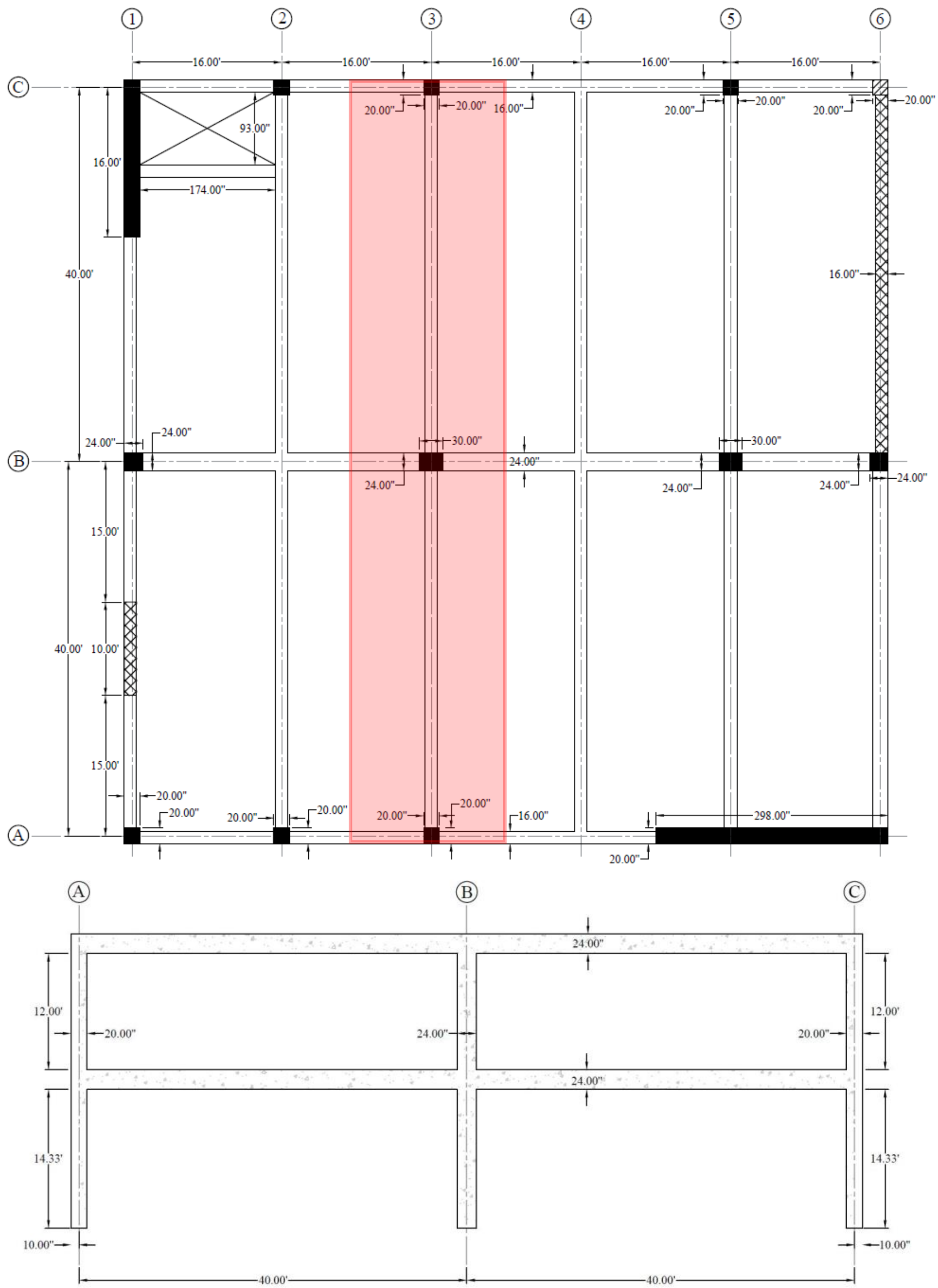


Figure 2 – Reinforced Concrete Continuous Beam (Grid 3)

1. Preliminary Member Sizing

Check the minimum beam depth requirement of ***ACI 318-14 (Table 9.3.1.1)*** to waive deflection computations. Using the minimum depth for non-prestressed beams in ***Table 9.3.1.1***.

End Span: $h = \dfrac{l}{18.5} = \dfrac{40 \times 12}{18.5} = 25.94 \text{ in.}$ ***ACI 318-14 (Table 9.3.1.1)***

Therefore, since h_{min} = 25.94 in. > h = 24 in., the preliminary beam depth does not satisfy the minimum depth requirement, and the beam deflection need to be checked.

2. Load and Load combination

Live Load, L: Calculate the live load reduction per ***ASCE/SEI 7-10***

$$L = L_o \times \left(0.25 + \frac{15}{\sqrt{K_{LL} A_T}}\right)$$ ***ASCE/SEI 7-10 (Eq. 4-1)***

Where:

L = reduced design live load per ft^2 of area supported by the member

L_o = unreduced design live load per ft^2 of area supported by the member = 50 psf

K_{LL} = live load element factor = 2 for interior beam ***ASCE/SEI 7-10 (Table 4-2)***

$A_T = \text{tributary area} = 40 \times 16 = 640 \text{ ft}^2$

$$L = 50 \times (0.25 + \frac{15}{\sqrt{2 \times 640}}) = 33.46 \text{ psf}$$

Which satisfies 0.5 x L_o requirement for members supporting one floor. ***ASCE/SEI 7-10 (4.7.2)***

33.46 psf > 0.5 x L_o = 25 psf

For the factored Load

$U = 1.2D + 1.6L$ ***ACI 318-14 (Eq. 5.3.1b)***

$$D = 0.150 \times \left(\frac{8}{12} \times 16 + \frac{24-8}{12} \times \frac{16}{12}\right) + 0.02 \times 16 = 2.19 \text{ kips/ft}$$

$$L = 0.03346 \times 16 = 0.54 \text{ kips/ft}$$

$$w_u = 1.2 \times 2.19 + 1.6 \times 0.54 = 3.48 \text{ kips/ft}$$

3. Structural Analysis by Moment Distribution

The T-beam will be analyzed by hand using the moment distribution method to determine design moment and shear values. Members stiffnesses, carry over factors COF, and fixed-end moments FEM for the beam and column members are determined as follows:

3.1. Flexural stiffness of beams, K_b

$$K_b = 4 \times \frac{E_c I_b}{L_b} = 4 \times \frac{3834.3 \times 40,634}{40 \times 12} = 1,298,357 \text{ kip-in.}$$

Where I_b is calculated for the beam as a T-beam section with b_{eff} equals to:

$$b_{eff} = \min \left\{ \begin{array}{l} b+2\times 8\times h = 16+2\times 8\times 8 = 144 \text{ in.} \\ b+2\times \dfrac{s_w}{2} = 16+2\times \dfrac{16\times 12-16/2-16/2}{2} = 192 \text{ in.} \\ b+2\times \dfrac{l_n}{8} = 16+2\times \dfrac{40\times 12-20/2-24/2}{8} = 130.5 \text{ in.} \end{array} \right\} = 130.5 \text{ in.}$$

ACI 318-14 (Table 6.3.2.1)

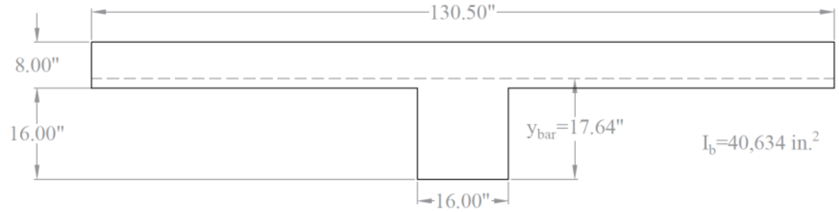


Figure 3 – Moment of inertia calculation for T-beam section

$$E_c = w_c^{1.5} \times 33\times\sqrt{f_c'} = 150^{1.5}\times 33\times\sqrt{4000} = 3834 \text{ ksi}$$

ACI 318-14 (19.2.2.1.a)

Carry-over factor COF = 0.5

$$\text{Fixed-end moment, } FEM = \frac{w_u \times L_b^2}{12} = \frac{3.48\times 40^2}{12} = 464.08 \text{ kip-ft}$$

3.2. Flexural stiffness of column members, K_c

For the Top Exterior Column:

$$COF_{c,top} = 0.50$$

$$K_{c,top} = 4\times\frac{E_c I_c}{L_c} = 4\times\frac{3834.3\times 13,333}{13\times 12} = 1,310,872 \text{ kip-in.}$$

$$\text{Where } I_c = \frac{c_2\times c_1^3}{12} = \frac{20\times 20^3}{12} = 13,333 \text{ in.}^4$$

$$E_c = w_c^{1.5} \times 33\times\sqrt{f_c'} = 150^{1.5}\times 33\times\sqrt{4000} = 3834 \text{ ksi}$$

ACI 318-14 (19.2.2.1.a)

L_c = 13 ft

For the Bottom Exterior Column:

$$COF_{c,bottom} = 0.50$$

$$K_{c,bottom} = 4\times\frac{E_c I_c}{L_c} = 4\times\frac{3834.3\times 13,333}{15.33\times 12} = 1,111,633 \text{ kip-in.}$$

$$\text{Where } I_c = \frac{c_2\times c_1^3}{12} = \frac{20\times 20^3}{12} = 13,333 \text{ in.}^4$$

L_c = 15.33 ft

<u>For the Top Interior Column:</u>

$$COF_{c,top} = 0.50$$

$$K_{c,top} = 4 \times \frac{E_c I_c}{L_c} = 4 \times \frac{3834.3 \times 34,560}{13 \times 12} = 3,397,780 \text{ kip-in.}$$

$$\text{Where } I_c = \frac{c_2 \times c_1^3}{12} = \frac{30 \times 24^3}{12} = 34,560 \text{ in.}^4$$

L_c = 13 ft

<u>For the Bottom Interior Column:</u>

$$COF_{c,bottom} = 0.50$$

$$K_{c,bottom} = 4 \times \frac{E_c I_c}{L_c} = 4 \times \frac{3834.3 \times 34,560}{15.33 \times 12} = 2,881,353 \text{ kip-in.}$$

$$\text{Where } I_c = \frac{c_2 \times c_1^3}{12} = \frac{30 \times 24^3}{12} = 34,560 \text{ in.}^4$$

L_c = 15.33 ft

3.3. Beam joint distribution factors, DF

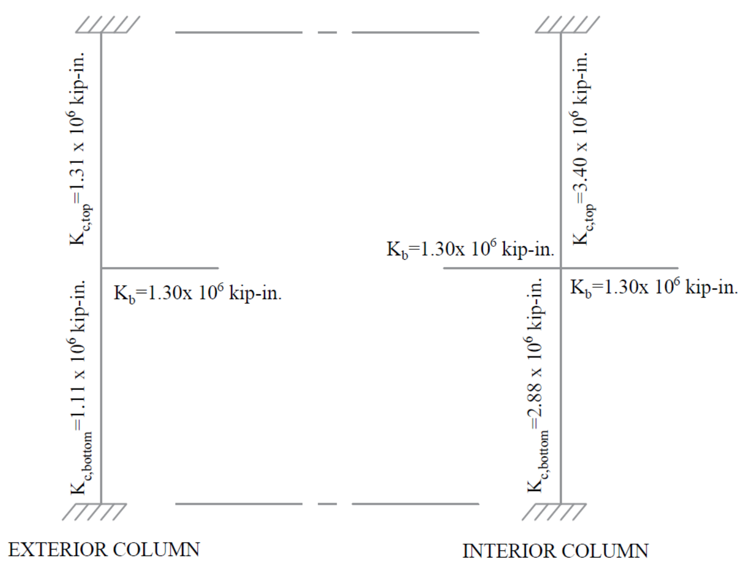
At exterior joint,

$$DF = \frac{1,298,357}{(1,298,357 + 1,310,872 + 1,111,633)} = 0.350$$

At interior joint,

$$DF = \frac{1,298,357}{(1,298,357 + 1,298,357 + 3,397,780 + 2,881,353)} = 0.150$$

COF for beams =0.5



<u>Figure 4 – beams and Columns Stiffnesses</u>

3.4. Moment Distribution

Determine negative and positive moments for the continuous beam using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. ***ACI 318-14 (6.4.3.2)***

$$\frac{L}{D} = \frac{0.54}{2.19} = 0.25 < \frac{3}{4}$$

Moment Distribution computations are shown in the following Table. Counterclockwise rotational moments acting on the member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_{u,midspan} = M_o - \frac{(M_{uL} + M_{uR})}{2}$$

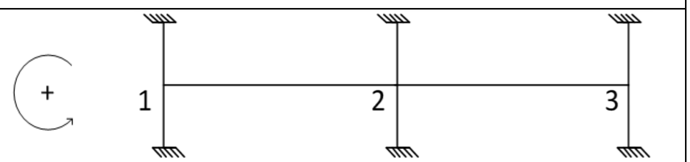
Where M_o is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example (see results comparison section).

Positive moment in span 1-2:

$$M_u = 3.48 \times \frac{40^2}{8} - \frac{302.15 + 545.04}{2} = 272.52 \text{ kip-ft}$$

Table 1 - Moment Distribution for Continuous Beam



Joint	1	2		3
Member	1-2	2-1	2-3	3-2
DF	0.35	0.15	0.15	0.35
COF	0.50	0.50	0.50	0.50
FEM	464.08	-464.08	464.08	-464.08
Dist	-161.93	0.00	0.00	161.93
CO	0.00	-80.97	80.97	0.00
Dist	0.00	0.00	0.00	0.00
M, k-ft	302.15	-545.04	545.04	-302.15
Midspan M, k-ft	272.52		272.52	

The ACI code allows the reduction of factored moments calculated by elastic theory at sections of maximum negative or maximum positive moment in any span of continuous flexural members for any assumed loading arrangement by a percentage equal to 1000 ε_t up to a maximum of 20 percent. ***ACI 318-14 (6.6.5.3)***

The moment redistribution is often utilized for the investigation of existing structures for conditions such as change of use, additional loading, or verifying adequacy for the latest design code. In these conditions, any reserve capacity from existing reinforcement layout at mid-span (or support) of a span may be utilized to compensate for the inadequacy of the support (or mid-span) of the same span.

The moment redistribution can also be utilized in the design of a new structure. One such example of its application may help reduce the negative moment at an interior support and corresponding top reinforcement while increasing the positive moment at mid-span. The advantage of this may be the alleviation of the congestion of rebar at support top regions.

The calculation of moment redistribution is a tedious process especially while considering live load patterning. The procedure gets far more complicated if point loads or partial line loads are present. The spBeam software program performs the moment redistribution calculations with speed and accuracy.

This example does not cover the moment redistribution. However, a detailed demonstration of this method can be found in "Continuous Beam Design with Moment Redistribution (ACI 318-14)" example.

3.5. Factored moments used for Design

Positive and negative factored moments for the continuous beam are plotted in the following Figure. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than 0.175 l from the centers of supports.

ACI 318-14 (8.11.6.1)

$$\frac{24 \text{ in.}}{12\times 2} = 1.00 \text{ ft} < 0.175\times 40 = 7.00 \text{ ft (use face of supporting location for interior column)}$$

$$\frac{20 \text{ in.}}{12\times 2} = 0.83 \text{ ft} < 0.175\times 40 = 7.00 \text{ ft (use face of supporting location for exterior column)}$$

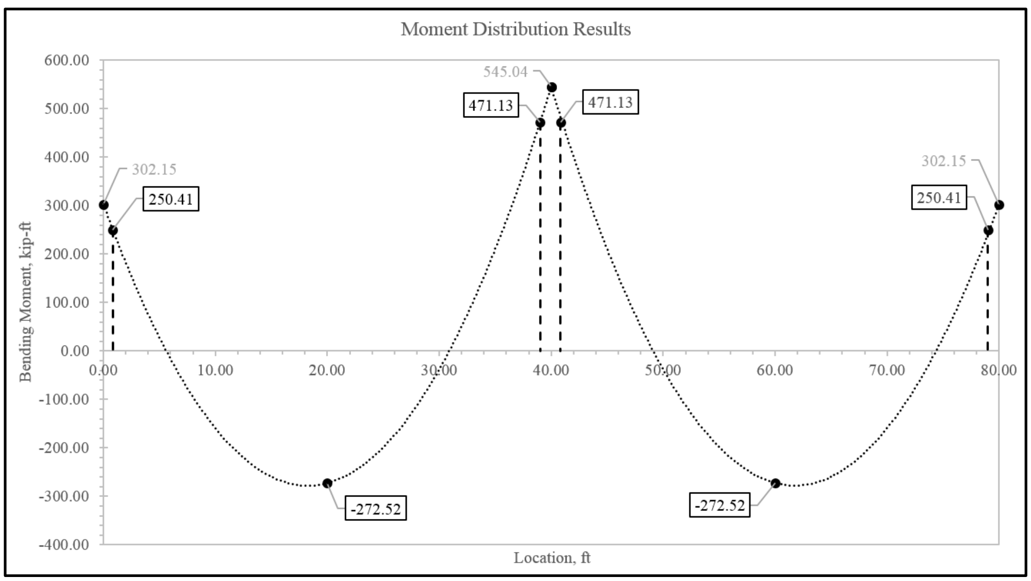


Figure 5 - Positive and Negative Design Moments for the Continuous Beam

4. Flexural Design

For this beam, the moment at the exterior face of the interior support governs the design as shown in the previous Figure.

Calculate the required reinforcement to resist the interior support negative moment:

$$M_u = 471.13 \text{ kip-ft}$$

Use #8 bars with 1.5 in. concrete cover per ***ACI 318-14 (Table 20.6.1.3.1)***. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, *d*, is calculated below:

$$d = 24 - \left(1.5 + 0.5 \times \frac{8}{8}\right) = 22 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the beam section (*jd*). In this example, tension-controlled section will be assumed so the reduction factor φ is equal to 0.9, and *jd* will be taken equal to *0.862d*. The assumptions will be verified once the area of steel is finalized.

$$jd = 0.862 \times d = 0.862 \times 22 = 18.96 \text{ in.}$$

$$b = 16 \text{ in.}$$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_u}{\varphi \times f_y \times jd} = \frac{471.13 \times 12{,}000}{0.9 \times 60{,}000 \times 18.96} = 5.52 \text{ in.}^2$$

Recalculate '*a*' for the actual A_s = 5.52 in.^{2}: $a = \dfrac{A_s \times f_y}{0.85 \times f'_c \times b} = \dfrac{5.52 \times 60{,}000}{0.85 \times 4{,}000 \times 16} = 6.09 \text{ in.}$

$$c = \frac{a}{\beta_1} = \frac{6.09}{0.85} = 7.17 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) \times d_t - 0.003 = \left(\frac{0.003}{7.17}\right) \times 22 - 0.003 = 0.0062 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\varphi \times f_y \times (d - a/2)} = \frac{471.13 \times 12{,}000}{0.9 \times 60{,}000 \times (22 - 6.09/2)} = 5.52 \text{ in.}^2$$

The minimum reinforcement shall not be less than

$$A_{s,\min} = \frac{3 \times \sqrt{f'_c}}{f_y} \times b_w \times d = \frac{3\sqrt{4{,}000}}{60{,}000} \times 16 \times 22 = 1.11 \text{ in.}^2$$ ***ACI 318-14 (9.6.1.2(a))***

And not less than

$$A_{s,\min} = \frac{200}{f_y} \times b_w \times d = \frac{200}{60{,}000} \times 16 \times 22 = 1.17 \text{ in.}^2$$ ***ACI 318-14 (9.6.1.2(b))***

$$\therefore A_{s,\min} = 1.17 \text{ in.}^2$$

Provide 7 - # 8 bars:

$$A_{s,prov} = 7 \times 0.79 = 5.53 \text{ in.}^2 > A_{s,req} = 5.52 \text{ in.}^2 \qquad \textbf{\textit{o.k.}}$$

The reinforcement is selected based on the closest $A_{s,provided}$ to $A_{s,required}$. However, moment redistribution can be used to reduce the A_s (section 3.4). Furthermore, other bar size and detailing options can be selected (e.g. 13 #6 or 10 #7 in one or two layers) to overcome any construction issues or preferences.

<u>Maximum spacing allowed:</u>

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

$$s = 15\left(\frac{40000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40000}{f_s}\right)$$ ***<u>ACI 318-14 (Table 24.3.2)</u>***

$$c_c = 1.5 \text{ in.}$$

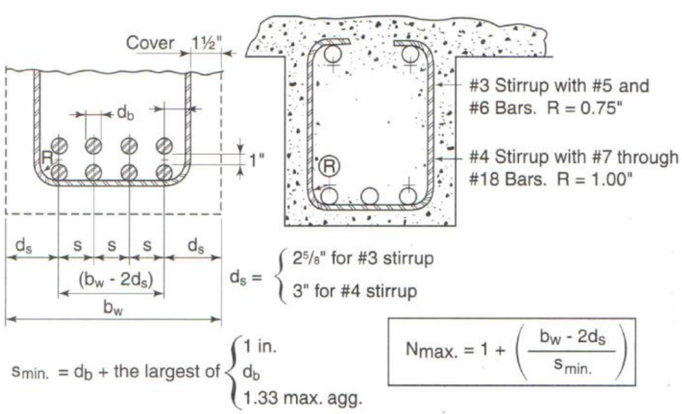
Use $f_s = \frac{2}{3} f_y = 40,000$ psi ***<u>ACI 318-14 (24.3.2.1)</u>***

$$s = 15 \times \left(\frac{40,000}{40,000}\right) - 2.5 \times 1.5 = 11.25 \text{ in. (Governs)}$$

$$s = 12 \times \left(\frac{40,000}{40,000}\right) = 12 \text{ in.}$$

$$s_{provided} = \frac{(b - 2 \times d_s)}{\#of\ bars - 1} = \frac{(45.8 - 2 \times 3)}{7 - 1} = 6.6 \text{ in.} < 11.25 \text{ in.} \qquad \textbf{\textit{o.k.}}$$

Where d_s = 3.0 in. for #4 stirrup as shown in the following Figure. ***<u>CRSI 2002 (Figure 12-9)</u>***



<u>Figure 6 - Maximum number of bars in beams</u>

If flanges of T-beams are in tension, part of the bonded flexural tension reinforcement shall be distributed over an effective flange width, but not wider than $l_n/10$. Where l_n is the length of clear span measured face-to-face of supports. ***<u>ACI 318-14 (24.3.2.1)</u>***

$$b = \min \begin{Bmatrix} b_{eff} = 130.5 \text{ in.} \\ \dfrac{l_n}{10} = \dfrac{40\times12-20/2-24/2}{10} = 45.8 \text{ in.} \end{Bmatrix} = 45.8 \text{ in.}$$

Check if $s_{provided}$ is greater than the minimum center to center spacing, s_{min} where

$$s_{\min} = d_b + \max \begin{Bmatrix} 1 \\ d_b \\ 1.33\times \max .agg. \end{Bmatrix}$$

CRSI 2002 (Figure 12-9)

Where the maximum aggregate size is ¾"

$$s_{\min} = 1.00 + \max \begin{Bmatrix} 1.00 \\ 1.00 \\ 1.33\times 0.75 = 1.00 \end{Bmatrix} = 1.00 + 1.00 = 2 \text{ in.}$$

Since the spacing provided is greater than 2 in. Therefore, 7-#8 bars are ***o.k.***

All the values in the following table are calculated based on the procedure outlined above.

Table 2 – Reinforcing Design Summary			
	End Span		
	Exterior Negative	Positive	Interior Negative
Design Moment, M_u (ft-kips)	250.41	272.52	471.13
Effective depth, d (in.)	22	22	22
$A_{s,req}$ (in.^{2})	2.71	2.78	5.52
$A_{s,min}$ (in.^{2})	1.17	1.17	1.17
Reinforcement	5-#8*	4-#8	7-#8
Spacing provided (in.)	10.0	3.3	6.6
* Number of bars governed by maximum allowable spacing			

5. Shear Design

From the following Figure, the shear value in end span at face of the interior support governs.

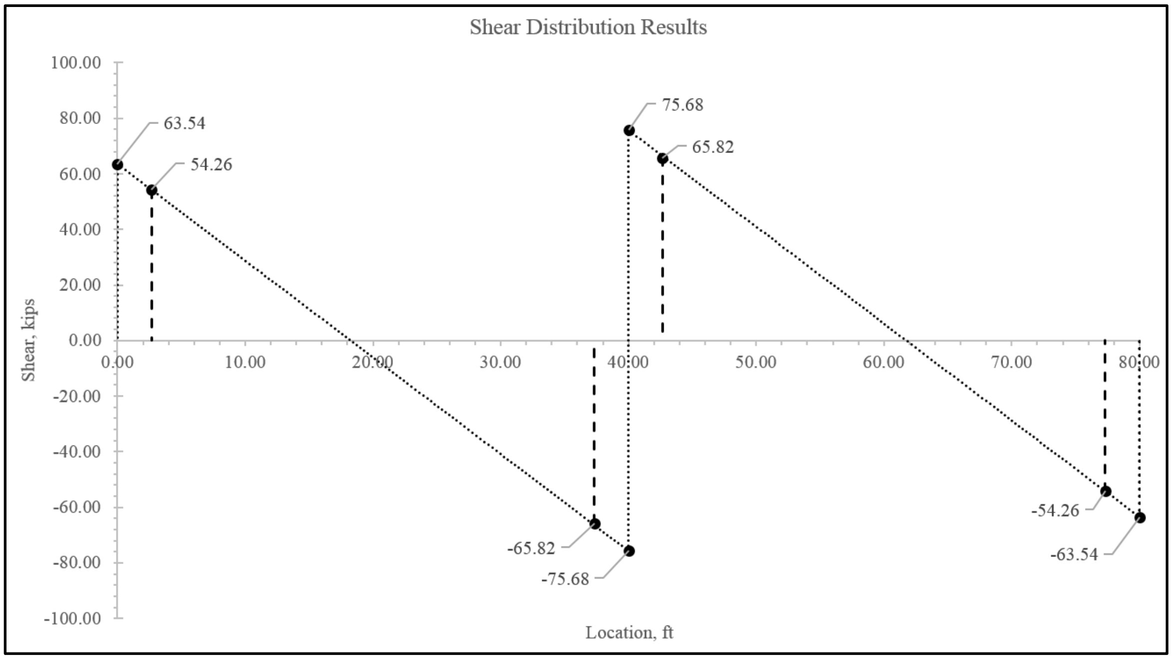


Figure 7 - Positive and Negative Design Shears for the Continuous Beam

$$V_u = \frac{-545.04 + 302.15 + \dfrac{w_u \times l^2}{2}}{l} = -75.68 \text{ kips}$$

The design shear at a distance, d, away from the face of support,

$$V_u = 75.68 - 3.48 \times \frac{24/2 + 22}{12} = 65.82 \text{ kips}$$

Shear strength provided by concrete

$$\phi V_c = \phi \times 2 \times \sqrt{f_c^{'}} \times b_w \times d$$ **ACI 318-14 (Eq. 22.5.5.1)**

$$\phi V_c = 0.75 \times 2 \times \sqrt{4000} \times 16 \times 22 = 33.39 \text{ kips}$$

Since $V_u > \phi V_c/2$, shear reinforcement is required.

Try # 4, Grade 60 two-leg stirrups (A_v = 0.40 in.^{2}).

The nominal shear strength required to be provided by shear reinforcement is

$$V_s = V_n - V_c = \frac{V_u}{\phi} - \frac{\phi V_c}{\phi} = \frac{65.82}{0.75} - \frac{33.39}{0.75} = 43.24 \text{ kips}$$

If V_s is greater than $8\sqrt{f_c'}b_w d$, then the cross-section has to be revised as ***ACI 318-14*** limits the shear capacity to be provided by stirrups to $8\sqrt{f_c'}b_w d$. **ACI 318-14 (22.5.1.2)**

$$8 \times \sqrt{f_c'} \times b_w \times d = 8 \times \sqrt{4,000} \times 16 \times 22 = 178.1 \text{ kips} \rightarrow \therefore \text{section is adequate}$$

$$\left(\frac{A_v}{s}\right)_{req} = \frac{V_u - \phi V_c}{\phi \times f_{yt} \times d} = \frac{65.82 - 33.39}{0.75 \times 60 \times 22} = 0.0328 \text{ in.}^2/\text{ in.}$$ **ACI 318-14 (22.5.10.5.3)**

$$s_{req} = \frac{A_v}{\left(\frac{A_v}{s}\right)_{req}} = \frac{0.40}{0.0328} = 12.21 \text{ in.}$$

$$\left(\frac{A_v}{s}\right)_{\min} = \max\begin{Bmatrix} \dfrac{0.75 \times \sqrt{f_c'} \times b_w}{f_{yt}} \\ \dfrac{50 \times b_w}{f_{yt}} \end{Bmatrix}$$ **ACI 318-14 (10.6.2.2)**

$$\left(\frac{A_v}{s}\right)_{\min} = \max\begin{Bmatrix} \dfrac{0.75 \times \sqrt{4000} \times 16}{60000} \\ \dfrac{50 \times 16}{60000} \end{Bmatrix} = \max\begin{Bmatrix} 0.0126 \\ 0.0133 \end{Bmatrix} = 0.0133 \text{ in.}^2/\text{in.} < \left(\frac{A_v}{s}\right)_{req} = 0.0328 \text{ in.}^2/\text{ in.}$$

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ***ACI 318-14 (9.7.6.2.2)***.

$$4 \times \sqrt{f_c'} \times b_w \times d = 4 \times \sqrt{4,000} \times 16 \times 22 = 89.05 \text{ kips} > V_s = 43.24 \text{ kips}$$

Therefore, maximum stirrup spacing shall be the smallest of *d/2* and *24 in.* **ACI 318-14 (Table 9.7.6.2.2)**

$$s_{\max} = lesser\ of \begin{bmatrix} d/2 \\ 24 \text{ in.} \end{bmatrix} = lesser\ of \begin{bmatrix} 22/2 \\ 24 \text{ in.} \end{bmatrix} = lesser\ of \begin{bmatrix} 11 \text{ in.} \\ 24 \text{ in.} \end{bmatrix} = 11 \text{ in.}$$

This value governs over the required stirrup spacing of 12.21 in which was based on the demand.

Therefore, s_{max} value is governed by the spacing limit per ***ACI 318-14 (9.7.6.2.2)***, and is equal to 11 in.

Use # 4 @ 10.5 in. stirrups

$$\phi V_n = \frac{\phi \times A_v \times f_{yt} \times d}{s} + \phi V_c$$ **ACI 318-14 (22.5.1.1 and 22.5.10.5.3)**

$$\phi V_n = \frac{0.75 \times 0.40 \times 60 \times 22}{10.5} + 33.39 = 37.71 + 33.39 = 71.11 \text{ kips} > V_u = 65.82 \text{ kips}$$ ***o.k.***

Compute where $\frac{V_u}{\varphi}$ is equal to $\frac{V_c}{2}$, and the stirrups can be stopped

$$x = \frac{\frac{V_u}{\phi} - \frac{V_c}{2}}{\frac{V_u}{\phi}} \times \frac{l}{2} = \frac{\frac{75.68}{0.75} - \frac{33.39}{0.75 \times 2}}{\frac{75.68}{0.75}} \times \frac{40 \times 12}{2} = 187 \text{ in.}$$

At interior end of the exterior span, use 19-# 4 @ 10.5 in. o.c., Place 1st stirrup 3 in. from the face of the column.

6. Deflection Control (Serviceability Requirements)

Since the preliminary beam depth did not meet minimum depth requirement, the deflection calculations are required. The calculation of immediate and time-dependent deflections are covered in detail in this section for illustration and comparison with spBeam model results for continuous T-beam.

6.1. Immediate (Instantaneous) Deflections

Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{full}$) is used to obtain immediate deflections of the continuous T-beam in this example. Other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. ***ACI 318-14 (24.2.3)***

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the beam. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g$$

ACI 318-14 (Eq. 24.2.3.5a)

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by Eq. (24.2.3.5a) unless obtained by a more comprehensive analysis.

I_e shall be permitted to be taken as the value obtained from Eq. (24.2.3.5a) at midspan for simple and continuous spans, and at the support for cantilevers. ***ACI 318-14 (24.2.3.7)***

For continuous one-way slabs and beams. I_e shall be permitted to be taken as the average of values obtained from Eq. (24.2.3.5a) for the critical positive and negative moment regions. ***ACI 318-14 (24.2.3.6)***

$$I_{e,avg} = 0.85\, I_e^{+} + 0.15\, I_e^{-} \text{ for end span}$$

spBeam Manual v5.00 (Eq. 2-104)

$$I_{e,avg} = 0.70\, I_e^{+} + 0.15\left(I_{e1}^{-} + I_{e2}^{-}\right) \text{ for end span}$$

spBeam Manual v5.00 (Eq. 2-103)

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously. The following Figure shows the maximum moments for the total service load level.

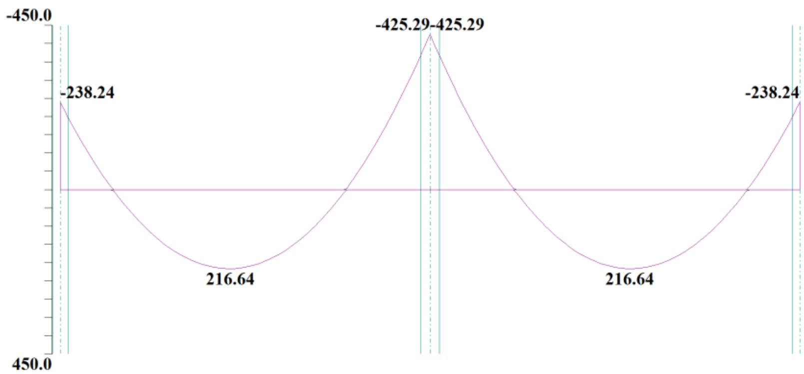


Figure 8 – Maximum Moments for the Total Service Load Level

For the negative moment region adjacent to the internal support:

The following calculations are based on considering the negative moment regions as rectangular sections. For T-beam, a T-shaped section can also be used to calculate I_{cr} in the negative moment regions. This approach will impact the calculations of I_g and M_{cr} where it has a slight impact on the instantaneous deflection results since the averaged effective moment of inertia is predominantly dependent on the properties of the mid-span region as can be seen later in this example. The hand calculations are continued with rectangular sections considered at the negative moment regions. A comparison between the two approaches is presented in "Design Results Comparison and Conclusions" section.

M_{cr} = cracking moment.

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474.34 \times 18,432}{12} \times \frac{1}{12 \times 1000} = 60.72 \text{ kip-ft}$$ ***ACI 318-14 (Eq. 24.2.3.5b)***

f_r = Modulus of rapture of concrete.

$$f_r = 7.5\lambda\sqrt{f_c'} = 7.5 \times 1.0 \times \sqrt{4000} = 474.34 \text{ psi}$$ ***ACI 318-14 (Eq. 19.2.3.1)***

I_g = Moment of inertia of the gross uncracked concrete section

$$I_g = \frac{b \times h^3}{12} = \frac{16 \times 24^3}{12} = 18,432 \text{ in.}^4$$

$$y_t = \frac{h}{2} = \frac{24}{2} = 12 \text{ in.}$$

I_{cr} = moment of inertia of the cracked section transformed to concrete. ***ACI 318-14 (2.2)***

The exterior span near the interior support is reinforced with 7 #8 bars located at 2 in. along the section from the top of the beam as determined previously.

E_c = Modulus of elasticity of concrete.

$$E_c = w_c^{1.5} 33\sqrt{f_c'} = 150^{1.5} \times 33 \times \sqrt{4000} = 3834 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n = \frac{E_s}{E_c} = \frac{29000000}{3834000} = 7.56$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B=\frac{b}{n\,A_s}=\frac{16}{7.56\times(7\times0.79)}=0.38\text{ in.}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd=\frac{\sqrt{2dB+1}-1}{B}=\frac{\sqrt{2\times22\times0.38+1}-1}{0.38}=8.42\text{ in.}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr}=\frac{b(kd)^3}{3}+nA_s(d-kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr}=\frac{16\times8.42^3}{3}+7.56\times(7\times0.79)\times(22-8.42)^2=10{,}897\text{ in.}^4$$

For the exterior span (span with one end continuous) with total service load level:

$$I_e^-=\left(\frac{M_{cr}}{M_a}\right)^3 I_g+\left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right]I_{cr}\text{ , since }M_{cr}=60.72\text{ ft-kips}<M_a=425.29\text{ ft-kips}$$ ***ACI 318-14 (24.2.3.5a)***

Where I_e^- is the effective moment of inertia for the critical negative moment region (near the support).

As mentioned earlier, midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan region is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg}=0.85\,I_e^+ +0.15\,I_e^-\text{ for end span}$$ ***spBeam Manual v5.00 (Eq. 2-104)***

$$I_{e,avg}=0.85\times(7{,}561)+0.15\times(10{,}919)=8{,}065\text{ in.}^4$$

Where:

I_e^- = The effective moment of inertia for the critical negative moment section near the support.

I_e^+ = The effective moment of inertia for the critical positive moment section (midspan).

The following Table provides a summary of the required parameters and calculated values needed for deflection calculation.

Table 3 – Averaged Effective Moment of Inertia Calculations (**Rectangular** Sections at Negative Moment Regions)

Span	zone	I_g, in.^{4}	I_{cr}, in.^{4}	M_a, ft-kip			M_{cr}, k-ft	I_e, in.^{4}			$I_{e,avg}$, in.^{4}		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	18432	8529	-191.38	-191.38	-238.24	-60.72	---	---	---	10011	10011	8065
	Midspan	39684	5100	174.04	174.04	216.64	89.77	9847	9847	7561			
	Right	18432	10897	-341.64	-341.64	-425.29	-60.72	10939	10939	10919			

Table 4 – Averaged Effective Moment of Inertia Calculations (**T-Shaped** Sections at Negative Moment Regions)													
Span	zone	I_g, in.^{4}	I_{cr}, in.^{4}	M_a, ft-kip			M_{cr}, k-ft	I_e, in.^{4}			$I_{e,avg}$, in.^{4}		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	39684	8529	-191.38	-191.38	-238.24	-240.36	---	---	---	11508	11508	8841
	Midspan	39684	5100	174.04	174.04	216.64	89.77	9847	9847	7561			
	Right	39684	10897	-341.64	-341.64	-425.29	-240.36	20922	20922	16094			

After obtaining the averaged effective moment of inertia, the maximum span deflections for the continuous beam can be obtained from anyone of the selected three procedures, other procedures can be used too.

For the case were the negative moment regions are considered as rectangular sections:

6.1.1. PCA Superposition Procedure:

PCA Notes on ACI 318-11

Flat Plate Design Example

For exterior span - service total load case:

$$\Delta_{fixed} = \frac{w \times l^4}{384 \times E_c \times I_{e,avg}}$$

Where:

Δ_{fixed} = Deflection of beam assuming fixed end condition.

w = 2.19 + 0.54 = 2.73 kip/ft

$I_{e,avg}$ = The averaged effective moment of inertia = 8,065 in.^{4}

$$E_c = w_c^{1.5} 33\sqrt{f_c'} = 150^{1.5} \times 33 \times \sqrt{4000} = 3834 \times 10^3 \text{ psi}$$

ACI 318-14 (19.2.2.1.a)

$$\Delta_{fixed} = \frac{2.73 \times 40^4 \times 12^3}{384 \times 3834 \times 8065} = 1.02 \text{ in.}$$

$$\theta_L = \frac{M_{net,L}}{\Sigma K_c}$$

Where:

θ_L = Rotation of the span left support.

$M_{net,L}$ = 236.99 kip-ft = Net negative moment of the left support.

$\sum K_c$ = column stiffness = (1.31+1.11) x 10^6 kip-in. = 2.42 x 10^6 kip-in. (calculated previously).

$$\theta_L = \frac{236.99}{2.42 \times 10^6 \times 12} = 1.18 \times 10^{-3} \text{ rad}$$

$$\Delta\theta_L = \theta_L \left(\frac{l}{8}\right)\left(\frac{I_g}{I_{e,avg}}\right)$$

Where:

$\Delta\theta_L$ = Midspan deflection due to rotation of left support.

$$\Delta\theta_L = 1.18\times10^{-3}\times\frac{40\times12}{8}\times\frac{18432}{8065} = 0.161 \text{ in.}$$

$$\theta_R = \frac{M_{net,R}}{\Sigma K_c} = \frac{0}{\Sigma K_c} = 0 \text{ rad}$$

$$\Delta\theta_R = \theta_R\left(\frac{l}{8}\right)\left(\frac{I_g}{I_{e,avg}}\right) = 0 \text{ in.}$$

$$\Delta_{Total} = \Delta_{fixed} + \Delta\theta_R + \Delta\theta_L$$

$$\Delta_{Total} = 1.02 + 0.0 + 0.161 = 1.181 \text{ in.}$$

Following the same procedure for dead service load level

$$\Delta_{DL} = 0.657 + 0.0 + 0.105 = 0.762 \text{ in.}$$

$$\Delta_{LL} = \Delta_{Total} - \Delta_{DL} = 1.18 - 0.76 = 0.42 \text{ in.} < \frac{l}{360} = \frac{40\times12}{360} = 1.33 \text{ in. } \textbf{\textit{o.k.}}$$ ***ACI 318-14 (Table 24.2.2)***

6.1.2. Simplified Superposition Procedure:

Reinforced Concrete Mechanics and Design – 7th edition – J. Wight (Section9-4)

For exterior span - service total load case:

$$M_o = \frac{w\times l^2}{8} = \frac{2.73\times40^2}{8} = 546 \text{ kip-ft}$$

$$M_m = M_o + \frac{M_1}{2} + \frac{M_2}{2} = 546 - \frac{236.99}{2} - \frac{425.29}{2} = 214.86 \text{ kip-ft}$$

$$\Delta_{Total} = \frac{5}{48}\times\frac{l^2}{E_c\times I_{e,avg}}\times\left(M_m + 0.1\times\left(M_1+M_2\right)\right)$$

$$\Delta_{Total} = \frac{5}{48}\times\frac{40^2}{3834\times8065}\times\left(214.86 - 0.1\times\left(236.99+425.29\right)\right) = 1.38 \text{ in.}$$

Where:

M_o = Simple span moment at midspan = $wl^2/8$

M_1 = Negative moment at the exterior support

M_2 = Negative moment at the interior support

Following the same procedure for dead service load level

$$\Delta_{DL} = 0.890 \text{ in.}$$

$$\Delta_{LL} = \Delta_{Total} - \Delta_{DL} = 1.380 - 0.890 = 0.490 \text{ in.} < \frac{l}{360} = \frac{40 \times 12}{360} = 1.33 \text{ in. } \textbf{o.k.}$$ ***ACI 318-14 (Table 24.2.2)***

6.1.3. PCA Simplified Procedure:

PCA Notes on ACI 318-11 (9.5)

This procedure is based on elastic equation given in the ACI 318-83 commentary on 9.5.2.4. For continuous beams, the midspan deflection may usually be used as an approximation of the maximum deflection:

$$\Delta = \frac{5}{48} \times K \times \frac{M_a \times l^2}{E_c \times I_{e,avg}}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(3))***

$$K = 1.2 - 0.2 \times \frac{M_o}{M_a}$$ ***PCA Notes on ACI 318-11 (Table 10-3)***

Where:

M_a = Midspan moment for service load level

M_o = Simple span moment at midspan = $wl^2/8$

For exterior span - service total load case:

M_a = 216.64 kip-ft

$$M_o = \frac{w \times l^2}{8} = \frac{2.73 \times 40^2}{8} = 546 \text{ kip-ft}$$

$$K = 1.2 - 0.2 \times \frac{546}{216.64} = 0.696$$

$$\Delta_{Total} = \frac{5}{48} \times 0.696 \times \frac{216.64 \times 40^2 \times 12}{3834 \times 8065} = 1.404 \text{ in.}$$

Following the same procedure for dead service load level

$$\Delta_{DL} = 0.910 \text{ in.}$$

$$\Delta_{LL} = \Delta_{Total} - \Delta_{DL} = 1.404 - 0.910 = 0.494 \text{ in.} < \frac{l}{360} = \frac{40 \times 12}{360} = 1.33 \text{ in. } \textbf{o.k.}$$ ***ACI 318-14 (Table 24.2.2)***

6.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) are estimated as follows. The results obtained from the PCA superposition procedure are used below, the same steps can be repeated to obtain the time-dependent deflections for the two other procedures.

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_{\Delta}) + [(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}]$$ **_CSA A23.3-04 (N9.8.2.5)_**

Where:

$(\Delta_{sust})_{Inst} =$ Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_{\Delta} = \frac{\xi}{1+50\rho'}$$ **_ACI 318-14 (24.2.4.1.1)_**

$(\Delta_{total})_{lt} =$ Time-dependent (long-term) total delfection, in.

$(\Delta_{total})_{Inst} =$ Total immediate (instantaneous) deflection, in.

For the exterior span

ξ = 2, consider the sustained load duration to be 60 months or more. **_ACI 318-14 (Table 24.2.4.1.3)_**

ρ' = 0, conservatively.

$$\lambda_{\Delta} = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2\times 0.762 = 1.52 \text{ in.}$$

$$\Delta_{cs} + \Delta_{LL} = 1.52 + 0.49 \simeq 2 \text{ in.} \leq \frac{l}{240} = \frac{40\times 12}{240} = 2 \text{ in. } \textbf{\textit{o.k.}}$$ **_ACI 318-14 (Table 24.2.2)_**

$$(\Delta_{total})_{lt} = 0.762\times(1+2)+(1.18-0.762) = 2.70 \text{ in.}$$

The previous deflection calculations are done considering a rectangular section in the negative moment regions. The same procedure can be followed to obtain the deflections values for the case where a T-shaped section is used in negative moment regions. Tables 9 and 10 in the comparison section show a results summary for both options.

7. Continuous Beam Analysis and Design – spBeam Software

spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318-14) and Canadian (CSA A23.3-14) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or T-shaped sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, spBeam provide users with options and flexibility to:

1. Design the beam as a rectangular cross-section or a T-shaped section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.

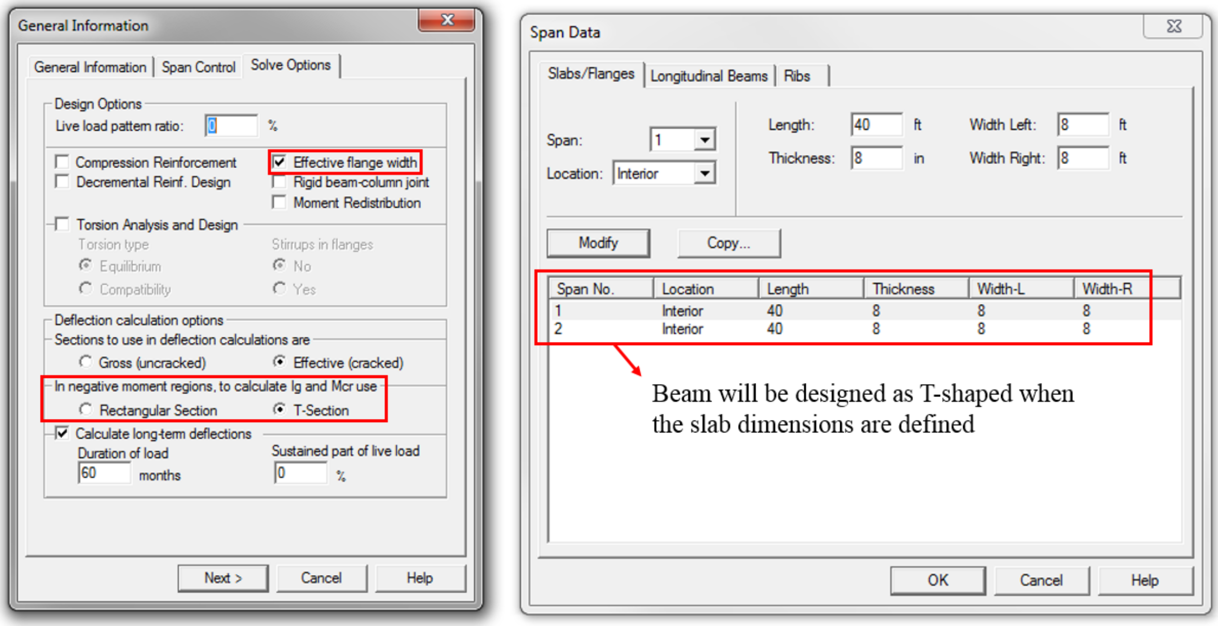


Figure 9 – spBeam Modeling and Solve Options

Also two other important options can be used:

4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the continuous beam on Grid 3 with effective flange width included in analysis, design and deflection calculations.

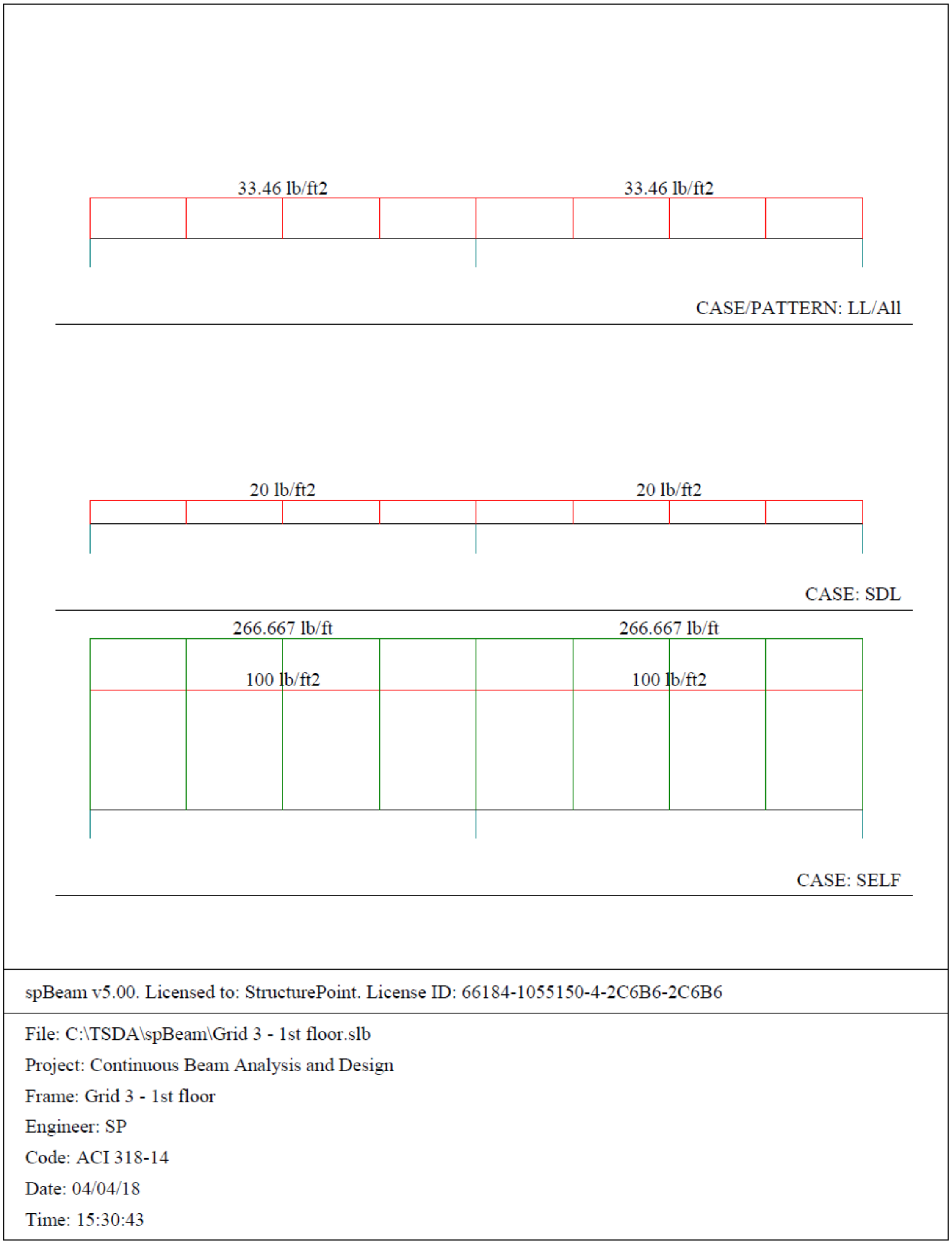
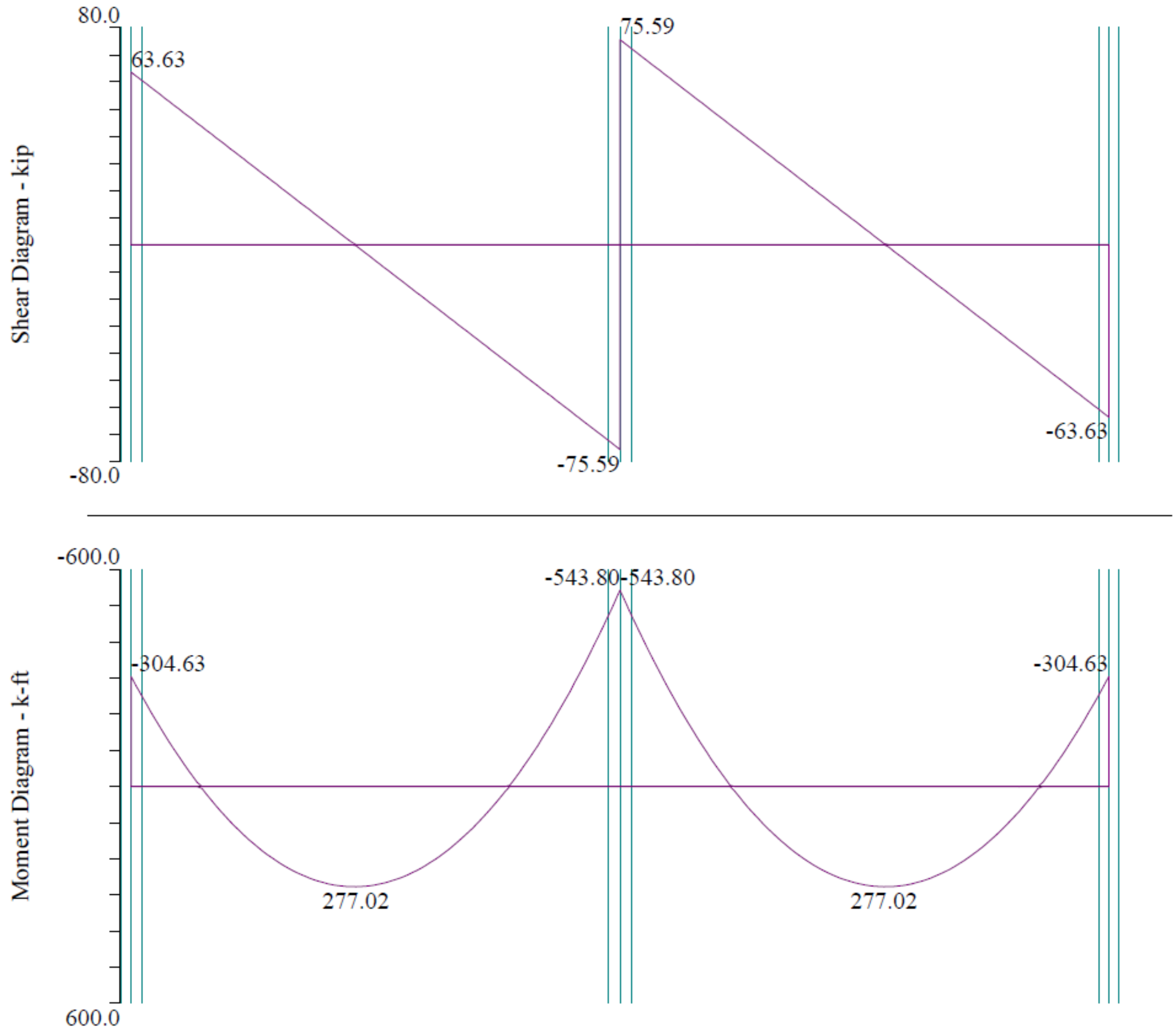


Figure 10 – Loading (spBeam)



spBeam v5.00. Licensed to: StructurePoint. License ID: 66184-1055150-4-2C6B6-2C6B6

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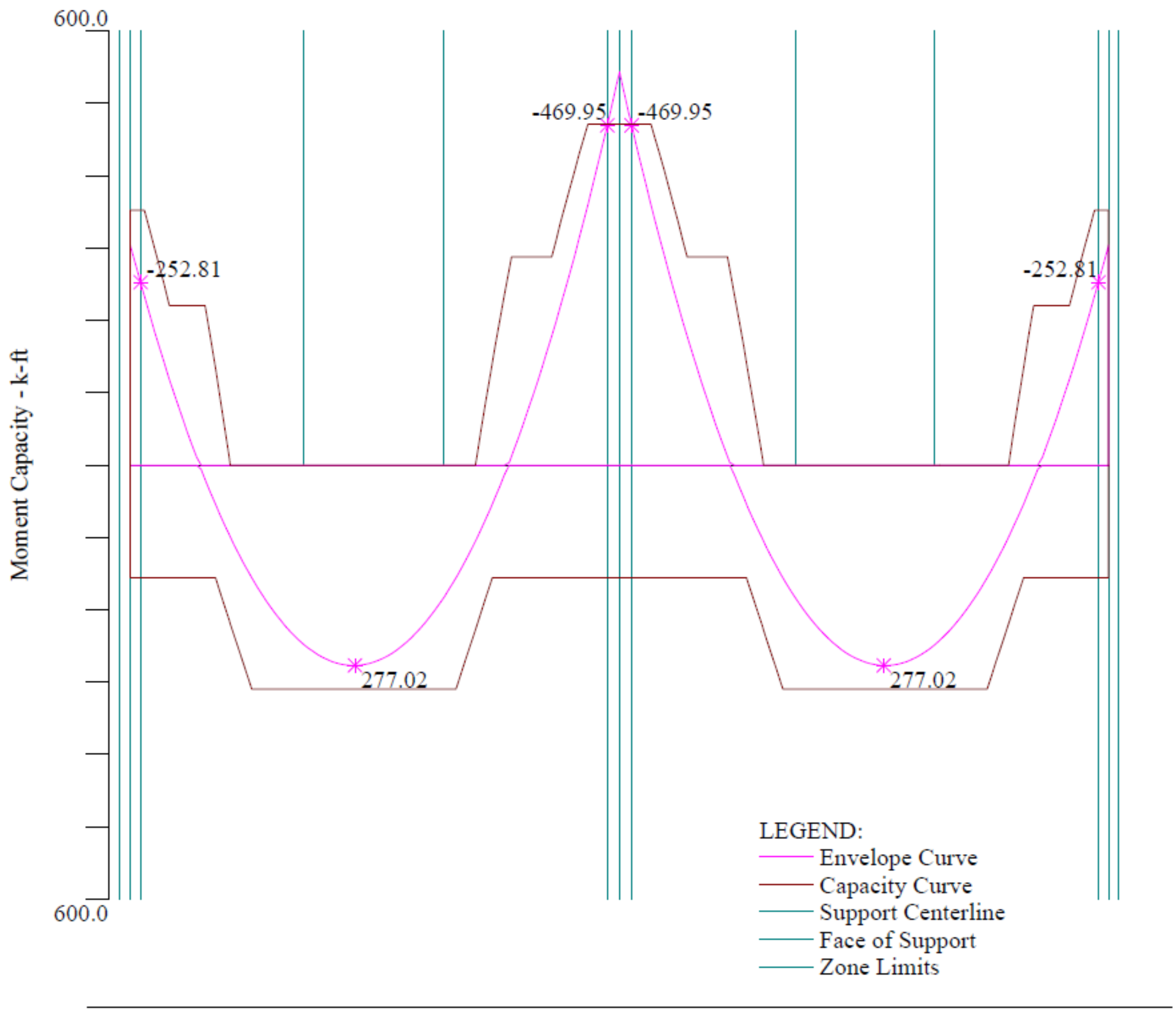
Project: Continuous Beam Analysis and Design
Frame: Grid 3 - 1st floor
Engineer: SP
Code: ACI 318-14
Date: 04/04/18
Time: 15:31:41

Figure 11 – Internal Forces (spBeam)



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Project: Continuous Beam Analysis and Design

Frame: Grid 3 - 1st floor

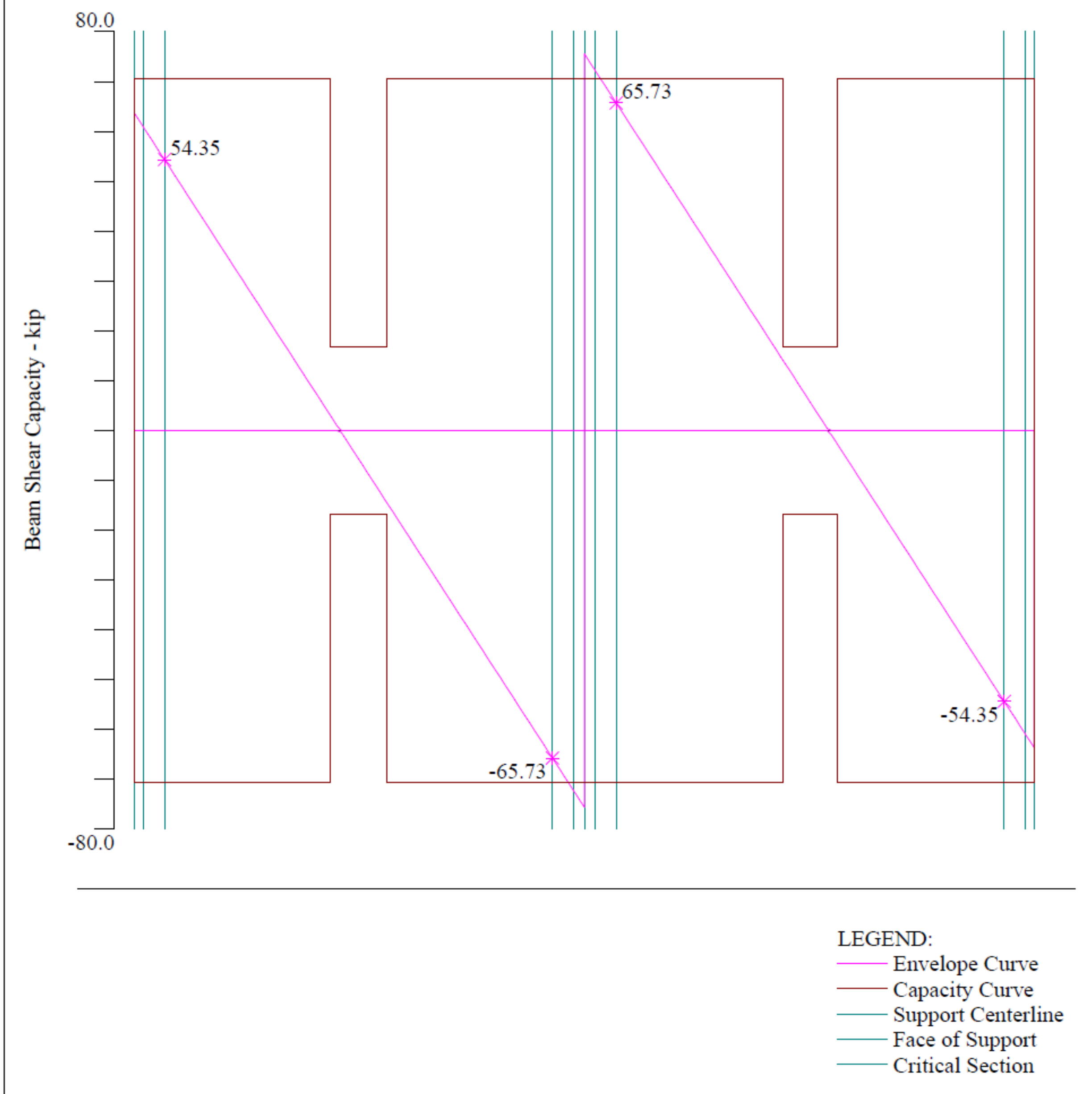
Engineer: SP

Code: ACI 318-14

Date: 04/04/18

Time: 16:13:53

Figure 12 – Moment Capacity Diagram (spBeam)



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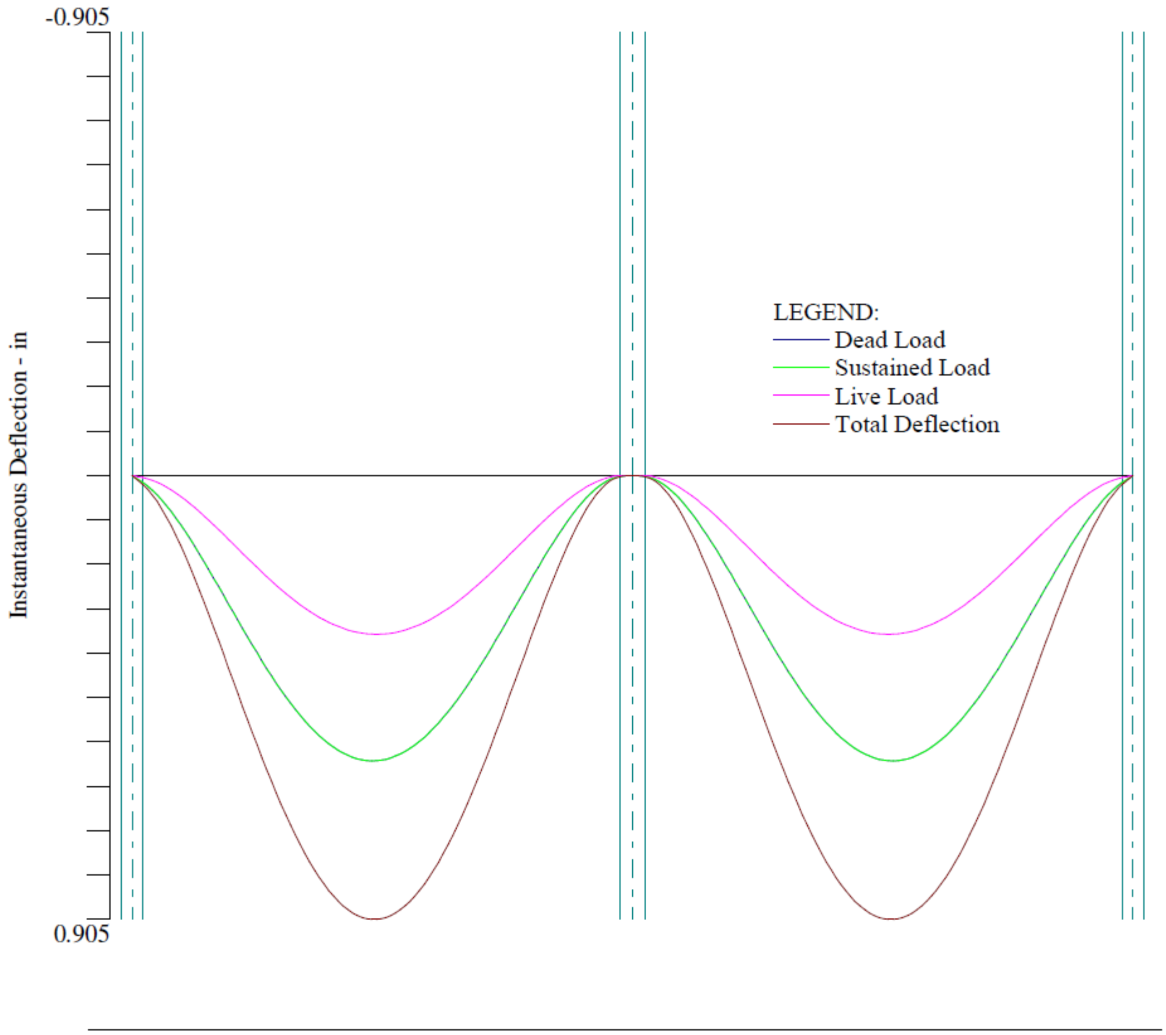
Frame: Grid 3 - 1st floor
Engineer: SP
Code: ACI 318-14
Date: 04/04/18
Time: 15:33:37

Figure 13 – Shear Capacity Diagram (spBeam)



spBeam v5.00. Licensed to: StructurePoint. License ID: 66184-1055150-4-2C6B6-2C6B6

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Project: Continuous Beam Analysis and Design
Frame: Grid 3 - 1st floor
Engineer: SP
Code: ACI 318-14
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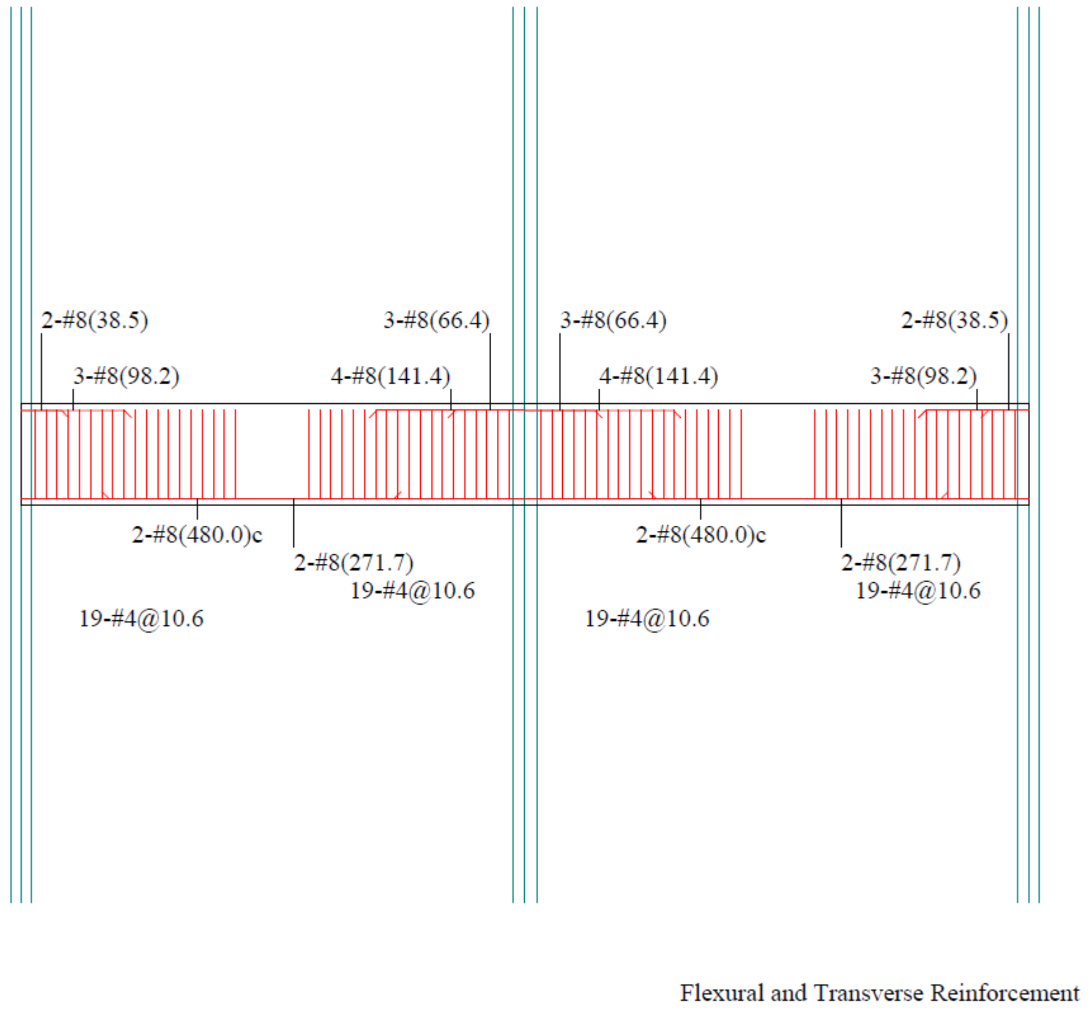
Time: 15:56:11

Figure 14 – Immediate Deflection Diagram (spBeam)



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Frame: Grid 3 - 1st floor
Engineer: SP
Code: ACI 318-14
Date: 04/04/18
Time: 15:58:33

Figure 15 – Longitudinal and Lateral Reinforcement (spBeam)

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spBeam v5.00 © StructurePoint                                                      04-12-2018, 11:38:01 AM
Licensed to: StructurePoint, License ID: 66184-1055150-4-2C6B6-2C6B6
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=================================================================================================
                                      spBeam v5.00 (TM)
                 A Computer Program for Analysis, Design, and Investigation of
                       Reinforced Concrete Beams and One-way Slab Systems
                           Copyright © 1992-2015, STRUCTUREPOINT, LLC
                                      All rights reserved
=================================================================================================
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==================================================================================================================
[1] INPUT ECHO
==================================================================================================================

General Information
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   File name: C:\TSDA\spBeam\BCs\Grid 3 - 1st floor.slb
   Project: Continuous Beam Analysis and Design
   Frame: Grid 3 - 1st floor
   Engineer: SP
   Code: ACI 318-14
   Reinforcement Database: ASTM A615
   Mode: Design
   Number of supports = 3
   Floor System: One-Way/Beam

   Live load pattern ratio = 0%
   Deflections are based on cracked section properties.
   In negative moment regions, Ig and Mcr include flange/slab contribution (if available)
   Long-term deflections are calculated for load duration of 60 months.
   0% of live load is sustained.
   Compression reinforcement calculations NOT selected.
   Default incremental rebar design selected.
   Moment redistribution NOT selected.
   Effective flange width calculations selected.
   Rigid beam-column joint NOT selected.
   Torsion analysis and design NOT selected.

Material Properties
===================
             Slabs|Beams       Columns
            ------------   ------------
   wc    =           150           150 lb/ft3
   f'c   =             4             4 ksi
   Ec    =        3834.3        3834.3 ksi
   fr    =       0.47434       0.47434 ksi

   fy    =            60 ksi, Bars are not epoxy-coated
   fyt   =            60 ksi
   Es    =         29000 ksi

Reinforcement Database
======================
   Units: Db (in), Ab (in^2), Wb (lb/ft)
```

Size	Db	Ab	Wb	Size	Db	Ab	Wb
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

```
Span Data
=========
```

Slabs

Units: L1, wL, wR (ft); t, bEff, Hmin (in)

Span	Loc	L1	t	wL	wR	bEff	Hmin
1	Int	40.000	8.00	8.000	8.000	120.00	0.00
2	Int	40.000	8.00	8.000	8.000	120.00	0.00

Ribs and Longitudinal Beams

Units: b, h, Sp (in)

	Ribs			Beams		Span
Span	b	h	Sp	b	h	Hmin
1	0.00	0.00	0.00	16.00	24.00	25.95 *b
2	0.00	0.00	0.00	16.00	24.00	25.95 *b

NOTES:
*b - Span depth is less than minimum. Deflection check required.

Support Data

Columns

Units: c1a, c2a, c1b, c2b (in); Ha, Hb (ft)

Supp	c1a	c2a	Ha	c1b	c2b	Hb	Red%
1	20.00	20.00	13.000	20.00	20.00	15.330	100
2	24.00	30.00	13.000	24.00	30.00	15.330	100
3	20.00	20.00	13.000	20.00	20.00	15.330	100

Boundary Conditions

Units: Kz (kip/in); Kry (kip-in/rad)

Supp	Spring Kz	Spring Kry	Far End A	Far End B
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed

Load Data

Load Cases and Combinations

Case	SELF	SDL	LL
Type	DEAD	DEAD	LIVE
U1	1.200	1.200	1.600

Area Loads

Units: Wa (lb/ft2)

Case/Patt	Span	Wa
SELF	1	100.00
	2	100.00
LL	1	33.46
	2	33.46
SDL	1	20.00
	2	20.00

Line Loads

Units: Wa, Wb (lb/ft), La, Lb (ft)

Case/Patt	Span	Wa	La	Wb	Lb
SELF	1	266.67	0.000	266.67	40.000
	2	266.67	0.000	266.67	40.000

Reinforcement Criteria

Slabs and Ribs

	Top bars		Bottom bars		
	Min	Max	Min	Max	
Bar Size	#5	#8	#5	#8	
Bar spacing	1.00	18.00	1.00	18.00	in
Reinf ratio	0.14	5.00	0.14	5.00	%
Cover	1.50		1.50		in

There is NOT more than 12 in of concrete below top bars.

Beams

	Top bars		Bottom bars		Stirrups	
	Min	Max	Min	Max	Min	Max
Bar Size	#8	#10	#8	#10	#4	#4

```
     Bar spacing    1.00    18.00     1.00    18.00       6.00    18.00 in
     Reinf ratio    0.14     5.00     0.14     5.00 %
     Cover          1.50              1.50          in
     Layer dist.    1.00              1.00          in
     No. of legs                                         2        2
     Side cover                                       1.50          in
     1st Stirrup                                      3.00          in
     There is NOT more than 12 in of concrete below top bars.
```

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[2] DESIGN RESULTS

Top Reinforcement

Units: Width (ft), Mmax (k-ft), Xmax (ft), As (in^2), Sp (in)

Span	Zone	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Left	4.00	252.81	0.833	1.173	6.358	2.742	10.677	5-#8 *5
	Midspan	4.00	0.00	19.917	0.000	6.358	0.000	0.000	---
	Right	4.00	469.95	39.000	1.173	6.358	5.507	7.118	7-#8
2	Left	4.00	469.95	1.000	1.173	6.358	5.507	7.118	7-#8
	Midspan	4.00	0.00	20.083	0.000	6.358	0.000	0.000	---
	Right	4.00	252.81	39.167	1.173	6.358	2.742	10.677	5-#8 *5

NOTES:
*5 - Number of bars governed by maximum allowable spacing.

Top Bar Details

Units: Length (ft)

	Left				Continuous		Right			
Span	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
1	3-#8	8.18	2-#8*	3.20	---		4-#8	11.78	3-#8*	5.53
2	4-#8	11.78	3-#8*	5.53	---		3-#8	8.18	2-#8*	3.20

NOTES:
* - Bar cut-off location does not meet ACI 318, 12.10.5.1. Revise location,
unless the requirements of either 12.10.5.2 or 12.10.5.3 are manually checked and satisfied.

Top Bar Development Lengths

Units: Length (in)

	Left				Continuous		Right			
Span	Bars	Length	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
1	3-#8	24.70	2-#8	24.70	---		4-#8	35.43	3-#8	35.43
2	4-#8	35.43	3-#8	35.43	---		3-#8	24.70	2-#8	24.70

Bottom Reinforcement

Units: Width (ft), Mmax (k-ft), Xmax (ft), As (in^2), Sp (in)

Span	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	1.33	277.02	18.390	1.173	47.685	2.825	3.569	4-#8
2	1.33	277.02	21.610	1.173	47.685	2.825	3.569	4-#8

Bottom Bar Details

Units: Start (ft), Length (ft)

Long Bars Short Bars

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Span	Bars	Start	Length	Bars	Start	Length
1	2-#8	0.00	40.00	2-#8*	6.96	22.64
2	2-#8	0.00	40.00	2-#8	10.40	22.64

NOTES:
* - Bar cut-off location does not meet ACI 318, 12.10.5.1. Revise location,
unless the requirements of either 12.10.5.2 or 12.10.5.3 are manually checked and satisfied.

Bottom Bar Development Lengths

Units: DevLen (in)

Span	Long Bars: Bars	Long Bars: DevLen	Short Bars: Bars	Short Bars: DevLen
1	2-#8	35.64	2-#8	35.64
2	2-#8	35.64	2-#8	35.64

Flexural Capacity

Units: x (ft), As (in^2), PhiMn, Mu (k-ft)

		Top						Bottom					
Span	x	AsTop	PhiMn-	Mu-	Comb	Pat	Status	AsBot	PhiMn+	Mu+	Comb	Pat	Status
1	0.000	3.95	-352.33	-304.63	U1	All	---	1.58	155.59	0.00	U1	All	---
	0.833	3.95	-352.33	-252.81	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	1.147	3.95	-352.33	-233.98	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	3.205	2.37	-220.69	-118.61	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	6.122	2.37	-220.69	0.00	U1	All	OK	1.58	155.59	19.69	U1	All	OK
	6.959	1.41	-134.29	0.00	U1	All	OK	1.58	155.59	53.92	U1	All	OK
	8.180	0.00	0.00	0.00	U1	All	OK	2.23	219.08	99.41	U1	All	OK
	9.930	0.00	0.00	0.00	U1	All	OK	3.16	309.54	155.59	U1	All	OK
	14.192	0.00	0.00	0.00	U1	All	OK	3.16	309.54	247.92	U1	All	OK
	18.390	0.00	0.00	0.00	U1	All	OK	3.16	309.54	277.02	U1	All	OK
	20.000	0.00	0.00	0.00	U1	All	OK	3.16	309.54	271.86	U1	All	OK
	25.642	0.00	0.00	0.00	U1	All	OK	3.16	309.54	182.78	U1	All	OK
	26.634	0.00	0.00	0.00	U1	All	OK	3.16	309.54	155.59	U1	All	OK
	28.218	0.00	0.00	0.00	U1	All	OK	2.32	227.63	105.19	U1	All	OK
	29.604	1.48	-141.39	0.00	U1	All	OK	1.58	155.59	53.90	U1	All	OK
	31.170	3.16	-288.06	-12.10	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	34.468	3.16	-288.06	-178.94	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	37.421	5.53	-471.58	-360.44	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	39.000	5.53	-471.58	-469.95	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	39.250	5.53	-471.58	-488.09	U1	All	---	1.58	155.59	0.00	U1	All	---
	40.000	5.53	-471.58	-543.80	U1	All	---	1.58	155.59	0.00	U1	All	---
2	0.000	5.53	-471.58	-543.80	U1	All	---	1.58	155.59	0.00	U1	All	---
	0.750	5.53	-471.58	-488.09	U1	All	---	1.58	155.59	0.00	U1	All	---
	1.000	5.53	-471.58	-469.95	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	2.579	5.53	-471.58	-360.44	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	5.532	3.16	-288.06	-178.94	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	8.830	3.16	-288.06	-12.10	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	10.396	1.48	-141.39	0.00	U1	All	OK	1.58	155.59	53.90	U1	All	OK
	11.782	0.00	0.00	0.00	U1	All	OK	2.32	227.63	105.19	U1	All	OK
	13.366	0.00	0.00	0.00	U1	All	OK	3.16	309.54	155.59	U1	All	OK
	14.358	0.00	0.00	0.00	U1	All	OK	3.16	309.54	182.78	U1	All	OK
	20.000	0.00	0.00	0.00	U1	All	OK	3.16	309.54	271.86	U1	All	OK
	21.610	0.00	0.00	0.00	U1	All	OK	3.16	309.54	277.02	U1	All	OK
	25.808	0.00	0.00	0.00	U1	All	OK	3.16	309.54	247.92	U1	All	OK
	30.070	0.00	0.00	0.00	U1	All	OK	3.16	309.54	155.59	U1	All	OK
	31.820	0.00	0.00	0.00	U1	All	OK	2.23	219.07	99.40	U1	All	OK
	33.041	1.41	-134.29	0.00	U1	All	OK	1.58	155.59	53.92	U1	All	OK
	33.878	2.37	-220.69	0.00	U1	All	OK	1.58	155.59	19.69	U1	All	OK
	36.795	2.37	-220.69	-118.61	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	38.853	3.95	-352.33	-233.98	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	39.167	3.95	-352.33	-252.81	U1	All	OK	1.58	155.59	0.00	U1	All	OK
	40.000	3.95	-352.33	-304.63	U1	All	---	1.58	155.59	0.00	U1	All	---

Longitudinal Beam Transverse Reinforcement Demand and Capacity

Section Properties

Units: d (in), Av/s (in^2/in), PhiVc (kip)

Span	d	(Av/s)min	PhiVc
1	22.00	0.0133	33.39
2	22.00	0.0133	33.39

Beam Transverse Reinforcement Demand

Units: Start, End, Xu (in), Vu (ft), Av/s (kip/in^2)

			Required				Demand	
Span	Start	End	Xu	Vu	Comb/Patt	Av/s	Av/s	
1	1.083	7.595	2.667	54.35	U1/All	0.0212	0.0212	
	7.595	12.524	7.595	37.20	U1/All	0.0038	0.0133	*8

```
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          12.524   17.452   12.524    20.04    U1/All   0.0000    0.0133 *8
          17.452   22.381   22.381    14.27    U1/All   0.0000    0.0000
          22.381   27.310   27.310    31.42    U1/All   0.0000    0.0133 *8
          27.310   32.238   32.238    48.58    U1/All   0.0153    0.0153
          32.238   38.750   37.167    65.73    U1/All   0.0327    0.0327

      2    1.250    7.762    2.833    65.73    U1/All   0.0327    0.0327
           7.762   12.690    7.762    48.58    U1/All   0.0153    0.0153
          12.690   17.619   12.690    31.42    U1/All   0.0000    0.0133 *8
          17.619   22.548   17.619    14.27    U1/All   0.0000    0.0000
          22.548   27.476   27.476    20.04    U1/All   0.0000    0.0133 *8
          27.476   32.405   32.405    37.20    U1/All   0.0038    0.0133 *8
          32.405   38.917   37.333    54.35    U1/All   0.0212    0.0212

   NOTES:
   *8 - Minimum transverse (stirrup) reinforcement governs.

   Beam Transverse Reinforcement Details
   -------------------------------------
   Units: spacing & distance (in).
   Span Size Stirrups (2 legs each unless otherwise noted)
   ---- ---- ---------------------------------------------
      1   #4 19 @ 10.6 + <-- 59.1 --> + 19 @ 10.6
      2   #4 19 @ 10.6 + <-- 59.1 --> + 19 @ 10.6

   Beam Transverse Reinforcement Capacity
   --------------------------------------
   Units: Start, End, Xu (ft), Vu, PhiVn (kip), Av/s (in^2/in), Av (in^2), Sp (in)
                                     ______________Required_______  _____________Provided______________
   Span     Start      End       Xu       Vu Comb/Patt    Av/s        Av       Sp     Av/s    PhiVn
   ---- --------- --------- --------- --------- --------- -------- -------- -------- -------- --------
      1    0.000    1.083    2.667    54.35    U1/All    -----    -----    -----    -----    -----
           1.083   17.452    2.667    54.35    U1/All   0.0212     0.40     10.6   0.0377    70.69
          17.452   22.381   22.381    14.27    U1/All   0.0000    -----    -----    -----    16.70
          22.381   38.750   37.167    65.73    U1/All   0.0327     0.40     10.6   0.0377    70.69
          38.750   40.000   37.167    65.73    U1/All    -----    -----    -----    -----    -----

      2    0.000    1.250    2.833    65.73    U1/All    -----    -----    -----    -----    -----
           1.250   17.619    2.833    65.73    U1/All   0.0327     0.40     10.6   0.0377    70.69
          17.619   22.548   17.619    14.27    U1/All   0.0000    -----    -----    -----    16.70
          22.548   38.917   37.333    54.35    U1/All   0.0212     0.40     10.6   0.0377    70.69
          38.917   40.000   37.333    54.35    U1/All    -----    -----    -----    -----    -----

Slab Shear Capacity
===================
   Units: b, d (in), Xu (ft), PhiVc, Vu(kip)
   Span        b        d   Vratio        PhiVc           Vu           Xu
   ---- -------- -------- -------- ------------ ------------ ------------
      1 --- Not checked ---
      2 --- Not checked ---

Material Takeoff
================
   Reinforcement in the Direction of Analysis
   ------------------------------------------
   Top Bars:        505.6 lb    <=>    6.32 lb/ft    <=>  0.395 lb/ft^2
   Bottom Bars:     669.0 lb    <=>    8.36 lb/ft    <=>  0.523 lb/ft^2
   Stirrups:        287.7 lb    <=>    3.60 lb/ft    <=>  0.225 lb/ft^2
   Total Steel:    1462.3 lb    <=>   18.28 lb/ft    <=>  1.142 lb/ft^2
   Concrete:        995.6 ft^3 <=>   12.44 ft^3/ft <=>  0.778 ft^3/ft^2
```

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[3] DEFLECTION RESULTS

Section Properties

Frame Section Properties

Units: Ig, Icr (in^4), Mcr (k-ft)

Span	Zone	M+ve Ig	M+ve Icr	M+ve Mcr	M-ve Ig	M-ve Icr	M-ve Mcr
1	Left	39684	4120	89.77	39684	8529	-240.36
	Midspan	39684	5100	89.77	39684	0	-240.36
	Right	39684	4120	89.77	39684	10897	-240.36
2	Left	39684	4120	89.77	39684	10897	-240.36
	Midspan	39684	5100	89.77	39684	0	-240.36
	Right	39684	4120	89.77	39684	8529	-240.36

NOTES: M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Frame Effective Section Properties

Units: Ie, Ie,avg (in^4), Mmax (k-ft)

			Load Level					
			Dead		Sustained		Dead+Live	
Span	Zone	Weight	Mmax	Ie	Mmax	Ie	Mmax	Ie
1	Middle	0.850	174.04	9847	174.04	9847	216.64	7561
	Right	0.150	-341.64	20922	-341.64	20922	-425.29	16094
	Span Avg	----	----	11508	----	11508	----	8841
2	Left	0.150	-341.64	20922	-341.64	20922	-425.29	16094
	Middle	0.850	174.04	9847	174.04	9847	216.64	7561
	Span Avg	----	----	11508	----	11508	----	8841

Instantaneous Deflections

Extreme Instantaneous Frame Deflections and Corresponding Locations

Units: Def (in), Loc (ft)

				Live			Total	
Span	Direction	Value	Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	0.582	---	0.324	0.324	0.582	0.905
		Loc	19.153	---	19.535	19.535	19.153	19.535
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
2	Down	Def	0.582	---	0.324	0.324	0.582	0.905
		Loc	20.847	---	20.465	20.465	20.847	20.465
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---

Long-term Deflections

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Long-term Deflection Factors

Time dependant factor for sustained loads = 2.000
Units: Astop, Asbot (in^2), b, d (in), Rho' (%), Lambda (-)

		M+ve					M-ve				
Span	Zone	Astop	b	d	Rho'	Lambda	Asbot	b	d	Rho'	Lambda
1	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Extreme Long-term Frame Deflections and Corresponding Locations

Units: Def (in), Loc (ft)

Span	Direction	Value	cs	cs+lu	cs+l	Total
1	Down	Def	1.164	1.487	1.487	2.069
		Loc	19.153	19.153	19.153	19.153
	Up	Def	---	---	---	---
		Loc	---	---	---	---
2	Down	Def	1.164	1.487	1.487	2.069
		Loc	20.847	20.847	20.847	20.847
	Up	Def	---	---	---	---
		Loc	---	---	---	---

NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

8. Design Results Comparison and Conclusions

The following tables show the comparison between hand results and spBeam model results.

<table>
<tr><th colspan="7">Table 5 - Comparison of Moments and Flexural Reinforcement</th></tr>
<tr><th rowspan="2">Location</th><th colspan="2">M_u^*, kip-ft</th><th colspan="2">$A_{s,req}$, in.</th><th colspan="2">Reinforcement</th></tr>
<tr><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th></tr>
<tr><td>Exterior Negative</td><td>250.41</td><td>252.81</td><td>2.71</td><td>2.74</td><td>5-#8</td><td>5-#8</td></tr>
<tr><td>Positive</td><td>272.52</td><td>277.02</td><td>2.78</td><td>2.83</td><td>4-#8</td><td>4-#8</td></tr>
<tr><td>Interior Negative</td><td>471.13</td><td>469.95</td><td>5.52</td><td>5.51</td><td>7-#8</td><td>7-#8</td></tr>
<tr><td colspan="7">* negative moments are taken at the faces of supports</td></tr>
</table>

<table>
<tr><th colspan="9">Table 6 - Comparison of Shear and lateral Reinforcement</th></tr>
<tr><th rowspan="2">Location</th><th colspan="2">V_u^*, kip</th><th colspan="2">$(A_v/s)_{req}$, in.^{2}/in.</th><th colspan="2">Reinforcement</th><th colspan="2">φV_n, kip</th></tr>
<tr><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th><th>Hand</th><th>spBeam</th></tr>
<tr><td>Interior Negative</td><td>65.82</td><td>65.73</td><td>0.0328</td><td>0.0327</td><td>19-#4@10.5</td><td>19-#4@10.6</td><td>71.11</td><td>70.69</td></tr>
<tr><td colspan="9">* Shear values are taken at distance d from the faces of supports</td></tr>
</table>

<table>
<tr><th colspan="9">Table 7 - Comparison of Section Properties (<u>Rectangular</u> Negative Moment Regions)</th></tr>
<tr><th rowspan="3">Location</th><th colspan="2">I_{cr}, in.^{4}</th><th colspan="6">$I_{e,avg}$, in.^{4}</th></tr>
<tr><th rowspan="2">Hand</th><th rowspan="2">spBeam</th><th colspan="3">Hand</th><th colspan="3">spBeam</th></tr>
<tr><th>DL</th><th>DL+LL_{sus}</th><th>Total</th><th>DL</th><th>DL+LL_{sus}</th><th>Total</th></tr>
<tr><td>Exterior Negative</td><td>8,529</td><td>8,529</td><td rowspan="3">10,011</td><td rowspan="3">10,011</td><td rowspan="3">8,065</td><td rowspan="3">10,011</td><td rowspan="3">10,011</td><td rowspan="3">8,065</td></tr>
<tr><td>Positive</td><td>5,100</td><td>5,100</td></tr>
<tr><td>Interior Negative</td><td>10,897</td><td>10,897</td></tr>
</table>

<table>
<tr><th colspan="9">Table 8 - Comparison of Section Properties (<u>T-shaped</u> Negative Moment Regions)</th></tr>
<tr><th rowspan="3">Location</th><th colspan="2">I_{cr}, in.^{4}</th><th colspan="6">$I_{e,avg}$, in.^{4}</th></tr>
<tr><th rowspan="2">Hand</th><th rowspan="2">spBeam</th><th colspan="3">Hand</th><th colspan="3">spBeam</th></tr>
<tr><th>DL</th><th>DL+LL_{sus}</th><th>Total</th><th>DL</th><th>DL+LL_{sus}</th><th>Total</th></tr>
<tr><td>Exterior Negative</td><td>8,529</td><td>8,529</td><td rowspan="3">11,508</td><td rowspan="3">11,508</td><td rowspan="3">8,841</td><td rowspan="3">11,508</td><td rowspan="3">11,508</td><td rowspan="3">8,841</td></tr>
<tr><td>Positive</td><td>5,100</td><td>5,100</td></tr>
<tr><td>Interior Negative</td><td>10,897</td><td>10,897</td></tr>
</table>

Table 9 - Comparison of Maximum Deflection (**Rectangular** Negative Moment Regions)		
Procedure	Δ_{inst}, in.	Δ_{lt}, in.
spBeam	0.98	2.29
PCA Superposition	1.18 (+20.4 %)	2.70
Simplified Superposition	1.38 (+40.8 %)	3.04
PCA Simplified	1.40 (+40.9 %)	3.22

Table 10 - Comparison of Maximum Deflection (**T-shaped** Negative Moment Regions)		
Procedure	Δ_{inst}, in.	Δ_{lt}, in.
spBeam	0.91	2.07
PCA Superposition	1.24 (+36.3 %)	2.77
Simplified Superposition	1.26 (+38.5 %)	2.84
PCA Simplified	1.28 (+40.7 %)	2.84

The results of all the hand calculations used illustrated above are in agreement with the automated exact results obtained from the spBeam program except for deflection (see section 8.2 for detailed discussion).

8.1. T-beam flange width considerations

In ACI 318-11 and earlier editions, the width of the slab effective as a T-beam flange was limited to one-fourth the beam span (center to center). In the 2014 revisions to ACI 318, the effective width was changed to become one-eighth of the clear span on each side of the beam web. This was intended to simplify Table 6.3.2.1 in the ACI 318-14 and according to the code commentary; this change has negligible impact on the designs. The use of one-fourth the span to limit the T-beam effective flange width has been used in earlier cycles of the ACI code and StructurePoint software was not changed to reflect this revision since the earlier provisions are more exact. A summary of the changes discussed here are given in the table below:

Table 11 – Comparison of b_{eff} Effects on Design Results								
Location	b_{eff}, in.		M_u^*, kip-ft		$A_{s,req}$, in.		Reinforcement	
	318-14 Hand	318-11 spBeam	318-14 Hand	318-11 spBeam	318-14 Hand	318-11 spBeam	318-14 Hand	318-11 spBeam
Exterior Negative	130.5	120.0	250.41	252.81	2.71	2.74	5-#8	5-#8
Positive			272.52	277.02	2.78	2.83	4-#8	4-#8
Interior Negative			471.13	469.95	5.52	5.51	7-#8	7-#8
* negative moments are taken at the faces of supports								

8.2. Deflection calculation methods:

Deflections calculations in reinforced concrete structures can be very tedious and time consuming because of the difficulty of accounting for the actual end boundary conditions in a building frame. As a result, numerous methods to estimate the deflection and the member stiffness have been presented in literature. It is important to note that these methods can only estimate deflections within an accuracy range of 20% to 40% as can be seen in Tables 9 and 10. It is important for the designer to be aware of this broad range of accuracy, especially in the modeling, design, and detailing of deflection-sensitive members.

spBeam uses elastic analysis (stiffness method) to obtain deflections along the beam span by discretizing the beam span into 110 elements. It also takes into account the adjacent spans effects, shape effects, supporting members stiffnesses above and below the beam, and cracked section effects based on the applied forces. This level of detail provides the maximum accuracy possible compared with other approximate methods used to calculate deflections.

In tables 9 and 10, the deflection value calculated by spBeam is lower than all the other values calculated by the approximate methods. This can be expected since approximate methods have a built-in conservatism to accommodate a wide range of applications and conditions. The designer can use spBeam and exploit its numerous features to get a closer deflection estimate and optimize the depth and size of the beam or slab under consideration.

9. Boundary condition effects on continuous beam deflections

Boundary conditions can have significant effects on the behavior of continuous beams especially deflections. This section focuses on investigating boundary conditions commonly found in building and used with continuous beams. In each section we will explore when to use each condition, and how to properly model them using spBeam.

The following discussion provides recommendations for each of the grids in the building floor system under consideration, observations on the impact of the condition, and the proper modeling method in spBeam.

9.1. Beam supported by columns- Grid 3

The length and size of the columns above and below the beam are modeled allowing spBeam to calculate the rotational stiffness for each member at the joint to correctly determine the beam end moments.

In the support date, the stiffness % of 100 indicates full utilization of the column and beam geometry. Whereas a value of 999 will impose a fixed support condition and a value of zero will impose a pinned support condition ignoring the supporting columns entirely.

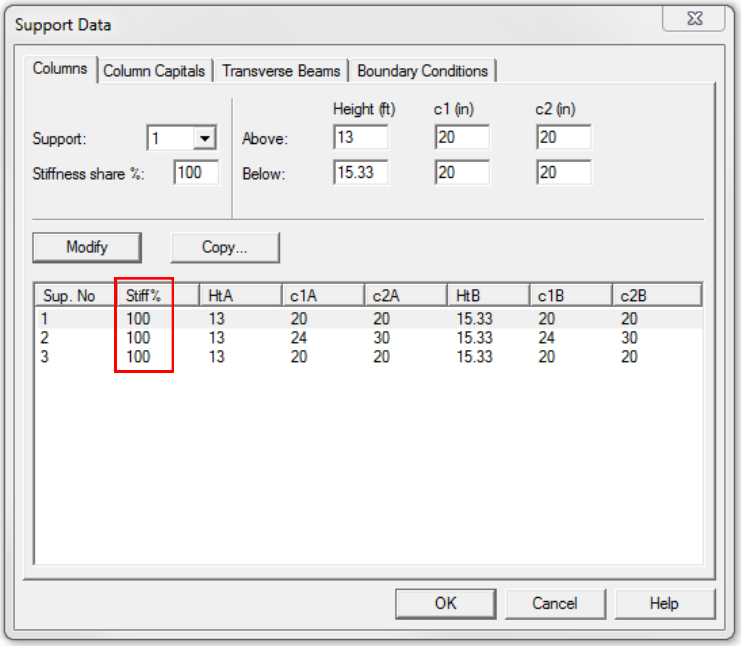


Figure 16 – Beam Supported by Columns – Defining Supports Geometry (spBeam)

9.2. Beam supported by transverse beams - Grid 4

To model transverse beam as a support, the cross-sectional dimensions and length **should not** be entered as shown below since this tab is reserved for two-way slab systems with beams between all supports.

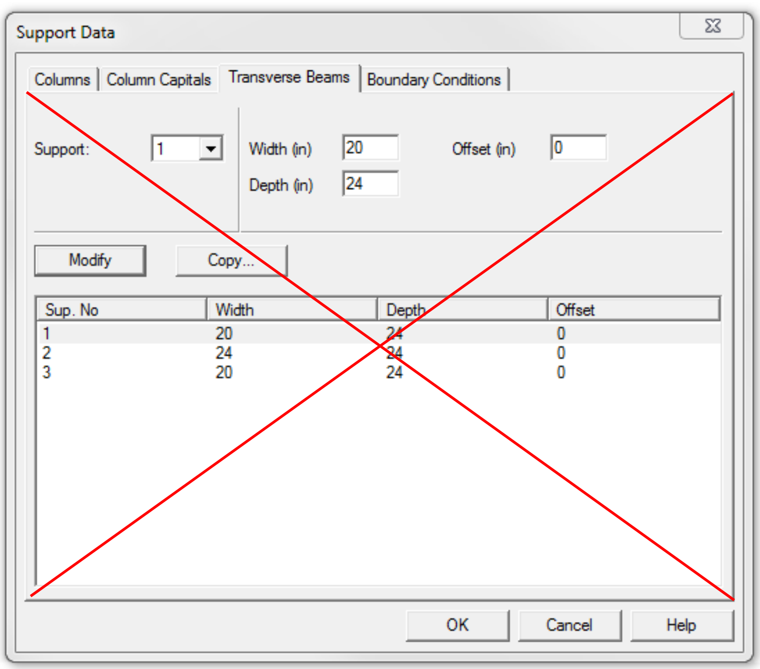


Figure 17 – Beam Supported by Transverse Beams – Defining Supports Geometry for two-way Slab systems (spBeam)

Alternately, the designer should enter a value for the rotational stiffness for the supporting transverse beams calculated by an external method if desired and to provide additional flexibility for special cases as the boundary conditions tab shown below.

The ACI code approximates the moment at transverse beam end support to 2/3 the moment at column end support. This can be used as a reference to find the rotational stiffness of transverse beams by trial and error.

ACI 318-14 (Table 6.5.2)

For Grid 4, adding the approximated rotational stiffnesses shown in the following Figure will result in (204.6 kip-ft) negative moment at the end support which is 2/3 the negative moment at the end support for Grid 3 (304.6 kip-ft).

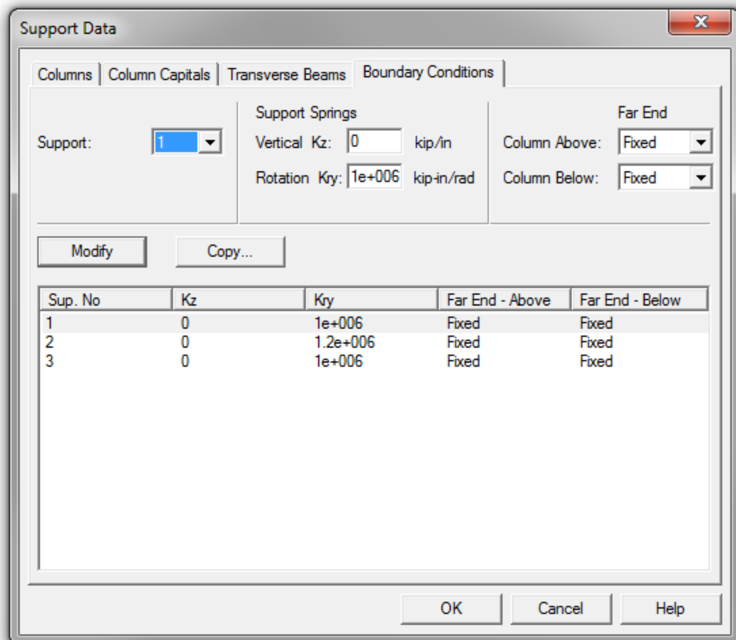


Figure 18 – Beam Supported by Transverse Beams – Defining Supports Stiffnesses (spBeam)

spBeam designs the beam for the moment at the center of the support when a rotation stiffness is defined. To design the beam for moments at the face of the supporting transverse beam, a dummy column can be defined with all parameters equals to zero except c_1 where it should be set equal to the width of the transverse beam. spBeam will obtain shear and moment values at critical sections and use them for design.

In Grid 2, a combination of the boundary conditions shown in grids 3 and 4 should be utilized where the end columns are modeled for support 1 and 3 and a transverse beam is modeled for support 2 as shown below.

9.3. Beam supported by transverse walls - Grid 5

In this model the beam is cast monolithically with the shear wall, the wall define as an elongated column.

9.4. Beam supported by masonry bearing walls - Grid C

The wall can be modeled as pin support with a stiffness percent of zero.

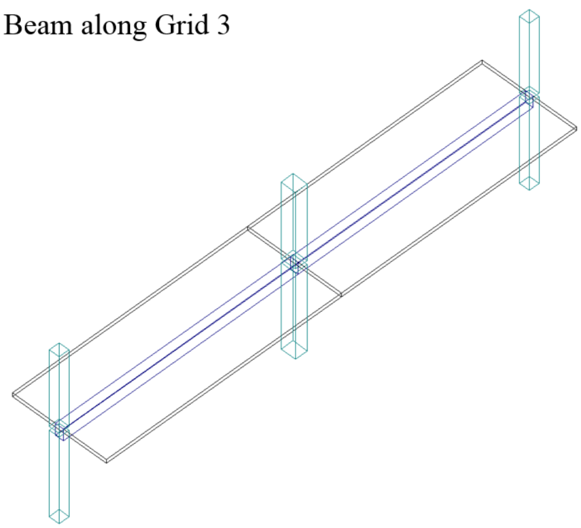
9.5. Beam supported by longitudinal walls - Grids 1, 6, and A

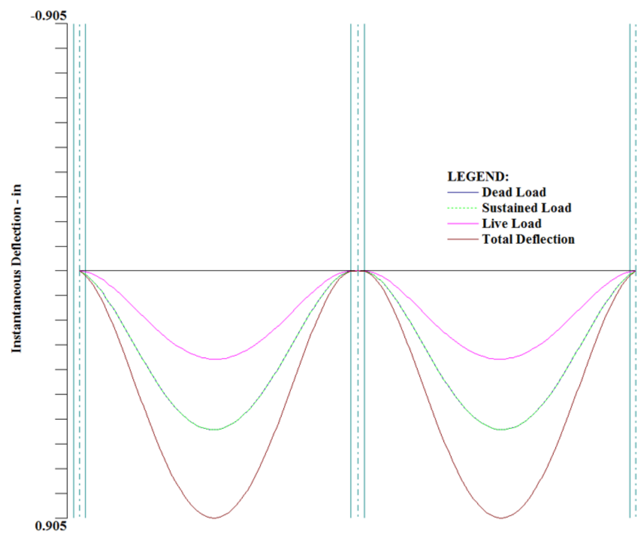
The beam can be modeled up to the face of the wall. At the face of the wall the beam should be restrained by a fixed support and the wall width can be ignored.

To illustrate the effects of boundary conditions, beams along grids 2, 3, 4 and 5 in Figure 1 are modeled using spBeam and deflections obtained from these models are compared.

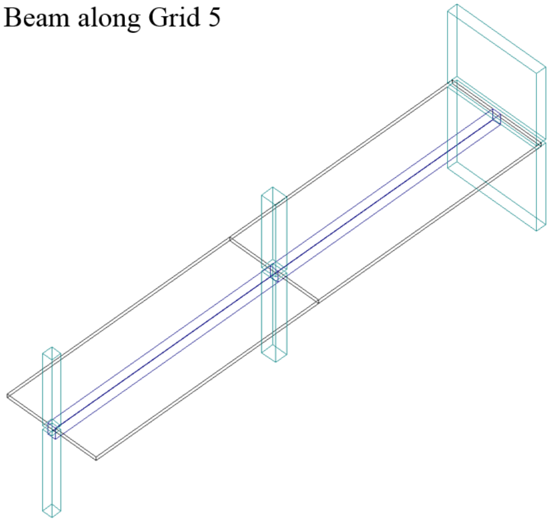
Table 12 - Comparison of Continuous Beams with Different Boundary Conditions (Deflections)				
Grid #	$(\Delta_{inst})_{total}$, in.	Δ_{cs}, in.	Δ_{lt}, in.	Notes
3	0.90	1.16	2.07	All columns are modeled with the exact geometry
5	0.69 (-23.3 %)	0.82 (-29.8 %)	1.51 (-27.0 %)	Transverse shear wall is modeled with the exact geometry for the right span
2*	0.90 (0.0 %)	1.16 (0.0 %)	2.07 (0.0 %)	The middle support is modeled as transverse beam using rotational stiffness with dummy column to obtain shear and moment values at the critical sections.
4	0.92 (+2.7 %)	1.31 (+12.5 %)	2.24 (+8.2 %)	All supports are modeled as transverse beams using rotational stiffness with dummy columns to obtain shear and moment values at the critical sections.
* The interior support boundary condition has slight effect on results due to symmetry for the case of continuous beam with two spans				

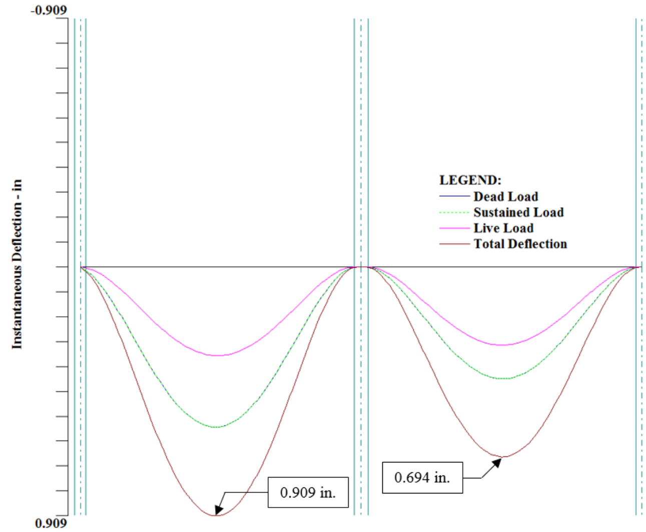
Beam along Grid 3



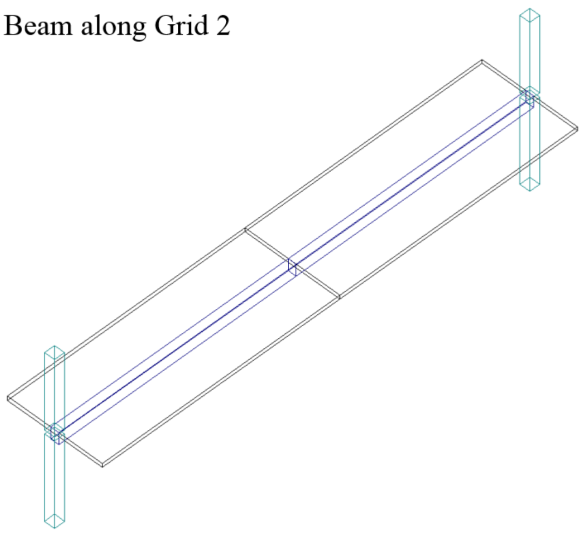


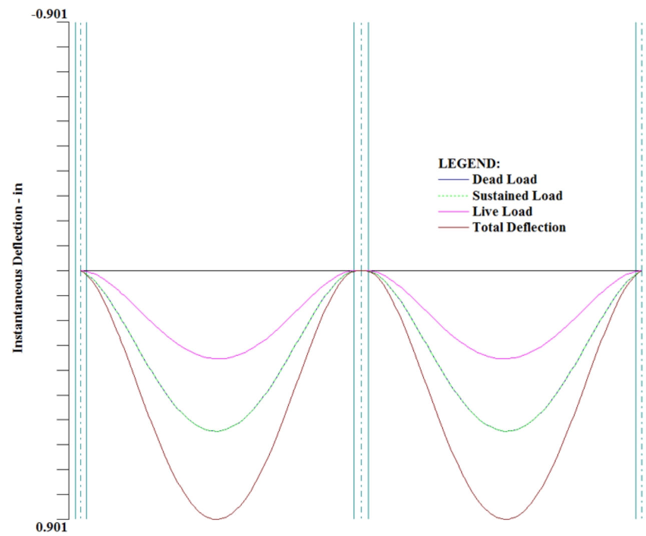
Beam along Grid 5





Beam along Grid 2





Beam along Grid 4