Reinforced Concrete Cantilever Retaining Wall Analysis and Design (ACI 318-14)
Reinforced Concrete Cantilever Retaining Wall Analysis and Design (ACI 318-14)

Reinforced concrete cantilever retaining walls consist of a relatively thin stem and a base slab. The stem may have constant thickness along the length or may be tapered based on economic and construction criteria. The base is divided into two parts, the heel and toe. The heel is the part of the base under the backfill. This system uses much less concrete than monolithic gravity walls, but require more design and careful construction. Cantilever retaining walls can be precast in a factory or formed on site and considered economical up to about 25 ft in height. This design example focuses on the analysis and design of a tapered cantilever retaining wall including a comparison with model results from the engineering software programs spWall and spMats. The retaining wall is fixed to the reinforced concrete slab foundation with a shear key for sliding resistance. The following figure and design data section will serve as input for detailed analysis and design.

Figure 1 – Cantilever Retaining Wall Dimensions
Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

- spWall Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2022
- spMats Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2020

Design Data

<table>
<thead>
<tr>
<th>Wall Stem Materials</th>
<th>Wall Foundation Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c' = 4,500$ psi</td>
<td>$f_c' = 4,500$ psi</td>
</tr>
<tr>
<td>$f_y = 60,000$ psi</td>
<td>$f_y = 60,000$ psi</td>
</tr>
<tr>
<td>$\gamma_c = 150$ pcf</td>
<td>$\gamma_c = 150$ pcf</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wall Stem Dimensions</th>
<th>Wall Foundation Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>Width = 1 ft strip</td>
</tr>
<tr>
<td>Height</td>
<td>Length = 9.75 ft</td>
</tr>
<tr>
<td>Thickness = 8 in. top</td>
<td>Thickness = 18 in.</td>
</tr>
<tr>
<td>= 16 in. bottom</td>
<td></td>
</tr>
</tbody>
</table>

Retaining Wall Loads

The following figure shows all the loads applied to the cantilever retaining wall where:

\[
W_1 = 0.67 \times 13.5 \times 150 = 1360 \text{ lb}
\]

\[
W_2 = 0.67 \times 0.5 \times 13.5 \times 150 = 680 \text{ lb}
\]

\[
W_3 = 9.75 \times 1.5 \times 150 = 2190 \text{ lb}
\]

\[
W_4 = 1.33 \times 1.25 \times 150 = 250 \text{ lb}
\]

\[
W_5 = 3.75 \times 2 \times 120 = 900 \text{ lb}
\]

\[
W_6 = 0.67 \times 0.5 \times 13.5 \times 120 = 540 \text{ lb}
\]

\[
W_6 = 4.67 \times 13.5 \times 120 = 7570 \text{ lb}
\]
Figure 2 – Applied Loads and Soil Pressure at Critical Sections
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1. Preliminary Design

The thickness of the footing is roughly estimated to calculate the required thickness of the stem at the critical section (stem bottom). With the bottom of the footing at 3.5 ft below grade and an estimated footing thickness of 1.5 ft, the free height of the stem is 13.5 ft. Using the information provided in Figures 1 and 2:

\[ P = 0.5 \times 0.333 \times 120 \times 13.5 \times (13.5 + 2 \times 3.33) = 5440 \text{ lb (at the stem bottom)} \]

\[ y = \frac{13.5^2 + 3 \times 13.5 \times 3.33}{3 \times (13.5 + 2 \times 3.33)} = 5.25 \text{ ft} \]

\[ M_u = P_y = 1.6 \times 5440 \times 5.25 = 45.7 \text{ ft-kip} \]

![Figure 3 – Bearing Pressure, Overturning and Sliding Loads](image)

The preliminary dimensions are selected using design aids from the reference Appendix A.

\[ \rho_{0.005} = 0.85 \times \beta_1 \times \frac{f_c}{f_y} \times \frac{0.003}{0.003 + 0.005} \]

Reference 1 (Table A.4)

\[ \rho_{0.005} = 0.85 \times 0.825 \times \frac{4500}{60000} \times \frac{0.003}{0.003 + 0.005} = 0.0197 \]
The reference recommends the use of a ratio of about 40% of the maximum ($\rho = 0.008$) for economy and ease of bar placement.

\[
\frac{M_u}{\phi \times b \times d^2} = 430
\]

\[d = \sqrt{\frac{45700 \times 12}{0.9 \times 12 \times 430}} = 10.9 \text{ in.}
\]

Using cover of 2 in. for members exposed to weather or in contact with ground. **ACI 318-14 (Table 20.6.1.3.1)**

And #8 bars ($d_b = 1$ in.), the minimum required thickness of the stem at the base equals:

\[
\text{minimum } t_{\text{stem,base}} = d_{\text{min}} + \text{cover} + \frac{d_b}{2} = 10.9 + 2 + \frac{1}{2} = 13.4 \text{ in.}
\]

Use $t_{\text{stem,base}} = 16$ in.

For Shear Check (at distance $d$ above the base):

\[
P = 0.5 \times 0.333 \times 120 \times 12.5 \times (12.5 + 2 \times 3.33) = 4800 \text{ lb}
\]

\[
V_u = 1.6 \times P = 1.6 \times 4800 = 7680 \text{ lb}
\]

\[
\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f_c} \times b \times d^2
\]

\[
\phi V_c = 0.75 \times 2 \times 1 \times \sqrt{4500 \times 12 \times 13.5^2} = 16300 \text{ lb} > V_u
\]

Stem thickness of 16 in. is adequate to resist the factored shear force.

The thickness of the foundation (base) is the same as or slightly larger than that at the bottom of the stem. Thus, the 18 in. selected earlier need not be revised. The stem thickness can be reduced by tapering one side only up to 8 in. at the top since the bending moment decreases with increasing distance from the wall base to zero at the top of the wall.
2. Wall Stability Checks

The wall has two failure modes: 1) Wall parts may not be strong enough to resist the acting forces, 2) the wall as a rigid body may be displaced or overturned by the earth pressure acting on it. The latter will be discussed in this section to ensure that the retaining wall is stable by checking stability against overturning, sliding, and allowable soil bearing pressure.

Note: two cases are being examined. Case 1 where surcharge load is applied to point a (see Figure 3), and Case 2 where surcharge load is applied to point b.

2.1. Wall Overturning Check

Case 1 governs for wall overturning since it generated the highest overturning with the least resistance.

Weights and moments about the front edge of the wall are shown in the following table (See figure 2 and design data section):

<table>
<thead>
<tr>
<th>Component</th>
<th>W, kips</th>
<th>x, ft</th>
<th>Mr, ft-kip</th>
</tr>
</thead>
<tbody>
<tr>
<td>W₁</td>
<td>1.36</td>
<td>4.08</td>
<td>5.55</td>
</tr>
<tr>
<td>W₂</td>
<td>0.68</td>
<td>4.67</td>
<td>3.18</td>
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<tr>
<td>W₃</td>
<td>2.19</td>
<td>4.88</td>
<td>10.69</td>
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<tr>
<td>W₄</td>
<td>0.25</td>
<td>4.42</td>
<td>1.11</td>
</tr>
<tr>
<td>W₅</td>
<td>0.90</td>
<td>1.88</td>
<td>1.69</td>
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<tr>
<td>W₆</td>
<td>0.54</td>
<td>4.86</td>
<td>2.62</td>
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<tr>
<td>W₇</td>
<td>7.57</td>
<td>7.42</td>
<td>56.17</td>
</tr>
<tr>
<td>Total</td>
<td>13.49</td>
<td></td>
<td>81.00</td>
</tr>
</tbody>
</table>

\[ P = 0.5 \times 0.333 \times 120 \times 15 \times (15 + 2 \times 3.33) = 6.49 \text{ kips} \]

\[ y = \frac{15^2 + 3 \times 15 \times 3.33}{3 \times (15 + 2 \times 3.33)} = 5.77 \text{ ft} \]

The overturning moment is equal to:

\[ M_o = P \times y = 6492 \times 5.77 = 37.46 \text{ ft-kip} \]

Factor of Safety against overturning:

\[ FOS_{overturning} = \frac{81.00}{37.46} = 2.16 > 1.5 \text{ (o.k.)} \]
2.2. Soil Bearing Pressure

The distance of the resultant force from the base slab front edge is:

\[ a = \frac{81.00 - 37.46}{13.49} = 3.23 \text{ ft} \approx \frac{9.75}{3} = 3.25 \text{ ft} \]

The resultant is barely outside the middle third of the foundation (it is assumed that the bearing pressure becomes zero exactly at the edge of the heel as shown in Figure 2). The maximum soil pressure at the toe is calculated as follows:

\[ q_1 = \frac{2 \times R}{3 \times a} \]

Reference 1 (Figure 16.5c)

\[ q_1 = \frac{2 \times 13.49 \times 1000}{3 \times 3.23} = 2784 \text{ psf} < q_{\text{allowable}} = 8000 \text{ psf (o.k.)} \]

\[ q_2 = 0 \]

Reference 1 (Figure 16.5c)

The soil pressure values calculated for Case 1. The soil pressure values for Case 2 do not govern for overturning and sliding. However, values calculated from Case 2 are needed for foundation flexural design as follows:

\[ q_1 = (4 \times l - 6 \times a) \frac{R}{l^2} \]

Reference 1 (Figure 16.5a)

\[ q_1 = 2710 \text{ psf} < q_{\text{allowable}} = 8000 \text{ psf (o.k.)} \]

\[ q_2 = (6 \times a - 2 \times l) \frac{R}{l^2} \]

Reference 1 (Figure 16.5a)

\[ q_2 = 492 \text{ psf} < q_{\text{allowable}} = 8000 \text{ psf (o.k.)} \]
2.3. Wall Sliding Check

Case 1 also governs for sliding since it produces the least pressure and corresponding friction resistance.

The coefficient of friction that applies for the length along the heel and key is 0.5, while the coefficient of friction for the length in front of the key is equal to the internal soil friction, that is, \( \tan 30^\circ = 0.577 \). More information about selecting the friction coefficient can be found in the reference in chapter 16 section 4. (for case where surcharge load is applied to point a):

Friction, toe:

\[
F_{\text{toe}} = 0.5 \times (2784 + 1713) \times 3.75 \times 0.577 = 4.87 \text{ kips}
\]

Friction, heel and key:

\[
F_{\text{heel and key}} = 0.5 \times 1713 \times 6 \times 0.5 = 2.57 \text{ kips}
\]

Passive earth pressure:

\[
P_{\text{passive}} = 0.5 \times 3.0 \times 120 \times (4.75 - 1.5)^2 = 1.90 \text{ kips}
\]

Note that the top 1.5 ft layer of soil is discounted in this check as unreliable.

Total:

\[
F_{\text{total}} = 4.87 + 2.57 + 1.90 = 9.34 \text{ kips}
\]

Factor of Safety against sliding:

\[
FOS_{\text{sliding}} = \frac{9.34}{6.49} = 1.44 \approx 1.5 \text{ (can be regarded as adequate)}
\]

Thus, the retaining wall with the selected geometry is externally stable.
3. Flexural Reinforcement Requirements

The required flexural reinforcement is traditionally calculated at three critical sections: at the stem base, the toe and heel at the face of the stem.

Calculate the required reinforcement to resist the moment at the stem base:

\[ M_u = 45.7 \text{ kip-ft} \]

Use #8 bars with 2.0 in. concrete cover per **ACI 318-14 (Table 20.6.1.3.1)**. The distance from extreme compression fiber to the centroid of longitudinal tension reinforcement, \( d \), is calculated below:

\[ d = 16 - (2 + 0.5 \times 1) = 13.5 \text{ in.} \]

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the beam section \( (jd) \). In this example, tension-controlled section will be assumed so the reduction factor \( \phi \) is equal to 0.9, and \( jd \) will be taken equal to 0.95\( d \). The assumptions will be verified once the area of steel is finalized.

\[ jd = 0.95 \times d = 0.95 \times 13.5 = 12.83 \text{ in.} \]

\[ b = 12 \text{ in.} \]

The required reinforcement at initial trial is calculated as follows:

\[ A_s = \frac{M_u}{\phi \times f_y \times jd} = \frac{45.7 \times 12,000}{0.9 \times 60,000 \times 12.83} = 0.79 \text{ in.}^2 \]

Recalculate ‘\( a \)’ for the actual \( A_s = 0.79 \text{ in.}^2 \):

\[ a = \frac{A_s \times f_y}{0.85 \times f'_c \times b} = \frac{0.79 \times 60,000}{0.85 \times 4500 \times 12} = 1.04 \text{ in.} \]

\[ c = \frac{a}{\beta_l} = \frac{1.04}{0.83} = 1.25 \text{ in.} \]

\[ \varepsilon_i = \left( \frac{0.003}{c} \right) \times d_i - 0.003 = \left( \frac{0.003}{1.25} \right) \times 13.5 - 0.003 = 0.0293 > 0.005 \]

Therefore, the assumption that section is tension-controlled is valid.

\[ A_s = \frac{M_u}{\phi \times f_y \times (d - a / 2)} = \frac{45.7 \times 12,000}{0.9 \times 60,000 \times (13.5 - 1.04 / 2)} = 0.78 \text{ in.}^2 \]

The minimum reinforcement shall not be less than

\[ A_{s,\text{min}} = \frac{3 \times f'_c}{f_y} \times b \times d = \frac{3 \times 4,500}{60,000} \times 12 \times 13.5 = 0.54 \text{ in.}^2 \]  **ACI 318-14 (9.6.1.2(a))**

And not less than

\[ A_{s,\text{min}} = \frac{200}{f_y} \times b \times d = \frac{200}{60,000} \times 12 \times 13.5 = 0.54 \text{ in.}^2 \]  **ACI 318-14 (9.6.1.2(b))**
\[ A_{s,\text{min}} = 0.54 \text{ in.}^2 \]

Maximum spacing allowed:

Check the requirement for distribution of flexural reinforcement to control flexural cracking:

\[ s = 15 \left( \frac{40000}{f_s} \right) - 2.5c_e \leq 12 \left( \frac{40000}{f_s} \right) \]

\[ c_e = 2.0 \text{ in.} \]

Use \( f_s = \frac{2}{3} f_y = 40,000 \text{ psi} \)

\textit{ACI 318-14 (Table 24.3.2)}

\[ s = 15 \times \left( \frac{40,000}{40,000} \right) - 2.5 \times 2.0 = 10 \text{ in.} \] (Governs)

\[ s = 12 \times \left( \frac{40,000}{40,000} \right) = 12 \text{ in.} \]

Provide #8 bars at 9 in. on centers.

Note that the stem bending moment decreases rapidly with increasing distance from the bottom. For this reason, only part of the main reinforcement is needed at higher elevations and alternate bars can be discontinued where no longer needed. More information about cutting bars in the stem are provided in the reference. All the values in the following table are calculated based on the procedure outlined above for the stem.

<table>
<thead>
<tr>
<th>Critical Section</th>
<th>Stem Base</th>
<th>Toe</th>
<th>Heel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Moment, ( M_u ) (ft-kips)</td>
<td>45.7</td>
<td>24.3</td>
<td>29.9</td>
</tr>
<tr>
<td>Effective depth, ( d ) (in.)</td>
<td>13.5</td>
<td>14.5</td>
<td>14.5</td>
</tr>
<tr>
<td>( A_{s,\text{req}} ) (in.(^2))</td>
<td>0.78</td>
<td>0.38</td>
<td>0.47</td>
</tr>
<tr>
<td>( A_{s,\text{min}} ) (in.(^2))</td>
<td>0.54</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>#8 @ 9 in.</td>
<td>#7 @ 12 in.</td>
<td>#7 @ 12 in.</td>
</tr>
</tbody>
</table>
4. Cantilever Retaining Wall Analysis and Design – spWall Software

spWall is a program for the analysis and design of reinforced concrete shear walls, tilt-up walls, precast walls, retaining walls, tank walls and Insulated Concrete Form (ICF) walls. It uses a graphical interface that enables the user to easily generate complex wall models. Graphical user interface is provided for:

- Wall geometry (including any number of openings and stiffeners)
- Material properties including cracking coefficients
- Wall loads (point, line, and area),
- Support conditions (including translational and rotational spring supports)

spWall uses the Finite Element Method for the structural modeling, analysis, and design of slender and non-slender reinforced concrete walls subject to static loading conditions. The wall is idealized as a mesh of rectangular plate elements and straight-line stiffener elements. Walls of irregular geometry are idealized to conform to geometry with rectangular boundaries. Plate and stiffener properties can vary from one element to another but are assumed by the program to be uniform within each element.

Six degrees of freedom exist at each node: three translations and three rotations relating to the three Cartesian axes. An external load can exist in the direction of each of the degrees of freedom. Sufficient number of nodal degrees of freedom should be restrained in order to achieve stability of the model. The program assembles the global stiffness matrix and load vectors for the finite element model. Then, it solves the equilibrium equations to obtain deflections and rotations at each node. Finally, the program calculates the internal forces and internal moments in each element. At the user’s option, the program can perform second order analysis. In this case, the program takes into account the effect of in-plane forces on the out-of-plane deflection with any number of openings and stiffeners.

In spWall, the required flexural reinforcement is computed based on the selected design standard (ACI 318-14 is used in this case study), and the user can specify one or two layers of wall reinforcement. In stiffeners and boundary elements, spWall calculates the required shear and torsion steel reinforcement. Wall concrete strength (in-plane and out-of-plane) is calculated for the applied loads and compared with the code permissible shear capacity.

For illustration purposes, the following figures provide a sample of the input modules and results obtained from an spWall model created for the cantilever retaining wall in this design example.
4.1. Cantilever Retaining Wall Model Input

Figure 4 – spWall Interface
Figure 5 – Assigning Soil Loads for Cantilever Retaining Wall (spWall)
Figure 6 – Solve and Mesh Options (spWall)
4.2. Cantilever Retaining Wall Result Contours

Figure 7 – Factored Axial Force Contour (spWall)
Figure 8 – Lateral Displacement Contour (Out-of-Plane) (spWall)
4.3. Cantilever Retaining Wall Cross-Sectional Forces

Figure 9 – Axial Load Diagram (spWall)
Figure 10 – Out-of-Plane Shear Diagram (spWall)
Figure 11 – Bending Moment Diagram (spWall)
Figure 12 – Required Vertical Reinforcement (spWall)

(Note: Minimum reinforcement value shown is based on the top wall stem thickness of 8” while the hand calculations show the minimum required at the wall stem base with 16” thickness)
4.4. Cantilever Retaining Wall Maximum Displacement

1. Results
1.1. Service
1.1.1. Nodal Displacements
1.1.1.1. 1.0D+1.0H
Coordinate System: Global

<table>
<thead>
<tr>
<th>Node</th>
<th>Dx</th>
<th>Dy</th>
<th>Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>271</td>
<td>0.000</td>
<td>0.000</td>
<td>0.154</td>
</tr>
<tr>
<td>272</td>
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<td>273</td>
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<tr>
<td>275</td>
<td>0.000</td>
<td>0.000</td>
<td>0.154</td>
</tr>
</tbody>
</table>

Figure 13 – Displacement at Critical Section (Service Combinations) (spWall)

1.2. Ultimate
1.2.1. Nodal Displacements
1.2.1.1. 1.2D+1.6H
Coordinate System: Global

<table>
<thead>
<tr>
<th>Node</th>
<th>Dx</th>
<th>Dy</th>
<th>Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>271</td>
<td>0.000</td>
<td>0.000</td>
<td>0.246</td>
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<td>272</td>
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<td>273</td>
<td>0.000</td>
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<tr>
<td>275</td>
<td>0.000</td>
<td>0.000</td>
<td>0.246</td>
</tr>
</tbody>
</table>

Figure 14 – Displacement at Critical Section (Ultimate Combinations) (spWall)

4.5. Cantilever Retaining Wall Cross-Sectional Forces at Stem Base

1.2.2. Wall Cross-Sectional Forces
1.2.2.1. 1.2D+1.6H
Coordinate System: Global
(+ ) Horizontal cross-section above Y-coordinate
(- ) Horizontal cross-section below Y-coordinate

<table>
<thead>
<tr>
<th>No.</th>
<th>Wall Crosssection</th>
<th>Y coordinate</th>
<th>X-Centroid</th>
<th>In-Plane Forces</th>
<th>Out-Of-Plane Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ft</td>
<td>ft</td>
<td>Vux kips</td>
<td>Vuz kips</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Nuy kips</td>
<td>Muz kip-ft</td>
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<tr>
<td></td>
<td></td>
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<td></td>
<td>Muz kip-ft</td>
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<td></td>
<td></td>
<td>0.00</td>
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</tbody>
</table>

Figure 15 – Wall Cross-Sectional Forces (spWall)
4.6. Cantilever Retaining Wall Reinforcement

1.3. Envelope
1.3.1. Plate Flexure Reinforcement

Coordinate System: Global

<table>
<thead>
<tr>
<th>Element</th>
<th>Curtains</th>
<th>Direction</th>
<th>Mu (x/y) kip-ft/ft</th>
<th>Nu (x/y) kips/ft Ld Comb.</th>
<th>As (x/y) in²/ft</th>
<th>Rho</th>
<th>Tie</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Horizontal</td>
<td>8.00</td>
<td>-0.40 0.90+1.6H</td>
<td>0.384</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vertical</td>
<td>46.16</td>
<td>-1.80 0.90+1.6H</td>
<td>0.777</td>
<td>0.40</td>
<td></td>
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<tr>
<td>2</td>
<td>1</td>
<td>Horizontal</td>
<td>9.34</td>
<td>-0.24 0.90+1.6H</td>
<td>0.384</td>
<td>0.20</td>
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<tr>
<td></td>
<td></td>
<td>Vertical</td>
<td>47.07</td>
<td>-1.78 0.90+1.6H</td>
<td>0.792</td>
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<tr>
<td>3</td>
<td>1</td>
<td>Horizontal</td>
<td>9.34</td>
<td>-0.24 0.90+1.6H</td>
<td>0.384</td>
<td>0.20</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Vertical</td>
<td>47.07</td>
<td>-1.78 0.90+1.6H</td>
<td>0.792</td>
<td>0.41</td>
<td></td>
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<tr>
<td>4</td>
<td>1</td>
<td>Horizontal</td>
<td>8.00</td>
<td>-0.40 0.90+1.6H</td>
<td>0.384</td>
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<tr>
<td></td>
<td></td>
<td>Vertical</td>
<td>46.16</td>
<td>-1.80 0.90+1.6H</td>
<td>0.777</td>
<td>0.40</td>
<td></td>
</tr>
</tbody>
</table>

Elements along the wall base

Figure 16 – Required Vertical Reinforcement (spWall)
5. Cantilever Retaining Wall Foundation Analysis and Design – spMats Software

spMats uses the Finite Element Method for the structural modeling, analysis and design of reinforced concrete slab systems or mat foundations subject to static loading conditions.

The slab, mat, or footing is idealized as a mesh of rectangular elements interconnected at the corner nodes. The same mesh applies to the underlying soil with the soil stiffness concentrated at the nodes. Slabs of irregular geometry can be idealized to conform to geometry with rectangular boundaries. Even though slab and soil properties can vary between elements, they are assumed uniform within each element. Piles and/or supporting soil are modeled as springs connected to the nodes of the finite element model.

For illustration purposes, the following figures provide a sample of the input modules and results obtained from an spMats model created for the cantilever retaining wall foundation in this design example.

5.1. Cantilever Retaining Wall Foundation Model Input

![Figure 17 – spMats Interface](image-url)
Figure 18 – Assigning Soil Lateral Moment for Cantilever Retaining Wall Foundation (spMats)
Figure 19 – Assigning Soil Toe Load for Cantilever Retaining Wall Foundation (spMats)
Figure 20 – Assigning Soil Heel Load for Cantilever Retaining Wall Foundation (spMats)
Figure 21 – Assigning Surcharge Load for Cantilever Retaining Wall Foundation (spMats)
Figure 22 – Assigning Wall Load for Cantilever Retaining Wall Foundation (spMats)
Figure 23 – Solve and Mesh Options (spMats)
5.2. Cantilever Retaining Wall Foundation Result Contours

Figure 24 – Vertical (Down) Displacement Contour (spMats)
Figure 25 – Vertical (Up) Displacement Contour (spMats)
(Note: figure indicates no uplift in the wall base)
Figure 26 – Soil Bearing Pressure Contour for Case 1 (spMats)
Figure 27 – Soil Bearing Pressure Contour for Case 2 (spMats)
Figure 28 – Moment Contour along X-Axis (Max for Toe) (spMats)
Figure 29 – Moment Contour along X-Axis (Max for Heel) (spMats)
5.3. Cantilever Retaining Wall Foundation Required Reinforcement

Figure 30 – Required Reinforcement Contour along X Direction (Bottom – Toe Design) (spMats)
(Note: minimum reinforcement governs)
Figure 31 – Required Reinforcement Contour along X Direction (Top – Heel Design) (spMats)
(Note: minimum reinforcement governs)
5.4. Soil Reactions / Pressure

1. Results
1.1. Service
1.1.1. Sum of Reactions

1.1.1.1. S2

NOTES:
Sum of all forces and moments with respect to center of gravity (X,Y) = (4.88, 0.50) ft

<table>
<thead>
<tr>
<th>Sum of Reactions</th>
<th>Fz</th>
<th>Mx</th>
<th>My</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>kips</td>
<td>kip-ft</td>
<td>kip-ft</td>
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<tr>
<td>Soil</td>
<td>13.24</td>
<td>0.00</td>
<td>22.16</td>
</tr>
<tr>
<td>Springs</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Piles</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Restraints</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Slaved nodes</td>
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<td>0.00</td>
<td>0.00</td>
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<tr>
<td>Total Reactions</td>
<td>13.24</td>
<td>0.00</td>
<td>22.16</td>
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<tr>
<td>Total loads</td>
<td>-13.24</td>
<td>0.00</td>
<td>-22.16</td>
</tr>
</tbody>
</table>

Figure 32 – Soil Service Reactions

1.1.2. Soil Disp. & Pressure

1.1.2.1. S2

NOTES:
[x] Indicates allowable pressure is exceeded.

<table>
<thead>
<tr>
<th>Element</th>
<th>Node</th>
<th>Disp, Dz</th>
<th>Pressure, Qz</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>in</td>
<td>ksf</td>
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<tr>
<td>83</td>
<td>128</td>
<td>-0.080</td>
<td>-2.678</td>
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<td>127</td>
<td>-0.082</td>
<td>-2.746</td>
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<tr>
<td>123</td>
<td>168</td>
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<tr>
<td></td>
<td>167</td>
<td>-0.002</td>
<td>-0.083</td>
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</table>

<table>
<thead>
<tr>
<th>Element</th>
<th>Node</th>
<th>Disp, Dz</th>
<th>Pressure, Qz</th>
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<tr>
<td></td>
<td>125</td>
<td>-0.002</td>
<td>-0.083</td>
</tr>
</tbody>
</table>

Figure 33 – Soil Bearing Pressure

Case 1

Case 2
5.5. Cantilever Retaining Wall Foundation Mesh Status

Since spMats is utilizing finite element analysis to model and design the foundation. It is useful to track the number of elements used in the model to optimize the model results (accuracy) and running time (processing stage). spMats provides mesh status to keep tracking the mesh sizing as a function of the number of elements, minimum and maximum element sizes, and maximum aspect ratio.

<table>
<thead>
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<tbody>
<tr>
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<td>Min. element size</td>
<td>0.16 ft</td>
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<tr>
<td>Max. element size</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Max. aspect ratio</td>
<td>1.52</td>
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</table>

Figure 34 – Mesh Status
6. Cantilever Retaining Wall Analysis and Design Results Comparison & Conclusions

| Table 3 - Cantilever Retaining Wall Flexural Results |
|---------------------------------|--------|----------|
| Method of Solution              | $M_u$, kip-ft/ft | $A_{s,req}$, in.$^2$/ft |
| Reference                       | 45.70  | 0.79     |
| Hand                            | 45.70  | 0.78     |
| spWall                          | 45.64  | 0.79     |

<table>
<thead>
<tr>
<th>Table 4 - Cantilever Retaining Wall Foundation Soil Bearing Pressure</th>
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</thead>
<tbody>
<tr>
<td>Method of Solution</td>
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<tr>
<td>---------------------</td>
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<tr>
<td>Reference</td>
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<td>Hand</td>
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<td>spMats</td>
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<table>
<thead>
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<th>Table 5 - Cantilever Retaining Wall Foundation Results</th>
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<tr>
<td>Reference</td>
</tr>
<tr>
<td>Hand</td>
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<tr>
<td>spMats</td>
</tr>
</tbody>
</table>

* the downward load of the earth fill over the toe is neglected by the reference
** the upward reaction of the soil is neglected by the reference

The results of all the hand calculations and the reference used illustrated above are in agreement with the automated exact results obtained from the spWall and spMats programs.

Note that the hand and reference considered the toe and heel as cantilever projecting outward and inward from the face of the stem, respectively. spMats provides the flexibility of modeling the foundation with the exact geometry and boundary conditions to achieve more accurate results leading to potential savings in the reinforcement required.

Some load cases were neglected by the reference for simplicity and to achieve a more conservative design. On the other hand, spMats take into account all the applied load cases and include them in the calculations of the required reinforcement for the toe and heel. Additional load combination can be easily employed in spMats to explore more loading scenarios to meet project criteria.