Reinforced Concrete Shear Wall Analysis and Design (CSA A23.3-14)
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A structural reinforced concrete shear wall in a 6-story building provides lateral and gravity load resistance for the applied load as shown in the figure below. Shear wall section and assumed reinforcement is investigated after analysis to verify suitability for the applied loads based on CSA A23.3-14 provisions, then compared with numerical analysis results obtained from spWall engineering software program from StructurePoint.

![Figure 1 – Reinforced Concrete Shear Wall Geometry and Loading](image)

\[
\begin{align*}
\text{D} &= 634.9 \text{ kN} \\
\text{L} &= 136.5 \text{ kN}
\end{align*}
\]
Code

Design of Concrete Structures (CSA A23.3-14) and Explanatory Notes on CSA Group standard A23.3-14 “Design of Concrete Structures”

References


spWall Engineering Software Program Manual v5.01, STRUCTUREPOINT, 2016

“Reinforced Concrete Shear Wall Analysis and Design (ACI 318-14)” Design Example, STRUCTUREPOINT, 2018

Design Data

\( f'_{c} = 40 \text{ MPa normal weight concrete (} w_c = 24 \text{ kN/m}^3) \)

\( f_y = 400 \text{ MPa} \)

Wind Pressure = 1.5 kPa

Uniformly distributed loads on the roof:

\( w_D = 5.5 \text{ kPa} \)
\( w_L = 1.5 \text{ kPa} \)

Uniformly distributed loads on a typical floor:

\( w_D = 6 \text{ kPa} \)
\( w_L = 2.4 \text{ kPa} \)

Height between floors = 4.2 m.

Assumed wall thickness = 200 mm

Wall length = 7 m.

Assumed vertical reinforcement: \( A_{v, \text{vertical}} = 10M @ 300 \text{ mm (one curtain)} \)

Assumed horizontal reinforcement: \( A_{h, \text{horizontal}} = 10M @ 250 \text{ mm (one curtain)} \)
1. Preliminary Member Sizing

\[
\begin{align*}
t &\geq \min \left\{ \frac{l_u}{25}, \frac{h_u}{25}, \frac{h_u}{150} \right\} = \min \left\{ \frac{7000}{25}, \frac{4200}{25}, \frac{150}{150} \right\} = \left\{ \frac{280}{25}, \frac{168}{25} \right\} = \{168\} = 168 \text{ mm}
\end{align*}
\]

Choose \( t = 200 \text{ mm} \) (first trial)

2. Load and Load Combination

For the factored load

\[
w_{u1} = 1.25 \times DL + 0.5 \times LL + 1.4 \times W
\]

\[
w_{u2} = 0.9 \times DL + 1.4 \times W
\]

Dead Loads

Tributary area: \( A_{\text{trib}} = 14 \times 6.5 = 91 \text{ m}^2 \)

On the roof and self-weight (per floor): \( 5.50 \times 91 + \left[ 0.2 \times 7.0 \times (4.2 - 0.2) \right] \times 24 = 634.90 \text{ kN} \)

On a typical floor and self-weight (per floor): \( 6.00 \times 91 + \left[ 0.2 \times 7.0 \times (4.2 - 0.2) \right] \times 24 = 680.40 \text{ kN} \)

Live Loads

On the roof: \( 1.5 \times 91 = 136.50 \text{ kN} \)

On a typical floor: \( 2.4 \times 91 = 218.40 \text{ kN} \)

Wind Loads (W)

At the roof level: \( W_{\text{roof}} = 1.5 \times \frac{4.2}{2} \times \frac{33}{2} = 51.98 \text{ kN} \)

At a typical floor level: \( W_{\text{typ}} = 1.5 \times 4.2 \times \frac{33}{2} = 103.95 \text{ kN} \)
3. Structural Analysis

For the combination: (1.25 × D + 0.5 × L + 1.4 × W)

\[ P_f = 1.25 \times (634.90 + 680.40 \times 5) + 0.5 \times (136.50 + 218.40 \times 5) = 5,660.38 \text{ kN} \]

\[ V_f = 1.4 \times (51.98 + 103.95 \times 5) = 800.42 \text{ kN} \]

\[ M_f = 1.4 \times [51.98 \times 4.2 \times 6 + 103.95 \times 4.2 \times (1 + 2 + 3 + 4 + 5)] = 11,002.07 \text{ kN-m} \]

For the combination: (0.90 × D + 1.4 × W)

\[ P_f = 0.9 \times (634.90 + 680.40 \times 5) = 3,633.21 \text{ kN} \]

\[ V_f = 1.4 \times (51.98 + 103.95 \times 5) = 800.42 \text{ kN} \]

\[ M_f = 1.4 \times [51.98 \times 4.2 \times 6 + 103.95 \times 4.2 \times (1 + 2 + 3 + 4 + 5)] = 11,002.07 \text{ kN-m} \]
4. Shear Wall Design

Since the wall thickness is less than 210 mm, only one layer of reinforcement is therefore required.  

The minimum horizontal reinforcement is  \( A_{h,\text{min}} = 0.002 \times A_c \).  

Use No. 10M bars.  

\[
s = \frac{100 \text{ mm}^2}{200 \text{ mm} \times 0.002} = 250 \text{ mm}
\]

\[
A_h \geq 0.06 \times \sqrt{f'_c} \times \frac{b_n \times s}{f_y}
\]

\[
\frac{A_h}{s} = \frac{100}{250} = 0.4 \geq \frac{0.06 \times \sqrt{40} \times 200}{400} = 0.19
\]

The effective shear depth, \( d_v \), of a wall need not be taken as less than 0.8\( l_w \).  

\[
d_v = 0.8 \times l_w = 0.8 \times 7000 = 5600 \text{ mm}
\]

The factored shear resistance shall be determined by  

\[
V_r = V_c + V_s + V_p = V_c
\]

However, \( V_r \) shall not exceed  

\[
V_{r,\text{max}} = 0.25 \times \phi \times f'_c \times b_n \times d_v
\]

\[
V_{r,\text{max}} = \frac{0.25 \times 0.65 \times 40 \times 200 \times 5600}{1000} = 7280 \text{ kN} \geq V_f = 800.42 \text{ kN}
\]

Shear strength provided by concrete  

\[
V_c = \phi \times \lambda \times \beta \times \sqrt{f'_c} \times b_n \times d_v
\]

\[
\beta = 0.18
\]

\[
\sqrt{f'_c} = \sqrt{40} = 6.33 < 8 \text{ MPa}
\]

\[
V_c = \frac{0.65 \times 1.0 \times 0.18 \times \sqrt{40} \times 200 \times 5600}{1000} = 828.77 \text{ kN}
\]

\[
V_f = 800.3 \text{ kN} \leq V_c = 828.77 \text{ kN} \rightarrow \text{No additional reinforcement required.}
\]

Use No. 10M @ 250 mm for horizontal reinforcement.
5. Flexural Design

For this wall, the moment at the base governs the design as shown in the previous Figure.

\[ M_f = 11002.07 \text{ kN-m} \]

The minimum area of vertical reinforcement distributed between concentrated reinforcements \( A_v \) at the wall ends is equal to:

\[ A_{vertical} = 0.0015 \times A_v \]

Use No. 10M bars.

\[ s = \frac{100 \text{ mm}^2}{200 \text{ mm} \times 0.0015} = 333.33 \text{ mm} \]

\[ s_{provided} = 300 \text{ mm} \]

Total number of vertical bars \( \frac{(7000 - 100)}{300} + 1 = 24 \)

\[ A_v = 24 \times 100 = 2400 \text{ mm}^2 \]

\[ \alpha = \frac{P_f}{\phi \times f_y \times l_w \times t} = \frac{5660.37}{0.65 \times 40 \times 7000 \times 200} = 0.1555 \]

\[ \omega = \frac{\phi \times A_v \times f_y}{\phi \times f_y \times l_w \times t} = \frac{0.85 \times 2400 \times 400}{0.65 \times 40 \times 7000 \times 200} = 0.0224 \]

\[ \alpha_i = 0.85 - 0.0015 \times f_y = 0.85 - 0.0015 \times 40 = 0.79 > 0.67 \]

\[ \beta_i = 0.97 - 0.0025 \times f_y = 0.97 - 0.0025 \times 40 = 0.87 > 0.67 \]

\[ c = \frac{\omega + \alpha}{2 \times \omega + \alpha_i \times \beta_i} = \frac{0.0224 + 0.1555}{2 \times 0.0224 + 0.1555 \times 0.87} = 0.243 \]

\[ M_r = 0.5 \times \phi \times A_v \times f_y \times l_w \times \left( 1 + \frac{P_f}{\phi \times A_v \times f_y} \right) \left( 1 - \frac{c}{l_w} \right) \]

\[ M_r = 0.5 \times 0.85 \times 2400 \times 400 \times 7000 \times \left( 1 + \frac{5660.38}{0.85 \times 2400 \times 400} \right) (1 - 0.243) \]

\[ M_r = 17158.74 \text{ kN-m} > M_f = 11002.07 \text{ kN-m} \]
Verification of the wall under the combination: \((0.90 \times D + 1.4 \times W)\)

\[
\alpha = \frac{P_f}{\phi_s \times f_c \times l_s \times t} = \frac{3633.21}{0.65 \times 40 \times 7000 \times 200} = 0.0998
\]

\(\omega = 0.0224, \ \alpha_1 = 0.79, \ \text{and} \ \beta_1 = 0.87\)

\[
\frac{c}{l_w} = \frac{\omega + \alpha}{2 \times \omega + \alpha_1 \times \beta_1} = \frac{0.0224 + 0.0998}{2 \times 0.0224 + 0.1555 \times 0.87} = 0.167
\]

\[
M_r = 0.5 \times \phi_s \times A_y \times f_y \times l_s \times \left(1 + \frac{P_f}{\phi_s \times A_y \times f_y} \right) \left(1 - \frac{c}{l_w} \right)
\]

\[
M_r = 0.5 \times 0.85 \times 2400 \times 400 \times 7000 \times \left(1 + \frac{3633.21}{0.85 \times 2400 \times 400} \right) \left(1 - 0.167 \right)
\]

\(M_r = 12972.43 \text{kN-m} > M_f = 11002.07 \text{kN-m}\)

Use 2-15M at each end, due to concentrated wind generated load.

Therefore, a 10M bar is replaced by 2-15M as illustrated in Figure 3.

Figure 2 – Reinforcement Details
6. Shear Wall Analysis and Design – spWall Software

spWall is a program for the analysis and design of reinforced concrete shear walls, tilt-up walls, precast wall and insulate concrete form (ICF) walls. It uses a graphical interface that enables the user to easily generate complex wall models. Graphical user interface is provided for:

- Wall geometry (including any number of openings and stiffeners)
- Material properties including cracking coefficients
- Wall loads (point, line, and area),
- Support conditions (including translational and rotational spring supports)

spWall uses the Finite Element Method for the structural modeling, analysis, and design of slender and non-slender reinforced concrete walls subject to static loading conditions. The wall is idealized as a mesh of rectangular plate elements and straight line stiffener elements. Walls of irregular geometry are idealized to conform to geometry with rectangular boundaries. Plate and stiffener properties can vary from one element to another but are assumed by the program to be uniform within each element.

Six degrees of freedom exist at each node: three translations and three rotations relating to the three Cartesian axes. An external load can exist in the direction of each of the degrees of freedom. Sufficient number of nodal degrees of freedom should be restrained in order to achieve stability of the model. The program assembles the global stiffness matrix and load vectors for the finite element model. Then, it solves the equilibrium equations to obtain deflections and rotations at each node. Finally, the program calculates the internal forces and internal moments in each element. At the user’s option, the program can perform second order analysis. In this case, the program takes into account the effect of in-plane forces on the out-of-plane deflection with any number of openings and stiffeners.

After the Finite Element Analysis (FEA) is completed in spWall, the required flexural reinforcement is computed based on the selected design standard (ACI 318-14 is used in this example), and the user can specify one or two layers of shear wall reinforcement. In stiffeners and boundary elements, spWall calculates the required shear and torsion steel reinforcement. Shear wall concrete strength (in-plane and out-of-plane) is calculated for the applied loads and compared with the code permissible shear capacity.

For illustration and comparison purposes, the following figures provide a sample of the input modules and the FEA results obtained from an spWall model created for the reinforced concrete shear wall in this example.
Figure 3 – Defining Loads for Shear Wall (spWall)
Figure 4 – Assigning Boundary Conditions for Shear Wall (spWall)
Figure 5 – Factored Axial Forces Contour Normal to Shear Wall Cross-Section (spWall)
Figure 6 – Shear Wall Lateral Displacement Contour (spWall)
Figure 7 – Shear Wall Axial Load Diagram (spWall)
Figure 8 – In-plane Shear Diagram (spWall)
Figure 9 – Shear Wall Moment Diagram (spWall)
Figure 10 – Shear Wall Vertical Reinforcement (spWALL)

Elements along the wall base

\[
\sum A_{s,i} = 5811 \text{ mm}^2/\text{m} \\
\text{Element width} = 0.5 \text{ m} \\
\sum A_{s,i} \times 0.5 \text{ m} = 2906 \text{ mm}^2
\]
### 7. Design Results Comparison and Conclusions

Table 1 – Comparison of Shear Wall Analysis and Design Results

<table>
<thead>
<tr>
<th>Solution</th>
<th>Wall Cross-Sectional Forces</th>
<th>$A_s$, required</th>
<th>$A_s$, provided</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_f$ (kN-m)</td>
<td>$N_f$ (kN)</td>
<td>$V_f$ (kN)</td>
</tr>
<tr>
<td>Hand</td>
<td>11,003</td>
<td>5,660</td>
<td>800</td>
</tr>
<tr>
<td>Reference</td>
<td>11,000</td>
<td>5,660</td>
<td>800</td>
</tr>
<tr>
<td>spWall</td>
<td>11,002</td>
<td>5,660</td>
<td>800</td>
</tr>
</tbody>
</table>

* Area of steel provided was selected based on the $A_s$, required reported in spWall.

The results of all the hand calculations and the reference used illustrated above are in very good agreement with the automated results obtained from the spWall FEA. It is worth noting that the minimum area of steel is governed by the minimum reinforcement ratio stipulated by the code. The same can be seen in spWall output for elements 3 through 14.
8. Conclusions & Observations

Engineers can evaluate several options when arriving at the reinforcing bar arrangement from an FEA model. The following conclusions and observations can be used to better understand designing and investigating shear walls using spWall:

1. In finite element analysis, selecting mesh size has a crucial impact on the results accuracy (as an example the amount and distribution of reinforcement). The mesh size should be optimized in a way that changing the element size has slight effect on the results obtained. However, the optimum element size is dependent on multiple parameters in the model which makes it difficult to find a generalized procedure to select the optimum size. Multiple studies conducted by StructurePoint showed that the element length should not be greater than 10% of the total wall length and a coarser mesh should be used with caution and engineering judgement.

2. spWall calculates the required area of steel for each element along the section. This area of steel is selected in a way that it should be enough to satisfy the strength requirements under a specific sets of extreme design forces. This approach will lead to placing most of the reinforcement at wall section ends, as was shown in this example, leading to the highest possible flexural capacity that can be achieved for the section with the same amount of steel. In practice, having a uniform distribution of reinforcement along the wall section is more common and the flexural capacity of the concrete wall is usually calculated based on it.

3. Concrete Shear walls can be analyzed and designed using simplified structural analysis approaches as the one used in this example. However, as the level of complexity of the wall increases, analyzing and designing shear walls using hand solution become more challenging and less effective. Computer software utilizing FEA (e.g. spWall) is an efficient solution to analyze and design concrete shear walls regardless of the level of complexity. spWall selects the minimum required area of steel with the optimum reinforcement distribution for the wall section in which the highest bending capacity of the wall section is achieved. spColumn software can be also utilized to obtain the wall interaction diagram to help better understand the behavior of the section selected. More information about this topic can be found in the Appendix of “Reinforced Concrete Shear Wall Analysis and Design (ACI 318-14)” design example.