Reinforced Concrete Tilt-Up Wall Panel with Opening Analysis and Design (ACI 551)
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Tilt-up is form of construction with increasing popularity owing to its flexibility and economics. Tilt-up concrete is essentially a precast concrete that is site cast instead of traditional factory cast concrete members. A structural reinforced concrete tilt-up wall panel with opening in a single-story warehouse (big-box) building provides gravity and lateral load resistance for the following applied loads from four roof joists bearing in wall pockets in addition to the wind:

- Roof dead load = 2.4 kip per joist
- Roof live load = 2.5 kip per joist
- Wind load = 27.2 psf

The assumed tilt-up wall panel section and reinforcement are investigated after analysis to verify suitability for the applied loads then compared with numerical analysis results obtained from spWall engineering software program from StructurePoint. Additionally, different modeling and analysis techniques using spWall engineering software program to investigate and design tilt-up wall panels with openings are discussed.

Figure 1 – Reinforced Concrete Tilt-Up Wall Panel Geometry (with 10 x 15 ft Door Opening)
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Code

Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary (ACI 318R-11)

Reference

Design Guide for Tilt-Up Concrete Panels, ACI 551.2R-15, 2015, Example B.2

spWall Engineering Software Program Manual v5.01, STRUCTUREPOINT, 2016

Design Data

- $f'_c = 4,000$ psi normal weight concrete ($w_c = 150$pcf)
- $f_y = 60,000$ psi
- Wall length = $l_c = 31$ ft – 1.5 ft = 29.5
- Assumed wall thickness = 8.75 in. (Note: reference example started with a thickness of 6.25 in. that was deemed not sufficient to meet tension control condition to use alternative design method).
- Assumed eccentricity = $e_{cc} = 3$ in.
- Assumed vertical reinforcement: 7 #6 (single layer) for the left leg (design strip)
  - 7 #6 (single layer) for the right leg (design strip)

Solution

The effect of openings on out-of-plane bending in tilt-up panels can be approximated in hand calculations by a simple, one-dimensional strip analysis that provides accuracy and economy for most designs. Where openings occur, the entire lateral and axial load, including self-weight above the critical section, is distributed to supporting legs or design strips at each side of the opening (sometimes referred to as wall piers). \(\text{ACI 551.2R-15 (7.2)}\)

The effective width of the strip should be limited to approximately 12 times the panel thickness to avoid localized stress concentrations along the edge of the opening. This limit is not mandated by ACI 318, but is included as a practical guideline where the opening width is less than one-half the clear vertical span. In most cases the tributary width for loads can be taken as the width of the strip plus one-half the width of adjacent openings. Tilt-up design strips should have constant properties for the full height and the reinforcement should not be cut off just above or below the opening. Thickened vertical or horizontal sections can be introduced within the panel where openings are large or where there are deep recesses on the exterior face. Some conditions may require ties around all vertical reinforcement bars in a vertical pilaster for the full height of the tilt-up panel. \(\text{ACI 551.2R-15 (7.2)}\)
Left Leg Analysis and Design

Figure 2 – Tilt-Up Design Strips Tributary Widths for Loads

1. Minimum Vertical Reinforcement

\[ \rho_v = \frac{A_{v,\text{vertical}}}{b \times h} = \frac{3.08}{(4 \times 12) \times 8.75} = 0.0073 \]

\[ \rho_{v,\min} = 0.0015 \]

\[ \rho_v = 0.0073 \geq \rho_{v,\min} = 0.0015 \text{ (o.k.)} \]

\[ s_{v,\max} = \text{smallest of} \left\{ \frac{3 \times h}{18 \text{ in.}} \right\} = \text{smallest of} \left\{ \frac{3 \times 8.75}{18 \text{ in.}} \right\} = \text{smallest of} \left\{ \frac{26.25 \text{ in.}}{18 \text{ in.}} \right\} = 18 \text{ in.} \]

\[ s_{v,\text{provided}} = \frac{4 \times 12}{7} = 6.86 \text{ in.} \leq s_{v,\max} = 18 \text{ in.} \text{ (o.k.)} \]

The design guide for tilt-up concrete panels ACI 551 states that tilt-up concrete walls can be analyzed using the provisions of Chapter 14 of the ACI 318-11. Most walls, and especially slender walls, are widely evaluated using the “Alternative design of slender walls” in Section 14.8. The same provisions are presented in ACI 318-14 but reorganized in different chapters and in slightly revised terminology. The method is applicable when the conditions summarized below are met:

- The cross section shall be constant over the height of the wall
- The wall can be designed as simply supported
- Maximum moments and deflections occurring at midspan
- The wall must be axially loaded
- The wall must be subjected to an out-of-plane uniform lateral load
- The wall shall be tension-controlled
- The reinforcement shall provide design strength greater than cracking strength

3. Tilt-Up Wall Structural Analysis

3.1. Applied loads

The tributary width for loads can be taken as the width of the strip plus one-half the width of adjacent openings.

Wall self-weight = \( \frac{8.75 \times 150 \times \left[ 4 \times \left( \frac{29.5}{2} + 1.5 \right) + 5 \times (31 - 15) \right]}{12} \times \frac{1 \text{ kip}}{1000 \text{ lb}} = 15.9 \text{ kip} \)

Joist loads are divided between the individual legs assuming an equivalent simply supported beam across the top of the panel with the supports at the centerline of each leg.

- \( P_{DL} = 4.5 \text{ kip (for the left leg)} \)
- \( P_{LL} = 4.7 \text{ kip (for the left leg)} \)

\( w = 27.2 \text{ lb/ft}^2 \)

3.2. Maximum wall forces

The calculation of maximum factored wall forces in accordance with 14.8.3 including moment magnification due to second order (P-Δ) effects is shown below (load combination \( U = 1.2 D \times 1.6 L_r \times 0.5 W \) is considered in this example):

\[ P_{uw} = 1.2 \times 4.5 + 1.6 \times 4.7 = 12.9 \text{ kip} \]
\[ P_{um} = 12.9 + 1.2 \times 15.9 = 32.0 \text{ kip} \]

\[ w_u = 0.5 \times 27.2 \times (4 + 5) \times \frac{1 \text{ kip}}{1000 \text{ lb}} = 0.122 \text{ kip/ft} \]

\[ M_u = \frac{M_{um}}{1 - \frac{5 \times P_{um} \times l^2}{8 \times 0.75 \times 48 \times E_c \times I_{cr}} } \quad \text{ACI 318-11 (Eq. 14-6)} \]

\[ M_{um} = \frac{w_u \times l^2}{8} + \frac{P_{um} \times e}{2} = \frac{0.122 \times (29.5)^2}{8} + \frac{12.9 \times 3}{2 \times 12} = 14.9 \text{ ft-kip} \]

Where \( M_{um} \) is the maximum factored moment at midheight of wall due to lateral and eccentric vertical loads, not including \( \Delta P \) effects. \quad \text{ACI 318-11 (14.8.3)}

\[ E_c = 57,000 \times \sqrt{f_c} = 57,000 \times \sqrt{4,000} = 3,605,000 \text{ psi} \quad \text{ACI 318-11 (8.5.1)} \]

\[ I_{cr} = n \times A_w \times (d - c)^2 + \frac{l_c \times c^3}{3} \quad \text{ACI 318-11 (Eq. 14-7)} \]

\[ n = \frac{E_s}{E_c} = \frac{29,000}{3,605} = 8.0 > 6.0 \quad \text{o.k.} \quad \text{ACI 318-11 (14.8.3)} \]

Calculate the effective area of longitudinal reinforcement in a slender wall for obtaining an approximate cracked moment of inertia.

\[ A_{se} = A_s + \frac{P_{um} \times h}{2 \times f_s \times d} = 3.08 + \frac{32.0 \times 8.75}{2 \times 60 \times (8.75/2)} = 3.61 \text{ in}^2 \quad \text{ACI 318-11 (R14.8.3)} \]

The following calculations are performed with the effective area of steel in lieu of the actual area of steel.

\[ a = \frac{A_{se} \times f_s}{0.85 \times f_s \times b} = \frac{3.61 \times 60}{0.85 \times 4 \times (4 \times 12)} = 1.33 \text{ in.} \]

\[ c = \frac{a}{\beta_i} = \frac{1.33}{0.85} = 1.56 \text{ in.} \]

\[ \frac{c}{d} = \frac{1.56}{4.375} = 0.356 < 0.375 \quad \therefore \text{tension-controlled} \quad \text{ACI 318-11 (R9.3.2.2)} \]

\[ \phi = 0.9 \quad \text{ACI 318-11 (9.3.2)} \]

\[ I_{cr} = 8.0 \times 3.61 \times (4.375 - 1.56)^2 + \frac{(4 \times 12) \times 1.56^3}{3} = 292 \text{ in}^4 \quad \text{ACI 318-11 (Eq. 14-7)} \]
\[ M_u = \frac{M_{uw}}{1 - \frac{P_{sn}}{0.75 \times K_b}} \]  

\[ K_b = \frac{48 \times E_s \times I_{cr}}{5 \times l_c^2} = \frac{48 \times 3605 \times 292}{5 \times (29.5 \times 12)^2} = 80.6 \text{ kip} \]

\[ M_u = \frac{14.9}{32.0} = 31.7 \text{ ft-kip} \]

3.3. Tension-controlled verification

\[ P_n = \frac{P_{sn}}{\phi} = \frac{32.0}{0.9} = 35.5 \text{ kips} \]

\[ a = \frac{A_{uw} \times f_y}{0.85 \times f'_c \times l_w} = \frac{P_n \times h}{2 \times d} + A_c \times f_y \left( \frac{0.85 \times f'_c \times l_w}{0.85 \times f'_c \times l_w} \right) = \frac{35.5 \times 8.75}{2 \times 4.375} + 3.08 \times 60 = 1.35 \text{ in.} \]

\[ c = \frac{a}{\beta} = \frac{1.35}{0.85} = 1.59 \text{ in.} \]

\[ \epsilon_i = \left( \frac{0.003}{c} \right) \times d_c - 0.003 = \left( \frac{0.003}{1.59} \right) \times 4.375 - 0.003 = 0.0053 > 0.0050 \]

Therefore, section is tension controlled

4. Tilt-Up Wall Cracking Moment Capacity (\( M_{cr} \))

Determine \( f_c = \) Modulus of rapture of concrete and \( I_c = \) Moment of inertia of the gross uncracked concrete section to calculate \( M_{cr} \)

\[ f_c = 7.5 \times f'_c = 7.5 \times 1.0 \times \sqrt{4,000} = 474.3 \text{ psi} \]

\[ I_c = \frac{l_c h^3}{12} = \frac{4 \times 12}{12} \times 8.75^3 = 2680 \text{ in.}^4 \]

\[ y_i = \frac{h}{2} = \frac{8.75}{2} = 4.375 \text{ in.} \]

\[ M_{cr} = \frac{f_c I_c}{y_i} = \frac{474.3 \times 2680}{4.375} \times \frac{1}{1000} \times \frac{1}{12} = 24.2 \text{ ft-kip} \]

5. Tilt-Up Wall Flexural Moment Capacity (\( \phi M_a \))

\[ M_a = A_y \times f_y \left( d - \frac{a}{2} \right) = 3.61 \times 60 \times \left( 4.375 - \frac{1.35}{2} \right) = 801.4 \text{ in.-kip} \]

It was shown previously that the section is tension controlled \( \Rightarrow \phi = 0.9 \)
6. Tilt-Up Wall Vertical Stress Check

\[
\phi M_u = \phi \times M_u = 0.9 \times 66.8 = 60.4 \text{ ft-kip} > M_u = 31.7 \text{ ft-kip} \quad \text{(o.k.)}
\]

\[
\phi M_n = 60.4 \text{ ft-kip} > M_{cr} = 24.4 \text{ ft-kip} \quad \text{(o.k.)}
\]

\[
\Delta_s = \frac{M_u}{0.75 \times K_0} = \frac{31.7 \times 12}{0.75 \times 80.6} = 6.29 \text{ in.}
\]

\[\text{ACI 318-11 (Eq. 14-5)}\]

7. Tilt-Up Wall Shear Stress Check

In-plane shear is not evaluated since in-plane shear forces are not applied in this example. Out-of-plane shear due to lateral load should be checked against the shear capacity of the wall. By inspection of the maximum shear forces \(f\), it can be determined that the maximum shear force is under 5 kip. The wall left leg (the weakest section) has a shear capacity approximately 50 kip and no detailed calculations are required by engineering judgement. (See figure 6 for detailed shear force diagram)

\[\text{ACI 318-11 (14.8.2.4)}\]

8. Tilt-Up Wall Mid-Height Deflection (\(\Delta_s\))

The maximum out-of-plane deflection (\(\Delta_s\)) due to service lateral and eccentric vertical loads, including \(P\Delta\) effects, shall not exceed \(l_s/150\). Where \(\Delta_s\) is calculated as follows:

\[
\Delta_s = \begin{cases} 
\frac{2}{3} \Delta_s + \frac{M_u - \frac{2}{3} M_{cr}}{M_u - \frac{2}{3} M_{cr}} \times \left( \Delta_s - \frac{2}{3} \Delta_{cr} \right) & \text{When } M_u > \frac{2}{3} M_{cr} \\
\frac{M_u}{M_{cr}} \Delta_{cr} & \text{When } M_u < \frac{2}{3} M_{cr} 
\end{cases}
\]

\[\text{ACI 318-11 (14.8.4)}\]

Where \(M_s\) is the maximum moment at mid-height of wall due to service lateral and eccentric vertical loads including \(P\Delta\) effects.

\[M_s = M_{sw} + P \Delta_s \]

\[
M_{sw} = \frac{w_s l_s^2}{8} + \frac{P_s e}{2} = \frac{0.7 \times 27.2}{1.6} \times (4 + 5) \times (29.5)^3 + \frac{(4.5) \times 3}{2} = 12.2 \text{ ft-kip}
\]

\[P_s = P_{DL} + \text{wall self-weight} = 4.5 + 15.9 = 20.4 \text{ kip} \]

\[
M_{cr} = \frac{f_{cr} I_s}{y_t} = 24.2 \text{ ft-kip} \quad \text{(as calculated perviously)}
\]

\[\text{ACI 318-11 (Eq. 9-9)}\]
\[ \Delta_s = \frac{5}{48} \times \frac{M_{as}}{E_s I_s} = \frac{5}{48} \times \frac{24.2 \times 12 \times (29.5 \times 12)^2}{3,605 \times 2680} = 0.392 \text{ in.} \quad (\text{ACI 318-11 Eq. 14-10}) \]

\( \Delta_s \) will be calculated by trial and error method since \( \Delta_s \) is a function of \( M_a \) and \( M_s \) is a function of \( \Delta_a \).

Assume \( M_{as} < \frac{2}{3} M_{cr} \)

Assume \( \Delta_a = \left( \frac{M_{as}}{M_{cr}} \right) \Delta_s = \left( \frac{12.2}{24.2} \right) \times 0.392 = 0.198 \text{ in.} \)

\[ M_{as} = M_{as} + P \Delta_s = 12.2 \times 12 + 20.4 \times 0.198 = 150.4 \text{ in.-kip} = 12.5 \text{ ft-kip} \]

\[ \Delta_s = \left( \frac{M_{as}}{M_{cr}} \right) \Delta_s = \frac{12.5}{24.2} \times 0.392 = 0.202 \text{ in.} \quad (\text{ACI 318-11 Eq. 14-9}) \]

No further iterations are required

\[ M_{as} = 12.5 \text{ ft-kip} < \frac{2}{3} M_{cr} = \frac{2}{3} \times 24.2 = 16.1 \text{ ft-kip} \quad (\text{o.k.}) \]

\[ \Delta_s = 0.202 \text{ in.} < \frac{l}{150} = \frac{29.5 \times 12}{150} = 2.36 \text{ in.} \quad (\text{o.k.}) \]

The wall left leg is adequate with 7 #6 vertical reinforcement and 8.75 in. thickness.

**Right Leg Analysis and Design**

Repeating the same process for the right leg (right design strip) leads to the following results:

\( P_{dk} = 5.1 \text{ kip} \) (for the right leg)

\( P_{ll} = 5.3 \text{ kip} \) (for the right leg)

\( w = 27.2 \text{ lb/ft}^2 \)

\( P_{sw} = 14.6 \text{ kip} \)

\( P_{aw} = 37.9 \text{ kip} \)

\( w_s = 0.150 \text{ kip/ft} \)

\( M_{as} = 18.1 \text{ ft-kip} \)

\( A_{as} = 3.71 \text{ in.}^2 \)

\( a = 0.91 \text{ in.} \)
\[ c = 1.07 \text{ in.} \]

\[ \frac{c}{d} = 0.245 < 0.375 \therefore \text{tension-controlled} \]

\[ I_{cr} = 355 \text{ in.}^4 \]

\[ K_h = 98.2 \text{ kip} \]

\[ M_s = 37.3 \text{ ft-kip} \]

\[ M_{yr} = 36.3 \text{ ft-kip} \]

\[ \phi M_n = 65.5 \text{ ft-kip} > M_{yr} = 37.3 \text{ ft-kip} \ (\text{o.k.}) \]

\[ \phi M_n = 65.5 \text{ ft-kip} > M_{cr} = 36.3 \text{ ft-kip} \ (\text{o.k.}) \]

\[ \Delta_u = 6.08 \text{ in.} \]

\[ \frac{P_{min}}{A_y} = 60.2 \text{ psi} < 0.06 \times f_c = 240 \text{ psi} \ (\text{o.k.}) \]

\[ M_n = 14.9 \text{ ft-kip} \]

\[ \Delta_{cr} = 0.392 \text{ in.} \]

\[ M_s = 15.2 \text{ ft-kip} < \frac{2}{3} M_{cr} = 24.2 \text{ ft-kip} \ (\text{o.k.}) \]

\[ \Delta_s = 0.164 \text{ in.} < \frac{I_{cr}}{150} = 2.36 \text{ in.} \ (\text{o.k.}) \]

The wall right leg is adequate with 7 #6 vertical reinforcement and 8.75 in. thickness.

9. **Analysis and Design of the Section between the Design Strips**

For the vertical reinforcement for the section between the design strips, minimum area of steel should be provided as follows:

\[ \rho_{l, \text{min}} = 0.0015 \]

ACI 318-11 (14.3.2)

Try single layer panel reinforcement of 9 #4.

\[ \rho_i = \frac{A_{i, \text{vertical}}}{b \times h} = \frac{9 \times 0.20}{(10 \times 12) \times 8.75} = 0.0017 \]

ACI 318-11 (2.1)

\[ \rho_i = 0.0017 \geq \rho_{l, \text{min}} = 0.0015 \ (\text{o.k.}) \]
\[ s_{t,\text{max}} = \text{smallest of } \left\{ \frac{3 \times h}{18 \text{ in.}} \right\} = \text{smallest of } \left\{ \frac{3 \times 8.75}{18 \text{ in.}} \right\} = \text{smallest of } \left\{ \frac{26.25 \text{ in.}}{18 \text{ in.}} \right\} = 18 \text{ in.} \quad \text{ACI 318-11 (7.6.5)} \]

\[ s_{t,\text{provided}} = \frac{10 \times 12}{9} = 13.3 \text{ in.} \leq s_{t,\text{max}} = 18 \text{ in.} \quad \text{o.k.} \]

10. Horizontal Reinforcement

\[ \rho_{h,\text{min}} = 0.00200 \quad \text{ACI 318-11 (14.3.3)} \]

Try single layer panel reinforcement of 33 #4.

\[ \rho_t = \frac{A_{\text{vertical}}}{b \times h} = \frac{33 \times 0.20}{(31 \times 12) \times 8.75} = 0.00203 \quad \text{ACI 318-11 (2.1)} \]

\[ \rho_h = 0.00203 \geq \rho_{h,\text{min}} = 0.00200 \quad \text{o.k.} \]

Additional reinforcement requirements are outlined in ACI 318-11 (14.3.7) for header and jambs of openings.

11. Tilt-Up Wall Panel Analysis and Design – spWall Software

spWall is a program for the analysis and design of reinforced concrete shear walls, tilt-up walls, precast walls and Insulate Concrete Form (ICF) walls. It uses a graphical interface that enables the user to easily generate complex wall models. Graphical user interface is provided for:

- Wall geometry (including any number of openings and stiffeners)
- Material properties including cracking coefficients
- Wall loads (point, line, and area),
- Support conditions (including translational and rotational spring supports)

spWall uses the Finite Element Method for the structural modeling, analysis, and design of slender and non-slender reinforced concrete walls subject to static loading conditions. The wall is idealized as a mesh of rectangular plate elements and straight line stiffener elements. Walls of irregular geometry are idealized to conform to geometry with rectangular boundaries. Plate and stiffener properties can vary from one element to another but are assumed by the program to be uniform within each element.

Six degrees of freedom exist at each node: three translations and three rotations relating to the three Cartesian axes. An external load can exist in the direction of each of the degrees of freedom. Sufficient number of nodal degrees of freedom should be restrained in order to achieve stability of the model. The program assembles the global stiffness matrix and load vectors for the finite element model. Then, it solves the equilibrium equations to obtain deflections and rotations at each node. Finally, the program calculates the internal forces and internal moments in each element. At the user’s option, the program can perform second order analysis. In this case, the
program takes into account the effect of in-plane forces on the out-of-plane deflection with any number of openings and stiffeners.

In spWall, the required flexural reinforcement is computed based on the selected design standard (ACI 318-11 is used in this example), and the user can specify one or two layers of wall reinforcement. In stiffeners and boundary elements, spWall calculates the required shear and torsion steel reinforcement. Wall concrete strength (in-plane and out-of-plane) is calculated for the applied loads and compared with the code permissible shear capacity.

For illustration and comparison purposes, the following figures provide a sample of the input modules and results obtained from an spWall model created for the reinforced concrete wall in this example.

Figure 3 –Defining Loads for Tilt-Up Wall Panel with Opening (spWall)
Figure 4 – Factored Axial Forces Contour Normal to Tilt-Up Wall Panel Design Strips Cross-Sections (spWall)
Figure 5 – Tilt-Up Wall Panel Service Lateral Displacement Contour (Out-of-Plane) (spWall)
Figure 6 – Out-of-plane Shear Diagram (spWall)
Figure 7 – Tilt-Up Wall Panel with Opening Moment Diagram (spWall)
### Service combinations | Displacements | S1

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<td>-1.64e-001</td>
</tr>
<tr>
<td>395</td>
<td>-2.59e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
<tr>
<td>396</td>
<td>-1.29e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
<tr>
<td>397</td>
<td>-1.06e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
<tr>
<td>398</td>
<td>1.29e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
<tr>
<td>399</td>
<td>2.59e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
<tr>
<td>400</td>
<td>3.88e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
<tr>
<td>401</td>
<td>5.18e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
<tr>
<td>402</td>
<td>6.47e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
<tr>
<td>403</td>
<td>7.77e-005</td>
<td>-1.91e-003</td>
<td>-1.64e-001</td>
</tr>
</tbody>
</table>

---

**Figure 8** – Tilt-Up Wall Panel with Opening Displacement at Critical Sections (Service Combinations) (**spWall**)

---

**Figure 9** – Tilt-Up Wall Panel with Opening Displacement at Critical Sections (Ultimate Combinations) (**spWall**)
12. Design Results Comparison and Conclusions

The model shown above was created in spWall taking into account the ACI 318-11 provisions (alternative design method) and ACI 551 recommendations regarding the analysis and design of tilt-up wall panels with openings in order to match the results presented in the reference. In this model the left and right design strips are modeled such that the entire lateral and axial load, including self-weight above the critical section, are distributed to the two strips at each side of the opening. The tributary width for loads was taken as the width of the strip plus one-half the width of the opening. The following table shows the comparison between hand and reference results with spWall model results.

<table>
<thead>
<tr>
<th>Solution</th>
<th>M\text{u} (kip-ft)</th>
<th>N\text{u} (kip)</th>
<th>D\text{z,service} (in.)</th>
<th>D\text{z,ultimate} (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Strip</td>
<td>Left</td>
<td>Right</td>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>Hand</td>
<td>31.70</td>
<td>37.30</td>
<td>32.00</td>
<td>37.90</td>
</tr>
<tr>
<td>Reference</td>
<td>31.70</td>
<td>37.30</td>
<td>32.00</td>
<td>37.90</td>
</tr>
<tr>
<td>spWall</td>
<td>32.07</td>
<td>37.71</td>
<td>32.00</td>
<td>37.88</td>
</tr>
</tbody>
</table>

The results of all the hand calculations and the reference used illustrated above are in agreement with the automated exact results obtained from the spWall program.
12.1. Comparison of Wall Modeling Methods

ACI 318 provides the alternative design method as a simple and accurate option for analysis and design of simple walls meeting the method conditions. Other methods such as finite element analysis can be used to address panels not meeting the numerous limitations of the alternative design method (cantilevered walls, variable thickness and width, walls with openings, non-standard boundary conditions, walls with high compressive loads, in-plane lateral loads, non-standard concentrated load position from attachments of piping, racking etc., concentrated out of plane loads).

The exact wall geometry and applied loads were modeled using spWall engineering software to investigate the differences between the simplified approximate method and the finite element method. For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spWall model created for the reinforced concrete wall in this example using exact wall geometry and applied loads.

It is very important to consider the wind load applied to the door opening and how it must be considered and applied in the model based on the door boundary condition. In this example, the door support reactions are assumed along the left and right side of the door opening. Load is modeled as an equivalent uniform line load applied along the right edge of the left leg and the left side of the right leg. The magnitude of this load is calculated as follows:

\[
W_{door} = 27.2 \times \frac{10}{2} \times \frac{1}{1000} = 0.136 \text{ kip/ft}
\]
Figure 11 – Factored Axial Forces Contour - Exact Geometry and Loads (spWall)
Figure 12 – Tilt-Up Wall Panel Service Lateral Displacement Contour (Out-of-Plane) - Exact Geometry and Loads

(spWall)
Figure 13 – Out-of-plane Shear Diagram - Exact Geometry and Loads (spWall)
Figure 14 – Tilt-Up Wall Panel with Opening Moment Diagram – Exact Geometry and Loads (spWall)
Figure 15 – Displacement at Critical Sections – Exact Geometry and Loads (Service Combinations) (spWall)

Figure 16 – Displacement at Critical Sections – Exact Geometry and Loads (Ultimate Combinations) (spWall)
Using the complete model with the exact wall geometry and applied loads compared with the simplified model of two equivalent design strips results in:

1. Reduction in the required moment capacity by 14%
2. Reduction in the out-of-plane displacements, at service and ultimate levels by 19% to 23% respectively.

The complete model, as shown in the following figure, displays a complete view of the torsional moment distribution indicating areas of torsional stress concentration at opening edges. This corresponds to the additional reinforcement requirements outlined in **ACI 318-11 (14.3.7)** for header and jambs of openings for improved serviceability.
Figure 18 – Tilt-Up Wall Panel with Opening Torsional Moment Contour (spWall)
12.2. **Tilt-up Wall Stiffness Reduction**

In column and wall analysis, section properties shall be determined by taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and the effect of load duration (creep effects). ACI 318 permits the use of reduced moment of inertia values of \(0.70 I_g\) for uncracked walls and \(0.35I_g\) for cracked walls.  

**ACI 318-11 (10.10.4.1)**

In SpWall program, these effects are accounted for where the user can input reduced moment of inertia using “cracking coefficient” values for plate and stiffener elements to effectively reduce stiffness. Cracking coefficients for out-of-plane (bending and torsion) and in-plane (axial and shear) stiffness can be entered for plate elements. Because the values of the cracking coefficients can have a large effect on the analysis and design results, the user must take care in selecting values that best represent the state of cracking at the particular loading stage. Cracking coefficients are greater than 0 and less than 1.

At ultimate loads, a wall is normally in a highly cracked state. The user could enter a value of out-of-plane cracking coefficient for plates of \(I_{cracked}/I_{gross}\) based on estimated values of \(A_s\). After the analysis and design, if the computed value of \(A_s\) greatly differs from the estimated value of \(A_s\), the analysis should be performed again with new values for the cracking coefficients. A factor 0.75 can be also used to reduce the calculated bending stiffness of the concrete section in accordance with ACI 318-11, Chapters 10 and 14. It is intended to account for variations in material properties and workmanship. This reduction factor in bending stiffness should be incorporated by all other alternate design methods to comply with the requirements of ACI 318 as ACI 551 committee stated.

At service loads, a wall may or may not be in a highly cracked state. For service load deflection analysis, a problem should be modeled with an out-of-plane cracking coefficient for plates of \((I_{effective}/I_{gross})\).

Based on the previous discussion, the ratio between \(I_{cr}\) and \(I_g\) including the reduction factor (0.75) can be used as the cracking coefficient for the out-of-plane case for the ultimate load combinations. In this example, \(I_{cr}\) and \(I_g\) were found to be equal to 292 in.\(^4\) and 2,680 in.\(^4\) for the left leg (design strip). Thus, the out-of-plane cracking coefficient for ultimate load combinations for the left leg can be found as follows:

\[
\alpha = \text{cracking coefficient} = \frac{0.75 \times I_{cr}}{I_g} = \frac{0.75 \times 292}{2,680} = 0.082
\]

For the service load combinations, it was found that \(M_a\) for the left leg equals to 12.5 ft-kip which is less than \(M_{cr} = 24.2\) ft-kip. That means the left leg section is uncracked and the cracking coefficient can be taken equal to 1.
12.3. Comparison of Load Type Effects

During the process of analyzing the tilt-up wall panels, the effect of load type on the wall behavior at the critical section was investigated in terms of out-of-plane deflection at service and ultimate level, required axial capacity, and required out-of-plane moment capacity.

Using equivalent uniform line load along the section width to represent the actual joists point loads has only a slight effect on the results obtained at the critical section (mid-height of the unbrace wall length). However, modeling point loads to reflect actual behavior and stress distribution is beneficial in cases where there are openings, variable thicknesses, changes in geometry, intermediate supports, and other variations from a simply supported wall with constant width and thickness.

12.4. Cracked Moment of Inertia Calculation Methods

The cracked moment of inertia for tilt-up wall panels can be calculated using different ACI 318 provisions. The following shows the commonly used provisions to calculate the cracked moment of inertia:

1. \(0.35 \, I_e\) for cracked walls and \(0.75 \, I_e\) for uncracked walls

2. When treating the wall as compression member:

\[
0.80 + 25 \frac{A_c}{A_e} \times \left(1 - \frac{M}{P_e \times h} - 0.5 \times \frac{P_c}{P_e} \right) \times I_e \leq 0.875 \times I_e
\]

**ACI 318-11 (10.10.4.1)**

**ACI 318-11 (Eq. 10-8)**
3. When treating the wall as flexural member:

\[
(0.10 + 25 \times \rho) \times \left(1.2 - 0.2 \times \frac{b_c}{d}\right) \times I_g \leq 0.5 \times I_g
\]

ACI 318-11 (Eq. 10-9)

4. Using the moment magnification procedure for nonsway frames:

\[
\frac{0.2 \times E_s \times I_g + E_i \times I_w}{(1 + \beta_{dss}) \times E_i}
\]

ACI 318-11 (Eq. 10-14)

5. Using the moment magnification procedure for nonsway frames:

\[
\frac{0.4 \times E_s \times I_g}{(1 + \beta_{dss}) \times E_i}
\]

ACI 318-11 (Eq. 10-15)

6. Using the alternative design method of slender walls:

\[
n \times A_w \times (d - c)^2 + \frac{I_w \times c^3}{3}
\]

ACI 318-11 (Eq. 14-7)

Equation 14-7 is used in this example to calculate the cracked moment of inertia for the wall section modeled in spWall. This is intended to best match the reference approach using the alternative design method to analyze and design the tilt-up wall panels.

The variation in the magnitude of \( I_{cr} \) has a significant effect on the analysis results and specifically the wall moments and displacement. In the following table a comparison of the resulting values based on variation of the \( I_{cr} \) is summarized for information.

<table>
<thead>
<tr>
<th>Method</th>
<th>( I_{cr} ), in.</th>
<th>Cracking coefficient (( \alpha )) for spWall</th>
<th>( M_u ), kip-ft</th>
<th>( D_{zz,service} ), in.</th>
<th>( D_{zz,ultimate} ), in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>Right</td>
<td>Left</td>
<td>Right</td>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>10.10.4.1</td>
<td>938</td>
<td>1407</td>
<td>0.350</td>
<td>0.350</td>
<td>17.03</td>
</tr>
<tr>
<td>Eq. 10-8</td>
<td>2345</td>
<td>3517</td>
<td>0.875</td>
<td>0.875</td>
<td>15.66</td>
</tr>
<tr>
<td>Eq. 10-9</td>
<td>607</td>
<td>715</td>
<td>0.227</td>
<td>0.178</td>
<td>18.49</td>
</tr>
<tr>
<td>Eq. 10-14</td>
<td>126</td>
<td>159</td>
<td>0.047</td>
<td>0.040</td>
<td>177.67</td>
</tr>
<tr>
<td>Eq. 10-15</td>
<td>133</td>
<td>200</td>
<td>0.050</td>
<td>0.050</td>
<td>109.04</td>
</tr>
<tr>
<td>Eq. 14-7</td>
<td>291</td>
<td>356</td>
<td>0.109</td>
<td>0.088</td>
<td>29.68</td>
</tr>
<tr>
<td>Eq. 14-7 with reduction factor (from 14-6)</td>
<td>218</td>
<td>267</td>
<td>0.081</td>
<td>0.066</td>
<td>32.41</td>
</tr>
</tbody>
</table>

From the table above the following can be observed:

1. The values above reveal the necessity to carefully select \( I_{cr} \) value (and the corresponding \( \alpha \) value) to ensure the wall moment capacity and estimated deflections are calculated with sufficient conservatism ensuring adequate strength and stability.
2. The $D_{z,\text{service}}$ values are unaffected by the method used to calculate $I_{cr}$ since the section is uncracked and the cracking coefficient $\alpha$ is taken as 1.

3. The $D_{z,\text{ultimate}}$ values are calculated however are not used in any calculations and the deflection limits are given for $D_{z,\text{service}}$ only.

4. The range of the cracking coefficient and the cracked moment of inertia values vary widely based on the equation used.

5. In this example the spWall model utilized the value of the cracked moment of inertia using the alternative design method equation Eq. 14-7 with reduction factor from 14-6.

13. Tilt-Up Wall Reinforcement and Cracking Coefficient Optimization

In the previous models, the cracking coefficients were selected based on the area of steel used by the reference and equation 14-7 with the reduction factor to best match the reference. The reinforcement selected in the reference is conservative and results in a higher cracking moment of inertia leading to lower values of reinforcement to be obtained by spWall.

To explore this topic in further details, the left leg (design strip) model results will be used. $I_{cr}$ for this model based on 7 #6 bars ($A_s = 3.08$ in.$^2$) vertical reinforcement was found to be equal to 292 in.$^4$ which leads to a 0.08172 cracking coefficient (the model outputs are highly dependent on and sensitive to the cracking coefficient and up to 4 significant figures is recommended). Using this value, the required area of steel of 1.18 in.$^2$ is less than the provided area of steel used to calculate the cracking coefficient by 61.7%. This is expected since the provided area of steel in reference example is much higher than the required ($\phi M_n = 60.4$ ft-kip $>> M_u = 31.7$ ft-kip).

The use of the required area of steel from this model in this case is insufficient because it is based on a high assumed value of the cracking coefficient. To confirm this, a model was reanalyzed using the new required area of steel (1.18 in.$^2$) to calculate the cracking coefficient (0.05276). spWall in this case shows that the model is failing and the following error will be provided:
In order to find the optimum required area of steel and the associated cracking coefficient for ultimate combinations using spWall, the following procedure should be followed:

1. Estimate the value of $A_s$.
2. Calculate $A_{se}$ using the following equation:

   \[ A_{se} = A_s + \frac{P_{u,s} \times h}{2 \times f_y \times d} \]

   Where $A_{se}$ is the effective area of longitudinal reinforcement in a slender wall.
3. Calculate $I_{cr}$ using the following equation:

   \[ I_{cr} = n \times A_{se} \times (d - c)^2 + \frac{l_w \times c^3}{3} \]

4. Calculate the cracking coefficient using the following equation:
\[ \alpha = \text{cracking coefficient} = \frac{0.75 \times I_{cr}}{I_g} \]

Where the 0.75 is bending stiffness reduction factor of the concrete section in accordance with ACI 318-11, Chapters 10 and 14. It is intended to account for variations in material properties and workmanship.

5. Run the first model in spWall using the initial cracking coefficient. After analysis and design, if the computed value of \( A_s (A_{s,n+1}) \) is greatly differs from the estimated value of \( A_s (A_{s,n}) \), the analysis should be performed again with new values of \( A_s \) and cracking coefficient until \( A_{s,n} \approx A_{s,n+1} \).

The following table shows the iteration stages to obtain the optimum area of steel for the left leg (design strip) wall of this example using the procedure described above:

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>( A_{s,n}, \text{in.}^2 )</th>
<th>( A_{s,n+1}, \text{in.}^2 )</th>
<th>Difference, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.080</td>
<td>1.180</td>
<td>61.7</td>
</tr>
<tr>
<td>2</td>
<td>1.220**</td>
<td>26.600</td>
<td>-2080.3</td>
</tr>
<tr>
<td>3</td>
<td>2.150</td>
<td>1.660</td>
<td>22.8</td>
</tr>
<tr>
<td>4</td>
<td>1.660</td>
<td>2.320</td>
<td>-39.8</td>
</tr>
<tr>
<td>5</td>
<td>1.905</td>
<td>1.920</td>
<td>-0.8</td>
</tr>
<tr>
<td>6</td>
<td>1.910</td>
<td>1.910</td>
<td>0.0</td>
</tr>
</tbody>
</table>

* Model wall reinforcement design failed
** The lowest wall reinforcement value that will produce a viable model

Using this procedure above for the left leg, we started with 3.080 in.\(^2\), the value used by the reference. After a few iterations with averaging of two consecutive reinforcement areas, it was found that the solution converged at 1.91 in.\(^2\) as the optimum reinforcement area. For illustration and comparison purposes, the following figures provide a sample of the results obtained from the spWall model created for the reinforced concrete wall with the optimum area of steel (1.91 in.\(^2\)).
Figure 21 – Factored Axial Forces Contour Normal to the Left Design Strip Cross-Section (spWall)
Figure 22 – Service Lateral Displacement Contour (Out-of-Plane) (spWall)
Figure 23 – Vertical Reinforcement Contour (in.²/ft) (spWall)
Figure 24 – Out-of-plane Shear Diagram (spWall)
Figure 25 – Tilt-Up Wall Panel with Opening Moment Diagram (spWall)
### Figure 26 – Lateral Displacement at Critical Sections (Service Combinations) (spWall)

<table>
<thead>
<tr>
<th>Node</th>
<th>Dx</th>
<th>Dy</th>
<th>Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>271</td>
<td>-6.47e-005</td>
<td>-2.38e-003</td>
<td>2.05e-001</td>
</tr>
<tr>
<td>272</td>
<td>-4.85e-005</td>
<td>-2.38e-003</td>
<td>2.04e-001</td>
</tr>
<tr>
<td>273</td>
<td>-3.23e-005</td>
<td>-2.38e-003</td>
<td>2.04e-001</td>
</tr>
<tr>
<td>275</td>
<td>-3.39e-015</td>
<td>-2.38e-003</td>
<td>2.04e-001</td>
</tr>
<tr>
<td>276</td>
<td>1.62e-005</td>
<td>-2.38e-003</td>
<td>2.04e-001</td>
</tr>
<tr>
<td>277</td>
<td>3.23e-005</td>
<td>-2.38e-003</td>
<td>2.04e-001</td>
</tr>
<tr>
<td>278</td>
<td>4.85e-005</td>
<td>-2.38e-003</td>
<td>2.04e-001</td>
</tr>
<tr>
<td>279</td>
<td>6.47e-005</td>
<td>-2.38e-003</td>
<td>2.05e-001</td>
</tr>
</tbody>
</table>

### Figure 27 – Lateral Displacement at Critical Sections (Ultimate Combinations) (spWall)

<table>
<thead>
<tr>
<th>Node</th>
<th>Dx</th>
<th>Dy</th>
<th>Dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>271</td>
<td>-1.06e-004</td>
<td>-3.74e-003</td>
<td>1.08e+001</td>
</tr>
<tr>
<td>272</td>
<td>-2.61e-005</td>
<td>-3.74e-003</td>
<td>1.08e+001</td>
</tr>
<tr>
<td>273</td>
<td>-5.08e-005</td>
<td>-3.74e-003</td>
<td>1.07e+001</td>
</tr>
<tr>
<td>275</td>
<td>-5.14e-015</td>
<td>-3.74e-003</td>
<td>1.07e+001</td>
</tr>
<tr>
<td>276</td>
<td>1.62e-005</td>
<td>-3.74e-003</td>
<td>1.07e+001</td>
</tr>
<tr>
<td>277</td>
<td>3.23e-005</td>
<td>-3.74e-003</td>
<td>1.08e+001</td>
</tr>
<tr>
<td>278</td>
<td>4.85e-005</td>
<td>-3.74e-003</td>
<td>1.08e+001</td>
</tr>
<tr>
<td>279</td>
<td>6.47e-005</td>
<td>-3.74e-003</td>
<td>1.08e+001</td>
</tr>
</tbody>
</table>
The hand calculation procedure shown earlier is repeated for the left leg based on the optimum area of steel ($A_s = 1.91$ in.$^2$) as follows:

$$P_{DL} = 4.5 \text{ kip (for the left leg)}$$

$$P_{LL} = 4.7 \text{ kip (for the left leg)}$$

$$w = 27.2 \text{ lb/ft}^2$$

$$P_{wa} = 12.9 \text{ kip}$$
\( P_{\text{in}} = 31.6 \text{ kip} \)

\( w_u = 0.122 \text{ kip/ft} \)

\( M_{uw} = 14.9 \text{ ft-kip} \)

\( A_{uw} = 2.44 \text{ in.}^2 \)

\( a = 0.90 \text{ in.} \)

\( c = 1.054 \text{ in.} \)

\( \frac{c}{d} = 0.241 < 0.375 \quad \text{tension-controlled} \)

\( I_{cr} = 235 \text{ in.}^4 \)

\( K_b = 64.9 \text{ kip} \)

\( M_u = 42.6 \text{ ft-kip} \)

\( M_{sa} = 24.2 \text{ ft-kip} \)

\( \phi M_u = 43.1 \text{ ft-kip} > M_u = 42.6 \text{ ft-kip} \quad \text{(o.k.)} \)

\( \phi M_u = 43.1 \text{ ft-kip} > M_{cr} = 24.4 \text{ ft-kip} \quad \text{(o.k.)} \)

\( \Delta_u = 10.5 \text{ in.} \)

\( \frac{P_{\text{min}}}{A_g} = 75.3 \text{ psi} < 0.06 \times f'_c = 240 \text{ psi} \quad \text{(o.k.)} \)

\( M_{sa} = 12.2 \text{ ft-kip} \)

\( \Delta_{sa} = 0.393 \text{ in.} \)

\( M_s = 12.5 \text{ ft-kip} < \frac{2}{3} M_{cr} = 24.2 \text{ ft-kip} \quad \text{(o.k.)} \)

\( \Delta_s = 0.203 \text{ in.} < \frac{l}{150} = 2.36 \text{ in.} \quad \text{(o.k.)} \)

The above calculations reveal a reduction in the cracked moment of inertia resulting in an increase in the \( M_u \) applied. Note that the moment capacity is now very close to the required moment.
The following table shows the comparison between hand results with spWall model results for the optimum area of steel.

Table 5 – Comparison of Analysis and Design Results for the Tilt-Up Wall with Optimum Area of Steel

<table>
<thead>
<tr>
<th>Solution</th>
<th>$M_u$ (kip-ft)</th>
<th>$N_u$ (kip)</th>
<th>$D_{z,\text{service}}$ (in.)</th>
<th>$D_{z,\text{ultimate}}$ (in.)</th>
<th>$A_{s,\text{required}}$ (in.$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand</td>
<td>42.6</td>
<td>31.6</td>
<td>0.203</td>
<td>10.5</td>
<td>1.91</td>
</tr>
<tr>
<td>spWall</td>
<td>44.0</td>
<td>32.0</td>
<td>0.204</td>
<td>10.7</td>
<td>1.91</td>
</tr>
</tbody>
</table>

After following the reinforcement optimization procedure, the results of all the hand calculations used above are in agreement with the automated exact results obtained from the spWall program including the required area of steel.