Shear Strength of Concrete Slabs with Openings
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Openings in concrete floor slab systems can have a significant effect on one-way and two-way shear strength of the slab. This effect is illustrated in this example for a concrete flat plate floor system shown in the figure below. The floor opening is cut to accommodate typical mechanical, electrical, and piping (MEP) systems needed for equipment housed within a building. Concrete floor openings are routinely placed on design drawings and are factored into the initial analysis and design calculations. However, when openings are needed in existing floor slabs after they have been placed into service, the investigation of the beam shear and punching shear must be done carefully to maintain the floor load carrying capacity and avoid costly and labor-intensive reinforcement of the floor slab. In this example a hypothetical floor opening is placed next to a column to simulate an opening needed for cable tray system in a data center building housing server racks. The 6 inch thick slab is considered with a superimposed dead load = 15 psf, and unfactored live load = 60 psf.
Figure 1 - Flat Plate Concrete Floor System with Openings
Figure 2 – Floor Opening Dimensions and Location
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Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference


Design Data

Slab Opening Dimensions:

\[ b = 12 \text{ in.} \quad h = 18 \text{ in.} \]

Slab Opening Location (centerline of opening to centerline of column):

\[ x = 12 \text{ in.} \quad y = 34 \text{ in.} \]

Slab Span length = 18 ft both direction

Superimposed Dead Load, \( SDL = 15 \text{ psf} \)

Live Load, \( LL = 60 \text{ psf} \)

\( f’ = 3000 \text{ psi} \) (for slab)

Slab thickness, \( t_s = 6 \text{ in.} \)
Solution
The effect of openings in slabs without beams on the concrete shear strength shall be considered when the opening is located:

\[ ACI \, 318-14 \, (8.5.4.2(d)) \]

1. Anywhere within a column strip of the slab system.
   (In this example, the opening is located within the intersection of two column strips)

2. Within 10 times the slab thickness from a concentrated load or reaction area (the least distance between the opening and the reaction area).
   In this example (10 x 6 in. = 60 in. > 12 in.).
   Thus, the effect of the opening should be evaluated (see Figure 2).

Slab opening effect is evaluated by reducing the perimeter of the critical section \( b_o \) by a length equal to the projection of the opening enclosed by two-lines extending from the centroid of the column and tangent to the opening. To demonstrate the opening effects, one-way and two-way shear checks are conducted for two cases: slab without openings, and slab with opening.

Figure 3 – Effect of Openings on Slabs Shear Strength (PCA Notes on ACI 318-11)
1. Slab One-Way (Beam) Shear Strength

1.1. Slab without Opening

Evaluate the average effective depth (see following Figure):

\[ d'_l = h_l - c_{\text{clear}} - d_b - \frac{d_b}{2} = 6 - 0.75 - 0.50 - \frac{0.50}{2} = 4.50 \text{ in.} \]

\[ d'_r = h_l - c_{\text{clear}} - \frac{d_b}{2} = 6 - 0.75 - \frac{0.50}{2} = 5.00 \text{ in.} \]

\[ d_{\text{avg}} = \frac{d'_l + d'_r}{2} = \frac{4.50 + 5.00}{2} = 4.75 \text{ in.} \]

Where:

- \( c_{\text{clear}} = 3/4 \) in. for \# 4 steel bar  \( \text{ACI 318-14 (Table 20.6.1.3.1)} \)
- \( d_b = 0.50 \) in. for \# 4 steel bar

![Figure 4 – Averaged Effective Depth Calculations](image)

Factored Dead Load: \[ q_{do} = 1.2 \times \left( \frac{6}{12} \times 150 + 15 \right) = 108 \text{ psf} \]

Factored Live Load: \[ q_{lu} = 1.6 \times 60 = 96 \text{ psf} \]  \( \text{ACI 318-14 (5.3.1)} \)

Total Factored Load: \[ q_u = 108 + 96 = 204 \text{ psf} \]

Check the adequacy of slab thickness for beam action (one-way shear)  \( \text{ACI 318-14 (22.5)} \)

Consider an 18 ft wide strip. The critical section for one-way shear is located at a distance (\( d_{\text{avg}} \)) from the face of support (see the following Figure). The tributary area for one-way shear is:
\[ A_{\text{Tributary}} = \text{larger of } \begin{bmatrix} 18 & 12 & 4.75 \\ 2 & 2 \times 12 & 12 \end{bmatrix} \times 18 \]

\[ = \text{larger of } \begin{bmatrix} 7.52 \text{ ft} \\ 8.10 \text{ ft} \end{bmatrix} \times 18 \text{ ft} = 145.88 \text{ ft}^2 \]

\[ V_a = q_o \times A_{Tributary} = 0.204 \times 145.88 = 29.80 \text{ kips} \]

\[ V_c = 2 \lambda \sqrt{f_c b_v d} \quad \text{ACI 318-14 (Eq. 22.5.5.1)} \]

Where \( \lambda = 1 \) for normal weight concrete

\[ \phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{3000} \times (18 \times 12) \times \frac{4.75}{1000} = 84.30 \text{ kips} > V_a = 29.80 \text{ kips} \]

Slab thickness of 6 in. is adequate for one-way shear.

Figure 5 – Critical Section for One-Way Shear (Slab without Opening)
1.2. Slab with Opening

Evaluate the average effective depth:

\[ d_{\text{avg}} = 4.75 \text{ in.} \quad (\text{calculated in Section 1.1}) \]

Total Factored Load: \( q_u = 204 \text{ psf} \quad (\text{calculated in Section 1.1}) \)  \( \text{ACI 318-14 (5.3.1)} \)

Check the adequacy of slab thickness for beam action (one-way shear)  \( \text{ACI 318-14 (22.5)} \)

\[ V_u = q_u \times A_{\text{Tributary}} = 0.204 \times 145.88 = 29.80 \text{ kips} \]

\[ V_c = 2\lambda \sqrt{f_{c}} b_n d \quad \text{ACI 318-14 (Eq. 22.5.5.1)} \]

Note that \( b_n \) is equal to tributary width of the slab minus the opening width as shown in the following Figure.

\( \lambda = 1 \) for normal weight concrete

\[ \phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{3000} \times (18 \times 12 - 18) \times \frac{4.75}{1000} = 77.30 \text{ kips} > V_u = 29.80 \text{ kips} \]

Slab thickness of 6 in. is adequate for one-way shear.

The one-way shear capacity of the slab was reduced by 8.33% due to the presence of the opening.

\[ \text{Figure 6 – Critical Section for One-Way Shear (Slab with Opening)} \]
2. Slab Two-Way (Punching) Shear Strength

2.1. Slab without Opening

\( b_1 = \) Dimension of the critical section \( b_o \) measured in the direction of the span for which moments are determined in ACI 318, Chapter 8 (see the following Figure).

\( b_2 = \) Dimension of the critical section \( b_o \) measured in the direction perpendicular to \( b_1 \) in ACI 318, Chapter 8 (see the following Figure).

\( b_o = \) The length of the critical perimeter.

For interior support:

\[ b_1 = c_1 + d_{avg} = 26 + 4.75 = 30.75 \text{ in.} \]

\[ b_2 = c_2 + d_{avg} = 12 + 4.75 = 16.75 \text{ in.} \]

\[ b_o = 2 \times (b_1 + b_2) = 2 \times (30.75 + 16.75) = 95 \text{ in.} \]

![Critical shear perimeter for interior column](image1)

![Critical shear perimeter for exterior column](image2)

![Critical shear perimeter for corner column](image3)

Figure 7 – Critical Shear Perimeters for Columns
Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (see the following Figure):

Tributary area for two-way shear is:

\[ A_{\text{tributary}} = (18 \times 18) \left( \frac{30.75}{12} \times \frac{16.75}{12} \right) = 320.42 \text{ ft}^2 \]

\[ V_a = q_a \times A_{\text{tributary}} = 0.204 \times 320.42 = 65.40 \text{ kips} \]

The two-way shear stress \( (v_u) \) can then be calculated as:

\[ v_u = \frac{V_a}{b_o \times d} + \frac{\gamma M_{\text{unb}} c_{sb}}{J_c} \quad \text{ACI 318-14 (R.8.4.4.2.3)} \]

Note that the Reference did not provide information about \( M_{\text{unb}} \) (unbalanced moment). Additionally, the value of \( M_{\text{unb}} \) for an interior support with equal adjacent spans is relatively small and can be neglected in the two-way punching shear calculations.

\[ v_u = \frac{V_a}{b_o \times d_{avg}} = \frac{65.40}{95.00 \times 4.75} = 144.86 \text{ psi} \]

\[ \phi v_c = \min \left\{ \phi \times 4 \times \frac{\sqrt{f_c}}{\alpha_d} \right\} \quad \text{ACI 318-14 (Table 22.6.5.2)} \]

\[ \phi v_c = \min \left\{ \phi \times \left( 2 + \frac{4}{\beta} \right) \times \frac{\sqrt{f_c}}{\alpha_d} \right\} \]

\[ \phi v_c = \min \left\{ \phi \times \left( \frac{a_d d}{b_o} + 2 \right) \times \frac{\sqrt{f_c}}{\alpha_d} \right\} \]

\[ \phi v_c = \min \left\{ \frac{0.75 \times 4 \times 1 \times \sqrt{3000}}{2 + \frac{4}{2.17}} \times 1 \times \sqrt{3000} \right\} = \min \left\{ \frac{164.32}{158.00} \right\} = 158.00 \text{ psi} \]

Since \( \phi v_c \geq v_u \) at the critical section, the slab has adequate two-way shear strength at this column.
Figure 8 – Critical Section for Two-Way Punching Shear (Slab without Opening)
2.2. Slab with Opening

Check the adequacy of the slab thickness for two-way (punching) shear at an interior column (see the following Figure):

Tributary area for two-way shear is:

\[ A_{\text{trib}} = 320.42 \text{ ft}^2 \]  
(calculated in Section 2.1)

\[ V_u = q_u \times A_{\text{trib}} = 65.40 \text{ kips} \]  
(calculated in Section 2.1)

The two-way shear stress \( v_u \) can then be calculated as:

Since the opening is located within 10 times the slab thickness from the column. The effect of openings in slabs on concrete shear strength shall be considered. (10 x 6 in. = 60 in. > 12 in.). Slab opening effect is evaluated by reducing the perimeter of the critical section \( b_o \) by a length equal to the projection of the opening enclosed by two-lines extending from the centroid of the column and tangent to the opening (see the following Figure).

\[ b_o = b_1 + b_2 + \frac{b_1 + b_2}{2} = 30.75 + 16.75 + 27.01 + 10.52 = 85.03 \text{ in.} \]

\[ v_u = \frac{V_u}{b_o \times d} = \frac{65.40}{58.03 \times 4.75} = 161.84 \text{ psi} \]

\[
\phi v_c = \min \left\{ \phi \times \left( \frac{2 + \frac{4}{\beta}}{b_o} \right) \times \lambda \times \sqrt{f'_c} \right\}
\]

\[
\phi v_c = \min \left\{ \phi \times \left( \frac{a_d d + 2}{b_o} \right) \times \lambda \times \sqrt{f'_c} \right\}
\]

\[ \phi v_c = \min \left\{ 0.75 \times 4 \times 1 \times \sqrt{3000} \right\} = \min \left\{ 164.32 \right\} = 158.00 \text{ psi} \]

Since \( \phi v_c < v_u \) at the critical section, the slab does not have adequate two-way shear strength at this column.

Note that the perimeter of the critical section was reduced by 10.49% and required two-way shear strength were increased by 11.73% due to the presence of the opening.
Figure 9 – Critical Section for Two-Way Shear (Slab with Opening)
3. Conclusions & Observations

The punching shear capacity of a concrete slab around the columns typically governs the thickness of the slab, thus, any openings at the intersection of column strips should be considered carefully. This is especially critical near corner and edge columns where the shear stresses in the slab are typically the highest.

If openings must be made in the intersection of column strips, to install a drainage pipe for example, the size of the opening should be no larger than 12 in. as recommended by PCA guideline (An Engineer’s Guide to: Openings in Concrete Floor Slabs).

Openings cut in the intersection of column strips should be evaluated carefully, since they reduce the critical section for resisting punching shear (as shown in this example). One possible exception to PCA guideline is when column capitals, commonly seen in older structures, are present to reduce shear stresses in the slab.

Openings located at the intersection of column and middle strips, are less critical, and small openings having a width less than 15% of the span length can often be made in this area.

The most favorable location for openings from a structural point of view is often the intersection of two middle strips. This is also often the least favorable location from an architectural point of view, however, because it's the most disruptive to the function of the space. The guidelines for openings in flat slabs generally follow the recommendations for flat plates, but the chances of accommodating larger openings in the intersection of two middle strips are increased due to the lower shear stresses in the region of the drop panels.

The following figure provides generic guidance for a designer with regards to permitted opening locations and dimensions. A designer needs to consider the distance between the opening and concentrated load or reaction area in addition to this generic guidance as outlined in ACI 318.  

ACI 318-14 (8.5.4.2)
Figure 10 – Permitted Openings in Slab Systems without Beams for $l_2 > l_1$ (PCA Notes on ACI 318-11)