



# Slenderness Effects for Columns in Non-Sway Frame - Moment Magnification Method (ACI 318-14)







#### Slender Concrete Column Design in Non-Sway Frame Buildings

Evaluate slenderness effects for columns in a non-sway multistory reinforced concrete frame by determining the adequacy of the square tied column shown below, which is an exterior first floor column. The design forces obtained from a first-order analysis are provided in the design data section below. The story height is 12 ft. it is assumed that the frame is braced sufficiently to prevent relative translation of its joints. Assume 40% of the factored axial load is sustained. Compare the calculated results with the values presented in the Reference and with exact values from <u>spColumn</u> engineering software program from <u>StructurePoint</u>.



#### Figure 1 - Reinforced Concrete Column Cross-Section



# Contents

Slenderness Effects and Sway or Non-sway Frame Designation	2
Determine Slenderness Effects	2
Moment Magnification – Non-Sway Frame	4
3.1. Calculation of Critical Load (Pc)	4
3.2. Calculation of Magnified Moment (Mc)	5
Column Design	6
Column Design - spColumn Software	6
Summary and Comparison of Design Results	.16
Conclusions & Observations	.17
7.1. General Observations	.17
7.2. Design Column Boundary Conditions in Slenderness Calculations	.17
	Slenderness Effects and Sway or Non-sway Frame Designation         Determine Slenderness Effects         Moment Magnification – Non-Sway Frame         3.1. Calculation of Critical Load (P <sub>c</sub> )         3.2. Calculation of Magnified Moment (M <sub>c</sub> )         Column Design         Column Design - spColumn Software         Summary and Comparison of Design Results         Conclusions & Observations         7.1. General Observations         7.2. Design Column Boundary Conditions in Slenderness Calculations



# Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

#### Reference

Reinforced Concrete Design, Eighth Edition, 2018, Wang C. et. al., Example 13.17.3.

# **Design Data**

 Concrete
  $f_c$ ' = 3000 psi

 Steel
  $f_y$  = 60000 psi

 Beams:
 h = 24 in.,
 b = 14 in.,
 l = 7 m

 Columns:
 h = 17 in.,
 b = 17 in.
 H = 12 ft

factored design forces obtained from first-order analysis from the reference:

Table 1 - Column factored loads				
Load Case	Axial Load,	Bending Moment, kip.ft		
	kip	Тор	Bottom	
Factored Load	1776*	105	0	
* Assume 40% of the axial load is sustained				



# 1. Slenderness Effects and Sway or Non-sway Frame Designation

Columns and stories in structures are considered as non-sway frames if the stability index for the story (Q) does not exceed 0.05. <u>ACI 318-14 (6.6.4.3)</u>

The reference assumed that the frame is a non-sway frame since Q value is less than 0.05.

#### 2. Determine Slenderness Effects

The reference decided to be consistent with the more conservative procedure provided by **ACI 318-14** (6.6.4.4.3) by taking k value equals to 1.0. However, the k value, in this example, is calculated based on the exact procedure for illustration purposes.

$$I_{column} = 0.7 \times \frac{c^4}{12} = 0.7 \times \frac{17^4}{12} = 4872 \text{ in.}^4$$

$$ACI 318-14 (6.6.3.1.1(a))$$

$$I_{beam} = 0.35 \times \frac{b \times h^3}{12} = 0.35 \times \frac{14 \times 24^3}{12} = 5645 \text{ in.}^4$$

$$ACI 318-14 (6.6.3.1.1(a))$$

$$E = 57,000 \times \sqrt{f_c} = 57,000 \times \sqrt{3000} = 3122 \text{ ksi}$$
 ACI 318-14 (19.2.2.1.b)

For columns:

$$\frac{E \times I_{column}}{l_c} = \frac{3122 \times 4872}{12 \times 12} = 8.8 \times 10^3 \text{ kip.ft}$$

 $\frac{E \times I_{beam}}{l_b} = \frac{3122 \times 5645}{30 \times 12} = 4.08 \times 10^3 \text{ kip.ft}$ 

For beams framing into the columns:

 $\Psi_B = \infty$  (Column was assumed hinged at base) Using Figure R6.2.5 from ACI 318-14  $\rightarrow k = 0.959$  as shown in the figure below for the exterior columns with one beam framing into them in the directions of analysis.

$$\Psi_{A} = \frac{\left(\sum \frac{EI}{l_{c}}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{2 \times 8.80 \times 10^{3}}{4.08 \times 10^{3}} = 4.32$$

$$ACI 318-14 (Figure R6.2.5)$$



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Figure 2 – Effective Length Factor (k) (Non-Sway Frame)

ACI 318-14 allows to neglect the slenderness in a non-sway frame if:

$$\frac{k \times l_u}{r} \le 34 + 12 \left(\frac{M_1}{M_2}\right)$$
ACI 318-14 (6.2.5b)

Where:

 $r = \text{radius of gyration} = (a) \sqrt{\frac{I_g}{A_g}} \quad or \quad (b) \ 0.3 \times c_1$   $r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{c^2}{12}} = \sqrt{\frac{17^2}{12}} = 4.91 \text{ in.}$   $\frac{0.959 \times (12 \times 12 - 24)}{4.91} = 23.45 < 34 - 12 \left(\frac{0}{105}\right) = 34 \quad \therefore \text{ slenderness can be neglected.}$ 

Even though it is not required to consider slenderness effects for this column, the moment magnification method will be shown for illustration.

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#### 3. Moment Magnification – Non-Sway Frame

$$M_{c2} = \delta M_2$$
 ACI 318-14 (6.6.4.5.1)

Where:

 $P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$ 

Where:

$$\delta = \text{magnification factor} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \ge 1.0$$
  
ACI 318-14 (6.6.4.5.2)
  
ACI 318-14 (6.6.4.5.2)

#### ACI 318-14 (6.6.4.4.4)

ACI 318-14 (6.6.4.4.2)

There are three options for calculating the effective flexural stiffness of slender concrete columns  $(EI)_{eff}$ . The second equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in <u>spColumn</u>. Further comparison of the available options is provided in "<u>Effective</u> <u>Flexural Stiffness for Critical Buckling Load of Concrete Columns</u>" technical note.

#### 3.1. Calculation of Critical Load (Pc)

 $(EI)_{eff} = \begin{cases} (a) & \frac{0.4E_c I_g}{1 + \beta_{dns}} \\ (b) & \frac{0.2E_c I_g + E_s I_{se}}{1 + \beta_{dns}} \\ (c) & \frac{E_c I}{1 + \beta_{dns}} \end{cases} \end{cases}$ 

$$I_{column} = \frac{c^4}{12} = \frac{17^4}{12} = 6960 \text{ in.}^4$$

$$\underline{ACI 318-14 (Table 6.6.3.1.1(a))}$$

$$E_c = 57,000 \times \sqrt{f_c} = 57,000 \times \sqrt{3000} = 3122 \text{ ksi}$$

$$\underline{ACI 318-14 (19.2.2.1.a)}$$

 $\beta_{dns}$  is the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination. <u>ACI 318-14 (6.6.4.4.4)</u>

In this example, it is assumed that 40% of the factored axial load is sustained.

$$\beta_{dns} = \frac{P_{u,sustained}}{P_u} = \frac{0.4 \times P_u}{P_u} = 0.40 < 1.00 \quad \rightarrow \quad \therefore \beta_{dns} = 0.40$$

With 10-#9 reinforcement equally distributed on two sides and 17 in. x 17 in. column section  $\rightarrow I_{se} = 360$  in.<sup>4</sup>.

$$(EI)_{eff} = \frac{0.2E_c I_g + E_s I_{se}}{1 + \beta_{dns}}$$
ACI 318-14 (6.6.4.4.4(b))





$$(EI)_{eff} = \frac{0.2 \times 3122 \times 6960 + 29,000 \times 360}{1+0.4} = 10.56 \times 10^6 \text{ kip-in.}^2$$

 $P_c = \frac{\pi^2 \times 10.56 \times 10^6}{\left(0.959 \times (12 - 2) \times 12\right)^2} = 7871 \text{ kip}$ 

3.2. Calculation of Magnified Moment (M<sub>c</sub>)

$C_m = 0.6 + 0.4 \frac{M_1}{M_2}$	<u>ACI 318-14 (6.6.4.5.3a)</u>
$C_m = 0.6 + 0.4 \left(\frac{0}{105}\right) = 0.6$	
$\delta = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \ge 1.0$	<u>ACI 318-14 (6.6.4.5.2)</u>
$\delta = \frac{0.6}{1 - \frac{525}{0.75 \times 7871}} = 0.66 < 1.00 \rightarrow \delta = 1.00$	
$M_{\min} = P_u \left( 0.6 + 0.03h \right)$	<u>ACI 318-14 (6.6.4.5.4)</u>

Where  $P_u = 525$  kip, and h = the section dimension in the direction being considered = 17 in.

 $M_{\min} = 525 \left( \frac{0.6 + 0.03 \times 17}{12} \right) = 48.56 \text{ kip.ft}$   $M_2 = 105 \text{ kip.ft} > M_{2,\min} = 48.56 \text{ kip.ft} \rightarrow M_2 = 105 \text{ kip.ft}$   $M_{c2} = \delta M_2$  ACI 318-14 (6.6.4.5.1)

 $M_{c2} = 1.00 \times 105 = 105$  kip.ft





#### 4. Column Design

Based on the factored axial loads and magnified moments considering slenderness effects, the capacity of the assumed column section (17 in.  $\times$  17 in. with 10 – #9 bars distributed on two sides) will be checked and confirmed to finalize the design. A column interaction diagram will be generated using strain compatibility analysis, the detailed procedure to develop column interaction diagram can be found in "Interaction Diagram - Tied Reinforced Concrete Column" example.



Figure 3 – Column Interaction Diagram

#### 5. Column Design - spColumn Software

<u>spColumn</u> program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames. For this column section, we ran in design mode with control points using the ACI 318-14. The graphical and text results are provided below for both input and output of the <u>spColumn</u> model.



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spColumn - Slenderness_Non-Sway_ACLco File Input Solve View Options Help	
• • • • • • + × • • •	General Information         Match         Match         Draign Code         AG 318-14         Urbaix         Projein Code         AG 318-14         Draign Code         Match         Projein Code         AG 318-14         Urbaix         Projein Code         Adout Xuaix         Design Code         With Landord         Divide Information         With Landord         Divide Information         OK         Cancel             Sides Different             Material Properties             With Landorg X; Till n         Deh (dong Y; Till n)             DK       Cancel
17 × 17 in 3.46% reinf. MATERIAL: ======== fc = 3 ksi Ec = 3122.02 ksi tc = 2.55 ksi Betal = 0.85 fy = 50 ksi Es = 29000 ksi SECTION: ======= Ag = 289 in^2 k = 6960.08 in^4 iy = 6960.08 in^4 iy = 6960.08 in^4 No = 0 in REINFORCEMENT:	Design Column         XAxis         Clear height:       10         R Nonsway frame         C Sway frame         Sway criteria         Compute K'actors         C Compute K'actors         C Input K'actors:         Kinet I       Kinet I         Copy to YAxis         OK       Cancel
anticle and a second and a seco	Service Loads         Saturation           Avial Load (kip)         XMoments (k, fit)         YMoments (k, fit)         Saturation of Load (kip)           Dead         555         105         0
Y-axis; ACI 318-14; Investigation	

Figure 4 – spColumn Model Input Wizard Windows





Figure 5 – Column Section Interaction Diagram about Y-Axis (spColumn)







spColumn v6.50 Computer program for the Strength Design of Reinforced Concrete Sections Copyright - 1988-2019, STRUCTUREPOINT, LLC. All rights reserved



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Page | 2 11/14/2019 1:56 PM

# Contents

1. General Information	3
2. Material Properties	3
2.1. Concrete	3
2.2. Steel	3
3. Section	3
3.1. Shape and Properties	3
3.2. Section Figure	4
4. Reinforcement	4
4.1. Bar Set: ASTM A615	4
4.2. Confinement and Factors	4
4.3. Arrangement	4
4.4. Bars Provided	5
5. Loading	5
5.1. Load Combinations	5
5.2. Service Loads	5
5.3. Sustained Load Factors	5
6. Slenderness	5
6.1. Sway Criteria	5
6.2. Columns	5
6.3. Y - Beams	5
7. Moment Magnification	6
7.1. General Parameters	6
7.2. Effective Length Factors	6
7.3. Magnification Factors: Y - axis	6
8. Factored Moments	6
8.1. Y - axis	6
9. Factored Loads and Moments with Corresponding Capacity Ratios	6
10. Diagrams	7
10.1. PM at θ=0 [deg]	7

# List of Figures

	•
Figure 1: C	Imp section
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1. General Information

File Name	C:\ACI 318-14 Exa\Slenderness_Non- Sway_ACI.col
Project	Pincheira Example 13.17.3
Column	Exterior
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	Y - axis
Slenderness	Considered
Column Type	Structural
Capacity Method	Critical capacity

# 2. Material Properties 2.1. Concrete

Туре	Standard	
f <sub>c</sub>	3	ksi
Ec	3122.02	ksi
fc	2.55	ksi
ε <sub>u</sub>	0.003	in/in
β1	0.85	

### 2.2. Steel

Туре	Standard	
fy	60	ksi
E₅	29000	ksi
ε <sub>yt</sub>	0.00206897	in/in

# 3. Section

#### 3.1. Shape and Properties

Туре	Rectangular	
Width	17	in
Depth	17	in
Ag	289	in²
l <sub>x</sub>	6960.08	in4
ly	6960.08	in <sup>4</sup>
٢ <sub>x</sub>	4.90748	in
ry	4.90748	in
X <sub>o</sub>	0	in
Yo	0	in

Page | 3 11/14/2019 1:56 PM





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#### 3.2. Section Figure



Figure 1: Column section

# 4. Reinforcement

### 4.1. Bar Set: ASTM A615

Bar	Diameter	Area	Bar	Diameter	Area	Bar	Diameter	Area
	in	in <sup>2</sup>		in	in <sup>2</sup>		in	in <sup>2</sup>
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

#### 4.2. Confinement and Factors

Confinement type	Tied	
For #10 bars or less	#3 ties	
For larger bars	#4 ties	
Capacity Reduction Factors		
Axial compression, (a)	0.8	
Tension controlled ¢, (b)	0.9	
Compression controlled $\phi$ , (c)	0.65	

#### 4.3. Arrangement

Pattern	Sides different
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	
Bars	





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Page | 5 11/14/2019 1:56 PM

Total steel area, A <sub>s</sub>	10.00	in²
Rho	3.46	%
Minimum clear spacing	1.87	in

#### 4.4. Bars Provided

		Bars	Cover
			in
Тор	2	#9	1.936
Bottom	2	#9	1.936
Left	3	#9	1.936
Right	3	#9	1.936

# 5. Loading

#### 5.1. Load Combinations

Combination	Dead	Live	Wind	EQ	Snow
U1	1.000	0.000	0.000	0.000	0.000

#### 5.2. Service Loads

No.	Load Case	Axial Load	Мх @ Тор	Mx @ Bottom	Му @ Тор	My @ Bottom
		kip	k-ft	k-ft	k-ft	k-ft
1	Dead	525.00	105.00	0.00	105.00	0.00
1	Live	0.00	0.00	0.00	0.00	0.00
1	Wind	0.00	0.00	0.00	0.00	0.00
1	EQ	0.00	0.00	0.00	0.00	0.00
1	Snow	0.00	0.00	0.00	0.00	0.00

#### 5.3. Sustained Load Factors

Load Case	Factor
	%
Dead	40
Live	0
Wind	0
EQ	0
Snow	0

# 6. Slenderness

### 6.1. Sway Criteria

Y-Axis	Non-sway column
1 / 003	Hon Sway column

#### 6.2. Columns

Column	Axis	Height	Width	Depth	I	fo	E.
		ft	in	in	in <sup>4</sup>	ksi	ksi
Design	Y	10	17	17	6960.08	3	3122.02
Above	Y	12	17	17	6960.08	3	3122.02
Below	Y	(no column specified)					

#### 6.3. Y - Beams

Beam	Length	Width	Depth	1	f'c	E₀
	ft	in	in	in <sup>4</sup>	ksi	ksi
Above Left	30	14	24	16128	3	3122.02



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Page | 6 11/14/2019 1:56 PM

Beam	Length	Width	Depth	1	fc	Ec
	ft	in	in	in <sup>4</sup>	ksi	ksi
Above Right	(no beam specified)					
Below Left	(no beam specified)					
Below Right	(no beam specified)					

# 7. Moment Magnification

### 7.1. General Parameters

Factors	Code defaults
Stiffness reduction factor, $\phi_{K}$	0.75
Cracked section coefficients, cl(beams)	0.35
Cracked section coefficients, cl(columns)	0.7
0.2 E <sub>c</sub> I <sub>g</sub> + E <sub>s</sub> I <sub>se</sub> (Y-axis)	1.48e+007 kip-in <sup>2</sup>
Minimum eccentricity, eymin	1.11 in

#### 7.2. Effective Length Factors

Axis	$\Psi_{top}$	$\Psi_{bottom}$	k (Nonsway)	k (Sway)	kl <sub>u</sub> /r
Y	4.512	999.000	0.960	(N/A)	23.48

#### 7.3. Magnification Factors: Y - axis

\* Slenderness need not be considered.

Load			At	t Ends			Along Length					
Combo		∑Pu	Pc	∑Pc	β <sub>ds</sub>	δ。	Pu	k'l <sub>u</sub> /r	Pc	$\beta_{dns}$	Cm	δ
		kip	kip	kip			kip		kip			
1	U1	(N/A)	(N/A)	(N/A)	(N/A)	(N/A)	525.00	(N/A)	7850.31	0.400	(N/A)	(N/A) *

# 8. Factored Moments

NOTE: Each loading combination includes the following cases: Top - At column top Bot - At column bottom

#### 8.1. Y - axis

Load				1 <sup>st</sup> Order			2 <sup>nd</sup> Orde	er	Ratio
Combo		M <sub>ns</sub>	Ms	Mu	M <sub>min</sub>	Mi	Mc	2 <sup>nd</sup> /1 <sup>st</sup>	
			k-ft	k-ft	k-ft	k-ft	k-ft	k-ft	
1	U1	Тор	105.00	(N/A)	105.00	(N/A)	M <sub>2</sub> = (N/A)	(N/A)	(N/A)
1	U1	Bot	0.00	(N/A)	0.00	(N/A)	M <sub>1</sub> = (N/A)	(N/A)	(N/A)

#### 9. Factored Loads and Moments with Corresponding Capacity Ratios

NOTE: Calculations are based on "Critical Capacity" Method. Each loading combination includes the following cases: Top - At column top Bot - At column bottom

Dot - At column bottom

No. Load			Demand		Capacity		Parameters at Capacity			Capacity
Combo			Pu	Muy	φPn	φM <sub>ny</sub>	NA Depth	٤t	ф	Ratio
			kip	k-ft	kip	k-ft	in			
1 1	U1	Тор	525.00	105.00	588.71	137.85	15.81	-0.00025	0.650	0.86
2 1	U1	Bot	525.00	0.00	681.95	0.00	18.48	-0.00065	0.650	0.77





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#### 10. Diagrams 10.1. PM at θ=0 [deg]

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0						
Project	Pincheira nle 13	17.3				
Column	Exterior					
Engineer	SP					
Code	ACI 318-14					
Bar Set	ASTM A615					
Units	English					
Run Option	Investigation					
Run Axis	Y - axis					
Slenderness	Considered					
Column Type	Structural					
Capacity Method	Critical capacity					
Materials		turi.				
r.	3	KSI				
Ec	3122.02	KSI				
	60	kei				
Ty	00	KSI				
<b>E</b> <sub>8</sub>	29000	KSI				
Section						
Туре	Rectangular					
Width	17	in				
Depth	17	in				
Ag	289	in <sup>2</sup>				
×	6960,08	in4				
y.	6960,08	in4				
Reinforcement						
Pattern	Sides different					
Bar layout	Rectangular					
Cover to	Longitudal bars					
Clear cover	_					
Bars						
Confinement type	Tied					
Total steel area, A <sub>s</sub>	10.00	in²				
Rho	3.46	%				
Min. clear spacing	1.87	in				





### 6. Summary and Comparison of Design Results

Analysis and design results from the hand calculations above are compared with the reference values and the exact values obtained from <u>spColumn</u> model.

Table 2 – Parameters for moment magnification of column in non-sway frame							
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$						M <sub>c</sub> , kip.ft	
Reference	1.000	10.50×10 <sup>6</sup>	7200	525	0.66	105	
Hand	0.959	10.56×10 <sup>6</sup>	7871	525	0.66	105	
spColumn	0.960	10.57×10 <sup>6</sup>	7850	525	N/A	105	

All the results of the hand calculations illustrated above are in precise agreement with the automated exact results obtained from the <u>spColumn</u> program.

The notes below are helpful to the <u>spColumn</u> user in creating the design model:

- The reference used the larger of the two equations provided by ACI 318-14 (6.6.4.4.4) to calculate (EI)<sub>eff</sub> since both (EI)<sub>eff</sub> equations are lower bounds. However, the hand solution and <u>spColumn</u> use the first equation since it provides an estimate that is dependent on the reinforcement configuration provided in the section. 2.
- 2. The reference used an approximate equation to calculate the radius of gyration (r) while the hand solution and <u>spColumn</u> use the exact equation to calculate r value.
- 3. The reference decided to use k = 1 in accordance with the more conservative procedure of ACI 318-14 (6.6.4.4.3). The hand solution and <u>spColumn</u> calculate the exact k value.
- 4.  $\delta_{ns}$  in the three methods of solution shown above need not be calculated since the slenderness effects need not be considered. The reference and hand solution show this value for illustration purposes.



#### 7. Conclusions & Observations

#### 7.1. General Observations

The analysis of the reinforced concrete section performed by <u>spColumn</u> conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames.

ACI 318 provides multiple options for calculating values of r and (EI)<sub>eff</sub> leading to variability in the determination of the adequacy of a column section. Engineers must exercise judgment in selecting suitable options to match their design condition. The <u>spColumn</u> program utilizes the exact methods whenever possible and allows user to override the calculated values with direct input based on their engineering judgment wherever it is permissible.

#### 7.2. Design Column Boundary Conditions in Slenderness Calculations

When the slenderness effects for a non-sway frame column is considered in creating a model using <u>spColumn</u>, the effective length factor can be computed by defining the properties of the columns and beams connected to the top and bottom of the design column. The following notes are helpful when using <u>spColumn</u> to calculate the k value for some of the special boundary conditions cases:

1. To model pin supports at the top and bottom of the design column:





2. To model fix supports at the top and bottom of the design column:



3. To model pin support at the top and fix support at the bottom of the design column:

Columns Above and Below       S3         Column Above       Column Below         Image: No column specified       Image: No column specified         Height (c/c):       12       ft         Width (along X):       17       in         Depth (along Y):       17       in         Concrete,       fc:       3       ksi         Ec:       3122.(       ksi       Ec:       3122.(         Copy to Column Below       Copy to Column Above       OK       Cancel	Y-Beams (perpendicular to Y)       23         Beam Location:       Above Right         Above Left       Above Right         Beam Below Left       Below Right         No beam specified       Copy From Beam Above         Span (c/c):       30       ft         Yidth:       14       in         Ec:       3122.02 ksi         Depth.       24       in         OK       Cancel	$\Psi_{\rm A} = 999$
→ "No column specified" for column above and column below	<ul> <li>Define a high moment of inertia value for one of the beams at the location of the fix support.</li> <li>"No beam specified" for beams at the location of the pin support.</li> </ul>	$\Psi_{\rm B} = 0$