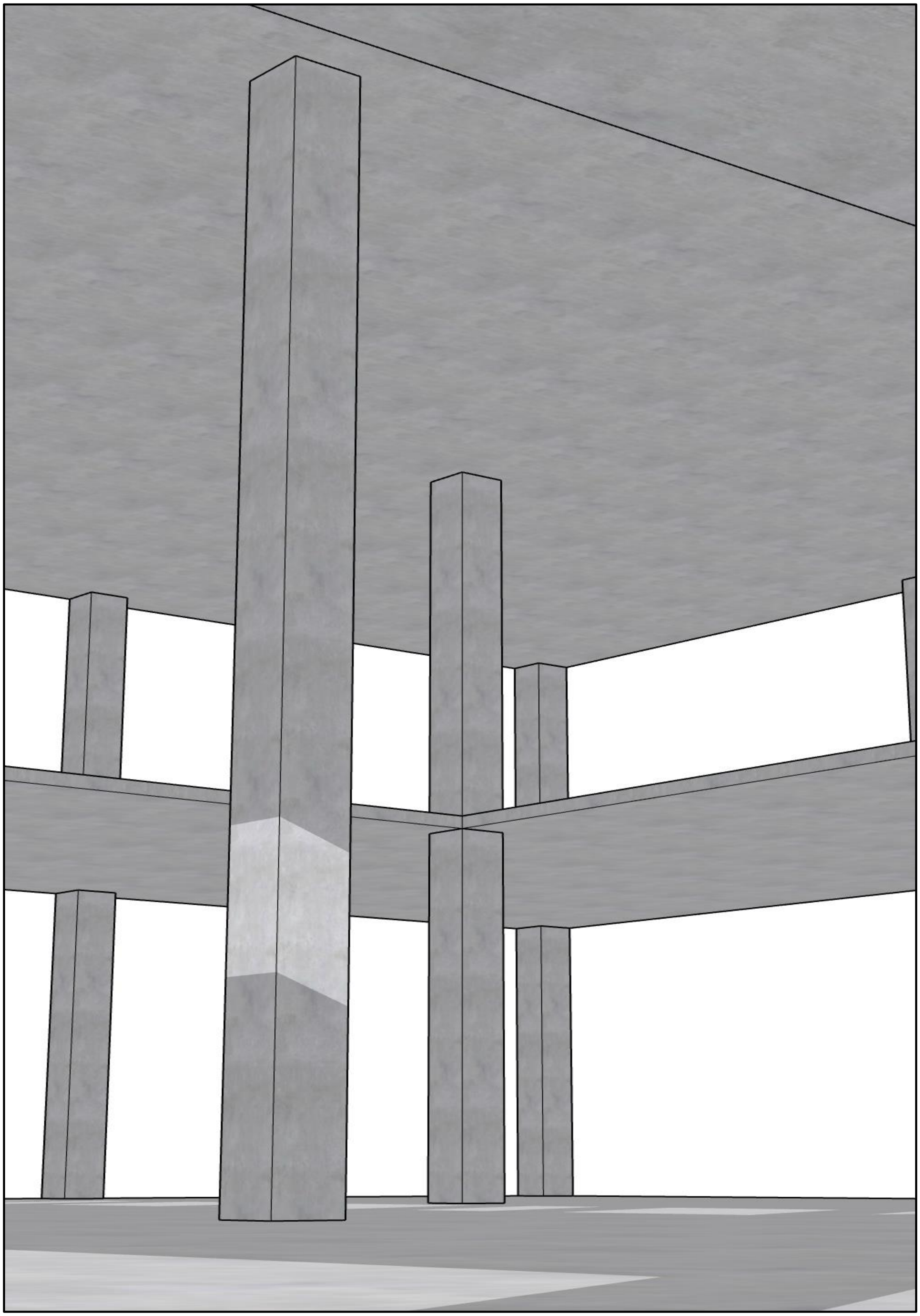
Slenderness Effects for Columns in Non-Sway Frame - Moment Magnification Method (CSA A23.3-19)



Slender Concrete Column Design in Non-Sway Frame Buildings

Evaluate slenderness effect for columns in a non-sway frame multistory reinforced concrete building (Q is computed to be much less than 0.05) by designing a two-story high column in the middle of an atrium opening at the second-floor level. The design forces obtained from a first-order analysis are provided in the design data section below. The story height is 4.3 m. it is assumed that the column only resists gravity loads. Compare the calculated results with exact values from spColumn engineering software program from StructurePoint.

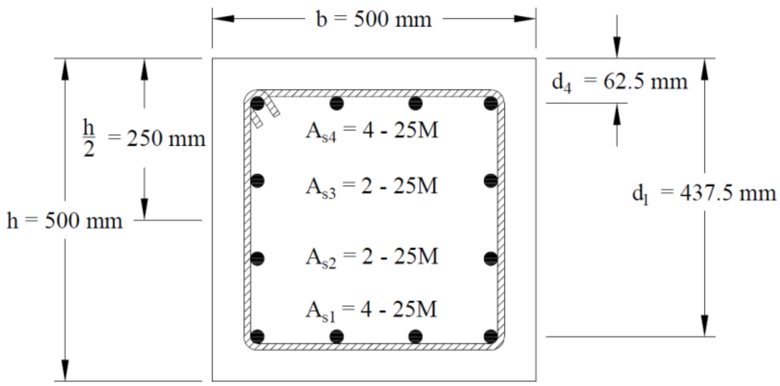


Figure 1 – Slender Reinforced Concrete Column Cross-Section

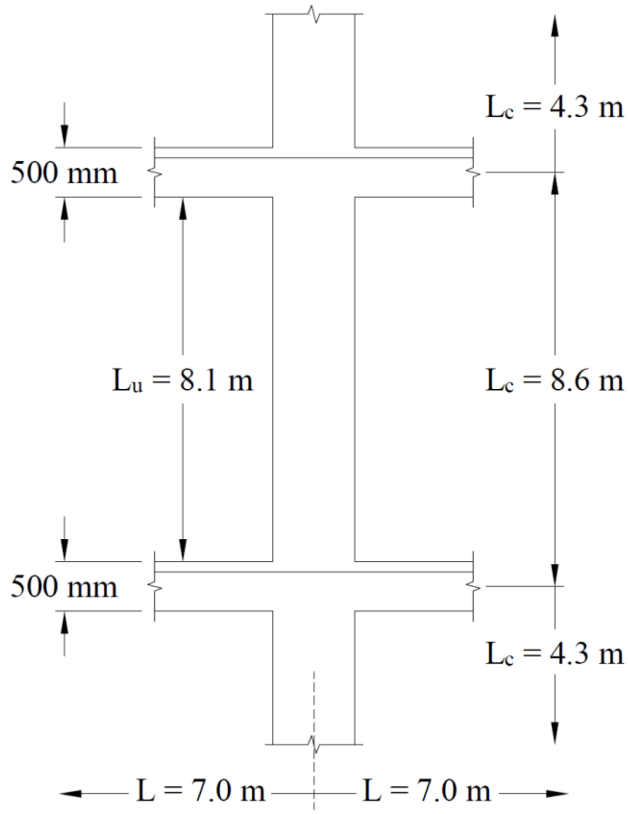


Figure 2 – Slender Reinforced Concrete Column Elevation

Contents

Code

Design of Concrete Structures (CSA A23.3-19)

Reference

Concrete Design Handbook, Fourth Edition, 2016, Cement Association of Canada (CAC), Example 8.1.

Design Data

Concrete $f_c' = 40$ MPa $\rho_c = 2400$ kg/m^3

Steel $f_y = 400$ MPa

Slab: $h_s = 150$ mm, $b_{eff} = 1800$ mm

Beams: h = 500 mm, $b_w = 400$ mm, $l = 7$ m

Columns: h = 500 mm, b = 500 mm

Additional service design forces obtained from first-order analysis will be considered in this example to outline and discuss the evolution of CSA A23.3 provisions in slenderness calculations for non-sway columns where the largest first-order moment, M_2, is less than the minimum moment, $M_{2,min}$:

Table 1 – Additional column service load cases

Load Case	Axial Load, kN	Bending Moment, kN.m	
		Top	Bottom
Dead, D	1776	48	-8
Live, L	1320	30	-5

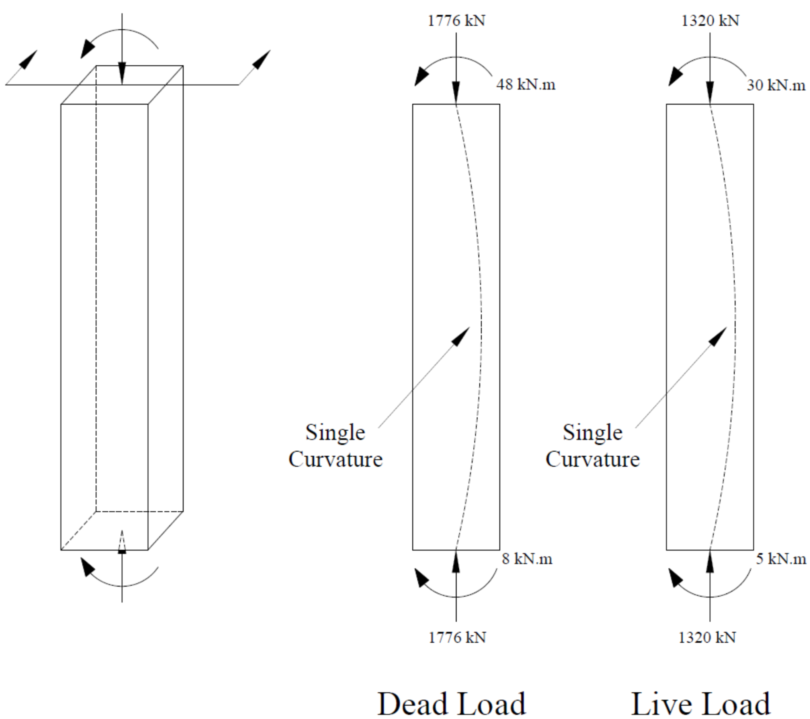


Figure 3 – Service Design Forces

1. Factored Axial Loads and Bending Moments

1.1. Load Combinations – Factored Loads

CSA A.23.3-19 (Annex C, Table C.1a)

Table 2 - Column factored loads							
CSA A23.3-14 Reference	No.	Load Combination	Axial Load, kN	Bending Moment, kN.m		$M_{Top,ns}$ kN.m	$M_{Bottom,ns}$ kN.m
				Top	Bottom		
Annex C Table C.1a	1	1.4D	2486	67.2	11.2	67.2	11.2
	2	1.25D + 1.5L	4200	105.0	17.5	105.0	17.5

2. Slenderness Effects and Sway or Non-sway Frame Designation

Columns and stories in structures are considered as non-sway frames if the stability index for the story (Q) does not exceed 0.05. ***CSA A.23.3-19 (10.14.4)***

The reference assumed that the Q value is much less than 0.05. Therefore, the frame is considered as a non-sway frame.

3. Effective Length Factor (k)

$$I_{column} = 0.7 \times \frac{c^4}{12} = 0.7 \times \frac{500^4}{12} = 3.65 \times 10^9 \text{ mm}^4$$ ***CSA A.23.3-19 (10.14.1.2)***

$$E_c = \left(3,300 \times \sqrt{f_c^{'}} + 6,900\right)\left(\frac{\gamma_c}{2,300}\right)^{1.5}$$ ***CSA A.23.3-19 (Eq. 8.1)***

$$E_c = \left(3,300 \times \sqrt{40} + 6,900\right)\left(\frac{2,400}{2,300}\right)^{1.5} = 29602 \text{ MPa}$$

For column being designed:

$$\frac{E_c \times I_{column}}{l_c} = \frac{29602 \times 3.65 \times 10^9}{8600} = 1.25 \times 10^{10} \text{ N.mm}$$

For other columns:

$$\frac{E_c \times I_{column}}{l_c} = \frac{29602 \times 3.65 \times 10^9}{4300} = 2.51 \times 10^{10} \text{ N.mm}$$

For beams framing into the columns:

$$\frac{E_b \times I_{beam}}{l_b} = \frac{29602 \times 2.70 \times 10^9}{7000} = 1.14 \times 10^{10} \text{ N.mm}$$

Where:

$$I_{beam} = 0.35 \times 7.7 \times 10^9 = 2.70 \times 10^9 \text{ mm}^4$$ ***CSA A.23.3-19 (10.14.1.2)***

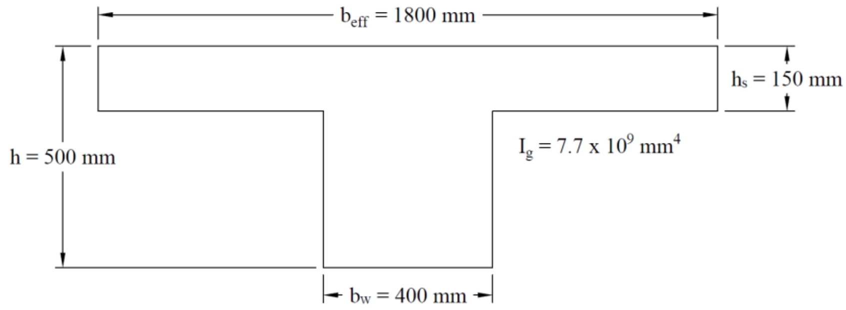


Figure 4 – Beam Cross-Section

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{1.25 + 2.51}{2 \times 1.14} = 1.65$$ ***CSA A.23.3-19 (Figure N.10.15.1)***

$$\Psi_B = \Psi_A = 1.65$$

Using Figure N10.15.1(a) from CSA A23.3-14 → k = 0.835 as shown in the figure below for the exterior column.

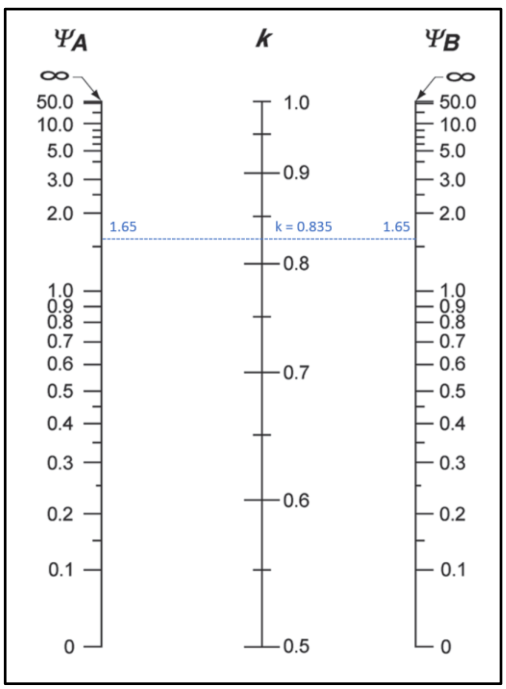


Figure 5 – Effective Length Factor (k) (Non-Sway Frame)

4. Check if Slenderness can be Neglected

CSA A23.3-19 allows to neglect the slenderness in a non-sway frame if:

$$\frac{k \times l_u}{r} \le \frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}}$$ ***CSA A.23.3-19 (Eq. 10.16)***

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{c^2}{12}} = \sqrt{\frac{500^2}{12}} = 144.34 \text{ mm}$$

$$\frac{k \times l_u}{r} = \frac{0.835 \times (8600-500)}{144.34} = 46.86$$

$$\frac{k \times l_u}{r} \le \frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}}$$ ***CSA A.23.3-19 (Eq. 10.16)***

per CSA A23.3-19:

- M_1/M_2 is not taken less than -0.5.
- M_1/M_2 shall be taken positive if the member is bent in single curvature **and**
- **shall be taken as 1.0 if M_2 is less than $M_{2,min}$**

Check minimum moment: ***CSA A23.3-19 (10.15.3.1)***

$$(M_2)_{\min} = P_f(15 + 0.03h)$$

$$(M_2)_{\min} = 4200 \times (15 + 0.03 \times 500)/1000 = 126 \text{ kN.m} > M_2 = 105 \text{ kN.m}$$

Since $M_2 < M_{2,min}$, M_1/M_2 ratio shall be taken as 1.0. ***CSA A.23.3-19 (10.15.2)***

$$\frac{M_1}{M_2} = 1.0$$

$$\frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}} = \frac{25-10(1.0)}{\sqrt{\frac{4200 \times 1000}{40 \times (500 \times 500)}}} \frac{25-10}{0.648} = 23.15$$

$$\frac{k \times l_u}{r} = 46.86 > \frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}} = 23.15 \quad \therefore \text{ slenderness can't be neglected.}$$

5. Moment Magnification – Non-Sway Frame

$$M_c = \frac{C_m M_2}{1-\dfrac{P_f}{\phi_m P_c}} \geq M_2$$ **CSA A23.3-19 (10.15.3.1)**

Where:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$ **CSA A23.3-19 (10.15.3.2)**

And, the member resistance factor would be $\phi_m = 0.75$ **CSA A23.3-19 (10.15.3.1)**

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ **CSA A23.3-19 (Eq. 10.18)**

Where:

$$(EI)_{eff} = \begin{Bmatrix} \text{(a)} \dfrac{0.2E_c I_g + E_s I_{st}}{1+\beta_d} \\ \text{(b)} \dfrac{0.4E_c I_g}{1+\beta_d} \end{Bmatrix}$$ **CSA A23.3-19 (10.15.3.1)**

There are two options for calculating the effective flexural stiffness of slender concrete columns $(EI)_{eff}$. The first equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

5.1. Calculation of Critical Load (P_c)

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{500^4/12}{500^2}} = 144.34 \text{ mm}$$ **CSA A23.3-19 (10.14.2)**

With 12 – 25M reinforcement equally distributed on all sides and 500 mm x 500 mm column section

$$I_{st} = 0.176 \times \rho_t \times b \times h^3 \times \gamma^2$$ **Concrete Design Handbook (Table 8.2(b))**

$$I_{st} = 0.176 \times \frac{12 \times 500}{500 \times 500} \times 500 \times 500^3 \times 0.75^2 = 1.485 \times 10^8 \text{ mm}^4$$

$$\beta_d = \frac{P_{f,sustained}}{P_f} = \frac{2220}{4200} = 0.529$$

$$(EI)_{eff} = \frac{0.2E_c I_g + E_s I_{st}}{1+\beta_d}$$ **CSA A23.3-19 (Eq. 10-19)**

$$(EI)_{eff} = \frac{0.2\times(29602)\times(5.21\times10^{9})+(200000)\times(1.485\times10^{8})}{1+0.529} = 3.96\times10^{13} \text{ N.mm}^2$$

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ **CSA A23.3-19 (Eq. 10.18)**

$$P_c = \frac{\pi^2\times(3.96\times10^{13})}{(0.835\times(8600-500))^2} = 8544 \text{ kN}$$

5.2. Calculation of Magnified Moment (M_c)

$$C_m = 0.6+0.4\frac{M_1}{M_2} \geq 0.4$$ **CSA A23.3-19 (10.15.3.2)**

CSA A23.3-04, clause 10.15.3.1 stated that *"M_2 in Equation 10.16 shall not be taken as less than* $P_f(15+0.03h)$ *about each axis separately."*

CSA A23.3-14, clause 10.15.3.1 stated that *"M_2 in Equation 10.17 shall not be taken as less than* $P_f(15+0.03h)$ *about each axis separately* ***with the member bent in single curvature with C_m taken as 1.0.****"*

The CSA A23.3-14, clause 10.15.3.1 provides unclear guidance implying the M_2 shall not be taken less than the minimum moment, $P_f(15+0.03h)$ with members bent in single curvature only. This provision is revised entirely and clarified in CSA A23.3-19 as follows to consistently require C_m = 1.0 in all cases where $M_{2,min}$ exceeds M_2.

CSA A23.3-19, clause 10.15.3.1 states that *"M_2 in Equation 10.17 shall not be taken as less than $M_{2,min}$ about each axis separately.* ***If $M_{2,min}$ exceeds M_2, C_m shall be taken as equal to 1.0.****"*

Check minimum moment: **CSA A23.3-19 (10.15.3.1)**

$$(M_2)_{\min} = P_f(15+0.03h) = 126 \text{ kN.m} > M_2 = 105 \text{ kN.m}$$

Therefore, $C_m = 1.0$ **CSA A23.3-19 (10.15.3.1)**

$$M_c = \frac{C_m M_2}{1-\dfrac{P_f}{\phi_m P_c}} \geq M_2$$ **CSA A23.3-19 (Eq. 10.17)**

$$M_c = \frac{1.0\times126}{1-\dfrac{4200}{0.75\times8544}} = \frac{1.0\times126}{1-0.655} = 2.9\times126 = 365.6 \text{ kN.m} \geq 126.0 \text{ kN.m}$$

The slenderness effects resulted in a 248% increase of the first-order moment.

6. Column Design

Based on the factored axial loads and magnified moments considering slenderness effects, the capacity of the assumed column section (500 mm × 500 mm with 12 – 25M bars distributed all sides equal) will be checked and confirmed to finalize the design. A column interaction diagram will be generated using strain compatibility analysis, the detailed procedure to develop column interaction diagram can be found in "Interaction Diagram - Tied Reinforced Concrete Column" example.

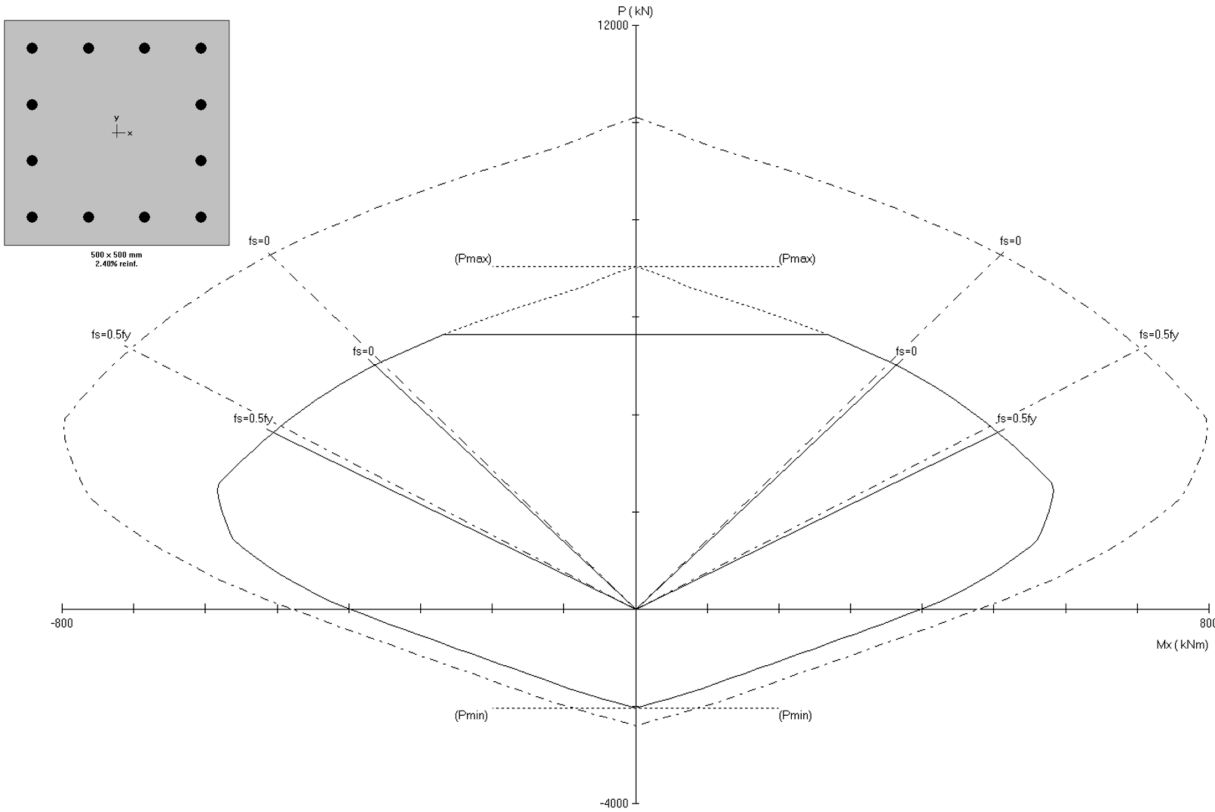
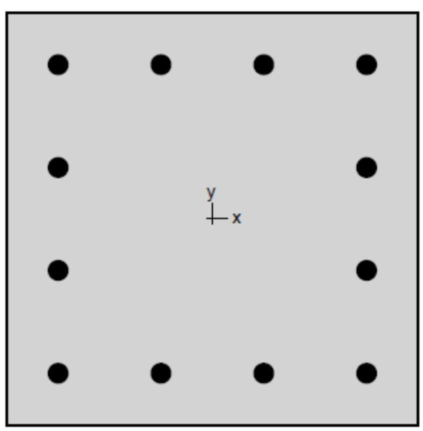


Figure 6 – Designed Column Interaction Diagram

7. Column Design - spColumn Software

spColumn (TM)

spColumn v7.00
Computer program for the Strength Design of Reinforced Concrete Sections



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List of Figures

1. General Information

File Name	C:...\Slenderness_Non-Sway Frame_CSA A23.3-19.col
Project	Design of Braced Column
Column	Interior
Engineer	SP
Code	CSA A23.3-19
Bar Set	CSA G30.18
Units	Metric
Run Option	Investigation
Run Axis	X - axis
Slenderness	Considered
Column Type	Structural
Capacity Method	Moment capacity

2. Material Properties

2.1. Concrete

Type	Standard
f'_c	40 MPa
E_c	29601.6 MPa
f_c	31.6 MPa
ε_u	0.0035 mm/mm
β_1	0.87

2.2. Steel

Type	Standard
f_y	400 MPa
E_s	200000 MPa
ε_{yt}	0.002 mm/mm

3. Section

3.1. Shape and Properties

Type	Rectangular
Width	500 mm
Depth	500 mm
A_g	250000 mm²
I_x	5.20833e+009 mm⁴
I_y	5.20833e+009 mm⁴
r_x	144.338 mm
r_y	144.338 mm
X_o	0 mm
Y_o	0 mm

3.2. Section Figure

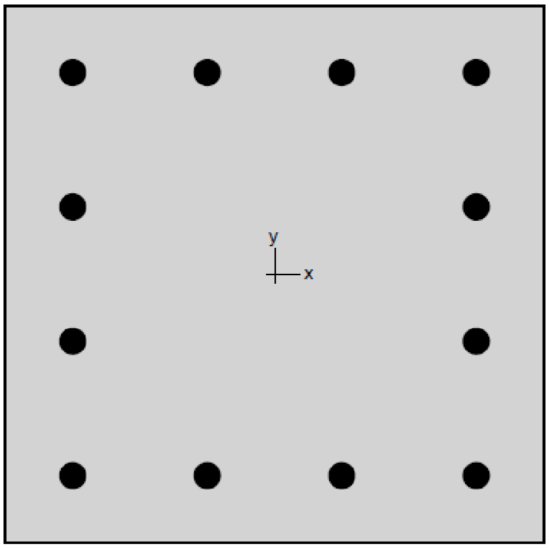


Figure 1: Column section

4. Reinforcement

4.1. Bar Set: CSA G30.18

Bar	Diameter mm	Area mm^2	Bar	Diameter mm	Area mm^2	Bar	Diameter mm	Area mm^2
#10	11.30	100.00	#15	16.00	200.00	#20	19.50	300.00
#25	25.20	500.00	#30	29.90	700.00	#35	35.70	1000.00
#45	43.70	1500.00	#55	56.40	2500.00			

4.2. Confinement and Factors

Confinement type	Tied
For #55 bars or less	#10 ties
For larger bars	#15 ties
Material Resistance Factors	
Axial compression, (a)	0.8
Steel (ϕ_s)	0.85
Concrete (ϕ_c)	0.65
Minimum dimension, h	500 mm

4.3. Arrangement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	50 mm

Bars	12 #25
Total steel area, A_s	6000 mm²
Rho	2.40 %
Minimum clear spacing	100 mm

5. Loading

5.1. Load Combinations

Combination	Dead	Live	Wind	EQ	Snow
U1	1.400	0.000	0.000	0.000	0.000
U2	1.250	1.500	0.000	0.000	0.000

5.2. Service Loads

No.	Load Case	Axial Load	Mx @ Top	Mx @ Bottom	My @ Top	My @ Bottom
		kN	kNm	kNm	kNm	kNm
1	Dead	1776.00	48.00	-8.00	0.00	0.00
1	Live	1320.00	30.00	-5.00	0.00	0.00
1	Wind	0.00	0.00	0.00	0.00	0.00
1	EQ	0.00	0.00	0.00	0.00	0.00
1	Snow	0.00	0.00	0.00	0.00	0.00

5.3. Sustained Load Factors

Load Case	Factor
	%
Dead	100
Live	0
Wind	0
EQ	0
Snow	0

6. Slenderness

6.1. Sway Criteria

X-Axis	Non-sway column

6.2. Columns

Column	Axis	Height	Width	Depth	I	f'c	Ec
		m	mm	mm	mm⁴	MPa	MPa
Design	X	8.1	500	500	5.20833e+009	40	29601.6
Above	X	4.3	500	500	5.20833e+009	40	29601.6
Below	X	4.3	500	500	5.20833e+009	40	29601.6

6.3. X - Beams

Beam	Length	Width	Depth	I	f'c	Ec
	m	mm	mm	mm⁴	MPa	MPa
Above Left	7	740	500	7.70833e+009	40	29601.6
Above Right	7	740	500	7.70833e+009	40	29601.6
Below Left	7	740	500	7.70833e+009	40	29601.6
Below Right	7	740	500	7.70833e+009	40	29601.6

7. Moment Magnification

7.1. General Parameters

Factors	Code defaults
Stiffness reduction factor, ϕ_K	0.75
Cracked section coefficients, cI(beams)	0.35
Cracked section coefficients, cI(columns)	0.7
$0.2\ E_c\ I_g + E_s\ I_{se}$ (X-axis)	6.05e+010 kNmm²
Minimum eccentricity, $e_{x\ min}$	30.00 mm
k'	$(P_f\ /\ (f'_c{}^*A_g))^{0.5}$

7.2. Effective Length Factors

Axis	Ψ_{top}	Ψ_{bottom}	k (Nonsway)	k (Sway)	kl_u/r
X	1.650	1.650	0.835	(N/A)	46.83

7.3. Magnification Factors: X - axis

Load Combo		At Ends $\sum P_f$ kN	P_c kN	$\sum P_c$ kN	β_{ds}	δ_s	Along Length P_f kN	$k'l_u/r$	P_c kN	β_{dns}	C_m	δ
1	U1	(N/A)	(N/A)	(N/A)	(N/A)	(N/A)	2486.40	(N/A)	6533.27	1.000	1.000	2.030
1	U2	(N/A)	(N/A)	(N/A)	(N/A)	(N/A)	4200.00	(N/A)	8548.21	0.529	1.000	2.899

8. Factored Moments

NOTE: Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

8.1. X - axis

Load Combo			1st Order M_{ns} kNm	M_s kNm	M_f kNm	M_{min} kNm	2nd Order	M_l kNm	M_c kNm	Ratio 2nd/1st
1	U1	Top	67.20	(N/A)	67.20	74.59	M_2=	67.20	151.44	(N/A)
1	U1	Bot	11.20	(N/A)	11.20	74.59	M_1=	11.20	151.44	(N/A)
1	U2	Top	105.00	(N/A)	105.00	126.00	M_2=	105.00	365.33	(N/A)
1	U2	Bot	17.50	(N/A)	17.50	126.00	M_1=	17.50	365.33	(N/A)

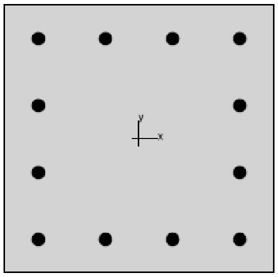
9. Factored Loads and Moments with Corresponding Capacity Ratios

NOTE: Calculations are based on "Moment Capacity" Method.
Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

No.	Load Combo			Demand P_f kN	M_{fx} kNm	Capacity P_r kN	M_{rx} kNm	Parameters at Capacity NA Depth mm	ε_t	Capacity Ratio
1	1	U1	Top	2486.40	151.44	2486.40	583.06	274	0.00209	0.26
2	1	U1	Bot	2486.40	151.44	2486.40	583.06	274	0.00209	0.26
3	1	U2	Top	4200.00	365.33	4200.00	455.21	379	0.00054	0.80
4	1	U2	Bot	4200.00	365.33	4200.00	455.21	379	0.00054	0.80

10. Diagrams

10.1. PM at θ=0 [deg]



500 x 500 mm

General Information

Project	Design of Braced Column
Column	Interior
Engineer	SP
Code	CSA A23.3-19
Bar Set	CSA G30.18
Units	Metric
Run Option	Investigation
Run Axis	X - axis
Slenderness	Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials

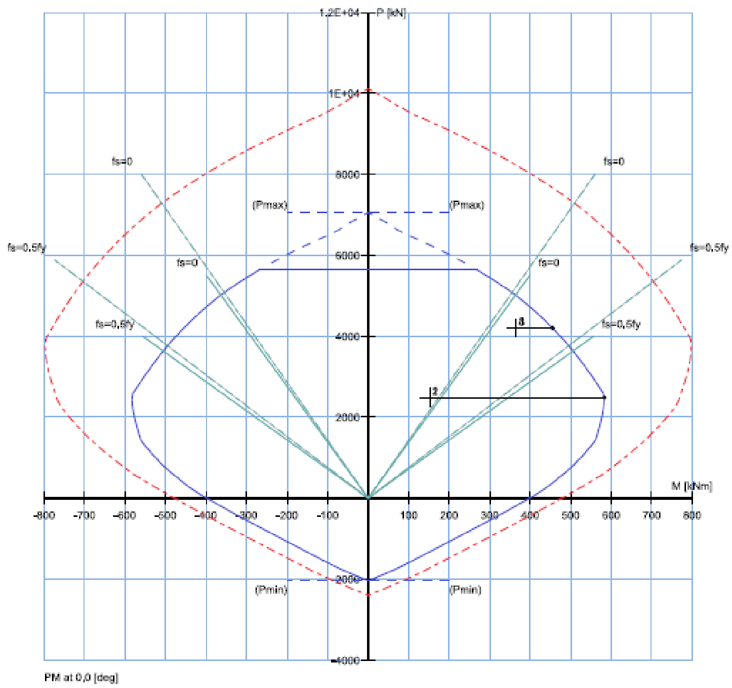
f'_c	40 MPa
E_c	29601.6 MPa
f_y	400 MPa
E_s	200000 MPa

Section

Type	Rectangular
Width	500 mm
Depth	500 mm
A_g	250000 mm²
I_x	5.20833e+009 mm⁴
I_y	5.20833e+009 mm⁴

Reinforcement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	50 mm
Bars	12 #25
Confinement type	Tied
Total steel area, A_s	6000 mm²
Rho	2.40 %
Min. clear spacing	100 mm



No.	Load Combo			P_f kN	M_{fx} kNm	P_r kN	M_{rx} kNm	Capacity Ratio
3	1	U2	Top	4200.0	365.3	4200.00	455.21	0.80
4	1	U2	Bot	4200.0	365.3	4200.00	455.21	0.80
1	1	U1	Top	2486.4	151.4	2486.40	583.06	0.26
2	1	U1	Bot	2486.4	151.4	2486.40	583.06	0.26

Max. Capacity Ratio: 0.80

8. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames.

CSA A23.3 provides multiple options for calculating values of $(EI)_{eff}$ and magnification factor leading to variability in the determination of the adequacy of a column section. Engineers must exercise judgment in selecting suitable options to match their design condition. The spColumn program utilizes the exact methods whenever possible and allows user to override the calculated values with direct input based on their engineering judgment wherever it is permissible.

In the case where the larger first-order moment (M_2) is less than the minimum moment ($M_{2,min}$) for a column in non-sway frame, significantly higher magnification is expected when using CSA A23.3-19 as compared to the prior editions of CSA A23.3. The first-order moment increased by 248% (compared to 93.5% increase when using CSA A23.3-14)[*] due to the adjustment on clause 10.15.3.1 in CSA A23.3-19 where C_m shall be taken as equal to 1.0 when $M_{2,min}$ exceeds M_2. [M_2 = 105 kN.m, $M_{2,min}$ = 126 kN.m, M_c = 365.6 kN.m (CSA A23.3-19) and M_c = 243.8 kN.m (CSA A23.3-14)[*]]

[*] Detailed calculations are shown in "Slenderness Effects for Columns in Non-Sway Frame - Moment Magnification Method (CSA A23.3-14)" design example.