Combined Axial Force and Biaxial Bending Interaction Diagram – Square Reinforced Concrete Column (ACI 318-14)
Combined Axial Force and Biaxial Bending Interaction Diagram - Square Reinforced Concrete Column (ACI 318-14)

Biaxial bending of columns occurs when the loading causes bending simultaneously about both principal axes. The commonly encountered case of such loading occurs in corner columns. Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load.

A uniaxial interaction diagram defines the load-moment strength along a single plane of a section under an axial load $P$ and a uniaxial moment $M$. The biaxial bending resistance of an axially loaded column can be represented schematically as a surface formed by a series of uniaxial interaction curves drawn radially from the $P$ axis. Data for these intermediate curves are obtained by varying the angle of the neutral axis (for assumed strain configurations) with respect to the major axes.

The difficulty associated with the determination of the strength of reinforced columns subjected to combined axial load and biaxial bending is primarily an arithmetic one. The bending resistance of an axially loaded column about a particular skewed axis is determined through iterations involving simple but lengthy calculations. These extensive calculations are compounded when optimization of the reinforcement or cross-section is sought.

This example demonstrates the determination of the design axial load capacity, $\phi P_n$, and the design $\phi M_{nx}$ and $\phi M_{ny}$ moments corresponding to the following case: The neutral axis position crosses the vertical axis of symmetry of the section ($y$-axis) at 10 in. below the top of the section, at an angle of $30^\circ$ counterclockwise from the $x$-axis of the cross section. The figure below shows the reinforced concrete square column cross section in consideration. We will compare the calculated values of the column axial strength and biaxial bending strength with the values from the reference and the exact values from spColumn engineering software program from StructurePoint. The steps to develop the three-dimensional failure surface (interaction diagram) using spColumn will be shown in detail as well.

Figure 1 – Reinforced Concrete Column Cross-Section
Contents

1. Concrete Column Biaxial Strength Calculations
   1.1. Location of Neutral Axis and Concrete Compression Force
   1.2. Strains and Forces Determination in Reinforcement Layers
   1.3. Calculation of $P_{\text{nl}}, M_{\text{nx}}$ and $M_{\text{ny}}$

2. Column Biaxial Bending Interaction Diagram – spColumn Software

3. Summary and Comparison of Design Results

4. Conclusions & Observations
Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference


spColumn Engineering Software Program Manual v6.50, StructurePoint, 2019

Design Data

\[ f'_c = 4000 \text{ psi} \]
\[ f_y = 60000 \text{ psi} \]

Cover = 2.4 in.

Column dimensions and reinforcement locations are shown in following figure.

Figure 2 – Reinforced Concrete Column Cross-Section and Reinforcement Locations
Solution

In a reinforced concrete column, the determination of the nominal axial load capacity, $P_n$, and the nominal $M_{nx}$ and $M_{ny}$ moments involves a trial-and-error process for calculating the neutral axis depth and angle $\alpha$. The reference provided the neutral axis depth and angle as an input (The neutral axis position crosses the vertical axis of symmetry of the section at 10 in. leading to $c = 12.66$ in. and an angle of $\alpha = 30.0^\circ$) for illustration.

The steps to calculate biaxial flexural strength of a reinforced concrete column for a given nominal axial strength and moment ratio of biaxial bending moments is discussed in details in “Combined Axial Force and Biaxial Bending Interaction Diagram - Rectangular Reinforced Concrete Column (ACI 318-14)” design example.

Figure 3 – Nominal Axial Load and Biaxial Flexural Strength Calculation Methods for a Reinforced Concrete Column.
1. Concrete Column Biaxial Strength Calculations

Figure 4 – Strains, Forces, and Moment Arms Diagram
1.1. **Location of Neutral Axis and Concrete Compression Force**

The trial-and-error process for calculating the neutral axis depth and angle $\alpha$ is not required in this example since these values are given by the reference ($c = 12.66$ in. and $\alpha = 30.0^\circ$). Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis and $\alpha$ is the angle of the neutral axis.

**ACI 318-14 (22.2.4.2)**

$$
\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207
$$

$$
\varepsilon_{cs} = (c - d_s) \times \frac{E_s}{c} = \left(12.66 - 18.578 \times \frac{0.003}{12.66}\right) = -0.00140 \text{ (Tension)} < \varepsilon_y \rightarrow \text{reinforcement has not yielded}
$$

$\therefore \phi = 0.65$

**ACI 318-14 (Table 21.2.2)**

$$a = \beta_1 \times c = 0.85 \times 12.66 = 10.761 \text{ in.}$$

**ACI 318-14 (22.2.4.1)**

$$\varepsilon_{cu} = 0.003$$

**ACI 318-14 (22.2.1)**

Where:

$a = \text{Depth of equivalent rectangular stress block}$

**ACI 318-14 (Table 22.2.4.3)**

$$\beta_1 = 0.85 - \frac{0.05 \times (f_y \times 4000)}{1000} = 0.85 - \frac{0.05 \times (4000 - 4000)}{1000} = 0.85$$

**ACI 318-14 (Table 22.2.4.3)**

$$C_y = 0.85 \times f_y \times A_{comp} = 0.85 \times 4000 \times 124.88 = 424.59 \text{ kip (Compression)}$$

**ACI 318-14 (22.2.4.1)**

Where (see the following figure):

$$A_{comp} = A_1 + A_2 = \left(\frac{1}{2} \times 9.24 \times 16\right) + (3.19 \times 16) = 124.88 \text{ in}^2$$

$$-\bar{x} = \left(\frac{A_1 \times x_1 + A_2 \times x_2}{A_1 + A_2}\right) = -8.00 = \left(\frac{73.84 \times 5.33 + 51.04 \times 8.00}{73.84 + 51.04}\right) - 8.00 = -1.58 \text{ in.}$$

$$-\bar{y} = \left(\frac{A_1 \times y_1 + A_2 \times y_2}{A_1 + A_2}\right) = -4.42 = \left(\frac{73.84 \times 6.153 + 51.04 \times 18.825}{73.84 + 51.04}\right) - 4.42 = 3.64 \text{ in.}$$
1.2. Strains and Forces Determination in Reinforcement Layers

The following shows the calculations of forces in the reinforcement layers with the extreme tension (at bar 5) and extreme compression (at bar 1) strains. The calculations for the rest of layers are shown the table at the end of this section.

For extreme tension reinforcement layer (at bar 5):

\[ \varepsilon_{t5} = -0.00140 \text{ (Tension) < } \varepsilon_y \rightarrow \text{reinforcement has not yielded} \]

\[ f_{t5} = \varepsilon_{t5} \times E_s = -0.00140 \times 29000000 = -40669 \text{ psi} \]

\[ F_{t5} = f_{t5} \times A_{s5} = -40669 \times (1 \times 0.79) = -32.13 \text{ kip (Tension)} \]

For extreme compression reinforcement layer (at bar 1):

\[ \varepsilon_{c1} = (c - d_i) \times \frac{E_s}{E_c} = (12.66 - 3.278) \times \frac{0.003}{12.66} = 0.00222 \text{ (Compression) > } \varepsilon_y \rightarrow \text{reinforcement has yielded} \]

\[ f_{c1} = f_c = 60000 \text{ psi} \]
The area of the reinforcement in this layer is included in the area used to compute $C_c$ ($a = 10.76$ in. > $d_1 = 3.28$ in.). As a result, it is necessary to subtract $0.85f_c'$ from $f_s$ before computing $F_s$.

$$F_s = f_s \times A_1 = (60000 - 0.85 \times 4000) \times (1 \times 0.79) = 44.71 \text{ kip (Compression)}$$

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement locations, results are summarized in the following table:

<table>
<thead>
<tr>
<th>Location</th>
<th>$d_i$, in.</th>
<th>$e_i$, in./in.</th>
<th>$f_i$, psi</th>
<th>$F_s$, kip</th>
<th>$C_c$, kip</th>
<th>Moment arm (x), in.</th>
<th>Moment arm (y), in.</th>
<th>Moment arm (z), kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>---</td>
<td>0.00300</td>
<td>---</td>
<td>---</td>
<td>424.59</td>
<td>1.58</td>
<td>55.90</td>
<td>3.64</td>
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<tr>
<td>Bar 1</td>
<td>3.278</td>
<td>0.00222</td>
<td>60000</td>
<td>44.71</td>
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<td>2.4</td>
<td>20.87</td>
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<td>6.078</td>
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<td>8.0</td>
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<td>13.6</td>
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<td>13.728</td>
<td>-0.00025</td>
<td>-7339</td>
<td>-5.8</td>
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<td>13.6</td>
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<td>Bar 5</td>
<td>18.578</td>
<td>-0.00140</td>
<td>-40669</td>
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<td>13.6</td>
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<td>-21427</td>
<td>-16.93</td>
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<td>0.00</td>
<td>13.6</td>
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<td>-0.00008</td>
<td>-2185</td>
<td>-1.73</td>
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<td>-0.81</td>
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<tr>
<td>Bar 8</td>
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<td>Axial Force and Biaxial Bending Moments Capacities</td>
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<td></td>
<td></td>
<td>$P_n$, kip</td>
<td>$M_{ny}$, kip-ft</td>
<td>$M_{nx}$, kip-ft</td>
<td>$\phi P_n$, kip</td>
<td>$\phi M_{ny}$, kip-ft</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>485.54</td>
<td>95.56</td>
<td>197.11</td>
<td>315.60</td>
<td>62.12</td>
</tr>
</tbody>
</table>

The area of the reinforcement in this layer has been included in the area used to compute $C_c$. As a result, 0.85$f_c'$ is subtracted from $f_s$ in the computation of $F_s$.

### 1.3. Calculation of $P_n$, $M_{ny}$ and $M_{nx}$

$$P_n = C_c + \sum F_i$$

$$\phi P_n = \phi \times P_n = 0.65 \times P_n$$

$$M_{ny} = C_c \left( \frac{b}{2} - x_c \right) + \sum_{i=1}^{n-10} \left( F_i \times \left( \frac{b}{2} - x_i \right) \right)$$

$$\phi M_{ny} = \phi \times M_{ny} = 0.65 \times M_{ny}$$

$$M_{nx} = C_c \left( \frac{b}{2} - y_c \right) + \sum_{i=1}^{n-10} \left( F_i \times \left( \frac{b}{2} - y_i \right) \right)$$

$$\phi M_{nx} = \phi \times M_{nx} = 0.65 \times M_{nx}$$

$\phi$ is a factor that accounts for the reduction in strength due to loading conditions.

$C_c$ is the characteristic compressive strength of concrete.

$F_i$ is the force from each reinforcement bar.

$x_i$, $y_i$, and $z_i$ are the distances of each reinforcement bar from the centroid of the cross-section.

$P_n$ is the axial force capacity.

$M_{ny}$ and $M_{nx}$ are the biaxial bending moments capacity.
2. **Column Biaxial Bending Interaction Diagram – spColumn Software**

   *spColumn* program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with “biaxial” option for “Run Axis” using the ACI 318-14.

   For biaxial runs, the values of maximum compressive axial load capacity and maximum tensile load capacity are computed. These two values set the range within which the moment capacities are computed for a predetermined number of axial load values. For each level of axial load, the section is rotated in 10-degree increments from 0 degrees to 360 degrees and the $M_x$ and $M_y$ moment capacities are computed. Thus, for each level of axial load, an $M_x$-$M_y$ contour is developed. Repeating this for the entire range of axial loads, the three-dimensional failure surface is computed. A three-dimensional visualization of the resulting entire nominal and factored failure surface is provided to support enhanced understanding of the section capacity.

   The “biaxial” feature allows the user to investigate the P-M interaction diagrams, the $M_x$-$M_y$ moment contour plots, as well as the 3D failure surface for even the most irregular column and shear wall sections quickly, simply, and accurately.

   In lieu of using program shortcuts, *spColumn* model editor was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.
Figure 6 – Generating spColumn Model
Figure 7 – spColumn Model Editor (spSection)
Figure 8 – Column Section Interaction Diagram at 26° (spColumn)
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<table>
<thead>
<tr>
<th>File Name</th>
<th>...Combined Axial Force and Biaxial Bending...</th>
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<tr>
<td>Project</td>
<td>Square Column - Biaxial</td>
</tr>
<tr>
<td>Column</td>
<td>Corner</td>
</tr>
<tr>
<td>Engineer</td>
<td>SP</td>
</tr>
<tr>
<td>Code</td>
<td>ACI 318-14</td>
</tr>
<tr>
<td>Bar Set</td>
<td>ASTM A615</td>
</tr>
<tr>
<td>Units</td>
<td>English</td>
</tr>
<tr>
<td>Run Option</td>
<td>Investigation</td>
</tr>
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<td>Biaxial</td>
</tr>
<tr>
<td>Slenderness</td>
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<tr>
<td>Column Type</td>
<td>Structural</td>
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<tr>
<td>Capacity Method</td>
<td>Critical capacity</td>
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2. Material Properties

2.1. Concrete

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<th>Type</th>
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<tbody>
<tr>
<td>$f_c$</td>
<td>4 ksi</td>
</tr>
<tr>
<td>$E_c$</td>
<td>3605 ksi</td>
</tr>
<tr>
<td>$f_y$</td>
<td>3.4 ksi</td>
</tr>
<tr>
<td>$e_y$</td>
<td>0.003 in/in</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.85</td>
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2.2. Steel

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<tr>
<td>$f_y$</td>
<td>60 ksi</td>
</tr>
<tr>
<td>$E_s$</td>
<td>29000 ksi</td>
</tr>
<tr>
<td>$e_s$</td>
<td>0.002068897 in/in</td>
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3. Section

3.1. Shape and Properties

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<th>Irregular</th>
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<tr>
<td>$A_c$</td>
<td>250 in²</td>
</tr>
<tr>
<td>$I_x$</td>
<td>5461.33 in⁴</td>
</tr>
<tr>
<td>$I_y$</td>
<td>5461.33 in⁴</td>
</tr>
<tr>
<td>$r_x$</td>
<td>4.6188 in</td>
</tr>
<tr>
<td>$r_y$</td>
<td>4.6188 in</td>
</tr>
<tr>
<td>$X_c$</td>
<td>0 in</td>
</tr>
<tr>
<td>$Y_c$</td>
<td>0 in</td>
</tr>
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</table>
3.2. Section Figure

![Figure 1: Column section](Image)

3.3. Exterior Points

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<th>X</th>
<th>Y</th>
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<tr>
<td>1</td>
<td>-8.0</td>
<td>-8.0</td>
</tr>
<tr>
<td>2</td>
<td>8.0</td>
<td>-8.0</td>
</tr>
<tr>
<td>3</td>
<td>8.0</td>
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4. Reinforcement

4.1. Bar Set: ASTM A615

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<tr>
<th>Bar</th>
<th>Diameter</th>
<th>Area</th>
<th>Bar</th>
<th>Diameter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3</td>
<td>0.38</td>
<td>0.11</td>
<td>#4</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>#6</td>
<td>0.75</td>
<td>0.44</td>
<td>#7</td>
<td>0.88</td>
<td>0.60</td>
</tr>
<tr>
<td>#9</td>
<td>1.13</td>
<td>1.00</td>
<td>#10</td>
<td>1.27</td>
<td>1.27</td>
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<tr>
<td>#14</td>
<td>1.69</td>
<td>2.25</td>
<td>#18</td>
<td>2.26</td>
<td>4.00</td>
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</table>

4.2. Confinement and Factors

- Confinement type: Tied
  - For #10 bars or less: #3 ties
  - For larger bars: #4 ties

- Capacity Reduction Factors
  - Axial compression, (a): 0.8
  - Tension controlled, (b): 0.9
  - Compression controlled, (c): 0.65
Two factored loads are applied to locate the nominal (point 1) and design (point 2) capacities of the section. In both points, the capacity ratio is calculated based on the design capacity causing point 1 to show 54% beyond design capacity.
6. Diagrams

6.1. PM at θ=26 [deg] [User]
### 6.2. MM at P=484.1 [kip] [User]

#### General Information
- **Project:** Square C, Biaxial
- **Column:** Corner
- **Code:** ACI 318-14
- **Bar Set:** ASTM A615
- **Units:** English
- **Run Option:** Investigation
- **Stress Code:** Biaxial
- **Slenderness:** Not Considered
- **Column Type:** Structural
- **Capacity Method:** Critical capacity

#### Materials
- $f_y = 4$ ksi
- $f_p = 36 ksi$
- $t = 60$ ksi
- $s_u = 29000$ ksi

#### Section
- **Type:** Irregular
- $A_s = 256$ in$^2$
- $I_x = 5461.33$ in$^4$
- $I_y = 5461.33$ in$^4$

#### Reinforcement
- **Pattern:** Irregular
- **Bar layout:** ---
- **Cover to:** ---
- **Clear cover:** ---
- **Bars:** ---
- **Confinement type:** Tied

- **Total steel area, $A_s = 6.32$ in$^2$**
- **Rho = 2.47 %**
- **Min. clear spacing = 4.80 in**

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<thead>
<tr>
<th>No.</th>
<th>$P_s$</th>
<th>$M_{ux}$</th>
<th>$M_{uy}$</th>
<th>$\phi P_s$</th>
<th>$M_{ux}$</th>
<th>$M_{uy}$</th>
<th>Capacity Ratio</th>
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<tbody>
<tr>
<td>1</td>
<td>484.1</td>
<td>197.2</td>
<td>95.7</td>
<td>358.41</td>
<td>124.60</td>
<td>60.46</td>
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Max. Capacity Ratio: 1.54
6.3. MM at P=314.7 [kip] [User]

**General Information**
- Project: Square C... Biaxial
- Column: Corner
- Engineer: SP
- Code: ACI 318-14
- Bar Set: ASTM A615
- Units: English
- Run Option: Investigation
- Run Axis: Biaxial
- Slenderness: Not Considered
- Column Type: Structural
- Capacity Method: Critical capacity

**Materials**
- $f_y$: 4 ksi
- $f_p$: 360 ksi
- $t_o$: 60 ksi
- $t_s$: 29000 ksi

**Section**
- Type: Irregular
- $A_s$: 256 in$^2$
- $L_s$: 5461.33 in
- $V_s$: 5461.33 in$^3$

**Reinforcement**
- Pattern: Irregular
- Bar layout: ---
- Cover to: ---
- Clear cover: ---
- Bars: ---
- Confinement type: Tied
- Total steel area, $A_s$: 6.32 in$^2$
- $Rho$: 2.47 %
- Min. clear spacing: 4.60 in

<table>
<thead>
<tr>
<th>No.</th>
<th>$P_a$</th>
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<th>$M_{uy}$</th>
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<tr>
<td>2</td>
<td>314.7</td>
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<td>314.68</td>
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Max. Capacity Ratio: 1.54
3. Summary and Comparison of Design Results

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<tbody>
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<td>c, in.</td>
<td>12.66</td>
<td>12.66</td>
<td>12.64</td>
</tr>
<tr>
<td>d₅, in.</td>
<td>18.58</td>
<td>18.58</td>
<td>18.58</td>
</tr>
<tr>
<td>εₛ, in./in.</td>
<td>0.00140</td>
<td>0.00140</td>
<td>0.00141</td>
</tr>
<tr>
<td>φₚₚ, kip</td>
<td>315.0</td>
<td>315.6</td>
<td>314.7</td>
</tr>
<tr>
<td>φₚₘₓ, kip-ft</td>
<td>128.3</td>
<td>128.1</td>
<td>128.2</td>
</tr>
<tr>
<td>φₚₘₙ, kip-ft</td>
<td>62.3</td>
<td>62.1</td>
<td>62.2</td>
</tr>
</tbody>
</table>

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.
4. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns may be subjected to biaxial bending ($M_x$ and $M_y$) due to lateral effects and unbalanced moments from both directions of analysis. This requires an investigation of the column P-$M_x=M_y$ interaction diagram in two directions simultaneously (axial force interaction with biaxial bending).

This example shows the calculations needed to obtain one point on the three-dimensional failure surface (biaxial $M_x-M_y$ interaction diagram). Generating the three-dimensional failure surface (interaction diagram) for a column section subjected to a combined axial force and biaxial bending moments is tedious and challenging for engineers and the use of a computer aid can save time and eliminate errors. StructurePoint’s spColumn program can, quickly, simply and accurately generate the three-dimensional failure surface (interaction diagram) for all commonly encountered column, beam or wall sections in addition to highly complex and irregular cross-sections.
Figure 9 – Interaction Diagram in Two Directions (Biaxial) (spColumn)
The spColumn 2D/3D viewer is a powerful tool especially for investigating interaction diagrams (failure surfaces) for columns and walls sections subjected to a combined axial force and biaxial bending moments. The viewer allows the user to view and analyze 2D interaction diagrams and contours along with 3D failure surfaces in a multi viewport environment. The following figure shows three views of:

1. P-M interaction diagram cut at angle of 26º
2. Mx-My interaction diagram cut at axial load of 314.7 kip in compression
3. A 3D failure surface (interaction diagram showing the points calculated in this example).

Figures (11-12) and (13) show 3D visualization of failure surface with a horizontal and vertical plane cut, respectively.
Figure 10 – 2D/3D Biaxial Interaction Diagram Viewer (spColumn)
Figure 11 – 3D Visualization of Failure Surface with a Horizontal Plane Cut at $P = 314.7$ kip (spColumn)
Figure 12 – 3D Visualization of Failure Surface with a Horizontal Plane Cut at P = 484.1 kip (spColumn)
Figure 13 – 3D Visualization of Failure Surface with a Vertical Plane Cut at 26° (spColumn)