

Post-Tensioning for Two-Way Flat Plate Construction

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Every major metropolitan area is getting a facelift. Old buildings are being renovated and converted, while new construction continues to add to the skyline. Downtown living is making its resurgence, and with this booming residential construction, two-way post-tensioned flat plates are the structural system of choice.

The post-tensioning system

The most common post-tensioning system for two-way slab building construction uses mono-strand unbonded tendons. In this type of construction, the prestressing steel is composed generally of high-strength, single-wire steel, wrapped with another six wires to form a seven-wire strand.

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Learning Objectives

Upon reading this article and completing the quiz, you should be able to understand the design process for post-tensioned two-way slabs and recognize constructibility issues involved when using a post-tensioning system.

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The common strand has a specified tensile strength, f_{pu} , of 270 kips per square inch (ksi), a nominal diameter of 1/2 inch, and an area of steel, A_{ps} , equaling 0.153 square inch (Figure 1). By design, unbonded tendons have a continuous plastic sheathing to prevent the strand from bonding with the concrete along its length. This sheathing serves as the bond breaker; provides protection during handling, shipping, and construction; and limits intrusion of corrosive elements. Corrosion-inhibiting grease coats the strands to reduce friction between the strand and the sheathing during stressing.

The force in a stressed tendon is transferred to the concrete via serrated wedges that lock into anchor plates provided at its ends. Anchors are classified as either live (stressing) ends or dead ends. Dead end anchors are embedded into the concrete and will not be stressed. These anchors are mounted to the tendon at the fabrication plant. Live end anchors are mounted and stressed in the field. Each tendon is stressed individually and has its own anchor plate (thus, *mono-strand*) with approximate dimensions of 2-1/4 inches by 5-1/4 inches. This small, ductile iron casting transfers 33 kips of force for a seven-wire, 1/2-inch-diameter, 270-ksi tendon in a concentrated area. Since this involves high local stresses, it is essential to place the anchors accurately, consolidate the surrounding concrete, limit or eliminate penetrations in the immediate vicinity, and sufficiently reinforce the anchorage zone to preserve its long-term integrity.

Analysis — Computers have increased the speed of post-tensioning design significantly, but it is still important to understand the concepts and calculations to arrive at an accurate output. When performing manual calculations, the Equivalent Frame Method (EFM) of the American Concrete Institute's Building Code Requirements for Structural Concrete (ACI 318-05) within Section 13.7 (excluding sections 13.7.7.4-5) often is used for the structural analysis of a post-tensioned, two-way, flat slab structure.



Figure 1: Tendon showing anchor, strand, and sheathing

EFM models a 3-D slab system as a series of equivalent 2-D frames along the support lines, taken longitudinally and transversely through the structure. Each equivalent frame then can be analyzed individually as an isolated plane frame, consisting of a row of columns or supports and

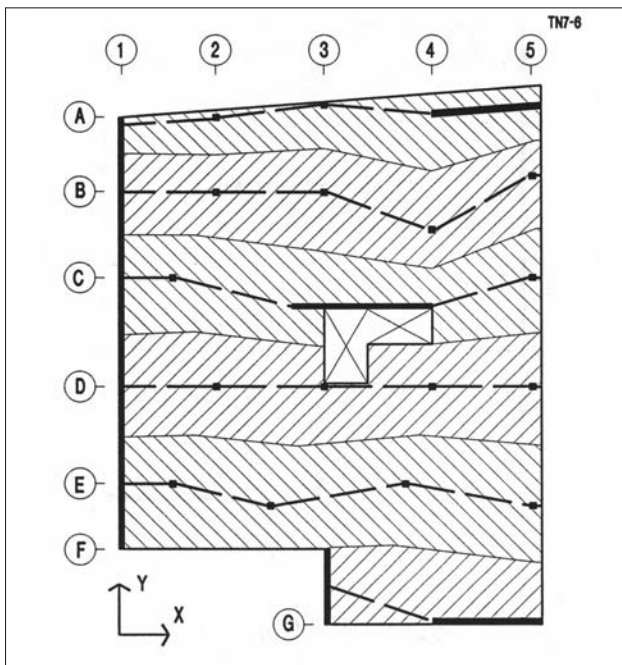


Figure 2: Tributary slab widths for equivalent frames (Aalami, B. and Bommer, A., *Design Fundamentals of Post-Tensioned Concrete Floors*, Post-Tensioning Institute, Phoenix, AZ, 1999)

the corresponding tributary slab width, bounded laterally by the centerlines of the adjacent slab panels (Figure 2). The analysis and design of post-tensioned, two-way slabs incorporate the full tributary slab width without the distribution of forces and reinforcement between column strips and middle strips, synonymous with mild reinforced two-way slab designs. This allowance eases the structural engineer's design process and ultimately simplifies construction.

Preliminary sizing — Before design can begin on the structure, there needs to be a starting point for the slab thickness. For common occupancy structures with live load to dead load (LL/DL) ratios less than 1.0, a preliminary slab thickness can be estimated using a longest span to thickness ratio, L/h , of 45 for floors and 48 for roofs. For example, an 8-inch-thick slab is typical for a 30-foot-long floor span (30 feet x (12 inches/foot) / 45 = 8 inches).

Because of post-tensioning's ability to balance loads and greatly reduce service load deflections, there is a 25 percent to 35 percent reduction in slab thickness in post-tensioned structures compared with mild reinforced structures. Therefore, in addition to the ability to span further, the reduced structural depth economizes material quantities for the slabs and consequently the columns and foundations.

Design for prestressing

The amount of prestressing in a slab system is guided by parameters and requirements given in ACI 318 as well as numerous other references, but the engineer has flexibility in adjusting the design for optimization of an individual project. Tendons for building construction usually are placed with a parabolic vertical profile to counteract a portion of the grav-

ity loads on the structure (Figure 3). This undulating profile places the center of gravity, CGS, of the tendon force, P , eccentric to the neutral axis of the concrete section creating the primary moment, $P \times e$. The eccentricity, e , is the distance from the tendon CGS to the section's neutral axis at any examined cross section.

The high and low points are governed by several parameters. The ends of the tendons usually are located at the section's neutral axis (mid-height for a slab), so as to not induce additional moment at the anchors. In building construction, minimum cover requirements per ACI 318 are sufficient for slabs not exposed to a corrosive environment. Fire protection often governs the minimum concrete cover of these structures. Fire resistance issues are addressed in model building codes, assigned at the local level for a given structure, and should be evaluated during the preliminary design. As it pertains to prestressed concrete two-way slabs, fire resistance requirements are dependent on the restrained versus unrestrained conditions. For most applications, the interior spans in the direction of frame design are considered restrained. That is, they are restrained against moving during a fire loading. However, the end spans for a flat slab system are considered unrestrained in the direction of the tendon design. The fire resistance provisions do not match the minimum cover requirements per ACI 318 for reinforcement protection, so it is necessary to reference the governing building code (most likely Section 720 of the International Code Council's 2003 International Building Code) for setting concrete cover parameters of the prestressed tendons. Typical two-hour bottom covers to the tendons for slabs are 3/4 inch for restrained, interior spans and 1-1/2 inches for unrestrained, exterior spans.

When the design is performed manually, the post-tensioning force typically is selected to balance a specified percentage of the floor self weight. Superimposed dead loads such as partitions, flooring, mechanical equipment, and live loads are not included, since they are not present at the time of stressing. Common load-balancing percentages are in the 65-percent to 80-percent range and should be kept relatively consistent between spans. Codes do not prescribe limitations for these percentages, but engineers still need to design to appropriate balancing loads to limit slab deflections and cracking.

The load-balancing effects reduce the amount of flexural stresses for ultimate requirements, helping economize member sizes and materials. Another advantage is the significantly reduced deflections. With a percentage of the dead

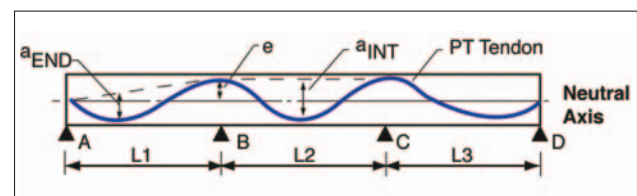
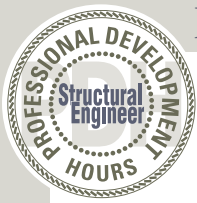


Figure 3: Tendon profile of a continuous post-tensioned beam.



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load being balanced by an upward uniform load, the result is little or no dead load deflection. The benefits are most noticeable in the long-term deflection calculations because the structure experiences deflection only from the remaining unbalanced dead load and the live load. These advantages are significant, but excessive use of post-tensioning can be detrimental. Note that it is not considered conservative to over-balance the gravity loads or over-precompress the section because unwanted camber and excessive cracking may occur. It is imperative to remember that post-tensioning is active reinforcement, exerting its load for the life of the structure.

The effect of a prestressing force on a member can be evaluated by replacing the tendon with equivalent, externally applied loads. The designed force in the tendons, P , will be a function of the designer-specified equivalent balancing load, w_b ; the span length, ℓ ; and its associated maximum drape, a . For a simply supported determinate span with a parabolic tendon profile, the formula for balancing a uniformly distributed load is $P = w_b \ell^2 / 8a$.

For a multispan indeterminate structure, the design process has additional considerations. Since the eccentricity and lengths may vary between spans, the prestressing force, P , needs to be determined for each span. The greatest force, P_{max} , typically is selected for the entire equivalent frame, but code requirements and other guidelines (discussed later) may influence the final effective force. Post-tensioning design is an iterative process to determine an optimized solution. From the selected equivalent frame force, the resulting balancing load in each span must be checked to ensure the percentages are within the range selected by the designer. The percentage is determined from w_b / w_{DL} , where $w_b = 8P_{max}a / \ell^2$.

If the balanced loads are above the acceptable limits in a given span, in lieu of adjusting the force, the engineer has the option to alter the tendon drape, a , to reduce the balanced load, w_b . It is more efficient to alter the drape by raising the bottom tendon ordinate while maintaining the top ordinates. By doing so, the only change in the construction process is to provide higher supporting chair heights, instead of altering the entire top steel configuration at the supports (Figure 4).

It can be difficult to balance a long end bay with the same force calculated for the same length or shorter adjacent interior bays. This is because of the reduction in eccentricity from the anchors being located at the neutral axis and the increase for bottom cover requirements at the end bay. Many times, the end bays will require additional single-span tendons for an increased force to achieve an appropriate balancing load.

Prestress losses

Although 1/2-inch-diameter, 270-ksi unbonded tendons can be stressed to 33 kips of force per ACI 318 Section 18.5.1 ($0.8 \times 270 \text{ ksi} \times 0.153 \text{ square inch} = 33 \text{ kips}$), they will not retain this maximum force for the life of the structure. The force calculated from the balancing load, w_b , is an effective value, $f_{se} \times A_{ps}$, where f_{se} , the reduced effective stress, is found

after calculating prestress losses. Instantaneous prestress losses arise from the seating of the wedges into the anchor, elastic shortening of the concrete, and friction along the length of the tendon. The long-term stress losses are caused by creep and shrinkage of the concrete and relaxation of the prestressing steel. Further explanation of prestress losses can be found in the Post-Tensioning Institute's Post-Tensioning Manual.

Common prestress losses range from 15 ksi to 20 ksi. Therefore, an effective force of 26.6 kips commonly is used for one, 1/2-inch-diameter, 270-ksi tendon calculated from the equation $A_{ps}(0.7f_{pu} - \text{losses})$. By assuming losses of 15 ksi, $(0.153 \text{ square inch})[(0.7 \times 270 \text{ ksi}) - 15 \text{ ksi}] = 26.6 \text{ kips}$. Specifying the effective force as a multiple of 26.6 kips will result in a more efficient and constructible design.

Additional design parameters and requirements

With the required effective tendon force calculated, the following code parameters and requirements may influence the final design value:

Limitation of the average prestress — For slabs not exposed to corrosive environments, the minimum average prestress is 125 pounds per square inch (psi) and the recommended maximum average prestress is 300 psi. Therefore, the prestress force is limited by $125 \text{ psi} \leq P/A \leq 300 \text{ psi}$, where A is the gross cross-sectional area of the total tributary slab width.

Limitation for service load stresses — The ACI 318 provisions limit tensile stresses in the concrete to control the development of flexural cracking. First introduced in the 2002 version, ACI 318-05 classifies flexural members based upon computed extreme fiber stress in tension, f_t , at service loads as Class U (Uncracked), Class T (Transition), or Class C (Cracked). Depending upon the classification, members are assumed to behave as cracked or uncracked sections for service load stress and deflection calculations. Per ACI 318-05 Section 18.3.3, prestressed two-way slabs are to be designed as Class U with $f_t \leq 6 \sqrt{f'_c}$ where f'_c is the specified concrete strength. This requirement permits service load stresses to be computed using uncracked section properties.

The remaining unbalanced service loads, $w_{net} = w_{DL+LL} - w_b$, create stresses in the concrete. Upon determining the resulting net moments, M_{net} , flexural stresses can be evaluated at the top and bottom fibers of each critical section, typically at the midspans and supports. Therefore, $f_t = -P/A + M_{net} c / I_g$.

These stresses then are compared against the maximum code permissible values for compressive stress, $f_c \leq 0.45 f'_c$

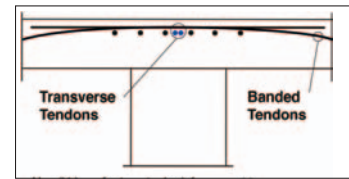


Figure 4: Recommended rebar and tendon layout at columns uses No. 4 bars for top steel reinforcement to match the 1/2-inch tendon diameter.



(certain conditions may allow $\leq 0.60 f'_c$) and tensile stress, $f_t \leq 6 \sqrt{f'_c}$.

Generally, the most economical design for flexural strength will be obtained by using the maximum permissible tensile stresses. Some limitation may result from serviceability restrictions, such as deflection.

Limitation for initial stresses — Compressive and tensile stresses also need to be checked at the time of stress transfer, also referred to as the initial stage. These stresses typically are computed with self weight and the forces induced by the prestressing, since live loads and superimposed dead loads will not be on the structure during the stressing procedure. Code limitations are in terms of the initial concrete compressive strength, f'_{ci} , at the time of stressing. For most designs, this value is a minimum of 3,000 psi, correlating to the bearing strength allowed in a typical anchor. The compressive stresses shall be no greater than $0.6 f'_{ci}$, and the tensile stresses are not to exceed $3 \sqrt{f'_{ci}}$. If the tensile stresses exceed this limit, additional mild reinforcing needs to be provided to resist the total tensile force in the concrete.

Minimum reinforcement requirements

Minimum bonded reinforcement must meet ACI 318-05 Section 18.9.3 requirements for two-way slab systems. When tensile stresses in the concrete at service loads are less than $2 \sqrt{f'_c}$ in the positive moment regions, Section 18.9.3.1 does not require bonded reinforcement in those areas. When the tensile stresses exceed $2 \sqrt{f'_c}$, the minimum area of bonded reinforcement for the positive moment regions is $A_s = N_c / 0.5 f_y$ (18.9.3.2), and the minimum lengths shall be 1/3 of the clear span, ℓ_n , and centered in the span (18.9.4.1). N_c is the tensile force in the concrete due to the service load combination, $DL+LL$, where DL is the dead load and LL is the live load, and f_y is the specified yield strength of mild reinforcement.

The minimum bonded reinforcement in the negative moment regions (top steel at supports) is computed by $A = 0.00075 A_{cf}$ where A_{cf} is the greater of the gross cross-sectional area of the slab panel on either side of the column in the direction of the design (18.9.3.3). This top steel shall be at least four bars and placed within 1.5 times the slab thickness from each face of the column. The minimum lengths for the negative moment reinforcement shall extend 1/6 of the clear span, ℓ_n , on each side of the support (18.9.4.2). For ease of placement at column locations, the use of No. 4 bars is recommended. Since the diameter of a No. 4 bar is equivalent to the typical 1/2-inch tendon diameter, the top mild reinforcement and post-tensioning tendons can be at the same elevation and placed on the same supporting chairs (Figure 4).

Compared to mild reinforced two-way slabs, post-tensioned slabs appear lightly reinforced. With the high strength of the continuous post-tensioning reinforcement, the slabs have minimal reinforcement congestion. This makes it easier and faster for the construction crew to place the reinforcement and the concrete. In addition, it facilitates inspection for compliance with construction documents.

Ultimate strength

The primary moment, $P \times e$, is not the only force generated with the post-tensioning. Secondary forces are induced in multispan indeterminate structures as a consequence of the constraint by the supports to the free movement of the prestressed member (Figure 5). The vertically profiled tendons cause a "lifting off" action at the interior support upon stressing and generate a secondary reaction. This reaction, in turn, creates a secondary moment in the concrete slab. Since the secondary moment is a result of the secondary reaction and no other applied load along the span, the moment will only vary linearly between the supports.

These secondary forces can impact calculated demand significantly in both the positive and negative moment regions and must be accounted for in design. Secondary moments are calculated by subtracting the primary moment, $P \times e$, from the total moment generated by the prestressing, M_{bal} , and are incorporated in the strength design, with a factor of 1.0 per Section 18.10.3 ACI 318-05.

The flexural design capacity of the slab, ϕM_n , must be checked starting with the minimum reinforcement required from 18.9.3. If the nominal strength is less than M_u / ϕ , where ϕ is the strength reduction factor, the minimum reinforcing

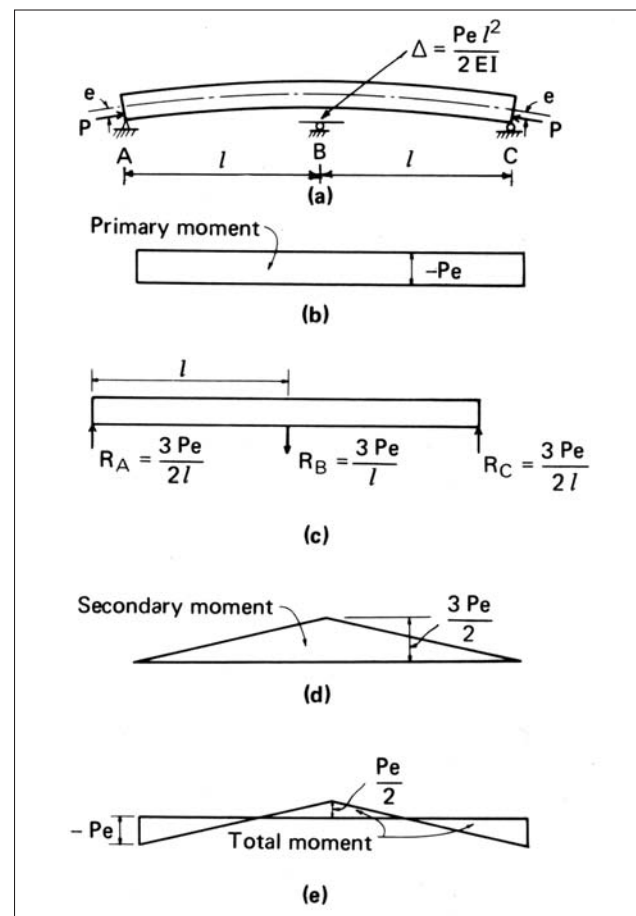


Figure 5: Primary and secondary moments (Post-Tensioning Institute, *Post-Tensioning Manual*, Fifth Edition, 1990)



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requirements do not govern and the reinforcement must be increased to meet the ultimate strength requirements.

Punching shear and deflection calculations also are important. Punching shear is critical and often can govern design criteria. Approved in the new ASTM A-1044 Standard Specification for Steel Stud Assemblies for Shear Reinforcement of Concrete, shear stud rails are a common form of shear reinforcement. Design guidelines for stud rails are found in ACI 421.1R-99: Shear Reinforcement for Slabs.

The layout

Designing and detailing with constructability in mind will improve the success of a post-tensioned concrete project and the relationship of the design-contractor team. The constructability aspect is arguably the most important component of a post-tensioned concrete project. Experience provides the best tool, but there are several code provisions and recommended guidelines to help facilitate construction through design.

Banded tendon distribution is the industry's common practice for two-way slab tendon arrangement (Figure 6). The code allows for grouping of tendons in one direction along the support lines with the transverse tendons spaced uniformly. This layout simplifies the construction sequence, reducing labor costs and construction time compared to the "basket weave" configuration. It also provides more flexibility in accommodating post-construction floor penetrations. Especially for an irregular column layout, the banded tendon arrangement makes it easier to visualize the load path to the supports.

In addition to the minimum amounts and placement requirements for the mild reinforcing, ACI 318 also gives provisions for the post-tensioning reinforcement. At least two tendons are required to pass through the critical shear area over columns in each direction. The uniformly spaced tendons must meet a minimum spacing per ACI 318-05 Section 7.6.7 and a maximum spacing of eight times the slab thickness or 5 feet.

For faster placement, tendons may be bundled as many as five strands flat per bundle. While maintaining the flat plane, the tendons can transition to meet the design parabolic profile. Horizontal curvature necessary to follow a non-linear column layout or to avoid an opening increases the risk for a blowout. Curves should be smooth and without kinks and limited to a minimum radius of curvature of 10 feet. The addition of hairpins along tight curves resists the lateral thrust from the tendons and helps minimize the blowout risk. Through the curve, the flat tendon bundle also should be splayed at a minimum 2-inch spacing and the tendons tied individually to the support bars, eliminating the opportunity of tendons to "roll over" one another during stressing.

Proper detailing of a post-tensioned structure has a large impact on constructability and serviceability issues. Because post-tensioned slabs shorten approximately 1 inch for every 100 feet of unrestrained slab, excessive restraint to that inherent movement will cause cracking. Mild reinforced concrete



Figure 6: Banded tendons follow column lines with transverse tendons spaced uniformly.

slabs require similar movement and restraint considerations, but the elastic shortening of post-tensioning slabs generates comparatively more movement. Restraint can be minimized by locating lateral resisting elements toward the center of the floor plate, where little movement occurs. When stiff supports are located away from the center of mass, detailing a slip connection at the slab interface will allow for some movement and help relieve restraint. Long, continuous sections of a building may require the incorporation of pour strips — a section of slab left open during the initial pour and filled in at a later date. These openings help reduce movement and restraint from early age volume changes, but they will not be effective if filled prematurely. Pour strips typically must be left open for 30 days, without any continuous reinforcement connecting the slab sections.

Along with pour strips, construction joints allow for interior stressing in zero-setback and subterranean construction. Placing construction joints between the 1/4 and 1/3 points of a span typically provides the best location for intermediate stressing, where the tendon elevation is naturally at or near the centroid of the slab.

Conclusion

Post-tensioning design is unique to each project and to each engineer. By understanding the concepts and knowing the criteria for design, the versatility of post-tensioning yields success in the design, construction, and economics of concrete buildings. Detailed, worked-out example problems that illustrate the material presented in this paper can be found on the Portland Cement Association's web site at www.cement.org/buildings/design_aids.asp and in the Post-Tensioning Institute's Post-Tensioning Manual. ■

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Quiz questions

1. Using the equivalent frame method (EFM), the tributary width for the slab is calculated using:
 - a) Engineering judgement
 - b) Eight times the slab thickness each side of column
 - c) 12 times the slab width each side of column
 - d) The dimension from the centerline of one bay to centerline of the opposite bay
2. Under typical floor live loads, a 28-foot by 26-foot column spacing would suggest a preliminary slab thickness of:
 - a) 6 inches
 - b) 7 inches
 - c) 7.5 inches
 - d) 8 inches
3. Typical loads to be balanced by the post-tensioning force include:
 - a) Dead load + live load
 - b) Dead load + percentage of live load
 - c) Dead load: self weight only
 - d) Dead load: self weight + superimposed
4. What is the minimum cover required for prestressing steel on an unrestrained exterior slab bays with a 2-hour fire rating ?
 - a) 3/4 inch
 - b) 1 inch
 - c) 1-1/2 inches
 - d) 2 inches
5. Which is a source of initial prestress loss at the time of stressing?
 - a) Friction
 - b) Relaxation of the steel
 - c) Creep of the concrete
 - d) All of the above
6. What is the recommended limit for maximum average slab precompression stress?
 - a) 400 psi
 - b) 300 psi
 - c) 200 psi
 - d) 125 psi
7. Per the ACI 318-05 provisions, what is the limit for service load tensile stresses for two-way slabs?
 - a) $6\sqrt{f'_c}$
 - b) $3\sqrt{f'_c}$
 - c) $2\sqrt{f'_c}$
 - d) $7.5\sqrt{f'_c}$
8. What is the load factor for the secondary moments to calculate the ultimate strength?
 - a) 1.6
 - b) 1.4
 - c) 1.2
 - d) 1.0
9. What is the minimum number of tendons required by ACI 318-05 to pass through the column critical section in each direction?
 - a) 1
 - b) 2
 - c) 3
 - d) 4
10. What is the minimum area of negative mild steel reinforcement at an interior column, 30-foot bays, and an 8-inch-thick slab?
 - a) 2.16 square inches
 - b) 2.80 square inches
 - c) 1.56 square inches
 - d) 3.24 square inches

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|---------------------------|----------------------------|
| 1) a b c d | 6) a b c d |
| 2) a b c d | 7) a b c d |
| 3) a b c d | 8) a b c d |
| 4) a b c d | 9) a b c d |
| 5) a b c d | 10) a b c d |

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Signature: _____ Date: _____