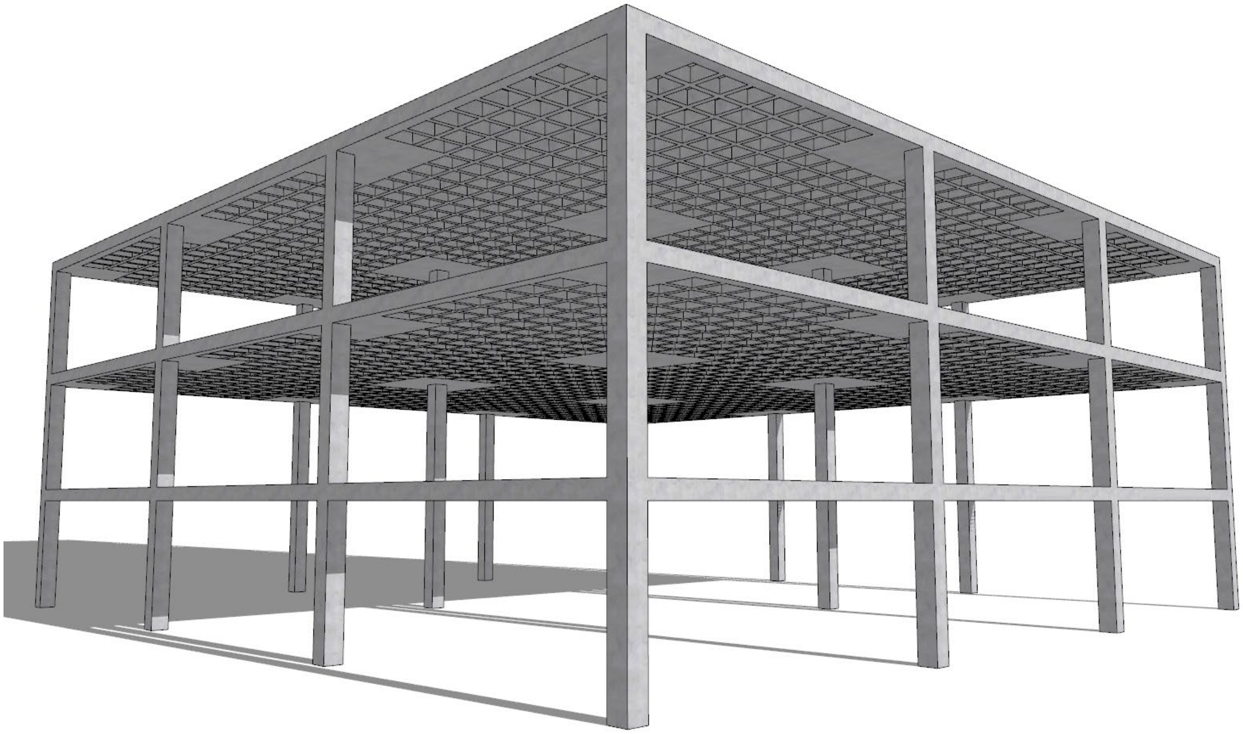
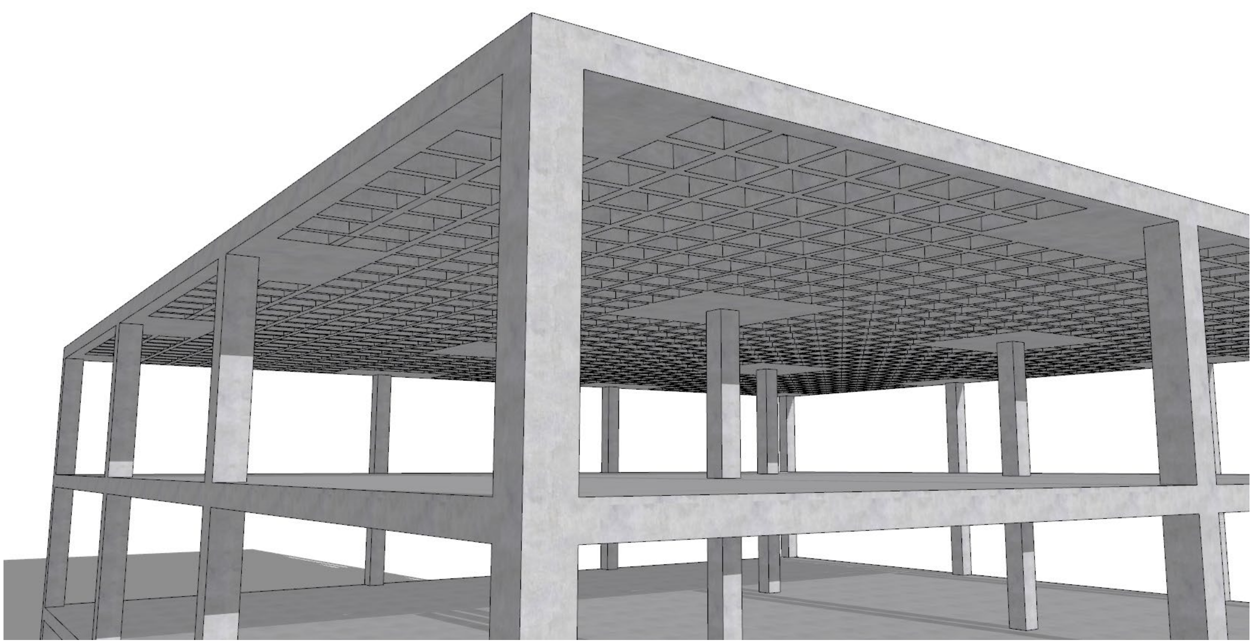
Two-Way Joist Concrete Slab Floor (Waffle Slab) System Analysis and Design (ACI 318-14)





Two-Way Joist Concrete Slab Floor (Waffle Slab) System Analysis and Design (ACI 318-14)

Design the concrete floor slab system shown below for an intermediate floor with partition weight of 50 psf, and unfactored live load of 100 psf. The lateral loads are independently resisted by shear walls. A flat plate system will be considered first to illustrate the impact longer spans and heavier applied loads. A waffle slab system will be investigated since it is economical for longer spans with heavy loads. The dome voids reduce the dead load and electrical fixtures can be fixed in the voids. Waffle system provides an attractive ceiling that can be left exposed when possible producing savings in architectural finishes. The Equivalent Frame Method (EFM) shown in ACI 318 is used in this example. The hand solution from EFM is also used for a detailed comparison with the model results of spSlab engineering software program from StructurePoint.

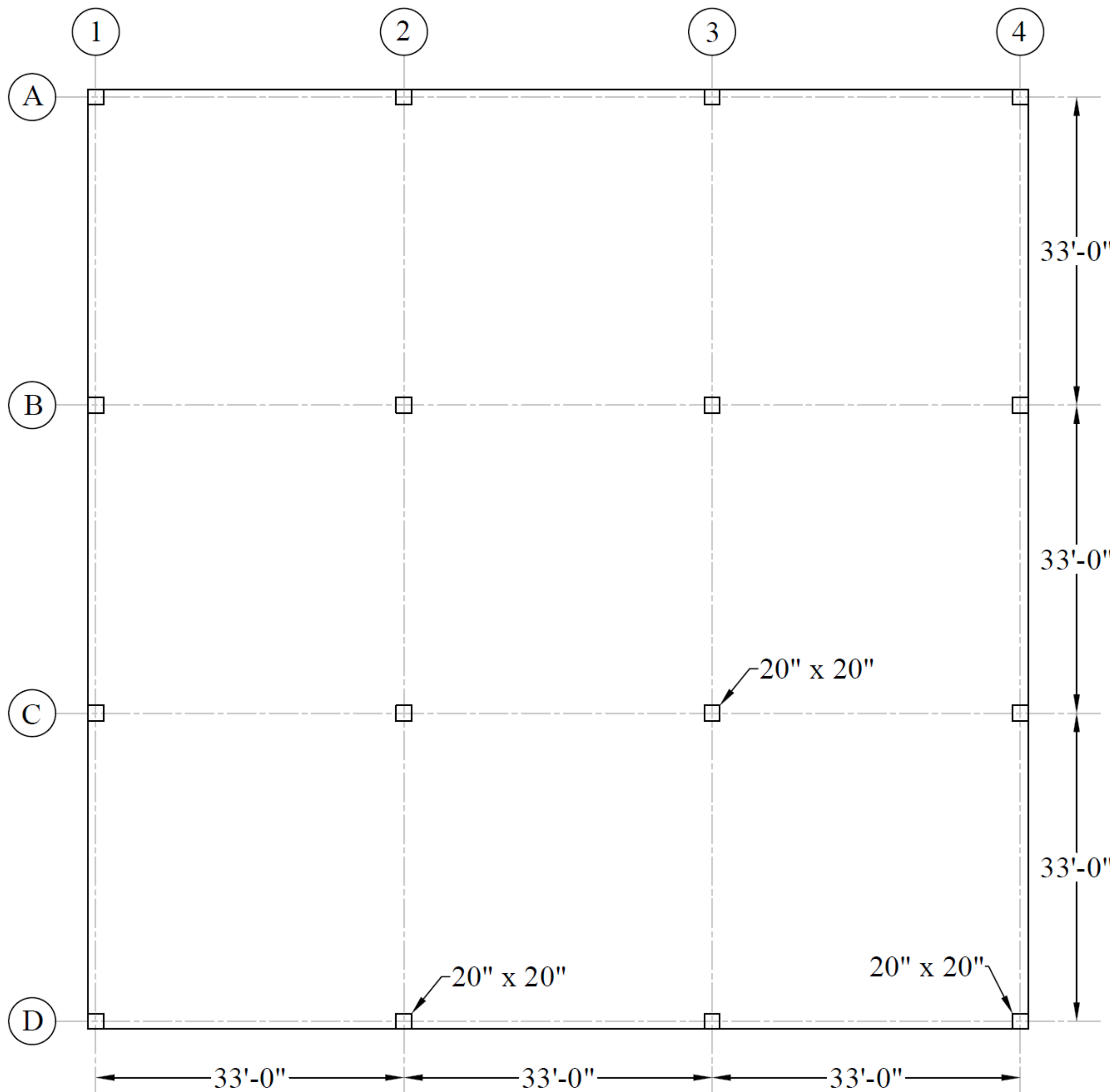


Figure 1 – Two-Way Flat Concrete Floor System

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

References

- Concrete Floor Systems (Guide to Estimating and Economizing), Second Edition, 2002 David A. Fanella, Portland Cement Association
- Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association.
- Simplified Design of Reinforced Concrete Buildings, Fourth Edition, 2011 Mahmoud E. Kamara and Lawrence C. Novak
- Control of Deflection in Concrete Structures (ACI 435R-95), American Concrete Institute
- Structural Concrete Theory and Design, 6th Edition, 2015, Nadim Hassoun and Akthem Al-Manaseer, Wiley, Example 17.12
- spSlab Engineering Software Program Manual v5.50, STRUCTUREPOINT, 2018
- "Two-Way Flat Plate Concrete Floor System Analysis and Design (ACI 318-14)" Design Example, STRUCTUREPOINT, 2023
- "Two-Way Flat Slab (Concrete Floor with Drop Panels) System Analysis and Design (ACI 318-14)" Design Example, STRUCTUREPOINT, 2023
- "Two-Way Concrete Floor Slab with Beams System Analysis and Design (ACI 318-14)" Design Example, STRUCTUREPOINT, 2024
- Contact Support@StructurePoint.org to obtain supplementary materials (spSlab models: Two-Way-Joist-Waffle-System-ACI-318-14.slb)

Design Data

Story Height = 13 ft (provided by architectural drawings)

Superimposed Dead Load, *SDL* = 50 psf for Frame walls, hollow concrete masonry unit wythe, 12 in. thick, 125 pcf unit density, with no grout ***ASCE/SEI 7-10 (Table C3-1)***

Live Load, *LL* = 100 psf for Recreational uses – Gymnasiums ***ASCE/SEI 7-10 (Table 4-1)***

f_c' = 5,000 psi (for slab)

f_c' = 6,000 psi (for columns)

f_y = 60,000 psi

1. Preliminary Member Sizing

1.1. Preliminary Flat Plate (without Joists)

1.1.1. Slab Minimum Thickness – Deflection

ACI 318-14 (8.3.1.1)

In lieu of detailed calculation for deflections, ACI 318 Code gives minimum slab thickness for two-way construction without interior beams in ***Table 8.3.1.1***.

For flat plate slab system, the minimum slab thickness per ***ACI 318-14*** are:

Exterior Panels: $h_s = \frac{l_n}{30} = \frac{376}{30} = 12.53$ in. ***ACI 318-14 (Table 8.3.1.1)***

But not less than 5 in. ***ACI 318-14 (8.3.1.1(a))***

Interior Panels: $h_s = \frac{l_n}{33} = \frac{376}{33} = 11.39$ in. ***ACI 318-14 (Table 8.3.1.1)***

But not less than 5 in. ***ACI 318-14 (8.3.1.1(a))***

Where l_n = length of clear span in the long direction = 33 × 12 – 20 = 376 in.

Use 13 in. slab for all panels (self-weight = 150 pcf × 13 in. /12 = 162.50 psf)

1.1.2. Slab Shear Strength – One Way Shear

Evaluate the average effective depth (Figure 2):

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 13 - 0.75 - 0.75 - \frac{0.75}{2} = 11.13 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 13 - 0.75 - \frac{0.75}{2} = 11.88 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{11.13 + 11.88}{2} = 11.50 \text{ in.}$$

Where:

c_{clear} = 3/4 in. for # 6 steel bar **_ACI 318-14 (Table 20.6.1.3.1)_**

d_b = 0.75 in. for # 6 steel bar

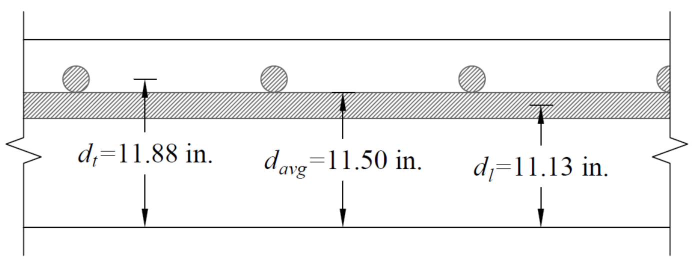


Figure 2 – Average Effective Depth for Flat Plate

Factored dead load, $q_{Du} = 1.20 \times (162.50 + 50.00) = 255.00$ psf

Factored live load, $q_{Lu} = 1.60 \times 100.00 = 160.00$ psf **_ACI 318-14 (5.3.1)_**

Total factored load, $q_u = 255.00 + 160.00 = 415.00$ psf

Check the adequacy of slab thickness for beam action (one-way shear) ***ACI 318-14 (22.5)***

At an interior column:

Consider a 12-in. wide strip. The critical section for one-way shear is located at a distance d, from the face of support (see Figure 3):

Tributary area for one-way shear is:

$$A_{Tributary} = \left[\frac{33}{2} - \frac{20}{2\times 12} - \frac{11.50}{12}\right] \times \frac{12}{12} = 14.71 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.415 \times 14.71 = 6.10 \text{ kips}$$

$$V_c = 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d$$ ***ACI 318-14 (Eq. 22.5.5.1)***

Where $\lambda = 1$ for normal weight concrete, more information can be found in "Concrete Type Classification Based on Unit Density" technical article.

$$\phi V_c = 0.75 \times 2.0 \times 1.0 \times \sqrt{5{,}000} \times 12 \times \frac{11.50}{1{,}000} = 14.64 \text{ kips} > V_u = 6.10 \text{ kips}$$

Slab thickness of 13 in. is adequate for one-way shear.

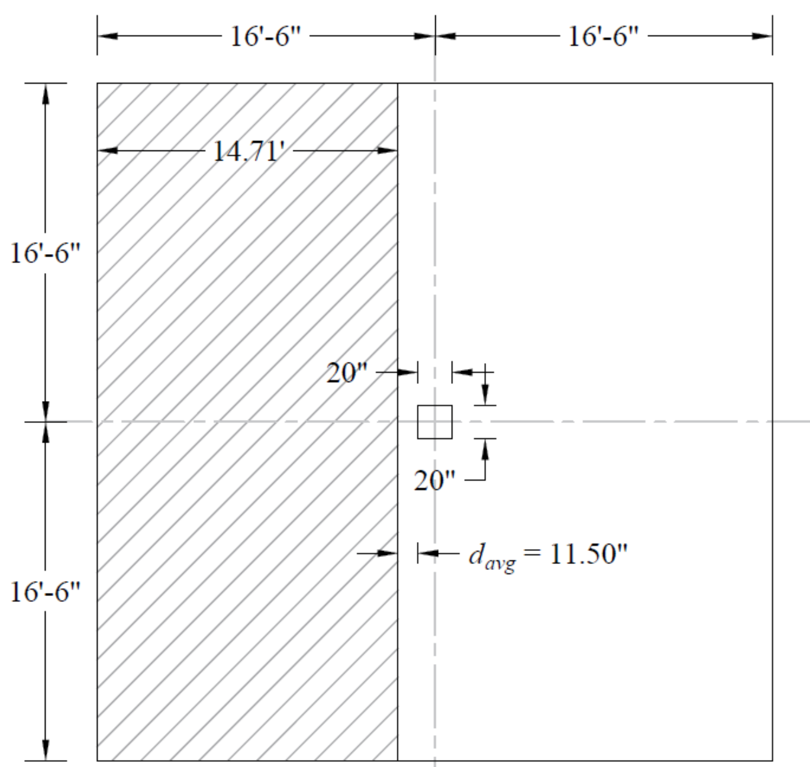


Figure 3 – Critical Section for One-Way Shear

1.1.3. Slab Shear Strength – Two-Way Shear

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 4):

Tributary area for two-way shear is:

$$A_{Tributary} = (33 \times 33) - \left(\frac{20+11.50}{12}\right)^2 = 1,082.11 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.415 \times 1,082.11 = 449.08 \text{ kips}$$

$$V_c = 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d \quad \text{(For square interior column)}$$ ***ACI 318-14 (Table 22.6.5.2(a))***

$$V_c = 4 \times 1 \times \sqrt{5,000} \times (4 \times (20+11.50)) \times \frac{11.50}{1,000} = 409.84 \text{ kips}$$

$$\phi V_c = 0.75 \times 409.84 = 307.38 \text{ kips} < V_u = 449.08 \text{ kips}$$

Slab thickness of 13 in. is not adequate for two-way shear. This is expected as the self-weight an applied loads are very challenging for a flat plate system.

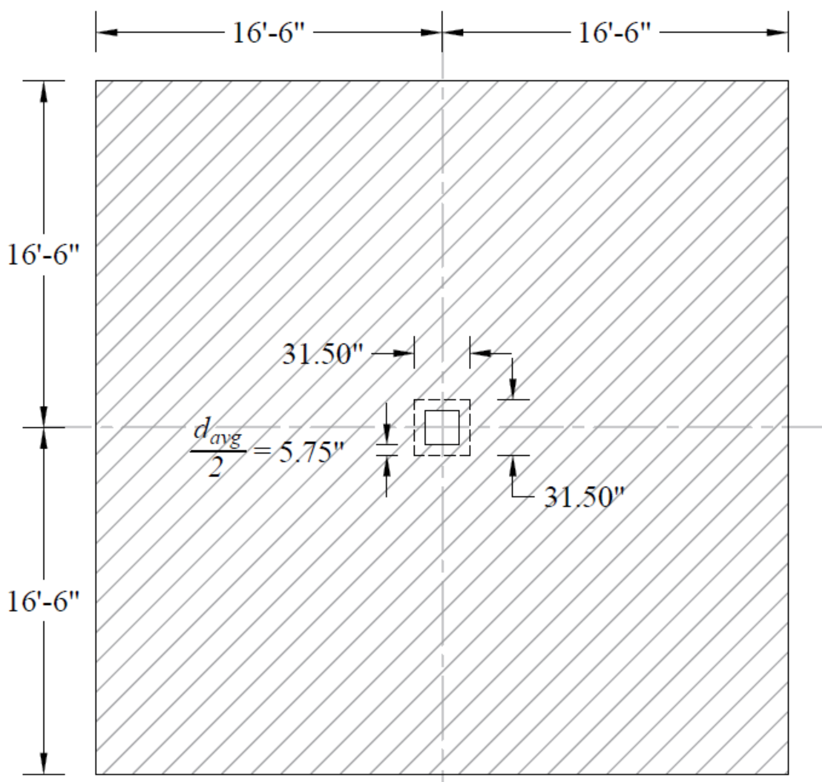


Figure 4 – Critical Section for Two-Way Shear

In this case, four options can be considered: 1) to increase the slab thickness further, 2) use headed shear reinforcement in the slab, 3) apply drop panels at columns, or 4) use two-way joist slab system. In this example, the latter option will be used to achieve better understanding for the design of two-way joist slab often called two-way ribbed slab or waffle slab.

Check the applicable joist dimensional limitations as follows:

1) Width of ribs shall be at least 4 in. at any location along the depth. ***ACI 318-14 (9.8.1.2)***

   Use ribs with 6 in. width.

2) Overall depth of ribs shall not exceed 3.5 times the minimum width. ***ACI 318-14 (9.8.1.3)***

   3.5 × 6 in. = 21 in. → Use ribs with 14 in. depth.

3) Clear spacing between ribs shall not exceed 30 in. ***ACI 318-14 (9.8.1.4)***

   Use 30 in. clear spacing.

4) Slab thickness (with removable forms) shall be at least the greater of: ***ACI 318-14 (8.8.3.1)***

   a) 1/12 clear distance between ribs = 1/12 × 30 = 2.50 in.

   b) 2 in.

   Use a slab thickness of 3.00 in. > 2.50 in.

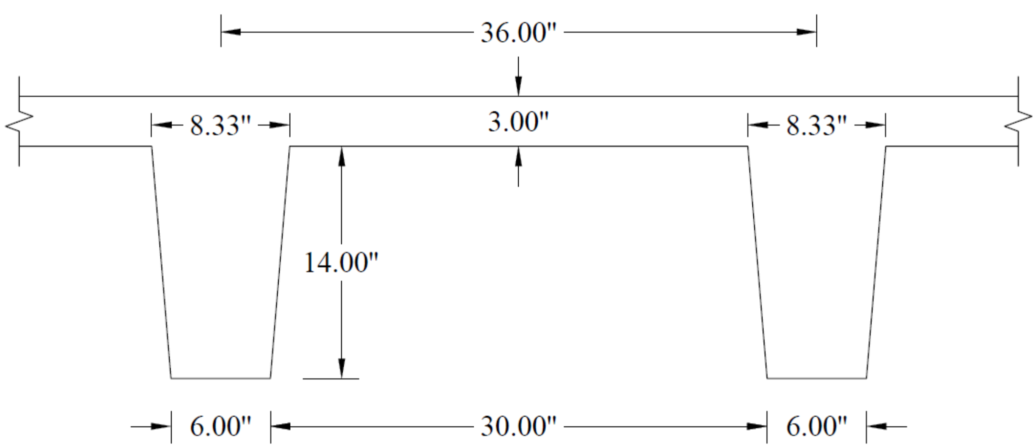


Figure 5 – Joists Dimensions

In waffle slabs a drop panel is automatically invoked to guarantee adequate two-way (punching) shear resistance at column supports. This is evident from the flat plate check conducted using 13 in. indicating insufficient punching shear capacity above. Check the drop panel dimensional limitations as follows:

1) The drop panel shall project below the slab at least one-fourth of the adjacent slab thickness.

ACI 318-14 (8.2.4(a))

Since the slab thickness (h_{MI} – calculated in page 9 of this document) is 12 in., the thickness of the drop panel should be at least:

$h_{dp,min} = 0.25 \times h_{MI} = 0.25 \times 12 = 3.00$ in.

Drop panel depth are also controlled by the rib depth (both at the same level). For nominal lumber size (2x),

$h_{dp} = h_{rib} = 14.00$ in. $> h_{dp,min} = 3.00$ in.

The total thickness including the actual slab and the drop panel thickness (h) = $h_s + h_{dp} = 3.00 + 14.00 =$ 17.00 in.

2) The drop panel shall extend in each direction from the centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.

ACI 318-14 (8.2.4(b))

$$l_{1,dp\_min} = \frac{1}{6} \times l_1 + \frac{1}{6} \times l_1 = \frac{1}{6} \times 33 + \frac{1}{6} \times 33 = 11.00 \text{ ft}$$

$$l_{2,dp\_min} = \frac{1}{6} \times l_2 + \frac{1}{6} \times l_2 = \frac{1}{6} \times 33 + \frac{1}{6} \times 33 = 11.00 \text{ ft}$$

Use $l_{1,dp} = l_{2,dp} = 12.00 \text{ ft} > l_{1,dp\_min} = l_{2,dp\_min} = 11.00 \text{ ft}$

Based on the previous discussion, Figure 6 shows the dimensions of the selected two-way joist system.

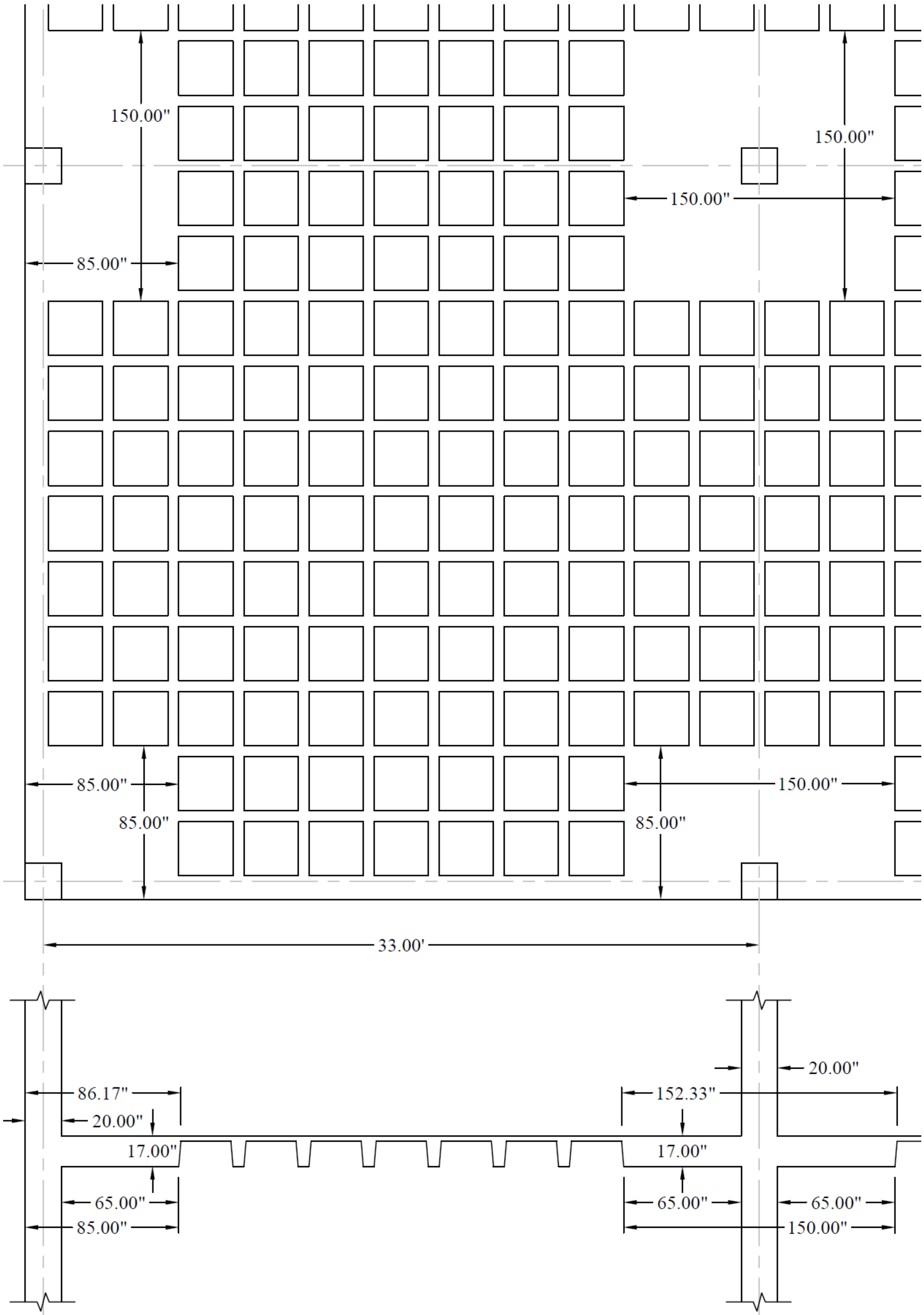


Figure 6 – Two-Way Joist (Waffle) Slab

1.2. Preliminary Two-Way Joist Slab (Waffle Slab)

For slabs with changes in thickness and subjected to bending in two directions, it is necessary to check shear at multiple sections as defined in the ***ACI 318-14***. The critical sections shall be located with respect to:

1) Edges or corners of columns. ***ACI 318-14 (22.6.4.1(a))***

2) Changes in slab thickness, such as edges of drop panels. ***ACI 318-14 (22.6.4.1(b))***

1.2.1. Slab Minimum Thickness – Deflection

ACI 318-14 (8.3.1.1)

In lieu of detailed calculation for deflections, ACI 318 Code gives minimum slab thickness for two-way construction without interior beams in ***Table 8.3.1.1***.

For this slab system, the minimum slab thicknesses per **ACI 318-14** are:

Exterior Panels: $h_s = \frac{l_n}{33} = \frac{376}{33} = 11.39 \text{ in.}$ ***ACI 318-14 (Table 8.3.1.1)***

But not less than 4 in. ***ACI 318-14 (8.3.1.1(b))***

Interior Panels: $h_s = \frac{l_n}{36} = \frac{376}{36} = 10.44 \text{ in.}$ ***ACI 318-14 (Table 8.3.1.1)***

But not less than 4 in. ***ACI 318-14 (8.3.1.1(b))***

Where l_n = length of clear span in the long direction = 33 × 12 – 20 = 376 in.

For the purposes of analysis and design, the ribbed slab will be replaced with a solid slab of equivalent moment of inertia, weight, punching shear capacity, and one-way shear capacity.

The equivalent thickness based on moment of inertia is used to find slab stiffness considering the ribs in the direction of the analysis only. The ribs spanning in the transverse direction are not considered in the stiffness computations. This thickness, h_{MI}, is given by:

$$h_{MI} = \left(\frac{12 \times I_{rib}}{b_{rib}}\right)^{1/3} = \left(\frac{12 \times 5134.87}{36}\right)^{1/3} = 12.00 \text{ in.}$$ ***spSlab Software Manual (Eq. 2-11)***

Where:

I_{rib} = Moment of inertia of one joist section between centerlines of ribs (see Figure 7).

b_{rib} = The center-to-center distance of two ribs (clear rib spacing plus rib width) (see Figure 7).

Since h_{MI} = 12.00 in. > h_{min} = 11.39 in., the deflection calculation can be neglected. However, the deflection calculation will be included in this example for comparison with the spSlab software results.

The drop panel depth for two-way joist (waffle) slab is set equal to the rib depth. The equivalent drop depth based on moment of inertia, d_{MI}, is given by:

$$d_{MI} = h_{MI} + h_{rib} = 12.00 + 5.00 = 17.00 \text{ in.}$$ ***spSlab Software Manual (Eq. 2-12)***

Where:

h_{rib} = 3.00 + 14.00 – 12.00 = 5.00 in.

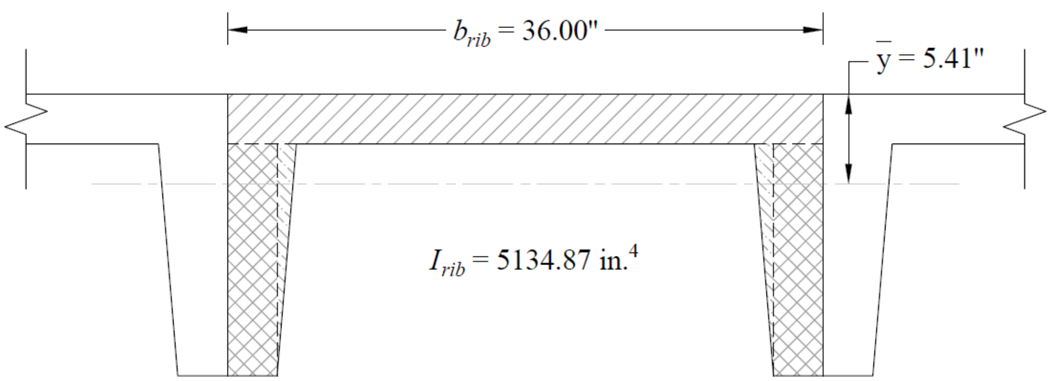


Figure 7 – Equivalent Thickness Based on Moment of Inertia

Find system self-weight using the equivalent thickness based on the weight of individual components (see the following Figure). This thickness, h_w, is given by:

$$h_w = \frac{V_{mod}}{A_{mod}} = \frac{66.037}{99.000} = 8.005 \text{ in.}$$ ***spSlab Software Manual (Eq. 2-10)***

Where:

V_{mod} = The Volume of one joist module (the transverse joists are included – 11 joists in the frame strip).

$$V_{mod} = V_{Longitudinal\ Joist} + V_{Transverse\ Joists} - V_{Intersection\ between\ Joists}$$

$$V_{Longitudinal\ Joist} = \left(\frac{6+8.33}{2} \times 14.00 + 3.00 \times 36.00\right) \times (33.00 \times 12.00) = 47.74 \text{ ft}^3$$

$$V_{Transverse\ Joists} = 11 \times \left(\frac{6+8.33}{2} \times 14.00\right) \times 36.00 = 22.99 \text{ ft}^3$$

$$V_{Intersection\ between\ Joists} = 11 \times \left(\frac{6^2+8.33^2}{2} \times 14.00\right) = 4.70 \text{ ft}^3$$

$V_{mod} = 47.74 + 22.99 - 4.70 = 66.04 \text{ ft}^3$

A_{mod} = The plan area of one joist module = 33 × 36/12 = 99.00 ft^2.

Self-weight for slab section without drop panel = 150 pcf × 8.00 in. /12 = 100.057 psf

Self-weight for slab section with drop panel = 150 pcf × (14.00 + 3.00 – 8.00) in. /12 = 112.443 psf

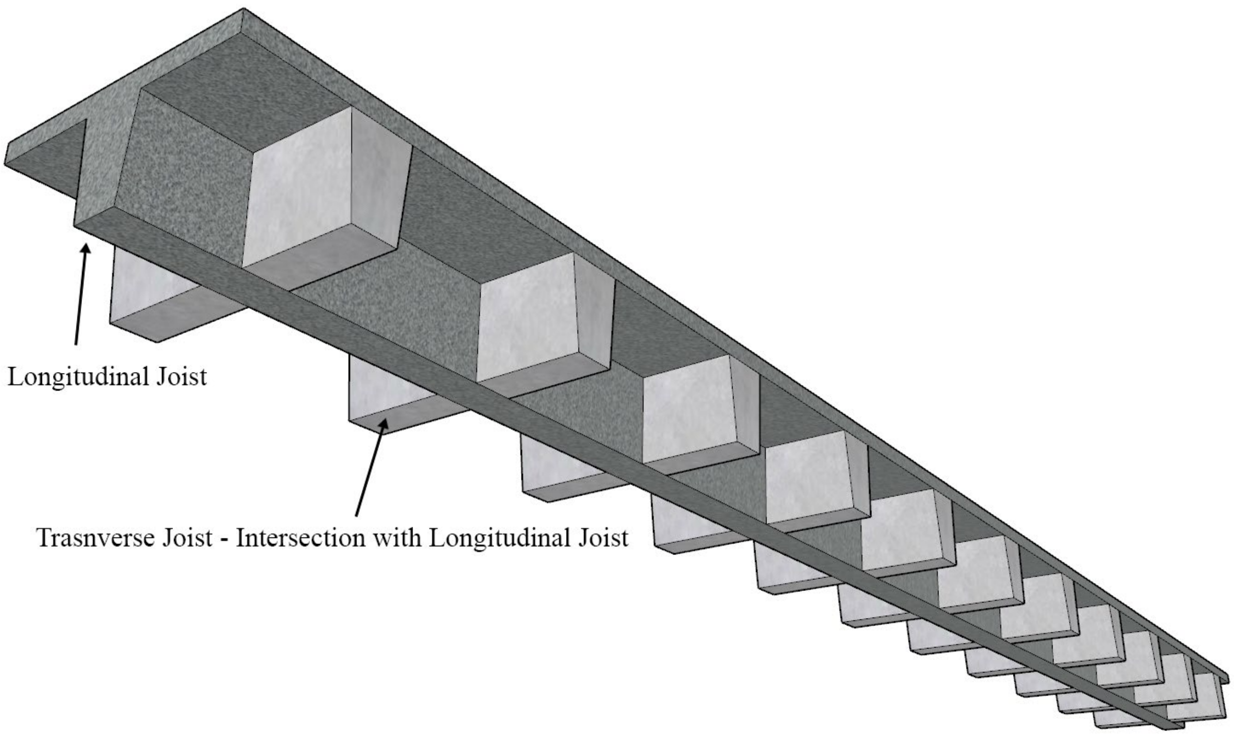


Figure 8 – Equivalent Thickness Based on the Weight of Individual Components

1.2.2. Slab Shear Strength – One Way Shear

For critical section at distance *d* from the edge of the column (slab section with drop panel):

Evaluate the average effective depth:

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 17.00 - 0.75 - 0.75 - \frac{0.75}{2} = 15.13 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 17.00 - 0.75 - \frac{0.75}{2} = 15.88 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{15.13 + 15.88}{2} = 15.50 \text{ in.}$$

Where:

c_{clear} = 3/4 in. for # 6 steel bar **_ACI 318-14 (Table 20.6.1.3.1)_**

d_b = 0.75 in. for # 6 steel bar

h_s = 17.00 in. = The drop depth (d_{MI})

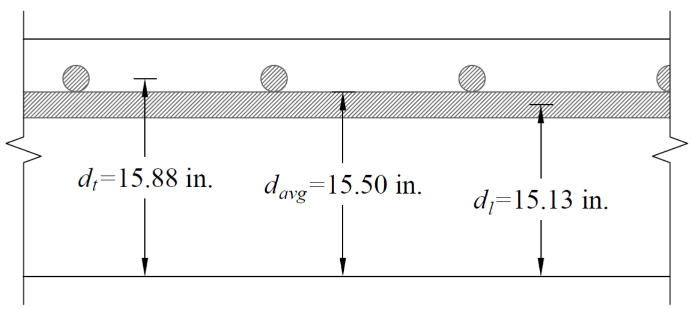


Figure 9 – Average Effective Depth for Slab Section with Drop Panel

Factored dead load, $q_{Du} = 1.20 \times (150 \times 17.00 / 12 + 50.00) = 315.00$ psf

Factored live load, $q_{Lu} = 1.60 \times 100.00 = 160.00$ psf **_ACI 318-14 (5.3.1)_**

Total factored load, $q_u = 315.00 + 160.00 = 475.00$ psf

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior column

ACI 318-14 (22.5)

Consider a 12-in. wide strip. The critical section for one-way shear is located at a distance *d*, from the edge of the column (see Figure 10)

Tributary area for one-way shear is:

$$A_{Tributary} = \left[\frac{33}{2} - \frac{20}{2\times12} - \frac{15.50}{12}\right]\times\frac{12}{12} = 14.38\text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.475\times14.38 = 6.83\text{ kips}$$

$$V_c = 2\times\lambda\times\sqrt{f_c'}\times b_w\times d$$

ACI 318-14 (Eq. 22.5.5.1)

Where $\lambda = 1$ for normal weight concrete

$$\phi V_c = 0.75\times2\times1.0\times\sqrt{5{,}000}\times12\times\frac{15.50}{1{,}000} = 19.73\text{ kips} > V_u = 6.83\text{ kips}$$

Slab thickness is adequate for one-way shear for the first critical section (from the edge of the column).

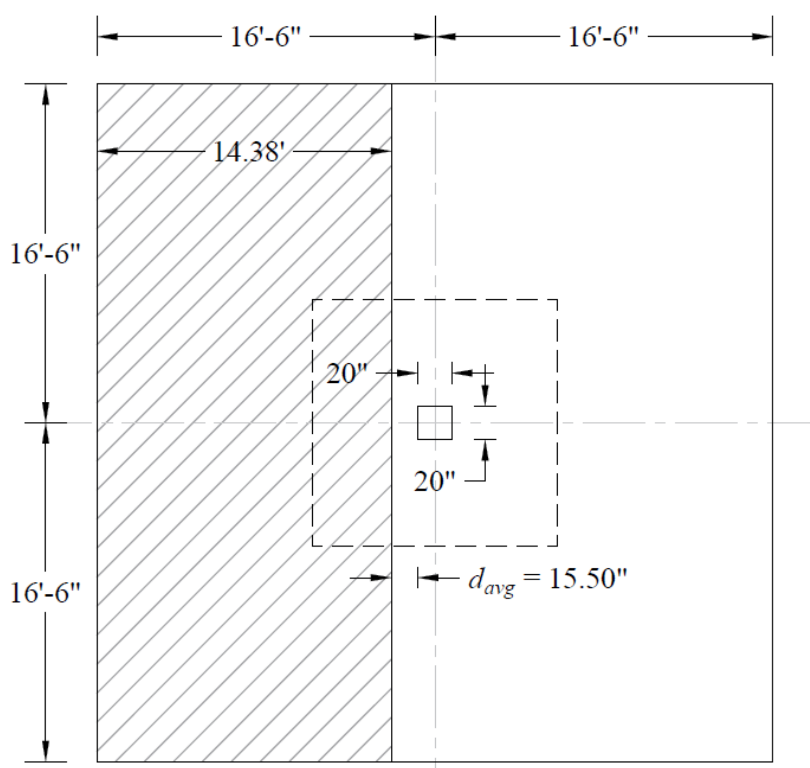


Figure 10 – Critical Section at Distance *d* from the Edge of the Column for One-Way Shear

For critical section at the edge of the drop panel (slab section without drop panel):

Evaluate the average effective depth:

$$d_l = h_s - c_{clear} - d_b - \frac{d_b}{2} = 12 - 0.75 - 0.75 - \frac{0.75}{2} = 10.13 \text{ in.}$$

$$d_t = h_s - c_{clear} - \frac{d_b}{2} = 12 - 0.75 - \frac{0.75}{2} = 10.88 \text{ in.}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{10.13 + 10.88}{2} = 10.50 \text{ in.}$$

Where:

c_{clear} = 3/4 in. for # 6 steel bar **_ACI 318-14 (Table 20.6.1.3.1)_**

d_b = 0.75 in. for # 6 steel bar

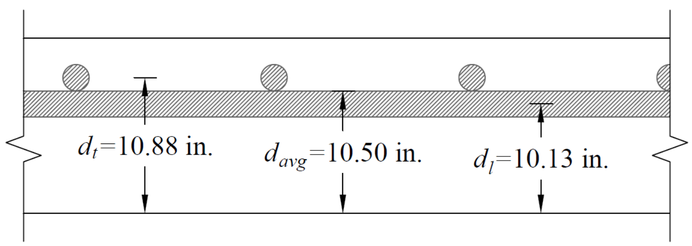


Figure 11 – Average Effective Depth for Slab Section without Drop Panel

Factored dead load, $q_{Du} = 1.20 \times (100.057 + 50.00) = 180.07$ psf

Factored live load, $q_{Lu} = 1.60 \times 100.00 = 160.00$ psf **_ACI 318-14 (5.3.1)_**

Total factored load, $q_u = 180.07 + 160.00 = 340.07$ psf

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior drop panel

ACI 318-14 (22.5)

Consider a 12-in. wide strip. The critical section for one-way shear is located at the face of the solid head (see Figure 12)

Tributary area for one-way shear is:

$$A_{Tributary} = \left[\frac{33}{2} - \frac{12}{2}\right] \times \frac{12}{12} = 10.50 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.340 \times 10.50 = 3.57 \text{ kips}$$

$$V_c = 2 \times \lambda \times \sqrt{f'_c} \times b_w \times d$$

ACI 318-14 (Eq. 22.5.5.1)

Where $\lambda = 1$ for normal weight concrete

$$\phi V_c = 0.75 \times 2.0 \times 1.0 \times \sqrt{5,000} \times 12 \times \frac{10.50}{1,000} = 13.36 \text{ kips} > V_u = 3.57 \text{ kips}$$

Slab thickness of 12 in. is adequate for one-way shear for the second critical section (at the edge of the drop panel).

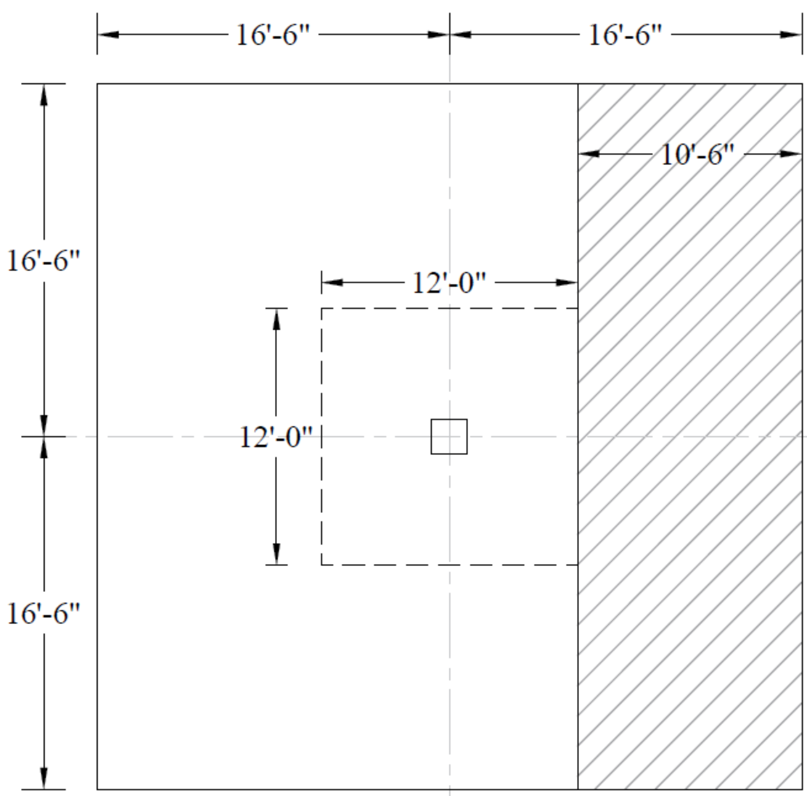


Figure 12 – Critical Section at the Face of the Drop Panel for One-Way Shear

1.2.3. Slab Shear Strength – Two-Way Shear

For critical section at distance *d*/2 from the edge of the column (slab section with drop panel):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 13):

Tributary area of two-way shear for the slab without the drop panel is:

$$A_{Tributary\_1} = (33\times33)-(12\times12)^2 = 945.00\ \text{ft}^2$$

Tributary area of two-way shear for the slab with the drop panel is:

$$A_{Tributary\_2} = (12\times12)-\left(\frac{20+15.50}{12}\right)^2 = 135.25\ \text{ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.340\times945.00+0.475\times135.25 = 385.61\ \text{kips}$$

$$V_c = 4\times\lambda\times\sqrt{f_c'}\times b_o\times d \quad \text{(For square interior column)}$$

ACI 318-14 (Table 22.6.5.2(a))

$$V_c = 4\times1.0\times\sqrt{5{,}000}\times\left(4\times\left(20+15.50\right)\right)\times\frac{15.50}{1{,}000} = 622.54\ \text{kips}$$

$$\phi V_c = 0.75\times622.54 = 466.90\ \text{kips} > V_u = 385.61\ \text{kips}$$

Slab thickness is adequate for two-way shear for the first critical section (from the edge of the column).

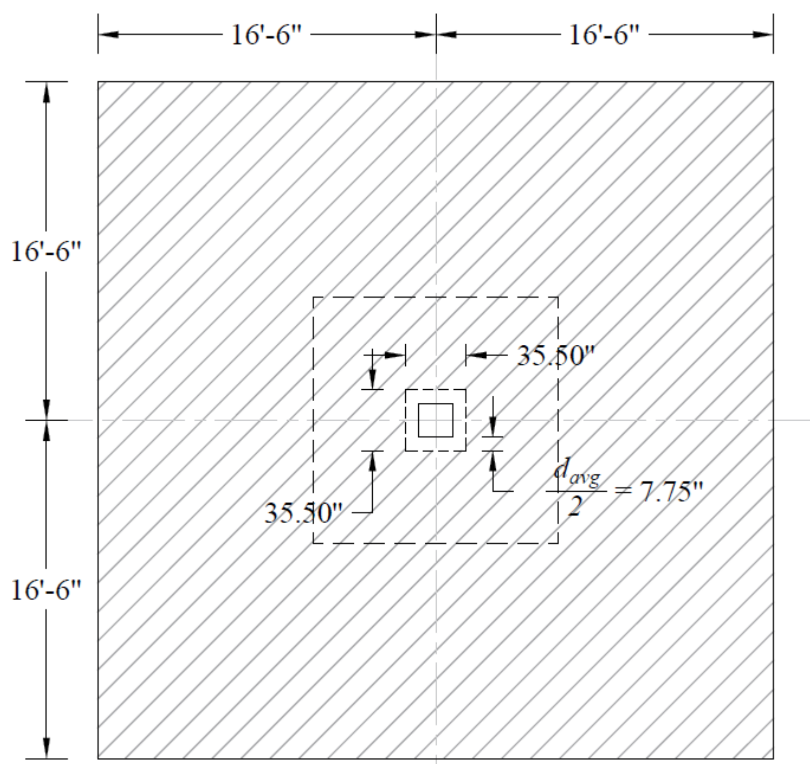


Figure 13 – Critical Section at *d*/2 from the Edge of the Column for Two-Way Shear

For critical section at the edge of the drop panel (slab section without drop panel):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior drop panel (Figure 14):

Tribulary area for two-way shear is:

$$A_{Tributary} = (33\times 33) - \left(12 + \frac{10.50}{12}\right)^2 = 923.23 \text{ ft}^2$$

$$V_u = q_u \times A_{Tributary} = 0.340\times 923.23 = 313.96 \text{ kips}$$

$$V_c = 4\times\lambda\times\sqrt{f'_c}\times b_o\times d \quad \text{(For square interior column)}$$ ***ACI 318-14 (Table 22.6.5.2(a))***

$$V_c = 4\times 1.0\times\sqrt{5,000}\times\left(4\times\left(144+10.50\right)\right)\times\frac{10.50}{1,000} = 1,835.37 \text{ kips}$$

$$\phi V_c = 0.75\times 1835.37 = 1,376.52 \text{ kips} > V_u = 313.96 \text{ kips}$$

Slab thickness of 12 in. is adequate for two-way shear for the second critical section (from the edge of the drop panel).

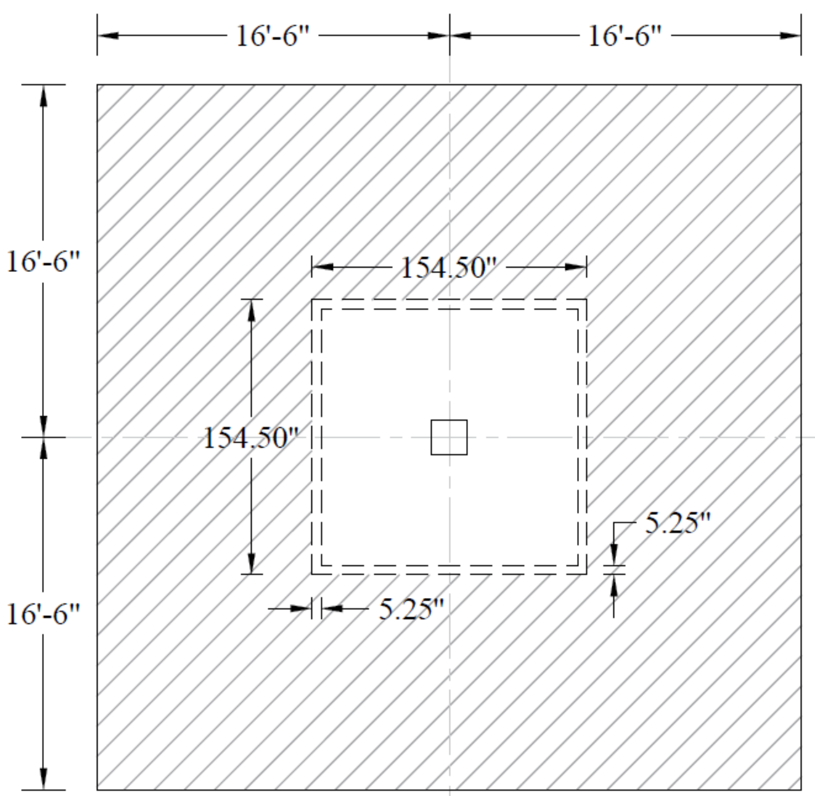


Figure 14 – Critical Section at *d*/2 from the Edge of the Drop Panel for Two-Way Shear

1.2.4. Column Dimensions - Axial Load

Check the adequacy of column dimensions for axial load:

For live load, superimposed dead load, and self-weight of the slab around an interior column:

$q_u = 340.07 \text{ psf}$ (see on page 14)

$A_{Tributary} = 33 \times 33 = 1{,}089.00 \text{ ft}^2$

For self-weight of additional slab thickness due to the presence of the drop panel around an interior column:

$q_u = 475.00 - 340.07 = 134.93 \text{ psf}$ (see on page 12 and on page 14)

$A_{Tributary} = 12 \times 12 = 144.00 \text{ ft}^2$

Assuming four story building

$$P_u = n \times q_u \times A_{Tributary} = 4 \times (0.340 \times 1{,}089.00 + 0.135 \times 144.00) = 1{,}559.06 \text{ kips}$$

Assume 20 in. square column with 12 – No. 11 vertical bars with design axial strength, $\phi P_{n,max}$ of

$$\phi P_{n,\max} = 0.80 \times \phi \times (0.85 \times f'_c \times (A_g - A_{st}) + f_y \times A_{st})$$ ***ACI 318-14 (22.4.2)***

$$\phi P_{n,\max} = 0.80 \times 0.65 \times (0.85 \times 6{,}000 \times (20 \times 20 - 12 \times 1.56) + 60{,}000 \times 12 \times 1.56) = 1{,}595.22 \text{ kips}$$

$$\phi P_{n,\max} = 1{,}595.22 \text{ kips} > P_u = 1{,}559.06 \text{ kips}$$

Column dimensions of 20 in. × 20 in. are adequate for axial load.

2. Flexural Analysis and Design

ACI 318 states that a slab system shall be designed by any procedure satisfying equilibrium and geometric compatibility, provided that strength and serviceability criteria are satisfied. Distinction of two-systems from one-way systems is given by ***ACI 318-14 (R8.10.2.3 & R8.3.1.2)***.

ACI 318 permits the use of Direct Design Method (DDM) and Equivalent Frame Method (EFM) for the gravity load analysis of orthogonal frames and is applicable to flat plates, flat slabs, and slabs with beams. The following sections outline the solution per EFM and spSlab software. For the solution per DDM, check the "Two-Way Flat Plate Concrete Floor System Analysis and Design (ACI 318-14)" example.

2.1. Equivalent Frame Method (EFM)

EFM is the most comprehensive and detailed procedure provided by the ACI 318 for the analysis and design of two-way slab systems where the structure is modeled by a series of equivalent frames (interior and exterior) on column lines taken longitudinally and transversely through the building.

The equivalent frame consists of three parts (for a detailed discussion of this method, refer to "Two-Way Flat Plate Concrete Floor System Analysis and Design (ACI 318-14)":

1) Horizontal slab-beam strip.

2) Columns or other vertical supporting members.

3) Elements of the structure (Torsional members) that provide moment transfer between the horizontal and vertical members.

2.1.1. Limitations for Use of Equivalent Frame Method

In EFM, live load shall be arranged in accordance with 6.4.3 which requires slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***ACI 318-14 (8.11.1.2 & 6.4.3)***

Complete analysis must include representative interior and exterior equivalent frames in both the longitudinal and transverse directions of the floor. ***ACI 318-14 (8.11.2.1)***

Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***ACI 318-14 (8.10.2.3)***

2.1.2. Frame Members of Equivalent Frame

Determine moment distribution factors and fixed-end moments for the equivalent frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors k, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a) Flexural stiffness of slab-beams at both ends, K_{sb}.

$$\frac{c_{N1}}{l_1}=\frac{20}{(33\times 12)}=0.051\ ,\ \frac{c_{N2}}{l_2}=\frac{20}{(33\times 12)}=0.051$$

Slab thickness = h = h_{MI} = 12.00 in. and drop thickness = d_{MI} – h_{MI} = 17.00 – 12.00 = 5.00 in.

$$\frac{\text{drop thickness}}{\text{slab thickness}}=\frac{5.00}{12.00}=0.4167$$

For c_{F1} = c_{F2}, stiffness factors, k_{NF} = k_{FN} = 5.541 ***PCA Notes on ACI 318-11 (Table A2 & A3)***

Thus, $K_{sb}=k_{NF}\times\frac{E_{cs}\times I_s}{l_1}=5.541\times\frac{E_{cs}\times I_s}{l_1}$ ***PCA Notes on ACI 318-11 (Table A2 & A3)***

$$K_{sb}=5.541\times\frac{4,287\times 10^3\times 57,024.00}{396.00}=3,420,614,448\text{ in.-lb}$$

Where, $I_s=\frac{l_2\times h^3}{12}=\frac{396\times(12)^3}{12}=57,024.00\text{ in.}^4$

$E_{cs}=w_c^{1.5}\times 33\times\sqrt{f'_c}=150^{1.5}\times 33\times\sqrt{5,000}=4,287\times 10^3\text{ psi}$ ***ACI 318-14 (19.2.2.1.a)***

Carry-over factor COF = 0.576 ***PCA Notes on ACI 318-11 (Table A2 & A3)***

Fixed-end moment $FEM=\sum_{i=1}^{n}m_{NFi}\times w_i\times l_1^2$ ***PCA Notes on ACI 318-11 (Table A2 & A3)***

Uniform load fixed end moment coefficient, m_{NF1} = 0.0913

Fixed end moment coefficient for (b-a) = 0.2 when a = 0, m_{NF2} = 0.0162

Fixed end moment coefficient for (b-a) = 0.2 when a = 0.8, m_{NF3} = 0.0020

b) Flexural stiffness of column members at both ends, K_c.

Referring to ***Table A7, Appendix 20A***,

For the Bottom Column (Below):

t_a = 3.00 / 2 + 14.00 = 15.50 in. t_b = 3.00 / 2 = 1.50 in.

H = 13 ft = 156.00 in. $H_c = H - t_a - t_b$ = 156.00 – 15.50 – 1.50 = 139.00 in.

$$\frac{t_a}{t_b} = \frac{15.50}{1.50} = 10.333 \qquad \frac{H}{H_c} = \frac{156.00}{139.00} = 1.122$$

Thus, k_{AB} = 6.178 and C_{AB} = 0.500 by interpolation.

$$K_{c,bottom} = \frac{6.178 \times E_{cc} \times I_c}{l_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_{c,bottom} = 6.178 \times \frac{4,696 \times 10^3 \times 13,333.33}{156} = 2,479,648,547 \text{ in.-lb}$$

Where, $I_c = \frac{c^4}{12} = \frac{(20)^4}{12} = 13,333.33 \text{ in.}^4$

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f_c'} = 150^{1.5} \times 33 \times \sqrt{6,000} = 4,696 \times 10^3 \text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

l_c = 13 ft = 156 in.

For the Top Column (Above):

$$\frac{t_b}{t_a} = \frac{1.50}{15.50} = 0.097 \qquad \frac{H}{H_c} = \frac{156.00}{139.00} = 1.122$$

Thus, k_{BA} = 4.624 and C_{BA} = 0.667 by interpolation.

$$K_c = \frac{4.624 \times E_{cc} \times I_c}{l_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_{c,top} = 4.624 \times \frac{4,696 \times 10^3 \times 13,333.33}{156} = 1,855,923,419 \text{ in.-lb}$$

c) Torsional stiffness of torsional members, K_t.

$$K_t = \frac{9 \times E_{cs} \times C}{\left[l_2 \times \left(1 - \frac{c_2}{l_2}\right)^3 \right]}$$ **ACI 318-14 (R.8.11.5)**

$$K_t = \frac{9 \times 4,287 \times 10^3 \times 15,213.92}{33 \times 12 \times \left(1 - \frac{20}{33 \times 12}\right)^3} = 1,731,665,695 \text{ in.-lb}$$

Where $C = \sum \left(1 - 0.63 \times \frac{x}{y}\right) \times \left(\frac{x^3 \times y}{3}\right)$ **ACI 318-14 (Eq. 8.10.5.2b)**

$$C = \left(1 - 0.63 \times \frac{17.00}{20.00}\right) \times \left(\frac{17.00^3 \times 20.00}{3}\right) = 15,213.92 \text{ in.}^4$$

$c_2 = 20$ in., $l_2 = 33$ ft $= 396$ in.

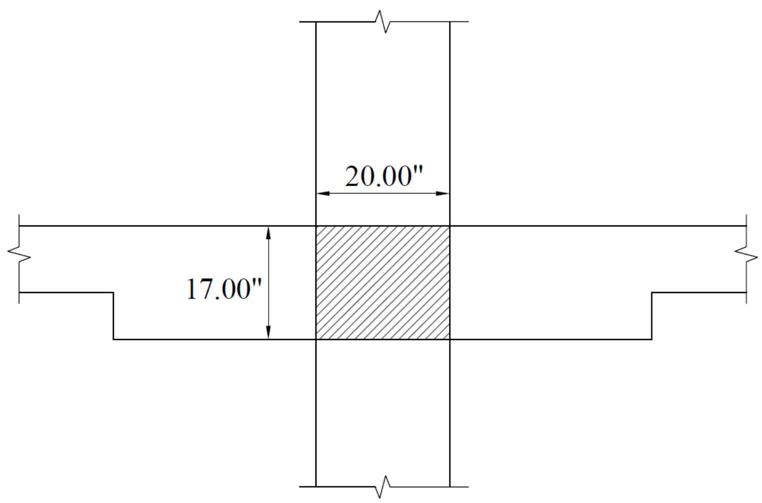


Figure 15 – Torsional Member

d) Equivalent column stiffness, K_{ec}.

$$K_{ec} = \frac{\sum K_c \times \sum K_t}{\sum K_c + \sum K_t}$$

$$K_{ec} = \frac{(2,479.65 + 1,855.92) \times (2 \times 1,731.67)}{[(2,479.65 + 1,855.92) + (2 \times 1,731.67)]} \times 10^6 = 1,925,337,678 \text{ in.-lb}$$

Where $\sum K_t$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

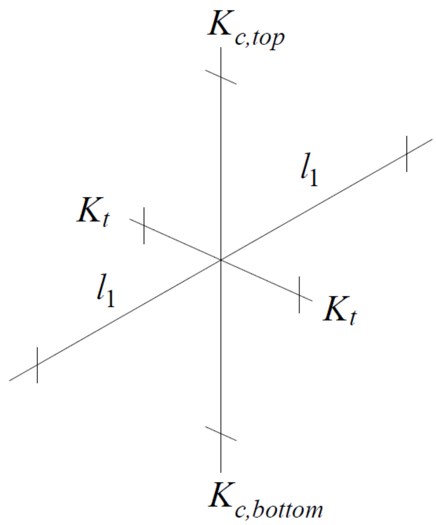


Figure 16 – Column and Edge of Slab

e) Slab-beam joint distribution factors, *DF*.

At exterior joint

$$DF = \frac{3,420.61}{(3,420.61 + 1,925.34)} = 0.640$$

At interior joint

$$DF = \frac{3,420.61}{(3,420.61 + 3,420.61 + 1,925.34)} = 0.390$$

COF for slab-beam = 0.576

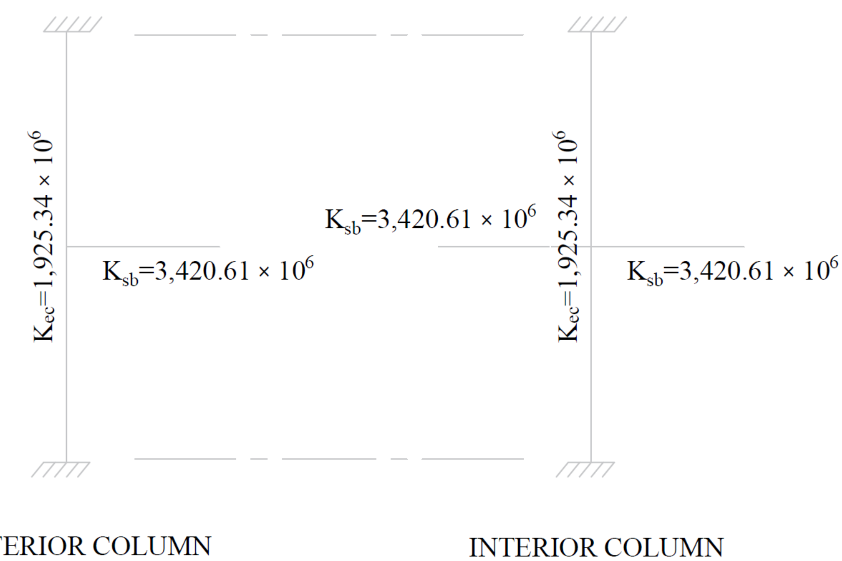


Figure 17 – Slab and Column Stiffness

2.1.3. Equivalent Frame Analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. ***ACI 318-14 (6.4.3.2)***

$$\frac{L}{D} = \frac{100}{(100+50)} = 0.67 < \frac{3}{4}$$

a) Factored load and Fixed-End Moments (FEM's).

For slab:

Factored dead load, $q_{Du} = 1.20 \times (100.00 + 50.00) = 180.00$ psf

Factored live load, $q_{Lu} = 1.60 \times 100.00 = 160.00$ psf ***ACI 318-14 (5.3.1)***

Total factored load, $q_u = q_{Du} + q_{Lu} = 340.00$ psf

For drop panels:

Factored dead load, $q_{Du} = 1.20 \times (150.00 \times 9.00 / 12) = 135.00$ psf

Factored live load, $q_{Lu} = 1.60 \times 0.00 = 0.00$ psf ***ACI 318-14 (5.3.1)***

Total factored load, $q_u = q_{Du} + q_{Lu} = 135.00$ psf

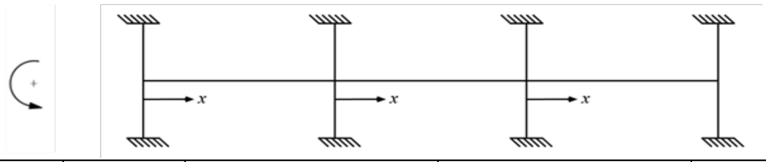
Fixed-end moment $FEM = \sum_{i=1}^{n} m_{NFi} \times w_i \times l_1^2$ ***PCA Notes on ACI 318-11 (Table A2 & A3)***

$$FEM = 0.0913 \times 0.340 \times 33 \times 33^2 + 0.0162 \times 0.135 \times 12 \times 33^2 + 0.0020 \times 0.135 \times 12 \times 33^2$$

$$FEM = 1{,}147.66 \text{ ft-kips}$$

b) Moment distribution. Computations are shown in the Table below Counterclockwise rotational moments acting on the member ends are taken as positive.

Table 1 - Moment Distribution for Equivalent Frame



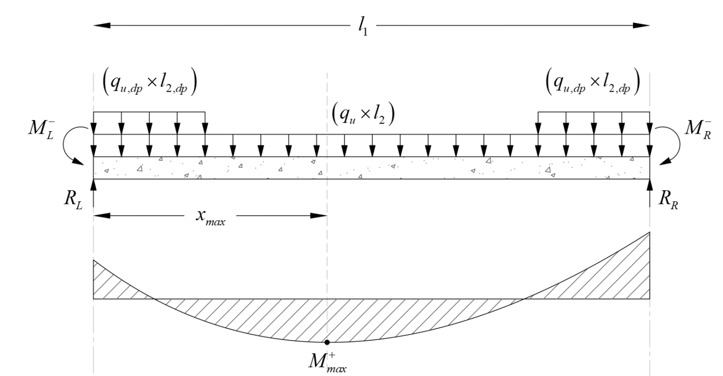
Joint	1	2		3		4
Member	**1-2**	**2-1**	**2-3**	**3-2**	**3-4**	**4-3**
DF	0.640	0.390	0.390	0.390	0.390	0.640
COF	0.576	0.576	0.576	0.576	0.576	0.576
FEM	1,147.66	-1,147.66	1,147.66	-1,147.66	1,147.66	-1,147.66
Dist	-734.33	0	0	0	0	734.33
CO	0	-422.89	0	0	422.89	0
Dist	0	165.01	165.01	-165.01	-165.01	0
CO	95.03	0	-95.03	95.03	0	-95.03
Dist	-60.80	37.08	37.08	-37.08	-37.08	60.80
CO	21.35	-35.01	-21.35	21.35	35.01	-21.35
Dist	-13.66	21.99	21.99	-21.99	-21.99	13.66
CO	12.67	-7.87	-12.67	12.67	7.87	-12.67
Dist	-8.10	8.01	8.01	-8.01	-8.01	8.10
CO	4.61	-4.67	-4.61	4.61	4.67	-4.61
Dist	-2.95	3.62	3.62	-3.62	-3.62	2.95
CO	2.09	-1.70	-2.09	2.09	1.70	-2.09
Dist	-1.33	1.48	1.48	-1.48	-1.48	1.33
CO	0.85	-0.77	-0.85	0.85	0.77	-0.85
Dist	-0.54	0.63	0.63	-0.63	-0.63	0.54
CO	0.36	-0.31	-0.36	0.36	0.31	-0.36
Dist	-0.23	0.26	0.26	-0.26	-0.26	0.23
CO	0.15	-0.13	-0.15	0.15	0.13	-0.15
Dist	-0.10	0.11	0.11	-0.11	-0.11	0.10
CO	0.06	-0.06	-0.06	0.06	0.06	-0.06
Dist	-0.04	0.05	0.05	-0.05	-0.05	0.04
CO	0.03	-0.02	-0.03	0.03	0.02	-0.03
Dist	-0.02	0.02	0.02	-0.02	-0.02	0.02
M^{-}_{max}	462.75	-1,382.84	1,248.72	-1,248.72	1,382.84	-462.75
V	166.97	-222.73	194.85	-194.85	222.73	-166.97
x_{max}	14.02		16.50		18.98	
M^{+}_{max}	668.33		307.76		668.33	

Maximum positive span moments are determined from the following equations:

$$M^{+}_{max} = M_1 + M_2 + M_3$$

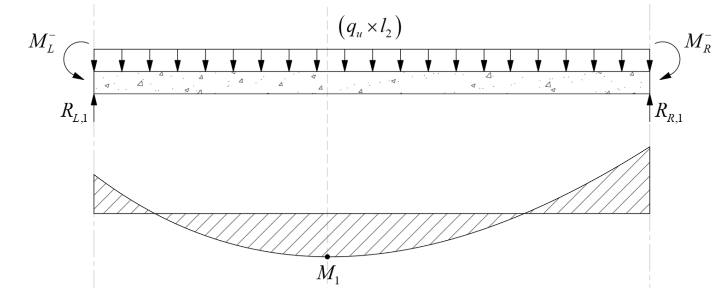
$$R_L = R_{L,1} + R_{L,2} + R_{L,3} \qquad R_R = R_{R,1} + R_{R,2} + R_{R,3}$$

$$x_{max} = \frac{l_1}{2} + \frac{M_L^- - M_R^-}{(q_u \times l_2) \times l_1}$$



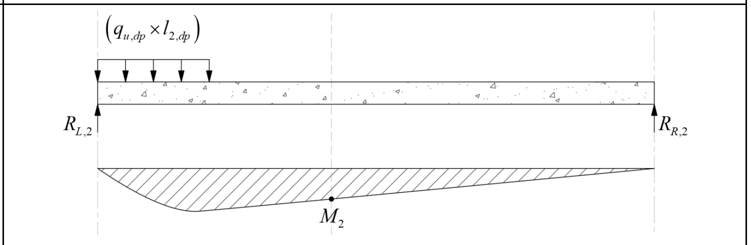
$$M_1 = \frac{(q_u \times l_2) \times l_1^2}{8} - \frac{M_L^- + M_R^-}{2} + \frac{(M_L^- - M_R^-)^2}{2 \times (q_u \times l_2) \times l_1^2}$$

$$R_{L,1} = \frac{(q_u \times l_2) \times l_1}{2} + \frac{M_L^- - M_R^-}{l_1} \qquad R_{R,1} = \frac{(q_u \times l_2) \times l_1}{2} - \frac{M_L^- - M_R^-}{l_1}$$



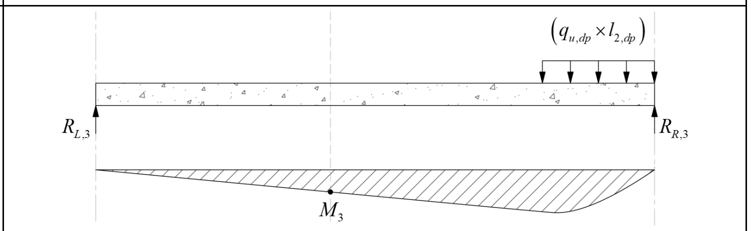
$$M_2 = R_{R,2} \times (l_1 - x_{max})$$

$$R_{L,2} = \frac{(q_{u,dp} \times l_{2,dp}) \times \frac{l_{1,dp}}{2}}{2 \times \ell_1} \times \left(2 \times \frac{9 \times l_{1,dp}}{11} + \frac{l_{1,dp}}{2}\right) \qquad R_{R,2} = \frac{(q_{u,dp} \times l_{2,dp}) \times \frac{l_{1,dp}}{2}}{2 \times l_1} \times \left(\frac{l_{1,dp}}{2}\right)$$



$$M_3 = R_{L,3} \times x_{max}$$

$$R_{L,3} = \frac{(q_{u,dp} \times l_{2,dp}) \times \frac{l_{1,dp}}{2}}{2 \times l_1} \times \left(\frac{l_{1,dp}}{2}\right) \qquad R_{R,3} = \frac{(q_{u,dp} \times l_{2,dp}) \times \frac{l_{1,dp}}{2}}{2 \times l_1} \times \left(2 \times \frac{9 \times l_{1,dp}}{11} + \frac{l_{1,dp}}{2}\right)$$



Maximum positive moment in spans 1-2 and 3-4:

$$M^{+}_{max} = M_1 + M_2 + M_3 = 639.17 + 16.78 + 12.38 = 668.33 \text{ ft-kips}$$

$$V_L = R_L = R_{L,1} + R_{L,2} + R_{L,3} = 157.25 + 8.84 + 0.88 = 166.97 \text{ kips}$$

$$V_R = R_R = R_{R,1} + R_{R,2} + R_{R,3} = 213.01 + 0.88 + 8.84 = 222.73 \text{ kips}$$

$$x_{max} = \frac{33}{2} + \frac{(462.75 - 1,382.84)}{(0.340 \times 33) \times 33} = 14.02 \text{ ft}$$

Where:

M_L^- = 462.75 ft-kips

M_R^- = 1,382.84 ft-kips

$$M_1 = \frac{(0.340 \times 33) \times 33^2}{8} - \frac{462.75 + 1,382.84}{2} + \frac{(462.75 - 1,382.84)^2}{2 \times (0.340 \times 33) \times 33^2} = 639.17 \text{ ft-kips}$$

$$M_2 = \frac{(0.135 \times 12) \times \left(\frac{12}{2}\right)}{2 \times 33} \times \left(\frac{12}{2}\right) \times (33 - 14.02) = 16.78 \text{ ft-kips}$$

$$M_3 = \frac{(0.135 \times 12) \times \left(\frac{12}{2}\right)}{2 \times 33} \times \left(\frac{12}{2}\right) \times 14.02 = 12.38 \text{ ft-kips}$$

And:

$$R_{L,1} = \frac{(0.340 \times 33) \times 33}{2} + \frac{(462.75 - 1,382.84)}{33} = 157.25 \text{ kips}$$

$$R_{L,2} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times \left(2 \times \frac{9 \times 33}{11} + 6\right) = 8.84 \text{ kips}$$

$$R_{L,3} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times (6) = 0.88 \text{ kips}$$

$$R_{R,1} = \frac{(0.340 \times 33) \times 33}{2} - \frac{(462.75 - 1,382.84)}{33} = 213.01 \text{ kips}$$

$$R_{R,2} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times (6) = 0.88 \text{ kips}$$

$$R_{R,3} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times \left(2 \times \frac{9 \times 33}{11} + 6\right) = 8.84 \text{ kips}$$

Maximum positive moment in span 2-3:

$$M^{+}_{max} = M_1 + M_2 + M_3 = 278.60 + 14.58 + 14.58 = 307.76 \text{ ft-kips}$$

$$V_L = R_L = R_{L,1} + R_{L,2} + R_{L,3} = 185.13 + 8.84 + 0.88 = 194.85 \text{ kips}$$

$$V_R = R_R = R_{R,1} + R_{R,2} + R_{R,3} = 185.13 + 0.88 + 8.84 = 194.85 \text{ kips}$$

$$x_{max} = \frac{33}{2} + \frac{(1,248.72 - 1,248.72)}{(0.340 \times 33) \times 33} = 16.50 \text{ ft}$$

Where:

$M_L^- = 1,248.72$ ft-kips

$M_R^- = 1,248.72$ ft-kips

$$M_1 = \frac{(0.340 \times 33) \times 33^2}{8} - \frac{1,248.72 + 1,248.72}{2} + \frac{(1,248.72 - 1,248.72)^2}{2 \times (0.340 \times 33) \times 33^2} = 278.60 \text{ ft-kips}$$

$$M_2 = \frac{(0.135 \times 12) \times \left(\frac{12}{2}\right)}{2 \times 33} \times \left(\frac{12}{2}\right) \times (33 - 16.50) = 14.58 \text{ ft-kips}$$

$$M_3 = \frac{(0.135 \times 12) \times \left(\frac{12}{2}\right)}{2 \times 33} \times \left(\frac{12}{2}\right) \times 16.50 = 14.58 \text{ ft-kips}$$

and:

$$R_{L,1} = \frac{(0.340 \times 33) \times 33}{2} + \frac{(1,248.72 - 1,248.72)}{33} = 185.13 \text{ kips}$$

$$R_{L,2} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times \left(2 \times \frac{9 \times 33}{11} + 6\right) = 8.84 \text{ kips}$$

$$R_{L,3} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times (6) = 0.88 \text{ kips}$$

$$R_{R,1} = \frac{(0.340 \times 33) \times 33}{2} - \frac{(1,248.72 - 1,248.72)}{33} = 185.13 \text{ kips}$$

$$R_{R,2} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times (6) = 0.88 \text{ kips}$$

$$R_{R,3} = \frac{(0.135 \times 12) \times 6}{2 \times 33} \times \left(2 \times \frac{9 \times 33}{11} + 6\right) = 8.84 \text{ kips}$$

2.1.4. Factored Moments Used for Design

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 18. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than 0.175 × l_1 from the centers of supports. ***ACI 318-14 (8.11.6.1)***

$$\frac{20 \text{ in.}}{12\times 2} = 0.83 \text{ ft} < 0.175\times 33 = 5.78 \text{ ft} \text{ (use face of support location)}$$

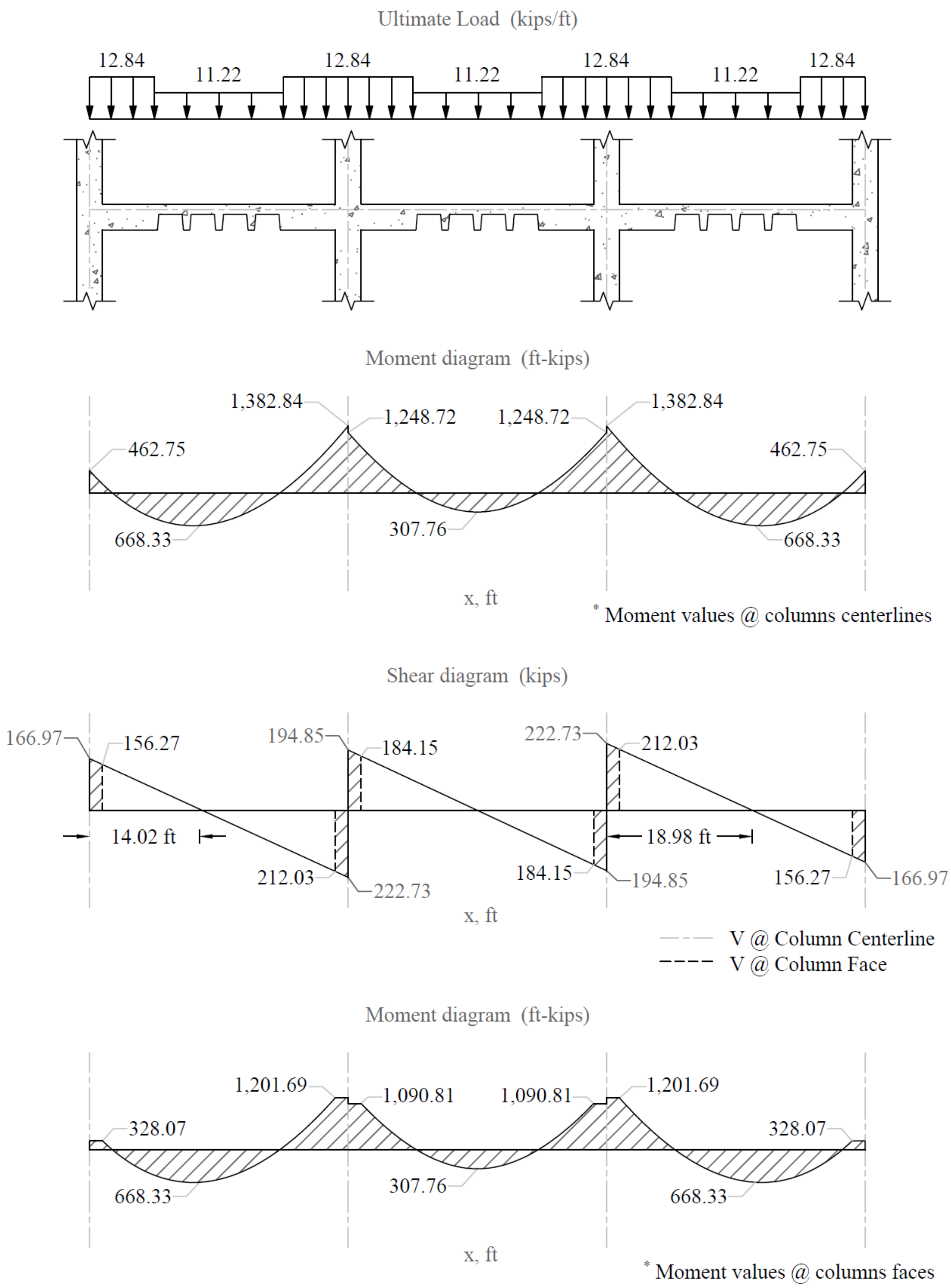


Figure 18 – Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)

2.1.5. Factored Moments in Slab-Beam Strip

a) Check whether the moments calculated above can take advantage of the reduction permitted by ***ACI 318-14 (8.11.6.5)***:

If the slab system analyzed using EFM within the limitations of ***ACI 318-14 (8.10.2)***, it is permitted by the ACI code to reduce the calculated moments obtained from EFM in such proportion that the absolute sum of the positive and average negative design moments need not exceed the total static moment M_o given by ***Equation 8.10.3.2*** in the ***ACI 318-14***.

Check Applicability of Direct Design Method:

1. There is a minimum of three continuous spans in each direction. ***ACI 318-14 (8.10.2.1)***
2. Successive span lengths are equal. ***ACI 318-14 (8.10.2.2)***
3. Long-to-Short ratio is 33/33 = 1.00 < 2.00. ***ACI 318-14 (8.10.2.3)***
4. Column are not offset. ***ACI 318-14 (8.10.2.4)***
5. Loads are gravity and uniformly distributed with service live-to-dead ratio of 0.67 < 2.00

   (Note: The self-weight of the drop panels is not uniformly distributed entirely along the span. However, the variation in load magnitude is small). ***ACI 318-14 (8.10.2.5 and 6)***
6. Check relative stiffness for slab panel. ***ACI 318-14 (8.10.2.7)***

   Slab system is without beams and this requirement is not applicable.

All limitation of ***ACI 318-14 (8.10.2)*** are satisfied and the provisions of ***ACI 318-14 (8.11.6.5)*** may be applied:

$$M_o = \frac{q_u \times l_2 \times l_n^2}{8} = \frac{0.340 \times 33 \times (33 - 20/12)^2}{8} = 1{,}376.94 \text{ ft-kips}$$ ***ACI 318-14 (Eq. 8.10.3.2)***

End spans: $668.33 + \frac{462.75 + 1{,}382.84}{2} = 1{,}591.13 \text{ ft-kips} > M_o$

Interior span: $307.76 + \frac{1{,}248.72 + 1{,}248.72}{2} = 1{,}556.48 \text{ ft-kips} > M_o$

To illustrate proper procedure, the interior span factored moments may be reduced as follows:

$$\text{Permissible reduction } = \frac{1,376.94}{1,556.48} = 0.885$$

Adjusted negative design moment = 1,248.72 × 0.885 = 1,104.68 ft-kips

Adjusted positive design moment = 307.76 × 0.885 = 272.26 ft-kips

$$M_o = 272.26 + \frac{1,104.68 + 1,104.68}{2} = 1,376.94 \text{ ft-kips}$$

ACI 318 allows the reduction of the moment values based on the previous procedure. Since the drop panels may cause gravity loads not to be uniform (Check limitation #5 and Figure 18), the moment values obtained from EFM will be used for comparison reasons.

b) Distribute factored moments to column and middle strips:

After the negative and positive moments have been determined for the slab-beam strip, the ACI code permits the distribution of the moments at critical sections to the column strips, beams (if any), and middle strips in accordance with the DDM. ***ACI 318-14 (8.11.6.6)***

Distribution of factored moments at critical sections is summarized in Table below.

Table 2 - Distribution of Factored Moments

Location		Slab-beam Strip	Column Strip		Middle Strip	
		Moment (ft-kips)	Percent	Moment (ft-kips)	Percent	Moment (ft-kips)
End Span	Exterior Negative	328.07	100	328.07	0	0.00
	Positive	668.33	60	401.00	40	267.33
	Interior Negative	1,201.69	75	901.27	25	300.42
Interior Span	Negative	1,090.81	75	818.10	25	272.70
	Positive	307.76	60	184.66	40	123.10

2.1.6. Flexural Reinforcement Requirements

a) Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – interior negative location is provided below:

M_u = 901.27 ft-kips

Use d = 15.88 in. (slab with drop panel where h = 17 in.)

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.90, and jd will be taken equal to 0.971× d. The assumptions will be verified once the area of steel in finalized.

Assume jd = 0.971× d = 15.41 in.

Column strip width, $b = \dfrac{33\times12}{2} = 198.00$ in.

Middle strip width, $b = 33\times12 - 198.00 = 198.00$ in.

$$A_s = \frac{M_u}{\phi\times f_y\times jd} = \frac{901.27\times12{,}000}{0.90\times60{,}000\times15.41} = 12.995\text{ in.}^2$$

Recalculate 'a' for the actual A_s = 12.995 in.^{2}:

$$a = \frac{A_s\times f_y}{0.85\times f'_c\times b} = \frac{12.995\times60{,}000}{0.85\times5{,}000\times198.00} = 0.927\text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.927}{0.85} = 1.090\text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right)\times d_t - 0.003 = \left(\frac{0.003}{1.090}\right)\times15.88 - 0.003 = 0.0407 \geq 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi\times f_y\times\left(d - \dfrac{a}{2}\right)} = \frac{901.27\times12{,}000}{0.90\times60{,}000\times\left(15.88 - \dfrac{0.927}{2}\right)} = 12.995\text{ in.}^2$$

Two values of thickness must be considered. The slab thickness in the column strip is 17.00 in. with the drop panel and 8.00 in. for the equivalent slab without the drop panel based on the system weight.

The weighted slab thickness:

$$h_w = \frac{17.00 \times (12) + 8 \times \left(\frac{33}{2} - 12\right)}{(12) + \left(\frac{33}{2} - 12\right)} = 14.55 \text{ in.}$$

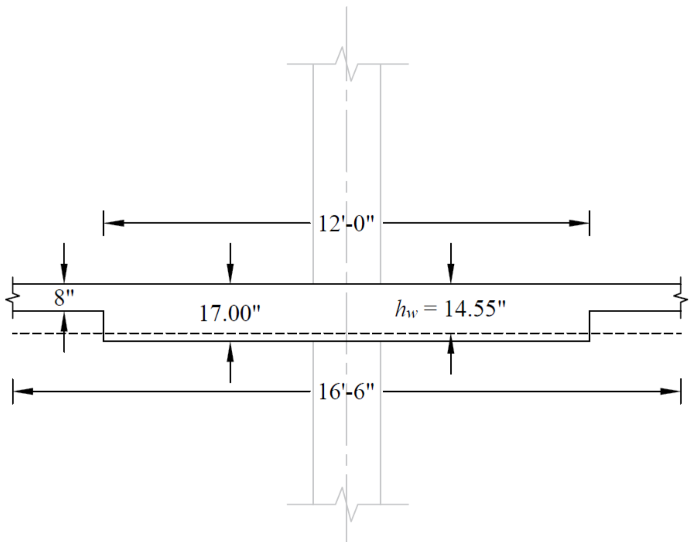


Figure 19 – The Weighted Slab Thickness

$$A_{s,\min} = 0.0018 \times b \times h_w$$ ***ACI 318-14 (24.4.3.2)***

$$A_{s,\min} = 0.0018 \times 198 \times 14.55 = 5.184 \text{ in.}^2 < 12.995 \text{ in.}^2$$

$$s_{\max} = \text{lesser of} \begin{bmatrix} 5h \\ 18 \text{ in.} \end{bmatrix} = \text{lesser of} \begin{bmatrix} 5 \times 3 = 15 \text{ in.} \\ 18 \text{ in.} \end{bmatrix} = 15.00 \text{ in.}$$ ***ACI 318-14 (24.4.3.3)***

Provide 30 – #6 bars with A_s = 13.20 in.^{2} and $s = \frac{198}{30} = 6.60 \text{ in.} \leq s_{\max} = 15.00 \text{ in.}$

<u>The flexural reinforcement calculation for the column strip of interior span – positive location is provided below:</u>

M_u = 184.66 ft-kips

Use d = 15.88 in. (slab with rib where h = 17 in.)

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the slab section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.90, and jd will be taken equal to 0.994× d. The assumptions will be verified once the area of steel in finalized.

Assume jd = 0.994× d = 15.78 in.

Column strip width, $b = \dfrac{33\times 12}{2} = 198.00$ in.

Middle strip width, $b = 33\times 12 - 198.00 = 198.00$ in.

$$A_s = \frac{M_u}{\phi\times f_y\times jd} = \frac{184.66\times 12{,}000}{0.90\times 60{,}000\times 15.78} = 2.600\ \text{in.}^2$$

Recalculate 'a' for the actual A_s = 2.600 in.^{2}:

$$a = \frac{A_s\times f_y}{0.85\times f'_c\times b} = \frac{2.600\times 60{,}000}{0.85\times 5{,}000\times 198.00} = 0.185\ \text{in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.185}{0.85} = 0.218\ \text{in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right)\times d_t - 0.003 = \left(\frac{0.003}{0.218}\right)\times 15.88 - 0.003 = 0.2154 \geq 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi\times f_y\times\left(d - \dfrac{a}{2}\right)} = \frac{184.66\times 12{,}000}{0.90\times 60{,}000\times\left(15.88 - \dfrac{0.185}{2}\right)} = 2.600\ \text{in.}^2$$

$A_{s,\min} = 0.0018\times b\times h_{eq}$ ***<u>ACI 318-14 (24.4.3.2)</u>***

$$A_{s,\min} = 0.0018 \times 198 \times 8.00 = 2.851 \text{ in.}^2 > 2.600 \text{ in.}^2$$

$$\therefore \text{ use } A_s = A_{s,\min} = 2.851 \text{ in.}^2$$

Since column strip has 5 ribs → provide 10 – #6 bars (2 bars / rib):

$$A_{s,provided} = 10 \times 0.44 = 4.40 \text{ in.}^2 > A_{s,required} = 2.851 \text{ in.}^2$$

Based on the procedure outlined above, values for all span locations are given in Table below.

Table 3 - Required Slab Reinforcement for Flexure [Equivalent Frame Method (EFM)]

Span Location		M_u (ft-kips)	b (in.)	d (in.)	$A_{s,req}$ (in.^{2})	$A_{s,min}$ (in.^{2})	Reinforcement Provided	$A_{s,provided}$ (in.^{2})
End Span								
Column Strip	Exterior Negative	328.07	198	15.88	4.641	5.184	14 – #6 * **	6.16
	Positive	401.00	198	15.81	5.709	2.851	10 – #7 (2 bars / rib)	6.00
	Interior Negative	901.27	198	15.88	12.995	5.184	30 – #6	13.20
Middle Strip	Exterior Negative	0.00	198	15.88	0	5.184	14 – #6 * **	6.16
	Positive	267.33	198	15.88	3.774	2.851	12 – #6 (2 bars / rib)	5.28
	Interior Negative	300.42	198	15.88	4.246	5.184	14 – #6 * **	6.16
Interior Span								
Column Strip	Positive	184.66	198	15.88	2.600	2.851	10 – #6 * (2 bars / rib)	4.40
Middle Strip	Positive	123.10	198	15.88	1.730	2.851	12 – #6 * (2 bars / rib)	5.28

* Design governed by minimum reinforcement.
** Number of bars governed by maximum allowable spacing.

b) Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

The factored slab moment resisted by the column ($\gamma_f \times M_{sc}$) shall be assumed to be transferred by flexure. Concentration of reinforcement over the column by closer spacing or additional reinforcement shall be used to resist this moment. The fraction of slab moment not calculated to be resisted by flexure shall be assumed to be resisted by eccentricity of shear. **_ACI 318-14 (8.4.2.3)_**

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_{sc}$ **_ACI 318-14 (8.4.2.3.1)_**

Where:

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{b_1}{b_2}}}$$ **_ACI 318-14 (8.4.2.3.2)_**

- b_1 = Dimension of the critical section b_o measured in the direction of the span for which moments are determined in ACI 318, Chapter 8 (see Figure 20).
- b_2 = Dimension of the critical section b_o measured in the direction perpendicular to b_1 in ACI 318, Chapter 8 (see Figure 20).
- b_b = Effective slab width = $c_2 + 3\times h$ **_ACI 318-14 (8.4.2.3.3)_**

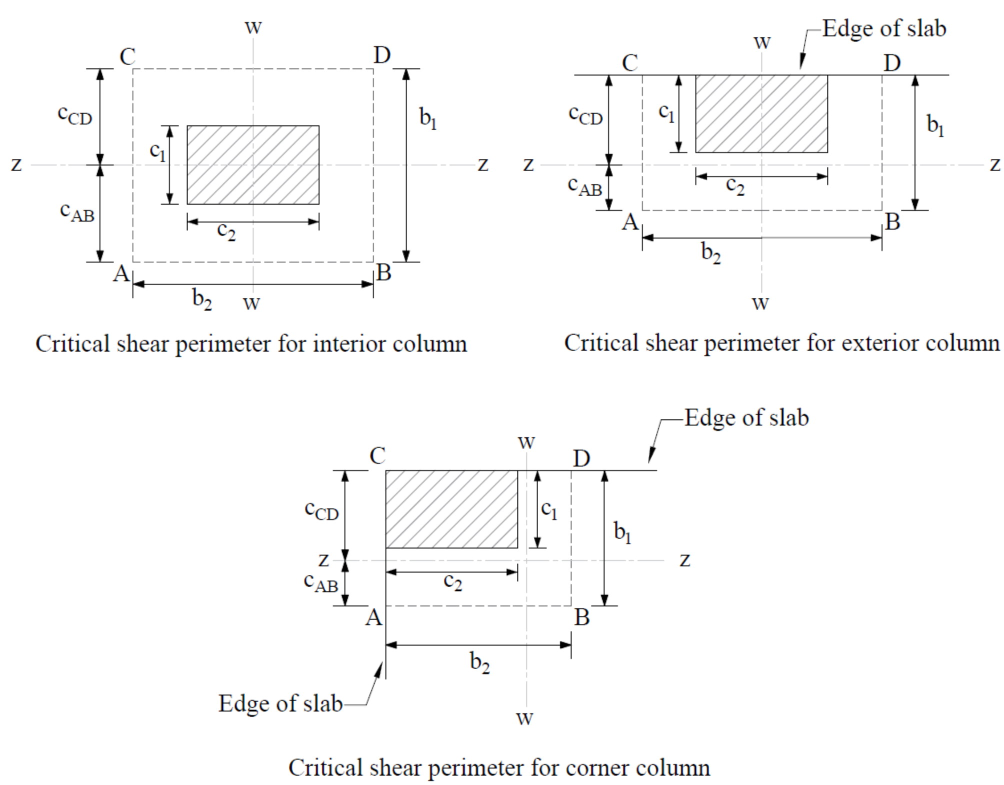


Figure 20 – Critical Shear Perimeters for Columns

For exterior support:

$$d = h - c_{clear} - \frac{d_b}{2} = 17.00 - 0.75 - \frac{0.75}{2} = 15.88 \text{ in.}$$

M_{sc} = 462.75 ft-kips

$A_{s(prov)}$ = 6.16 in.^{2}

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{15.88}{2} = 27.94 \text{ in.}$$

$$b_2 = c_2 + d = 20 + 15.88 = 35.88 \text{ in.}$$

$$b_b = c_2 + 3 \times h = 20 + 3 \times 17.00 = 71.00 \text{ in.}$$

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{27.94}{35.88}}} = 0.630$$

$$A_s = \frac{0.85 \times f'_c \times b_b}{f_y} \times \left(d - \sqrt{d^2 - \frac{2 \times \gamma_f \times M_{sc}}{\phi \times 0.85 \times f'_c \times b_b}} \right)$$

$$A_s = \frac{0.85 \times 5{,}000 \times 71.00}{60{,}000} \times \left(15.88 - \sqrt{15.88^2 - \frac{2 \times 0.630 \times 462.75}{0.90 \times 0.85 \times 5{,}000 \times 71.00}} \right) = 4.188 \text{ in.}^2$$

However, the area of steel provided to resist the flexural moment within the effective slab width, b_b:

$$A_{s,provided\ within\ bb} = A_{s,provided} \times \frac{b_b}{b} = 4.188 \times \frac{71.00}{198} = 2.209 \text{ in.}^2$$

Then, the required additional reinforcement at exterior column for moment transfer between slab and column:

$$A_{s,additional} = A_s - A_{s,provided\ within\ bb} = 4.188 - 2.209 = 1.979 \text{ in.}^2$$

Provide 5 – #6 additional bars with A_s = 2.20 in.^{2}

Based on the procedure outlined above, values for all supports are given in Table below.

Table 4 - Additional Slab Reinforcement required for moment transfer between slab and column (EFM)

Span Location		M_{sc}* (ft-kips)	γ_f	$\gamma_f M_{sc}$ (ft-kips)	b_b (in.)	d (in.)	A_s req'd within b_b (in.^{2})	A_s prov. For flexure within b_b (in.^{2})	Add'l Reinf.
End Span									
Column Strip	Exterior Negative	462.75	0.630	291.35	71.00	15.88	4.188	2.209	5-#6
	Interior Negative	134.12	0.600	80.47	71.00	15.88	2.029	4.733	-

* M_{sc} is taken at the centerline of the support in Equivalent Frame Method solution.

2.1.7. Factored Moments in Columns

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the support columns above and below the slab-beam in proportion to the relative stiffness of the support columns. Referring to Figure 18, the unbalanced moment at the exterior and interior joints are:

Exterior Joint = + 462.75 ft-kips

Joint 2 = - 1,382.84 + 1,248.72 = -134.12 ft-kips

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced slab moments (M_{sc}) to the exterior and interior columns are shown in the following Figure.

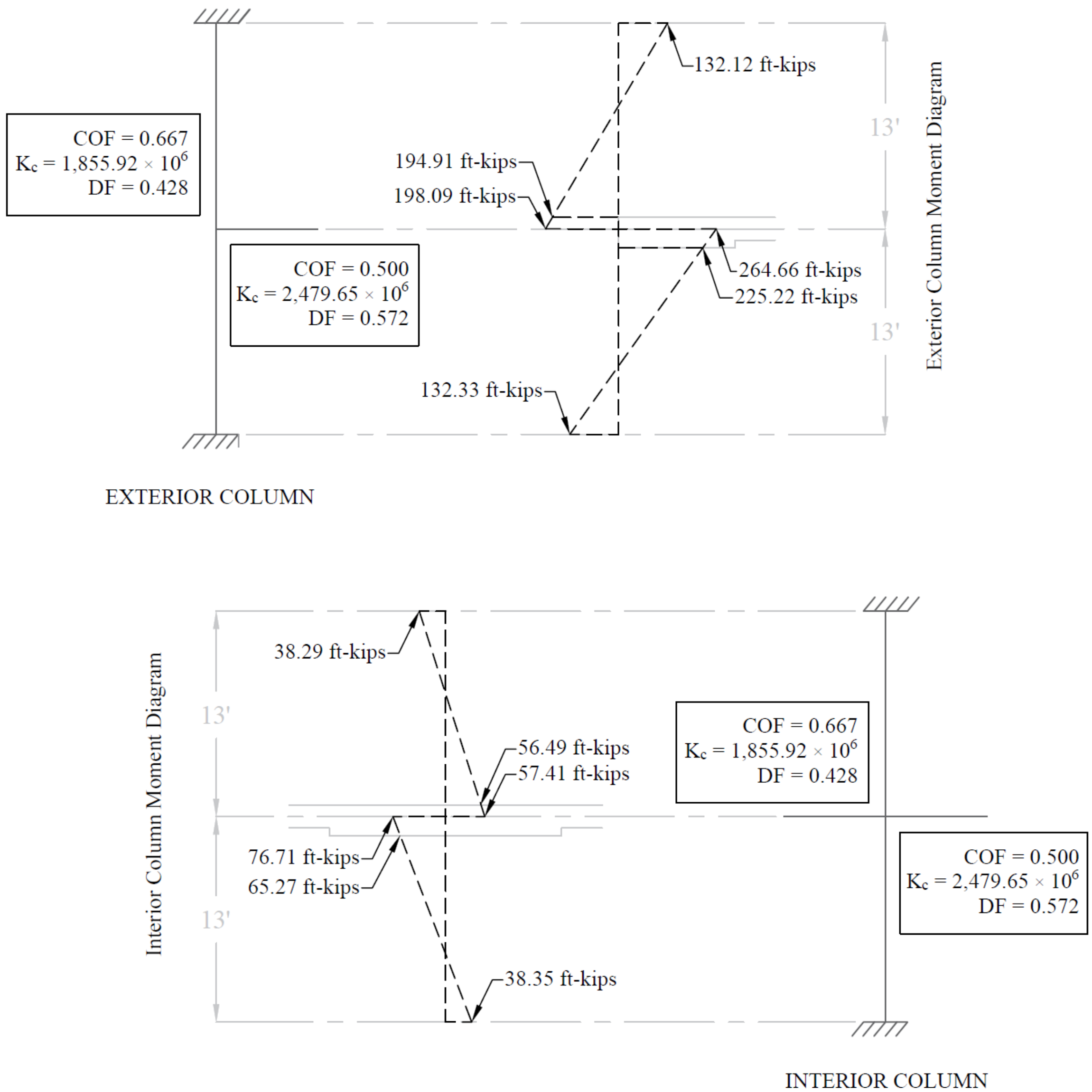


Figure 21 – Column Moments (Unbalanced Moments from Slab-Beam)

In summary:

For Top column (Above):

$M_{col,Exterior}$= 194.91 ft-kips

$M_{col,Interior}$ = 56.49 ft-kips

For Bottom column (Below):

$M_{col,Exterior}$= 225.22 ft-kips

$M_{col,Interior}$ = 65.27 ft-kips

The moments determined above are combined with the factored axial loads (for each story) and factored moments in the transverse direction for design of column sections. The moment values at the face of interior, exterior, and corner columns from the unbalanced moment values are shown in the following Table.

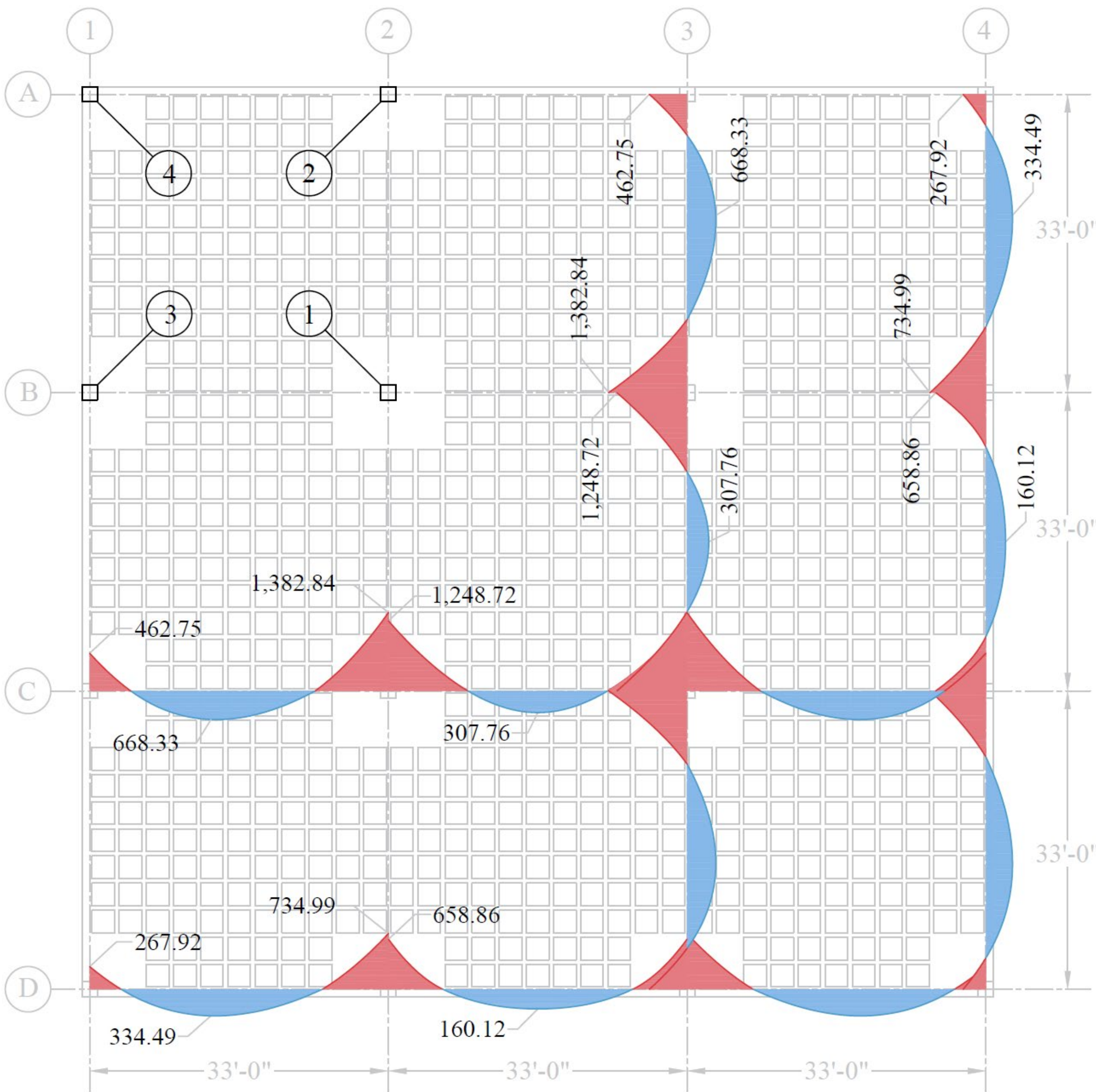


Figure 22 – Moment Diagrams (kips-ft)

Table 5 - Factored Moments in Columns			
M_u (kips-ft)	Column Location		
	Interior	Exterior	Corner
M_{ux}	65.27	225.22	225.22
M_{uy}	65.27	65.27	225.22

3. Design of Columns by spColumn

This section includes the design of interior, edge, and corner columns using spColumn software. The preliminary dimensions for these columns were calculated previously in section one. The reduction of live load per ***ASCE 7-10*** will be ignored in this example. However, the detailed procedure to calculate the reduced live loads is explained in the "One-Way Wide Module (Skip) Joist Concrete Floor System Design (ACI 318-14)" example.

3.1. Determination of Factored Loads

Assume 4 story building

Interior Column:

Tributary area for interior column for live load, superimposed dead load, and self-weight of the slab is

$A_{Tributary} = (33 \times 33) = 1{,}089.00 \text{ ft}^2$

Tributary area for interior column for self-weight of additional slab thickness due to the presence of the drop panel is

$A_{Tributary} = (12 \times 12) = 144.00 \text{ ft}^2$

- $P_u = 4 \times q_u \times A_{Tributary} = 4 \times (0.340 \times 1{,}089.00 + 0.135 \times 144.00) = 1{,}558.80$ kips
- $M_{u,x} = 65.27$ ft-kips (see the previous Table)
- $M_{u,y} = 65.27$ ft-kips (see the previous Table)

Edge (Exterior) Column:

Tributary area for exterior column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left(\frac{33}{2} + \frac{20/2}{12}\right) \times 33 = 572.00 \text{ ft}^2$$

Tributary area for exterior column for self-weight of additional slab thickness due to the presence of the drop panel is

$$A_{Tributary} = \left(\frac{12}{2}+\frac{20/2}{12}\right)\times 12 = 82.00 \text{ ft}^2$$

- P_u = 4 × q_u × $A_{Tributary}$ = 4 × (0.340 × 572.00 + 0.135 × 82.00) = 822.20 kips
- $M_{u,x}$ = 225.22 ft-kips (see the previous Table)
- $M_{u,y}$ = 65.27 ft-kips (see the previous Table)

Corner Column:

Tributary area for corner column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \left(\frac{33}{2}+\frac{20/2}{12}\right)\times\left(\frac{33}{2}+\frac{20/2}{12}\right) = 300.44 \text{ ft}^2$$

Tributary area for corner column for self-weight of additional slab thickness due to the presence of the drop panel is

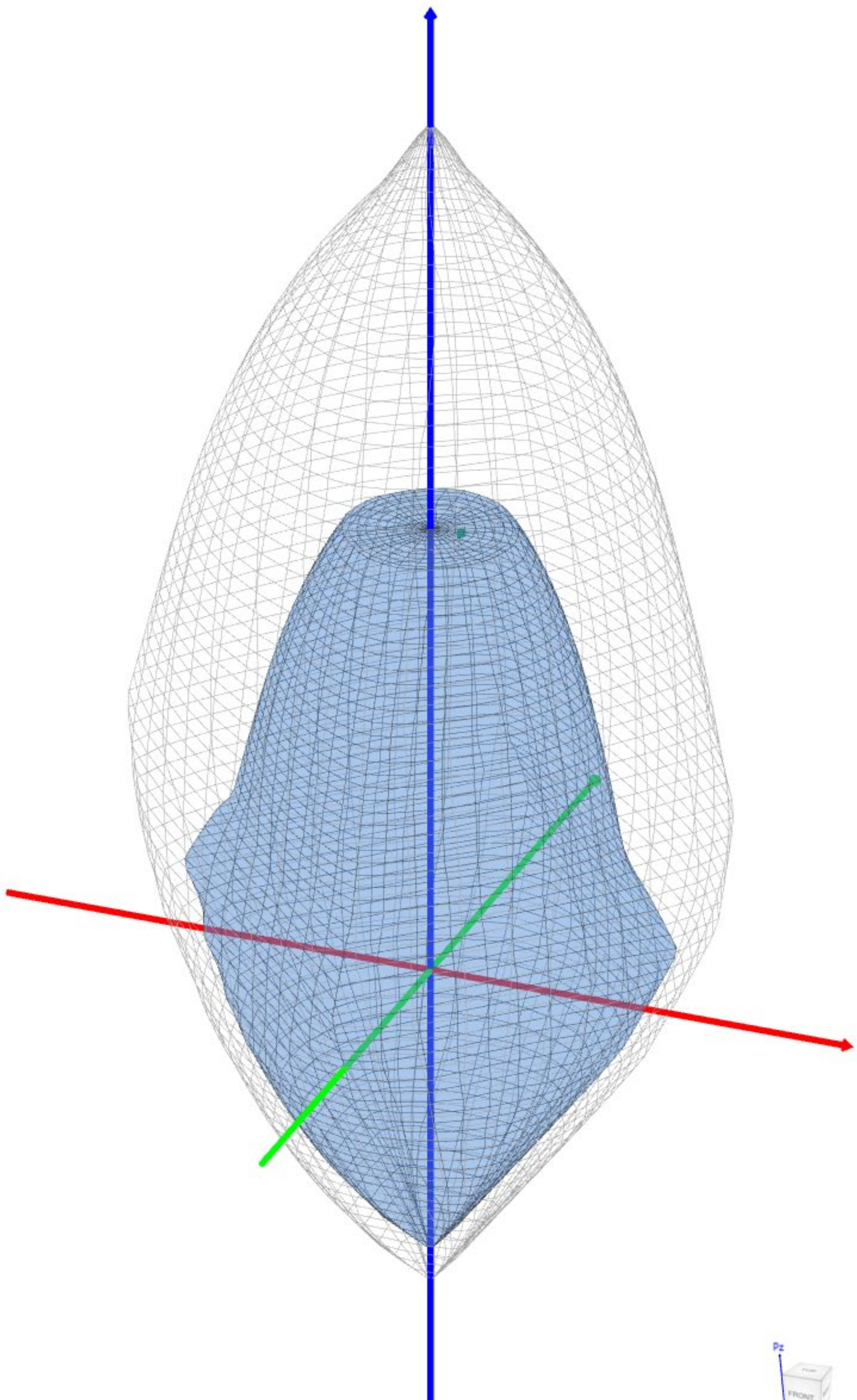
$$A_{Tributary} = \left(\frac{12}{2}+\frac{20/2}{12}\right)\times\left(\frac{12}{2}+\frac{20/2}{12}\right) = 46.69 \text{ ft}^2$$

- P_u = 4 × q_u × $A_{Tributary}$ = 4 × (0.340 × 300.44 + 0.135 × 46.69) = 433.82 kips
- $M_{u,x}$ = 225.22 ft-kips (see the previous Table)
- $M_{u,y}$ = 225.22 ft-kips (see the previous Table)

The factored loads are then input into spColumn to construct the axial load – moment interaction diagram.

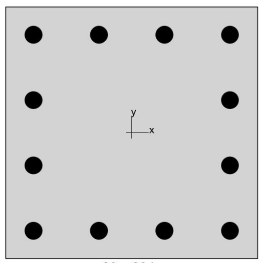
3.2. Moment Interaction Diagram

Interior Column:

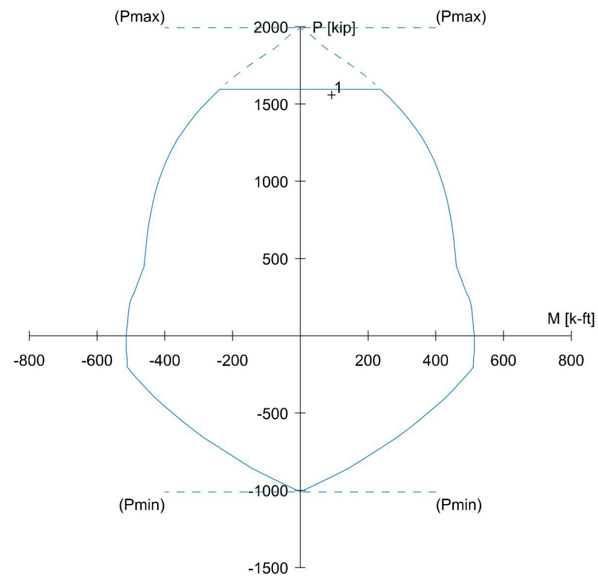


STRUCTUREPOINT - spColumn v10.10 (TM)
Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3
F:\StructurePoint\spColumn\Interior Column.colx

Page | 1
2/9/2024
3:04 PM



20 x 20 in



PM at 45.0 [deg]

General Information	
Project	Waffle Slab
Column	Interior
Engineer	StructurePoint
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials	
f'_c	6 ksi
E_c	4415.21 ksi
f_y	60 ksi
E_s	29000 ksi

Section	
Type	Rectangular
Width	20 in
Depth	20 in
A_g	400 in^2
I_x	13333.3 in^4
I_y	13333.3 in^4

Reinforcement	
Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	1.5 in
Bars	12 #11
Confinement type	Tied
Total steel area, A_s	18.72 in^2
Rho	4.68 %
Min. clear spacing	3.79 in

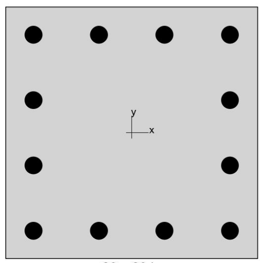
No.	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	Capacity
	kip	k-ft	k-ft	kip	k-ft	k-ft	Ratio
1	1558.8	65.3	65.3	1558.80	180.38	180.38	0.36

Max. Capacity Ratio: 0.36

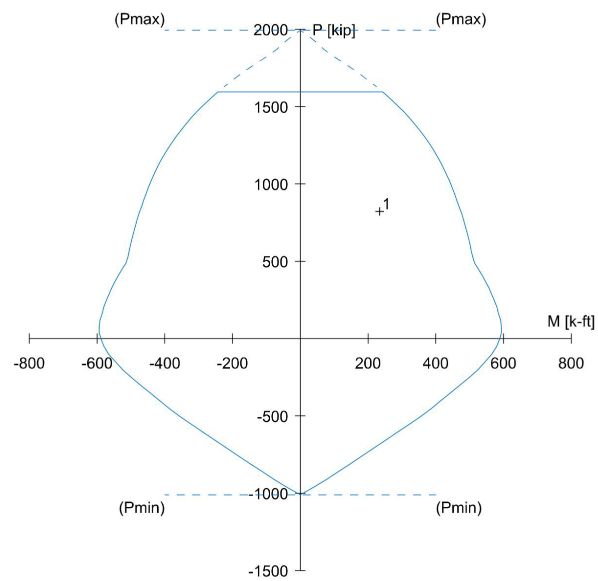
Edge Column:

STRUCTUREPOINT - spColumn v10.10 (TM)
Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3
F:\StructurePoint\spColumn\Exterior Column.colx

Page | 1
2/9/2024
3:06 PM



20 x 20 in



PM at 16.0 [deg]

General Information

Project	Waffle Slab
Column	Exterior
Engineer	StructurePoint
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials

f'_c	6 ksi
E_c	4415.21 ksi
f_y	60 ksi
E_s	29000 ksi

Section

Type	Rectangular
Width	20 in
Depth	20 in
A_g	400 in²
I_x	13333.3 in⁴
I_y	13333.3 in⁴

Reinforcement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudinal bars
Clear cover	1.5 in
Bars	12 #11
Confinement type	Tied
Total steel area, A_s	18.72 in²
Rho	4.68 %
Min. clear spacing	3.79 in

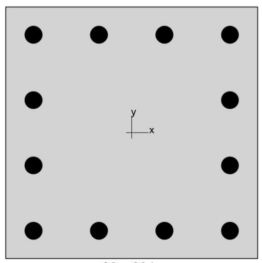
No.	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	Capacity
	kip	k-ft	k-ft	kip	k-ft	k-ft	Ratio
1	822.2	225.2	65.3	822.20	455.46	132.00	0.49

Max. Capacity Ratio: 0.49

Corner Column:

STRUCTUREPOINT - spColumn v10.10 (TM)
Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3
F:\StructurePoint\spColumn\Corner Column.colx

Page | 1
2/9/2024
3:07 PM



20 x 20 in

General Information

Project	Waffle Slab
Column	Corner
Engineer	StructurePoint
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials

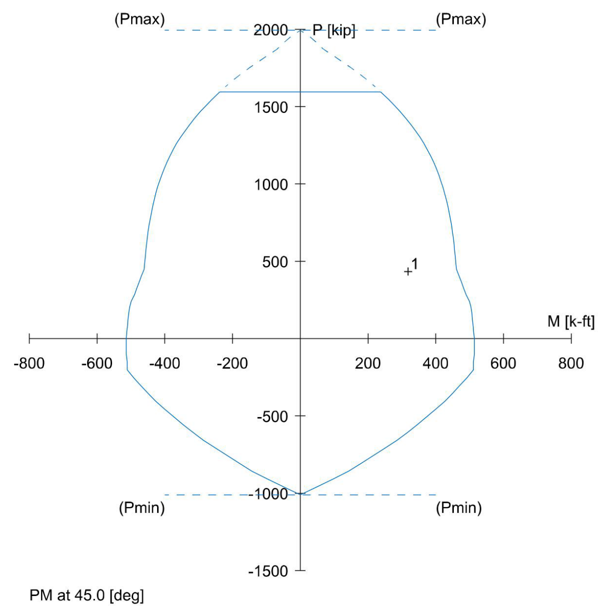
f'_c	6 ksi
E_c	4415.21 ksi
f_y	60 ksi
E_s	29000 ksi

Section

Type	Rectangular
Width	20 in
Depth	20 in
A_g	400 in²
I_x	13333.3 in⁴
I_y	13333.3 in⁴

Reinforcement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Longitudinal bars
Clear cover	1.5 in
Bars	12 #11
Confinement type	Tied
Total steel area, A_s	18.72 in²
Rho	4.68 %
Min. clear spacing	3.79 in



No.	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	Capacity
	kip	k-ft	k-ft	kip	k-ft	k-ft	Ratio
1	433.8	225.2	225.2	433.82	326.46	326.46	0.69

Max. Capacity Ratio: 0.69

4. Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with ACI 318 Chapter 22.

4.1. One-Way (Beam Action) Shear Strength

ACI 318-14 (22.5)

One-way shear is critical at a distance *d* from the face of the column as shown in Figure 3. Figure 24 and Figure 26 show the factored shear forces (V_u) at the critical sections around each column and each drop panel, respectively. In members without shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$\phi V_n = \phi V_c + \phi V_s = \phi V_c ,\ (\phi V_s = 0)$$ ***ACI 318-14 (Eq. 22.5.1.1)***

Where:

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f_c'} \times b_w \times d$$ ***ACI 318-14 (Eq. 22.5.5.1)***

One-way shear capacity is calculated assuming the shear cross-section area consisting of the drop panel (if any), the ribs, and the slab portion above them, decreased by concrete cover. For such section the equivalent shear width for single rib is calculated from the formula:

$$b_v = b + \frac{d}{12}$$ ***spSlab Software Manual (Eq. 2-13)***

Where:

b = rib width, in.

d = distance from extreme compression fiber to tension reinforcement centroid.

4.1.1. At Distance *d* from the Supporting Column

$$d = h - c_{clear} - \frac{d_b}{2} = 17.00 - 0.75 - \frac{0.75}{2} = 15.88 \text{ in.}$$ for middle span with #6 reinforcement.

$$b_v = 6.00 + \frac{15.88}{12} = 7.32 \text{ in.}$$

Where $\lambda = 1$ for normal weight concrete

$$b = l_{2,drop} + n_{ribs} \times b_v = 12 \times 12 + 7 \times 7.32 = 195.26 \text{ in.}$$ (see the following Figure)

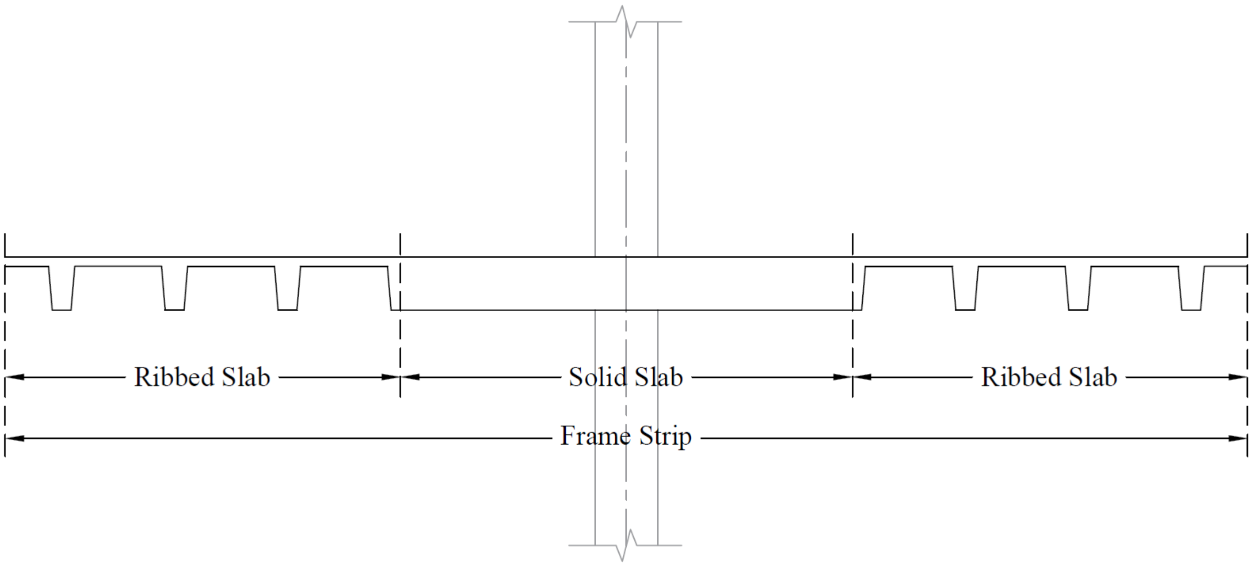


Figure 23 – Frame Strip Cross Section (at Distance *d* from the Face of the Supporting Column)

The one-way shear capacity for the ribbed slab portions shown in Figure 23 is permitted to be increased by 10%. ***ACI 318-14 (9.8.1.5)***

$$\phi V_c = \left(\phi V_c\right)_{\text{Solid Slab}} + 1.10 \times \left(\phi V_c\right)_{\text{Ribbed Slab}}$$

$$\phi V_c = 0.75 \times 2.0 \times 1.0 \times \frac{\sqrt{5,000}}{1,000} \times (12 \times 12) \times 15.88 + 1.10 \times 0.75 \times 2.0 \times 1.0 \times \frac{\sqrt{5,000}}{1,000} \times (7 \times 7.32) \times 15.88$$

$$\phi V_c = 337.41 \text{ kips}$$

Because $\phi V_c \geq V_u$ at all the critical sections, the slab has adequate one-way shear strength.

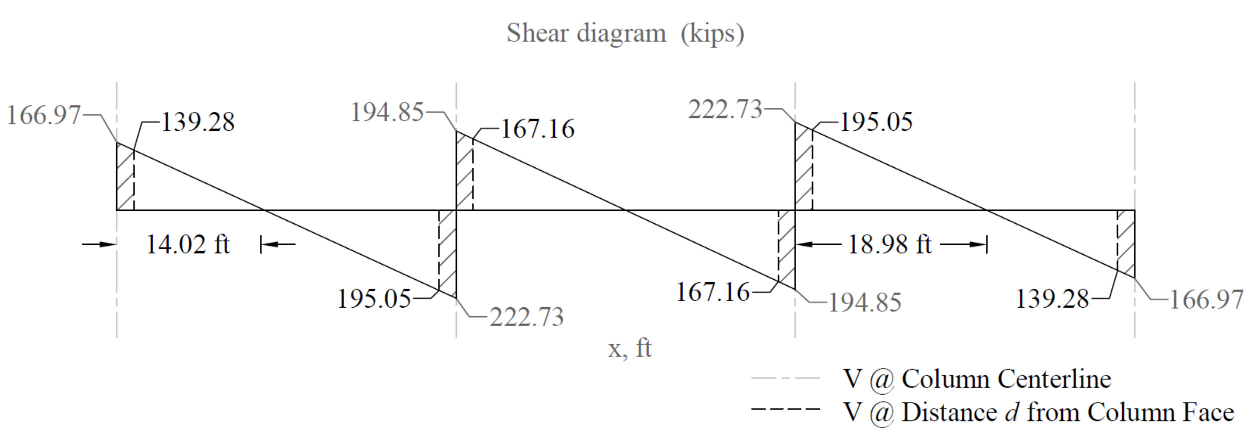


Figure 24 – One-way Shear at Critical Sections (at Distance *d* from the Face of the Supporting Column)

4.1.2. At the Face of the Drop Panel

$$d = h - c_{clear} - \frac{d_b}{2} = 17.00 - 0.75 - \frac{0.75}{2} = 15.88 \text{ in. for middle span with \#6 reinforcement.}$$

$$b_v = 6.00 + \frac{15.88}{12} = 7.32 \text{ in.}$$

Where $\lambda = 1$ for normal weight concrete

$b = n_{ribs} \times b_v = 11 \times 7.32 = 80.55$ in. (see the following Figure)

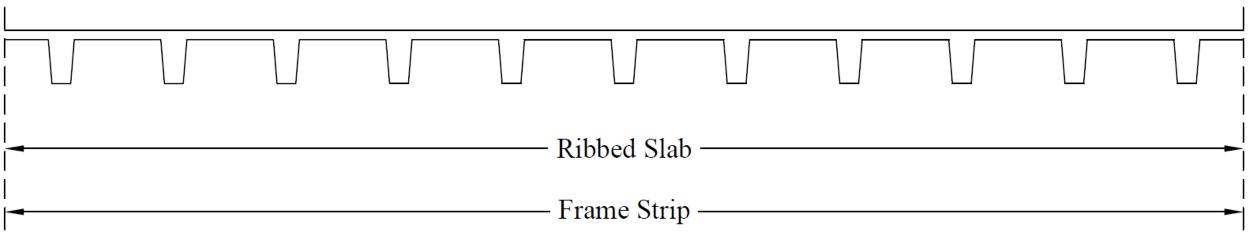


Figure 25 – Frame Strip Cross Section (at Distance *d* from the Face of the Drop Panel)

The one-way shear capacity for the ribbed slab portions shown in Figure 25 is permitted to be increased by 10%. ***ACI 318-14 (9.8.1.5)***

$$\phi V_c = 1.10 \times (\phi V_c)_{\text{Ribbed Slab}} = 1.10 \times 0.75 \times 2.0 \times 1.0 \times \frac{\sqrt{5,000}}{1,000} \times (11 \times 7.32) \times 15.88 = 149.20 \text{ kips}$$

Because $\phi V_c \geq V_u$ at all the critical sections, the slab has adequate one-way shear strength.

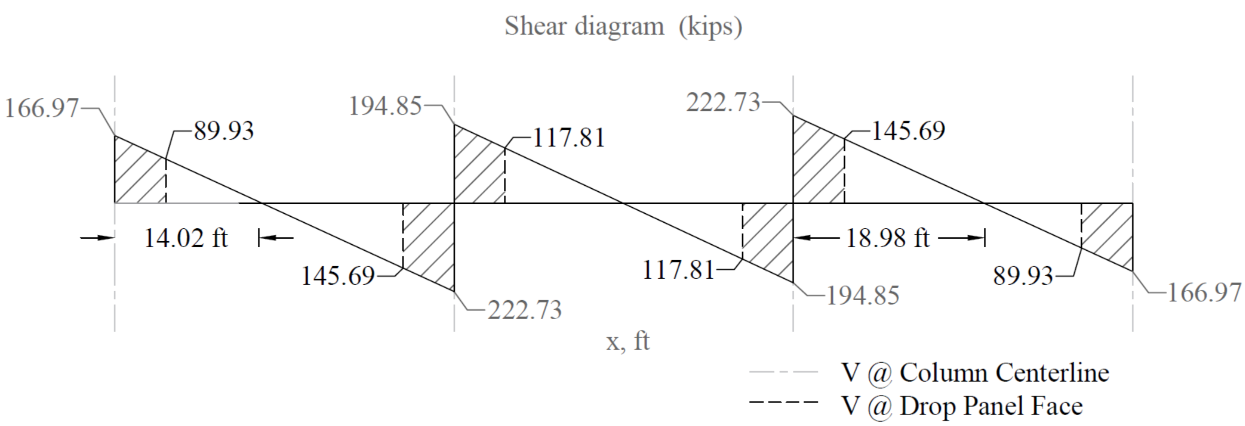


Figure 26 – One-Way Shear at Critical Sections (at the Face of the Drop Panel)

4.2. Two-Way (Punching) Shear Strength

ACI 318-14 (22.6)

4.2.1. Around the Columns Faces

Two-way shear is critical on a rectangular section located at *d*/2 away from the face of the column as shown in Figure 20.

a) Exterior column:

The factored shear force (V_u) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section (*d/2* away from column face).

$$V_u = V - q_u \times (b_1 \times b_2) = 166.97 - 0.475 \times \left(\frac{27.94 \times 35.88}{144}\right) = 163.66 \text{ kips}$$

The factored unbalanced moment used for shear transfer, M_{unb}, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$M_{unb} = M - V_u \times \left(b_1 - c_{AB} - \frac{c_1}{2}\right) = 462.75 - 163.66 \times \left(\frac{27.94 - 8.51 - \frac{20}{2}}{12}\right) = 334.13 \text{ ft-kips}$$

For the exterior column in Figure 18 the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{b_1^2}{2 \times b_1 + b_2} = \frac{27.94^2}{2 \times 27.94 + 35.88} = 8.51 \text{ in.}$$

Where:

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{15.88}{2} = 27.94 \text{ in.} \qquad b_2 = c_2 + d = 20 + 15.88 = 35.88 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2 \times \left(\frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 \times d \times c_{AB}^2$$

$$J_c = 2 \times \left(\frac{27.94 \times 15.88^3}{12} + \frac{15.88 \times 27.94^3}{12} + (27.94 \times 15.88) \times \left(\frac{27.94}{2} - 8.51\right)^2\right) + 35.88 \times 15.88 \times 8.51^2$$

$$J_c = 143{,}997.36 \text{ in.}^4$$

$\gamma_v = 1 - \gamma_f = 1 - 0.630 = 0.370$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

Where:

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{b_1}{b_2}}}$$ ***ACI 318-14 (8.4.2.3.2)***

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{27.94}{35.88}}} = 0.630$$

The length of the critical perimeter for the exterior column:

$$b_o = 2 \times b_1 + b_2 = 2 \times 27.94 + 35.88 = 91.75 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{163.66 \times 1,000}{91.75 \times 15.88} + \frac{0.370 \times (334.13 \times 12 \times 1,000) \times 8.51}{143,997.36} = 112.36 + 87.74 = 200.10 \text{ psi}$$

$$v_c = \min \begin{Bmatrix} 4 \times \lambda \times \sqrt{f'_c} \\ \left(2 + \frac{4}{\beta}\right) \times \lambda \times \sqrt{f'_c} \\ \left(\frac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f'_c} \end{Bmatrix}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min \begin{Bmatrix} 4 \times 1 \times \sqrt{5,000} \\ \left(2 + \frac{4}{1}\right) \times 1 \times \sqrt{5,000} \\ \left(\frac{30 \times 15.88}{91.75} + 2\right) \times 1 \times \sqrt{5,000} \end{Bmatrix} = \min \begin{Bmatrix} 282.84 \\ 424.26 \\ 508.46 \end{Bmatrix} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75 \times 282.84 = 212.13 \text{ psi}$$

Because $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

b) <u>Interior column:</u>

$$V_u = V - q_u \times (b_1 \times b_2) = (222.73 + 194.85) - 0.475 \times \left(\frac{35.88 \times 35.88}{144}\right) = 413.34 \text{ kips}$$

$$M_{unb} = M - V_u \times \left(b_1 - c_{AB} - \frac{c_1}{2}\right) = (1,382.84 - 1,248.72) - 413.34 \times (0) = 134.12 \text{ ft-kips}$$

For the interior column in Figure 18, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{35.88}{2} = 17.94 \text{ in.}$$

Where:

$b_1 = c_1 + d = 20 + 15.88 = 35.88 \text{ in.}$ $\qquad$ $b_2 = c_2 + d = 20 + 15.88 = 35.88 \text{ in.}$

The polar moment J_c of the shear perimeter is:

$$J_c = 2 \times \left(\frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left(\frac{b_1}{2} - c_{AB}\right)^2\right) + 2 \times b_2 \times d \times c_{AB}{}^2$$

$$J_c = 2 \times \left(\frac{35.88 \times 15.88^3}{12} + \frac{15.88 \times 35.88^3}{12} + (35.88 \times 15.88) \times (0)^2\right) + 2 \times 35.88 \times 15.88 \times 17.94^2$$

$$J_c = 512,571.48 \text{ in.}^4$$

$\gamma_v = 1 - \gamma_f = 1 - 0.600 = 0.400$ **<u>*ACI 318-14 (Eq. 8.4.4.2.2)*</u>**

Where:

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{b_1}{b_2}}}$$ **<u>*ACI 318-14 (8.4.2.3.2)*</u>**

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \times \sqrt{\frac{35.88}{35.88}}} = 0.600$$

The length of the critical perimeter for the interior column:

$$b_o = 2 \times (b_1 + b_2) = 2 \times (35.88 + 35.88) = 143.50 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{413.34 \times 1,000}{143.50 \times 15.88} + \frac{0.400 \times (134.12 \times 12 \times 1,000) \times 17.94}{512,571.48} = 181.44 + 22.53 = 203.97 \text{ psi}$$

$$v_c = \min \begin{Bmatrix} 4 \times \lambda \times \sqrt{f'_c} \\ \left(2 + \frac{4}{\beta}\right) \times \lambda \times \sqrt{f'_c} \\ \left(\frac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f'_c} \end{Bmatrix}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min \begin{Bmatrix} 4 \times 1 \times \sqrt{5,000} \\ \left(2 + \frac{4}{1}\right) \times 1 \times \sqrt{5,000} \\ \left(\frac{40 \times 15.88}{143.50} + 2\right) \times 1 \times \sqrt{5,000} \end{Bmatrix} = \min \begin{Bmatrix} 282.84 \\ 424.26 \\ 454.32 \end{Bmatrix} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75 \times 282.84 = 212.13 \text{ psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

c) **Corner column:**

In this example, interior equivalent frame strip was selected where it only has exterior and interior supports (no corner supports are included in this strip). However, the two-way shear strength of corner supports usually governs. Thus, the two-way shear strength for the corner column in this example will be checked for educational purposes. Same procedure is used to find the reaction and factored unbalanced moment used for shear transfer at the centroid of the critical section for the corner support for the exterior equivalent frame strip.

$$V_u = V - q_u \times (b_1 \times b_2) = 94.32 - 0.475 \times \left(\frac{27.94 \times 27.94}{144}\right) = 91.75 \text{ kips}$$

$$M_{unb} = M - V_u \times \left(b_1 - c_{AB} - \frac{c_1}{2}\right) = 265.55 - 91.75 \times \left(\frac{27.94 - 6.98 - \frac{20}{2}}{12}\right) = 181.81 \text{ ft-kips}$$

For the corner column in Figure 18, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{b_1^2}{2 \times (b_1 + b_2)} = \frac{27.94^2}{2 \times (27.94 + 27.94)} = 6.98 \text{ in.}$$

Where:

$$b_1 = c_1 + \frac{d}{2} = 20 + \frac{15.88}{2} = 27.94 \text{ in.} \qquad b_2 = c_2 + \frac{d}{2} = 20 + \frac{15.88}{2} = 27.94 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = \left(\frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 \times d \times c_{AB}^2$$

$$J_c = \left(\frac{27.94 \times 15.88^3}{12} + \frac{15.88 \times 27.94^3}{12} + (27.94 \times 15.88) \times \left(\frac{27.94}{2} - 6.98\right)^2\right) + 27.94 \times 15.88 \times 6.98^2$$

$$J_c = 81{,}430.82 \text{ in.}^4$$

$\gamma_v = 1 - \gamma_f = 1 - 0.600 = 0.400$ ***ACI 318-14 (Eq. 8.4.4.2.2)***

Where:

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{b_1}{b_2}}}$$ ***ACI 318-14 (8.4.2.3.2)***

$$\gamma_f = \frac{1}{1+\frac{2}{3}\times\sqrt{\frac{27.94}{27.94}}} = 0.600$$

The length of the critical perimeter for the corner column:

$$b_o = b_1 + b_2 = 27.94 + 27.94 = 55.88 \text{ in.}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d} + \frac{\gamma_v \times M_{unb} \times c_{AB}}{J_c}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{91.75\times 1,000}{55.88\times 15.88} + \frac{0.400\times(181.81\times 12\times 1,000)\times 6.98}{81,430.82} = 103.43 + 74.85 = 178.28 \text{ psi}$$

$$v_c = \min\begin{Bmatrix} 4\times\lambda\times\sqrt{f'_c} \\ \left(2+\frac{4}{\beta}\right)\times\lambda\times\sqrt{f'_c} \\ \left(\frac{\alpha_s\times d}{b_o}+2\right)\times\lambda\times\sqrt{f'_c} \end{Bmatrix}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min\begin{Bmatrix} 4\times 1\times\sqrt{5,000} \\ \left(2+\frac{4}{1}\right)\times 1\times\sqrt{5,000} \\ \left(\frac{20\times 15.88}{55.88}+2\right)\times 1\times\sqrt{5,000} \end{Bmatrix} = \min\begin{Bmatrix} 282.84 \\ 424.26 \\ 543.22 \end{Bmatrix} = 282.84 \text{ psi}$$

$$\phi v_c = 0.75\times 282.84 = 212.13 \text{ psi}$$

Since $\phi v_c > v_u$ at the critical section, the slab has adequate two-way shear strength at this joint.

4.2.2. Around Drop Panels

Two-way shear is critical on a rectangular section located at $d/2$ away from the face of the drop panel.

The factored shear force (V_u) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section ($d/2$ away from column face).

Note: For simplicity, it is conservative to deduct only the self-weight of the slab and joists in the critical section from the shear reaction in punching shear calculations. This approach is also adopted in the spSlab program for the punching shear check around the drop panels.

a) Exterior drop panel:

$$V_u = V - q_u \times (b_1 \times b_2) = 166.97 - 0.340 \times \left(\frac{89.94 \times 159.88}{144}\right) = 133.02 \text{ kips}$$

d value is used in the calculation of v_u is given by (see the following Figure).

$$d = \frac{(\text{\# of ribs within the drop panel width}) \times h \times b_v}{\text{the drop panel width}}$$ ***spSlab Software Manual (Eq. 2-14)***

$$d = \frac{4 \times 17 \times 7.32}{150} = 3.32 \text{ in.}$$

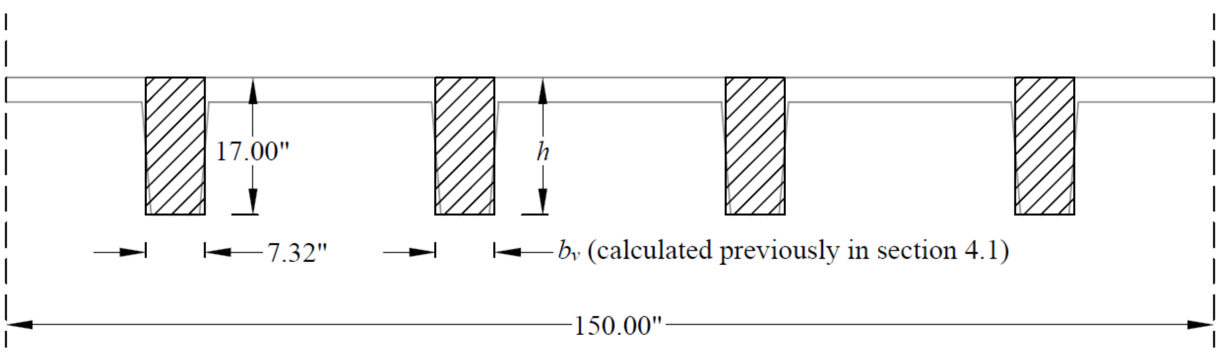


Figure 27 – Equivalent Thickness based on Shear Area Calculation)

The length of the critical perimeter for the exterior drop panel:

$$b_o = 2 \times 89.94 + 159.88 = 339.75 \text{ in.}$$

Where:

$$b_1 = \frac{c_1}{2} + \frac{d}{2} + \frac{l_{1,dp}}{2} = \frac{20}{2} + \frac{15.88}{2} + \frac{12 \times 12}{2} = 89.94 \text{ in.} \qquad b_2 = d + l_{2,dp} = 15.88 + 12 \times 12 = 159.88 \text{ in.}$$

$$c_{AB}=\frac{b_1^{\ 2}}{2\times b_1+b_2}=\frac{89.94^2}{2\times 89.94+159.88}=23.81\text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c=2\times\left(\frac{b_1\times d^3}{12}+\frac{d\times b_1^{\ 3}}{12}+\left(b_1\times d\right)\left(\frac{b_1}{2}-c_{AB}\right)^2\right)+b_2\times d\times c_{AB}^{\ \ 2}$$

$$J_c=2\times\left(\frac{89.94\times 3.32^3}{12}+\frac{3.32\times 89.94^3}{12}+\left(89.94\times 3.32\right)\times\left(\frac{89.94}{2}-23.81\right)^2\right)+159.88\times 3.32\times 23.81^2$$

$$J_c=971{,}273.30\text{ in.}^4$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u=\frac{V_u}{b_o\times d}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u=\frac{133.02\times 1{,}000}{339.75\times 3.32}=117.94\text{ psi}$$

The two-way shear capacity for the ribbed slab is permitted to be increased by 10%.

ACI 318-14 (9.8.1.5)

$$v_c=\min\left\{\begin{array}{l}1.10\times 4\times\lambda\times\sqrt{f_c'}\\ 1.10\times\left(2+\frac{4}{\beta}\right)\times\lambda\times\sqrt{f_c'}\\ 1.10\times\left(\frac{\alpha_s\times d}{b_o}+2\right)\times\lambda\times\sqrt{f_c'}\end{array}\right\}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c=\min\left\{\begin{array}{l}1.10\times 4\times 1\times\sqrt{5{,}000}\\ 1.10\times\left(2+\frac{4}{1}\right)\times 1\times\sqrt{5{,}000}\\ 1.10\times\left(\frac{30\times 3.32}{339.75}+2\right)\times 1\times\sqrt{5{,}000}\end{array}\right\}=\min\left\{\begin{array}{l}311.13\\ 466.69\\ 178.36\end{array}\right\}=178.36\text{ psi}$$

$$\phi v_c=0.75\times 178.36=133.77\text{ psi}$$

In waffle slab design where the drop panels create a large critical shear perimeter, the factor (b_o/d) has limited contribution and is traditionally neglected for simplicity and conservatism. This approach is adopted in this calculation and in the spSlab program (spSlab software manual, Eq. 2-46).

The two-way shear capacity for the ribbed slab is permitted to be increased by 10%.

ACI 318-14 (9.8.1.5)

$$v_c = 1.10 \times 2 \times \lambda \times \sqrt{f'_c}$$ ***spSlab Software Manual (Eq. 2-46)***

$$v_c = 1.10 \times 2 \times 1 \times \sqrt{5{,}000} = 155.56 \text{ psi}$$

$$\phi v_c = 0.75 \times 155.56 = 116.67 \text{ psi}$$

Since $\phi v_c < v_u$ at the critical section, the slab does not have adequate two-way shear strength around this drop panel.

b) Interior drop panel:

$$V_u = V - q_u \times (b_l \times b_2) = 222.73 + 194.85 - 0.340 \times \left(\frac{159.88 \times 159.88}{144}\right) = 357.23 \text{ kips}$$

The length of the critical perimeter for the interior drop panel:

$$b_o = 2 \times (159.88 + 159.88) = 639.50 \text{ in.}$$

Where:

$$b_1 = d + l_{1,dp} = 15.88 + 12 \times 12 = 159.88 \text{ in.} \qquad b_2 = d + l_{2,dp} = 15.88 + 12 \times 12 = 159.88 \text{ in.}$$

$$c_{AB} = \frac{b_1}{2} = \frac{159.88}{2} = 79.94 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2 \times \left(\frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left(\frac{b_1}{2} - c_{AB}\right)^2\right) + 2 \times b_2 \times d \times c_{AB}{}^2$$

$$J_c = 2 \times \left(\frac{159.88 \times 3.32^3}{12} + \frac{3.32 \times 159.88^3}{12} + (159.88 \times 3.32) \times (0)^2\right) + 2 \times 159.88 \times 3.32 \times 79.94^2$$

$$J_c = 9{,}044{,}800.03 \text{ in.}^4$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{357.23 \times 1{,}000}{639.50 \times 3.32} = 168.27 \text{ psi}$$

The two-way shear capacity for the ribbed slab is permitted to be increased by 10%.
ACI 318-14 (9.8.1.5)

$$v_c = \min\begin{Bmatrix} 1.10\times4\times\lambda\times\sqrt{f'_c} \\ 1.10\times\left(2+\dfrac{4}{\beta}\right)\times\lambda\times\sqrt{f'_c} \\ 1.10\times\left(\dfrac{\alpha_s\times d}{b_o}+2\right)\times\lambda\times\sqrt{f'_c} \end{Bmatrix}$$ **_ACI 318-14 (Table 22.6.5.2)_**

$$v_c = \min\begin{Bmatrix} 1.10\times4\times1\times\sqrt{5,000} \\ 1.10\times\left(2+\dfrac{4}{1}\right)\times1\times\sqrt{5,000} \\ 1.10\times\left(\dfrac{40\times3.32}{639.50}+2\right)\times1\times\sqrt{5,000} \end{Bmatrix} = \min\begin{Bmatrix} 311.13 \\ 466.69 \\ 171.71 \end{Bmatrix} = 171.71 \text{ psi}$$

$$\phi v_c = 0.75\times171.71 = 128.79 \text{ psi}$$

$$v_c = 1.10\times2\times\lambda\times\sqrt{f'_c}$$ **_spSlab Software Manual (Eq. 2-46)_**

$$v_c = 1.10\times2\times1\times\sqrt{5,000} = 155.56 \text{ psi}$$

$$\phi v_c = 0.75\times155.56 = 116.67 \text{ psi}$$

Since $\phi v_c < v_u$ at the critical section, the slab does not have adequate two-way shear strength around this drop panel.

c) Corner drop panel:

$$V_u = V - q_u \times (b_l \times b_2) = 94.32 - 0.340 \times \left(\frac{89.94 \times 89.94}{144}\right) = 75.22 \text{ kips}$$

The length of the critical perimeter for the corner drop panel:

$$b_o = 89.94 + 89.94 = 179.88 \text{ in.}$$

Where:

$$b_1 = \frac{l_{1,dp}}{2} + \frac{d}{2} + \frac{c_1}{2} = \frac{12 \times 12}{2} + \frac{15.88}{2} + \frac{20}{2} = 89.94 \text{ in.} \qquad b_2 = \frac{l_{2,dp}}{2} + \frac{d}{2} + \frac{c_2}{2} = \frac{12 \times 12}{2} + \frac{15.88}{2} + \frac{20}{2} = 89.94 \text{ in.}$$

$$c_{AB} = \frac{b_1^2}{2 \times (b_1 + b_2)} = \frac{89.94^2}{2 \times (89.94 + 89.94)} = 22.48 \text{ in.}$$

The polar moment J_c of the shear perimeter is:

$$J_c = \left(\frac{b_1 \times d^3}{12} + \frac{d \times b_1^3}{12} + (b_1 \times d) \times \left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 \times d \times c_{AB}^{\;2}$$

$$J_c = \left(\frac{89.94 \times 3.32^3}{12} + \frac{3.32 \times 89.94^3}{12} + (89.94 \times 3.32) \times \left(\frac{89.94}{2} - 22.48\right)^2\right) + 89.94 \times 3.32 \times 22.48^2$$

$$J_c = 503{,}407.36 \text{ in.}^4$$

The two-way shear stress (v_u) can then be calculated as:

$$v_u = \frac{V_u}{b_o \times d}$$ ***ACI 318-14 (R.8.4.4.2.3)***

$$v_u = \frac{75.22 \times 1{,}000}{179.88 \times 3.32} = 125.97 \text{ psi}$$

The two-way shear capacity for the ribbed slab is permitted to be increased by 10%.

ACI 318-14 (9.8.1.5)

$$v_c = \min \begin{Bmatrix} 1.10 \times 4 \times \lambda \times \sqrt{f_c'} \\ 1.10 \times \left(2 + \dfrac{4}{\beta}\right) \times \lambda \times \sqrt{f_c'} \\ 1.10 \times \left(\dfrac{\alpha_s \times d}{b_o} + 2\right) \times \lambda \times \sqrt{f_c'} \end{Bmatrix}$$ ***ACI 318-14 (Table 22.6.5.2)***

$$v_c = \min \begin{Bmatrix} 1.10 \times 4 \times 1 \times \sqrt{5,000} \\ 1.10 \times \left(2 + \dfrac{4}{1}\right) \times 1 \times \sqrt{5,000} \\ 1.10 \times \left(\dfrac{20 \times 3.32}{179.88} + 2\right) \times 1 \times \sqrt{5,000} \end{Bmatrix} = \min \begin{Bmatrix} 311.13 \\ 466.69 \\ 184.27 \end{Bmatrix} = 184.27 \text{ psi}$$

$$\phi v_c = 0.75 \times 184.27 = 138.21 \text{ psi}$$

$$v_c = 1.10 \times 2 \times \lambda \times \sqrt{f_c'}$$ ***spSlab Software Manual (Eq. 2-46)***

$$v_c = 1.10 \times 2 \times 1 \times \sqrt{5,000} = 155.56 \text{ psi}$$

$$\phi v_c = 0.75 \times 155.56 = 116.67 \text{ psi}$$

Since $\phi v_c < v_u$ at the critical section, the slab does not have adequate two-way shear strength around this drop panel.

To mitigate the deficiency in two-way shear capacity an evaluation of possible options is required:

1. Increase the thickness of the slab system
2. Increasing the dimensions of the drop panels (length and/or width)
3. Increasing the concrete strength
4. Reduction of the applied loads
5. Reduction of the panel spans
6. Using less conservative punching shear allowable (gain of 5-10%)
7. Refine the deduction of drop panel weight from the shear reaction (gain of 2-5%)

This example will be continued without the required modification discussed above to continue the illustration of the analysis and design procedure.

5. Serviceability Requirements (Deflection Check)

Since the slab thickness was selected to meet the minimum slab thickness tables in ACI 318-14, the deflection calculations of immediate and time-dependent deflections are not required. They are shown below for illustration purposes and comparison with spSlab software results.

5.1. Immediate (Instantaneous) Deflections

The calculation of deflections for two-way slabs is challenging even if linear elastic behavior can be assumed. Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D + L_{Full}$) is used to obtain immediate deflections of the two-way slab in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. ***ACI 318-14 (24.2.3)***

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the slab. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr} \leq I_g$$ ***ACI 318-14 (Eq. 24.2.3.5a)***

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously in this document. These moments are shown in Figure 28.

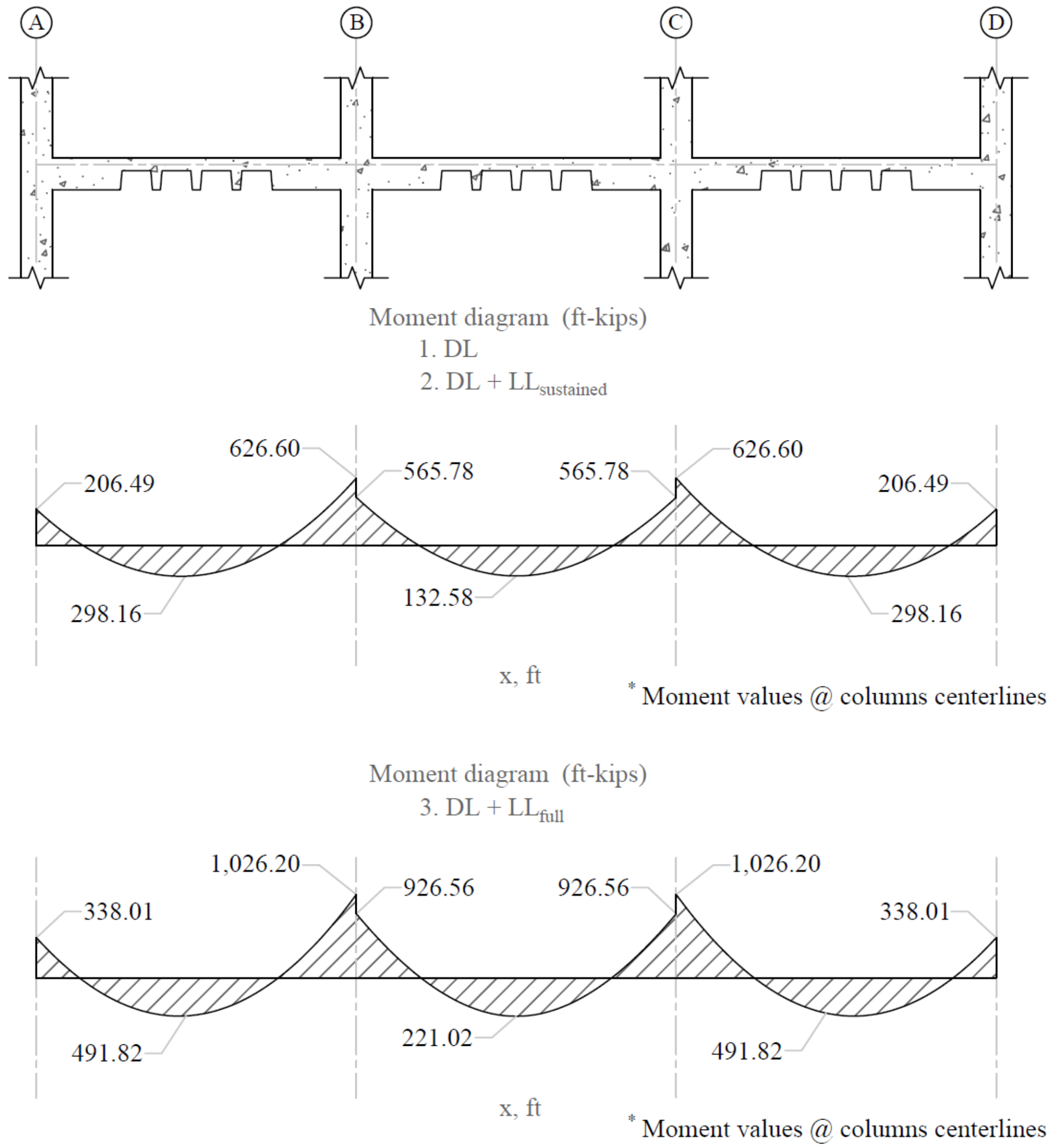


Figure 28 – Maximum Moments for the Three Service Load Levels

For positive moment (midspan) section:

M_{cr} = cracking moment.

$$M_{cr} = \frac{f_r \times I_g}{y_t} = \frac{530.33 \times 60,254.73}{11.41} \times \frac{1}{12 \times 1,000} = 233.46 \text{ ft-kips}$$ ***ACI 318-14 (Eq. 24.2.3.5b)***

f_r = Modulus of rapture of concrete.

$$f_r = 7.5 \times \lambda \times \sqrt{f'_c} = 7.5 \times 1.0 \times \sqrt{5,000} = 530.33 \text{ psi}$$ ***ACI 318-14 (Eq. 19.2.3.1)***

I_g = Moment of inertia of the gross uncracked concrete section. See the following Figure.

$$I_g = I_{g/rib} \times \#\ of\ ribs = 5{,}477.70 \times 11 = 60{,}254.73\text{ in.}^2$$

y_t = Distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.

$$y_t = h_{rib} - y_{bar} = 17.00 - 5.59 = 11.41\text{ in.}$$

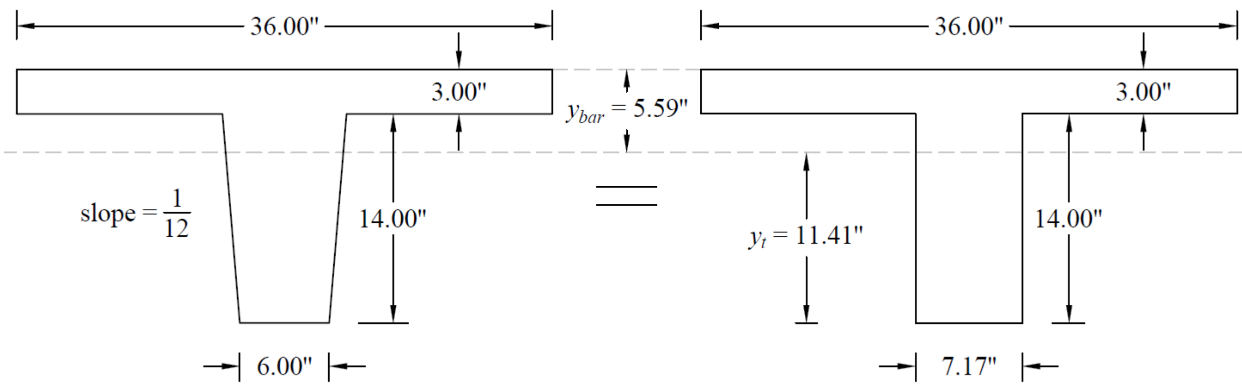


Figure 29 – Equivalent Gross Section for One Rib – Positive Moment Section

I_{cr} = moment of inertia of the cracked section transformed to concrete. ***PCA Notes on ACI 318-11 (9.5.2.2)***

As calculated previously, the positive reinforcement for the middle span frame strip is 22 #6 bars located at 1.125 in. along the section from the bottom of the slab. The Figure below shows all the parameters needed to calculate the moment of inertia of the cracked section transformed to concrete at midspan.

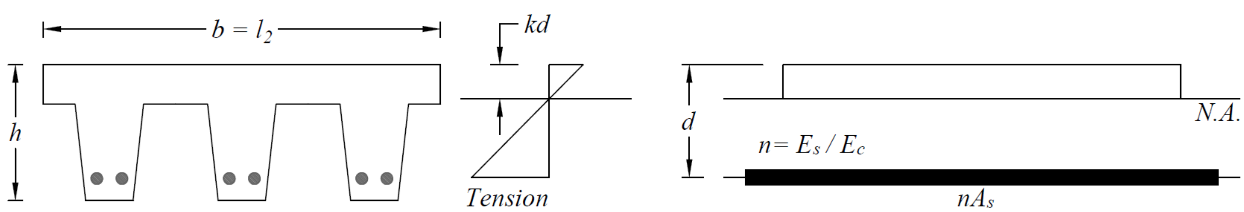


Figure 30 – Cracked Transformed Section – Positive Moment Section

E_{cs} = Modulus of elasticity of slab concrete.

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{5{,}000} = 4{,}287 \times 10^3\text{ psi}$$ ***ACI 318-14 (19.2.2.1.a)***

$$n = \frac{E_s}{E_{cs}} = \frac{29{,}000{,}000}{4{,}287{,}000} = 6.76$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$B = \frac{b}{n \times A_s} = \frac{33 \times 12}{6.76 \times (22 \times 0.44)} = 6.05\text{ in.}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd = \frac{\sqrt{2 \times d \times B + 1} - 1}{B} = \frac{\sqrt{2 \times 15.88 \times 6.05 + 1} - 1}{6.05} = 2.13 \text{ in.}$$ <u>***PCA Notes on ACI 318-11 (Table 10-2)***</u>

$$I_{cr} = \frac{b \times (kd)^3}{3} + n \times A_s \times (d - kd)^2$$ <u>***PCA Notes on ACI 318-11 (Table 10-2)***</u>

$$I_{cr} = \frac{33 \times 12 \times (2.13)^3}{3} + 6.76 \times (22 \times 0.44) \times (15.88 - 2.13)^2 = 13,646.72 \text{ in.}^4$$

<u>For negative moment section (near the interior support of the end span):</u>

The negative reinforcement for the end span frame strip near the interior support is 45 #6 bars located at 1.125 in. along the section from the top of the slab.

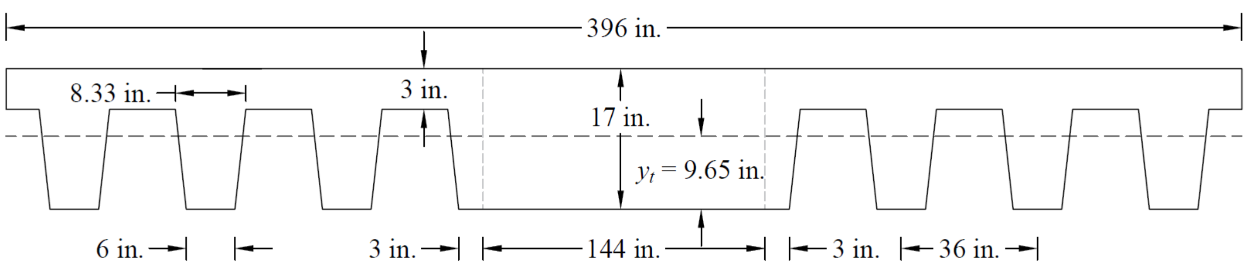
$$M_{cr} = \frac{f_r \times I_g}{y_t} = \frac{530.33 \times 103,622.30}{9.65} \times \frac{1}{12 \times 1,000} = 474.40 \text{ ft-kips}$$ <u>***ACI 318-14 (Eq. 24.2.3.5b)***</u>

$$f_r = 7.5 \times \lambda \times \sqrt{f'_c} = 7.5 \times 1.0 \times \sqrt{5,000} = 530.33 \text{ psi}$$ <u>***ACI 318-14 (Eq. 19.2.3.1)***</u>

$I_g = 103,622.30 \text{ in.}^2$ (See the <u>following Figure</u>)

Note: A lower value of I_g (60,254.73 in.^{4}) excluding the drop panel is conservatively adopted in calculating waffle slab deflection by the spSlab software.

$y_t = 9.65 \text{ in.}$



<u>Figure 31 – Gross Section – Negative Moment Section</u>

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f'_c} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$ <u>***ACI 318-14 (19.2.2.1.a)***</u>

$$n = \frac{E_s}{E_{cs}} = \frac{29,000,000}{4,287,000} = 6.76$$ <u>***PCA Notes on ACI 318-11 (Table 10-2)***</u>

$$B = \frac{b_{total}}{n \times A_s} = \frac{194.17}{6.76 \times (45 \times 0.44)} = 1.45 \text{ in.}^{-1}$$ <u>***PCA Notes on ACI 318-11 (Table 10-2)***</u>

Where $b_{total} = 144.00 + 7 \times 7.17 = 194.17$ in. (See Figure 31 and Figure 32)

$$kd = \frac{\sqrt{2 \times d \times B + 1} - 1}{B} = \frac{\sqrt{2 \times 15.88 \times 1.45 + 1} - 1}{1.45} = 4.04 \text{ in.}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{b_{total} \times (kd)^3}{3} + n \times A_s \times (d - kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{194.17 \times (4.04)^3}{3} + 6.76 \times (45 \times 0.44) \times (15.88 - 4.04)^2 = 23,028.31 \text{ in.}^4$$

Note: A lower value of I_{cr} (18,722.37 in.^{4}) excluding the drop panel is conservatively adopted in calculating waffle slab deflection by the spSlab software.

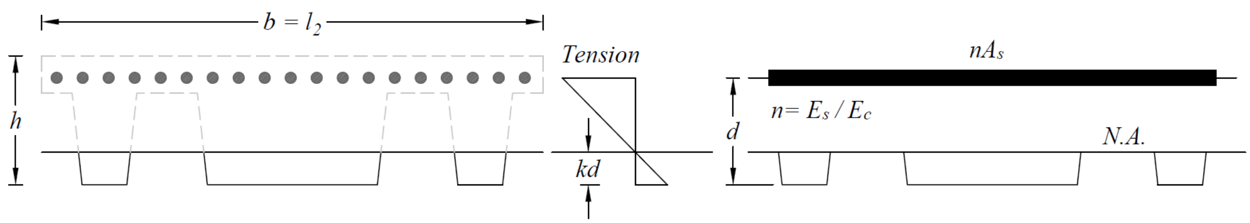


Figure 32 – Cracked Transformed Section – Negative Moment Section

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by Eq. (24.2.3.5a) unless obtained by a more comprehensive analysis.

I_e shall be permitted to be taken as the value obtained from Eq. (24.2.3.5a) at midspan for simple and continuous spans, and at the support for cantilevers. ***ACI 318-14 (24.2.3.7)***

For continuous one-way slabs and beams. I_e shall be permitted to be taken as the average of values obtained from Eq. (24.2.3.5a) for the critical positive and negative moment sections. ***ACI 318-14 (24.2.3.6)***

For the middle span (span with two ends continuous) with service load level ($D + LL_{full}$):

Since $M_{cr} = 474.40$ ft-kips < $M_a = 926.56$ ft-kips

$$I_e^- = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr}$$ ***ACI 318-14 (24.2.3.5a)***

Where I_e^- is the effective moment of inertia for the critical negative moment section (near the support).

$$I_e^- = \left(\frac{474.40}{926.56}\right)^3 \times 103{,}622.30 + \left[1 - \left(\frac{474.40}{926.56}\right)^3\right] \times 23{,}028.31 = 33{,}845.29 \text{ in.}^4$$

$$I_e^+ = I_g = 60{,}254.73 \text{ in.}^4 \text{ , since } M_{cr} = 233.46 \text{ ft-kips} > M_a = 221.02 \text{ ft-kips}$$

Where I_e^+ is the effective moment of inertia for the critical positive moment section (midspan).

Since midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan section is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. Both the midspan stiffness (I_e^+) and averaged span stiffness ($I_{e,avg}$) can be used in the calculation of immediate (instantaneous) deflection.

The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.70 \times I_e^+ + 0.15 \times \left(I_{e,l}^- + I_{e,r}^-\right) \text{ for interior span}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(2))***

$$I_{e,avg} = 0.85 \times I_e^+ + 0.15 \times I_e^- \text{ for end span}$$ ***PCA Notes on ACI 318-11 (9.5.2.4(1))***

However, these expressions lead to improved results only for continuous prismatic members. The drop panels in this example result in non-prismatic members and the following expressions are recommended according to ACI 318-89:

$$I_{e,avg} = 0.50 \times I_e^+ + 0.25 \times \left(I_{e,l}^- + I_{e,r}^-\right) \text{ for interior span}$$ ***ACI 435R-95 (2.14)***

For the middle span (span with two ends continuous) with service load level ($D + LL_{full}$):

$$I_{e,avg} = 0.50 \times 60{,}254.73 + 0.25 \times \left(33{,}845.29 + 33{,}845.29\right) = 47{,}050.01 \text{ in.}^4$$

$$I_{e,avg} = 0.50 \times I_e^+ + 0.50 \times I_e^- \text{ for end span}$$ ***ACI 435R-95 (2.14)***

For the end span (span with one end continuous) with service load level ($D + LL_{full}$):

$$I_{e,avg} = 0.50 \times 20{,}378.52 + 0.50 \times 30{,}990.47 = 25{,}684.49 \text{ in.}^4$$

Where:

- $I_{e,l}^-$ = The effective moment of inertia for the critical negative moment section near the left support.
- $I_{e,r}^-$ = The effective moment of inertia for the critical negative moment section near the right support.
- I_e^+ = The effective moment of inertia for the critical positive moment section (midspan).

Note: The prismatic member equations excluding the effect of the drop panel are conservatively adopted in calculating waffle slab deflection by spSlab.

The following Table provides a summary of the required parameters and calculated values needed for deflections for exterior and interior spans.

Table 6 - Averaged Effective Moment of Inertia Calculations													
For Frame Strip													
Span	zone	I_g (in.^{4})	I_{cr} (in.^{4})	M_a (ft-kips)			M_{cr} (k-ft)	I_e (in.^{4})			$I_{e,avg}$ (in.^{4})		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	103,622	15,504	206.49	206.49	338.01	474.40	103,622	103,622	103,622	47,520	47,520	25,684
	Midspan	60,255	15,603	298.16	298.16	491.82	233.46	37,037	37,037	20,379			
	Right	103,622	23,028	626.60	626.60	1,026.20	474.40	58,003	58,003	30,990			
Int	Left	103,622	23,028	565.78	565.78	926.56	474.40	70,538	70,538	33,845	65,396	65,396	47,050
	Midspan	60,255	13,647	132.58	132.58	221.02	233.46	60,255	60,255	60,255			
	Right	103,622	23,028	565.78	565.78	926.56	474.40	70,538	70,538	33,845			

Deflections in two-way slab systems shall be calculated taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. For immediate deflections in two-way slab systems, the midpanel deflection is computed as the sum of deflection at midspan of the column strip or column line in one direction (Δ_{cx} or Δ_{cy}) and deflection at midspan of the middle strip in the orthogonal direction (Δ_{mx} or Δ_{my}). Figure 33 shows the deflection computation for a rectangular panel. The average Δ for panels that have different properties in the two direction is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)

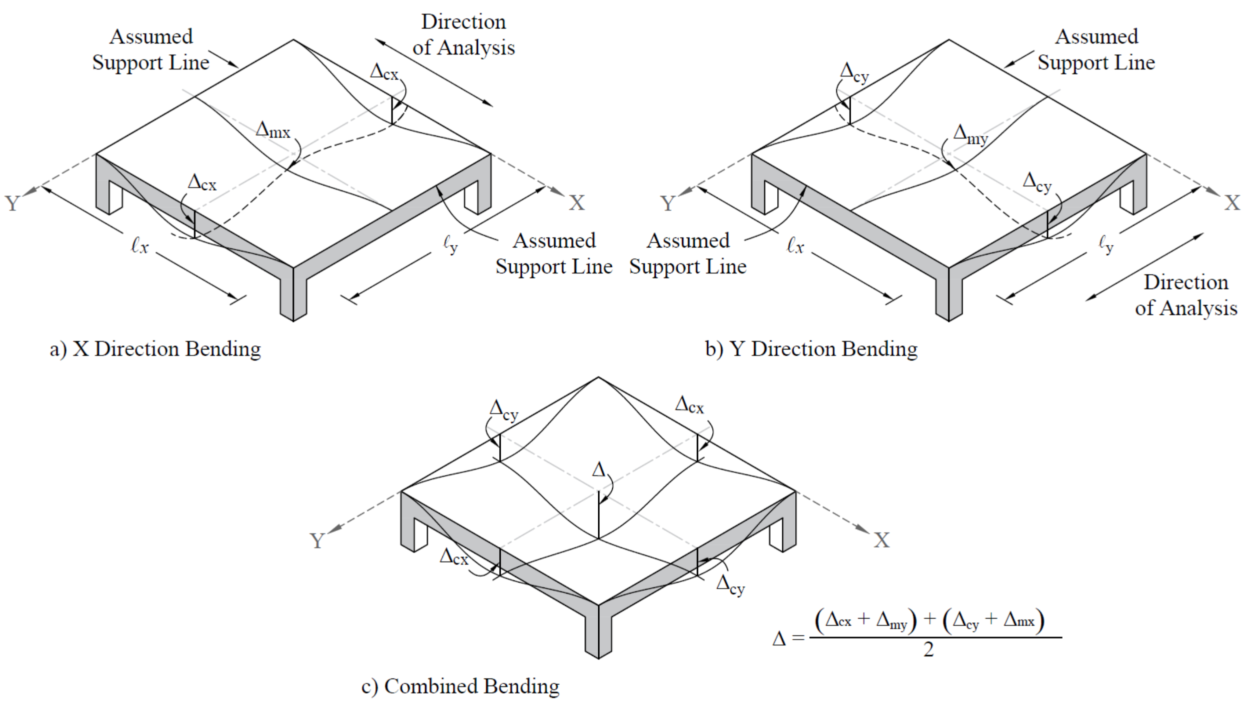


Figure 33 – Deflection Computation for a Rectangular Panel

To calculate each term of the previous equation, the following procedure should be used. Figure 34 shows the procedure of calculating the term Δ_{cx}. Same procedure can be used to find the other terms.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \left(\frac{I_{frame}}{I_c}\right)_g$$

$$LDF_c = \frac{LDF^+ + \dfrac{LDF_l^- + LDF_R^-}{2}}{2}$$

$$\Delta_{frame,fixed} = \frac{w \times l^4}{384 \times E_c \times I_{frame,averaged}}$$

$$\Delta\theta_{c,R} = \theta_{c,R} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame}$$

$$\theta_{c,R} = \frac{\left(M_{net,R}\right)_{frame}}{K_{ec}}$$

$$\Delta\theta_{c,L} = \theta_{c,L} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame}$$

$$\theta_{c,L} = \frac{\left(M_{net,L}\right)_{frame}}{K_{ec}}$$

Figure 34 – Δ_{cx} Calculation Procedure

For end span - service dead load case:

$$\Delta_{frame,fixed} = \frac{w \times l^4}{384 \times E_c \times I_{frame,averaged}}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 10)

Where:

Δ_{frame_fixed} = Deflection of frame strip assuming fixed-end condition.

$$w_{SDL+slab} = \left(50.00 + 150 \times \frac{8}{12}\right) \times 33 = 4,950.00 \ \frac{\text{lb}}{\text{ft}}$$

$$w_{drop\ panel} = \left(150 \times \frac{17}{12}\right) \times 12 = 2,550.00 \ \frac{\text{lb}}{\text{ft}}$$

$$w = \frac{4,950.00 \times (33-12) + (4,950.00 + 2,550.00) \times (12 - 20/12)}{(33 - 20/12)} = 5,790.96 \ \frac{\text{lb}}{\text{ft}}$$

$$E_{cs} = w_c^{1.5} \times 33 \times \sqrt{f_c'} = 150^{1.5} \times 33 \times \sqrt{5,000} = 4,287 \times 10^3 \text{ psi}$$

ACI 318-14 (19.2.2.1.a)

$I_{frame,averaged}$ = The averaged effective moment of inertia ($I_{e,avg}$) for the frame strip for service dead load case from Table 6 = 47,520.23 in.^{4}

$$\Delta_{frame,fixed} = \frac{5,790.96 \times (33 - 20/12)^4 \times 12^3}{384 \times (4,287 \times 10^3) \times 47,520.23} = 0.1233 \text{ in.}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \left(\frac{I_{frame}}{I_c}\right)_g$$ **PCA Notes on ACI 318-11 (9.5.3.4 Eq. 11)**

LDF_c is the load distribution factor for the column strip. The load distribution factor for the column strip can be found from the following equation:

$$LDF_c = \frac{LDF^+ + \frac{LDF_l^- + LDF_R^-}{2}}{2}$$ **spSlab Software Manual (Eq. 2-114)**

And the load distribution factor for the middle strip can be found from the following equation:

$$LDF_m = 1 - LDF_c$$ **spSlab Software Manual (Eq. 2-115)**

Taking for example the end span where highest deflections are expected, the LDF for exterior negative region (LDF_L^-), interior negative region (LDF_R^-), and positive region (LDF_L^+) are 1.00, 0.75, and 0.60, respectively (From Table 2 of this document). Thus, the load distribution factor for the column strip for the end span is given by:

$$LDF_c = \frac{0.6 + \frac{1.0 + 0.75}{2}}{2} = 0.738$$

$I_{c,g}$ = The gross moment of inertia (I_g) for the column strip for service dead load = 28,289.32 in.^{4}

$$\Delta_{c,fixed} = 0.738 \times 0.1233 \times \frac{60,254.73}{28,289.32} = 0.1937 \text{ in.}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$ **PCA Notes on ACI 318-11 (9.5.3.4 Eq. 12)**

Where:

$\theta_{c,L}$ = Rotation of the span left support

$(M_{net,L})_{frame}$ = 206.49 ft-kips = Net frame strip negative moment of the left support

K_{ec} = Effective column stiffness = 1,925.34 × 10^6 in.-lb (calculated above).

$$\theta_{c,L} = \frac{206.49 \times 12 \times 1,000}{1,925.34 \times 10^6} = 0.00129 \text{ rad}$$

$$\Delta\theta_{c,L} = \theta_{c,L} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame}$$ **PCA Notes on ACI 318-11 (9.5.3.4 Eq. 14)**

Where:

$\Delta\theta_{c,L}$ = Midspan deflection due to rotation of left support.

$(I_g / I_e)_{frame}$ = Gross-to-effective moment of inertia ratio for frame strip.

$$\Delta\theta_{c,L} = 0.00129 \times \frac{33 \times 12 - 20}{8} \times \frac{60,254.73}{47,520.23} = 0.0767 \text{ in.}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}} = \frac{(626.60 - 565.78) \times 12 \times 1,000}{1,925.34 \times 10^6} = 0.00038 \text{ rad}$$

Where:

$\theta_{c,R}$ = rotation of the end span right support.

$(M_{net,R})_{frame}$ = Net frame strip negative moment of the right support.

$$\Delta\theta_{c,R} = \theta_{c,R} \times \left(\frac{l}{8}\right) \times \left(\frac{I_g}{I_e}\right)_{frame} = 0.00038 \times \frac{33 \times 12 - 20}{8} \times \frac{60,254.73}{47,520.23} = 0.0226 \text{ in.}$$

Where:

$\Delta\theta_{c,R}$ = Midspan deflection due to rotation of right support.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 9)

$$\Delta_{cx} = 0.1937 + 0.0767 + 0.0226 = 0.2930 \text{ in.}$$

Following the same procedure, Δ_{mx} can be calculated for the middle strip. This procedure is repeated for the equivalent frame in the orthogonal direction to obtain Δ_{cy}, and Δ_{my} for the end and middle spans for the other load levels ($D + LL_{sus}$ and $D + LL_{full}$).

Since this example has square panels, $\Delta_{cx} = \Delta_{cy}$= 0.2930 in., and $\Delta_{mx} = \Delta_{my}$= 0.1682 in.

The average Δ for the corner panel is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2} = (\Delta_{cx} + \Delta_{my}) = (\Delta_{cy} + \Delta_{mx}) = 0.2930 + 0.1682 = 0.4612 \text{ in.}$$

Table 7 - Immediate (Instantaneous) Deflections in the x-direction

Column Strip

Span	LDF	D						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx}, (in.)
Ext	0.738	0.1233	0.1937	0.00129	0.00038	0.0767	0.0226	0.2930
Int	0.675	0.0896	0.1288	-0.00038	-0.00038	-0.0164	-0.0164	0.0960

Span	LDF	$D+LL_{sus}$						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx} (in.)
Ext	0.738	0.1233	0.1937	0.00129	0.00038	0.0767	0.0226	0.2930
Int	0.675	0.0896	0.1288	-0.00038	-0.00038	-0.0164	-0.0164	0.0960

Span	LDF	$D+LL_{full}$						
		$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{c\text{-}fixed}$ (in.)	θ_{c1} (rad)	θ_{c2} (rad)	$\Delta\theta_{c1}$ (in.)	$\Delta\theta_{c2}$ (in.)	Δ_{cx} (in.)
Ext	0.738	0.3581	0.5625	0.00211	0.00062	0.2323	0.0685	0.8633
Int	0.675	0.1955	0.2811	-0.00062	-0.00062	-0.0374	-0.0374	0.2063

Span	LDF	LL
		Δ_{cx} (in.)
Ext	0.738	0.5703
Int	0.675	0.1103

Middle Strip

LDF	D						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.263	0.1233	0.0689	0.00129	0.00038	0.0767	0.0226	0.1682
0.325	0.0896	0.0620	-0.00038	-0.00038	-0.0164	-0.0164	0.0292

LDF	$D+LL_{sus}$						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.263	0.1233	0.0689	0.00129	0.00038	0.0767	0.0226	0.1682
0.325	0.0896	0.0620	-0.00038	-0.00038	-0.0164	-0.0164	0.0292

LDF	$D+LL_{full}$						
	$\Delta_{frame\text{-}fixed}$ (in.)	$\Delta_{m\text{-}fixed}$ (in.)	θ_{m1} (rad)	θ_{m2} (rad)	$\Delta\theta_{m1}$ (in.)	$\Delta\theta_{m2}$ (in.)	Δ_{mx} (in.)
0.263	0.3581	0.2002	0.00211	0.00062	0.2323	0.0685	0.5010
0.325	0.1955	0.1353	-0.00062	-0.00062	-0.0374	-0.0374	0.0606

LDF	LL
	Δ_{mx} (in.)
0.263	0.3328
0.325	0.0314

5.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) may be estimated as follows:

$$\Delta_{cs} = \lambda_\Delta \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_\Delta) + \left[(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}\right]$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst}$ = Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_\Delta = \frac{\xi}{1+50\times\rho'}$$ ***ACI 318-14 (24.2.4.1.1)***

$(\Delta_{total})_{lt}$ = Time-dependent (long-term) total deflection, in.

$(\Delta_{total})_{Inst}$ = Total immediate (instantaneous) deflection, in.

For the exterior span

ξ = 2, consider the sustained load duration to be 60 months or more. ***ACI 318-14 (Table 24.2.4.1.3)***

ρ' = 0, conservatively.

$$\lambda_\Delta = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2\times 0.2930 = 0.5859 \text{ in.}$$

$$(\Delta_{total})_{lt} = 0.2930\times(1+2)+(0.8633-0.2930) = 1.4492 \text{ in.}$$

The following Table shows long-term deflections for the exterior and interior spans for the analysis in the x-direction, for column and middle strips.

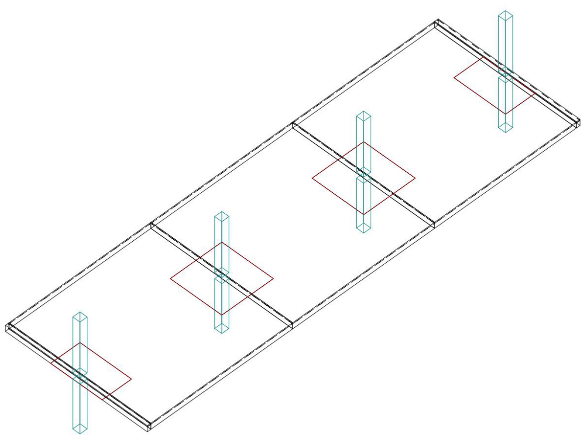
Table 8 - Long-Term Deflections					
Column Strip					
Span	**$(\Delta_{sust})_{Inst}$ (in.)**	**λ_Δ**	**Δ_{cs} (in.)**	**$(\Delta_{total})_{Inst}$ (in.)**	**$(\Delta_{total})_{lt}$ (in.)**
Exterior	0.2930	2	0.5859	0.8633	1.4492
Interior	0.0960	2	0.1920	0.2063	0.3983
Middle Strip					
Exterior	0.1682	2	0.3365	0.5010	0.8374
Interior	0.0292	2	0.0584	0.0606	0.1189

6. spSlab Software Program Model Solution

spSlab program utilizes the Equivalent Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems with drop panels. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept ***(ACI 318-14 (R8.11.4))***.

spSlab Program models the equivalent frame as a design strip. The design strip is, then, separated by spSlab into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips.

spSlab v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems



Contents

1. Input Echo

1.1. General Information

File Name	F:\...\Two-Way-Joist-Waffle-System-ACI-318-14.slb
Project	Two-Way Joist (Waffle) System
Frame	Interior Frame
Engineer	SP
Code	ACI 318-14
Reinforcement Database	ASTM A615
Mode	Design
Number of supports =	4 + Left cantilever + Right cantilever
Floor System	Two-Way

1.2. Solve Options

Live load pattern ratio = 0%
Minimum free edge distance for punching shear = 4 times slab thickness.
Circular critical section around circular supports used (if possible).
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
User-defined slab strip widths NOT selected.
User-defined distribution factors NOT selected.
One-way shear in drop panel selected.
Distribution of shear to strips NOT selected.
Beam T-section design NOT selected.
Longitudinal beam contribution in negative reinforcement design over support NOT selected.
Transverse beam contribution in negative reinforcement design over support NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150	lb/ft³
f'_c	5	ksi
E_c	4286.8	ksi
f_r	0.53033	ksi

1.3.2. Concrete: Columns

w_c	150	lb/ft³
f'_c	6	ksi
E_c	4696	ksi
f_r	0.58095	ksi

1.3.3. Reinforcing Steel

f_y	60	ksi
f_{yt}	60	ksi

E_s	29000 ksi
Epoxy coated bars	No

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

1.5. Span Data

1.5.1. Slabs

Notes:
Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

Span	Loc	L1	t	wL	wR	L2L	L2R	H_{min}
		ft	in	ft	ft	ft	ft	in
1	Int	0.833	3.00	16.500	16.500	33.000	33.000	--- LC *i
2	Int	33.000	3.00	16.500	16.500	33.000	33.000	2.50
3	Int	33.000	3.00	16.500	16.500	33.000	33.000	2.50
4	Int	33.000	3.00	16.500	16.500	33.000	33.000	2.50
5	Int	0.833	3.00	16.500	16.500	33.000	33.000	--- RC *i

1.5.2. Ribs and Longitudinal Beams

Span	Ribs			Beams		
	b	h	Sp	b	h	Offset
	in	in	in	in	in	in
1	6.00	14.00	30.00	0.00	0.00	0.00
2	6.00	14.00	30.00	0.00	0.00	0.00
3	6.00	14.00	30.00	0.00	0.00	0.00
4	6.00	14.00	30.00	0.00	0.00	0.00
5	6.00	14.00	30.00	0.00	0.00	0.00

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	20.00	20.00	13.000	20.00	20.00	13.000	100
2	20.00	20.00	13.000	20.00	20.00	13.000	100
3	20.00	20.00	13.000	20.00	20.00	13.000	100
4	20.00	20.00	13.000	20.00	20.00	13.000	100

1.6.2. Drop Panels

Support	h	Ll	Lr	Wl	Wr
	in	ft	ft	ft	ft
1	0.00	0.833	6.000	6.000	6.000
2	0.00	6.000	6.000	6.000	6.000
3	0.00	6.000	6.000	6.000	6.000
4	0.00	6.000	0.833	6.000	6.000

1.6.3. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kip/in	kip-in/rad		
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	SELF	Dead	Live
Type	DEAD	DEAD	LIVE
U1	1.200	1.200	1.600

1.7.2. Area Loads

Case/Patt	Span	Wa
		lb/ft^2
SELF	1	100.06
	2	100.06
	3	100.06
	4	100.06
	5	100.06
Dead	1	50.00
	2	50.00
	3	50.00
	4	50.00
	5	50.00
Live	1	100.00
	2	100.00
	3	100.00
	4	100.00
	5	100.00

1.7.3. Line Loads

Case/Patt	Span	Wa	La	Wb	Lb
		lb/ft	ft	lb/ft	ft
SELF	1	1349.32	0.000	1349.32	0.833
	2	1349.32	0.000	1349.32	6.000
	2	1349.32	27.000	1349.32	33.000
	3	1349.32	0.000	1349.32	6.000
	3	1349.32	27.000	1349.32	33.000
	4	1349.32	0.000	1349.32	6.000
	4	1349.32	27.000	1349.32	33.000
	5	1349.32	0.000	1349.32	0.833

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#6	#8	#6	#8
Bar spacing	in	1.00	18.00	1.00	18.00

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	in	0.75		0.75	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#5	#8	#5	#8	#3	#5
Bar spacing	in	1.00	18.00	1.00	18.00	6.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	1.50		1.50			
Layer dist.	in	1.00		1.00			
No. of legs						2	6
Side cover	in					1.50	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

2. Design Results*

*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

2.1. Strip Widths and Distribution Factors

Notes:
*Used for bottom reinforcement. **Used for top reinforcement.

Span	Strip	Width			Moment Factor		
		Left **	Right **	Bottom *	Left **	Right **	Bottom *
		ft	ft	ft	ft	ft	ft
1	Column	16.50	16.50	16.50	1.000	1.000	0.600
	Middle	16.50	16.50	16.50	0.000	0.000	0.400
2	Column	16.50	16.50	16.50	1.000	0.750	0.600
	Middle	16.50	16.50	16.50	0.000	0.250	0.400
3	Column	16.50	16.50	16.50	0.750	0.750	0.600
	Middle	16.50	16.50	16.50	0.250	0.250	0.400
4	Column	16.50	16.50	16.50	0.750	1.000	0.600
	Middle	16.50	16.50	16.50	0.250	0.000	0.400
5	Column	16.50	16.50	16.50	1.000	1.000	0.600
	Middle	16.50	16.50	16.50	0.000	0.000	0.400

2.2. Top Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Span	Strip	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars	
			ft	k-ft	ft	in^2	in^2	in^2	in		
1	Column	Left	16.50	0.41	0.241	2.853	10.120	0.006	14.143	14-#6	*3 *5
		Midspan	16.50	1.32	0.447	2.853	10.120	0.018	14.143	14-#6	*3 *5
		Right	16.50	3.09	0.687	5.184	51.614	0.043	14.143	14-#6	*3 *5

Span	Strip	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
			ft	k-ft	ft	in²	in²	in²	in	
	Middle	Left	16.50	0.00	0.000	2.853	12.144	0.000	14.143	14-#6 *3 *5
		Midspan	16.50	0.00	0.344	2.853	12.144	0.000	14.143	14-#6 *3 *5
		Right	16.50	0.00	0.687	2.853	12.144	0.000	14.143	14-#6 *3 *5
2	Column	Left	16.50	323.84	0.833	5.184	51.614	4.594	14.143	14-#6 *3 *5
		Midspan	16.50	0.00	16.500	0.000	10.120	0.000	0.000	---
		Right	16.50	907.33	32.167	5.184	51.614	13.208	6.387	31-#6
	Middle	Left	16.50	1.39	2.063	2.853	12.144	0.019	14.143	14-#6 *3 *5
		Midspan	16.50	0.00	16.500	0.000	12.144	0.000	0.000	---
		Right	16.50	302.44	32.167	2.853	12.144	4.482	14.143	14-#6 *5
3	Column	Left	16.50	823.68	0.833	5.184	51.614	11.945	6.387	31-#6
		Midspan	16.50	0.00	16.500	0.000	10.120	0.000	0.000	---
		Right	16.50	823.68	32.167	5.184	51.614	11.945	6.387	31-#6
	Middle	Left	16.50	274.56	0.833	2.853	12.144	4.046	14.143	14-#6 *5
		Midspan	16.50	0.00	16.500	0.000	12.144	0.000	0.000	---
		Right	16.50	274.56	32.167	2.853	12.144	4.046	14.143	14-#6 *5
4	Column	Left	16.50	907.33	0.833	5.184	51.614	13.208	6.387	31-#6
		Midspan	16.50	0.00	16.500	0.000	10.120	0.000	0.000	---
		Right	16.50	323.84	32.167	5.184	51.614	4.595	14.143	14-#6 *3 *5
	Middle	Left	16.50	302.44	0.833	2.853	12.144	4.482	14.143	14-#6 *5
		Midspan	16.50	0.00	16.500	0.000	12.144	0.000	0.000	---
		Right	16.50	1.39	30.937	2.853	12.144	0.019	14.143	14-#6 *3 *5
5	Column	Left	16.50	3.10	0.146	5.184	51.614	0.043	14.143	14-#6 *3 *5
		Midspan	16.50	1.32	0.386	2.853	10.120	0.018	14.143	14-#6 *3 *5
		Right	16.50	0.41	0.593	2.853	10.120	0.006	14.143	14-#6 *3 *5
	Middle	Left	16.50	0.00	0.146	2.853	12.144	0.000	14.143	14-#6 *3 *5
		Midspan	16.50	0.00	0.490	2.853	12.144	0.000	14.143	14-#6 *3 *5
		Right	16.50	0.00	0.833	2.853	12.144	0.000	14.143	14-#6 *3 *5

2.3. Top Bar Details

NOTES:
* - Bar cut-off location does not meet ACI 318, 12.10.5.1. Revise location, unless the requirements of either 12.10.5.2 or 12.10.5.3 are manually checked and satisfied.

		Left				Continuous		Right			
Span	Strip	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
			ft		ft		ft		ft		ft
1	Column	---		---		14-#6	0.83	---		---	
	Middle	---		---		14-#6	0.83	---		---	
2	Column	12-#6	11.17	2-#6	7.10	---		16-#6	11.17	15-#6 *	7.10
	Middle	14-#6	7.73	---		---		14-#6	10.21	---	
3	Column	16-#6	11.21	15-#6 *	7.10	---		16-#6	11.21	15-#6 *	7.10
	Middle	14-#6	11.21	---		---		14-#6	11.21	---	
4	Column	16-#6	11.17	15-#6 *	7.10	---		12-#6	11.17	2-#6	7.10
	Middle	14-#6	10.21	---		---		14-#6	7.73	---	

Span	Strip	Left Bars	Left Length (ft)	Left Bars	Left Length (ft)	Continuous Bars	Continuous Length (ft)	Right Bars	Right Length (ft)	Right Bars	Right Length (ft)
5	Column	---		---		14-#6	0.83	---		---	
	Middle	---		---		14-#6	0.83	---		---	

2.4. Top Bar Development Lengths

Span	Strip	Left Bars	Left DevLen (in)	Left Bars	Left DevLen (in)	Continuous Bars	Continuous DevLen (in)	Right Bars	Right DevLen (in)	Right Bars	Right DevLen (in)
1	Column	---		---		14-#6	12.00	---		---	
	Middle	---		---		14-#6	12.00	---		---	
2	Column	12-#6	18.99	2-#6	18.99	---		16-#6	24.65	15-#6	24.65
	Middle	14-#6	12.00	---		---		14-#6	18.52	---	
3	Column	16-#6	22.29	15-#6	22.29	---		16-#6	22.29	15-#6	22.29
	Middle	14-#6	16.72	---		---		14-#6	16.72	---	
4	Column	16-#6	24.65	15-#6	24.65	---		12-#6	18.99	2-#6	18.99
	Middle	14-#6	18.52	---		---		14-#6	12.00	---	
5	Column	---		---		14-#6	12.00	---		---	
	Middle	---		---		14-#6	12.00	---		---	

2.5. Bottom Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.

Span	Strip	Width (ft)	M_{max} (k-ft)	X_{max} (ft)	$A_{s,min}$ (in^2)	$A_{s,max}$ (in^2)	$A_{s,req}$ (in^2)	Sp_{Prov} (in)	Bars	
1	Column	16.50	0.00	0.344	0.000	66.794	0.000	0.000	---	
	Middle	16.50	0.00	0.344	0.000	66.794	0.000	0.000	---	
2	Column	16.50	400.59	14.000	2.853	66.531	5.703	3.823	10-#7	
	Middle	16.50	267.06	14.000	2.853	66.794	3.770	3.938	12-#6	
3	Column	16.50	180.35	16.500	2.853	66.794	2.539	3.938	10-#6	*3
	Middle	16.50	120.24	16.500	2.853	66.794	1.690	3.938	12-#6	*3
4	Column	16.50	400.59	19.000	2.853	66.531	5.703	3.823	10-#7	
	Middle	16.50	267.06	19.000	2.853	66.794	3.770	3.938	12-#6	
5	Column	16.50	0.00	0.490	0.000	66.794	0.000	0.000	---	
	Middle	16.50	0.00	0.490	0.000	66.794	0.000	0.000	---	

2.6. Bottom Bar Details

Span	Strip	Long Bars Bars	Long Bars Start (ft)	Long Bars Length (ft)	Short Bars Bars	Short Bars Start (ft)	Short Bars Length (ft)	Waffle Ribs	Waffle Bars/Rib	Waffle A_s/Rib (in^2)
1	Column	---			---			---		
	Middle	---			---			---		
2	Column	10-#7	0.00	33.00	---			5	2-#7	1.200

Span	Strip	Long Bars			Short Bars			Waffle		
		Bars	Start	Length	Bars	Start	Length	Ribs	Bars/Rib	A_s/Rib
			ft	ft		ft	ft			in²
	Middle	12-#6	0.00	33.00	---			6	2-#6	0.880
3	Column	10-#6	0.00	33.00	---			5	2-#6	0.880
	Middle	12-#6	0.00	33.00	---			6	2-#6	0.880
4	Column	10-#7	0.00	33.00	---			5	2-#7	1.200
	Middle	12-#6	0.00	33.00	---			6	2-#6	0.880
5	Column	---			---			---		
	Middle	---			---			---		

2.7. Bottom Bar Development Lengths

Span	Strip	Long Bars		Short Bars	
		Bars	DevLen	Bars	DevLen
			in		in
1	Column	---		---	
	Middle	---		---	
2	Column	10-#7	39.00	---	
	Middle	12-#6	18.18	---	
3	Column	10-#6	14.69	---	
	Middle	12-#6	12.00	---	
4	Column	10-#7	39.00	---	
	Middle	12-#6	18.18	---	
5	Column	---		---	
	Middle	---		---	

2.8. Flexural Capacity

Span	Strip	Top							Bottom					
		x	$A_{s,top}$	ΦM_n-	M_u-	Comb	Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb	Pat	Status
		ft	in²	k-ft	k-ft				in²	k-ft	k-ft			
1	Column	0.000	6.16	-399.88	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.241	6.16	-399.88	-0.41	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.417	6.16	-399.88	-1.11	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.447	6.16	-432.18	-1.32	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.687	6.16	-432.18	-3.09	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.833	6.16	-432.18	-4.46	U1	All	---	0.00	0.00	0.00	U1	All	---
	Middle	0.000	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.241	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.417	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.447	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.687	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.833	6.16	-406.57	0.00	U1	All	---	0.00	0.00	0.00	U1	All	---
2	Column	0.000	6.16	-432.18	-461.26	U1	All	---	6.00	421.16	0.00	U1	All	---
		0.833	6.16	-432.18	-323.84	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		5.518	6.16	-432.18	0.00	U1	All	OK	6.00	421.16	159.35	U1	All	OK
		6.000	5.89	-413.70	0.00	U1	All	OK	6.00	421.16	186.15	U1	All	OK
		6.000	5.89	-384.13	0.00	U1	All	OK	6.00	421.16	186.19	U1	All	OK

		Top							Bottom					
Span	Strip	x	$A_{s,top}$	ΦM_n-	M_u-	Comb	Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb	Pat	Status
		ft	in^2	k-ft	k-ft				in^2	k-ft	k-ft			
		7.100	5.28	-347.67	0.00	U1	All	OK	6.00	421.16	241.15	U1	All	OK
		9.591	5.28	-347.67	0.00	U1	All	OK	6.00	421.16	335.67	U1	All	OK
		11.173	0.00	0.00	0.00	U1	All	OK	6.00	421.16	374.01	U1	All	OK
		11.800	0.00	0.00	0.00	U1	All	OK	6.00	421.16	384.55	U1	All	OK
		14.000	0.00	0.00	0.00	U1	All	OK	6.00	421.16	400.59	U1	All	OK
		16.500	0.00	0.00	0.00	U1	All	OK	6.00	421.16	379.22	U1	All	OK
		21.200	0.00	0.00	0.00	U1	All	OK	6.00	421.16	225.08	U1	All	OK
		21.827	0.00	0.00	0.00	U1	All	OK	6.00	421.16	193.28	U1	All	OK
		23.881	7.04	-450.44	0.00	U1	All	OK	6.00	421.16	70.55	U1	All	OK
		25.900	7.04	-450.44	-103.71	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		27.000	10.57	-616.27	-224.26	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		27.000	10.58	-732.25	-224.33	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		27.954	13.64	-935.78	-336.00	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		32.167	13.64	-935.78	-907.33	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		32.375	13.64	-935.78	-938.65	U1	All	---	6.00	421.16	0.00	U1	All	---
		33.000	13.64	-935.78	-1034.21	U1	All	---	6.00	421.16	0.00	U1	All	---
	Middle	0.000	6.16	-406.57	3.05	U1	All	---	5.28	372.72	0.00	U1	All	---
		0.833	6.16	-406.57	0.00	U1	All	OK	5.28	372.72	0.00	U1	All	OK
		2.063	6.16	-406.57	-1.39	U1	All	OK	5.28	372.72	0.00	U1	All	OK
		6.727	6.16	-406.57	0.00	U1	All	OK	5.28	372.72	148.95	U1	All	OK
		7.727	0.00	0.00	0.00	U1	All	OK	5.28	372.72	179.27	U1	All	OK
		11.800	0.00	0.00	0.00	U1	All	OK	5.28	372.72	256.36	U1	All	OK
		14.000	0.00	0.00	0.00	U1	All	OK	5.28	372.72	267.06	U1	All	OK
		16.500	0.00	0.00	0.00	U1	All	OK	5.28	372.72	252.81	U1	All	OK
		21.200	0.00	0.00	0.00	U1	All	OK	5.28	372.72	150.05	U1	All	OK
		22.792	0.00	0.00	0.00	U1	All	OK	5.28	372.72	92.79	U1	All	OK
		24.335	6.16	-406.57	0.00	U1	All	OK	5.28	372.72	26.38	U1	All	OK
		32.167	6.16	-406.57	-302.44	U1	All	OK	5.28	372.72	0.00	U1	All	OK
		33.000	6.16	-406.57	-357.07	U1	All	---	5.28	372.72	0.00	U1	All	---
3	Column	0.000	13.64	-935.78	-942.14	U1	All	---	4.40	311.22	0.00	U1	All	---
		0.833	13.64	-935.78	-823.68	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		5.242	13.64	-935.78	-308.27	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		6.000	10.95	-757.29	-238.55	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		6.000	10.95	-617.63	-238.50	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		7.100	7.04	-450.44	-146.47	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		9.351	7.04	-450.44	0.00	U1	All	OK	4.40	311.22	8.22	U1	All	OK
		11.208	0.00	0.00	0.00	U1	All	OK	4.40	311.22	86.05	U1	All	OK
		11.800	0.00	0.00	0.00	U1	All	OK	4.40	311.22	105.95	U1	All	OK
		16.500	0.00	0.00	0.00	U1	All	OK	4.40	311.22	180.35	U1	All	OK
		21.200	0.00	0.00	0.00	U1	All	OK	4.40	311.22	105.95	U1	All	OK
		21.792	0.00	0.00	0.00	U1	All	OK	4.40	311.22	86.05	U1	All	OK
		23.649	7.04	-450.44	0.00	U1	All	OK	4.40	311.22	8.22	U1	All	OK
		25.900	7.04	-450.44	-146.47	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		27.000	10.95	-617.63	-238.50	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		27.000	10.95	-757.29	-238.55	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		27.758	13.64	-935.78	-308.27	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		32.167	13.64	-935.78	-823.68	U1	All	OK	4.40	311.22	0.00	U1	All	OK
		33.000	13.64	-935.78	-942.14	U1	All	---	4.40	311.22	0.00	U1	All	---
	Middle	0.000	6.16	-406.57	-314.05	U1	All	---	5.28	372.72	0.00	U1	All	---
		0.833	6.16	-406.57	-274.56	U1	All	OK	5.28	372.72	0.00	U1	All	OK
		9.815	6.16	-406.57	0.00	U1	All	OK	5.28	372.72	19.91	U1	All	OK
		11.208	0.00	0.00	0.00	U1	All	OK	5.28	372.72	57.37	U1	All	OK
		11.800	0.00	0.00	0.00	U1	All	OK	5.28	372.72	70.63	U1	All	OK

		Top							Bottom					
Span	**Strip**	**x**	**$A_{s,top}$**	**ΦM_n-**	**M_u-**	**Comb**	**Pat**	**Status**	**$A_{s,bot}$**	**ΦM_n+**	**M_u+**	**Comb**	**Pat**	**Status**
		ft	in^2	k-ft	k-ft				in^2	k-ft	k-ft			
		16.500	0.00	0.00	0.00	U1	All	OK	5.28	372.72	120.24	U1	All	OK
		21.200	0.00	0.00	0.00	U1	All	OK	5.28	372.72	70.63	U1	All	OK
		21.792	0.00	0.00	0.00	U1	All	OK	5.28	372.72	57.37	U1	All	OK
		23.185	6.16	-406.57	0.00	U1	All	OK	5.28	372.72	19.91	U1	All	OK
		32.167	6.16	-406.57	-274.56	U1	All	OK	5.28	372.72	0.00	U1	All	OK
		33.000	6.16	-406.57	-314.05	U1	All	---	5.28	372.72	0.00	U1	All	---
4	Column	0.000	13.64	-935.78	-1034.21	U1	All	---	6.00	421.16	0.00	U1	All	---
		0.625	13.64	-935.78	-938.65	U1	All	---	6.00	421.16	0.00	U1	All	---
		0.833	13.64	-935.78	-907.33	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		5.046	13.64	-935.78	-336.00	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		6.000	10.58	-732.25	-224.33	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		6.000	10.57	-616.27	-224.26	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		7.100	7.04	-450.44	-103.71	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		9.119	7.04	-450.44	0.00	U1	All	OK	6.00	421.16	70.55	U1	All	OK
		11.173	0.00	0.00	0.00	U1	All	OK	6.00	421.16	193.28	U1	All	OK
		11.800	0.00	0.00	0.00	U1	All	OK	6.00	421.16	225.08	U1	All	OK
		16.500	0.00	0.00	0.00	U1	All	OK	6.00	421.16	379.22	U1	All	OK
		19.000	0.00	0.00	0.00	U1	All	OK	6.00	421.16	400.59	U1	All	OK
		21.200	0.00	0.00	0.00	U1	All	OK	6.00	421.16	384.55	U1	All	OK
		21.827	0.00	0.00	0.00	U1	All	OK	6.00	421.16	374.01	U1	All	OK
		23.409	5.28	-347.67	0.00	U1	All	OK	6.00	421.16	335.67	U1	All	OK
		25.900	5.28	-347.67	0.00	U1	All	OK	6.00	421.16	241.15	U1	All	OK
		27.000	5.89	-384.13	0.00	U1	All	OK	6.00	421.16	186.18	U1	All	OK
		27.000	5.89	-413.70	0.00	U1	All	OK	6.00	421.16	186.15	U1	All	OK
		27.482	6.16	-432.18	0.00	U1	All	OK	6.00	421.16	159.35	U1	All	OK
		32.167	6.16	-432.18	-323.84	U1	All	OK	6.00	421.16	0.00	U1	All	OK
		33.000	6.16	-432.18	-461.27	U1	All	---	6.00	421.16	0.00	U1	All	---
	Middle	0.000	6.16	-406.57	-357.07	U1	All	---	5.28	372.72	0.00	U1	All	---
		0.833	6.16	-406.57	-302.44	U1	All	OK	5.28	372.72	0.00	U1	All	OK
		8.665	6.16	-406.57	0.00	U1	All	OK	5.28	372.72	26.38	U1	All	OK
		10.208	0.00	0.00	0.00	U1	All	OK	5.28	372.72	92.79	U1	All	OK
		11.800	0.00	0.00	0.00	U1	All	OK	5.28	372.72	150.05	U1	All	OK
		16.500	0.00	0.00	0.00	U1	All	OK	5.28	372.72	252.81	U1	All	OK
		19.000	0.00	0.00	0.00	U1	All	OK	5.28	372.72	267.06	U1	All	OK
		21.200	0.00	0.00	0.00	U1	All	OK	5.28	372.72	256.36	U1	All	OK
		25.273	0.00	0.00	0.00	U1	All	OK	5.28	372.72	179.26	U1	All	OK
		26.273	6.16	-406.57	0.00	U1	All	OK	5.28	372.72	148.95	U1	All	OK
		30.937	6.16	-406.57	-1.39	U1	All	OK	5.28	372.72	0.00	U1	All	OK
		32.167	6.16	-406.57	0.00	U1	All	OK	5.28	372.72	0.00	U1	All	OK
		33.000	6.16	-406.57	3.05	U1	All	---	5.28	372.72	0.00	U1	All	---
5	Column	0.000	6.16	-432.18	-4.46	U1	All	---	0.00	0.00	0.00	U1	All	---
		0.146	6.16	-432.18	-3.10	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.386	6.16	-399.88	-1.32	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.417	6.16	-399.88	-1.12	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.593	6.16	-399.88	-0.41	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.833	6.16	-399.88	0.00	U1	Odd	OK	0.00	0.00	0.00	U1	All	OK
	Middle	0.000	6.16	-406.57	0.00	U1	All	---	0.00	0.00	0.00	U1	All	---
		0.146	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.386	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.417	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.593	6.16	-406.57	0.00	U1	All	OK	0.00	0.00	0.00	U1	All	OK
		0.833	6.16	-406.57	0.00	U1	Odd	OK	0.00	0.00	0.00	U1	All	OK

2.9. Slab Shear Capacity

Span	b	d	V_{ratio}	ΦV_c	V_u	X_u
	in	in		kip	kip	ft
1	80.55	15.87	1.000	149.20	10.70	0.00
	195.26	15.88	1.000	337.41	10.70	0.00
2	195.22	15.81	1.000	336.01	138.99	2.15
	80.49	15.81	1.000	148.50	146.11	27.00
	195.22	15.81	1.000	336.01	195.53	30.85
3	195.26	15.88	1.000	337.41	167.19	2.16
	80.55	15.87	1.000	149.20	117.83	6.00
	195.26	15.88	1.000	337.41	167.19	30.84
4	195.22	15.81	1.000	336.01	195.53	2.15
	80.49	15.81	1.000	148.50	146.11	6.00
	195.22	15.81	1.000	336.01	138.99	30.85
5	195.26	15.88	1.000	337.41	0.00	0.83
	80.55	15.87	1.000	149.20	0.00	0.83

2.10. Flexural Transfer of Negative Unbalanced Moment at Supports

Support	Width	Width-c	d	M_{unb}	Comb	Patt	γ_f	$A_{s,req}$	$A_{s,prov}$	Add Bars
	in	in	in	k-ft				in^2	in^2	
1	71.00	71.00	15.88	453.76	U1	All	0.630	4.105	2.209	5-#6
2	71.00	71.00	15.88	135.09	U1	All	0.600	1.143	4.891	---
3	71.00	71.00	15.88	135.09	U1	All	0.600	1.143	4.891	---
4	71.00	71.00	15.88	453.76	U1	All	0.630	4.105	2.209	5-#6

2.11. Punching Shear Around Columns

2.11.1. Critical Section Properties

Support	Type	b_1	b_2	b_0	d_{avg}	CG	$c_{(left)}$	$c_{(right)}$	A_c	J_c
		in	in	in	in	in	in	in	in^2	in^4
1	Rect	27.94	35.88	91.75	15.87	9.43	19.43	8.51	1456.5	1.4399e+005
2	Rect	35.88	35.88	143.50	15.88	0.00	17.94	17.94	2278.1	5.1257e+005
3	Rect	35.88	35.88	143.50	15.88	0.00	17.94	17.94	2278.1	5.1257e+005
4	Rect	27.94	35.88	91.75	15.88	-9.43	8.51	19.43	1456.5	1.4399e+005

2.11.2. Punching Shear Results

Support	V_u	v_u	M_{unb}	Comb	Patt	γ_v	v_u	ΦV_c
	kip	psi	k-ft				psi	psi
1	174.86	120.1	316.33	U1	All	0.370	203.1	212.1
2	414.86	182.1	-135.09	U1	All	0.400	204.8	212.1
3	414.86	182.1	135.09	U1	All	0.400	204.8	212.1
4	174.87	120.1	-316.33	U1	All	0.370	203.1	212.1

2.12. Punching Shear Around Drops

2.12.1. Critical Section Properties

Support	Type	b_1	b_2	b_0	d_{avg}	CG	$c_{(left)}$	$c_{(right)}$	A_c	J_c
		in	in	in	in	in	in	in	in^2	in^4
1	Rect	89.94	159.88	339.75	3.32	56.13	66.13	23.81	1127	9.705e+005
2	Rect	159.88	159.88	639.50	3.32	0.00	79.94	79.94	2121.3	9.0377e+006
3	Rect	159.88	159.88	639.50	3.32	0.00	79.94	79.94	2121.3	9.0377e+006
4	Rect	89.94	159.88	339.75	3.32	-56.13	23.81	66.13	1127	9.705e+005

2.12.2. Punching Shear Results

Support	V_u kip	Comb	Pat	v_u psi	ΦV_c psi	
1	143.28	U1	All	127.1	116.7	*EXCEEDED
2	357.54	U1	All	168.5	116.7	*EXCEEDED
3	357.54	U1	All	168.5	116.7	*EXCEEDED
4	143.28	U1	All	127.1	116.7	*EXCEEDED

2.13. Material TakeOff

2.13.1. Reinforcement in the Direction of Analysis

Top Bars	3456.8 lb	<=>	34.34 lb/ft	<=>	1.041 lb/ft²
Bottom Bars	3629.1 lb	<=>	36.05 lb/ft	<=>	1.092 lb/ft²
Stirrups	0.0 lb	<=>	0.00 lb/ft	<=>	0.000 lb/ft²
Total Steel	7085.9 lb	<=>	70.39 lb/ft	<=>	2.133 lb/ft²
Concrete	2215.9 ft³	<=>	22.01 ft³/ft	<=>	0.667 ft³/ft²

3. Deflection Results: Summary

3.1. Section Properties

3.1.1. Frame Section Properties

Notes:
M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Span	Zone	M_{+ve} I_g in⁴	I_{cr} in⁴	M_{cr} k-ft	M_{-ve} I_g in⁴	I_{cr} in⁴	M_{cr} k-ft
1	Left	60255	0	233.46	60255	13128	-476.06
	Midspan	60255	0	233.46	60255	13128	-476.06
	Right	60255	0	233.46	60255	13128	-476.06
2	Left	60255	12200	233.46	60255	13128	-476.06
	Midspan	60255	15599	233.46	60255	0	-476.06
	Right	60255	12200	233.46	60255	18722	-476.06
3	Left	60255	10861	233.46	60255	18722	-476.06
	Midspan	60255	13647	233.46	60255	0	-476.06
	Right	60255	10861	233.46	60255	18722	-476.06
4	Left	60255	12200	233.46	60255	18722	-476.06
	Midspan	60255	15599	233.46	60255	0	-476.06
	Right	60255	12200	233.46	60255	13128	-476.06
5	Left	60255	0	233.46	60255	13128	-476.06
	Midspan	60255	0	233.46	60255	13128	-476.06
	Right	60255	0	233.46	60255	13128	-476.06

3.1.2. Frame Effective Section Properties

Span	Zone	Weight	Load Level: Dead M_{max} k-ft	Dead I_e in⁴	Sustained M_{max} k-ft	Sustained I_e in⁴	Dead+Live M_{max} k-ft	Dead+Live I_e in⁴
1	Right	1.000	-2.19	60255	-2.19	60255	-3.33	60255
	Span Avg	----	----	60255	----	60255	----	60255
2	Middle	0.850	298.16	37035	298.16	37035	491.82	20376
	Right	0.150	-626.60	36936	-626.60	36936	-1026.20	22869
	Span Avg	----	----	37020	----	37020	----	20750
3	Left	0.150	-565.78	43465	-565.78	43465	-926.56	24356
	Middle	0.700	132.58	60255	132.58	60255	221.02	60255

Span	Zone	Weight	Load Level Dead M_{max} k-ft	Dead I_e in⁴	Sustained M_{max} k-ft	Sustained I_e in⁴	Dead+Live M_{max} k-ft	Dead+Live I_e in⁴
	Right	0.150	-565.78	43465	-565.78	43465	-926.56	24356
	Span Avg	----	----	55218	----	55218	----	49485
4	Left	0.150	-626.60	36936	-626.60	36936	-1026.20	22869
	Middle	0.850	298.16	37035	298.16	37035	491.82	20376
	Span Avg	----	----	37020	----	37020	----	20750
5	Left	1.000	-2.19	60255	-2.19	60255	-3.34	60255
	Span Avg	----	----	60255	----	60255	----	60255

3.1.3. Strip Section Properties at Midspan

Notes:
Load distirubtion factor, LDL, averages moment distribution factors listed in Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

Span	Column Strip I_g in⁴	Column Strip LDF	Column Strip Ratio	Middle Strip I_g in⁴	Middle Strip LDF	Middle Strip Ratio
1	28289.3	0.800	1.704	28289.3	0.200	0.426
2	28289.3	0.738	1.571	28289.3	0.262	0.559
3	28289.3	0.675	1.438	28289.3	0.325	0.692
4	28289.3	0.738	1.571	28289.3	0.262	0.559
5	28289.3	0.800	1.704	28289.3	0.200	0.426

3.2. Instantaneous Deflections

3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.253	---	0.374	0.374	0.253	0.627
		Loc	ft	15.000	---	15.500	15.500	15.000	15.250
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.070	---	0.070	0.070	0.070	0.140
		Loc	ft	16.500	---	16.500	16.500	16.500	16.500
	Up	Def	in	-0.004	---	-0.001	-0.001	-0.004	-0.005
		Loc	ft	1.571	---	1.325	1.325	1.571	1.325
4	Down	Def	in	0.253	---	0.374	0.374	0.253	0.627
		Loc	ft	18.000	---	17.500	17.500	18.000	17.750
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

3.2.2. Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	in	---	---	---	---	---	---

					Live			Total	
Span	**Direction**	**Value**	**Units**	**Dead**	**Sustained**	**Unsustained**	**Total**	**Sustained**	**Dead+Live**
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.337	---	0.531	0.531	0.337	0.867
		Loc	ft	15.500	---	15.750	15.750	15.500	15.750
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.116	---	0.107	0.107	0.116	0.222
		Loc	ft	16.500	---	16.500	16.500	16.500	16.500
	Up	Def	in	-0.003	---	-0.001	-0.001	-0.003	-0.004
		Loc	ft	1.325	---	1.079	1.079	1.325	1.079
4	Down	Def	in	0.337	---	0.531	0.531	0.337	0.867
		Loc	ft	17.500	---	17.250	17.250	17.500	17.250
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

3.2.3. Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

					Live			Total	
Span	**Direction**	**Value**	**Units**	**Dead**	**Sustained**	**Unsustained**	**Total**	**Sustained**	**Dead+Live**
1	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	in	0.189	---	0.254	0.254	0.189	0.443
		Loc	ft	14.500	---	14.750	14.750	14.500	14.750
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
3	Down	Def	in	0.039	---	0.043	0.043	0.039	0.082
		Loc	ft	16.500	---	16.500	16.500	16.500	16.500
	Up	Def	in	-0.005	---	-0.002	-0.002	-0.005	-0.006
		Loc	ft	2.310	---	1.571	1.571	2.310	1.817
4	Down	Def	in	0.189	---	0.254	0.254	0.189	0.443
		Loc	ft	18.500	---	18.250	18.250	18.500	18.250
	Up	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
5	Down	Def	in	---	---	---	---	---	---
		Loc	ft	---	---	---	---	---	---
	Up	Def	in	-0.017	---	-0.017	-0.017	-0.017	-0.034
		Loc	ft	0.833	---	0.833	0.833	0.833	0.833

3.3. Long-term Deflections

3.3.1. Long-term Column Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span	Zone	M_{+ve} $A_{s,top}$	b	d	Rho'	Lambda	M_{-ve} $A_{s,bot}$	b	d	Rho'	Lambda
		in²	in	in	%		in²	in	in	%	
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

3.3.2. Long-term Middle Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span	Zone	M_{+ve} $A_{s,top}$	b	d	Rho'	Lambda	M_{-ve} $A_{s,bot}$	b	d	Rho'	Lambda
		in²	in	in	%		in²	in	in	%	
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

3.3.3. Extreme Long-term Column Strip Deflections and Corresponding Locations

Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.033	-0.050	-0.050	-0.067
		Loc	ft	0.000	0.000	0.000	0.000
2	Down	Def	in	0.673	1.204	1.204	1.540
		Loc	ft	15.500	15.750	15.750	15.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
3	Down	Def	in	0.231	0.338	0.338	0.454
		Loc	ft	16.500	16.500	16.500	16.500
	Up	Def	in	-0.005	-0.006	-0.006	-0.009
		Loc	ft	1.325	1.079	1.079	1.325
4	Down	Def	in	0.673	1.204	1.204	1.540
		Loc	ft	17.500	17.250	17.250	17.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
5	Down	Def	in	---	---	---	---

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
		Loc	ft	---	---	---	---
	Up	Def	in	-0.033	-0.050	-0.050	-0.067
		Loc	ft	0.833	0.833	0.833	0.833

3.3.4. Extreme Long-term Middle Strip Deflections and Corresponding Locations

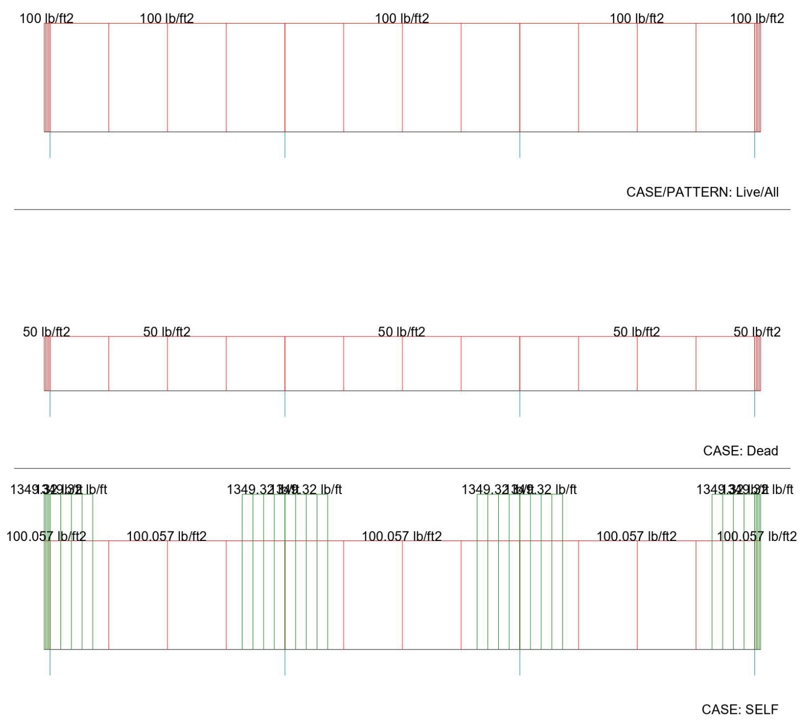
Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.

Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.033	-0.050	-0.050	-0.067
		Loc	ft	0.000	0.000	0.000	0.000
2	Down	Def	in	0.378	0.632	0.632	0.821
		Loc	ft	14.500	14.500	14.500	14.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
3	Down	Def	in	0.078	0.121	0.121	0.160
		Loc	ft	16.500	16.500	16.500	16.500
	Up	Def	in	-0.009	-0.011	-0.011	-0.015
		Loc	ft	2.310	2.063	2.063	2.063
4	Down	Def	in	0.378	0.632	0.632	0.821
		Loc	ft	18.500	18.500	18.500	18.500
	Up	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
5	Down	Def	in	---	---	---	---
		Loc	ft	---	---	---	---
	Up	Def	in	-0.033	-0.050	-0.050	-0.067
		Loc	ft	0.833	0.833	0.833	0.833

4. Diagrams

4.1. Loads



spSlab v5.50. Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3

File: F:\StructurePoint\spSlab\Two-Way-Joist-Waffle-System-ACI-318-14.slb

Project: Two-Way Joist (Waffle) System

Frame: Interior Frame

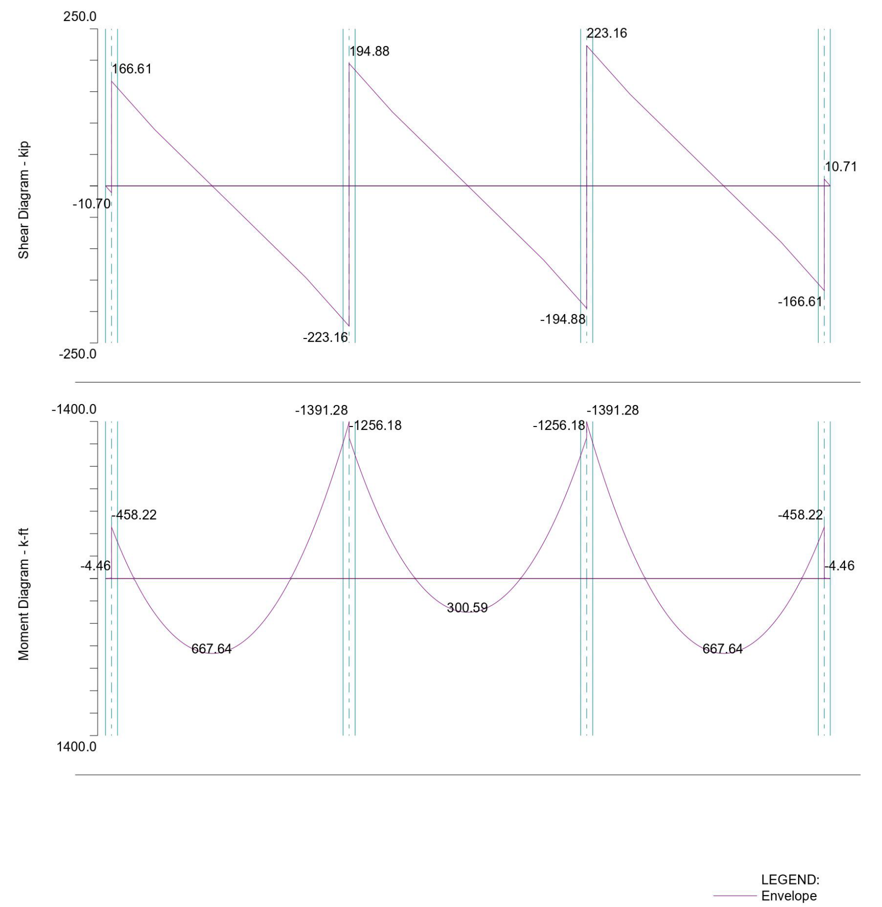
Engineer: SP

Code: ACI 318-14

Date: 02/09/24

Time: 17:46:15

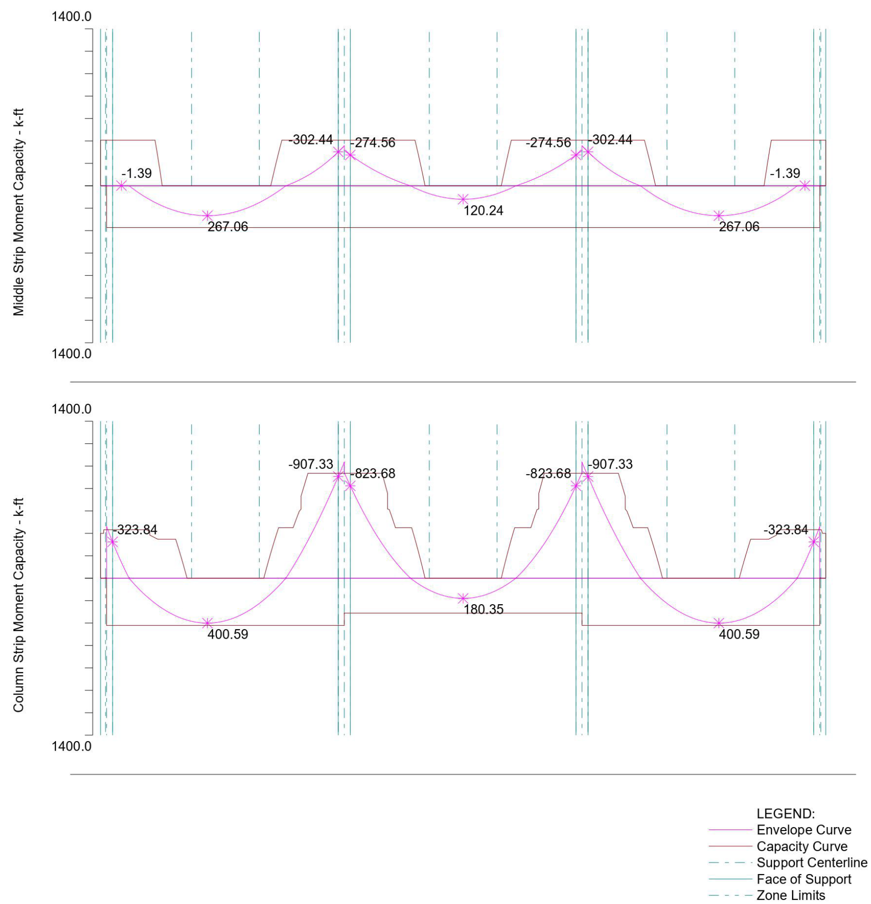
4.2. Internal Forces




```
spSlab v5.50. Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3
```

```
File: F:\StructurePoint\spSlab\Two-Way-Joist-Waffle-System-ACI-318-14.slb
Project: Two-Way Joist (Waffle) System
Frame: Interior Frame
Engineer: SP
Code: ACI 318-14
Date: 02/09/24
Time: 17:46:15
```

4.3. Moment Capacity



spSlab v5.50. Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3

File: F:\StructurePoint\spSlab\Two-Way-Joist-Waffle-System-ACI-318-14.slb

Project: Two-Way Joist (Waffle) System

Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

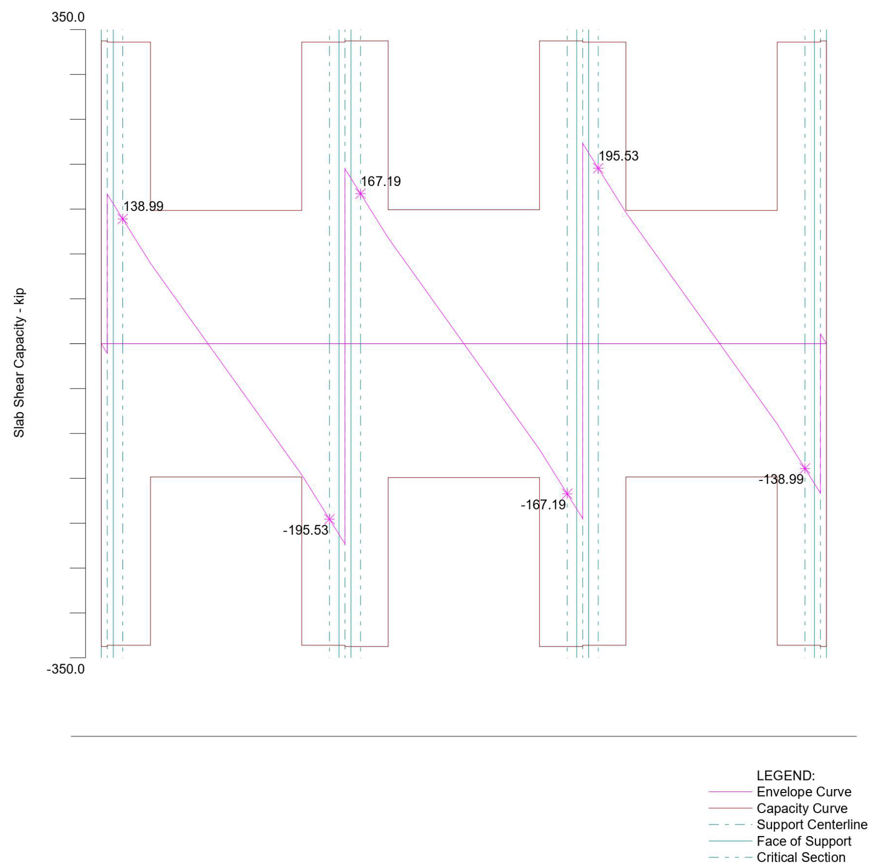
Date: 02/09/24

Time: 17:46:15

STRUCTUREPOINT - spSlab v5.50
Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3
F:\StructurePoint\spSlab\Two-Way-Joist-Waffle-System-ACI-318-14.slb

Page | **22**
2/9/2024
5:46 PM

4.4. Shear Capacity



spSlab v5.50. Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3

File: F:\StructurePoint\spSlab\Two-Way-Joist-Waffle-System-ACI-318-14.slb

Project: Two-Way Joist (Waffle) System

Frame: Interior Frame

Engineer: SP

Code: ACI 318-14

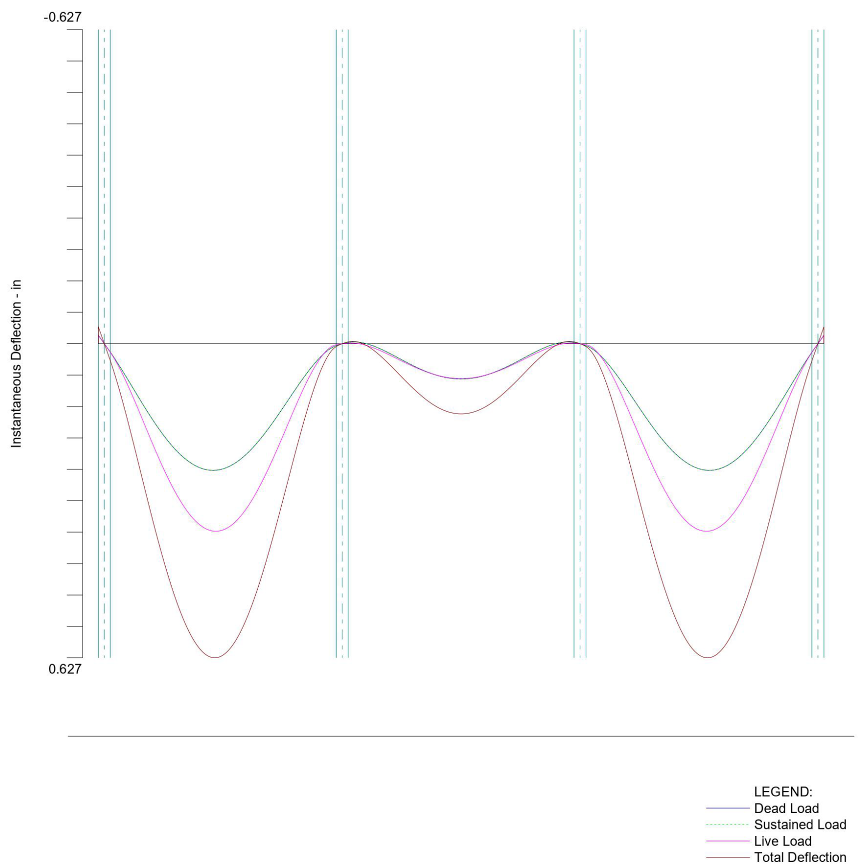
Date: 02/09/24

Time: 17:46:15

STRUCTUREPOINT - spSlab v5.50
Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3
F:\StructurePoint\spSlab\Two-Way-Joist-Waffle-System-ACI-318-14.slb

Page | **23**
2/9/2024
5:46 PM

4.5. Deflection



spSlab v5.50. Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3

File: F:\StructurePoint\spSlab\Two-Way-Joist-Waffle-System-ACI-318-14.slb

Project: Two-Way Joist (Waffle) System

Frame: Interior Frame

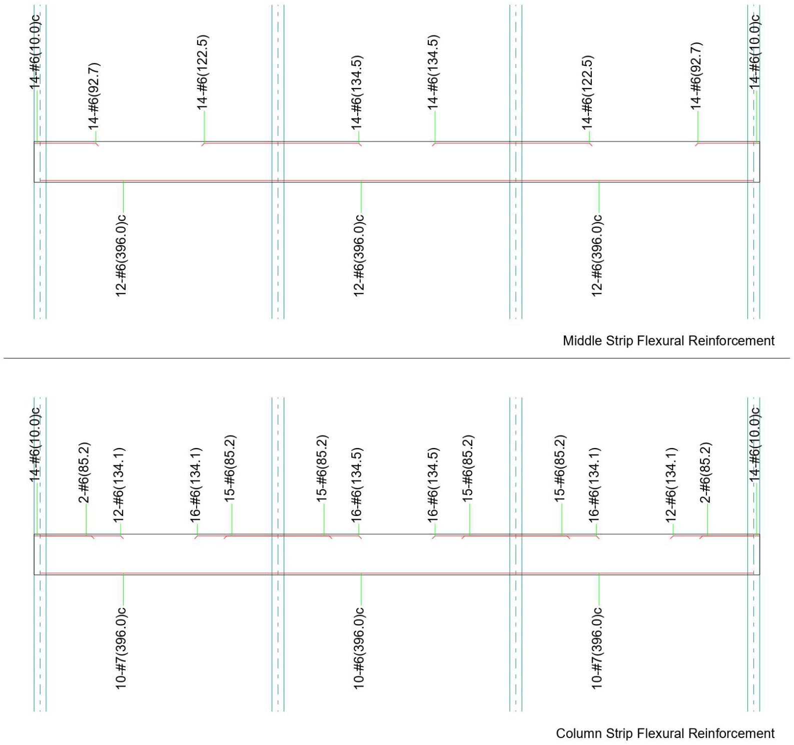
Engineer: SP

Code: ACI 318-14

Date: 02/09/24

Time: 17:46:15

4.6. Reinforcement




```
spSlab v5.50. Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-241C3-241C3
```

```
File: F:\StructurePoint\spSlab\Two-Way-Joist-Waffle-System-ACI-318-14.slb
Project: Two-Way Joist (Waffle) System
Frame: Interior Frame
Engineer: SP
Code: ACI 318-14
Date: 02/09/24
Time: 17:46:15
```

7. Summary and Comparison of Design Results

Table 9 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution (ft-kips)				
		Hassoun (DDM)**	**Hand (EFM)**	**spSlab**
Exterior Span				
Column Strip	**Exterior Negative***	370.00	328.07	323.84
	Positive	444.00	401.00	400.59
	Interior Negative*	748.00	901.27	907.33
Middle Strip	**Exterior Negative***	---	0.00	0.00
	Positive	---	267.33	267.06
	Interior Negative*	---	300.42	302.44
Interior Span				
Column Strip	**Interior Negative***	---	818.10	823.68
	Positive	---	184.66	180.35
Middle Strip	**Interior Negative***	249.00	272.70	274.56
	Positive	296.00	123.10	120.24

* Negative moments are taken at the faces of supports

** Direct Design Method does not distinguish between interior and exterior spans nor explicitly address the effect of column contribution at joints

Table 10 - Comparison of Reinforcement Results										
Span Location		**Reinforcement Provided for Flexure**			**Additional Reinforcement Provided for Unbalanced Moment Transfer**			**Total Reinforcement Provided**		
		Hassoun	**Hand**	**spSlab**	**Hassoun**	**Hand**	**spSlab**	**Hassoun**	**Hand**	**spSlab**
Exterior Span										
Column Strip	**Exterior Negative**	14-#6	14-#6	14-#6	---	5-#6	5-#6	14-#6	19-#6	19-#6
	Positive	10-#8 2 bars / rib	10-#7 2 bars / rib	10-#7 2 bars / rib	---	n/a	n/a	10-#8 2 bars / rib	10-#7 2 bars / rib	10-#7 2 bars / rib
	Interior Negative	28-#6	30-#6	31-#6	---	---	---	28-#6	30-#6	31-#6
Middle Strip	**Exterior Negative**	10-#6	14-#6	14-#6	---	n/a	n/a	10-#6*	14-#6	14-#6
	Positive	12-#7 2 bars / rib	12-#6 2 bars / rib	12-#6 2 bars / rib	---	n/a	n/a	12-#7 2 bars / rib	12-#6 2 bars / rib	12-#6 2 bars / rib
	Interior Negative	10-#6	14-#6	14-#6	---	n/a	n/a	10-#6*	14-#6	14-#6
Interior Span										
Column Strip	**Positive**	10-#7 2 bars / rib	10-#6 2 bars / rib	10-#6 2 bars / rib	---	n/a	n/a	10-#7 2 bars / rib	10-#6 2 bars / rib	10-#6 2 bars / rib
Middle Strip	**Positive**	10-#6 2 bars / rib	12-#6 2 bars / rib	12-#6 2 bars / rib	---	n/a	n/a	10-#6 2 bars / rib	12-#6 2 bars / rib	12-#6 2 bars / rib
* Max spacing requirement exceeded (not checked)										

Table 11 - Comparison of One-Way (Beam Action) Shear Check Results								
Span	**V_u @ d (kips)**		**V_u @ drop panel (kips)**		**ϕV_c @ d (kips)**		**ϕV_c @ drop panel (kips)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	195.05	195.53	145.69	146.11	336.01	336.01	148.50	148.50
Interior	167.16	167.19	117.81	117.83	337.41	337.41	149.20	149.20
* One-way shear check is not provided in the reference (Hassoun and Al-Manaseer)								

Table 12 - Comparison of Two-Way (Punching) Shear Check Results (around Columns Faces)

Support	b_1 (in.)		b_2 (in.)		b_o (in.)		V_u (kips)		c_{AB} (in.)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	27.94	27.94	35.88	35.88	91.75	91.75	163.66	174.86	8.51	8.51
Interior	35.88	35.88	35.88	35.88	143.50	143.50	413.34	414.86	17.94	17.94
Corner	27.94	27.94	27.94	27.94	55.88	55.87	91.75	92.43	6.98	6.98

Support	J_c (in.^{4})		γ_v		M_{unb} (ft-kips)		v_u (psi)		ϕv_c (psi)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	143,997	143,990	0.370	0.370	334.13	316.33	200.1	203.1	212.1	212.1
Interior	512,571	512,570	0.400	0.400	134.12	135.09	204.0	204.8	212.1	212.1
Corner	81,431	81,428	0.400	0.400	181.81	181.19	178.3	178.8	212.1	212.1

Table 13 - Comparison of Two-Way (Punching) Shear Check Results (around Drop Panels)

Support	b_1 (in.)		b_2 (in.)		b_o (in.)		V_u (kips)		c_{AB} (in.)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	89.94	89.94	159.88	159.88	339.75	339.75	133.02	143.28	23.81	23.81
Interior	159.88	159.88	159.88	159.88	639.50	639.50	357.23	357.54	79.94	79.94
Corner	89.94	89.94	89.94	89.94	179.88	179.87	75.22	75.17	22.48	22.48

Support	J_c (in.^{4})		γ_v		M_{unb} (ft-kips)		v_u (psi)		ϕv_c (psi)	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	971,273	970,500	N.A.	N.A.	N.A.	N.A.	117.9	127.1	116.7	116.7
Interior	9,044,800	9,037,700	N.A.	N.A.	N.A.	N.A.	168.3	168.5	116.7	116.7
Corner	503,407	503,010	N.A.	N.A.	N.A.	N.A.	126.0	126.0	116.7	116.7

General notes:

1. Red values are exceeding permissible shear capacity
2. Hand Calculation fail to capture analysis details possible in spSlab like accounting for the exact value of the moments and shears at supports and including the loads for the small slab section extending beyond the supporting column centerline.
3. Shear stresses from spSlab are higher than hand calculations since it considers the load effects beyond the column centerline known in the model as right/left cantilevers. This small increase is often neglected in simplified hand calculations like the one used here.

Table 14 - Comparison of Immediate Deflection Results (in.)								
Column Strip								
Span	**D**		**D+LL$_{sus}$**		**D+LL$_{full}$**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	0.293	0.337	0.293	0.337	0.863	0.867	0.570	0.531
Interior	0.096	0.116	0.096	0.116	0.206	0.222	0.110	0.107
Middle Strip								
Span	**D**		**D+LL$_{sus}$**		**D+LL$_{full}$**		**LL**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	0.168	0.189	0.168	0.189	0.501	0.443	0.333	0.254
Interior	0.029	0.039	0.029	0.039	0.061	0.082	0.031	0.043

Table 15 - Comparison of Time-Dependent Deflection Results						
Column Strip						
Span	**λ_Δ**		**Δ_{cs} (in.)**		**Δ_{total} (in.)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	0.586	0.673	1.449	1.540
Interior	2.0	2.0	0.192	0.231	0.398	0.454
Middle Strip						
Span	**λ_Δ**		**Δ_{cs} (in.)**		**Δ_{total} (in.)**	
	Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior	2.0	2.0	0.337	0.378	0.837	0.821
Interior	2.0	2.0	0.058	0.078	0.119	0.160

In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the spSlab model. The deflection results from spSlab are, however, more conservative than hand calculations for two main reasons explained previously: 1) Values of I_g and I_{cr} at the negative section exclude the stiffening effect of the drop panel and 2) The $I_{e,avg}$ used by spSlab considers equations for prismatic members.

8. Conclusions & Observations

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in ***ACI 318-14 Chapter 8 (8.2.1)***.

Direct Design Method (DDM) is an approximate method and is applicable to two-way slab concrete floor systems that meet the stringent requirements of ***ACI 318-14 (8.10.2)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Equivalent Frame Method (EFM) does not have the limitations of Direct Design Method. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StucturePoint's spSlab software program solution utilizes the Equivalent Frame Method to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable ACI 318-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
8.10.2.1	Minimum of three continuous spans in each direction	☑		
8.10.2.2	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
8.10.2.3	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
8.10.2.4	Column offset shall not exceed 10% of the span in direction of offset from either axis between centerlines of successive columns	☑		
8.10.2.5	All loads shall be due to gravity only	☑		
8.10.2.5	All loads shall be uniformly distributed over an entire panel (q_u)	☑		
8.10.2.6	Unfactored live load shall not exceed two times the unfactored dead load	☑		
8.10.2.7	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑		
8.7.4.2	Structural integrity steel detailing	☑	☑	☑
8.5.4	Openings in slab systems	☑	☑	☑
8.2.2	Concentrated loads	Not permitted	☑	☑
8.11.1.2	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
R8.10.4.5*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)		Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb}).