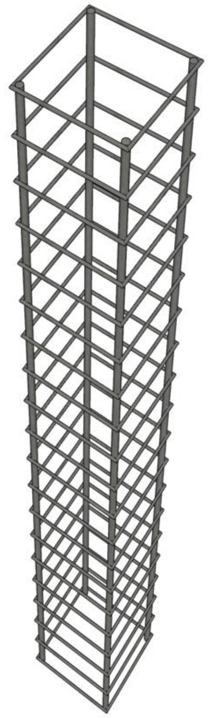
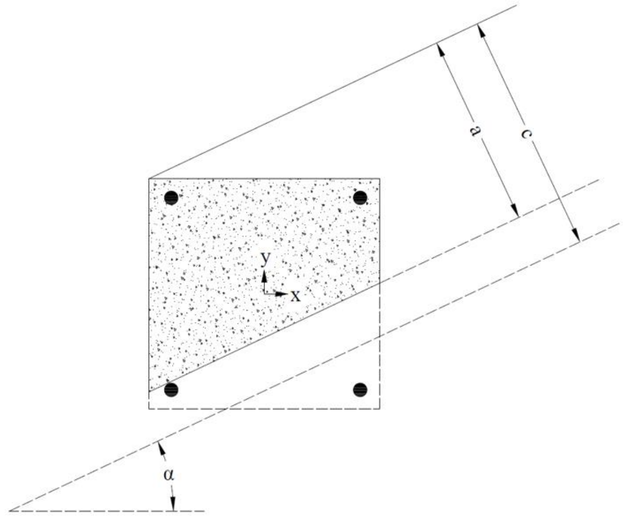
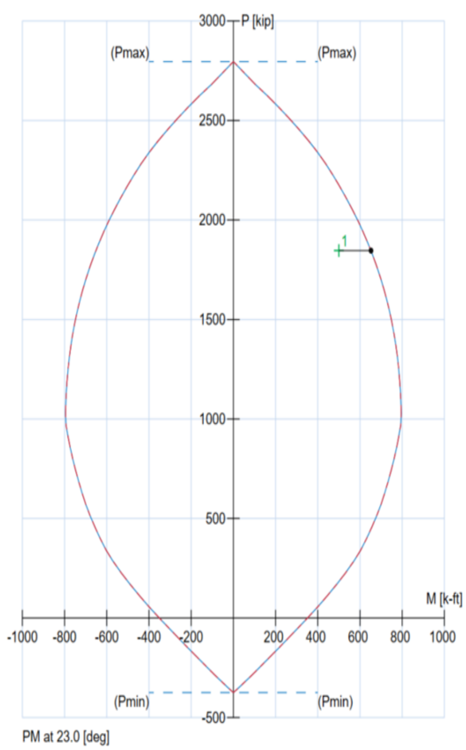
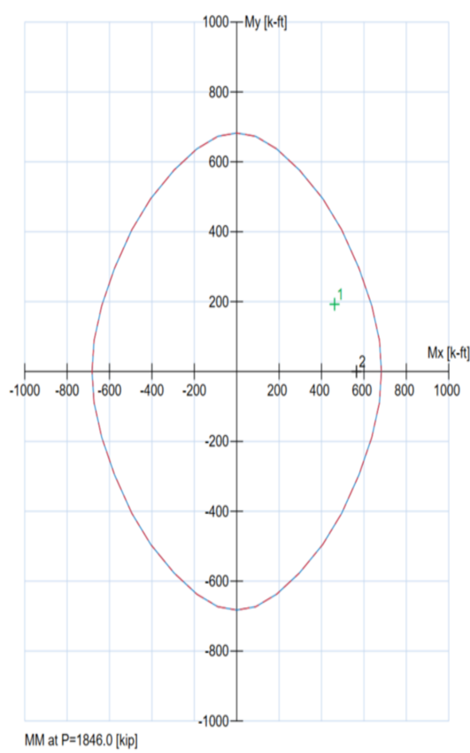
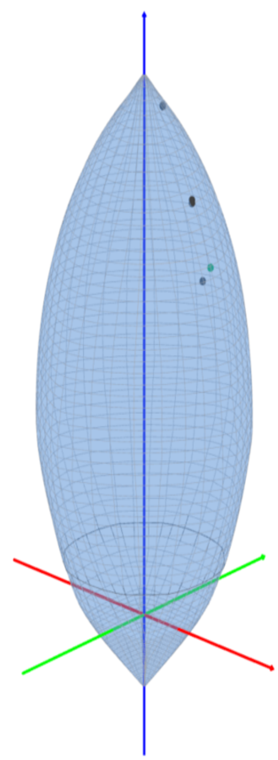
Manual Design Procedure for Columns with Biaxial Bending (ACI 318-11/14/19)











Designing Columns for Biaxial Bending using Manual Design Procedure (ACI 318-11/14/19)

Biaxial bending of columns occurs when the loading causes bending simultaneously about both principal axes. The commonly encountered case of such loading occurs in corner columns. Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load.

A uniaxial interaction diagram defines the load-moment strength along a single plane of a section under an axial load P and a uniaxial moment M. The biaxial bending resistance of an axially loaded column can be represented schematically as a surface formed by a series of uniaxial interaction curves drawn radially from the P axis. Data for these intermediate curves are obtained by varying the angle of the neutral axis (for assumed strain configurations) with respect to the major axes.

The difficulty associated with the determination of the strength of reinforced columns subjected to combined axial load and biaxial bending is primarily an arithmetic one. The bending resistance of an axially loaded column about a particular skewed axis is determined through iterations involving simple but lengthy calculations. These extensive calculations are compounded when optimization of the reinforcement or cross-section is sought.

This example demonstrates the determination of a column section capacity (P_n, M_{nx} and M_{ny}) required to resist the following factored load and moments: P_u = 1200 kips, M_{ux} = 300 kip-ft, and M_{uy} = 125 kip-ft utilizing the most commonly used approximate manual design procedures and the exact design procedure. The following approximate procedures are discussed in this example:

1. Bresler Reciprocal Load Method
2. Bresler Load Contour Method
3. PCA Load Contour Method

Figure 1 shows the reinforced concrete square column cross section under consideration. We will compare the calculated approximate values with the exact hand calculated values and automated results obtained from the spColumn engineering software program from StructurePoint formerly the PCA engineering Software Group The steps to develop the three-dimensional failure surface (3D interaction diagram) using spColumn will be shown in detail as well.

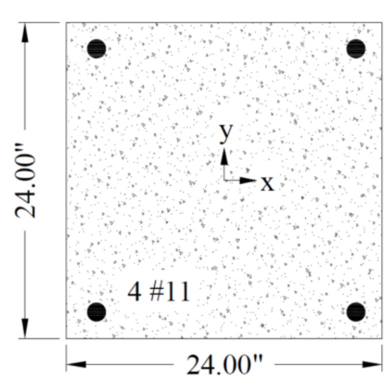


Figure 1 – Reinforced Concrete Column Cross-Section

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary (ACI 318R-11)

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association, Example 7.8 (this reference will be referred as "PCA Notes" in the rest of this document)

spColumn Engineering Software Program Manual v7.00, StructurePoint, 2019 (with support for ACI 318-19)

Pannell, F. N., "The Design of Biaxially Loaded Columns by Ultimate Load Methods," *Magazine of Concrete Research*, London, July 1960, pp. 103-104.

Pannell, F. N., "Failure Surfaces for Members in Compression and Biaxial Bending," *ACI Journal, Proceedings* Vol. 60, January 1963, pp. 129-140.

Bresler, Boris, "Design Criteria for Reinforced Columns under Axial Load and Biaxial Bending," *ACI Journal, Proceedings* Vol. 57, November 1960, pp. 481-490, discussion pp. 1621-1638.

Furlong, Richard W., "Ultimate Strength of Square Columns under Biaxially Eccentric Loads," *ACI Journal, Proceedings* Vol. 58, March 1961, pp. 1129-1140.

Meek, J. L., "Ultimate Strength of Columns with Biaxially Eccentric Loads," *ACI Journal, Proceedings* Vol., 60, August 1963, pp. 1053-1064.

Aas-Jakobsen, A., "Biaxial Eccentricities in Ultimate Load Design," *ACI Journal, Proceedings* Vol. 61, March 1964, pp. 293-315.

Ramamurthy, L. N., "Investigation of the Ultimate Strength of Square and Rectangular Columns under Biaxially Eccentric Loads," Symposium on Reinforced Concrete Columns, American Concrete Institute, Detroit, MI, 1966, pp. 263-298.

Capacity of Reinforced Rectangular Columns Subject to Biaxial Bending, Publication EB011D, Portland Cement Association, Skokie, IL, 1966.

Biaxial and Uniaxial Capacity of Rectangular Columns, Publication EB031D, Portland Cement Association, Skokie, IL, 1967.

Design Data

f_c' = 5000 psi

f_y = 60000 psi

Cover = 2.0 in.

Reinforcement = 4 #11

Column dimensions and reinforcement locations are shown in Figure 2.

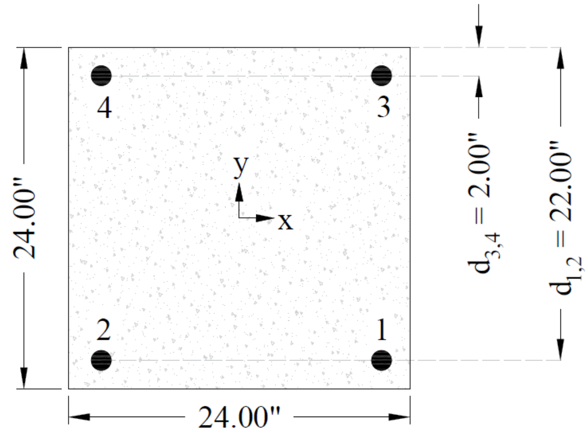


Figure 2 – Reinforced Concrete Column Cross-Section and Reinforcement Locations

Solution – Manual Design Procedure

The nominal strength of a section under biaxial bending and compression is a function of three variables P_n, M_{nx} and M_{ny} which may be expressed in terms of an axial load acting at eccentricities $e_x = M_{ny}/P_n$ and $e_y = M_{nx}/P_n$ as shown in Figure 3. A failure surface may be described as a surface produced by plotting the failure load P_n as a function of its eccentricities e_x and e_y, or of its associated bending moments M_{ny} and M_{nx}. Three types of failure surfaces have been defined. The basic surface S_1 is defined by a function which is dependent upon the variables P_n, e_x and e_y, as shown in Figure 4(a). a reciprocal surface can be derived from S_1 in which the reciprocal of the nominal axial load P_n is employed to produce the surface S_2 $(1/P_n, e_x, e_y)$ as illustrated in Figure 4(b). the third type of failure surface, shown in Figure 4(c), is obtained by relating the nominal axial load P_n to the moments M_{nx} and M_{ny} to produce surface S_3 (P_n, M_{nx}, M_{ny}). Failure surface S_3 is the three-dimensional extension of the uniaxial interaction diagram previously described. A number of investigators have made approximations for both the S_2 and S_3 failure surfaces for use in design and analysis. An explanation of these methods used in current practice is given below.

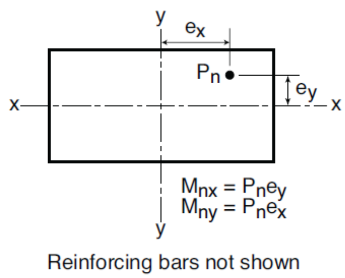


Figure 3 – Notation for Biaxial Loading

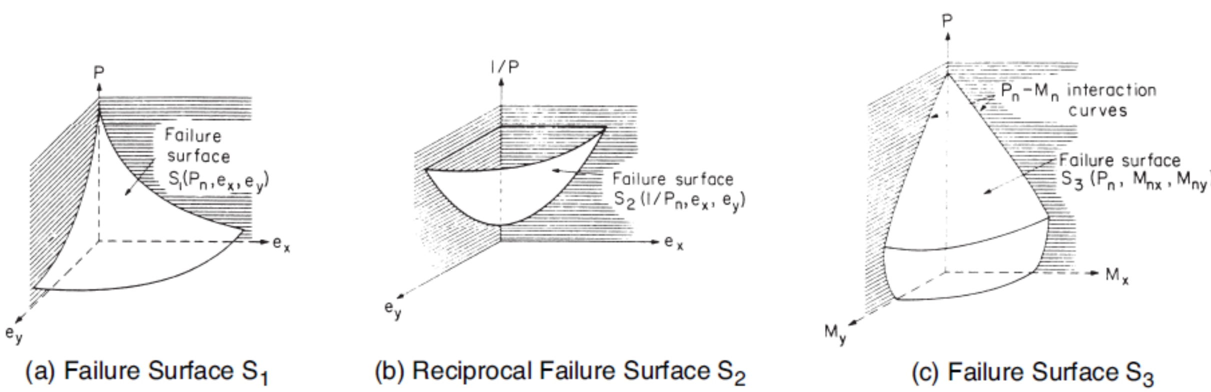


Figure 4 – Failure Surfaces

The following outlines the approximate manual design procedure proposed by the reference to estimate the capacity of column or wall sections subjected to combined axial load and biaxial bending moments. The procedure is comprised of four key steps where the fourth step covers the approximate biaxial methods.

STEP A) Choose the value of β at 0.65 or use **Figures 7-15 and 7-16 in Chapter 7 in PCA Notes** to make an estimate.

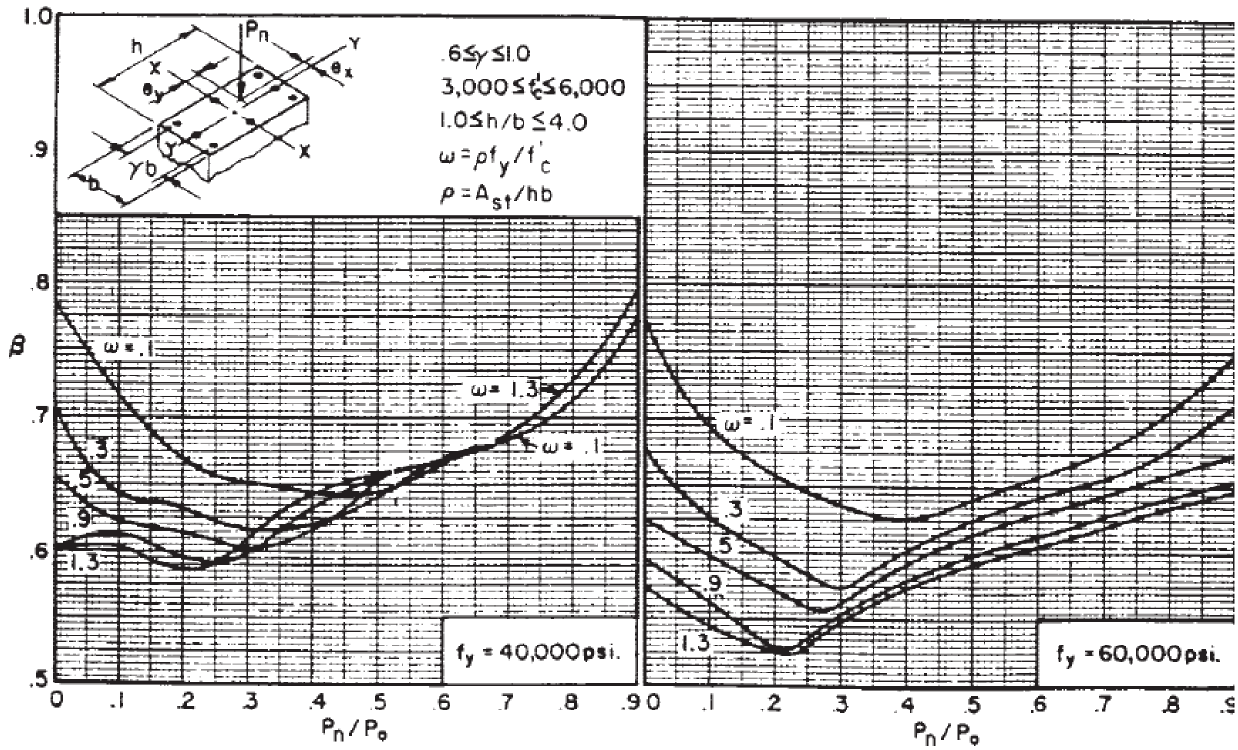


Figure 5 – Sample of Figure 7-15 (Biaxial Design Constants - 4 Bar Arrangement)

For this example, assume β = 0.65 and compression-controlled behavior

$\therefore \phi = 0.65$ ***PCA Notes (Chapter 7 Eq. 17)***

The minimum required strengths equal to:

$$P_{n\_req} = \frac{P_u}{\phi} = \frac{1200}{0.65} = 1846 \text{ kips}$$

$$M_{nx\_req} = \frac{M_{ux}}{\phi} = \frac{300}{0.65} = 461.5 \text{ kip-ft}$$

$$M_{ny\_req} \frac{M_{uy}}{\phi} = \frac{125}{0.65} = 192.3 \text{ kip-ft}$$

STEP B) If M_{ny_req}/M_{nx_req} is greater than b/h, use **Eq. (17) in Chapter 7 in PCA Notes** to calculate an approximate equivalent uniaxial moment strength M_{noy_req}. If M_{ny_req}/M_{nx_req} is less than b/h, use **Eq. (20) in Chapter 7 in PCA Notes** to calculate an approximate equivalent uniaxial moment strength M_{nox_req}.

$$M_{nx\_req} \times \frac{b}{h} \times \left(\frac{1-\beta}{\beta}\right) + M_{ny\_req} \approx M_{noy\_req}$$ **PCA Notes (Chapter 7 Eq. 17)**

$$M_{nx\_req} + M_{ny\_req} \times \frac{h}{b} \times \left(\frac{1-\beta}{\beta}\right) \approx M_{nox\_req}$$ **PCA Notes (Chapter 7 Eq. 20)**

$$\frac{M_{ny\_req}}{M_{nx\_req}} = \frac{192.3}{461.5} = 0.42 < \frac{b}{h} = \frac{24}{24} = 1.00 \rightarrow \text{Use Eq. 20}$$

$$M_{nox\_req} \approx 461.5 + 192.3 \times \frac{24}{24} \times \left(\frac{1-0.65}{0.65}\right) = 565.1 \text{ kip-ft}$$ **PCA Notes (Chapter 7 Eq. 20)**

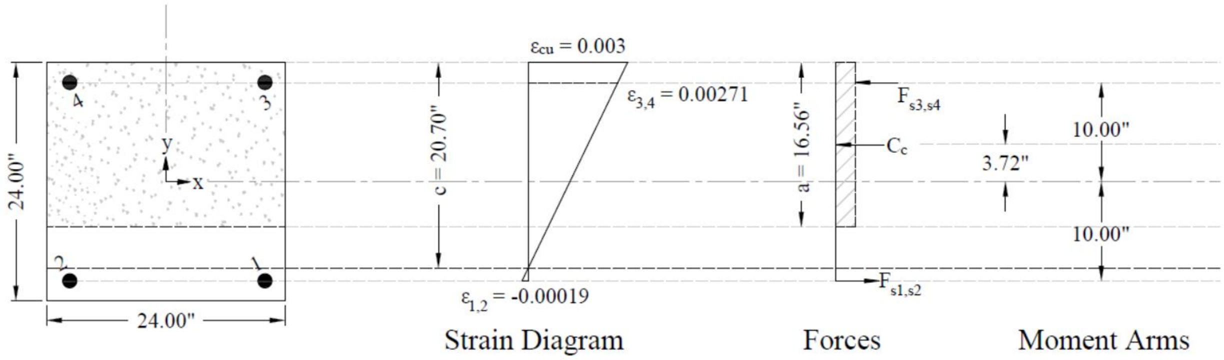
STEP C) Design the section using any suitable approximate or exact method for uniaxial bending with axial load to provide an axial load strength P_{n_req} and an equivalent uniaxial moment strength M_{noy_req} or M_{nox_req}.

In this example, the exact calculations of column strength for uniaxial bending with axial load is used. Complete details about this procedure is provided in "Interaction Diagram – Tied Reinforced Concrete Column" design example. Using the same procedure explained in the design example (Summarized in Figure 6), a 24 in. square column with 4 #11 bars provides the following capacities:

$P_n = 1846 = P_{n\_req}$

$M_{nox} = 682.8$ kip-ft > $M_{nox\_req} = 565.1$ kip-ft

∴ The section is adequate with this reinforcement for (P_{n_req}, M_{nox_req}).



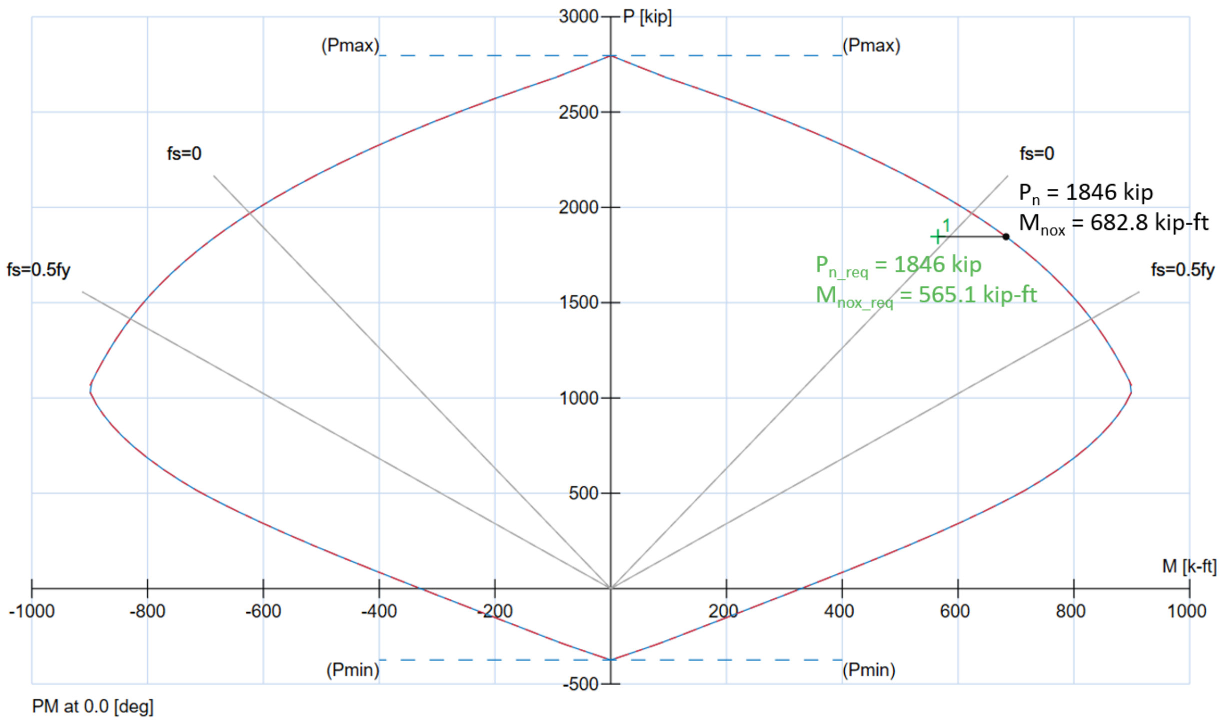


Figure 6 – Column Section Capacity Interaction Diagram (spColumn)

STEP D) Verify the suitability of the chosen section by any one of the following four methods:

1. Bresler Reciprocal Load Method.
2. Bresler Load Contour Method.
3. PCA Load Contour Method.
4. Exact Biaxial Bending-Axial Interaction Method.

1. Bresler Reciprocal Load Method

1.1. Theory

This method approximates the ordinate $1/P_n$ on the surface S_2 ($1/P_n$, e_x, e_y) by a corresponding ordinate $1/P_n'$ on the plane S'_2 ($1/P_n'$, e_x, e_y), which is defined by the characteristic points A, B and C, as indicated in Figure 7. For any particular cross-section, the value P_o (corresponding to point C) is the load strength under pure axial compression; P_{ox} (corresponding to point B) and P_{oy} (corresponding to point A) are the load strengths under uniaxial eccentricities e_y and e_x, respectively. Each point on the true surface is approximated by a different plane; therefore, the entire surface is approximated using an infinite number of planes.

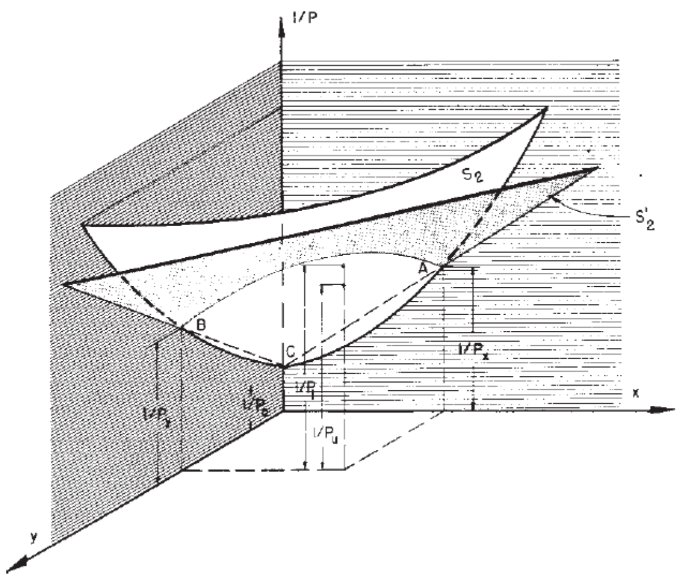


Figure 7 – Reciprocal Load Metho

The general expression for axial load strength for any values of e_x and e_y is as follows:

$$P_n = \frac{1}{\dfrac{1}{P_{ox}} + \dfrac{1}{P_{oy}} - \dfrac{1}{P_o}}$$

PCA Notes (Chapter 7 Eq. 7)

where

P_{ox} = Maximum uniaxial load strength of the column with a moment of $M_{nx\_req} = P_{n\_req} \times e_y$

P_{oy} = Maximum uniaxial load strength of the column with a moment of $M_{ny\_req} = P_{n\_req} \times e_x$

P_o = Maximum axial load strength with no applied moments

This equation is simple in form and the variables are easily determined. Axial load strengths P_o, P_{ox}, and P_{oy} are determined using the method presented in "Interaction Diagram – Tied Reinforced Concrete Column" design example. Experimental results have shown the above equation to be reasonably accurate when flexure does not govern design. The equation should only be used when:

$$P_n \geq 0.1 \times f_c^{'} \times A_g$$ ***PCA Notes (Chapter 7 Eq. 8)***

1.2. Calculations

Check if flexure does not govern design:

$$P_n \geq 0.1 \times f_c^{'} \times A_g$$ ***PCA Notes (Chapter 7 Eq. 8)***

$$P_n = 1846 \text{ kip} \geq 0.1 \times f_c^{'} \times A_g = 0.1 \times 5000 \times (24 \times 24) = 288 \text{ kip}$$ ***(O.K.)***

P_o, P_{ox}, and P_{oy} need to be determined to use this method:

$$P_o = 0.85 \times f_c^{'} \times (A_g - A_{st}) + f_y \times A_{st}$$

$$P_o = 0.85 \times 5000 \times (24 \times 24 - 4 \times 1.56) + 60000 \times (4 \times 1.56) = 2796 \text{ kip}$$

P_{ox} is the uniaxial load strength when only M_{nx_req} acts on the column. As shown in Figure 8, for M_{nx_req} = 461.5 kip-ft, P_{ox} equals to 2241 kip.

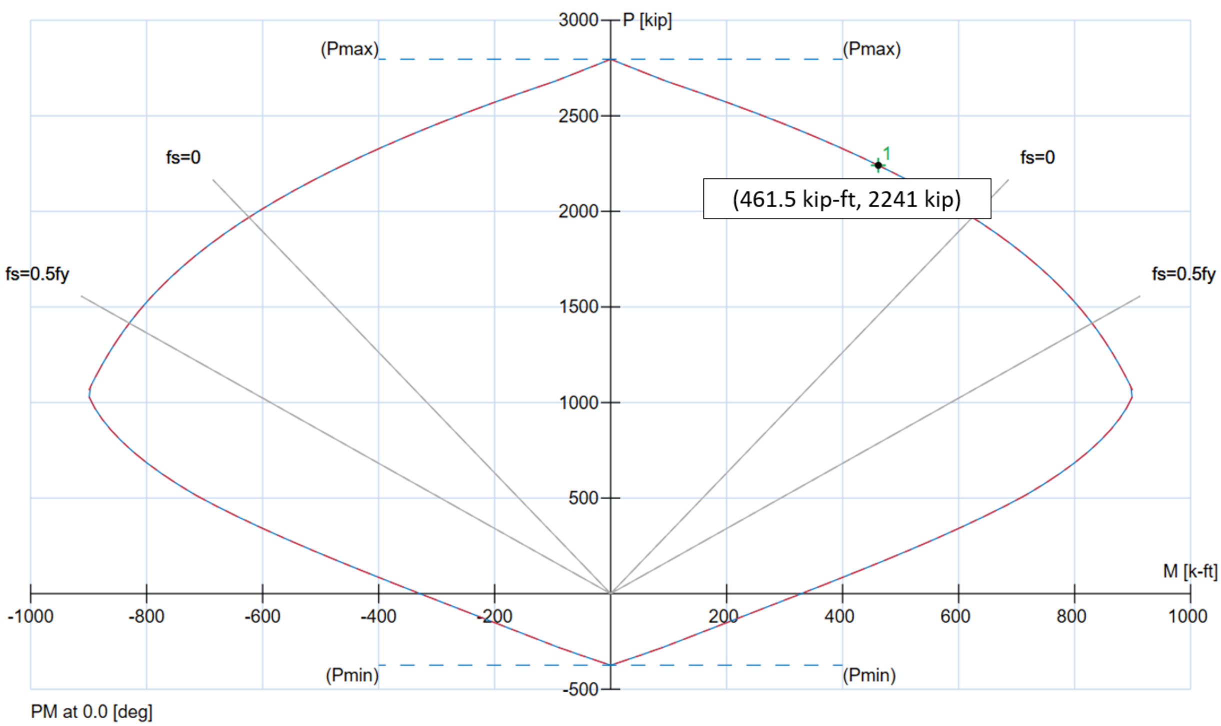


Figure 8 – Column Uniaxial Interaction Diagram (P_{nox} and M_{nx_req})

Similarly, P_{oy} is the uniaxial load strength when only M_{ny_req} acts on the column. As shown in Figure 9, for M_{ny_req} = 192.3 kip-ft, P_{oy} equals to 2579.5 kip.

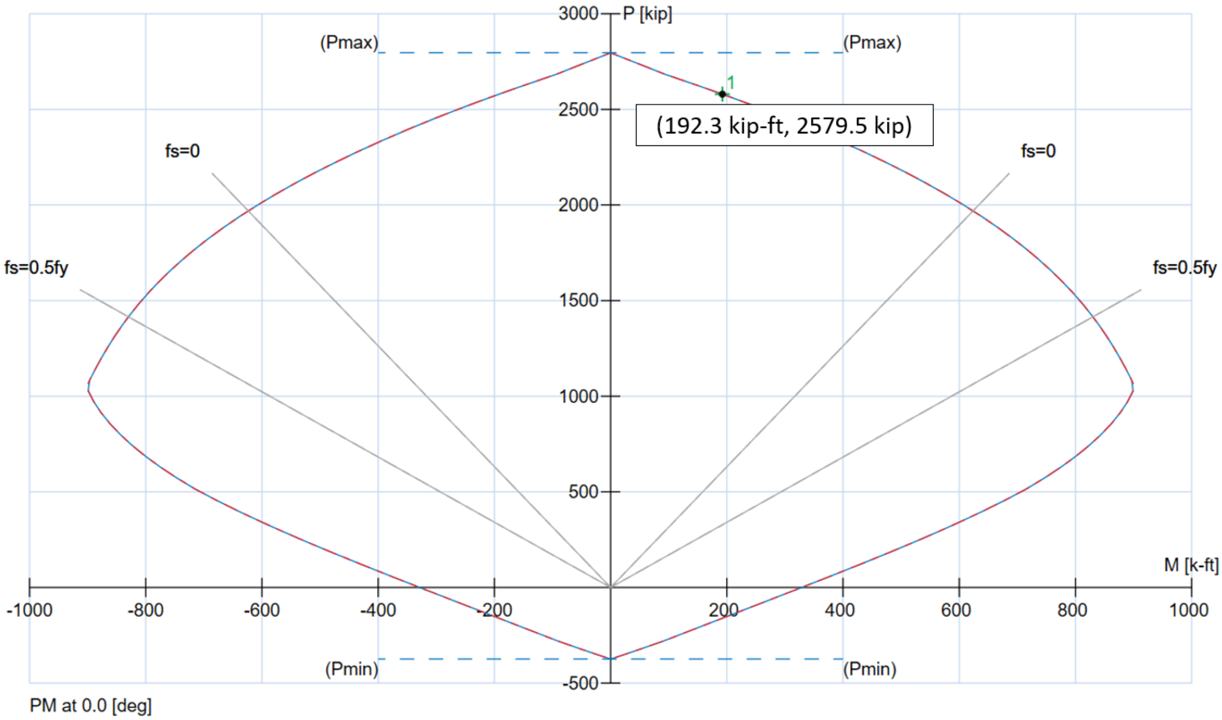


Figure 9 – Column Uniaxial Interaction Diagram (P_{noy} and M_{ny_req})

Note that both P_{ox} (2241 kip) and P_{oy} (2579.5 kip) are greater than the balanced axial force (1049.2 kip), so that the section is compression-controlled.

Using the values above, the axial load strength equals to:

$$P_n = \frac{1}{\dfrac{1}{P_{ox}} + \dfrac{1}{P_{oy}} - \dfrac{1}{P_o}}$$

PCA Notes (Chapter 7 Eq. 7)

$$P_n = \frac{1}{\dfrac{1}{2241.0} + \dfrac{1}{2579.5} - \dfrac{1}{2796.0}} = 2100.0 \text{ kip} > P_n = 1846 \text{ kip}$$

(O.K.)

2. Bresler Load Contour Method

2.1. Theory

In this method, the surface S_3 (P_n, M_{nx}, M_{ny}) is approximated by a family of curves corresponding to constant values of P_n. These curves, as illustrated in Figure 10, may be regarded as "load contours".

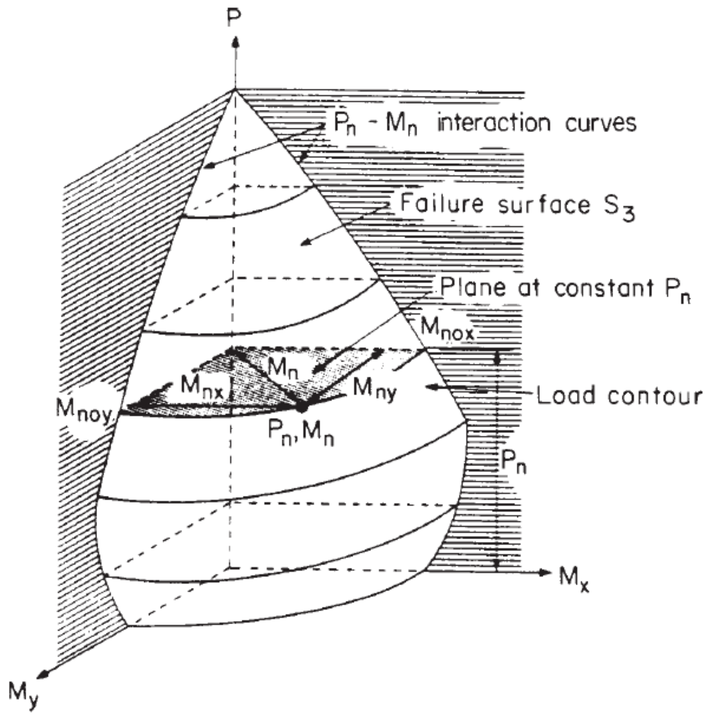


Figure 10 – Bresler Load Contours for Constant P_n on Failure Surface S_3

The general expression for these curves can be approximated by a nondimensional interaction equation of the form:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha}+\left(\frac{M_{ny}}{M_{noy}}\right)^{\beta}=1.0$$

PCA Notes (Chapter 7 Eq. 9)

where

M_{nx} = The nominal biaxial moment strength in the direction of x axis

M_{ny} = The nominal biaxial moment strength in the direction of y axis

M_{nox} = The nominal uniaxial moment strength about the x-axis

M_{noy} = The nominal uniaxial moment strength about the y-axis

Note that (M_{nx} and M_{ny}) are the vectorial equivalent of the nominal uniaxial moment M_n. The values of the exponents α and β are a function of the amount, distribution and location of reinforcement, the dimensions of the column, and the strength and elastic properties of the steel and concrete. Bresler indicates that it is reasonably accurate to assume that α = β; therefore, the previous equation becomes (shown graphically in Figure 11):

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\alpha} = 1.0$$ ***PCA Notes (Chapter 7 Eq. 10)***

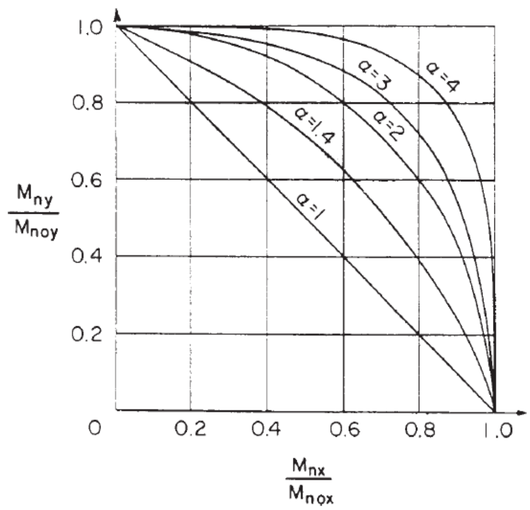


Figure 11 – Interaction Curves for Bresler Load Contour Method

When using the previous equation or figure, it is still necessary to determine the α value for the cross-section being designed. Bresler indicated that, typically, α varied from 1.15 to 1.55, with a value of 1.5 being reasonably accurate for most square and rectangular sections having uniformly distributed reinforcement.

With α set at unity, the interaction equation becomes linear:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} = 1.0$$ ***PCA Notes (Chapter 7 Eq. 11)***

The previous equation would always yield conservative results since it underestimates the column capacity, especially for high axial loads or low percentages of reinforcement. It should only be used when:

$$P_n < 0.1 \times f_c^{'} \times A_g$$ ***PCA Notes (Chapter 7 Eq. 12)***

2.2. Calculations

The reference conservatively selected α = 1.0 due to a lack of available data. Check if flexure does not govern design:

$$P_n < 0.1 \times f_c^{'} \times A_g$$ ***PCA Notes (Chapter 7 Eq. 12)***

$$P_n = 1846 \text{ kip} > 0.1 \times f_c^{'} \times A_g = 0.1 \times 5000 \times (24 \times 24) = 288 \text{ kip}$$ ***(No Good)***

Although $P_n > 0.1 \times f_c^{'} \times A_g$, the reference decided to carry out the necessary calculations for illustration purposes.

Since the section is symmetrical:

$$M_{nox} = M_{noy} = 682.8 \text{ kip-ft (Figure 6)}$$

Using the interaction equation becomes linear:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} = 1.0$$ ***PCA Notes (Chapter 7 Eq. 11)***

$$\frac{M_{nx\_req}}{M_{nox}} + \frac{M_{ny\_req}}{M_{noy}} = \frac{461.5}{682.8} + \frac{192.3}{682.8} = 0.96 < 1.0 = \frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}}$$ ***(O.K.)***

3. PCA Load Contour Method

3.1. Theory

The PCA approach described below was developed as an extension of the Bresler Load Contour Method. The Bresler interaction equation was chosen as the most viable method in terms of accuracy, practicality, and simplification potential.

A typical Bresler load contour for a certain P_n is shown in the following Figure 12. In the PCA method, point B is defined such that the nominal biaxial moment strengths M_{nx} and M_{ny} at this point are in the same ratio as the uniaxial moment strengths M_{nox} and M_{noy}. Therefore, at point B (M_{nx} / M_{ny} = M_{nox} / M_{noy}).

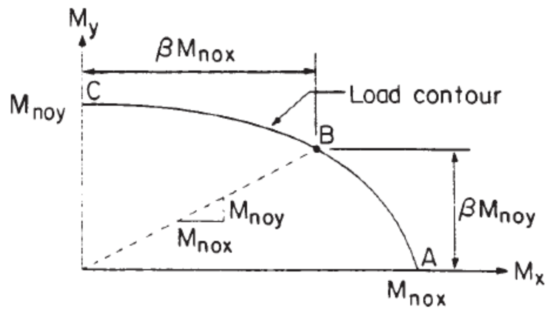


Figure 12 – Load Contour of Failure Surface S_3 along Plane of Constant P_n

When the load contour of Figure 12 is nondimensionalized, it takes the form shown in Figure 13, and the point B will have x and y coordinates of β. When the bending resistance is plotted in terms of the dimensionless parameters P_n/P_o, M_{nx}/M_{nox}, M_{ny}/M_{noy} (the latter two designated as the relative moments), the generated failure surface S4 (P_n/P_o, M_{nx}/M_{nox}, M_{ny}/M_{noy}) assumes the typical shape shown in Figure 14. The advantage of expressing the behavior in relative terms is that the contours of the surface (Figure 13) - i.e., the intersection formed by planes of constant P_n/P_o and the surface - can be considered for design purposes to be symmetrical about the vertical plane bisecting the two coordinate planes. Even for sections that are rectangular or have unequal reinforcement on the two adjacent faces, this approximation yields values sufficiently accurate for design.

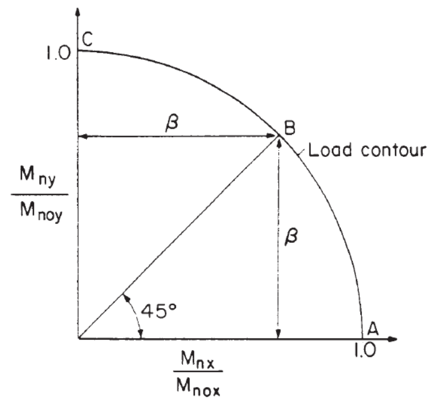


Figure 13 – Nondimensional Load Contour at Constant P_n

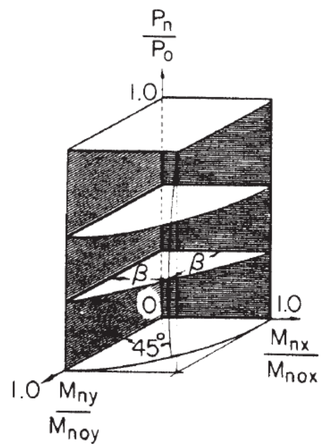


Figure 14 – Nondimensional Load Contour at Constant P_n

The reference obtained the relationship between α from Eq. (10) and β by substituting the coordinates of point B from Figure 11 into Eq. (10), and solving for α in terms of β. This yields:

$$\alpha = \frac{\log 0.5}{\log \beta}$$

Thus, Eq. (10) may be written as:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)} = 1.0$$

PCA Notes (Chapter 7 Eq. 14)

For design convenience, a plot of the curves generated by Eq. (14) for nine values of β are given in Figure 15. Note that when β = 0.5, its lower limit, Eq. (14) is a straight line joining the points at which the relative moments equal 1.0 along the coordinate planes. When β = 1.0, its upper limit, Eq. (14) is two lines, each of which is parallel to one of the coordinate planes.

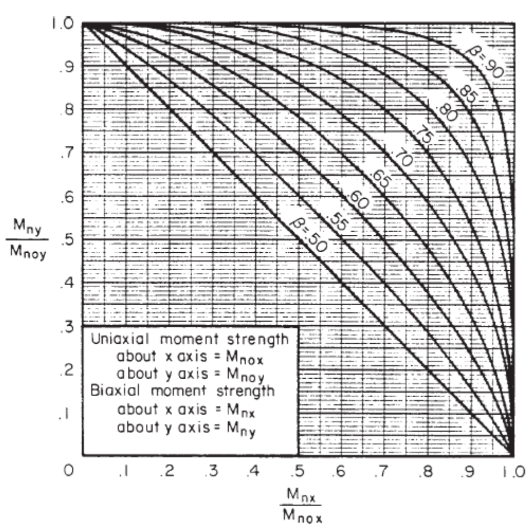
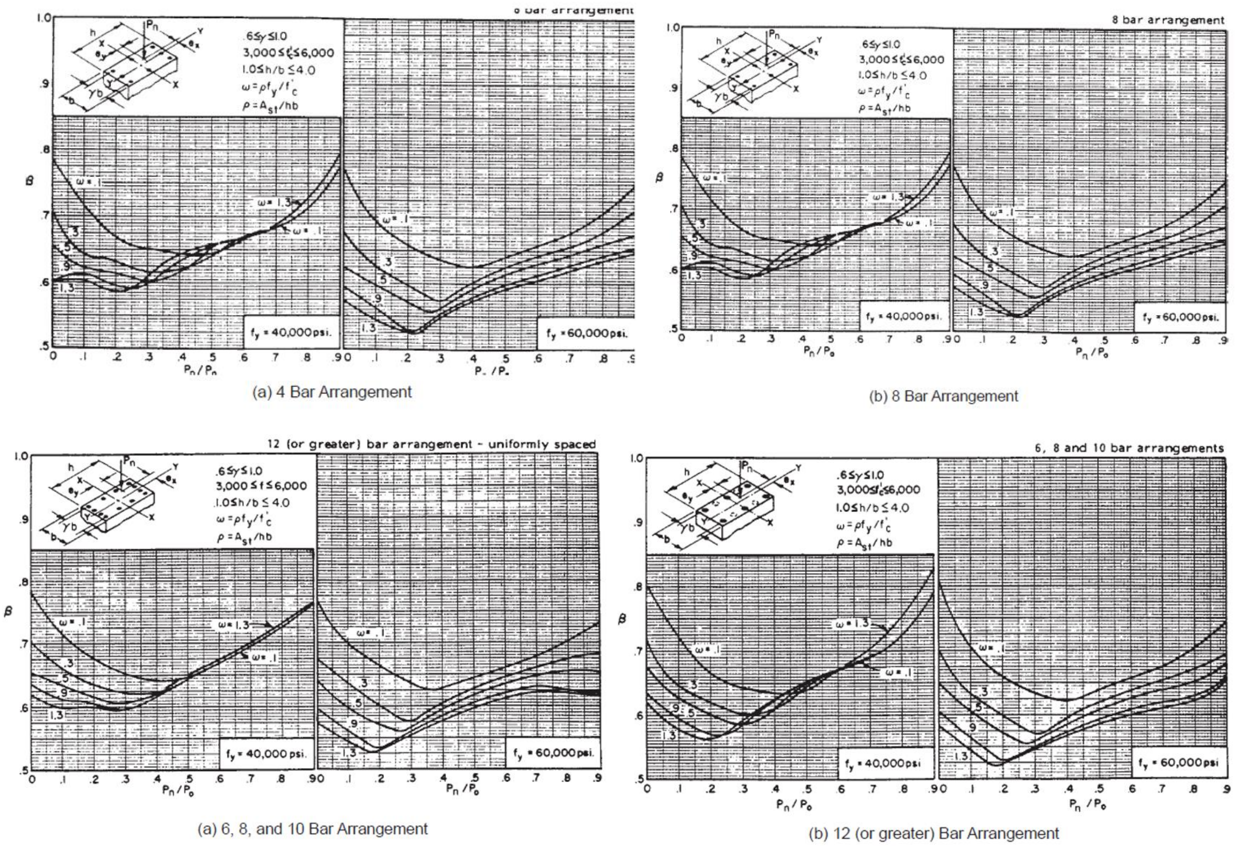


Figure 15 – Biaxial Moment Strength Relationship

β is dependent primarily on the ratio P_n/P_o and to a lesser, though still significant extent, on the bar arrangement, the reinforcement index ω and the strength of the reinforcement as shown in Figure 16.



(a) 4 Bar Arrangement

(b) 8 Bar Arrangement

(a) 6, 8, and 10 Bar Arrangement

(b) 12 (or greater) Bar Arrangement

Figure 16 – Biaxial Design Constants

Rapid and easy convergence to a satisfactory section can be achieved by approximating the curves in Figure 15 by two straight lines intersecting at the 45 degree line, as shown in Figure 17.

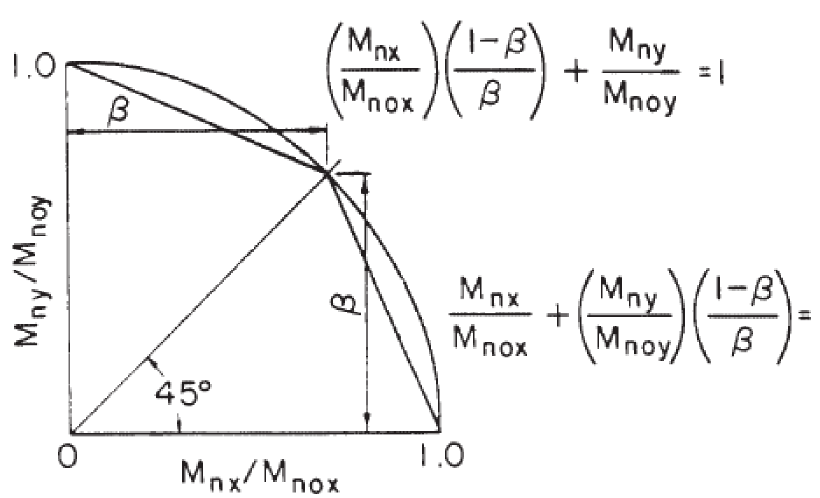


Figure 17 – Bilinear Approximation of Nondimensionalized Load Contour

By simple geometry, it can be shown that the equation of the upper lines is:

$$\frac{M_{nx}}{M_{nox}} \times \left(\frac{1-\beta}{\beta}\right) + \frac{M_{ny}}{M_{noy}} = 1.0 \quad \text{for} \quad \frac{M_{ny}}{M_{nx}} > \frac{M_{noy}}{M_{nox}}$$ ***PCA Notes (Chapter 7 Eq. 15)***

which can be restated for design convenience as follows:

$$M_{nx} \times \left(\frac{M_{noy}}{M_{nox}}\right) \times \left(\frac{1-\beta}{\beta}\right) + M_{ny} = M_{noy}$$ ***PCA Notes (Chapter 7 Eq. 16)***

For rectangular sections with reinforcement equally distributed on all faces, Eq. (16) can be approximated by:

$$M_{nx} \times \left(\frac{b}{h}\right) \times \left(\frac{1-\beta}{\beta}\right) + M_{ny} \approx M_{noy}$$ ***PCA Notes (Chapter 7 Eq. 17)***

The equation of the lower line of Figure 17 is:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} \times \left(\frac{1-\beta}{\beta}\right) = 1.0 \quad \text{for} \quad \frac{M_{ny}}{M_{nx}} < \frac{M_{noy}}{M_{nox}}$$ ***PCA Notes (Chapter 7 Eq. 18)***

$$M_{nx} + M_{ny} \times \left(\frac{M_{nox}}{M_{noy}}\right) \times \left(\frac{1-\beta}{\beta}\right) = M_{nox}$$ ***PCA Notes (Chapter 7 Eq. 19)***

For rectangular sections with reinforcement equally distributed on all faces,

$$M_{nx} + M_{ny} \times \left(\frac{h}{b}\right) \times \left(\frac{1-\beta}{\beta}\right) \approx M_{nox}$$ ***PCA Notes (Chapter 7 Eq. 20)***

In design Eqs. (17) and (20), the ratio b/h or h/b must be chosen and the value of β must be assumed. For lightly loaded columns, β will generally vary from 0.55 to about 0.70. Hence, a value of 0.65 for β is generally a good initial choice in a biaxial bending analysis.

3.2. Calculations

P_o, M_{nox}, M_{noy} and β need to be determined to use this method:

$$P_o = 0.85 \times f_c^{'} \times \left(A_g - A_{st}\right) + f_y \times A_{st}$$

$$P_o = 0.85 \times 5000 \times \left(24 \times 24 - 4 \times 1.56\right) + 60000 \times \left(4 \times 1.56\right) = 2796 \text{ kip}$$

Since the section is symmetrical:

$$M_{nox} = M_{noy} = 682.8 \text{ kip-ft (Figure 6)}$$

β can be determined using P_o, ρ_g and the biaxial design constants figure:

$$\frac{P_n}{P_o} = \frac{1846}{2796} = 0.66$$

$$\omega = \frac{\rho_g \times f_y}{f_c^{'}} = \frac{0.011 \times 60000}{5000} = 0.13 \text{, where } \rho_g = \frac{A_{st}}{b \times h} = \frac{4 \times 1.56}{24 \times 24} = 0.011$$

β = 0.66 (Figure 18):

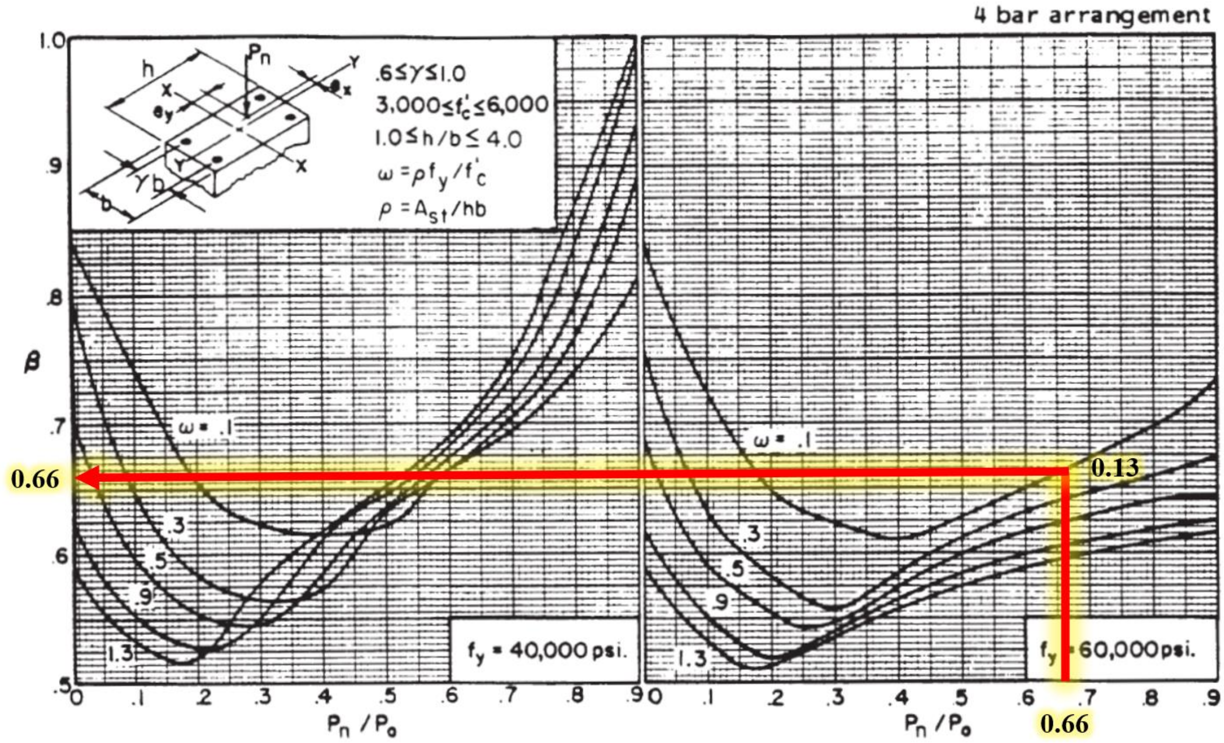


Figure 18 – Biaxial Design Constants (4 Bar Arrangement)

Using the accurate equation (equation 14):

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)}+\left(\frac{M_{ny}}{M_{noy}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)}=1.0$$ ***PCA Notes (Chapter 7 Eq. 14)***

$$\left(\frac{M_{nx\_req}}{M_{nox}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)}+\left(\frac{M_{ny\_req}}{M_{noy}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)}<\left(\frac{M_{nx}}{M_{nox}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)}+\left(\frac{M_{ny}}{M_{noy}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)}$$

$$\left(\frac{461.5}{682.8}\right)^{\left(\frac{\log 0.5}{\log 0.66}\right)}+\left(\frac{192.3}{682.8}\right)^{\left(\frac{\log 0.5}{\log 0.66}\right)}=0.52+0.12=0.64<1.0$$ ***(O.K.)***

Using bilinear approximation (Eqs. 15 and 18):

$$\frac{M_{ny\_req}}{M_{nx\_req}}=\frac{192.3}{461.5}=0.42<\frac{M_{noy}}{M_{nox}}=\frac{682.8}{682.8}=1.00$$

$\therefore$ Eq. 18 should be used

$$\frac{M_{nx}}{M_{nox}}+\frac{M_{ny}}{M_{noy}}\times\left(\frac{1-\beta}{\beta}\right)=1.0 \quad \text{for} \quad \frac{M_{ny}}{M_{nx}}<\frac{M_{noy}}{M_{nox}}$$ ***PCA Notes (Chapter 7 Eq. 18)***

$$\frac{M_{nx\_req}}{M_{nox}}+\frac{M_{ny\_req}}{M_{noy}}\times\left(\frac{1-\beta}{\beta}\right)<\frac{M_{nx}}{M_{nox}}+\frac{M_{ny}}{M_{noy}}\times\left(\frac{1-\beta}{\beta}\right)$$

$$\frac{461.5}{682.8}+\frac{192.3}{682.8}\times\left(\frac{1-0.66}{0.66}\right)=0.676+0.145=0.82<1.0$$ ***(O.K.)***

4. Exact Biaxial Bending-Axial Interaction Method

4.1. Theory

In a reinforced concrete column, the exact calculations of section capacity (axial force biaxial moment strengths) involves a trial-and-error process for calculating the neutral axis depth and angle α. The steps to calculate biaxial flexural strength of a rectangular reinforced concrete column for a given nominal axial strength are as follows:

1. Assuming a value for the angle of the neutral axis (α) and the neutral axis depth (c) and calculating the strain values in each reinforcement layer
2. Calculating the forces values in the concrete (C_c)and reinforcement layers (F_{si})
3. Calculating P_n, M_{nx} and M_{ny} using the following equations

$$P_n = C_c + \sum F_s$$

$$M_{nx} = C_c \times \left(\frac{h}{2} - \bar{y}_c \right) + \sum_{i=1}^{n=10} \left(F_{si} \times \left(\frac{h}{2} - y_i \right) \right)$$

$$M_{ny} = C_c \times \left(\frac{b}{2} - \bar{x}_c \right) + \sum_{i=1}^{n=10} \left(F_{si} \times \left(\frac{b}{2} - x_i \right) \right)$$

The procedure above should be repeated until the calculated P_n is equal to the given P_n .

The difficulty associated with the determination of the strength of reinforced columns subjected to combined axial load and biaxial bending is primarily an arithmetic one. The bending resistance of an axially loaded column about a particular skewed axis is determined through iterations involving simple but lengthy calculations. These extensive calculations are compounded when optimization of the reinforcement or cross-section is sought.

4.2. Calculations

Figure 19 summarizes the exact calculations procedure to determine column section capacity subjected to axial load and biaxial bending moments. This procedure is illustrated in detail in several design examples published by StructurePoint:

- Combined Axial Force and Biaxial Bending Interaction Diagram - Square Column
- Combined Axial Force and Biaxial Bending Interaction Diagram - Rectangular Column
- C-Shaped Concrete Core Wall Biaxial Bending Interaction Diagram

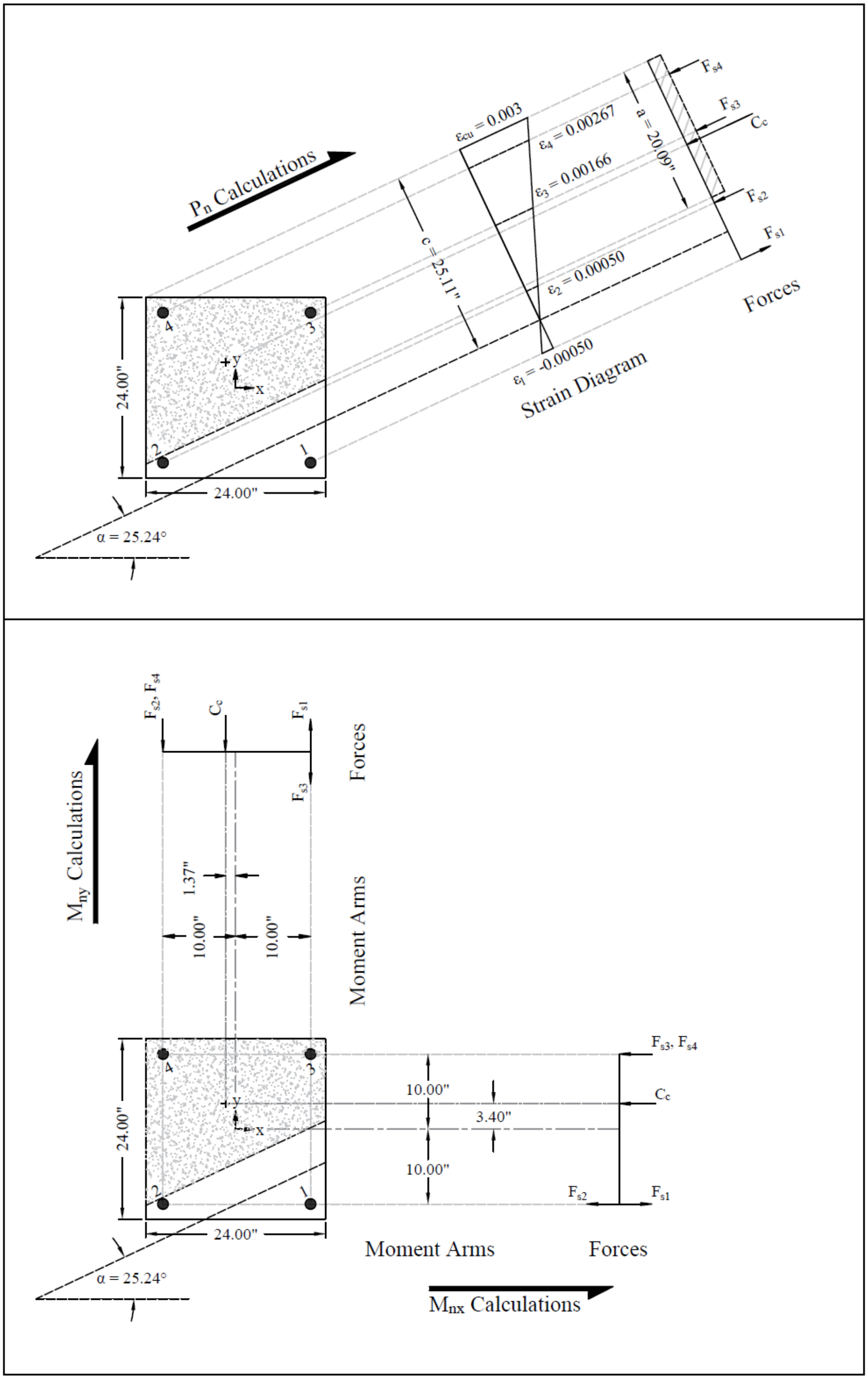


Figure 19 – Column Biaxial Bending – Axial Interaction Exact Method.

Following the procedure described above:

$$c = 25.11 \text{ in.}$$

$$\alpha = 25.24°$$

$$P_n = C_c + \sum F_s = 1846 \text{ kip} = P_{n\_req} \qquad \textbf{\textit{(O.K.)}}$$

$$M_{nx} = C_c \times \left(\frac{h}{2} - \bar{y}_c\right) + \sum_{i=1}^{n}\left(F_{si} \times \left(\frac{h}{2} - y_i\right)\right) = 608 \text{ kip-ft} > M_{nx\_req} = 461.5 \text{ kip-ft} \qquad \textbf{\textit{(O.K.)}}$$

$$M_{ny} = C_c \times \left(\frac{b}{2} - \bar{x}_c\right) + \sum_{i=1}^{n}\left(F_{si} \times \left(\frac{b}{2} - x_i\right)\right) = 245.4 \text{ kip-ft} > M_{ny\_req} = 192.3 \text{ kip-ft} \qquad \textbf{\textit{(O.K.)}}$$

5. Column Biaxial Bending Interaction Diagram – spColumn Software

Formerly pcaColumn, the spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with **"biaxial"** option for "Run Axis" using the ACI 318.

For biaxial runs, the values of maximum compressive axial load capacity and maximum tensile load capacity are computed. These two values set the range within which the moment capacities are computed for a predetermined number of axial load values. For each level of axial load, the section is rotated in 10-degree increments from 0 degrees to 360 degrees and the M_x and M_y moment capacities are computed. Thus, for each level of axial load, an M_x-M_y contour is developed. Repeating this for the entire range of axial loads, the three-dimensional failure surface is computed. A three-dimensional visualization of the resulting entire nominal and factored failure surface is provided to support enhanced understanding of the section capacity.

The **"biaxial"** feature allows the user to investigate the P-M interaction diagrams, the M_x-M_y moment contour plots, as well as the 3D failure surface for even the most irregular column and shear wall sections quickly, simply, and accurately.

In lieu of using program shortcuts, spColumn model editor was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

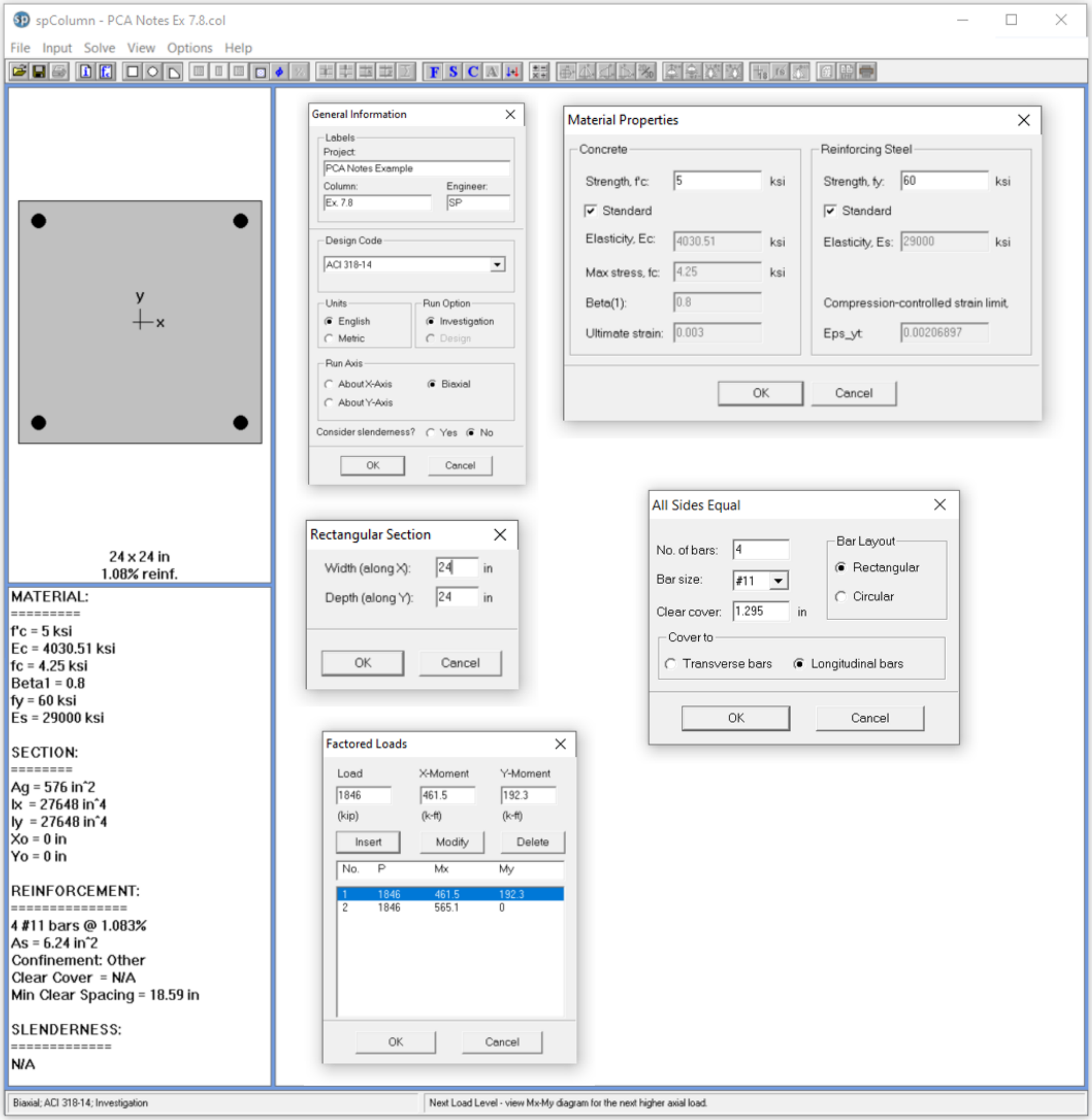


Figure 20 – Generating spColumn Model

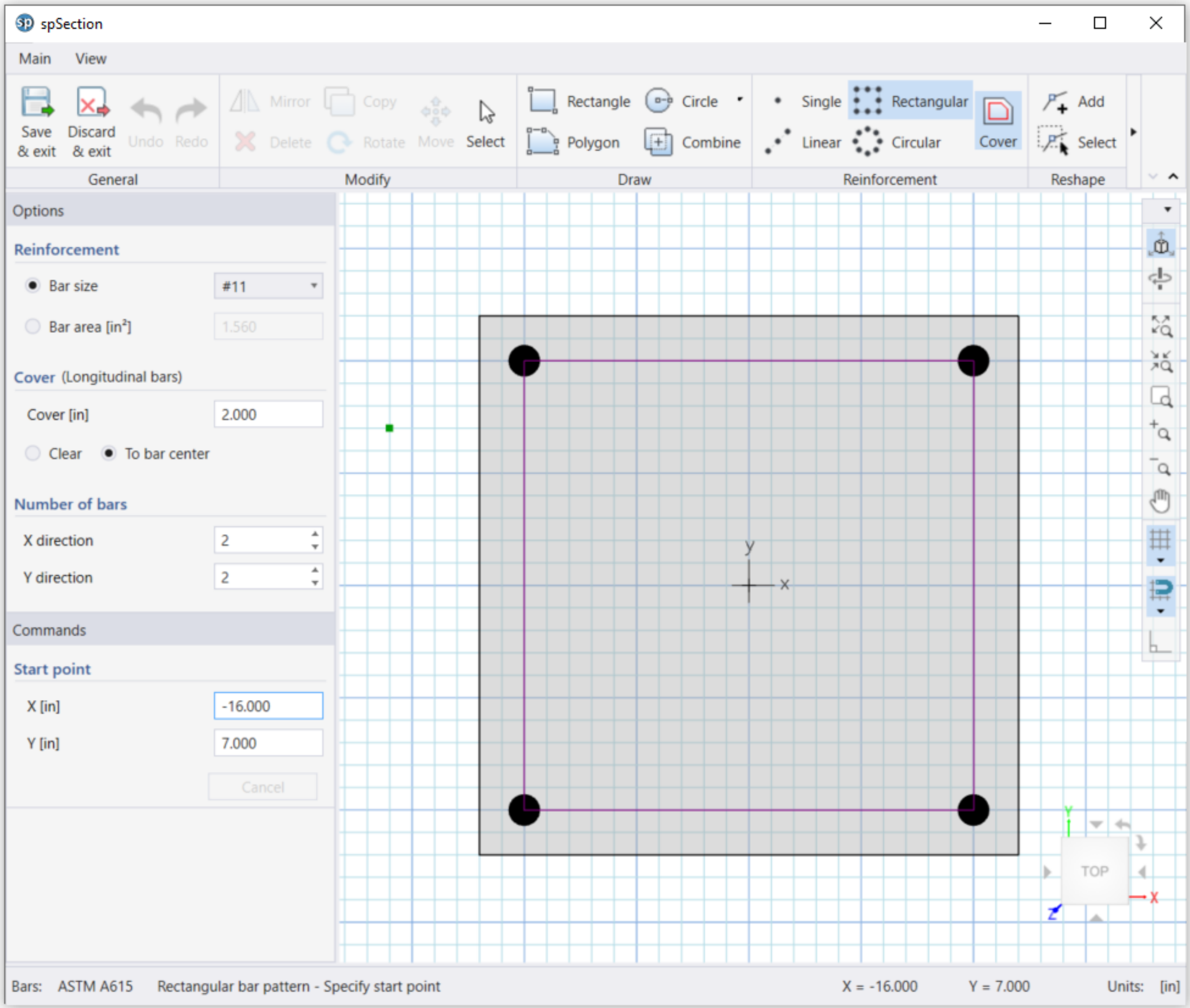


Figure 21 – spColumn Model Editor (spSection)

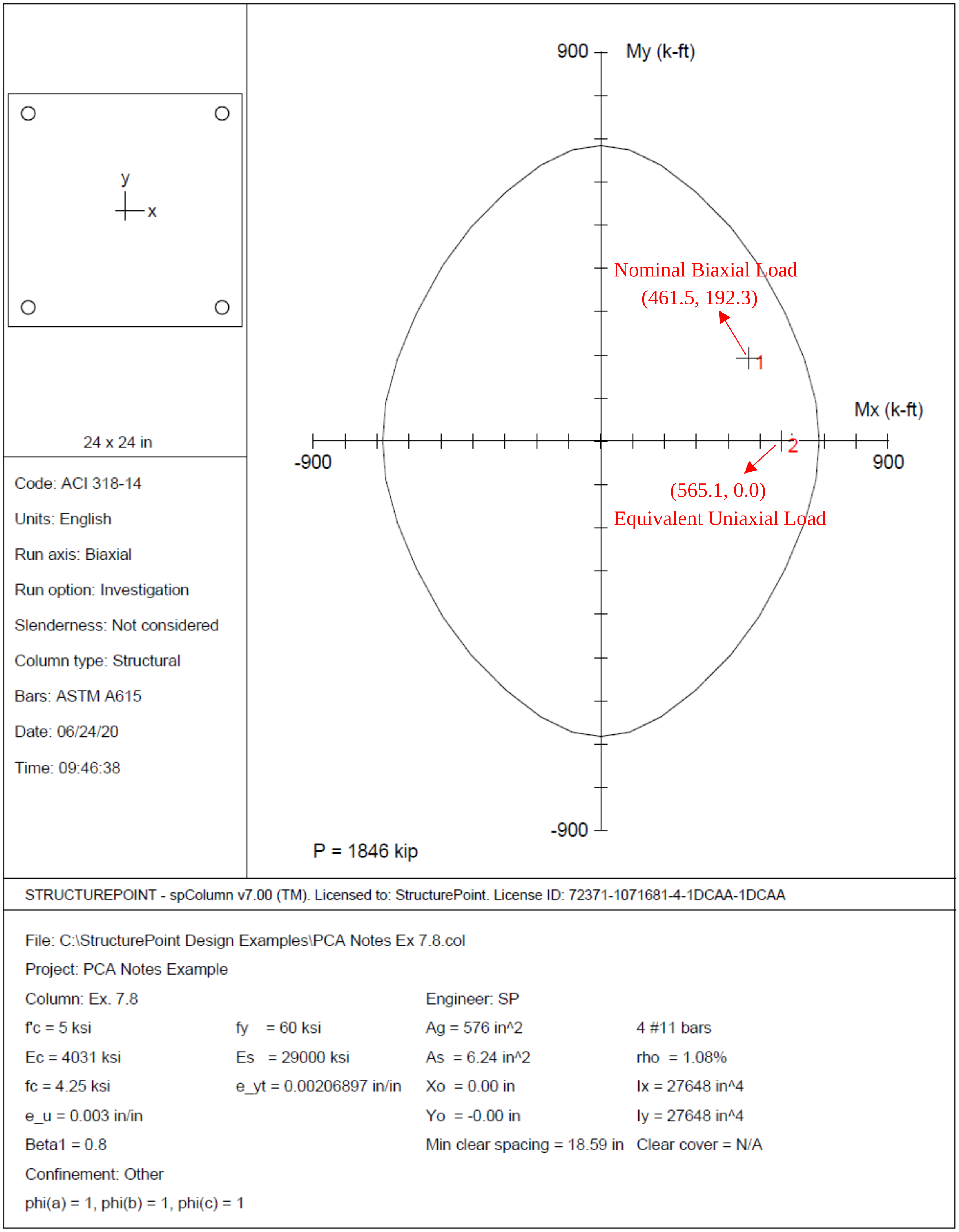


Figure 22 – Column Section M_{nx}-M_{ny} Contour at P_n = 1846 kip (spColumn)

spColumn v7.00
Computer program for the Strength Design of Reinforced Concrete Sections
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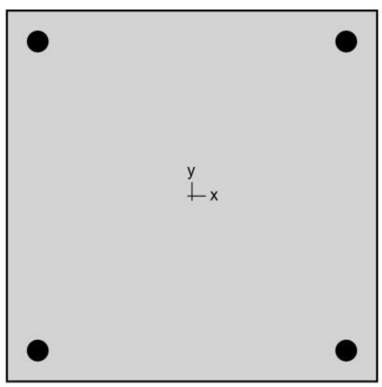

StructurePoint

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Contents

List of Figures

1. General Information

File Name	C:\StructurePoint Design ...\PCA Notes Ex 7.8.col
Project	PCA Notes Example
Column	Ex. 7.8
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

2. Material Properties

2.1. Concrete

Type	Standard	
f'_c	5	ksi
E_c	4030.51	ksi
f_c	4.25	ksi
ε_u	0.003	in/in
β_1	0.8	

2.2. Steel

Type	Standard	
f_y	60	ksi
E_s	29000	ksi
ε_{yt}	0.00206897	in/in

3. Section

3.1. Shape and Properties

Type	Irregular	
A_g	576	in^2
I_x	27648	in^4
I_y	27648	in^4
r_x	6.9282	in
r_y	6.9282	in
X_o	0	in
Y_o	0	in

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3.2. Section Figure

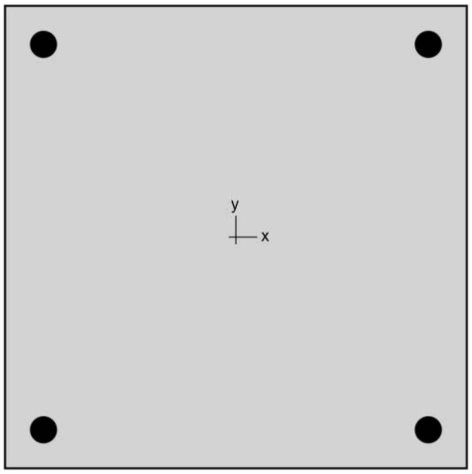

Figure 1: Column section

3.3. Exterior Points

Points	X in	Y in	Points	X in	Y in	Points	X in	Y in
1	-12.0	-12.0	2	12.0	-12.0	3	12.0	12.0
4	-12.0	12.0						

4. Reinforcement

4.1. Bar Set: ASTM A615

Bar	Diameter in	Area in²	Bar	Diameter in	Area in²	Bar	Diameter in	Area in²
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

4.2. Confinement and Factors

Confinement type	Other
For #10 bars or less	#3 ties
For larger bars	#4 ties
Capacity Reduction Factors	
Axial compression, (a)	1
Tension controlled ϕ, (b)	1
Compression controlled ϕ, (c)	1

Capacity reduction factors set to 1.0 in spColumn input for illustration purposes

4.3. Arrangement

Pattern	Irregular
Bar layout	---
Cover to	---
Clear cover	---
Bars	---
Total steel area, A_s	6.24 in²
Rho	1.08 %
Minimum clear spacing	18.59 in

4.4. Bars Provided

Area	X	Y	Area	X	Y	Area	X	Y
in²	in	in	in²	in	in	in²	in	in
1.56	-10.0	10.0	1.56	10.0	10.0	1.56	-10.0	-10.0
1.56	10.0	-10.0						

5. Factored Loads and Moments with Corresponding Capacity Ratios

NOTE: Calculations are based on "Moment Capacity" Method.

No.	Demand			Capacity			Parameters at Capacity			Capacity
	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	NA Depth	ε_t	ϕ	Ratio
	kip	k-ft	k-ft	kip	k-ft	k-ft	in			
1	1846.00	461.50	192.30	1846.00	601.97	250.83	25.11	0.00050	1.000	0.77
2	1846.00	565.10	0.00	1846.00	682.79	0.00	20.70	0.00019	1.000	0.83

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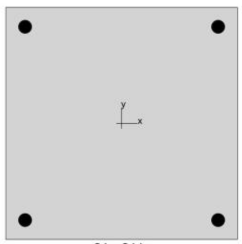
6. Diagrams

6.1. PM at θ=0 [deg]



24 x 24 in

General Information

Project	PCA Notes Example
Column	Ex. 7.8
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	Biaxial
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials

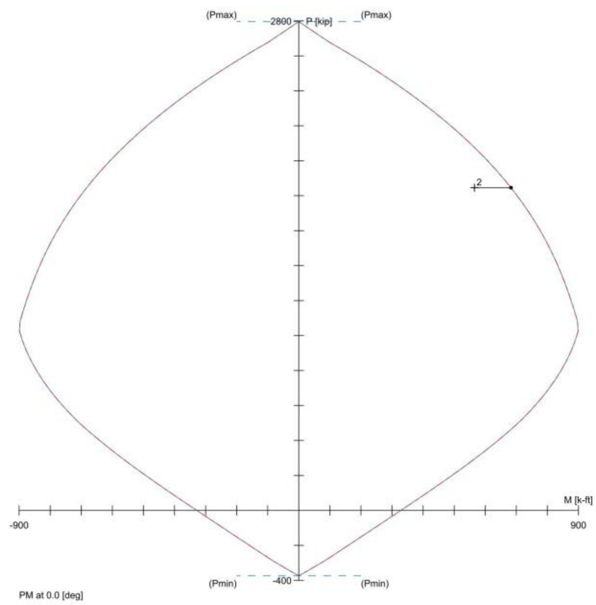
f'_c	5 ksi
E_c	4030.51 ksi
f_y	60 ksi
E_s	29000 ksi

Section

Type	Irregular
A_g	576 in²
I_x	27648 in⁴
I_y	27648 in⁴

Reinforcement

Pattern	Irregular
Bar layout	---
Cover to	---
Clear cover	---
Bars	---
Confinement type	Other
Total steel area, A_s	6.24 in²
Rho	1.08 %
Min. clear spacing	18.59 in



No.	P_u	M_{ux}	M_{uy}	ϕP_n	ϕM_{nx}	ϕM_{ny}	Capacity
	kip	k-ft	k-ft	kip	k-ft	k-ft	Ratio
2	1846.0	565.1	0.0	1846.00	682.79	0.00	0.83

Max. Capacity Ratio: 0.83

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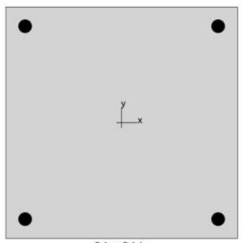
6.2. MM at P=1846 [kip]



24 x 24 in

General Information	
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Materials	
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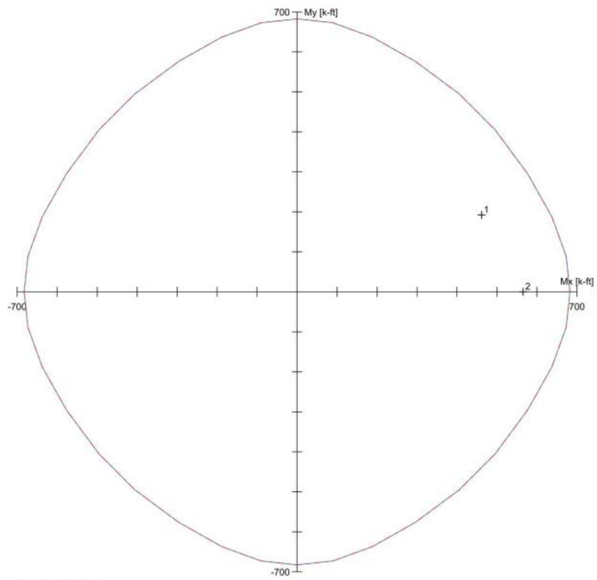
Section	
Type	Irregular
A_g	576 in²
I_x	27648 in⁴
I_y	27648 in⁴

Reinforcement	
Pattern	Irregular
Bar layout	---
Cover to	---
Clear cover	---
Bars	---
Confinement type	Other
Total steel area, A_s	6.24 in²
Rho	1.08 %
Min. clear spacing	18.59 in



MM at P=1846.0 [kip]

No.	P_u kip	M_{ux} k-ft	M_{uy} k-ft	ϕP_n kip	ϕM_{nx} k-ft	ϕM_{ny} k-ft	Capacity Ratio
2	1846.0	565.1	0.0	1846.00	682.79	0.00	0.83
1	1846.0	461.5	192.3	1846.00	601.97	250.83	0.77

Max. Capacity Ratio: 0.83

6. Summary and Comparison of Results

Figure 23 shows M_x-M_y contour generated by spColumn (exact solution). The approximate results obtained by Bresler Load Contour Method and PCA Load Contour Method are superimposed for comparison. There is very good agreement between the spColumn solution and the PCA curve while the Bresler curve yields straight segments given the conservative assumption of ($\alpha = 1.0$).

The nominal biaxial load (P_n, M_{nx}, M_{ny}) is represented by Point 1 shown in the diagram. It is located inside all three curves indicating the column section is adequate according to all methods. Point 2 represents the equivalent uniaxial strength (P_n, M_{nox}).

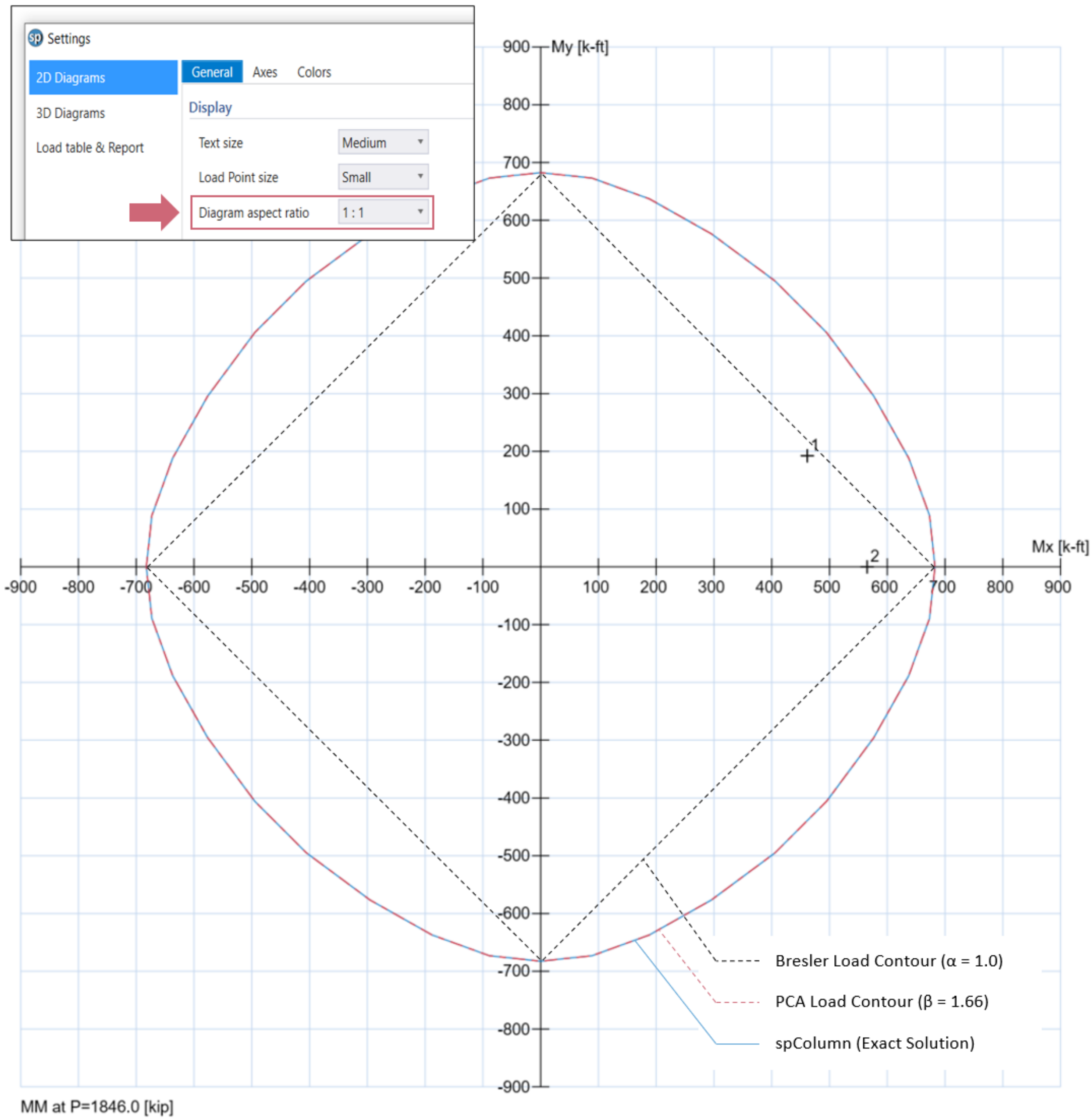


Figure 23 – Column Section M_{nx}-M_{ny} Contour at P_n = 1846 kip (spColumn 2D/3D Viewer)

7. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns may be subjected to biaxial bending (M_x and $M_{y)}$ due to lateral effects and unbalanced moments from both directions of analysis. This requires an investigation of the column P-M_x-M_y interaction diagram (3D failure surface) in two directions simultaneously (axial force interaction with biaxial bending).

This example shows a detailed comparison between exact hand solution, spColumn software results and the most commonly used approximate methods needed to design reinforced concrete column subjected to a combined axial force and biaxial bending moments. The approximate methods provide conservative solution in most cases and require tedious calculations to obtain the equivalent uniaxial capacities and other parameters. Additionally, performing exact solution manually to design column section subjected to a combined axial force and biaxial bending moments is tedious and challenging for engineers and the use of a computer aid can save time and eliminate errors. StucturePoint's spColumn program can, quickly, simply and accurately design and generate the three-dimensional failure surface (interaction diagram) for all commonly encountered column, beam or wall sections in addition to highly complex and irregular cross-sections.

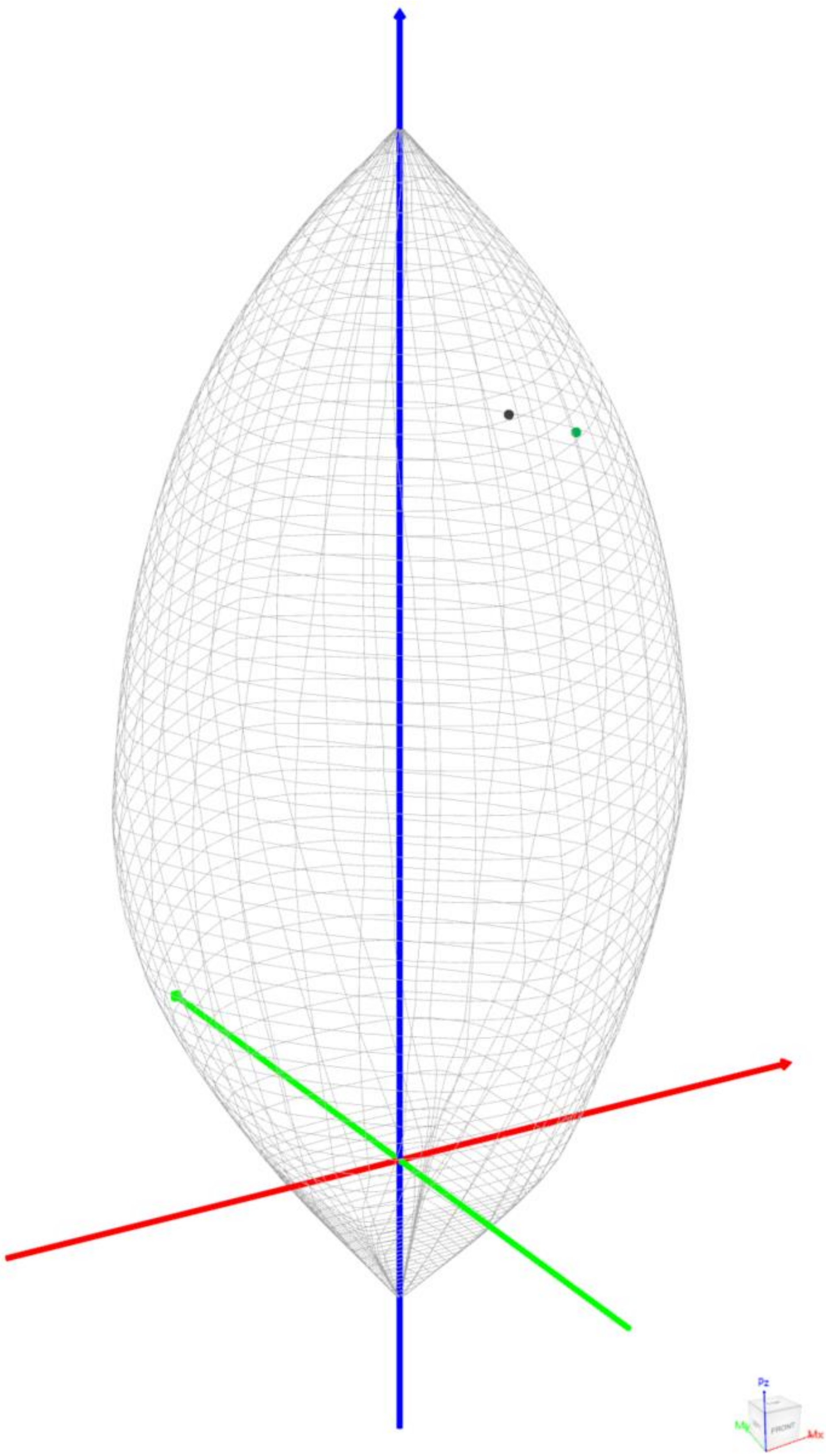


Figure 24 – Interaction Diagram in Two Directions (Biaxial) (spColumn)

The spColumn viewer is a powerful tool especially for investigating interaction diagrams (failure surfaces) for columns and walls sections subjected to a combined axial force and biaxial bending moments. The viewer allows the user to view and analyze 2D interaction diagrams and contours along with 3D failure surfaces in a multi viewport environment. Figure 25 shows three views of:

1. P-M interaction diagram cut at angle of 23º
2. M_x-M_y interaction diagram cut at axial load of 1846.0 kip in compression
3. A 3D failure surface (interaction diagram showing the points calculated in this example).

Figure 26 and Figure 27 show 3D visualization of failure surface with a horizontal and vertical plane cut, respectively.

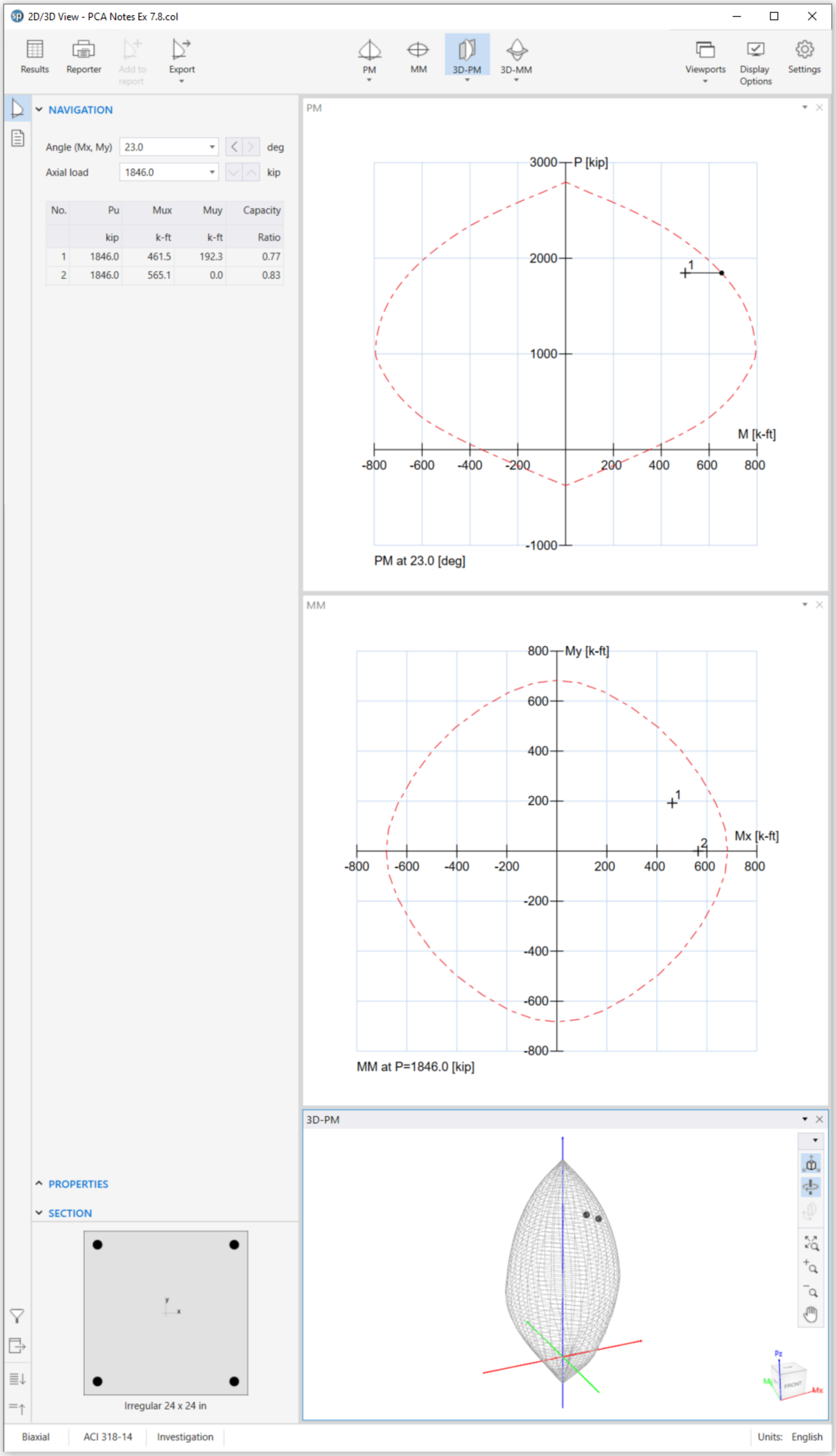


Figure 25 – 2D/3D Biaxial Interaction Diagram Viewer (spColumn)

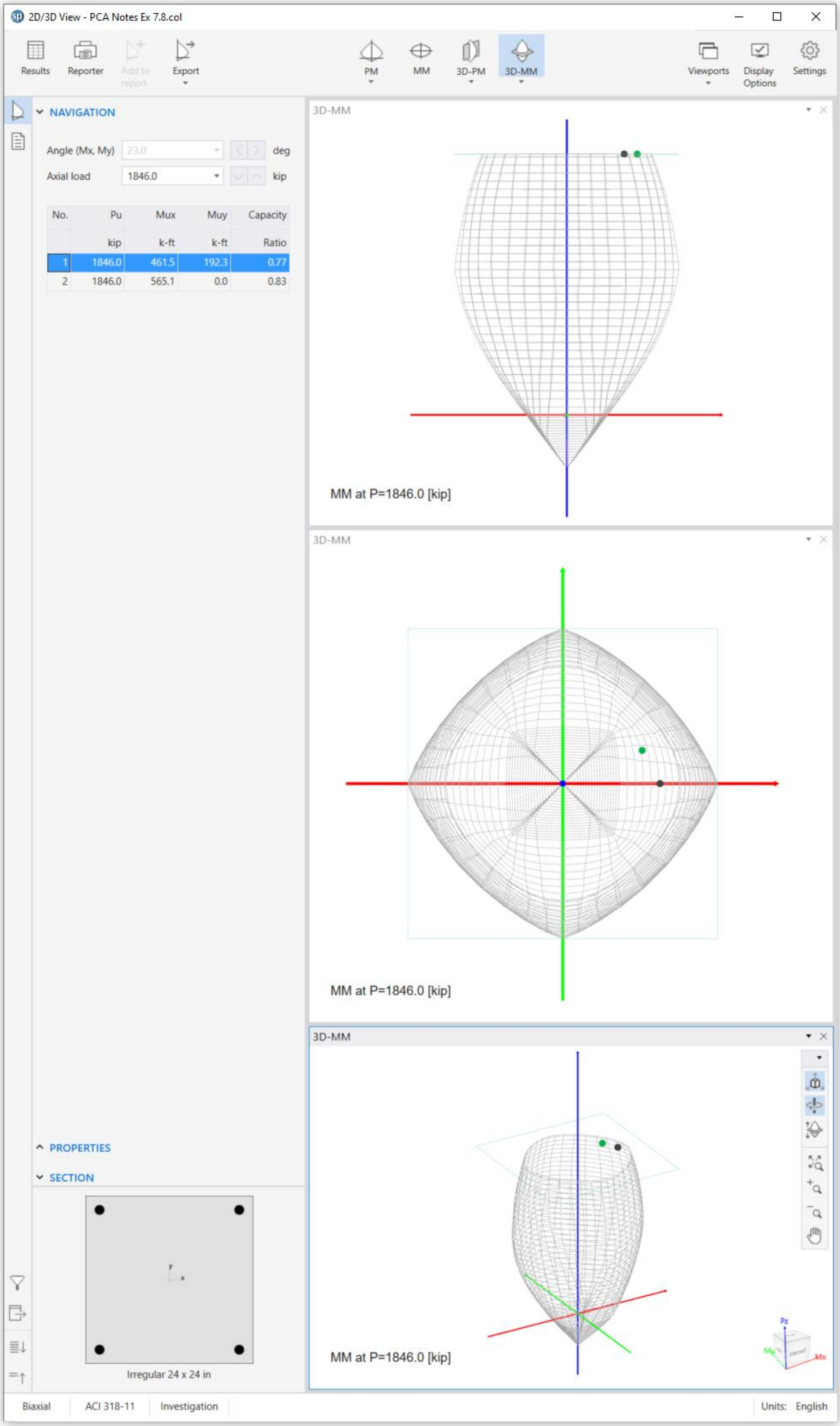


Figure 26 – 3D Visualization of Failure Surface with a Horizontal Plane Cut at P = 1846.0 kip (spColumn)

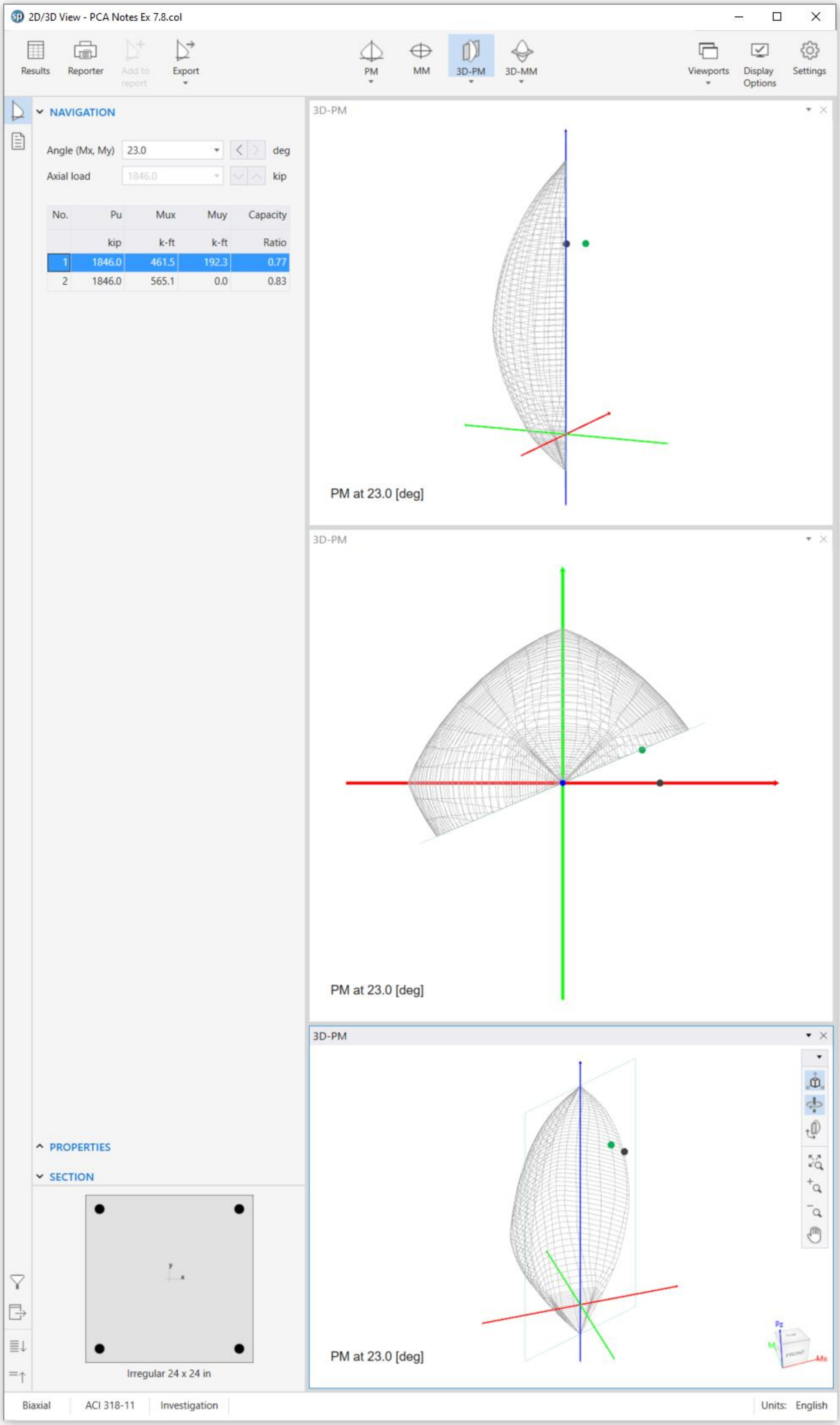


Figure 27 – 3D Visualization of Failure Surface with a Vertical Plane Cut at 23º (spColumn)