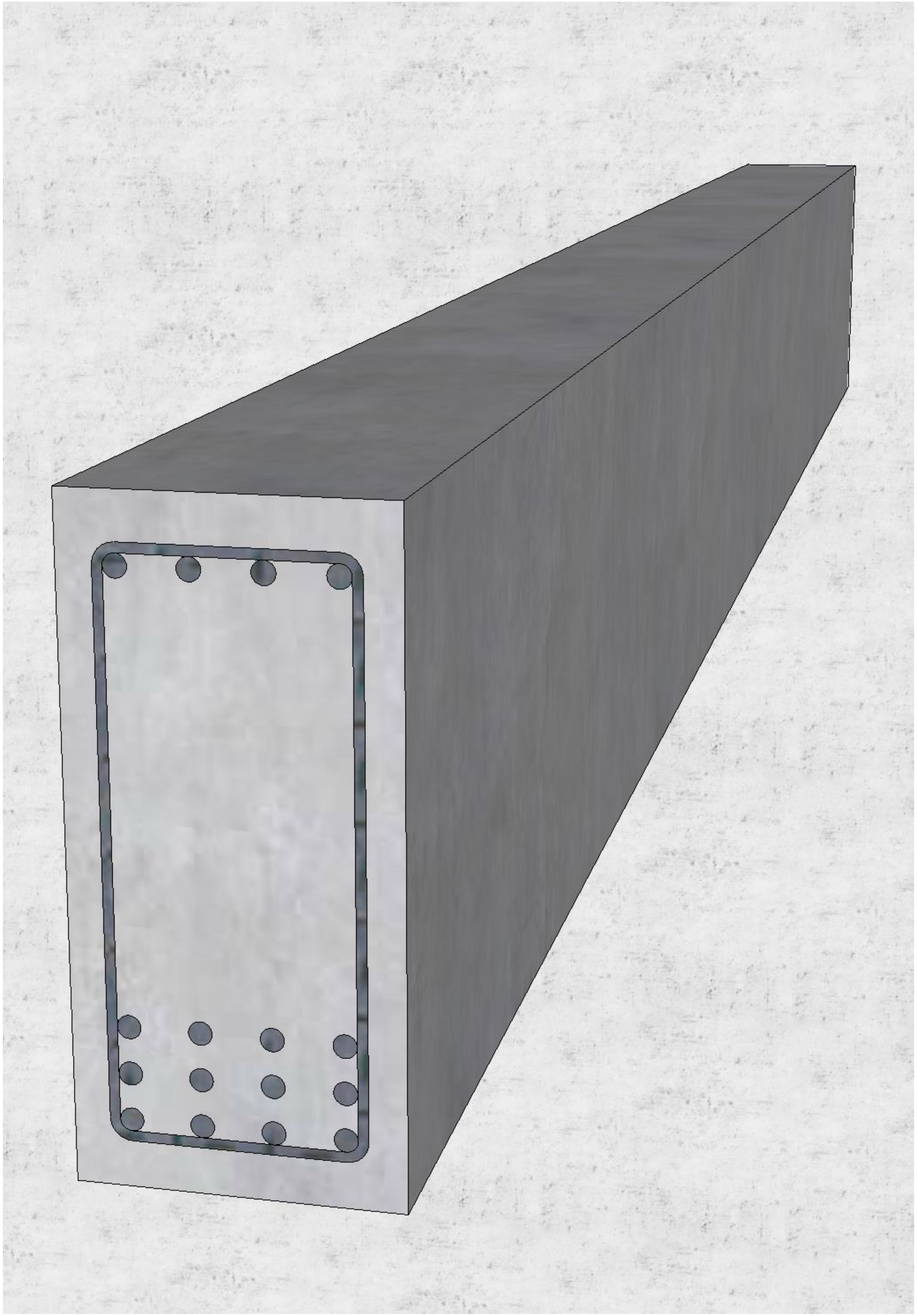
Doubly Reinforced Concrete Beam Design (ACI 318-14)



Doubly Reinforced Concrete Beam Design (ACI 318-14)

Determine the required reinforcement steel area for a concrete beam carrying service dead and live loads. First, check if singly reinforced beam section is suitable. If not, try doubly reinforced concrete beam section by adding compression reinforcement. It is desired that the section for this cantilever beam be tension controlled. Compare the calculated values in the Reference and the hand calculations with values obtained by [spBeam] engineering software program from [StructurePoint].

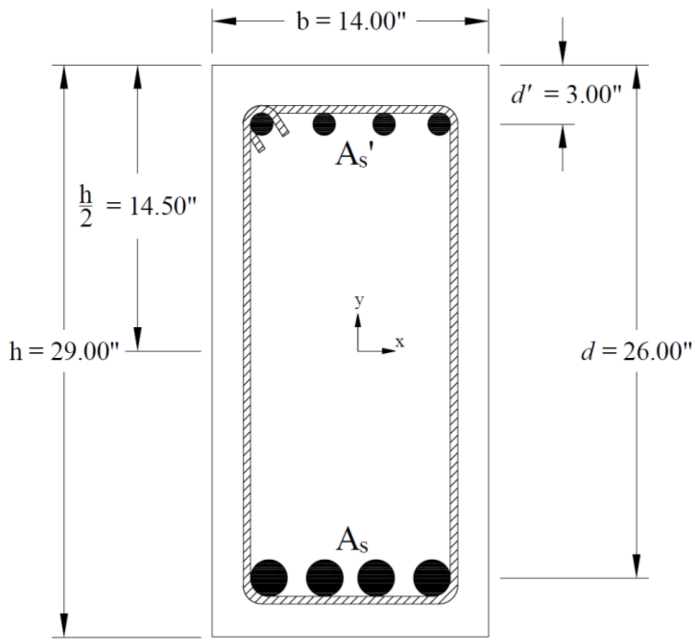


Figure 1 – Doubly Reinforced Concrete Beam Cross-Section

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

Reinforced Concrete Design, 8 th Edition, 2018, Wang et. al., Oxford University Press, Example 3.11.1

spBeam Engineering Software Program Manual v5.50, StructurePoint, 2018

Design Data

f_c' = 5000 psi

f_y = 60,000 psi

Cover = 3 in. to the center of the reinforcement

Beam cross-section: 14 in. x 29 in.

M_{DL} = 234 kip-ft

M_{LL} = 414 kip-ft

Solution

This cantilever beam is subjected to a concentrated moment at the free end producing a constant positive moment (tension at bottom, compression at top) along the span. This simple configuration is ideal to illustrate the steps required for doubly reinforced beam design and match the reference design scenario. The first step in the solution is to determine the nominal flexural strength and maximum reinforcement allowed for a tension-controlled singly reinforced section. If the required capacity exceeds the maximum strength of the singly reinforced section, the addition of compression reinforcement will be considered.

1. Required Nominal Strength

$$M_u = 1.2 \times M_{DL} + 1.6 \times M_{LL} = 1.2 \times 234 + 1.6 \times 414 = 943 \text{ kip-ft}$$ ***ACI 318-14 (5.3.1)***

Assuming a tension-controlled section → 0.090

$$M_{n,required} = \frac{M_u}{\phi} = \frac{943}{0.9} = 1048 \text{ kip-ft}$$

2. Nominal Flexural Strength

2.1. Singly Reinforced Beam Section

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

$$\varepsilon_s = 0.005 \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.90$$ ***ACI 318-14 (Table 21.2.2)***

$\varepsilon_{cu} = 0.003$ **_ACI 318-14 (22.2.2.1)_**

$$c = \frac{d_t}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{26}{0.005 + 0.003} \times 0.003 = 9.75 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-14 (22.2.2.4.2)

$a = \beta_1 \times c = 0.80 \times 9.75 = 7.80 \text{ in.}$ **_ACI 318-14 (22.2.2.4.1)_**

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times \left(f_c^{'} \times 4000\right)}{1000} = 0.85 - \frac{0.05 \times \left(5000 \times 4000\right)}{1000} = 0.80$$ **_ACI 318-14 (Table 22.2.2.4.3)_**

$C_c = 0.85 \times f_c^{'} \times a \times b = 0.85 \times 5{,}000 \times 7.80 \times 14 = 464.0 \text{ kip}$ **_ACI 318-14 (22.2.2.4.1)_**

$f_s = f_y = 60{,}000 \text{ psi}$

The maximum area of steel in a tension-controlled, singly reinforced section is:

$$\text{T}_s = f_y \times A_s \rightarrow A_s = \frac{\text{T}_s}{f_y} = \frac{464}{60} = 7.74 \text{ in.}^2$$

The corresponding nominal moment is:

$$M_n = C_c \times \left(d - \frac{a}{2}\right) = 464 \times \left(26 - \frac{7.80}{2}\right) = 855 \text{ kip-ft} < M_{n,required} = 1048 \text{ kip-ft}$$

Therefore, compression reinforcement is needed to increase the amount of tension reinforcement enough to achieve the required strength while maintaining $\varepsilon_t \geq 0.005$ (tension-controlled limit).

2.2. Doubly Reinforced Beam Section

$M_{ns'} = 1048 - 855 = 193 \text{ kip-ft}$

$$M_{ns'} = C_s \times \left(d - d'\right) \rightarrow C_s = \frac{M_{ns'}}{\left(d - d'\right)} = \frac{193 \times 12}{\left(26 - 3\right)} = 101 \text{ kips}$$

$$\varepsilon_{s'} = \frac{c - d'}{c} \times \varepsilon_{cu} = \frac{9.75 - 3}{9.75} \times 0.003 = 0.0021 > \varepsilon_y = 0.00207$$

Since $\varepsilon_s^{'} > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$\therefore f_s^{'} = f_y = 60{,}000 \text{ psi}$

The area of the reinforcement in this layer has been included in the area (*ab*) used to compute C_c. As a result, it is necessary to subtract $0.85 f_c'$ from f_s' before computing A_s':

$$C_s = \left(f_s' - 0.85 f_c' \right) \times A_s' \rightarrow A_s' = \frac{C_s}{\left(f_s' - 0.85 f_c' \right)} = \frac{101}{\left(60 - 0.85 \times 5 \right)} = 1.81 \text{ in.}^2$$

$$T_s = C_c + C_s = 464 + 101 = 565 \text{ kip}$$

$$T_s = f_y \times A_s \rightarrow A_s = \frac{T_s}{f_y} = \frac{565}{60} = 9.42 \text{ in.}^2$$

9.42 in.^{2} is the amount of tension reinforcement that correspond exactly to the tension-controlled limit for the beam section with compression reinforcement of 1.81 in.^{2}

3. Doubly Reinforced Concrete Beam Design – spBeam Software

spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318) and Canadian (CSA A23.3) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

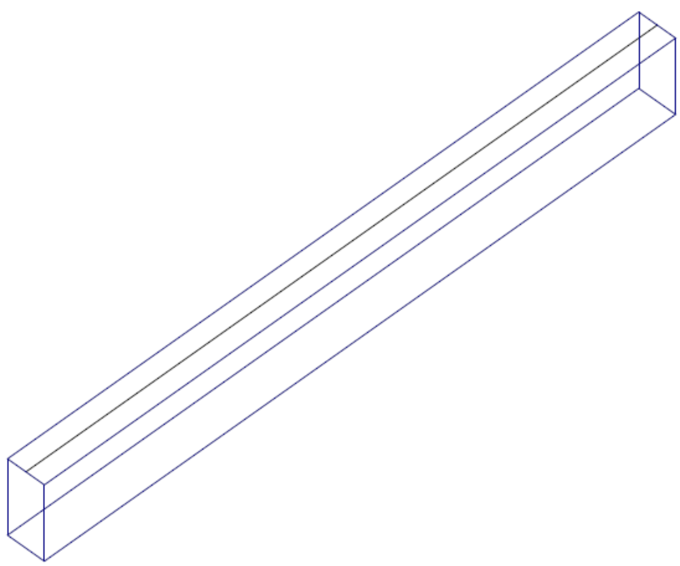
spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or T-shaped sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, spBeam provide users with options and flexibility to:

1. Design the beam as a rectangular cross-section or a T-shaped section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.
4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia
6. Design the beam as singly or doubly reinforced section.

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the beam covered in this design example.

spBeam v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams and One-way Slab Systems



Contents

1. Input Echo

1.1. General Information

File Name	C:\ACI 318-14 Examp...\Doubly Reinforced Beam.slb
Project	Doubly Reinforced Beam
Frame	
Engineer	SP
Code	ACI 318-14
Reinforcement Database	User Defined
Mode	Design
Number of supports =	2
Floor System	One-Way/Beam

1.2. Solve Options

Live load pattern ratio = 0%
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations selected.
Default incremental rebar design selected.
Moment redistribution NOT selected.
Effective flange width calculations NOT selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150 lb/ft^3
f'_c	5 ksi
E_c	4286.8 ksi
f_r	0.53033 ksi

1.3.2. Concrete: Columns

w_c	150 lb/ft^3
f'_c	5 ksi
E_c	4286.8 ksi
f_r	0.53033 ksi

1.3.3. Reinforcing Steel

f_y	60 ksi
f_{yt}	60 ksi
E_s	29000 ksi
Epoxy coated bars	No

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#1	0.01	0.45	0.01	#2	0.02	2.35	0.02

1.5. Span Data

1.5.1. Slabs

Span	Loc	L1	t	wL	wR	H_{min}
		ft	in	ft	ft	in
1	Int	20.000	0.00	0.583	0.583	0.00

1.5.2. Ribs and Longitudinal Beams

Span	Ribs			Beams		Span
	b	h	Sp	b	h	H_{min}
	in	in	in	in	in	in
1	0.00	0.00	0.00	14.00	29.00	15.00

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	0.00	0.00	0.000	0.00	0.00	0.000	999
2	0.00	0.00	0.000	0.00	0.00	0.000	100

1.6.2. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kip/in	kip-in/rad		
1	0	0	Fixed	Fixed
2	1e-010	0	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.200	1.600

1.7.2. Point Moments

Case/Patt	Span	Wa	La
		k-ft	ft
Dead	1	234.00	20.000
Live	1	414.00	20.000

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#2	#2	#2	#2
Bar spacing	in	10.00	18.00	1.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00

STRUCTUREPOINT - spBeam v5.50
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	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Clear Cover	in	1.50		1.50	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#1	#1	#2	#2	#1	#2
Bar spacing	in	1.00	14.00	1.00	14.00	6.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	3.00		2.99			
Layer dist.	in	1.00		1.00			
No. of legs						2	6
Side cover	in					1.50	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

2. Design Results

2.1. Top Reinforcement

Span	Zone	Width	M_{max}	X_{max}	A'_s	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
		ft	k-ft	ft	in²	in²	in²	in²	in	
1	Left	1.17	0.00	0.000	0.000	1.287	7.735	1.681	3.643	4-#1
	Midspan	1.17	0.00	10.000	0.000	1.287	7.735	1.681	3.643	4-#1
	Right	1.17	0.00	20.000	0.000	1.287	7.735	1.681	3.643	4-#1

2.2. Bottom Reinforcement

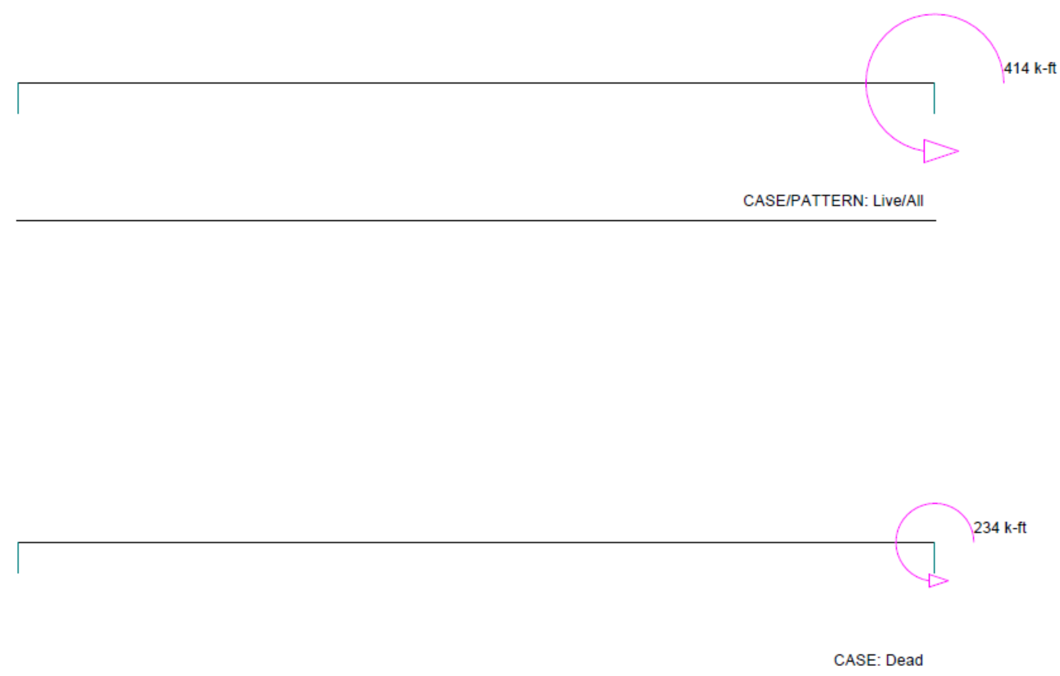
Span	Width	M_{max}	X_{max}	A'_s	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
	ft	k-ft	ft	in²	in²	in²	in²	in	
1	1.17	943.20	20.000	1.681	1.287	7.735	9.416	3.641	4-#2

2.3. Flexural Capacity

Span	Top						Bottom				
	x	$A_{s,top}$	ΦM_n-	M_u-	Comb Pat	Status	$A_{s,bot}$	ΦM_n+	M_u+	Comb Pat	Status
	ft	in²	k-ft	k-ft			in²	k-ft	k-ft		
1	0.000	1.81	-206.58	0.00	U1 All	OK	9.42	943.29	943.20	U1 All	OK
	7.000	1.81	-206.58	0.00	U1 All	OK	9.42	943.29	943.20	U1 All	OK
	10.000	1.81	-206.58	0.00	U1 All	OK	9.42	943.29	943.20	U1 All	OK
	13.000	1.81	-206.58	0.00	U1 All	OK	9.42	943.29	943.20	U1 All	OK
	20.000	1.81	-206.58	0.00	U1 All	OK	9.42	943.29	943.20	U1 All	OK

3. Diagrams

3.1. Loads

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Engineer: SP
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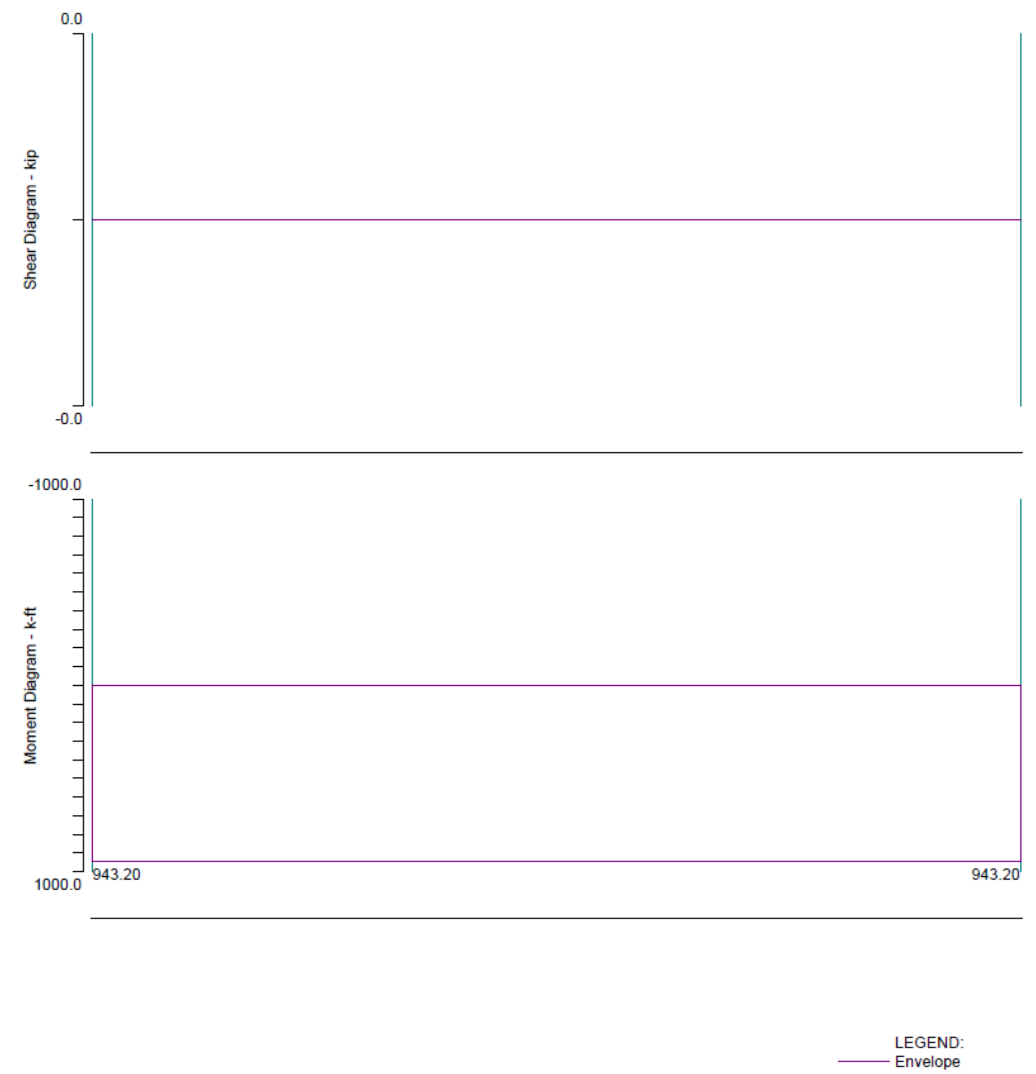
3.2. Internal Forces



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Engineer: SP

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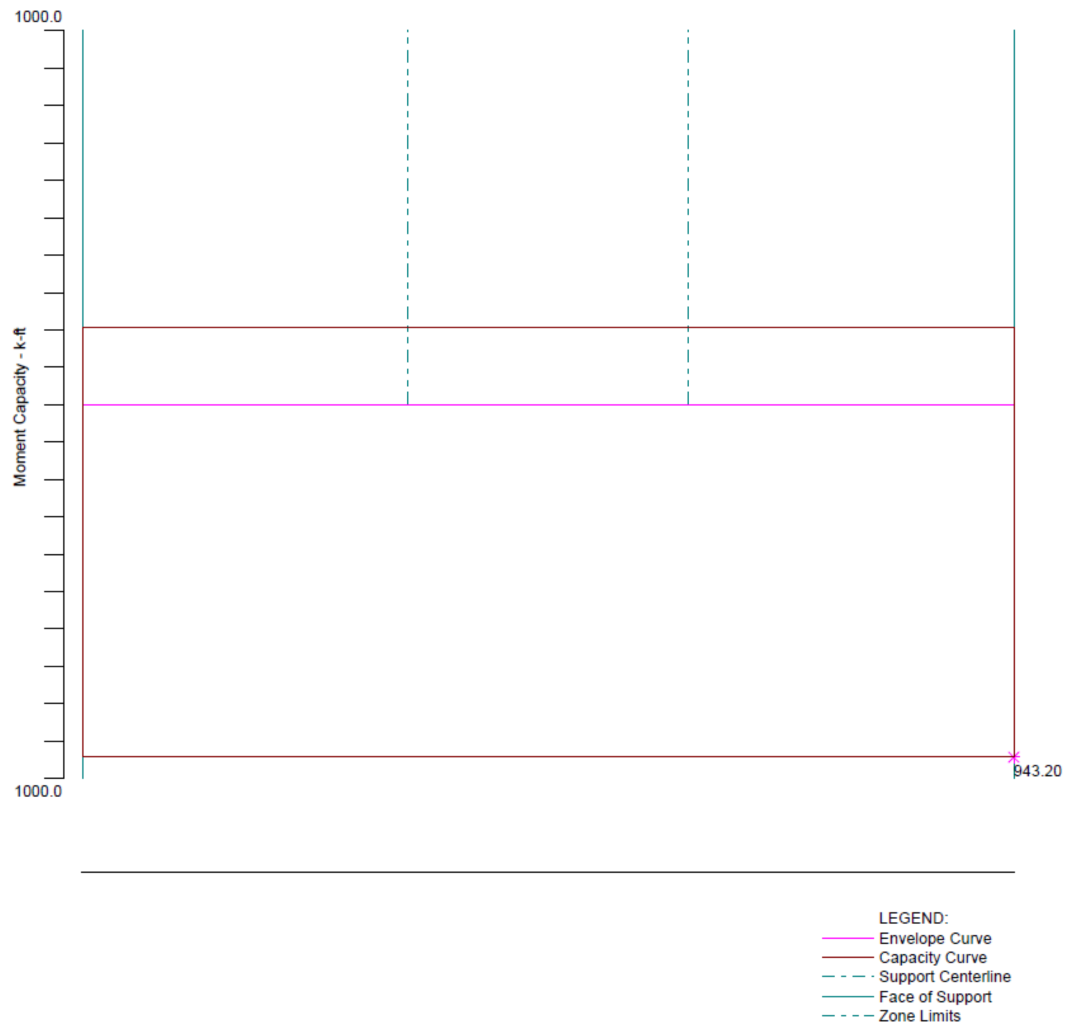
3.3. Moment Capacity



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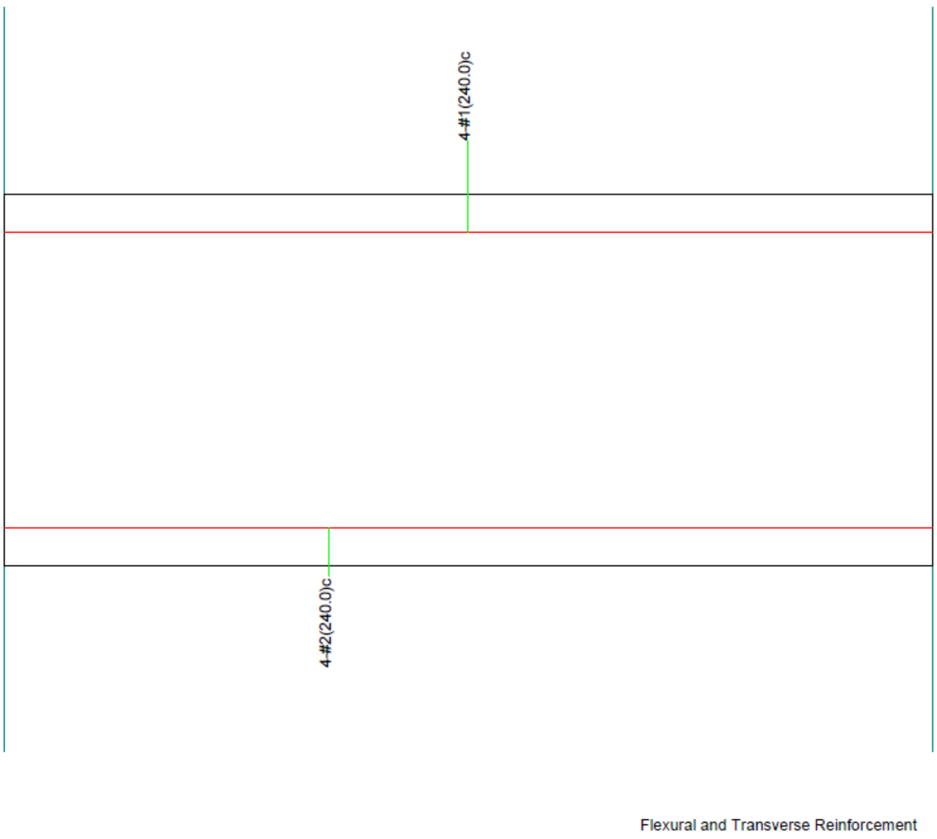
3.4. Reinforcement



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Engineer: SP

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4. Comparison of Design Results

Table 1 - Comparison of Results							
Method	**M_u, kip-ft**	**$A_{s,max}$, in.^{2}**	**$A_{s',req}$, in.^{2}**	**$A_{s,req}$, in.^{2}**	**$A_{s',provided}$, in.^{2}**	**$A_{s,provided}$, in.^{2}**	**ϕM_n, kip-ft**
Reference	943	7.74	1.81	9.42	---	---	---
Hand	943	7.74	1.81	9.42	---	---	---
spBeam	943	7.74	1.68*	9.42	1.81	9.42	943

* spBeam reports the required top tension reinforcement. However, the program calculates the capacity of doubly reinforced beam section using the required top compression reinforcement.

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spBeam program.

5. Conclusions & Observations

As shown in this example, using compression reinforcement helps in changing the beam failure mode from compression (brittle failure mode) to tension (ductile failure mode). The following shows other applications where the use of doubly reinforced beam sections can be helpful:

The use of compression reinforcement in beams reduces the long-term deflections of a beam subjected to sustained loads. Creep of the concrete in the compression zone transfers load from the concrete to the compression steel, reducing the stress in the concrete. Because of the lower compression stress in the concrete, it creeps less, leading to a reduction in sustained-load deflections.

In seismic regions or if moment redistribution is desired, doubly reinforced beam sections can be helpful since the beam ductility increases when compression reinforcement is used. The strain in the tension reinforcement at failure increases since the depth of the compression stress block decreases, resulting in more ductile behavior.

Compression reinforcement maybe be used for fabrication purposes. It is customary to provide small bars in the corners of the stirrups to hold the stirrups in place in the form and also to help anchor the stirrups. Such reinforcement may have a small effect on strength but can be considered in spBeam for investigation purposes.