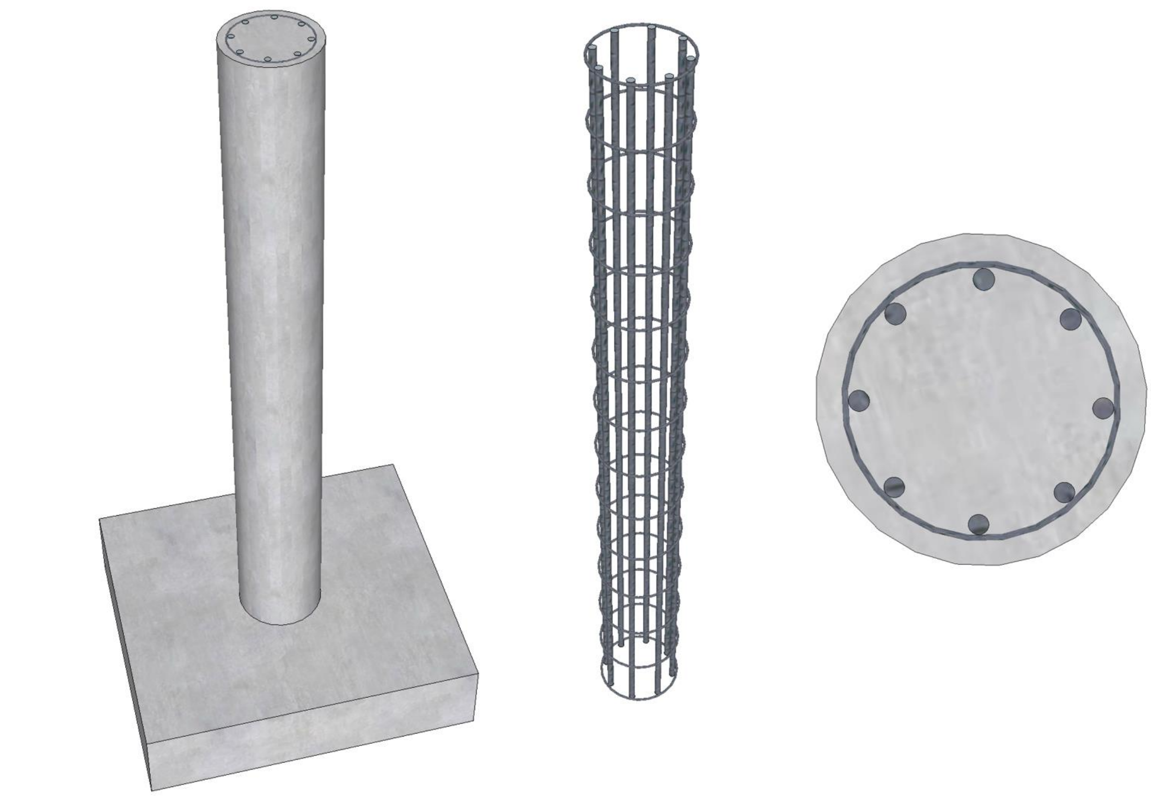
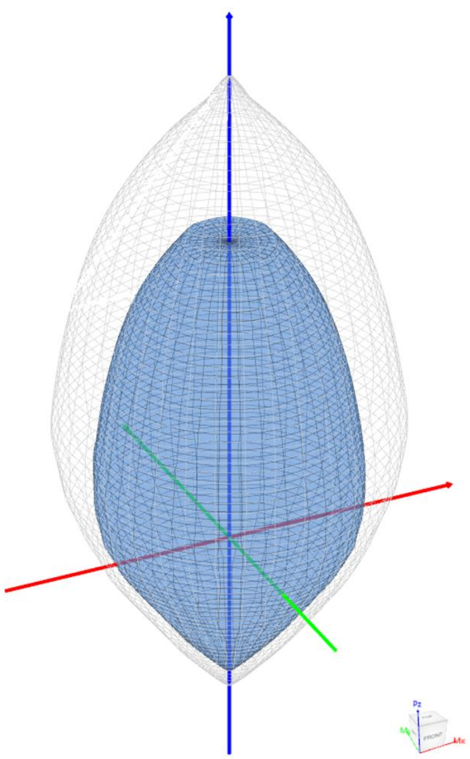
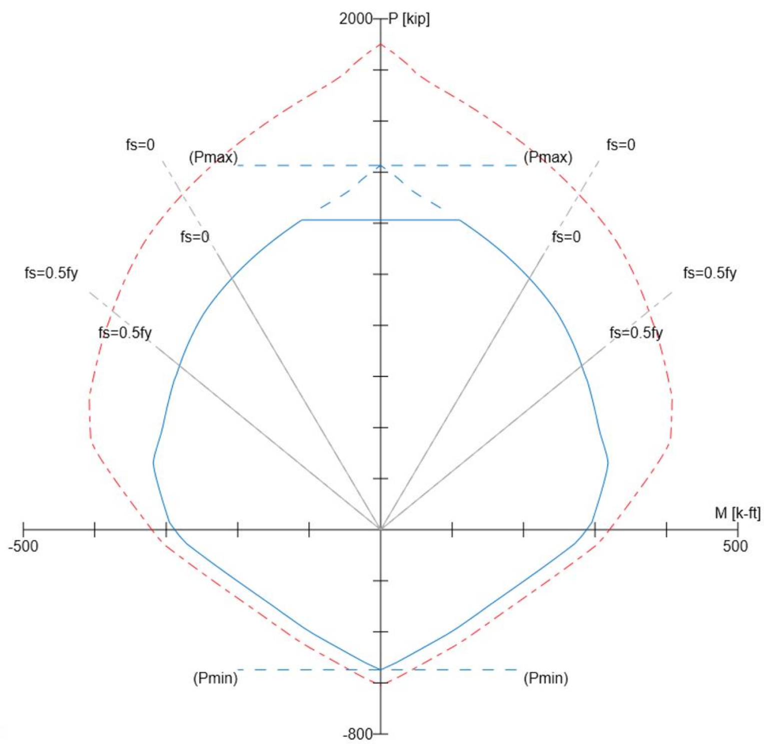
Interaction Diagram - Circular Spiral Reinforced Concrete Column (ACI 318-19)







Interaction Diagram - Circular Spiral Reinforced Concrete Column (ACI 318-19)

Develop an interaction diagram for the circular concrete column shown in the figure below about the x-axis. Determine seven control points on the interaction diagram and compare the calculated values with the Reference and exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint.

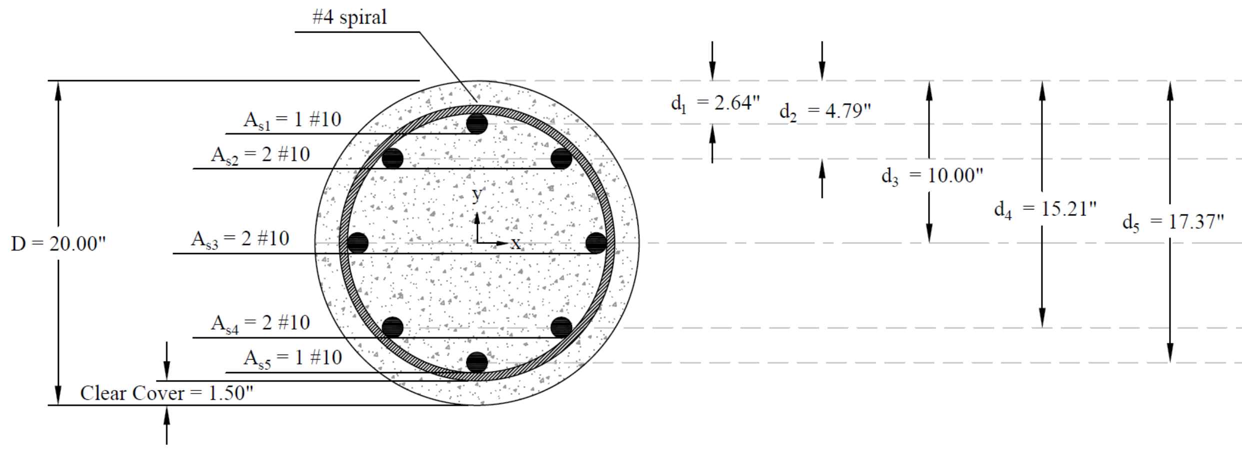


Figure 1 – Circular Reinforced Concrete Column Cross-Section

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Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

- Reinforced Concrete Design, 9th Edition, 2021, Pincheira J. et. al., Oxford University Press, Example 10.18.1
- spColumn Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2021
- "Interaction Diagram - Tied Reinforced Concrete Column Design Strength (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram - Tied Reinforced Concrete Column with High-Strength Reinforcing Bars (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram - Barbell Concrete Shear Wall Unsymmetrical Boundary Elements (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram – Building Elevator Reinforced Concrete Core Wall Design Strength (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022

Design Data

f_c' = 5000 psi

f_y = 60000 psi

Clear Cover = 1.5 in.

Column Diameter = 20 in.

Stirrups, longitudinal reinforcement and reinforcement locations are shown in Figure 1 and Table 1.

Table 1 - Reinforcement Configuration

Layer, i	d_i, in	n_i	A_{si}, in^2	nA_{si}, in^2
1	2.64	1	1.27	1.27
2	4.79	2	1.27	2.54
3	10.00	2	1.27	2.54
4	15.21	2	1.27	2.54
5	17.37	1	1.27	1.27
			$A_{st} = \Sigma\, n_i A_{si}$	10.16

Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following seven control points:

Point 1: Maximum compression

Point 2: Bar stress near tension face equal to zero, (f_s = 0)

Point 3: Bar stress near tension face equal to 0.5 f_y (f_s = 0.5 f_y)

Point 4: Bar stress near tension face equal to f_y (f_s = f_y)

Point 5: Bar strain near tension face equal to ε_y + 0.003

Point 6: Pure bending

Point 7: Maximum tension

Several terms are used to facilitate the following calculations:

A_{comp} = concrete area in compression, in^2.

$\bar{y}$ = geometric centroid location along the y-axis, in.

P_o = nominal axial compressive strength, kip

ϕP_o = factored axial compressive strength, kip

$\phi P_{n,max}$ = maximum (allowable) factored axial compressive strength, kip

c = distance from the fiber of maximum compressive strain to the neutral axis, in.

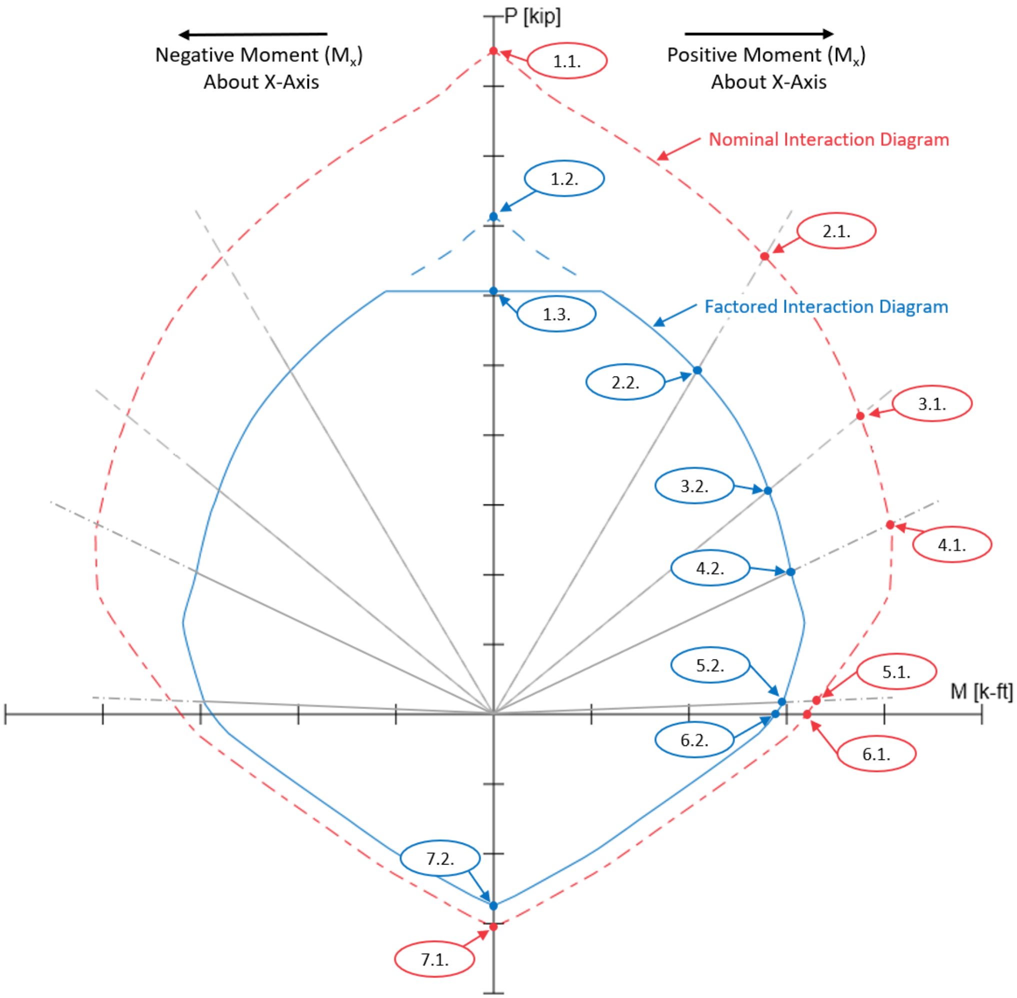
a = depth of equivalent rectangular stress block, in.

C_c = compression force in equivalent rectangular stress block, kip

$\varepsilon_{s,i}$ = strain value in reinforcement layer, in./in.

C_s = compression force in reinforcement, kip

T_s = tension force in reinforcement, kip



1.1. Maximum Nominal Axial Strength in Compression (P_o)
2.1. Nominal Control Point at Zero Stress in Tension Reinforcement
3.1. Nominal Control Point at Tension Reinforcement Stress = $0.5f_y$
4.1. Nominal Control Point at Tension Reinforcement Stress = f_y
5.1. Nominal Control Point at Tension Reinforcement Stress = ε_y + 0.003 in./in.
6.1. Nominal Control Point at Pure Bending
7.1. Maximum Nominal Axial Strength in Tension

1.2. Maximum Factored Axial Strength in Compression
1.3. Allowable Factored Axial Strength in Compression ($\phi P_{n,max}$)
2.2. Factored Control Point at Zero Stress in Tension Reinforcement
3.2. Factored Control Point at Tension Reinforcement Stress = $0.5f_y$
4.2. Factored Control Point at Tension Reinforcement Stress = f_y
5.2. Factored Control Point at Tension Reinforcement Stress = ε_y + 0.003 in./in.
6.2. Factored Control Point at Pure Bending
7.2. Maximum Factored Axial Strength in Tension

Figure 2 – Circular Column Section Interaction Diagram Control Points

1. Maximum Compression

1.1. Nominal axial compressive strength at zero eccentricity

$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$$ ***ACI 318-19 (22.4.2.2)***

$$P_o = 0.85 \times 5000 \times \left(\frac{\pi}{4} \times 20^2 - 8 \times 1.27 \right) + 60000 \times 8 \times 1.27 = 1902 \text{ kips}$$

1.2. Factored axial compressive strength at zero eccentricity

Since this column is a spiral column with steel strain in compression:

$$\phi = 0.75$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_o = 0.75 \times 1902 = 1426.2 \text{ kips}$$

Since the section is regular (symmetrical) about the x-axis, the moment capacity associated with the maximum axial compressive strength is equal to zero.

$$M_o = \phi M_o = 0.00 \text{ kip-ft}$$

1.3. Maximum (allowable) factored axial compressive strength

$$\phi P_{n,max} = 0.85 \times \phi P_o = 0.85 \times 1426.2 = 1212.3 \text{ kips}$$ ***ACI 318-19 (Table 22.4.2.1)***

2. Bar Stress Near Tension Face Equal to Zero, (ε_s = f_s = 0)

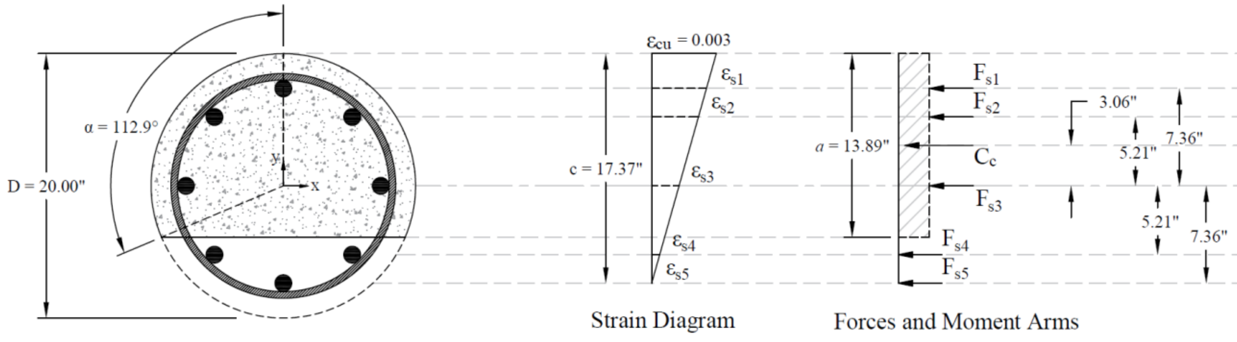


Figure 3 – Strains, Forces, and Moment Arms (ε_t = f_s = 0)

Strain ε_s is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. ***ACI 318-19 (10.7.5.2.1 and 2)***

For Concrete:

$$c = d_5 = 17.37 \text{ in.}$$

$$\varepsilon_{s5} = 0 < \varepsilon_y = \frac{F_y}{E_s} = \frac{60}{29000} = 0.00207$$

$$\therefore \phi = 0.75$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.
ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 17.37 = 13.89 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

a = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$C_c = 0.85 \times f_c' \times A_{comp} = 0.85 \times 5000 \times 232.9 = 989.9 \text{ kip (Compression)}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\theta = \cos^{-1}\left(\frac{\frac{D}{2} - a}{\frac{D}{2}}\right) = \cos^{-1}\left(\frac{\frac{20}{2} - 13.89}{\frac{20}{2}}\right) = 112.9°$$

$$A_{comp} = D^2 \times \frac{\theta - \sin(\theta) \times \cos(\theta)}{4}$$

$$A_{comp} = 20^2 \times \frac{\left(112.9° \times \frac{\pi}{180°}\right) - \sin(112.9°) \times \cos(112.9°)}{4} = 232.9 \text{ in.}^2$$

$$\bar{y} = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{comp}} = \frac{20^3 \times \sin^3(112.9°)}{12 \times 232.9} = 2.24 \text{ in.}$$

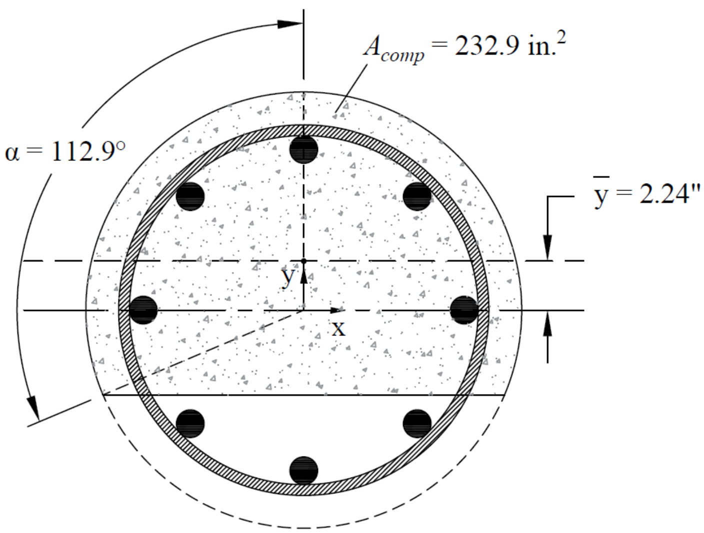


Figure 4 – Cracked Column Section Properties ($\varepsilon_t = f_s = 0$)

For Reinforcement:

$$\varepsilon_{s4} = (c - d_4) \times \frac{\varepsilon_{cu}}{c} = (17.37 - 15.21) \times \frac{0.003}{17.37} = 0.00037 \text{ (Compression)} < \varepsilon_y = \frac{F_y}{E_s} = \frac{60}{29000} = 0.00207$$

$$f_{s5} = 0 \text{ psi} \rightarrow \text{F}_{s5} = f_{s5} \times A_{s5} = 0 \text{ kip}$$

Since $\varepsilon_{s4} < \varepsilon_y \rightarrow$ reinforcement has not yielded

$$\therefore f_{s4} = \varepsilon_{s4} \times E_s = 0.00037 \times 29000000 = 10808 \text{ psi}$$

The area of the reinforcement in this layer is not included in the area (*ab*) used to compute C_c. As a result, it is NOT necessary to subtract $0.85f_c'$ from f_{s4} before computing F_{s4}:

$F_{s4} = f_{s4} \times A_{s4} = 10808 \times (2 \times 1.27) = 27.45 \text{ kip}$ (Compression)

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

Table 2 - Strains and internal force resultants ($\varepsilon_s = f_s = 0$)							
Layer	d, in.	ε, in./in.	f_s, psi	F_s, kip	C_c, kip	Moment arm (r), in.	Moment, kip-ft
Concrete	---	0.00300	---	---	989.86	2.24	184.55
1	2.64	0.00254	60000	70.80*	---	7.37	43.46
2	4.79	0.00217	60000	141.61*	---	5.21	61.45
3	10.00	0.00127	36899	82.93*	---	0.00	0.00
4	15.21	0.00037	10808	27.45	---	-5.21	-11.91
5	17.37	0.00000	0	0.00	---	-7.37	0.00
Axial Force and Bending Moment			P_n, kip	1312.64		M_n, kip-ft	277.54
			ϕP_n, kip	984.48		ϕM_n, kip-ft	208.16

* The area of the reinforcement in this layer has been included in the area (*ab*) used to compute C_c. As a result, $0.85f_c'$ is subtracted from f_s in the computation of F_s.

Where:

$P_n = C_c + \sum F_s$ (+) = Compression (-) = Tension

$\phi P_n = \phi \times P_n = 0.75 \times P_n$

$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n=5}\left(F_{si} \times \left(\frac{h}{2} - d_i\right)\right)$ (+) = Counter Clockwise (-) = Clockwise

$\phi M_n = \phi \times M_n = 0.75 \times M_n$

3. Bar Stress Near Tension Face Equal to 0.5 f_y, (f_s = 0.5 f_y)

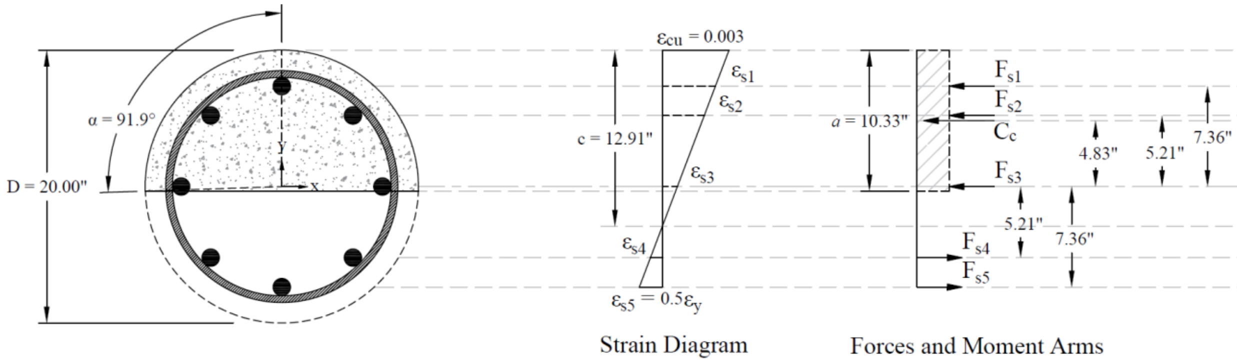


Figure 5 – Strains, Forces, and Moment Arms (f_s = 0.5 f_y)

For Concrete:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_{s5} = -\frac{\varepsilon_y}{2} = -\frac{0.00207}{2} = -0.00103 \text{ (Tension)} < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$$\varepsilon_{s5} < \varepsilon_y$$

$$\therefore \phi = 0.75$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_5}{\varepsilon_{s5} + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{17.37}{0.00103 + 0.003} \times 0.003 = 12.91 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 12.91 = 10.33 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

a = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$C_c = 0.85 \times f_c' \times A_{comp} = 0.85 \times 5000 \times 163.7 = 695.63 \text{ kip (Compression)}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\theta = \cos^{-1}\left(\frac{\frac{D}{2}-a}{\frac{D}{2}}\right) = \cos^{-1}\left(\frac{\frac{20}{2}-10.33}{\frac{20}{2}}\right) = 91.9°$$

$$A_{comp} = D^2 \times \frac{\theta - \sin(\theta)\times\cos(\theta)}{4}$$

$$A_{comp} = 20^2 \times \frac{\left(91.9°\times\frac{\pi}{180°}\right) - \sin(91.9°)\times\cos(91.9°)}{4} = 163.7 \text{ in.}^2$$

$$\bar{y} = \frac{D^3 \times \sin^3(\theta)}{12\times A_{comp}} = \frac{20^3\times\sin^3(91.9°)}{12\times 163.7} = 4.07 \text{ in.}$$

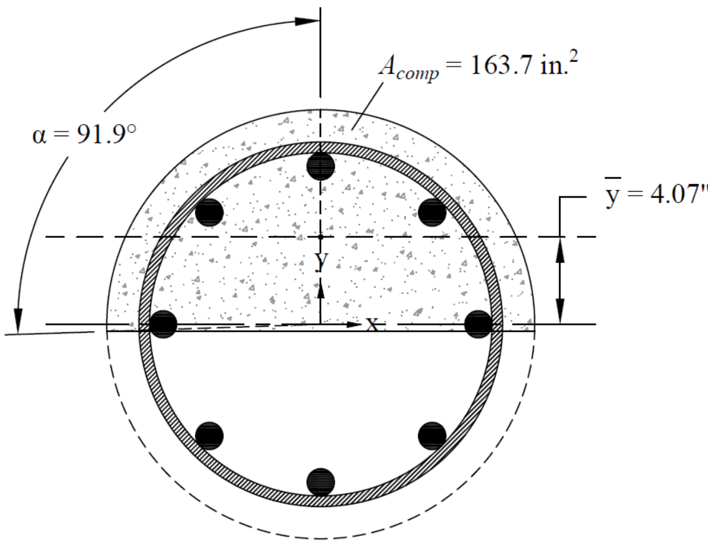


Figure 6 – Cracked Column Section Properties ($f_s = 0.5\ f_y$)

For Reinforcement:

$$\varepsilon_{s5} = -\frac{\varepsilon_y}{2} = -\frac{0.00207}{2} = -0.00103 \text{ (Tension)} < \varepsilon_y \rightarrow \text{reinforcement has not yielded}$$

$$\therefore f_{s5} = \varepsilon_{s5}\times E_s = -0.00053\times 29000000 = -30000 \text{ psi}$$

The area of the reinforcement in this layer is not included in the area (*ab*) used to compute C_c. As a result, it is NOT necessary to subtract $0.85f_c'$ from f_{s5} before computing F_{s5}:

$$F_{s5} = f_{s5}\times A_{s5} = -30000\times(1\times 1.27) = -38.1 \text{ kip (Tension)}$$

$$\varepsilon_{s4} = (c - d_4) \times \frac{\varepsilon_{cu}}{c} = (12.91 - 15.21) \times \frac{0.003}{12.91} = -0.00053 \text{ (Tension)} < \varepsilon_y \rightarrow \text{reinforcement has not yielded}$$

$$\therefore\ f_{s4} = \varepsilon_{s4} \times E_s = -0.00053 \times 29000000 = -15465 \text{ psi}$$

The area of the reinforcement in this layer is not included in the area (ab) used to compute C_c. As a result, it is NOT necessary to subtract $0.85 f_c'$ from f_{s4} before computing F_{s4}:

$$F_{s4} = f_{s4} \times A_{s4} = -15465 \times (2 \times 1.27) = -39.28 \text{ kip (Tension)}$$

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

Table 3 - Strains and internal force resultants ($f_s = 0.5\ f_y$)							
Layer	d, in.	ε, in./in.	f_s, psi	F_s, kip	C_c, kip	Moment arm (r), in.	Moment, kip-ft
Concrete	---	0.00300	---	---	695.63	4.07	235.73
1	2.64	0.00239	60000	70.80*	---	7.37	43.46
2	4.79	0.00189	54712	128.17*	---	5.21	55.62
3	10.00	0.00068	19623	39.05*	---	0.00	0.00
4	15.21	-0.00053	-15465	-39.28	---	-5.21	17.05
5	17.37	-0.00103	-30000	-38.1	---	-7.37	23.38
Axial Force and Bending Moment			P_n, kip	856.27		M_n, kip-ft	375.24
			ϕP_n, kip	642.20		ϕM_n, kip-ft	281.43

* The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, $0.85 f_c'$ is subtracted from f_s in the computation of F_s.

Where:

$$P_n = C_c + \sum F_s \qquad \text{(+) = Compression} \qquad \text{(-) = Tension}$$

$$\phi P_n = \phi \times P_n = 0.75 \times P_n$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n=5} \left(F_{si} \times \left(\frac{h}{2} - d_i\right)\right) \qquad \text{(+) = Counter Clockwise} \qquad \text{(-) = Clockwise}$$

$$\phi M_n = \phi \times M_n = 0.75 \times M_n$$

4. Bar Stress Near Tension Face Equal to f_y, ($f_s = f_y$)

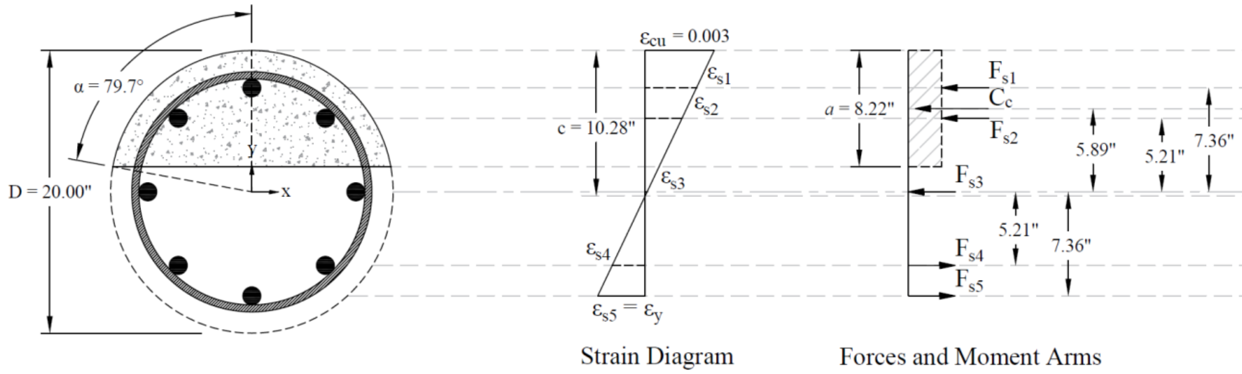


Figure 7 – Strains, Forces, and Moment Arms ($f_s = f_y$)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for ϕ for columns in which ϕ increases from 0.75 for spiral columns (or 0.65 for tied columns) up to 0.90.

For Concrete:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

$$\varepsilon_{s5} = -\varepsilon_y = -0.00207 \text{ (Tension)} = \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.75$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_5}{\varepsilon_{s5} + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{17.37}{0.00207 + 0.003} \times 0.003 = 10.28 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 10.28 = 8.22 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

a = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$C_c = 0.85 \times f_c' \times A_{comp} = 0.85 \times 5000 \times 121.7 = 517.24 \text{ kip (Compression)}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\theta = \cos^{-1}\left(\frac{\frac{D}{2} - a}{\frac{D}{2}}\right) = \cos^{-1}\left(\frac{\frac{20}{2} - 8.22}{\frac{20}{2}}\right) = 79.8°$$

$$A_{comp} = D^2 \times \frac{\theta - \sin(\theta) \times \cos(\theta)}{4}$$

$$A_{comp} = 20^2 \times \frac{\left(79.8° \times \frac{\pi}{180°}\right) - \sin(79.8°) \times \cos(79.8°)}{4} = 121.7 \text{ in.}^2$$

$$\bar{y} = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{comp}} = \frac{20^3 \times \sin^3(79.8°)}{12 \times 121.7} = 5.22 \text{ in.}$$

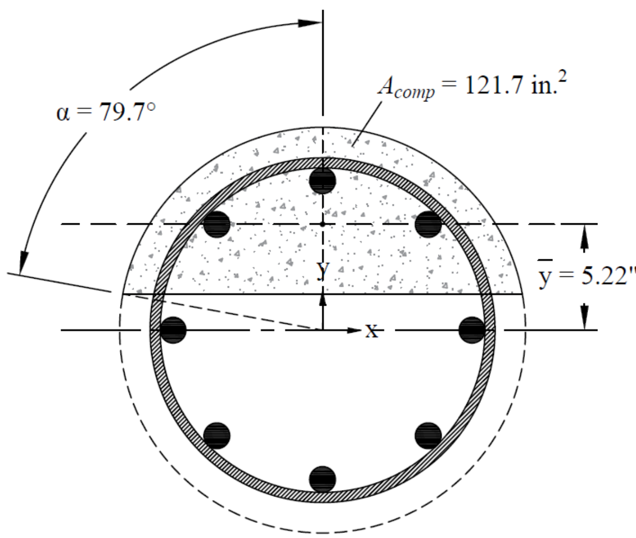


Figure 8 – Cracked Column Section Properties ($f_s = f_y$)

For Reinforcement:

$$\varepsilon_{s5} = -\varepsilon_y = -0.00207 \text{ (Tension)} = \varepsilon_y \rightarrow \text{reinforcement has yielded}$$

$$\therefore f_{s5} = f_y = -60000 = -60000 \text{ psi}$$

The area of the reinforcement in this layer is not included in the area (ab) used to compute C_c. As a result, it is NOT necessary to subtract $0.85f_c'$ from f_{s5} before computing F_{s5}:

$$F_{s5} = f_{s5} \times A_{s5} = -60000 \times (1 \times 1.27) = -76.2 \text{ kip (Tension)}$$

$$\varepsilon_{s4} = (c - d_4) \times \frac{\varepsilon_{cu}}{c} = (10.28 - 15.21) \times \frac{0.003}{10.28} = -0.00144 \text{ (Tension)} < \varepsilon_y \rightarrow \text{reinforcement has not yielded}$$

$$\therefore\ f_{s4} = \varepsilon_{s4} \times E_s = -0.00144 \times 29000000 = -41739 \text{ psi}$$

The area of the reinforcement in this layer is not included in the area (ab) used to compute C_c. As a result, it is NOT necessary to subtract $0.85f_c'$ from f_{s4} before computing F_{s4}:

$$\text{F}_{s4} = f_{s4} \times A_{s4} = -41739 \times (2 \times 1.27) = -106.02 \text{ kip (Tension)}$$

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

Table 4 - Strains and internal force resultants ($f_s = f_y$)							
Layer	d, in.	ε, in./in.	f_s, psi	F_s, kip	C_c, kip	Moment arm (r), in.	Moment, kip-ft
Concrete	---	0.00300	---	---	517.24	5.22	225.00
1	2.64	0.00223	60000	70.8*	---	7.37	43.46
2	4.79	0.00160	46433	107.14*	---	5.21	46.50
3	10.00	0.00008	2347	5.96*	---	0.00	0.00
4	15.21	-0.00144	-41739	-106.02	---	-5.21	46.01
5	17.37	-0.00207	-60000	-76.20	---	-7.37	46.77
Axial Force and Bending Moment			P_n, kip	518.93		M_n, kip-ft	407.73
			ϕP_n, kip	389.20		ϕM_n, kip-ft	305.80
* The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, $0.85f_c'$ is subtracted from f_s in the computation of F_s.							

Where:

$$P_n = C_c + \sum F_s \qquad (+) = \text{Compression} \qquad (-) = \text{Tension}$$

$$\phi P_n = \phi \times P_n = 0.75 \times P_n$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n=5} \left(F_{si} \times \left(\frac{h}{2} - d_i\right)\right) \qquad (+) = \text{Counter Clockwise} \qquad (-) = \text{Clockwise}$$

$$\phi M_n = \phi \times M_n = 0.75 \times M_n$$

5. Bar Strain Near Tension Face Equal to ε_y + 0.003 in./in., (ε_s = 0.00507 in./in.)

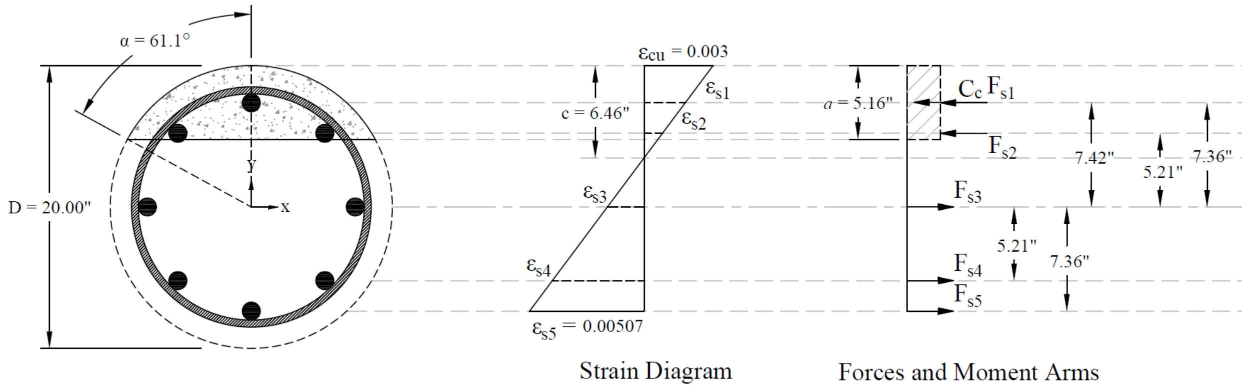


Figure 9 – Strains, Forces, and Moment Arms (ε_s = 0.00507 in./in.)

In ACI 318-19 provisions, this control point corresponds to the tension-controlled strain limit of ε_y + 0.003 (used to be 0.005 in ACI 318-14). It is the strain at the tensile limit of the transition zone for ϕ, used to define a tension-controlled section. Additional resources concerning code provision changes in ACI 318-19 can be found in "ACI 318-19 Code Revisions Impact on StructurePoint Software" technical article.

For Concrete:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_{s5} = \varepsilon_y + 0.003 = 0.00207 + 0.003 = -0.00507 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.90$$ **ACI 318-19 (Table 21.2.2)**

$$\varepsilon_{cu} = 0.003$$ **ACI 318-19 (22.2.2.1)**

$$c = \frac{d_5}{\varepsilon_{s5} + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{17.37}{0.00507 + 0.003} \times 0.003 = 6.46 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 6.46 = 5.16 \text{ in.}$$ **ACI 318-19 (22.2.2.4.1)**

Where:

a = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ **ACI 318-19 (Table 22.2.2.4.3)**

$$C_c = 0.85 \times f'_c \times A_{comp} = 0.85 \times 5000 \times 64.29 = 273.24 \text{ kip (Compression)}$$ **ACI 318-19 (22.2.2.4.1)**

Where:

$$\theta = \cos^{-1}\left(\frac{\frac{D}{2}-a}{\frac{D}{2}}\right) = \cos^{-1}\left(\frac{\frac{20}{2}-5.16}{\frac{20}{2}}\right) = 61.1°$$

$$A_{comp} = D^2 \times \frac{\theta - \sin(\theta)\times\cos(\theta)}{4}$$

$$A_{comp} = 20^2 \times \frac{\left(61.1° \times \frac{\pi}{180°}\right) - \sin(61.1°)\times\cos(61.1°)}{4} = 64.29 \text{ in.}^2$$

$$\bar{y} = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{comp}} = \frac{20^3 \times \sin^3(61.1°)}{12 \times 64.29} = 6.95 \text{ in.}$$

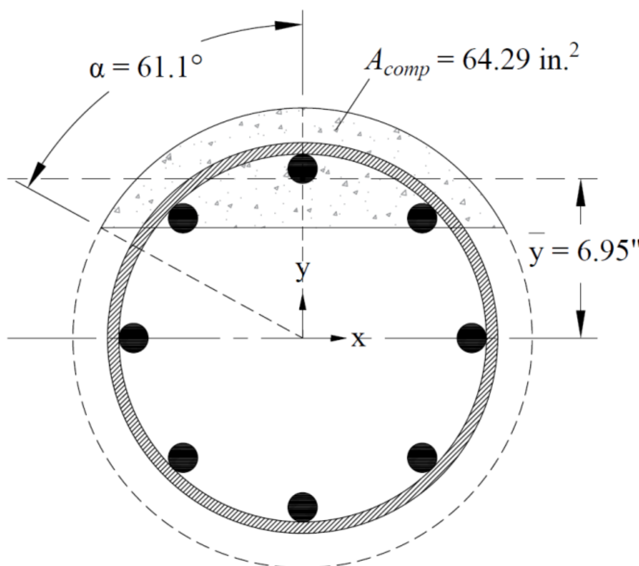


Figure 10 – Cracked Column Section Properties (ε_s = 0.00507 in./in.)

For Reinforcement:

$\varepsilon_{s5} = -0.00507$ (Tension) $> \varepsilon_y \rightarrow$ reinforcement has yielded

$\therefore f_{s5} = f_y = -60000$ psi

The area of the reinforcement in this layer is not included in the area (ab) used to compute C_c. As a result, it is NOT necessary to subtract $0.85f_c'$ from f_{s5} before computing F_{s5}:

$$F_{s5} = f_{s5} \times A_{s5} = -60000 \times (1 \times 1.27) = -76.2 \text{ kip (Tension)}$$

$$\varepsilon_{s4} = (c - d_4) \times \frac{\varepsilon_{cu}}{c} = (6.46 - 15.21) \times \frac{0.003}{6.46} = -0.00407 \text{ (Tension)} > \varepsilon_y \rightarrow \text{reinforcement has yielded}$$

$$\therefore f_{s4} = f_y = -60000 \text{ psi}$$

The area of the reinforcement in this layer is not included in the area (ab) used to compute C_c. As a result, it is NOT necessary to subtract $0.85f_c'$ from f_{s4} before computing F_{s4}:

$$F_{s4} = f_{s4} \times A_{s4} = -60000 \times (2 \times 1.27) = -152.4 \text{ kip (Tension)}$$

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

Table 5 - Strains and internal force resultants (ε_s = 0.00507 in./in.)							
Layer	d, in.	ε, in./in.	f_s, psi	F_s, kip	C_c, kip	Moment arm (r), in.	Moment, kip-ft
Concrete	---	0.00300	---	---	273.24	6.95	158.36
1	2.64	0.00178	51492	60.00*	---	7.37	36.82
2	4.79	0.00077	22423	46.16*	---	5.21	20.03
3	10.00	-0.00165	-47754	-121.29	---	0.00	0.00
4	15.21	-0.00407	-60000	-152.4	---	-5.21	66.14
5	17.37	-0.00507	-60000	-76.20	---	-7.37	46.77
Axial Force and Bending Moment			P_n, kip	29.50		M_n, kip-ft	328.13
			ϕP_n, kip	26.55		ϕM_n, kip-ft	295.31

* The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, $0.85f_c'$ is subtracted from f_s in the computation of F_s.

Where:

$$P_n = C_c + \sum F_s$$ (+) = Compression (-) = Tension

$$\phi P_n = \phi \times P_n = 0.90 \times P_n$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n=5} \left(F_{si} \times \left(\frac{h}{2} - d_i\right)\right)$$ (+) = Counter Clockwise (-) = Clockwise

$$\phi M_n = \phi \times M_n = 0.90 \times M_n$$

6. Pure Bending

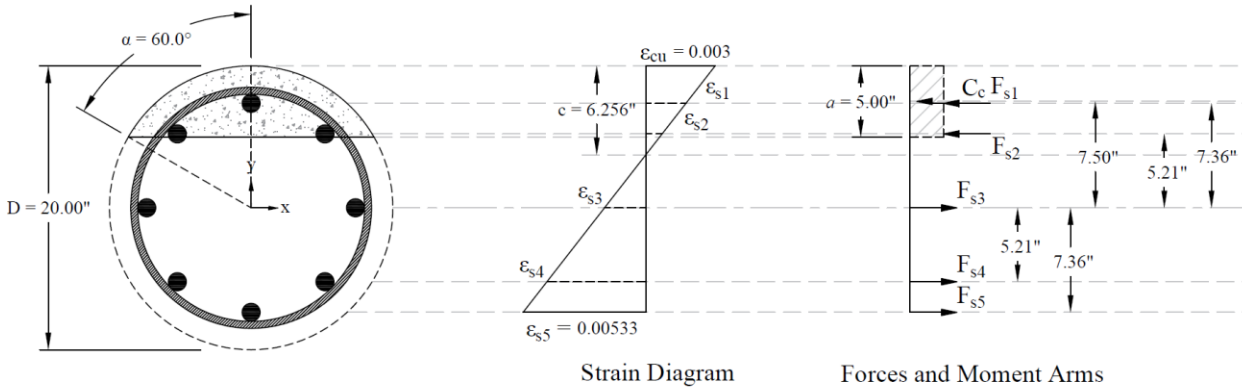


Figure 11 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the nominal axial load capacity, P_n, is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

Try $c = 6.256$ in.

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_{s5} = (c - d_5) \times \frac{\varepsilon_{cu}}{c} = (6.256 - 17.36) \times \frac{0.003}{6.256} = -0.00533 \text{ (Tension)} > \varepsilon_y \rightarrow \text{reinforcement has yielded}$$

$$\varepsilon_{s5} > 0.005$$

$\therefore \phi = 0.9$ ***ACI 318-19 (Table 21.2.2)***

$a = \beta_1 \times c = 0.80 \times 6.256 = 5.00$ in. ***ACI 318-19 (22.2.2.4.1)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-19 (22.2.2.1)***

Where:

a = Depth of equivalent rectangular stress block

$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$ ***ACI 318-19 (Table 22.2.2.4.3)***

$C_c = 0.85 \times f'_c \times A_{comp} = 0.85 \times 5000 \times 61.5 = 261.38$ kip (Compression) ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\theta = \cos^{-1}\left(\frac{\frac{D}{2}-a}{\frac{D}{2}}\right) = \cos^{-1}\left(\frac{\frac{20}{2}-5.00}{\frac{20}{2}}\right) = 60.0°$$

$$A_{comp} = D^2 \times \frac{\theta - \sin(\theta)\times\cos(\theta)}{4}$$

$$A_{comp} = 20^2 \times \frac{\left(60.0° \times \frac{\pi}{180°}\right) - \sin(60.0°)\times\cos(60.0°)}{4} = 61.5 \text{ in.}^2$$

$$\bar{y} = \frac{D^3 \times \sin^3(\theta)}{12 \times A_{comp}} = \frac{20^3 \times \sin^3(60.0°)}{12 \times 61.5} = 7.05 \text{ in.}$$

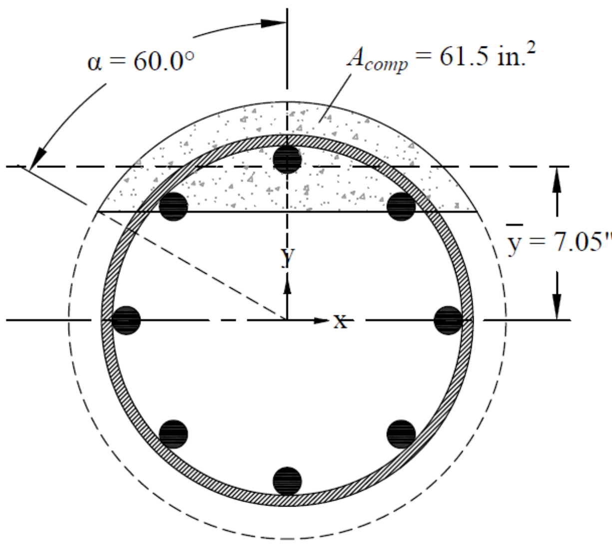


Figure 12 – Cracked Column Section Properties (Pure Moment)

$$\varepsilon_{s5} = -0.00533 \text{ (Tension)} > \varepsilon_y \rightarrow \text{reinforcement has yielded}$$

$$\therefore f_{s5} = f_y = -60000 \text{ psi}$$

The area of the reinforcement in this layer is not included in the area (ab) used to compute C_c. As a result, it is NOT necessary to subtract $0.85f_c'$ from f_{s5} before computing F_{s5}:

$$F_{s5} = f_{s5} \times A_{s5} = -60000 \times (1 \times 1.27) = -76.2 \text{ kip (Tension)}$$

$$\varepsilon_{s4} = (c - d_4) \times \frac{\varepsilon_{cu}}{c} = (6.256 - 15.21) \times \frac{0.003}{6.256} = -0.00429 \text{ (Tension)} > \varepsilon_y \rightarrow \text{reinforcement has yielded}$$

$$\therefore f_{s4} = f_y = -60000 \text{ psi}$$

The area of the reinforcement in this layer is not included in the area (ab) used to compute C_c. As a result, it is NOT necessary to subtract $0.85f_c'$ from f_{s4} before computing F_{s4}:

$$\mathrm{F}_{s4} = f_{s4} \times A_{s4} = -60000 \times (2 \times 1.27) = -152.4 \text{ kip} \;\; \text{(Tension)}$$

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

Table 6 - Strains and internal force resultants (Pure Moment)

Layer	d, in.	ε, in./in.	f_s, psi	F_s, kip	C_c, kip	Moment arm (r), in.	Moment, kip-ft
Concrete	---	0.00300	---	---	261.38	7.05	153.51
1	2.64	0.00174	50356	58.55*	---	7.37	35.94
2	4.79	0.00070	20357	40.91*	---	5.21	17.75
3	10.00	-0.00180	-52066	-132.25	---	0.00	0.00
4	15.21	-0.00429	-60000	-152.4	---	-5.21	66.14
5	17.37	-0.00533	-60000	-76.20	---	-7.37	46.77
Axial Force and Bending Moment			P_n, kip	0.00		M_n, kip-ft	320.11
			ϕP_n, kip	0.00		ϕM_n, kip-ft	288.09

* The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, $0.85f_c'$ is subtracted from f_s in the computation of F_s.

Where:

$$P_n = C_c + \sum F_s \qquad (+) = \text{Compression} \qquad (-) = \text{Tension}$$

$$\phi P_n = \phi \times P_n = 0.90 \times P_n$$

Since $P_n = \phi P_n = 0\,\text{kip}$, the assumption that c = 3.25 in. is correct.

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n=5}\left(F_{si} \times \left(\frac{h}{2} - d_i\right)\right) \qquad (+) = \text{Counter Clockwise} \qquad (-) = \text{Clockwise}$$

$$\phi M_n = \phi \times M_n = 0.90 \times M_n$$

7. Maximum Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

$$P_{nt} = f_y \times \sum_{i=1}^{n=5} n_i A_{si} = 60,000 \times (8 \times 1.27) = -609.60 \text{ kip (Tension)}$$ ***ACI 318-19 (22.4.3.1)***

$$\phi = 0.90$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_{nt} = 0.90 \times 609.60 = -548.64 \text{ kip (Tension)}$$

Since the section is symmetrical

$$M_n = \phi M_n = 0 \text{ kip.ft}$$

8. Column Interaction Diagram - spColumn Software

spColumn is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode was used with no loads (the program will only report control points) and no slenderness considerations using ACI 318-19.

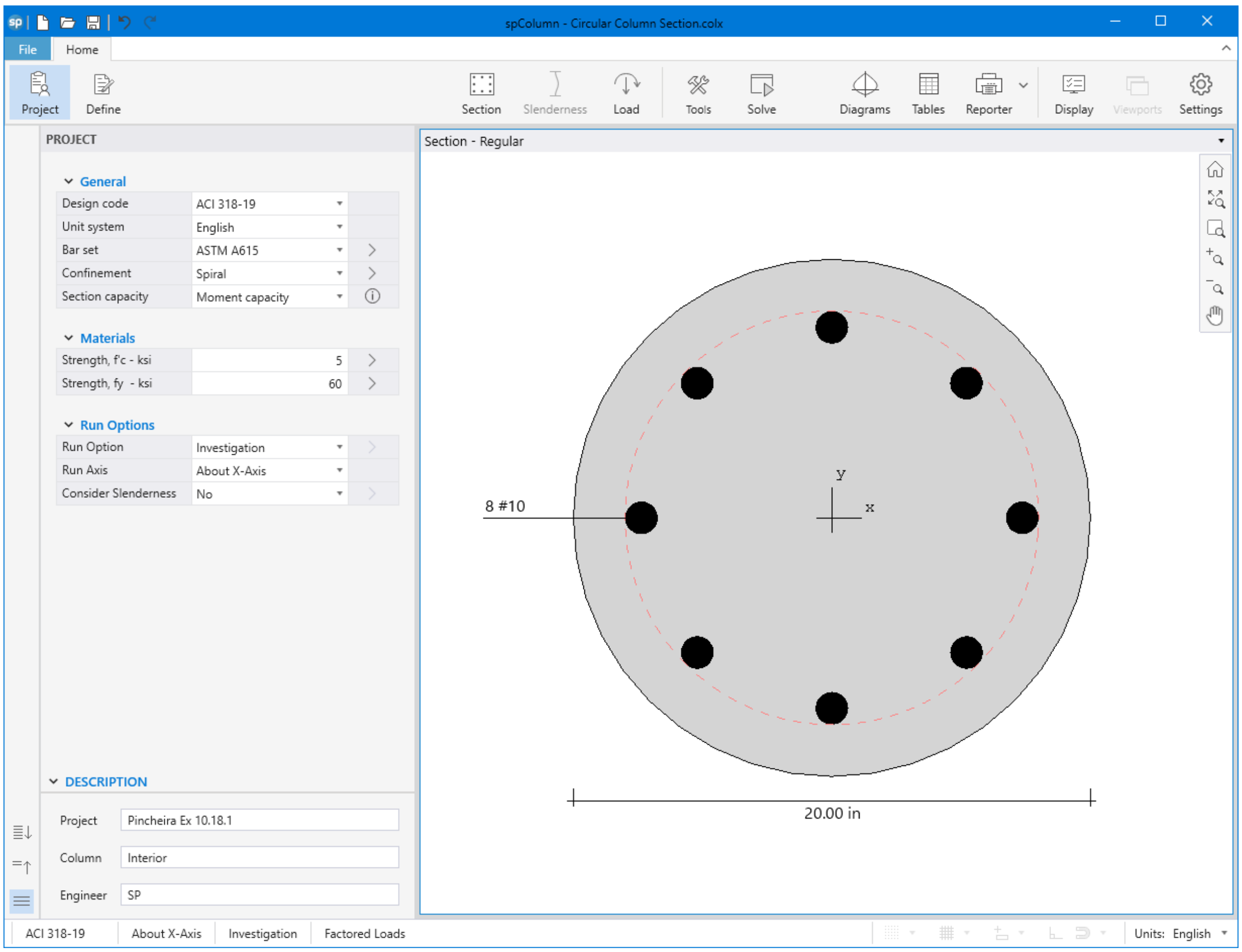


Figure 13 – spColumn Interface

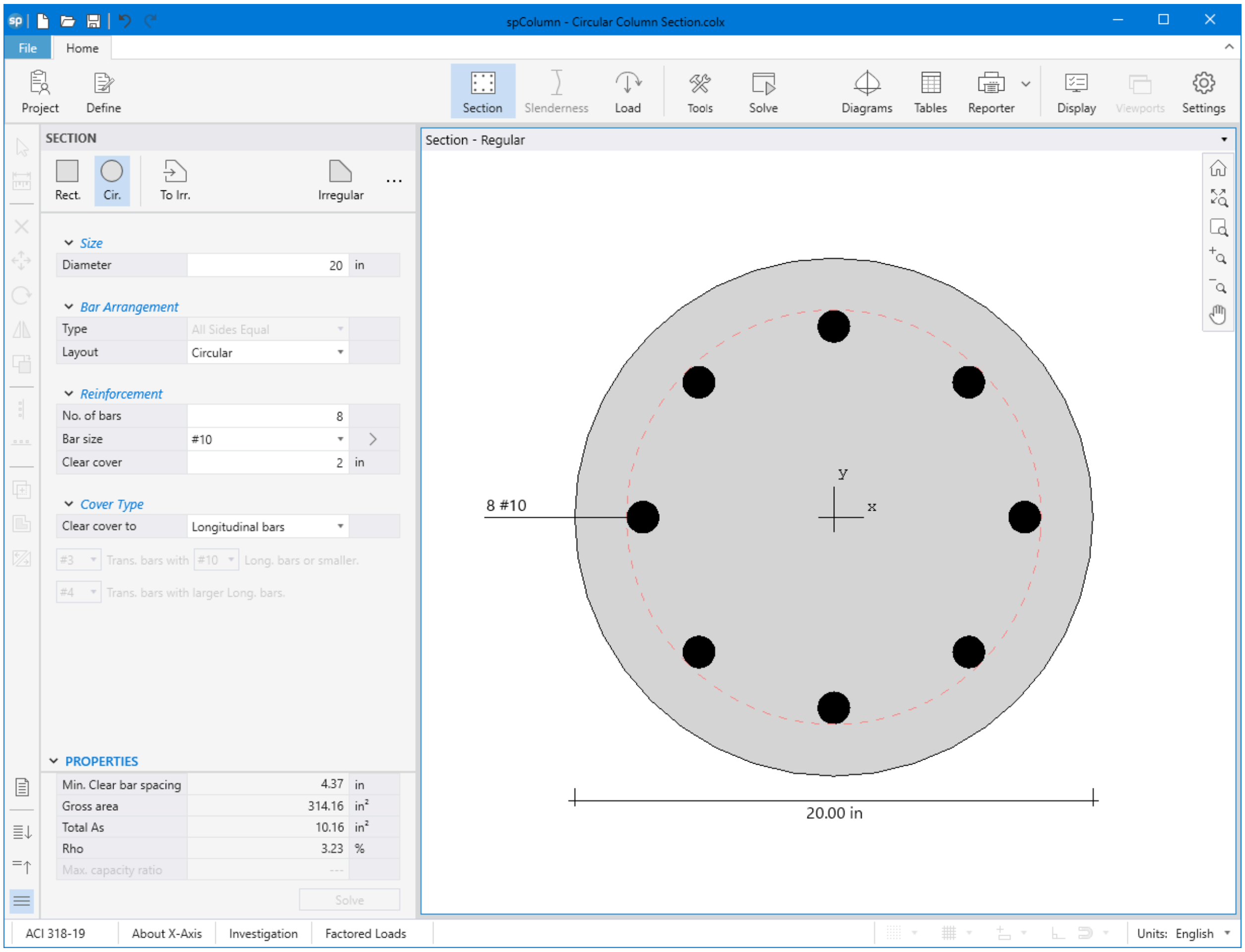


Figure 14 – spColumn Model Editor

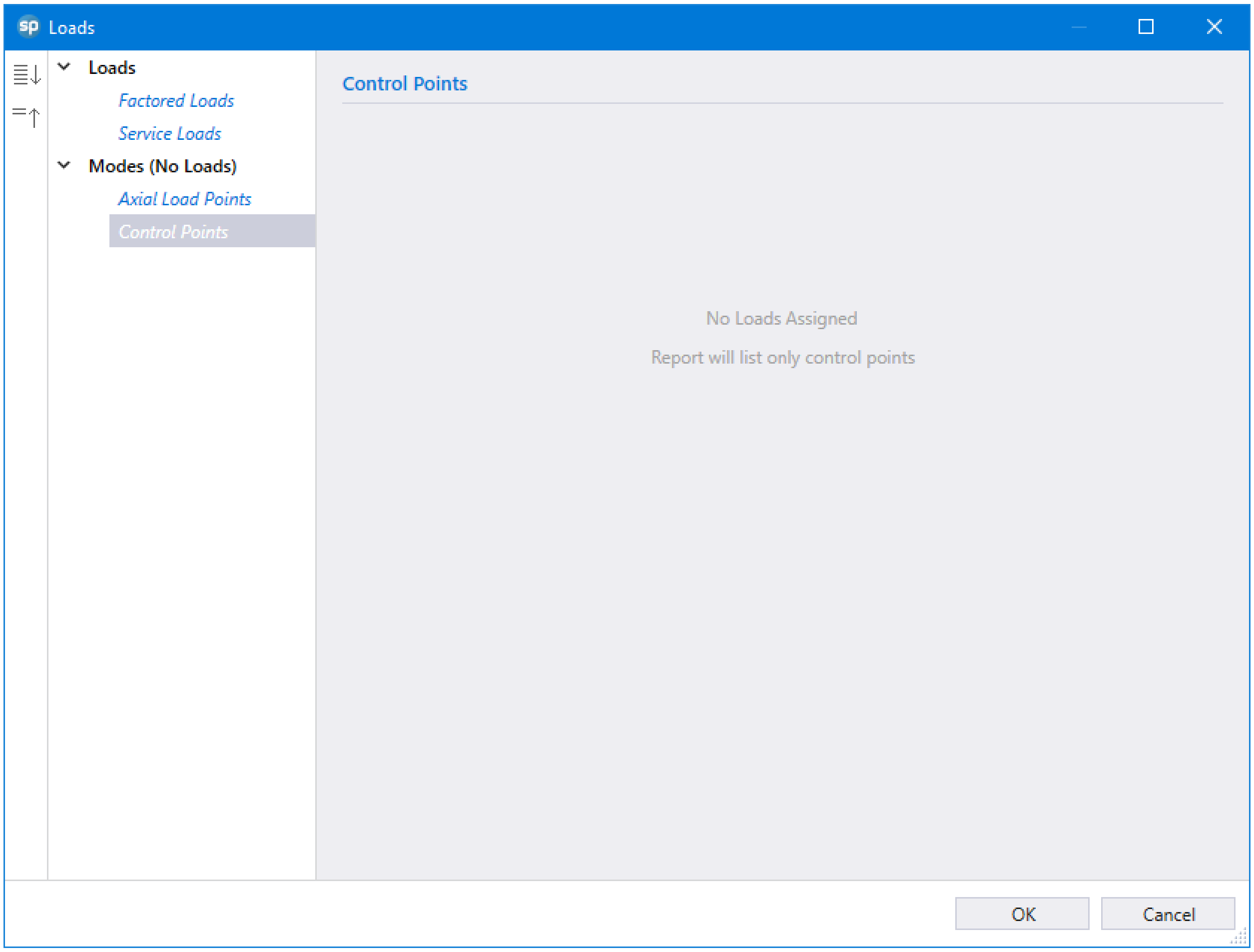


Figure 15 – Defining Loads / Modes (spColumn)

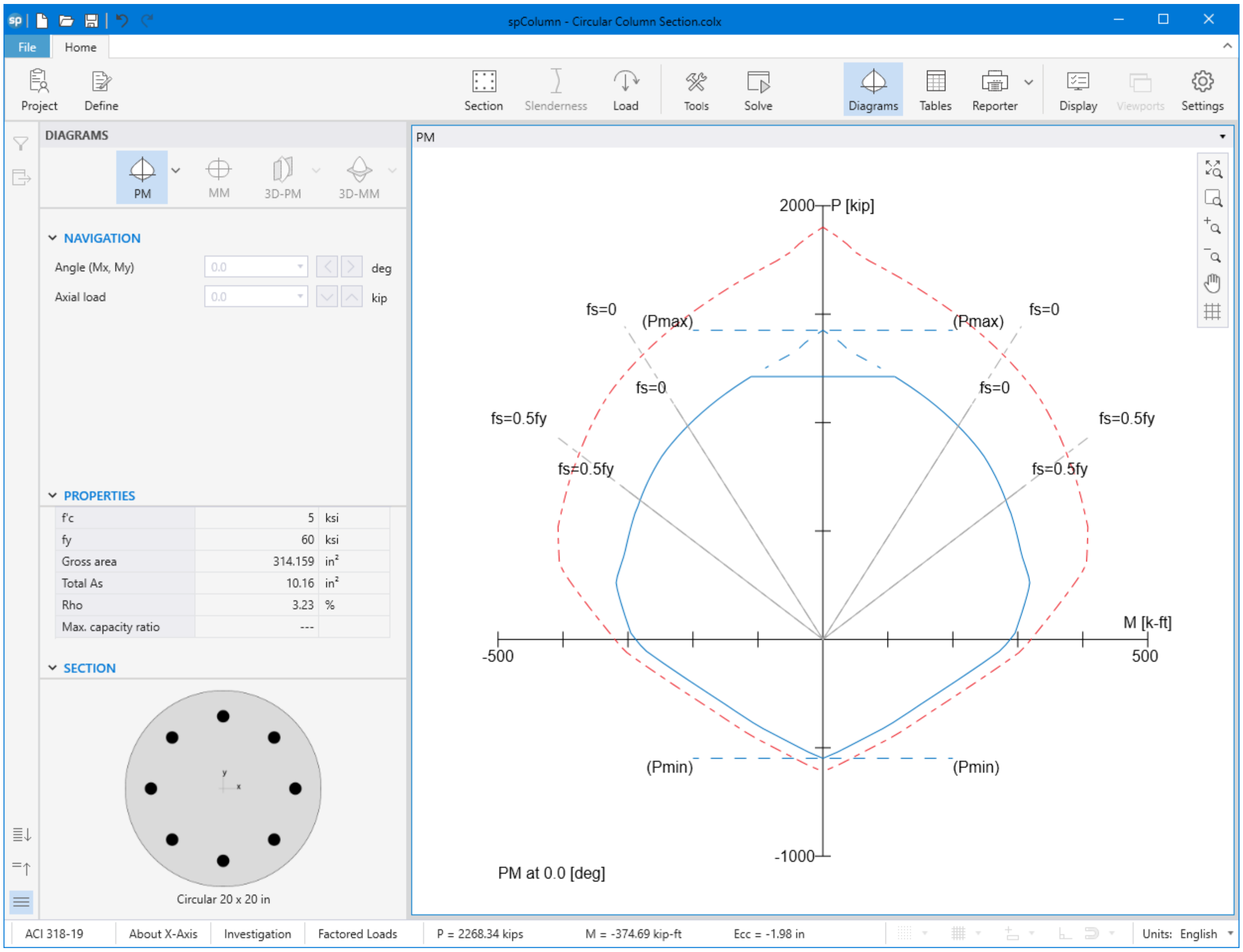


Figure 16 – Column P-M Interaction Diagram about the X-Axis (spColumn)

spColumn (TM)

spColumn v10.00 (TM)
Computer program for the Strength Design of Reinforced Concrete Sections

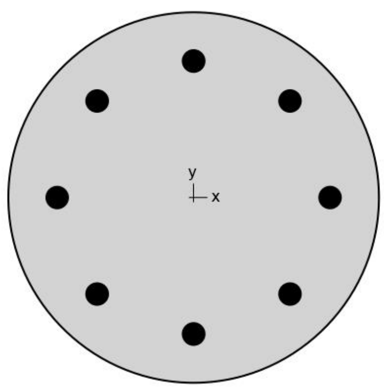


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Figure 1: Column section 4

1. General Information

File Name	E:\StructurePoint...\Circular Column Section.colx
Project	Pincheira Ex 10.18.1
Column	Interior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

2. Material Properties

2.1. Concrete

Type	Standard	
f'_c	5	ksi
E_c	4030.51	ksi
f_c	4.25	ksi
ε_u	0.003	in/in
β_1	0.8	

2.2. Steel

Type	Standard	
f_y	60	ksi
E_s	29000	ksi
ε_{ty}	0.00206897	in/in

3. Section

3.1. Shape and Properties

Type	Circular	
Diameter	20	in
A_g	314.159	in²
I_x	7853.98	in⁴
I_y	7853.98	in⁴
r_x	5	in
r_y	5	in
X_o	0	in
Y_o	0	in

3.2. Section Figure

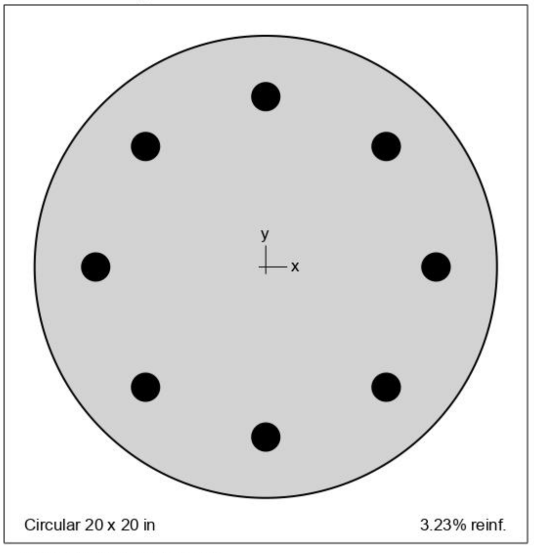


Figure 1: Column section

4. Reinforcement

4.1. Bar Set: ASTM A615

Bar	Diameter	Area	Bar	Diameter	Area	Bar	Diameter	Area
	in	in²		in	in²		in	in²
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

4.2. Confinement and Factors

Confinement type	Spiral
For #10 bars or less	#3 ties
For larger bars	#4 ties
Capacity Reduction Factors	
Axial compression, (a)	0.85
Tension controlled ϕ, (b)	0.9
Compression controlled ϕ, (c)	0.75

4.3. Arrangement

Pattern	All sides equal
Bar layout	Circular
Cover to	Longitudal bars
Clear cover	2 in
Bars	8 #10

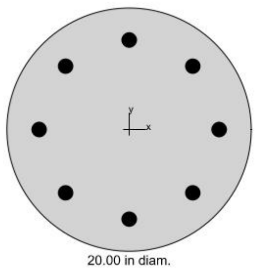
Total steel area, A_s	10.16 in²
Rho	3.23 %
Minimum clear spacing	4.37 in

5. Control Points

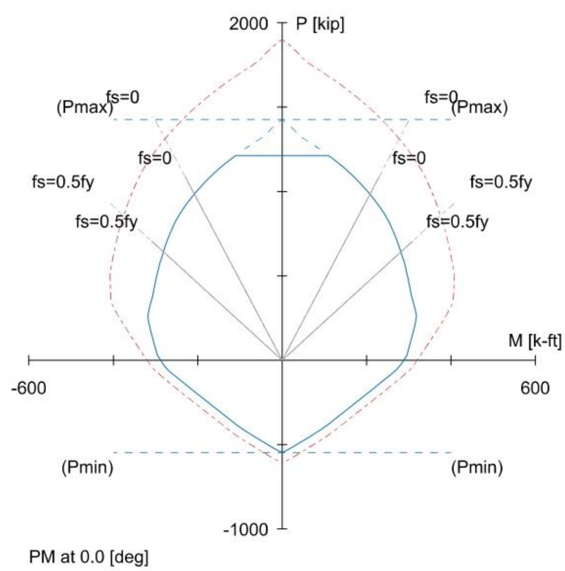
About	Point	P	X-Moment	Y-Moment	NA Depth	d_t Depth	ε_t	ϕ
		kip	k-ft	k-ft	in	in		
X	@ Max compression	1426.2	0.00	0.00	55.95	17.36	-0.00207	0.75000
X	@ Allowable comp.	1212.3	110.60	0.00	21.50	17.36	-0.00058	0.75000
X	@ f_s = 0.0	984.5	208.16	0.00	17.36	17.36	0.00000	0.75000
X	@ f_s = 0.5 f_y	642.2	281.43	0.00	12.91	17.36	0.00103	0.75000
X	@ Balanced point	389.2	305.80	0.00	10.28	17.36	0.00207	0.75000
X	@ Tension control	26.6	295.31	0.00	6.46	17.36	0.00507	0.90000
X	@ Pure bending	0.0	288.10	0.00	6.26	17.36	0.00533	0.90000
X	@ Max tension	-548.6	0.00	0.00	0.00	17.36	9.99999	0.90000
-X	@ Max compression	1426.2	0.00	0.00	55.95	17.36	-0.00207	0.75000
-X	@ Allowable comp.	1212.3	-110.60	0.00	21.50	17.36	-0.00058	0.75000
-X	@ f_s = 0.0	984.5	-208.16	0.00	17.36	17.36	0.00000	0.75000
-X	@ f_s = 0.5 f_y	642.2	-281.43	0.00	12.91	17.36	0.00103	0.75000
-X	@ Balanced point	389.2	-305.80	0.00	10.28	17.36	0.00207	0.75000
-X	@ Tension control	26.6	-295.31	0.00	6.46	17.36	0.00507	0.90000
-X	@ Pure bending	0.0	-288.10	0.00	6.26	17.36	0.00533	0.90000
-X	@ Max tension	-548.6	0.00	0.00	0.00	17.36	9.99999	0.90000

6. Diagrams

6.1. PM at θ=0 [deg]



General Information	
Project	Pincheira Ex 10.18.1
Column	Interior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity
Materials	
f'_c	5 ksi
E_c	4030.51 ksi
f_y	60 ksi
E_s	29000 ksi
Section	
Type	Circular
Diameter	20 in
A_g	314.159 in²
I_x	7853.98 in⁴
I_y	7853.98 in⁴
Reinforcement	
Pattern	All sides equal
Bar layout	Circular
Cover to	Longitudal bars
Clear cover	2 in
Bars	8 #10
Confinement type	Spiral
Total steel area, A_s	10.16 in²
Rho	3.23 %
Min. clear spacing	4.37 in



9. Summary and Comparison of Design Results

Table 7 - Comparison of Results (Balanced Point)			
Parameter	Reference	Hand	spColumn
c, in.	10.28	10.28	10.28
d_5, in.	17.36	17.36	17.36
ε_{s5}, in./in.	0.00207	0.00207	0.00207
P_n, kip	519	519	519
M_n, kip-ft	408	408	408

Table 8 - Comparison of Results				
Control Point	ϕP_n, kip		ϕM_n, kip-ft	
	Hand	spColumn	Hand	spColumn
Max compression	1426.2	1426.2	0.00	0.00
Allowable compression	1212.3	1212.3	---	---
f_s = 0.0	984.5	984.5	208.16	208.16
f_s = 0.5 f_y	642.2	642.2	281.43	281.43
Balanced point	389.2	389.2	305.80	305.80
Tension control	26.6	26.6	295.31	295.31
Pure bending	0.0	0.0	288.09	288.10
Max tension	-548.6	-548.6	0.00	0.00

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.

10. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to M_x and M_y due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint's spColumn program can also evaluate column sections in biaxial mode to produce the results shown in the following Figures for the column section in this example.

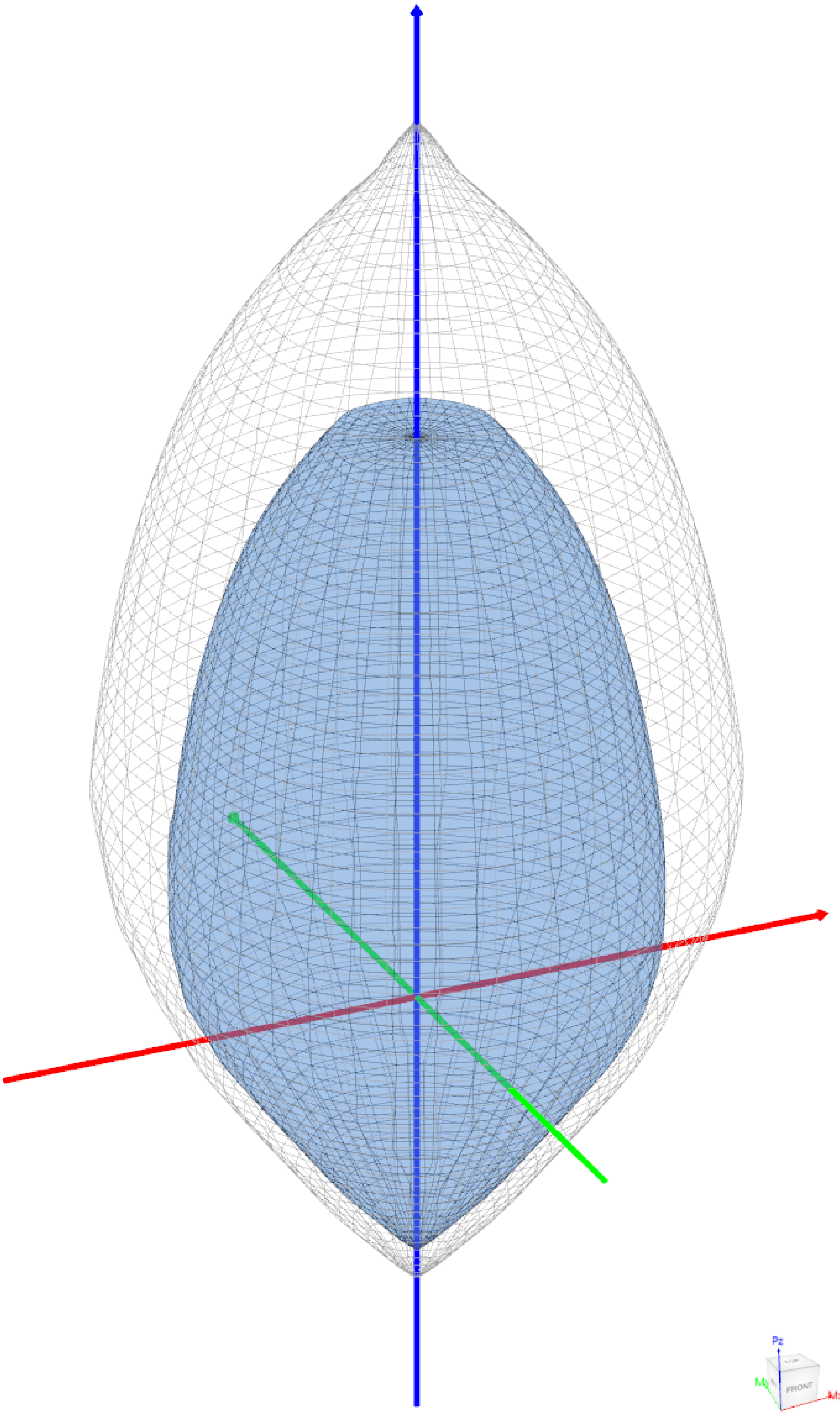


Figure 17 – Nominal & Design Interaction Diagram in Two Directions (Biaxial) (spColumn)

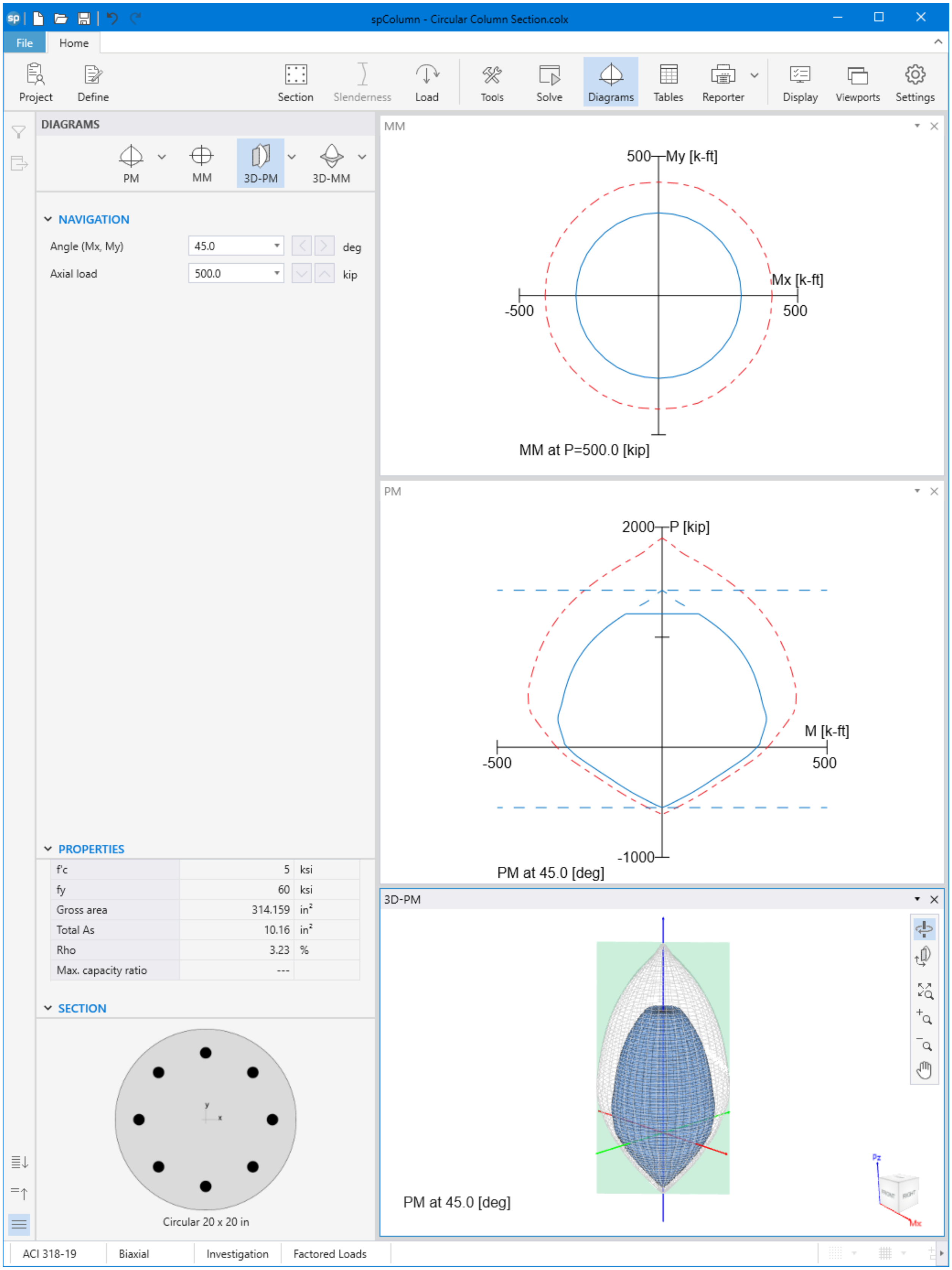


Figure 18 – Circular Column Interaction Diagram and 3D failure Surface Viewer (spColumn)

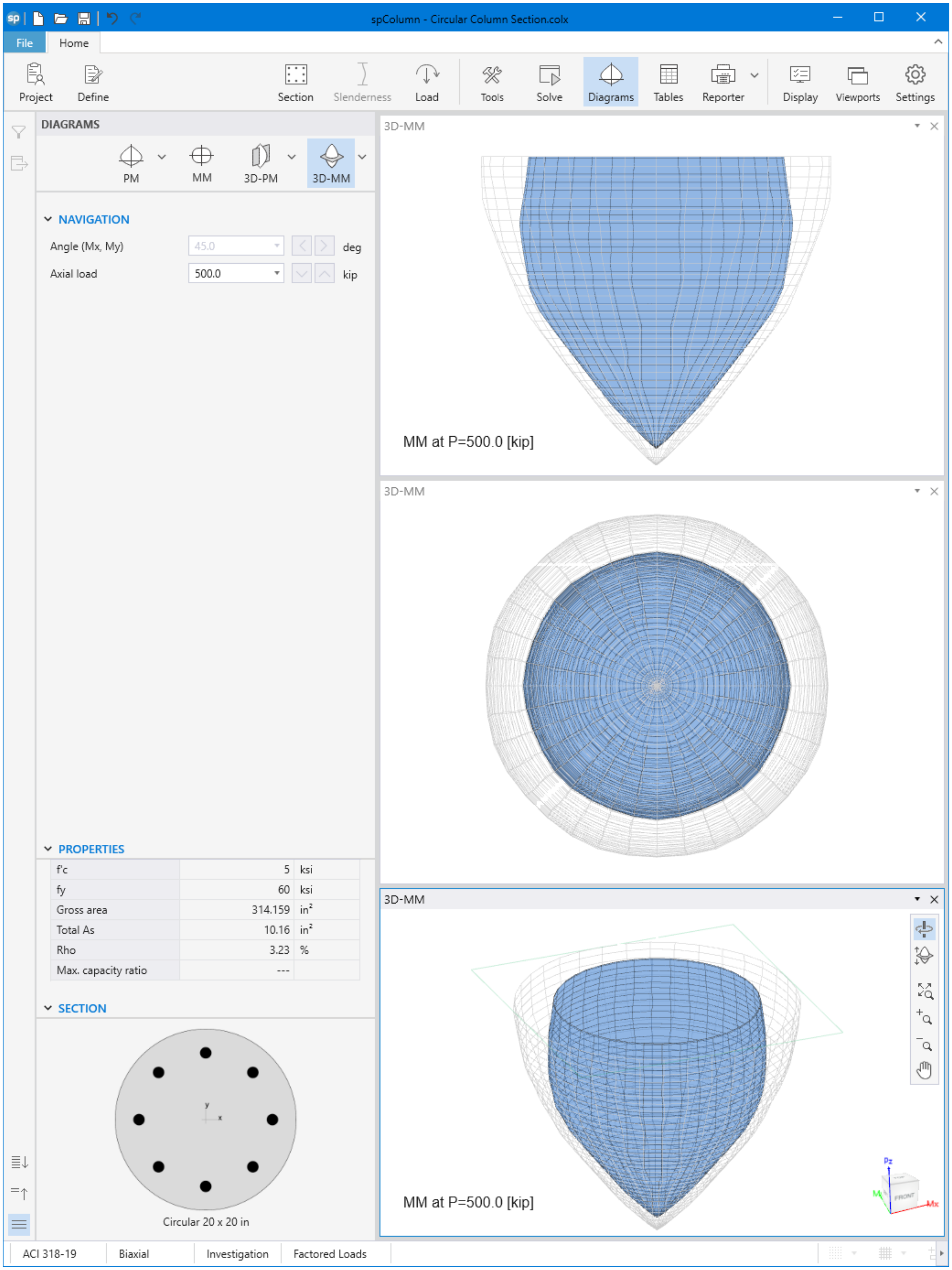


Figure 19 – Circular Column 3D Failure Surface with a Horizontal Plane Cut at P = 500 kip (spColumn)

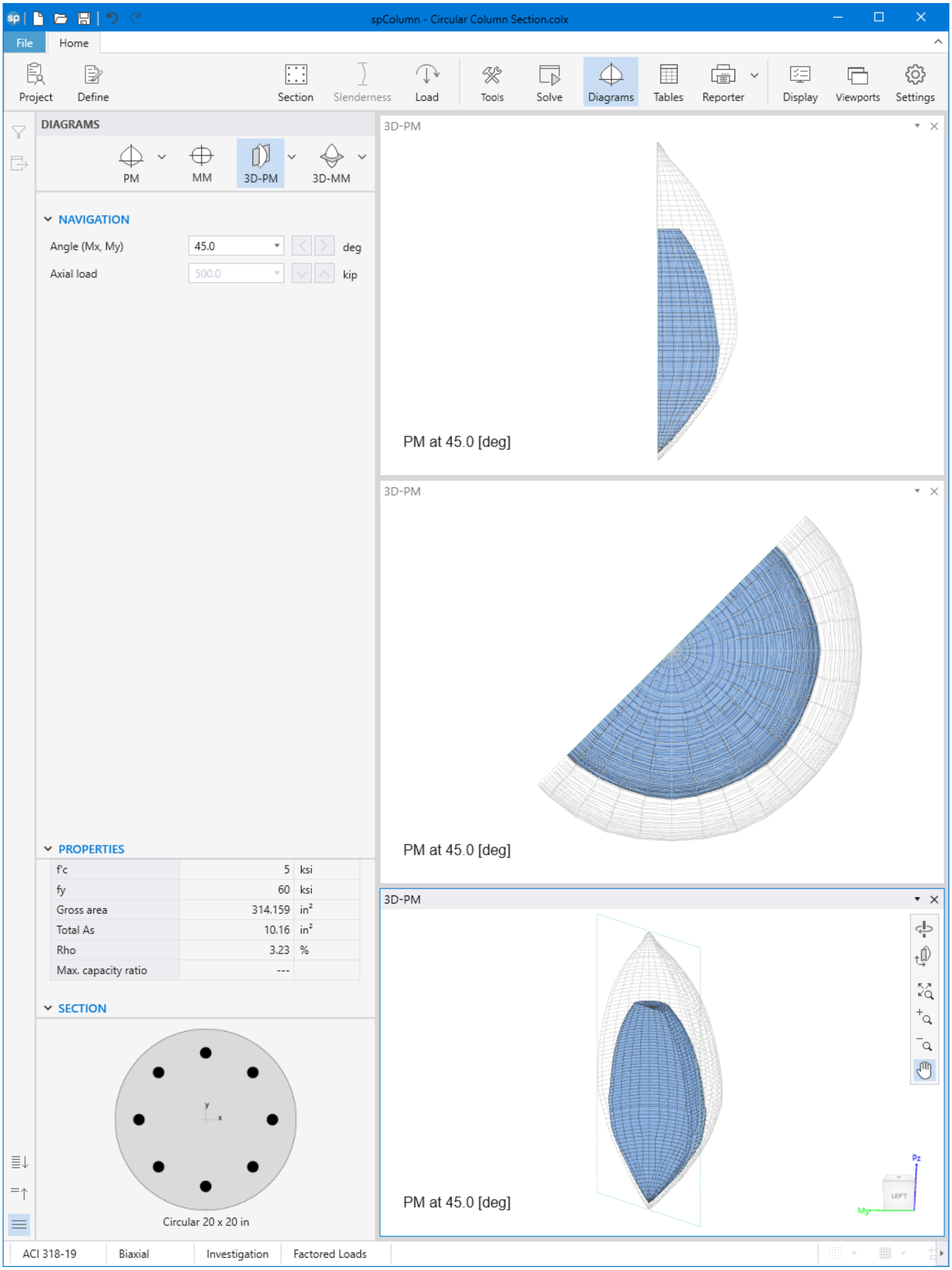


Figure 20 – Circular Column 3D Failure Surface with a Vertical Plane Cut at 45° (spColumn)