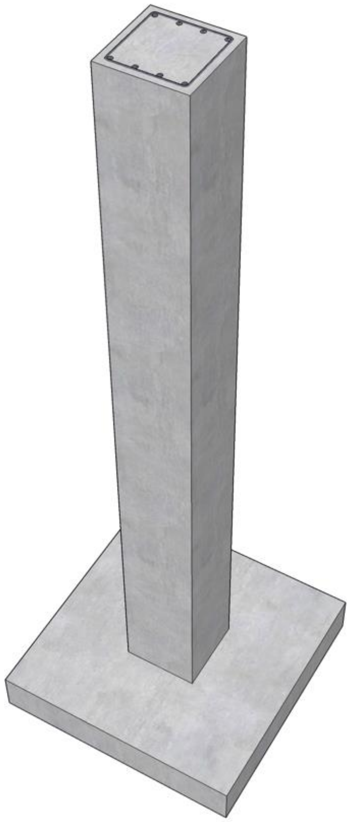
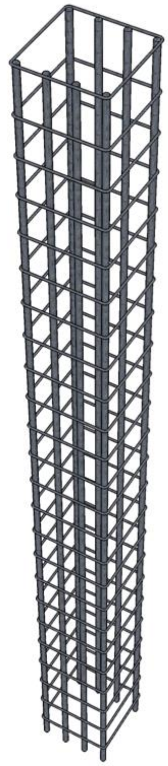
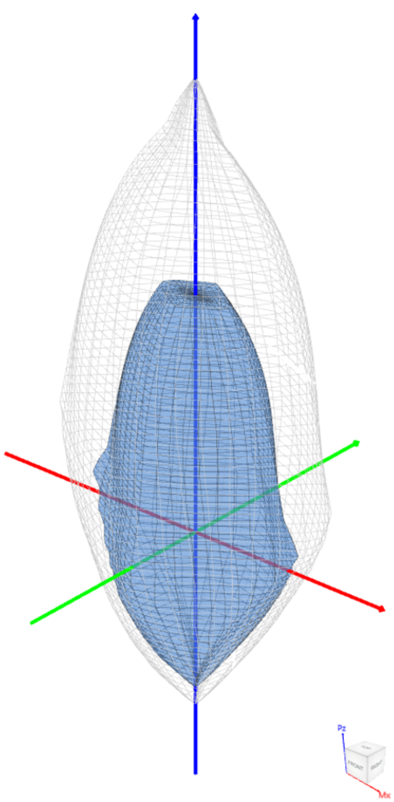
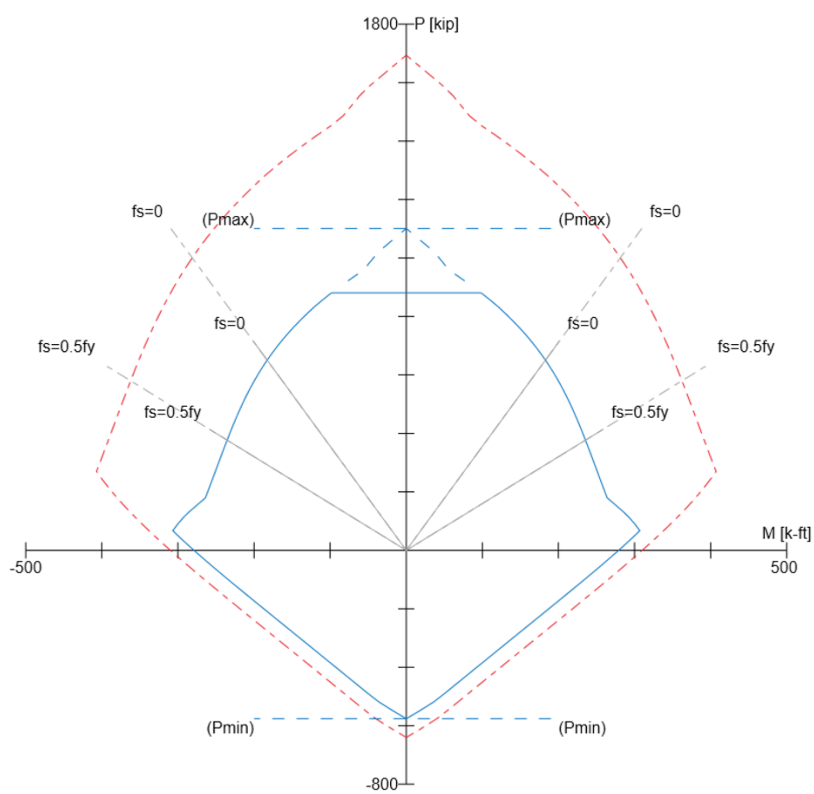
Interaction Diagram - Tied Reinforced Concrete Column with High-Strength Reinforcing Bars (ACI 318-19)









Interaction Diagram - Tied Reinforced Concrete Column with High-Strength Reinforcing Bars (ACI 318-19)

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using ACI 318-19. Determine seven control points on the interaction diagram and compare the calculated values with exact values from the complete interaction diagram generated by spColumn engineering software program from StructurePoint. High Strength Reinforcing Bars (HSRB) with Grade 80 steel (f_y = 80 ksi) is being used to assist with congestion of reinforcement at columns/beams joints.

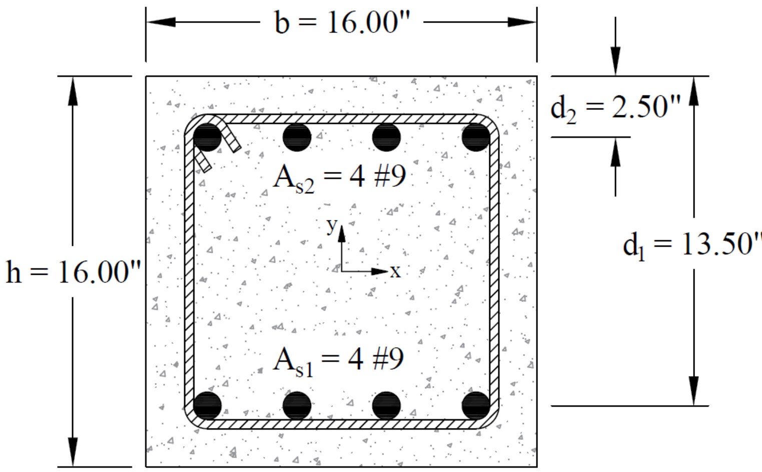


Figure 1 – Reinforced Concrete Column Cross-Section

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

- Reinforced Concrete Mechanics and Design, 8th Edition, 2021, James Wight, Pearson
- "Column Design with High-Strength Reinforcing Bars per ACI 318-19" Technical Article, STRUCTUREPOINT, 2019
- "Column Design Capacity Comparison with High Strength Reinforcing Bars per ACI 318-14 and ACI 318-19" Technical Article, STRUCTUREPOINT, 2019
- "ACI 318-19 Code Revisions Impact on StructurePoint Software" Technical Article, STRUCTUREPOINT, 2019
- spColumn Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2021
- "Interaction Diagram - Tied Reinforced Concrete Column Design Strength (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram - Circular Spiral Reinforced Concrete Column (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram - Barbell Concrete Shear Wall Unsymmetrical Boundary Elements (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Interaction Diagram – Building Elevator Reinforced Concrete Core Wall Design Strength (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022

Design Data

f_c' = 5000 psi

f_y = 80,000 psi

Cover = 2.5 in. to the center of the reinforcement

Column 16 in. x 16 in.

Top reinforcement = 4 #9

Bottom reinforcement = 4 #9

Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following seven control points:

Point 1: Maximum compression

Point 2: Bar stress near tension face of member equal to zero, (f_s = 0)

Point 3: Bar stress near tension face of member equal to 0.5 f_y (f_s = 0.5 f_y)

Point 4: Bar stress near tension face of member equal to f_y (f_s = f_y)

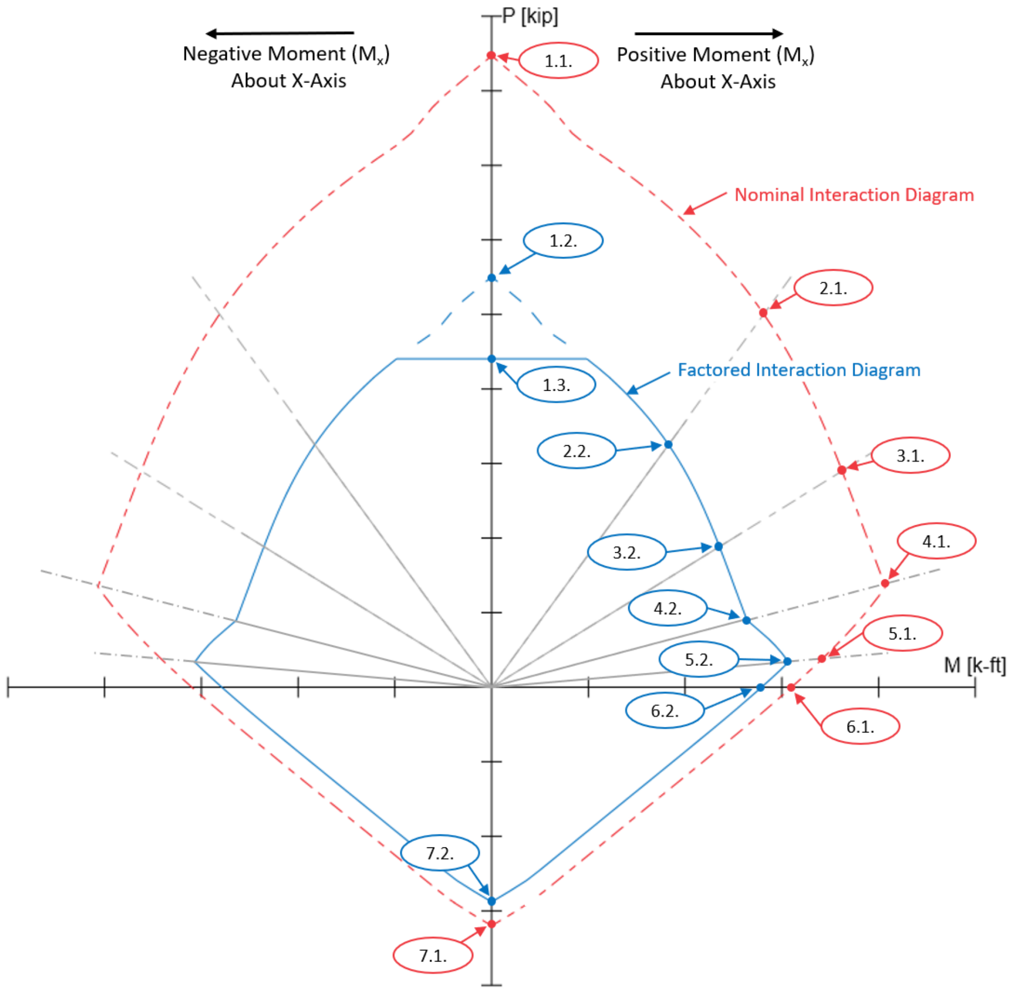
Point 5: Bar strain near tension face of member equal to ε_y + 0.003

Point 6: Pure bending

Point 7: Maximum tension

Several terms are used to facilitate the following calculations:

P_o	= nominal axial compressive strength, kip
ϕP_o	= factored axial compressive strength, kip
$\phi P_{n,max}$	= maximum (allowable) factored axial compressive strength, kip
c	= distance from the fiber of maximum compressive strain to the neutral axis, in.
a	= depth of equivalent rectangular stress block, in.
C_c	= compression force in equivalent rectangular stress block, kip
ε_s	= strain value in reinforcement, in./in.
C_s	= compression force in reinforcement, kip
T_s	= tension force in reinforcement, kip



1.1. Maximum Nominal Axial Strength in Compression (P_o)
2.1. Nominal Control Point at Zero Stress in Tension Reinforcement
3.1. Nominal Control Point at Tension Reinforcement Stress = $0.5f_y$
4.1. Nominal Control Point at Tension Reinforcement Stress = f_y
5.1. Nominal Control Point at Tension Reinforcement Stress = ε_y + 0.003 in./in.
6.1. Nominal Control Point at Pure Bending
7.1. Maximum Nominal Axial Strength in Tension

1.2. Maximum Factored Axial Strength in Compression
1.3. Allowable Factored Axial Strength in Compression ($\phi P_{n,max}$)
2.2. Factored Control Point at Zero Stress in Tension Reinforcement
3.2. Factored Control Point at Tension Reinforcement Stress = $0.5f_y$
4.2. Factored Control Point at Tension Reinforcement Stress = f_y
5.2. Factored Control Point at Tension Reinforcement Stress = ε_y + 0.003 in./in.
6.2. Factored Control Point at Pure Bending
7.2. Maximum Factored Axial Strength in Tension

Figure 2 – Tied Column Section Interaction Diagram Control Points

1. Maximum Compression

1.1. Nominal axial compressive strength at zero eccentricity

$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$$ ***ACI 318-19 (22.4.2.2)***

$$P_o = 0.85 \times 5000 \times (16 \times 16 - 8 \times 1.00) + 80000 \times 8 \times 1.00 = 1694 \text{ kips}$$

1.2. Factored axial compressive strength at zero eccentricity

Since this column is a tied column with steel strain in compression:

$$\phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_o = 0.65 \times 1694 = 1101.1 \text{ kips}$$

Since the section is regular (symmetrical) about the x-axis, the moment capacity associated with the maximum axial compressive strength is equal to zero.

$$M_o = \phi M_o = 0.00 \text{ kip-ft}$$

1.3. Maximum (allowable) factored axial compressive strength

$$\phi P_{n,max} = 0.80 \times \phi P_o = 0.80 \times 1101.1 = 880.9 \text{ kips}$$ ***ACI 318-19 (Table 22.4.2.1)***

2. Bar Stress Near Tension Face Equal to Zero, ($\varepsilon_s = f_s = 0$)

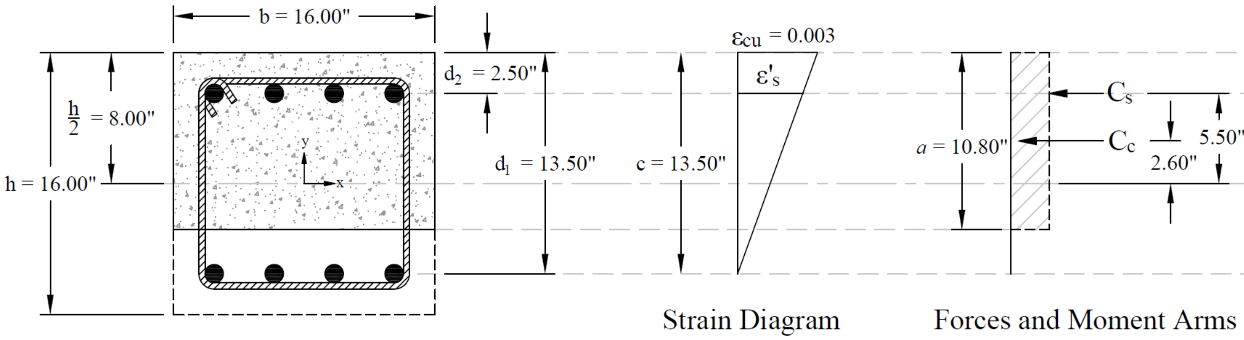


Figure 3 – Strains, Forces, and Moment Arms ($\varepsilon_t = f_s = 0$)

Strain ε_s is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. ***ACI 318-19 (10.7.5.2.1 and 2)***

2.1. c, *a*, and strains in the reinforcement

$$c = d_1 = 13.5 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 13.5 = 10.80 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

a = Depth of equivalent rectangular stress block

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s = 0$$

$$\therefore \phi = 0.65$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon_s' = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (13.50 - 2.5) \times \frac{0.003}{13.50} = 0.00244 \text{ (Compression)} < \varepsilon_y = \frac{F_y}{E_s} = \frac{80}{29000} = 0.00276$$

2.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5{,}000 \times 10.80 \times 16 = 734.4 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = 0 \text{ psi} \rightarrow \mathrm{T}_s = f_s \times A_{s1} = 0 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s' = \varepsilon_s' \times E_s = 0.00244 \times 29000000 = 70889 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\mathrm{C}_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(80000 - 0.85 \times 5000\right) \times 4 = 266.6 \text{ kip}$$

2.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 734.4 + 266.6 - 0 = 1001.0 \text{ kip}$$

$$\phi P_n = 0.65 \times 1001.0 = 650.6 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 734.4 \times \left(\frac{16}{2} - \frac{10.80}{2}\right) + 266.6 \times \left(\frac{16}{2} - 2.5\right) + 0 \times \left(13.50 - \frac{16}{2}\right) = 281.29 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 281.3 = 182.84 \text{ kip.ft}$$

3. Bar Stress Near Tension Face Equal to 0.5 f_y, (f_s = 0.5 f_y)

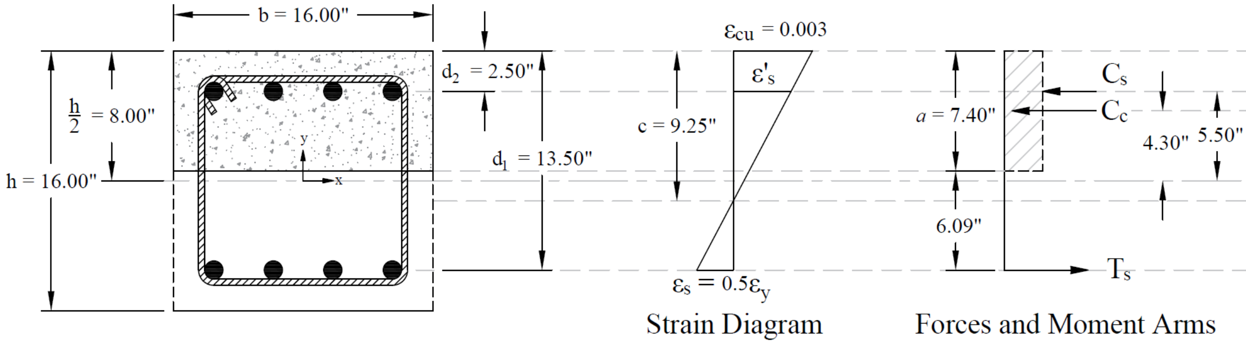


Figure 4 – Strains, Forces, and Moment Arms (f_s = 0.5 f_y)

3.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{80}{29000} = 0.00276$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.00276}{2} = 0.00138 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$\therefore \phi = 0.65$ ***ACI 318-19 (Table 21.2.2)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00138 + 0.003} \times 0.003 = 9.25 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$a = \beta_1 \times c = 0.80 \times 9.25 = 7.40$ in. ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon'_s = (c - d_2) \times \frac{0.003}{c} = (9.25 - 2.5) \times \frac{0.003}{9.25} = 0.00219 \text{ (Compression)} < \varepsilon_y$$

3.2. Forces in the concrete and steel

$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 5000 \times 7.40 \times 16 = 503.1$ kip ***ACI 318-19 (22.2.2.4.1)***

$f_s = \varepsilon_s \times E_s = 0.00138 \times 29000000 = 40000$ psi

$$\text{T}_s = f_s \times A_{s1} = 40000 \times 4 = 160 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s' = \varepsilon_s' \times E_s = 0.00219 \times 29000000 = 63481 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\text{C}_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(63481 - 0.85 \times 5000\right) \times 4 = 237.0 \text{ kip}$$

3.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 503.1 + 237.0 - 160.0 = 580.0 \text{ kip}$$

$$\phi P_n = 0.65 \times 580.0 = 377.0 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 503.1 \times \left(\frac{16}{2} - \frac{7.40}{2}\right) + 237.0 \times \left(\frac{16}{2} - 2.5\right) + 160.0 \times \left(13.50 - \frac{16}{2}\right) = 362.23 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 362.2 = 235.45 \text{ kip.ft}$$

4. Bar Stress Near Tension Face Equal to f_y, ($f_s = f_y$)

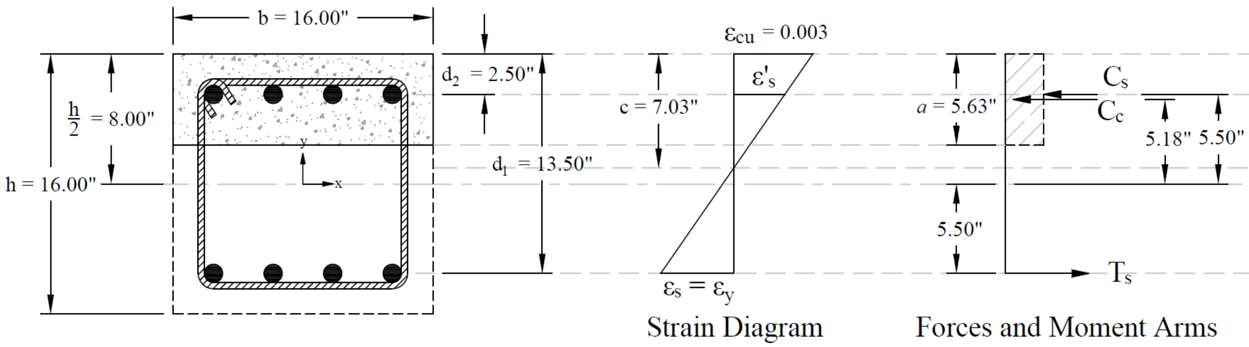


Figure 5 – Strains, Forces, and Moment Arms ($f_s = f_y$)

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement. It also marks the start of the transition zone for ϕ for columns in which ϕ increases from 0.65 (or 0.75 for spiral columns) up to 0.90.

4.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{80}{29000} = 0.00276$$

$\varepsilon_s = \varepsilon_y = 0.00276 \rightarrow$ tension reinforcement has yielded

$\therefore \phi = 0.65$ ***ACI 318-19 (Table 21.2.2)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00276 + 0.003} \times 0.003 = 7.03 \text{ in.}$$

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$a = \beta_1 \times c = 0.80 \times 7.03 = 5.63 \text{ in.}$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (7.03 - 2.5) \times \frac{0.003}{7.03} = 0.00193 \text{ (Compression)} < \varepsilon_y$$

4.2. Forces in the concrete and steel

$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5000 \times 5.63 \times 16 = 382.6 \text{ kip}$ **ACI 318-19 (22.2.2.4.1)**

$f_s = f_y = 80000 \text{ psi}$

$\mathrm{T}_s = f_y \times A_{s1} = 80000 \times 4 = 320 \text{ kip}$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$\therefore f_s' = \varepsilon_s' \times E_s = 0.00193 \times 29000000 = 56074 \text{ psi}$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$\mathrm{C}_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(56074 - 0.85 \times 5000\right) \times 4 = 207.3 \text{ kip}$

4.3. ϕP_n and ϕM_n

$P_n = C_c + C_s - T_s = 382.6 + 207.3 - 320.0 = 269.9 \text{ kip}$

$\phi P_n = 0.65 \times 269.9 = 175.4 \text{ kip}$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 382.6 \times \left(\frac{16}{2} - \frac{5.63}{2}\right) + 207.3 \times \left(\frac{16}{2} - 2.5\right) + 320.0 \times \left(13.50 - \frac{16}{2}\right) = 407.05 \text{ kip.ft}$$

$\phi M_n = 0.65 \times 407.0 = 264.58 \text{ kip.ft}$

5. Bar Strain Near Tension Face Equal to ε_y + 0.003, (ε_s = 0.00576 in./in.)

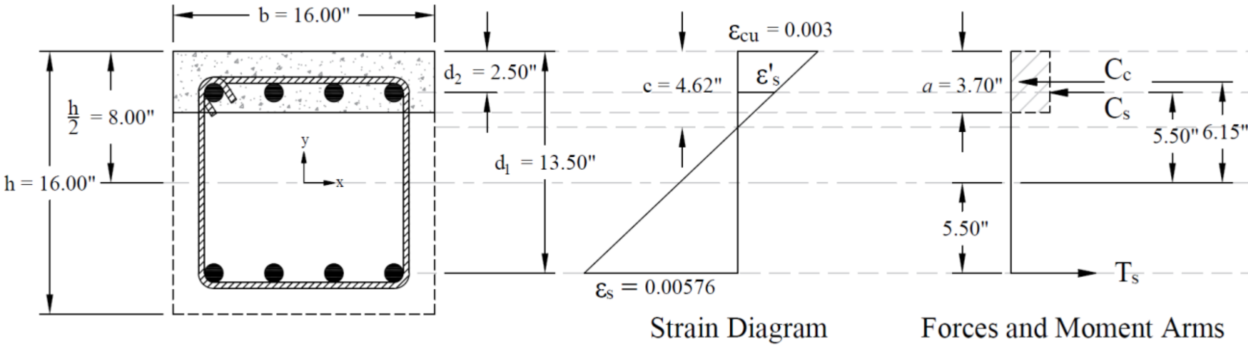


Figure 6 – Strains, Forces, and Moment Arms (ε_s = 0.00576 in./in.)

In ACI 318-19 provisions, this control point corresponds to the tension-controlled strain limit of ε_y + 0.003 (used to be 0.005 in ACI 318-14). It is the strain at the tensile limit of the transition zone for ϕ, used to define a tension-controlled section. Additional resources concerning code provision changes in ACI 318-19 can be found in "ACI 318-19 Code Revisions Impact on StructurePoint Software" technical article.

5.1. c, *a*, and strains in the reinforcement

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{80}{29000} = 0.00276$$

$$\varepsilon_s = \varepsilon_y + 0.003 = 0.00276 + 0.003 = 0.00576 > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.9$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{13.50}{0.00576 + 0.003} \times 0.003 = 4.62 \text{ in.}$$

Where *c* is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 4.62 = 3.70 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c} = (4.62 - 2.5) \times \frac{0.003}{4.62} = 0.00138 \text{ (Compression)} < \varepsilon_y$$

5.2. Forces in the concrete and steel

$$C_c = 0.85 \times f_c' \times a \times b = 0.85 \times 5000 \times 3.70 \times 16 = 251.5 \text{ kip}$$ **_ACI 318-19 (22.2.2.4.1)_**

$$f_s = f_y = 80000 \text{ psi}$$

$$T_s = f_y \times A_{s1} = 80000 \times 4 = 320 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s' = \varepsilon_s' \times E_s = 0.00138 \times 29000000 = 39963 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (*ab*) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$C_s = \left(f_s' - 0.85 f_c'\right) \times A_{s2} = \left(39963 - 0.85 \times 5000\right) \times 4 = 142.9 \text{ kip}$$

5.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 251.5 + 142.9 - 320 = 74.4 \text{ kip}$$

$$\phi P_n = 0.90 \times 74.4 = 67.0 \text{ kip}$$

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_s \times \left(\frac{h}{2} - d_2\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 251.5 \times \left(\frac{16}{2} - \frac{3.70}{2}\right) + 142.9 \times \left(\frac{16}{2} - 2.5\right) + 320 \times \left(13.50 - \frac{16}{2}\right) = 341.07 \text{ kip.ft}$$

$$\phi M_n = 0.90 \times 341.1 = 306.96 \text{ kip.ft}$$

6. Pure Bending

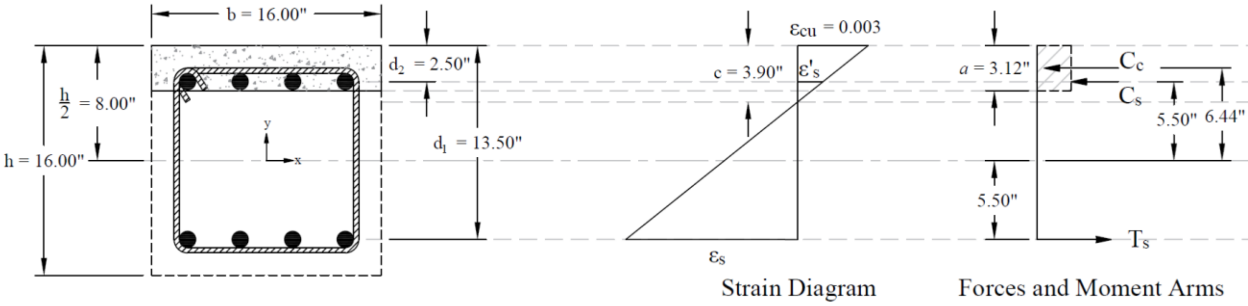


Figure 7 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the nominal axial load capacity, P_n, is equal to zero. Iterative procedure is used to determine the nominal moment capacity as follows:

6.1. c, *a*, and strains in the reinforcement

Try $c = 3.899$ in.

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$$a = \beta_1 \times c = 0.80 \times 3.899 = 3.119 \text{ in.}$$ ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f_c' - 4000)}{1000} = 0.85 - \frac{0.05 \times (5000 - 4000)}{1000} = 0.80$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{80}{29000} = 0.00276$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.003}{c}$$

$$\varepsilon_s = (13.50 - 3.899) \times \frac{0.003}{3.899} = 0.00739 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\therefore \phi = 0.9$$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon_s' = (c - d_2) \times \frac{0.003}{c}$$

$$\varepsilon_s' = (3.899 - 2.5) \times \frac{0.003}{3.899} = 0.00108 \text{ (Compression)} < \varepsilon_y \rightarrow \text{compression reinforcement has not yielded}$$

6.2. Forces in the concrete and steel

$$C_c = 0.85\times f_c'\times a\times b = 0.85\times 5000\times 3.119\times 16 = 212.1 \text{ kip}$$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = f_y = 80000 \text{ psi}$$

$$\text{T}_s = f_y\times A_{s1} = 80000\times 4 = 320 \text{ kip}$$

Since $\varepsilon_s' < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore\ f_s' = \varepsilon_s'\times E_s = 0.00108\times 29000000 = 31216 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (*ab*) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\text{C}_s = \left(f_s' - 0.85 f_c'\right)\times A_{s2} = \left(31216 - 0.85\times 5000\right)\times 4 = 107.9 \text{ kip}$$

6.3. ϕP_n and ϕM_n

$$P_n = C_c + C_s - T_s = 212.1 + 107.9 - 320 = 0 \text{ kip} \rightarrow \phi P_n = 0.0 \text{ kip}$$

The assumption that c = 3.899 in. is correct

$$M_n = C_c\times\left(\frac{h}{2}-\frac{a}{2}\right) + C_s\times\left(\frac{h}{2}-d_2\right) + T_s\times\left(d_1-\frac{h}{2}\right)$$

$$M_n = 212.1\times\left(\frac{16}{2}-\frac{3.119}{2}\right) + 107.1\times\left(\frac{16}{2}-2.5\right) + 320\times\left(13.50-\frac{16}{2}\right) = 309.94 \text{ kip.ft}$$

$$\phi M_n = 0.90\times 209.9 = 278.95 \text{ kip.ft}$$

7. Pure Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

7.1. P_{nt} and ϕP_{nt}

$$P_{nt} = f_y \times (A_{s1} + A_{s2}) = 80000 \times (4+4) = 640.0 \text{ kip}$$ ***ACI 318-19 (22.4.3.1)***

$$\phi = 0.9$$ ***ACI 318-19 (Table 21.2.2)***

$$\phi P_{nt} = 0.90 \times 640 = 576.0 \text{ kip}$$

7.2. M_n and ϕM_n

Since the section is symmetrical

$$M_n = \phi M_n = 0.00 \text{ kip.ft}$$

8. Column Interaction Diagram - spColumn Software

spColumn is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode was used with no loads (the program will only report control points) and no slenderness considerations using ACI 318-19.

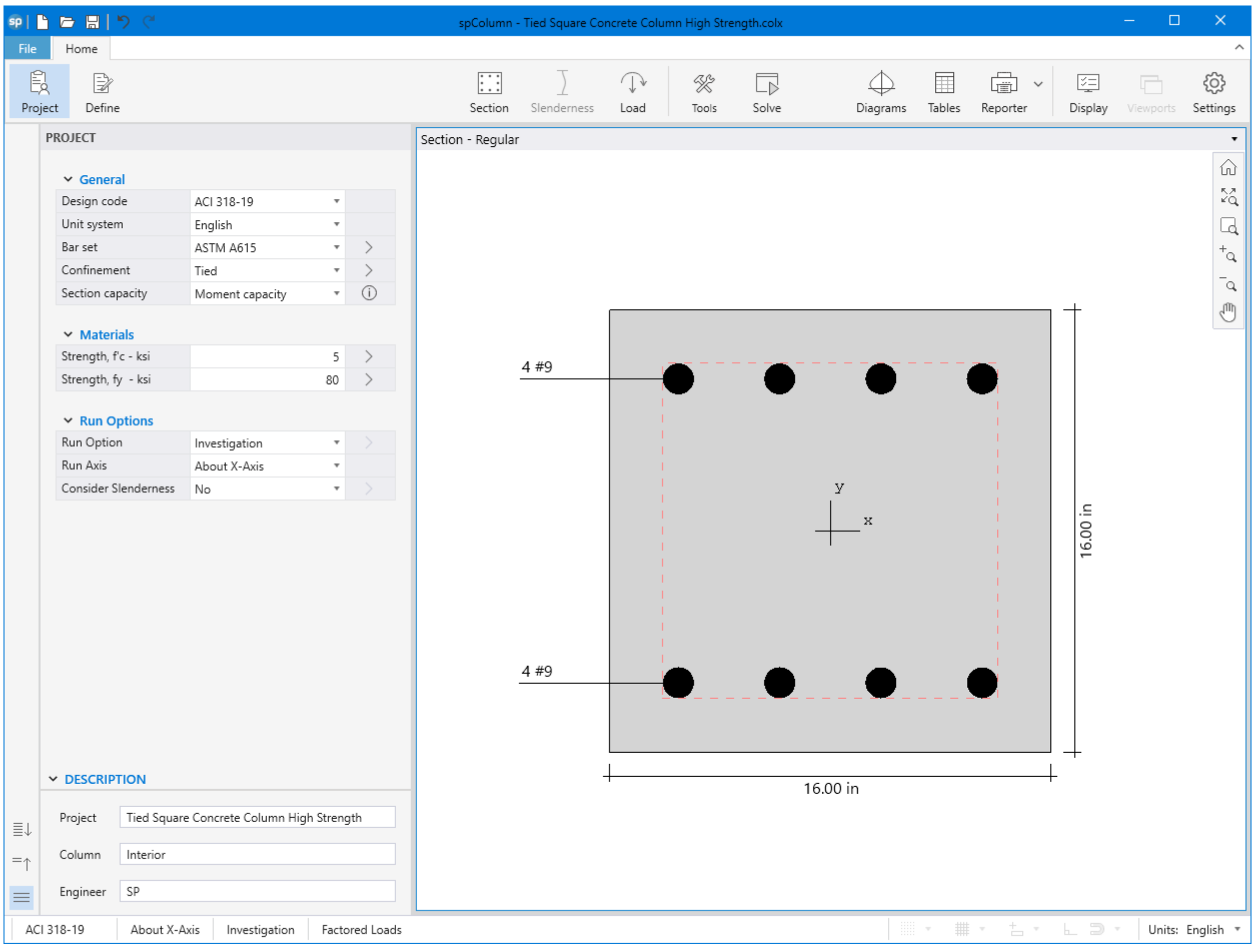


Figure 8 – spColumn Interface

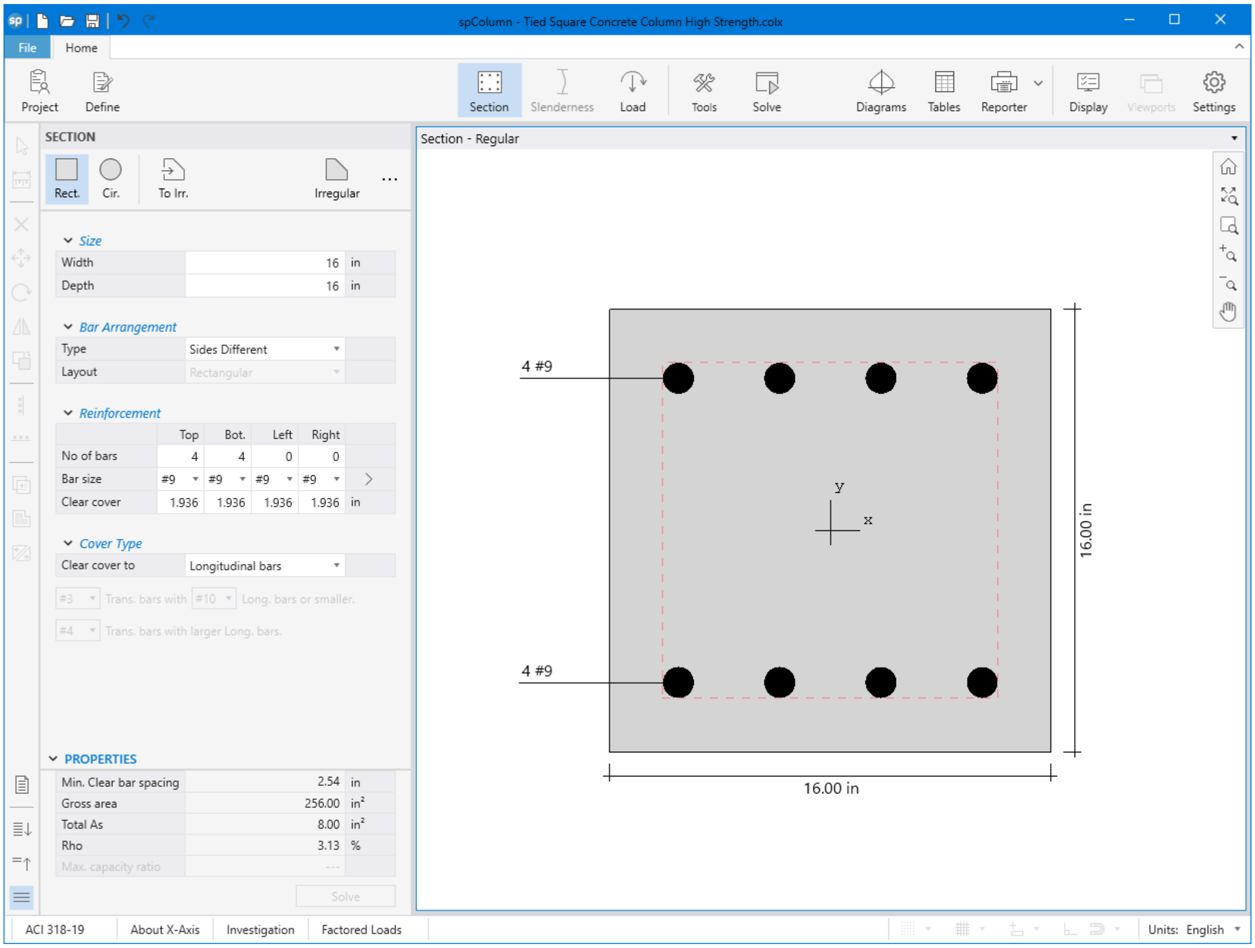


Figure 9 – spColumn Model Editor

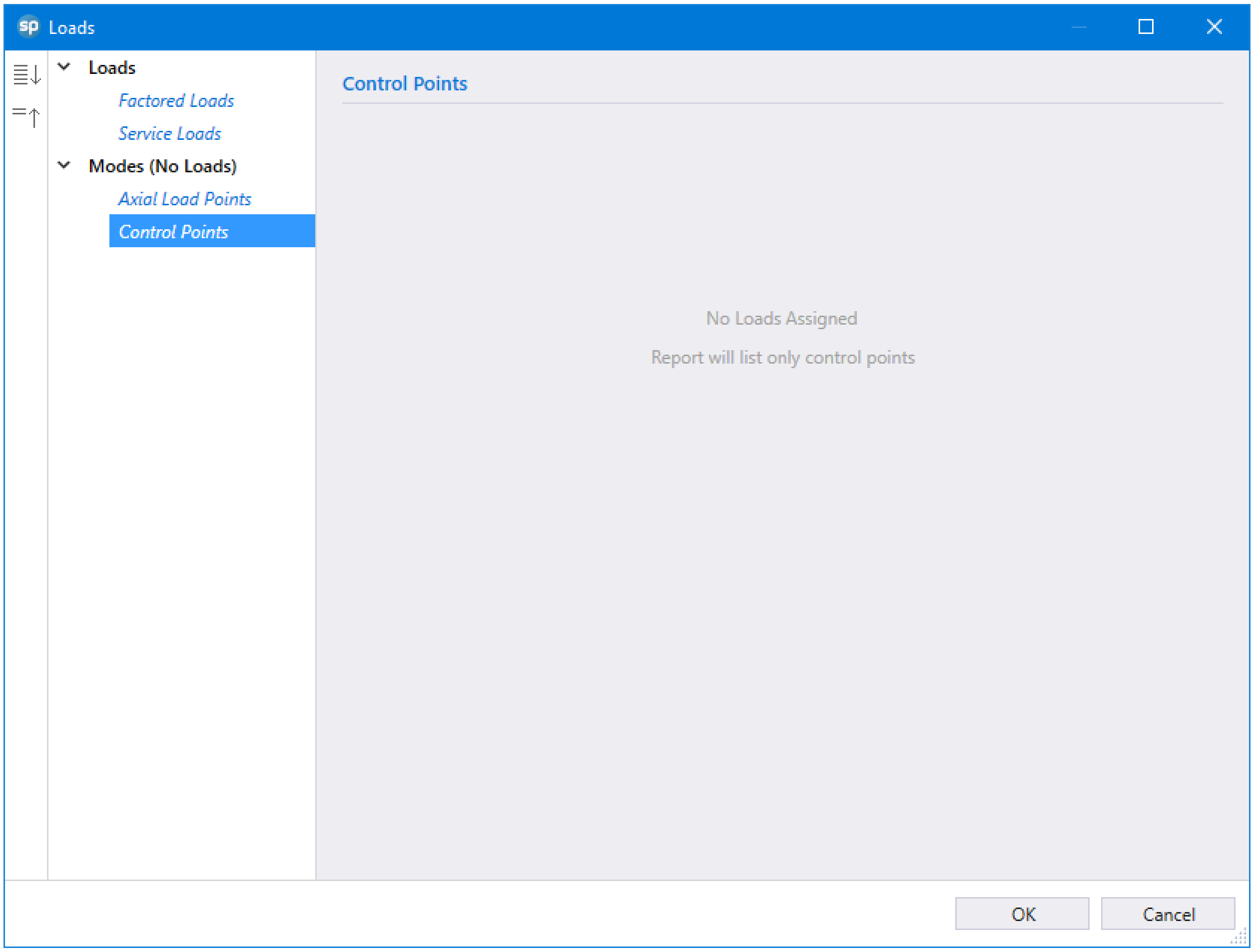


Figure 10 – Defining Loads / Modes (spColumn)

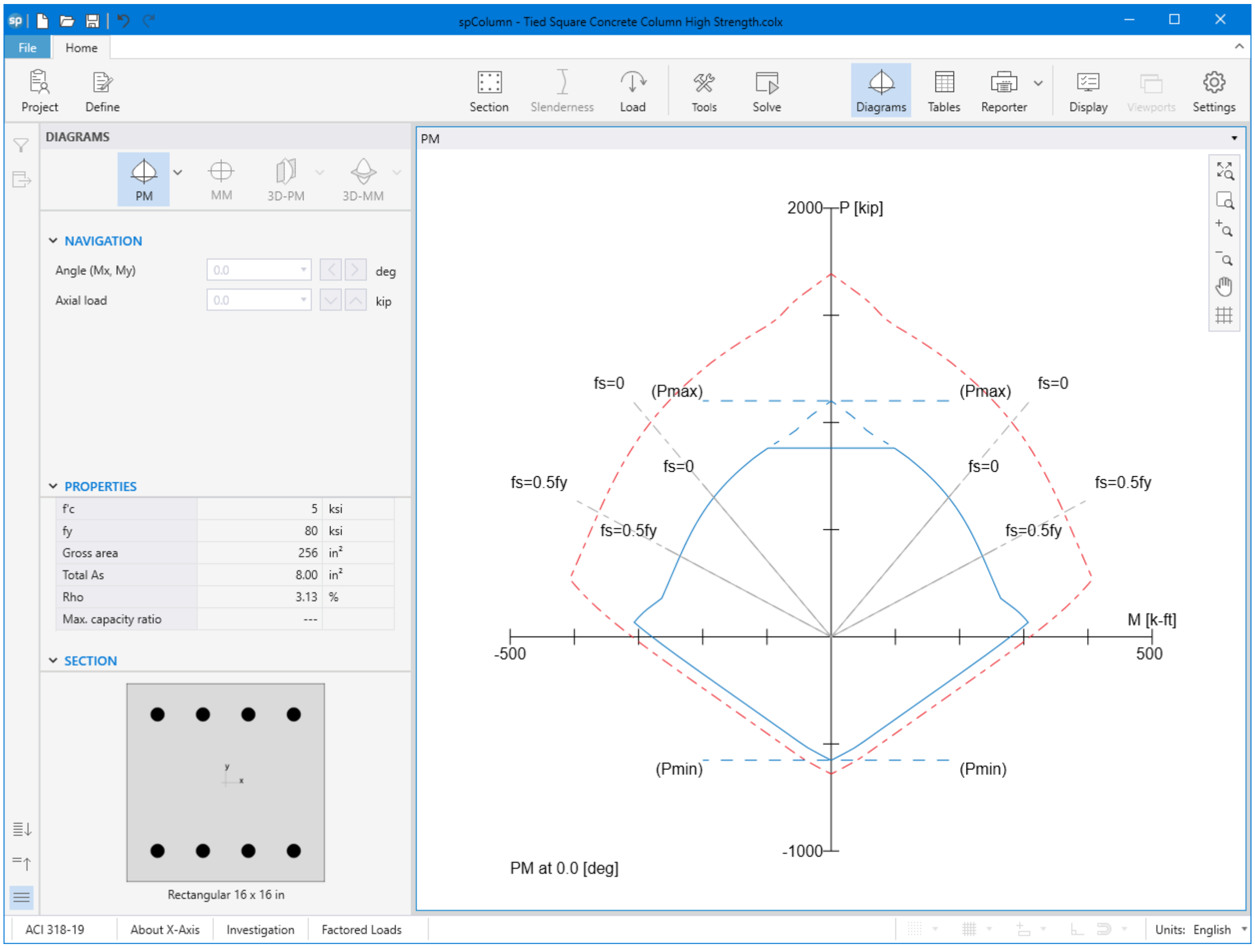
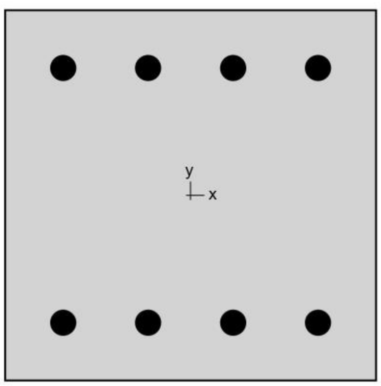


Figure 11 – Column Section Interaction Diagram about the X-Axis (spColumn)

spColumn v10.00 (TM)
Computer program for the Strength Design of Reinforced Concrete Sections



StructurePoint

Contents

List of Figures

1. General Information

File Name	...\Tied Square Concrete Column High Strength....
Project	Tied Square Concrete Column High Strength
Column	Interior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

2. Material Properties

2.1. Concrete

Type	Standard	
f'_c	5	ksi
E_c	4030.51	ksi
f_c	4.25	ksi
ε_u	0.003	in/in
β_1	0.8	

2.2. Steel

Type	Standard	
f_y	80	ksi
E_s	29000	ksi
ε_{ty}	0.00275862	in/in

3. Section

3.1. Shape and Properties

Type	Rectangular	
Width	16	in
Depth	16	in
A_g	256	in²
I_x	5461.33	in⁴
I_y	5461.33	in⁴
r_x	4.6188	in
r_y	4.6188	in
X_o	0	in
Y_o	0	in

3.2. Section Figure

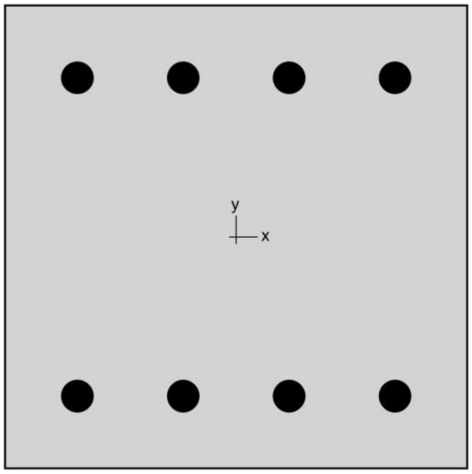


Figure 1: Column section

4. Reinforcement

4.1. Bar Set: ASTM A615

Bar	Diameter	Area	Bar	Diameter	Area	Bar	Diameter	Area
	in	in^2		in	in^2		in	in^2
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

4.2. Confinement and Factors

Confinement type	Tied
For #10 bars or less	#3 ties
For larger bars	#4 ties
Capacity Reduction Factors	
Axial compression, (a)	0.8
Tension controlled ϕ, (b)	0.9
Compression controlled ϕ, (c)	0.65

4.3. Arrangement

Pattern	Sides different
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	---
Bars	---

Total steel area, A_s	8.00 in²
Rho	3.13 %
Minimum clear spacing	2.54 in

4.4. Bars Provided

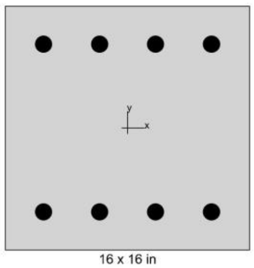
		Bars	Clear cover
			in
Top	4	#9	1.936
Bottom	4	#9	1.936
Left	0	#9	1.936
Right	0	#9	1.936

5. Control Points

About	Point	P	X-Moment	Y-Moment	NA Depth	d_t Depth	ε_t	ϕ
		kip	k-ft	k-ft	in	in		
X	@ Max compression	1101.1	0.00	0.00	167.79	13.50	-0.00276	0.65000
X	@ Allowable comp.	880.9	98.36	0.00	18.33	13.50	-0.00079	0.65000
X	@ f_s = 0.0	650.6	182.84	0.00	13.50	13.50	0.00000	0.65000
X	@ f_s = 0.5 f_y	377.0	235.45	0.00	9.25	13.50	0.00138	0.65000
X	@ Balanced point	175.4	264.58	0.00	7.03	13.50	0.00276	0.65000
X	@ Tension control	67.0	306.96	0.00	4.62	13.50	0.00576	0.90000
X	@ Pure bending	0.0	278.96	0.00	3.90	13.50	0.00739	0.90000
X	@ Max tension	-576.0	0.00	0.00	0.00	13.50	9.99999	0.90000
-X	@ Max compression	1101.1	0.00	0.00	167.79	13.50	-0.00276	0.65000
-X	@ Allowable comp.	880.9	-98.36	0.00	18.33	13.50	-0.00079	0.65000
-X	@ f_s = 0.0	650.6	-182.84	0.00	13.50	13.50	0.00000	0.65000
-X	@ f_s = 0.5 f_y	377.0	-235.45	0.00	9.25	13.50	0.00138	0.65000
-X	@ Balanced point	175.4	-264.58	0.00	7.03	13.50	0.00276	0.65000
-X	@ Tension control	67.0	-306.96	0.00	4.62	13.50	0.00576	0.90000
-X	@ Pure bending	0.0	-278.96	0.00	3.90	13.50	0.00739	0.90000
-X	@ Max tension	-576.0	0.00	0.00	0.00	13.50	9.99999	0.90000

6. Diagrams

6.1. PM at θ=0 [deg]



General Information

Project	Tied Squ... Strength
Column	Interior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Column Type	Structural
Capacity Method	Moment capacity

Materials

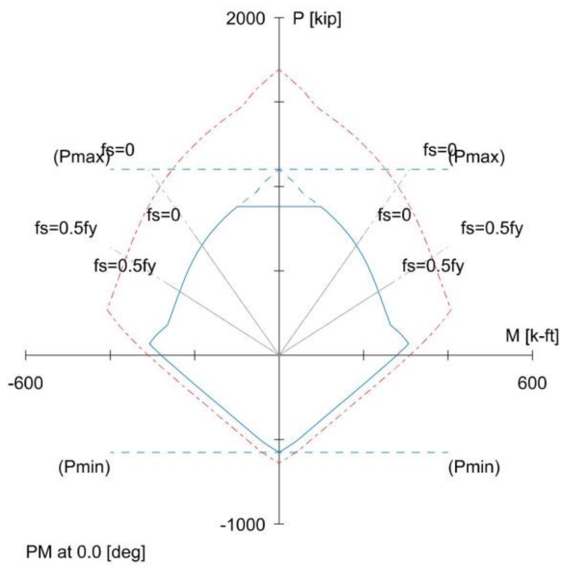
f'_c	5 ksi
E_c	4030.51 ksi
f_y	80 ksi
E_s	29000 ksi

Section

Type	Rectangular
Width	16 in
Depth	16 in
A_g	256 in²
I_x	5461.33 in⁴
I_y	5461.33 in⁴

Reinforcement

Pattern	Sides different
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	---
Bars	---
Confinement type	Tied
Total steel area, A_s	8.00 in²
Rho	3.13 %
Min. clear spacing	2.54 in



9. Summary and Comparison of Design Results

Table 1 - Comparison of Results				
Support	**ϕP_n, kip**		**ϕM_n, kip-ft**	
	Hand	**spColumn**	**Hand**	**spColumn**
Max compression	1101.1	1101.1	0.00	0.00
Allowable compression	880.9	880.9	---	---
f_s = 0.0	650.6	650.6	182.84	182.84
f_s = 0.5 f_y	377.0	377.0	235.45	235.45
Balanced point	175.4	175.4	264.58	264.58
Tension control	67.0	67.0	306.96	306.96
Pure bending	0.0	0.0	278.95	278.96
Max tension	576.0	576.0	0.00	0.00

In all of the hand calculations illustrated above for this column with HSRB, the results are in precise agreement with the automated exact results obtained from the spColumn program.

10. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

ACI 318-19 introduced new provisions for high-strength reinforcing bars (HSRB) with 80 ksi and 100 ksi strengths. Table 21.2.2 in ACI 318-19 defines the strength reduction factor ϕ, for tension-controlled sections as an expression of f_y, for all reinforcement grades. Previously in ACI 318-14 Fig. R21.2.2b, the tension-controlled strain limit was set to 0.005. Therefore, beginning with the 2019 Code, the expression (ε_{ty} + 0.003) defines the lower limit on ε_t for tension-controlled behavior. The new limit leads to a constant transition zone range from ε_{ty} to ε_{ty} + 0.003.

The Figure below shows factored P-M interaction diagrams for a column section with Gr 80 reinforcement per ACI 318-14 where the tension-controlled limit was 0.005 and per ACI 318-19 where the tension-controlled limit for Gr 80 is 0.00576 (ε_{ty} + 0.003). The change in the tension-controlled limit leads to the reduction of axial load and moment capacities in the transition zone for this column section designed in accordance with ACI 318-19.

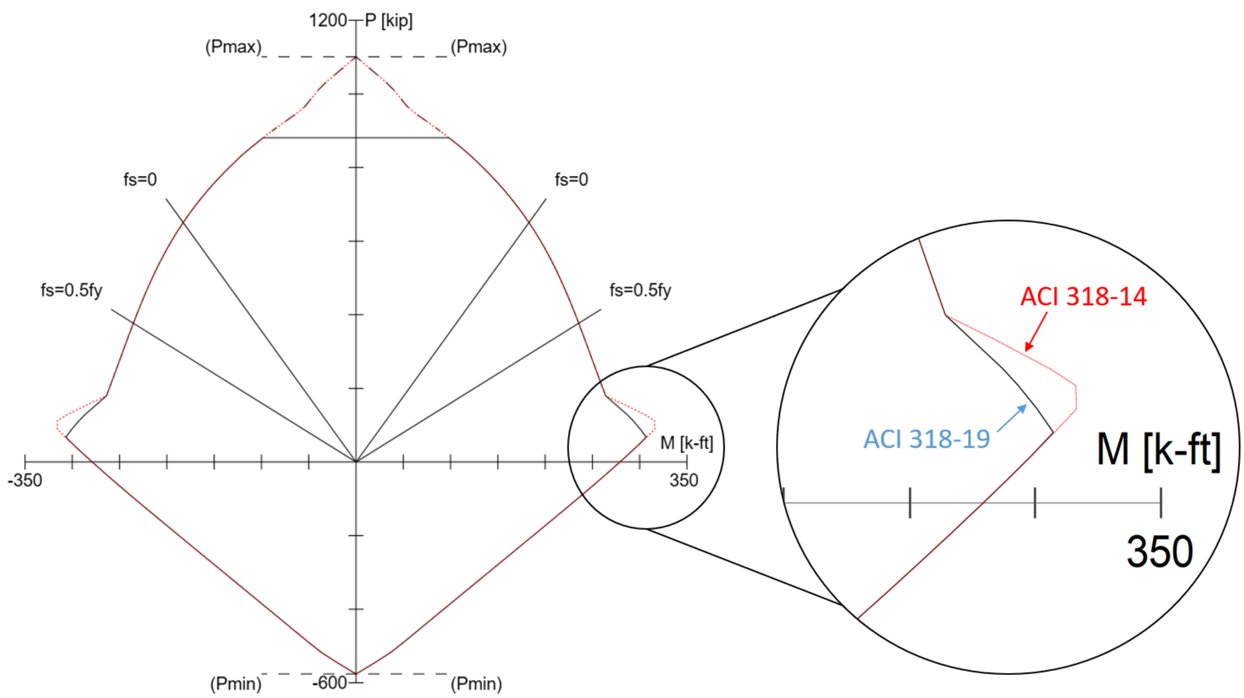


Figure 12 – Design (Factored) Interaction Diagrams using ACI 318-14 and ACI 318-19 (spColumn)

The Figure below shows factored interaction diagrams for a column section with Gr 60 and Gr 80 reinforcement per ACI 318-19. The factored moment capacity of a column with Gr 80 reinforcement is greater than that of a column with Gr 60 reinforcement with the exception of the transition zone region of a column with Gr 60 reinforcement.

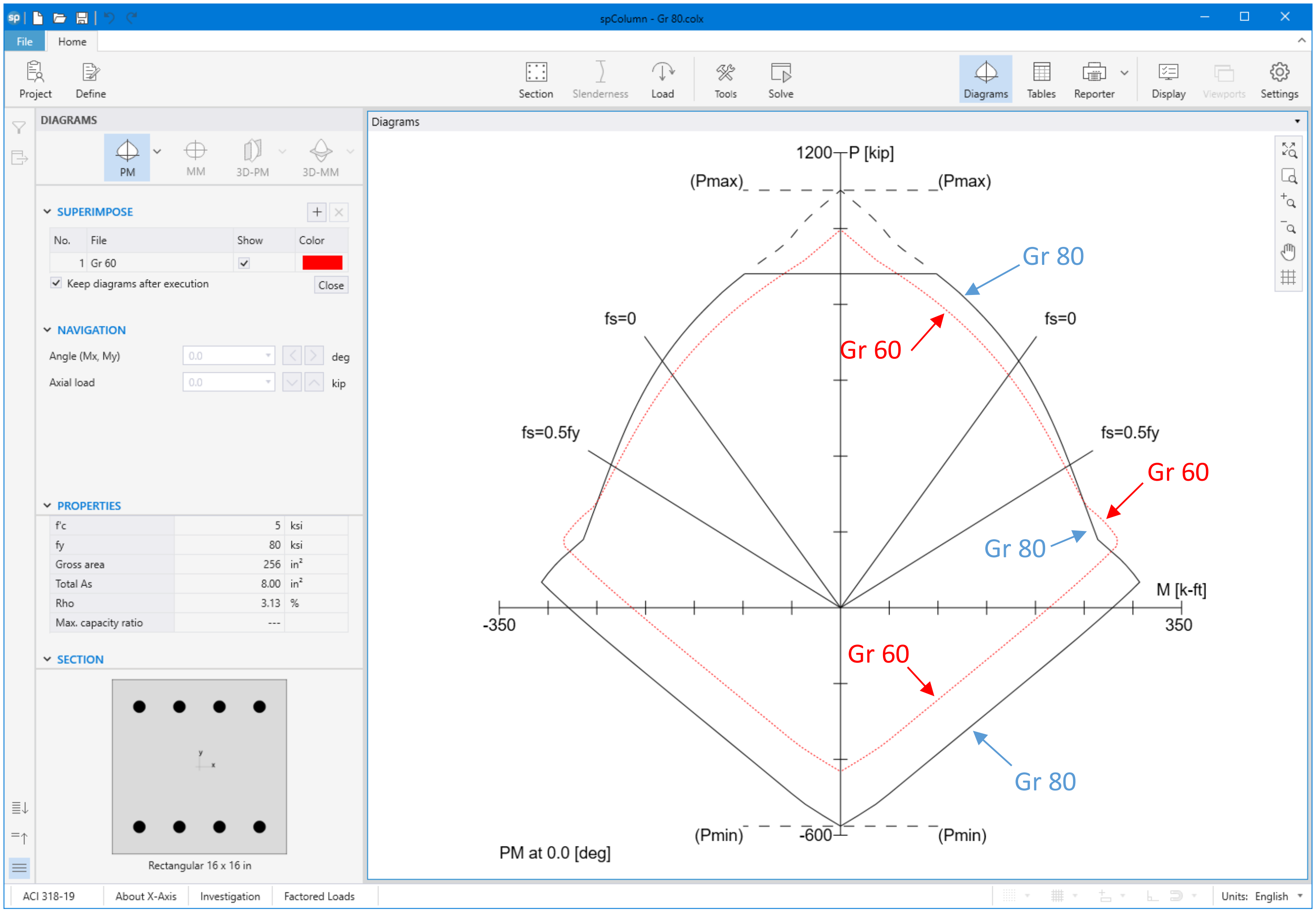


Figure 13 – Design (Factored) Interaction Diagrams using Gr 60 and Gr 80 (spColumn)

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where f_s = f_y).

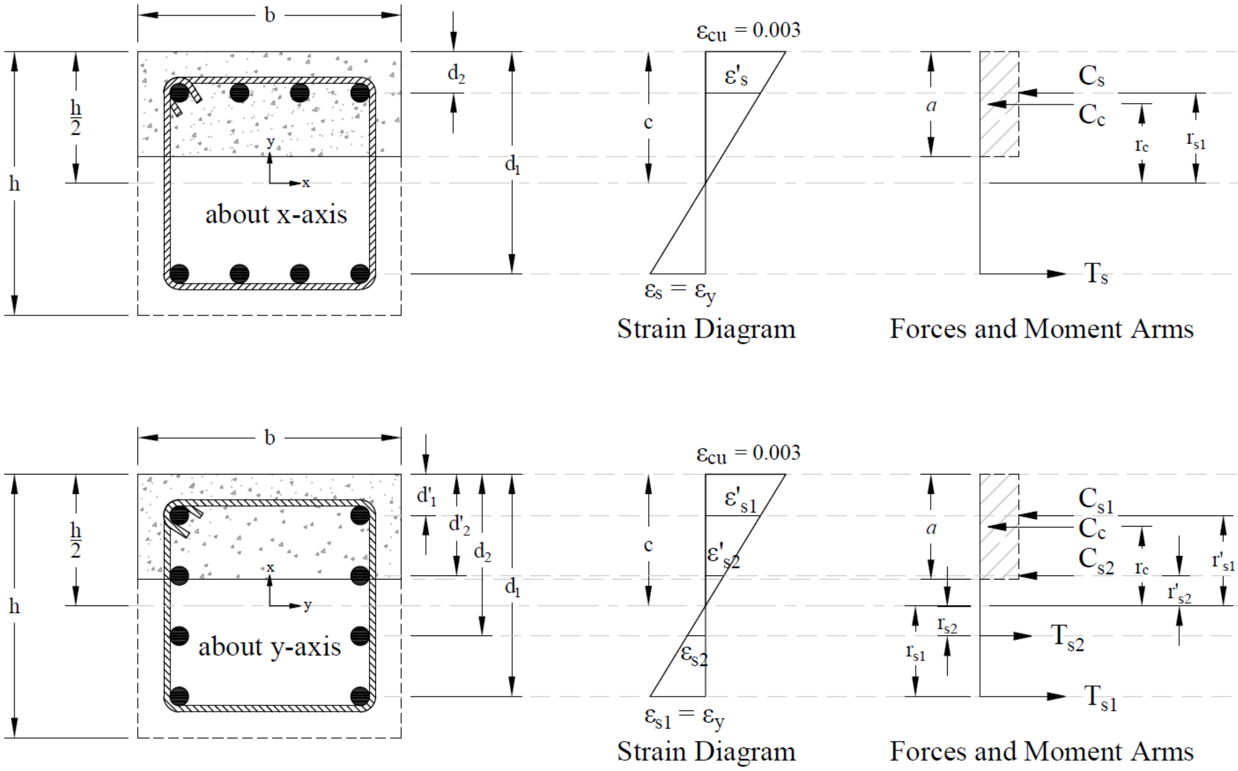


Figure 14 – Strains, Forces, and Moment Arms (f_s = - f_y Moments About x- and y-axis)

When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure. Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

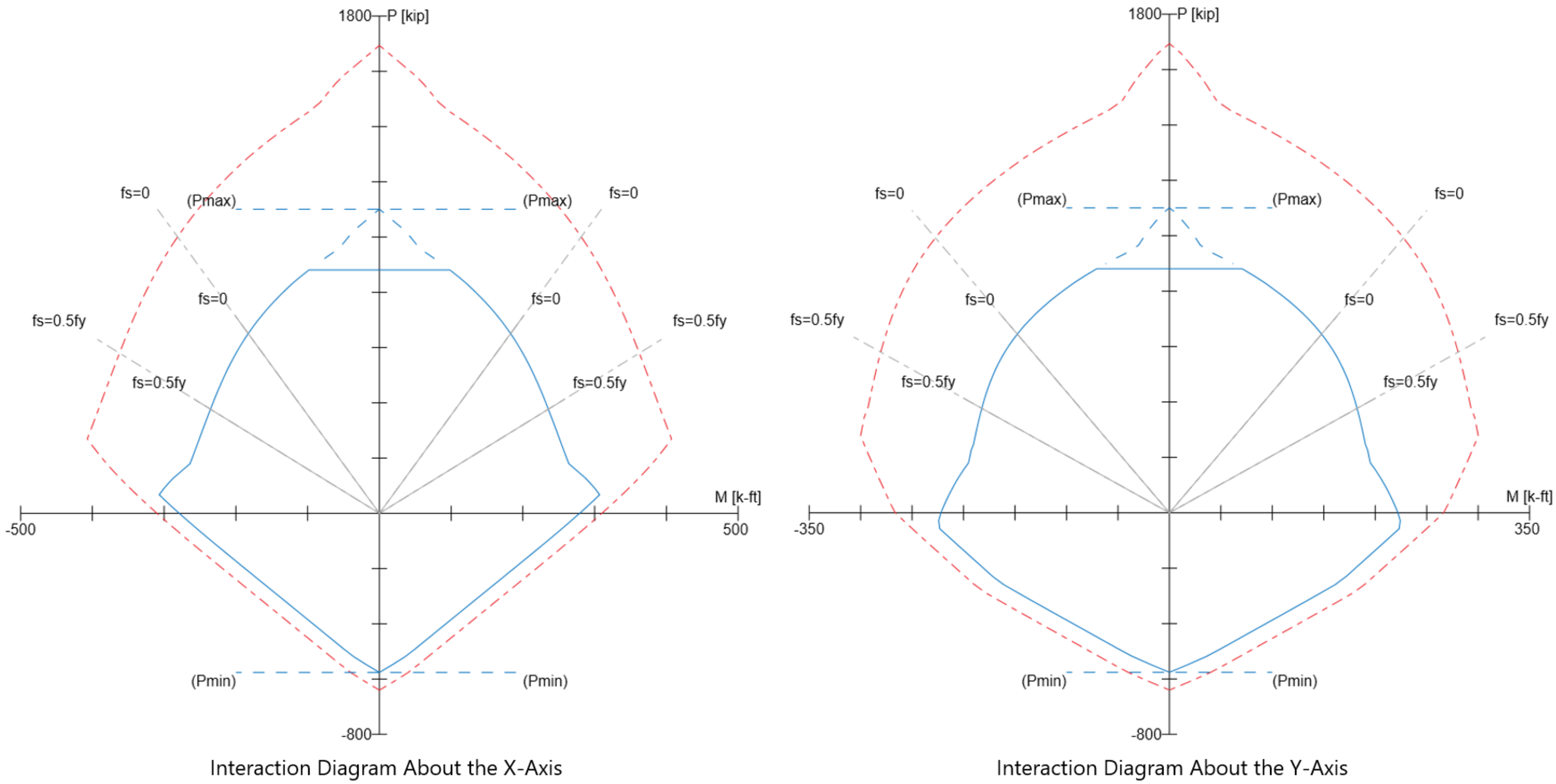


Figure 15 – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns are subjected to M_x and M_y due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StucturePoint's spColumn program can also investigate column and wall sections in biaxial mode to produce the results shown in the following Figure for the column section in this example. In biaxial run mode, M_x and M_y diagrams at each axial force level can be viewed in 2D and 3D views.

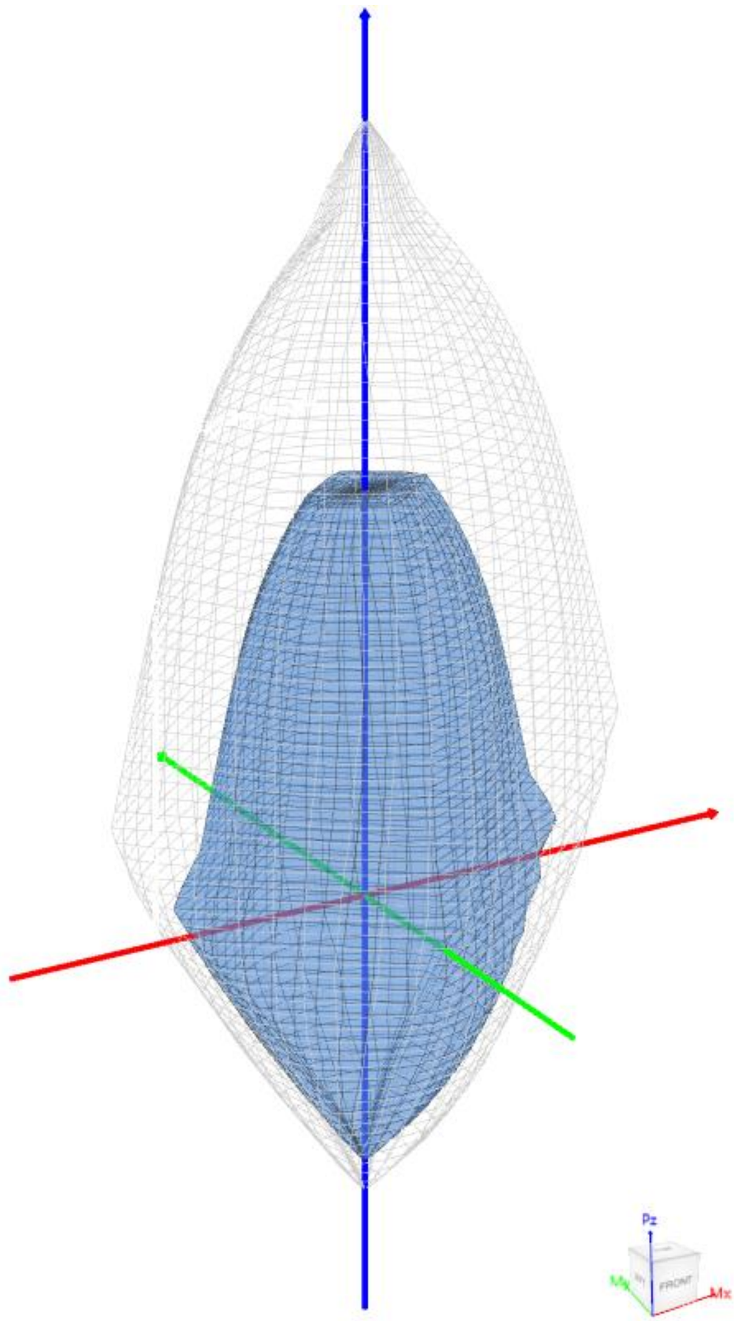


Figure 16 – Nominal & Design 3D Failure Surfaces (Biaxial) (spColumn)

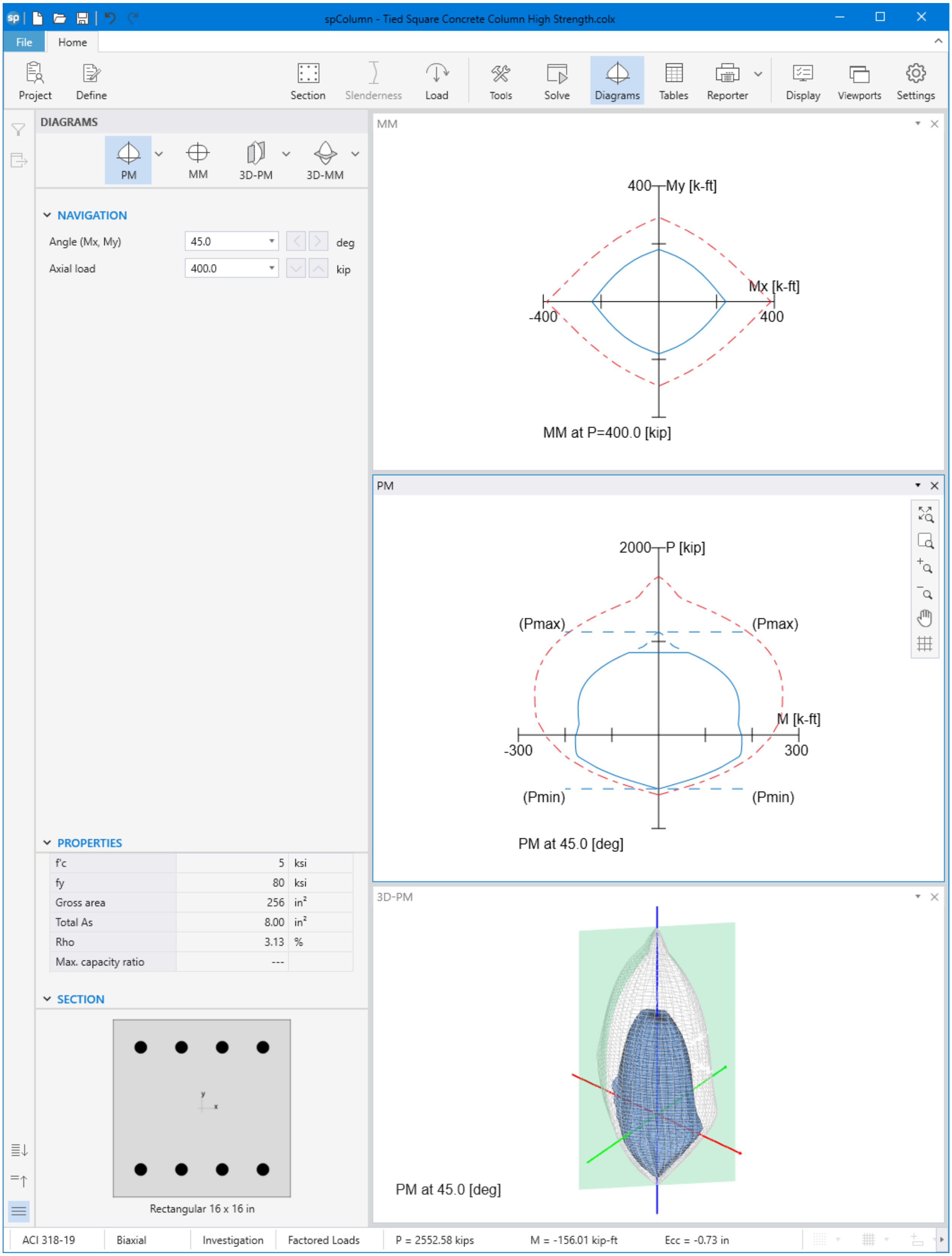


Figure 17 – Tied Column Interaction Diagram and 3D failure Surface Viewer (spColumn)

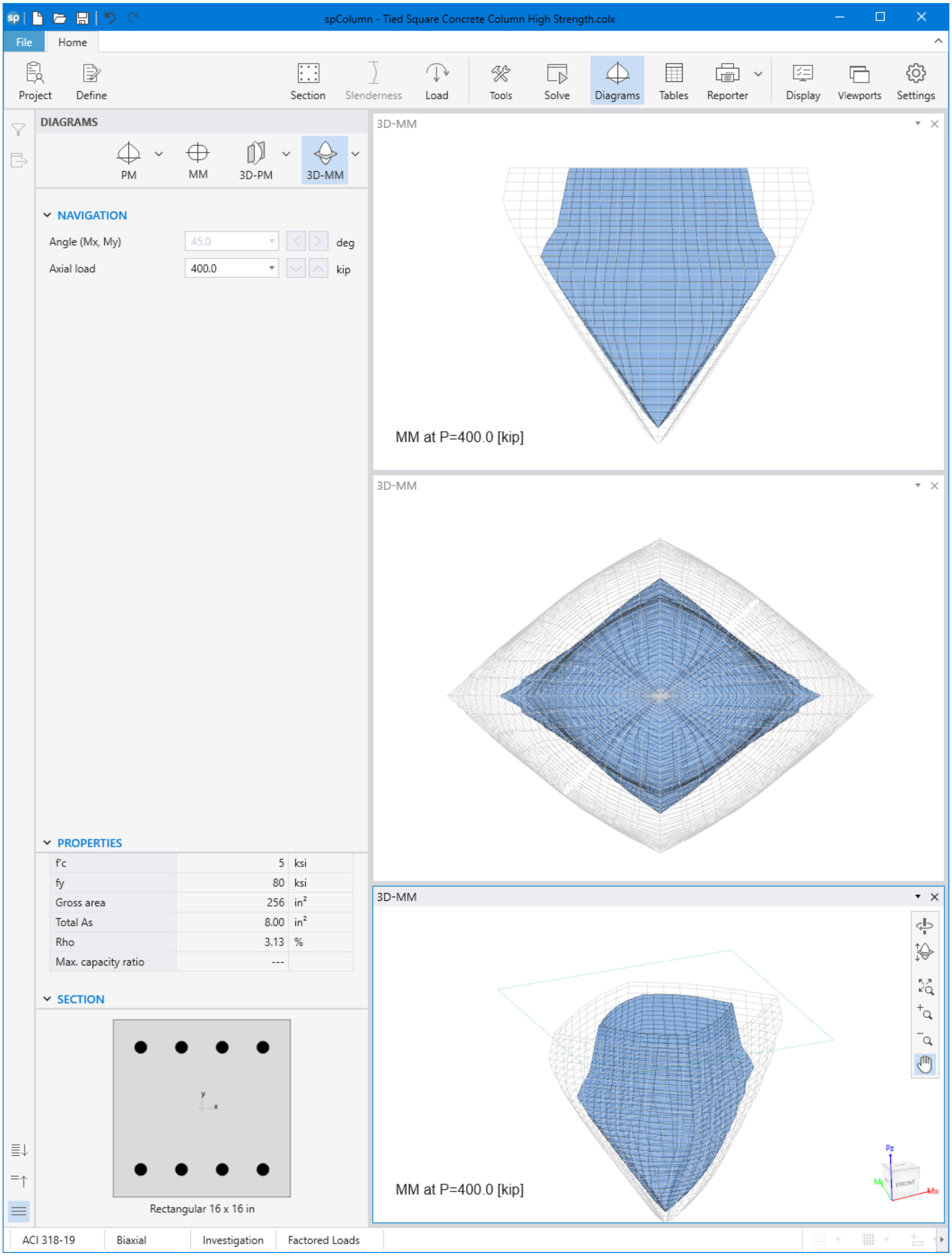


Figure 18 – Tied Column 3D Failure Surface with a Horizontal Plane Cut at P = 400 kip (spColumn)

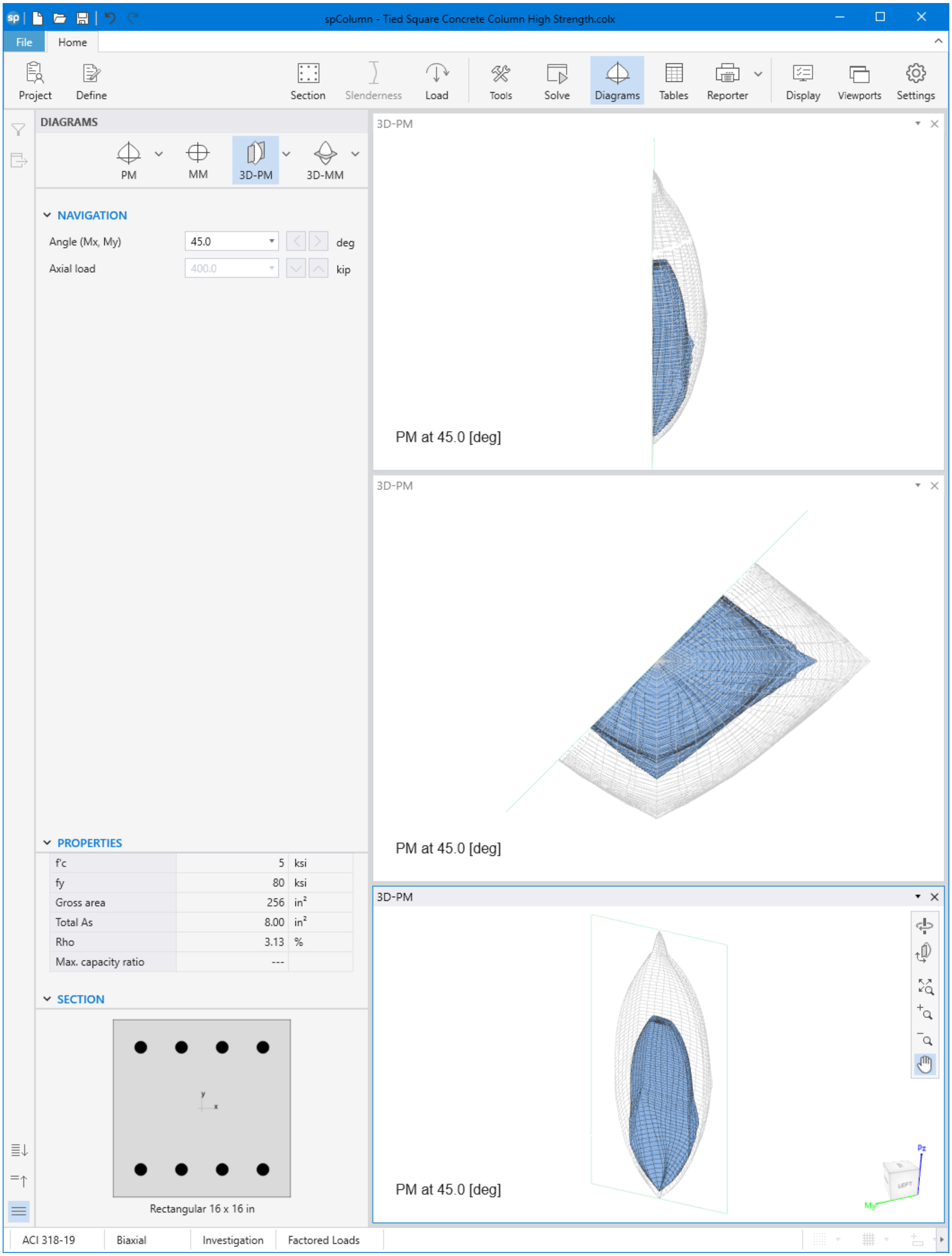


Figure 19 – Tied Column 3D Failure Surface with a Vertical Plane Cut at 45º (spColumn)