concrete information

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Design of Deep Girders

Section 1. Introduction

The customary design procedure for sections subject to bending is based on the assumption of straight-line stress distribution in the section. This assumption represents fairly closely the conditions that exist as long as the behavior of both steel and concrete is essentially elastic and the height-to-span ratio is relatively small.

In deep girders, the curve of stress distribution is not straight but may take on radically different shapes. Such deep girders were analyzed according to the elastic theory by Franz Dischinger and the results of his mathematical investigation were presented in *Contribution to the Theory of Wall-Like Girders*, published by International Association for Bridge and Structural Engineering, 1932, in the first volume of the *Publications* by that Association. For mathematical derivations reference is made to that paper.

The results of Dischinger's work are presented in this article to which special studies and numerical examples have been added. The data and the procedures illustrated apply to design of deep wall-like members such as encountered in bins, hoppers and foundation walls. The proposed deep-beam design is applicable for height-to-span ratios of, say, $\frac{2}{5}$ or more for continuous girders and $\frac{4}{5}$ or more for single-span girders. For smaller ratios, the straight-line stress distribution is usually satisfactory as a basis for design.

Section 2. Continuous Girder, Uniform Load

Fig. 1 illustrates a continuous girder with load uniformly distributed along the bottom edge of the girder. The height-to-span ratio, H/L, is denoted as β and the support-to-span ratio, C/L, is denoted as ϵ. These two ratios are used as parameters in all calculations and charts. Dischinger's analysis was carried out for the following values: $\beta = \frac{1}{2}, \frac{2}{3}$, 1, and infinity; $\epsilon = \frac{1}{20}, \frac{1}{10}, \frac{1}{5}$, and $\frac{1}{2}$. For all combinations of β and ϵ,

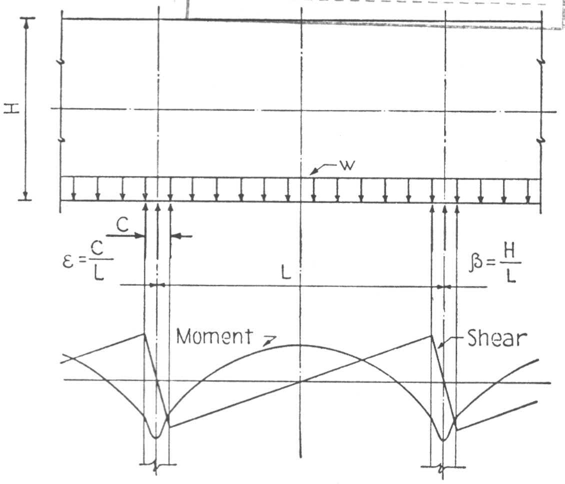

Fig. 1. Typical uniform loading and ratios for design of deep girders

except for $\beta =$ infinity which represents a special condition, the bending stresses were determined at the two edges and at the intermediate eighth-points. The computed stress values were tabulated in Dischinger's article for sections at mid-span and at centerline of intermediate supports. His tabulated values have been used for drawing the charts in this article. For uniform load the stresses are the same regardless of the elevation at which the load is applied to the girder.

For the loading in Fig. 1, stresses are presented in Fig. 2 for the section at mid-span. The horizontal distance from the vertical base line to the curves represents the stress coefficient which when multiplied by w/b (b being the width of the girder) results in the unit stress. Stresses to the right of the base line are compression and stresses to the left are tension. The curves are drawn for the ratio of $\epsilon = \frac{1}{10}$ but all the curves for $\epsilon = \frac{1}{5}, \frac{1}{10}$ and $\frac{1}{20}$ are so nearly alike for the conditions in Fig. 2 that the curves for $\epsilon = \frac{1}{10}$ may be used with good accuracy for all three ϵ-values. The

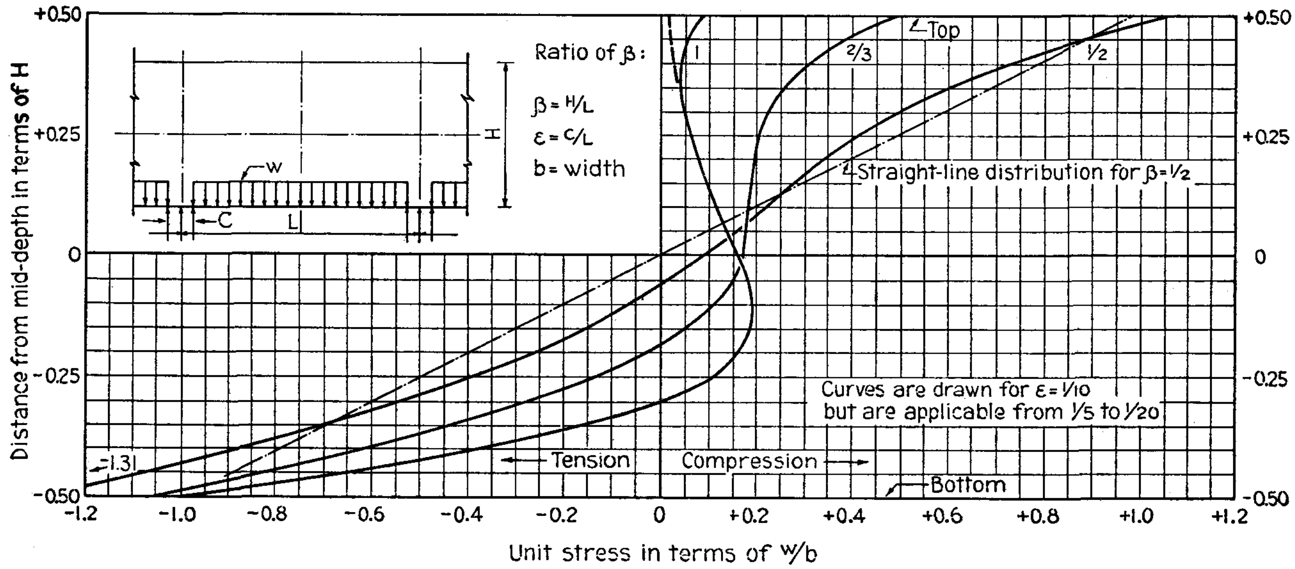

Fig. 2. Moment stresses at mid-span of girders having uniform loading.

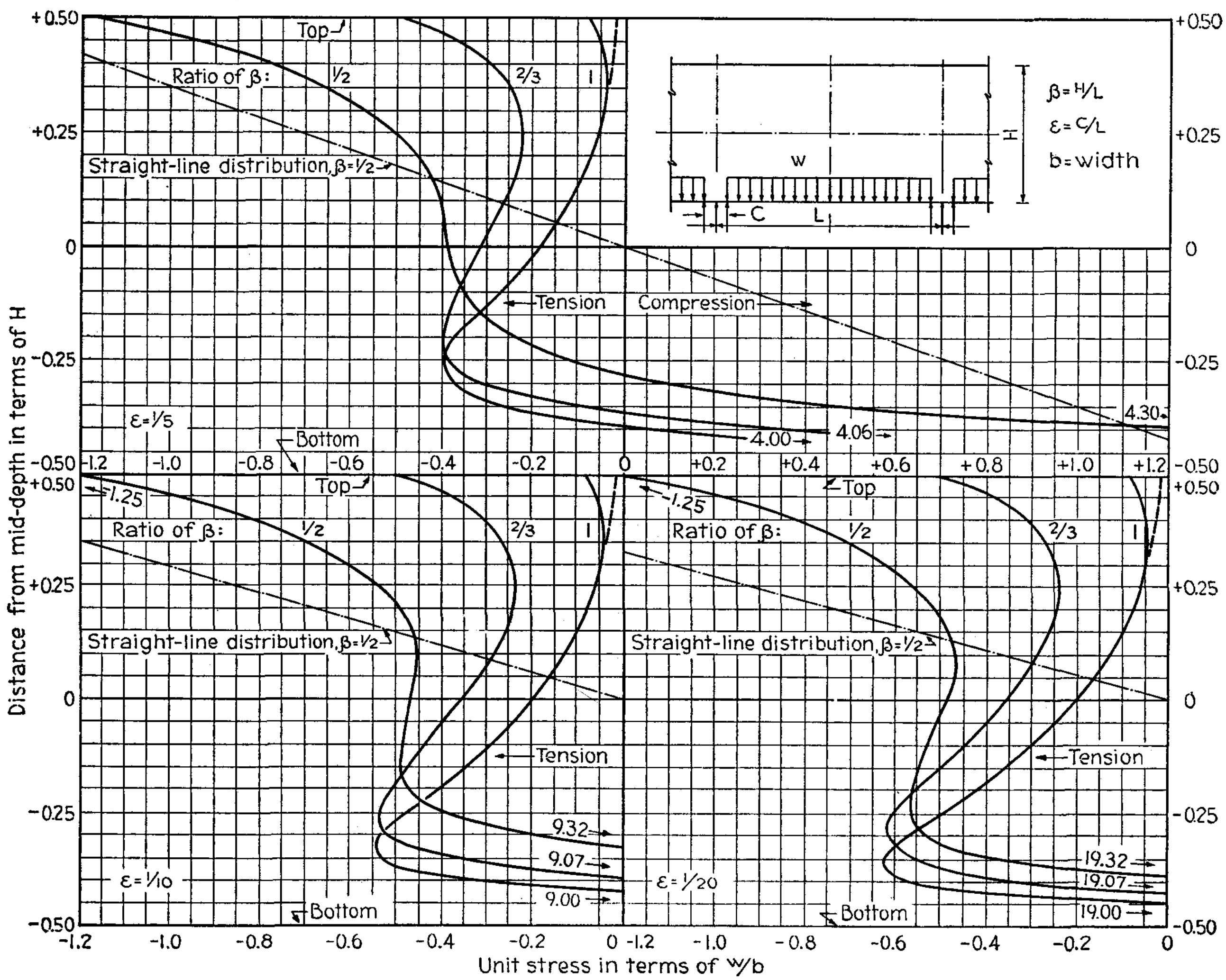

Fig. 3. Moment stresses at centerline of support of girders having uniform loading.

special value of $\epsilon = \frac{1}{2}$ will be discussed in Section 3.

Attention is first directed to the curve for $\beta = \frac{1}{2}$ which extends from $+1.07$ at top to -1.31 at bottom. The adjacent dash-line represents the straight-line stress distribution which is also drawn for $\beta = \frac{1}{2}$. It is seen that the customary design for shallow beams remains applicable almost all the way up to $\beta = \frac{1}{2}$ as far as stresses at mid-span are concerned. Note that in these studies concrete is assumed effective in tension.

When β increases from $\frac{1}{2}$ to 1, the shape of the stress curve undergoes a radical change. The compressive stress decreases rapidly at the top, the neutral axis approaches the lower fifth-point, and the tensile stress at the bottom decreases, but comparatively slowly, from $-1.31w/b$ to $-1.00w/b$. When $\beta=1$, the maximum compressive stress occurs near the lower third-point and the stress is zero at the lower fifth-point. Increasing H above the value of L has practically no effect upon the stresses within the depth equal to L, the only noticeable change being as indicated by the short dash-line curve which represents stresses within the depth of L for β = infinity. Note that the line marked "Top" in Figs. 2 and 3 indicates top of girder for $\beta=\frac{1}{2}$, $\frac{2}{3}$, and 1, but for β = infinity (the dash-line) it represents the line a distance of L above the bottom just as it does for $\beta=1$.

Moment stresses at centerline of support for continuous girders with uniform load are plotted in Fig. 3. The curves are shown in their entirety for $\epsilon=\frac{1}{5}$ whereas only the tensile part of the curves are shown for $\epsilon = \frac{1}{10}$ and $\frac{1}{20}$. The coefficients for maximum compressive stress are recorded on the diagram. The general characteristics of the curves are the same as described in connection with Fig. 2. However, the curves for stresses at support in Fig. 3 deviate from the straight-line distribution somewhat more than in Fig. 2, the neutral axis is closer to the bottom and the bottom stresses are much greater than in Fig. 2. Note that for $\beta=1$ the maximum tensile stress occurs near the lower quarter-point. This is where the tensile steel is most effective but some nominal reinforcement may be provided above that point as will be discussed later.

The external moments involved in the stress determination may be derived in the manner usually applied for continuous beams on supports the stiffness of which is assumed equal to zero, an assumption that is fairly accurate on account of the relatively large stiffness of the deep girders. For the loading shown in Figs. 2 and 3 the moment at mid-span of a typical interior span is:

$$\frac{1}{24}(wL)L - \frac{1}{4}(wL)L\frac{\epsilon}{2} + \frac{1}{12}(wL)L\left(3\frac{\epsilon}{2} - 2\frac{\epsilon^2}{4}\right) = \frac{1}{24}wL^2(1-\epsilon^2)$$

The moment at centerline of typical interior support is:

$$\frac{1}{12}(wL)L - \frac{1}{12}(wL)L\left(3\frac{\epsilon}{2} - 2\frac{\epsilon^2}{4}\right) = \frac{1}{24}wL^2(1-\epsilon)(2-\epsilon)$$

Concrete tensile stresses computed by means of the data in Figs. 2 and 3 may not be so large as to cause the concrete to crack. Yet, since it is customary to disregard tensile stresses in concrete, Dischinger proceeded to draw to a large scale curves such as those in Figs. 2 and 3 from which he determined the resultant, T, of all the concrete tensile stresses on each section, and also the distance, d_0, between the resultant of tensile stresses and the bottom of the girder. The results are presented in Figs. 4 and 5 which may be used in conjunction with the moments computed by the above formulas to determine the area of tensile steel and its position.

The total tension determined as the resultant of the concrete tensile stresses is, of course, somewhat different from the tension that exists in the steel when the concrete is assumed to carry no tension. For illustration, when the straight-line distribution applies, the total tension is $3M/2H$ before concrete cracks but it is M/jd when tension in concrete is disregarded. Since $\frac{2}{3}H$ is usually smaller than jd, the tension is greater when computed as $3M/2H$. Similarly, when the straight-line distribution no longer applies it can usually be anticipated that the total tension in the steel, computed as the sum of all tensile stresses in the uncracked section, will give conservative results.

The curves in Fig. 4 are drawn through the points marked, which are determined as described above for values of $\beta=\frac{1}{2}$, $\frac{2}{3}$, and 1. For ratios smaller than $\frac{1}{2}$, the straight-line distribution gives $M = T \times \frac{2}{3}H = \frac{2}{3}LT\beta = \frac{2}{3}L(\text{Coef.} \times wL)\beta = \frac{2}{3}wL^2(\text{Coef.})\beta$. With the coefficient and β being ordinate and abscissa, respectively, the equation represents a hyperbola hav-

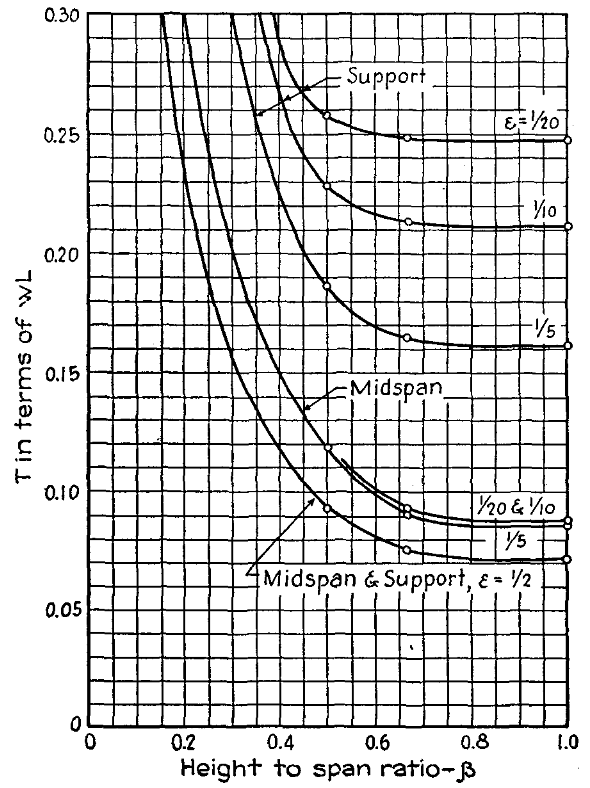

Fig. 4. Resultant, T, of tensile stresses in girders having uniform loading.

ing the coordinate axes as asymptotes. Such hyperbolas are drawn in Fig. 4 and short transition curves are inserted between the hyperbolas and the curves derived from the deep-beam theory. In this way a check is obtained on the latter theory, especially for values of β close to $\frac{1}{2}$, but also in general in regard to the consistency of the data for deep girders. The fact that values of T are included for shallow beams does not imply that Fig. 4 is to be used for $\beta < \frac{2}{5}$.

Fig. 4 illustrates that for values greater than 0.7 the value of T becomes constant and is not appreciably affected by further increase in girder depth. The latter is a significant point because it shows a discrepancy from the usual concept based on straightline distribution, according to which the internal forces decrease when the height increases.

The variation in the ratio of ϵ in Fig. 4 illustrates the effect of change in width of support. This change is seen to have negligible effect upon T at mid-span for $\epsilon = \frac{1}{5}$, $\frac{1}{10}$, and $\frac{1}{20}$ but a pronounced effect upon T at centerline of support. Curves for $\epsilon = \frac{1}{2}$ are included in Fig. 4 although the presentation of data for this ratio is deferred to Section 3.

The position of the resultant of the tensile stresses as determined by Dischinger is shown in Fig. 5 for uniform loading. At mid-span the distance d_0 from T to the bottom of the girder is $0.06L$ for all ratios of β greater than 0.4. Theoretically this is where reinforcement at mid-span should be placed, or if bars are in several layers, the center of gravity of all tensile steel should be at that position. In most instances, however, the tensile steel at mid-span should be placed as close to the bottom as permissible since the stress curves show that this is where it is most effective.

At the support, the position of tensile steel warrants careful study. For illustration, for values of ϵ as small as $\frac{1}{20}$ the center of gravity of tensile stresses is a distance of $d_0 = 0.32L$ above the bottom edge for virtually all β-ratios above 0.5. Turning to Fig. 3 it is seen that the tensile steel for $\beta = \frac{1}{2}$ is most effective at the top edge, that is, at a height of $0.50L$; whereas for $\beta = 1$ the tensile steel is most effective at a height of $0.15L$. In most instances it is probably desirable to have some reinforcement distributed throughout the entire tensile zone, but in any case it is advisable to give careful attention to both Figs. 3 and 5 when placing bars for negative moments at supports.

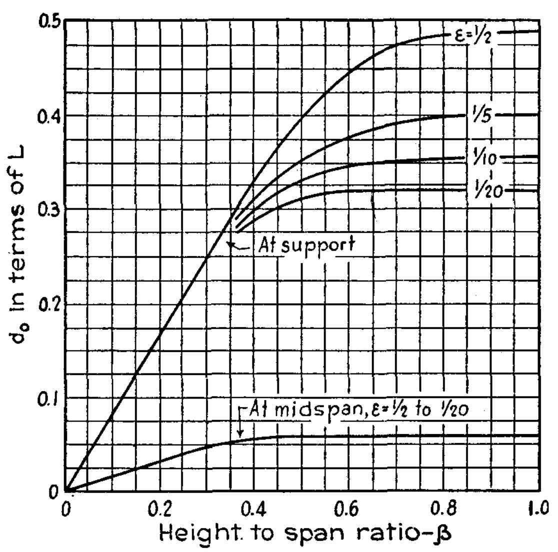

Fig. 5. Distance, d_0, from resultant of tensile stresses to bottom edge of girders having uniform loading.

Section 3. Single-Span Girder with Uniform Loading

Fig. 6(a) shows a continuous girder of the type discussed in Section 2 but for the specific case in which the length of support equals one-half of the theoretical span measured between centerlines of supports, that is, for $\epsilon = \frac{1}{2}$. Since the length of support equals the clear span, the condition is that of a beam which is loaded alternately upward and downward with equal loads but which has no other supports. It is apparent that the moment is zero at alternate vertical dash-and-dot lines as indicated in Fig. 6(a). There is no assurance that the bending stresses are zero everywhere in those sections, but the fact that the total bending moment is zero indicates that portions of the beam with length of $L/2$ between points of inflection may

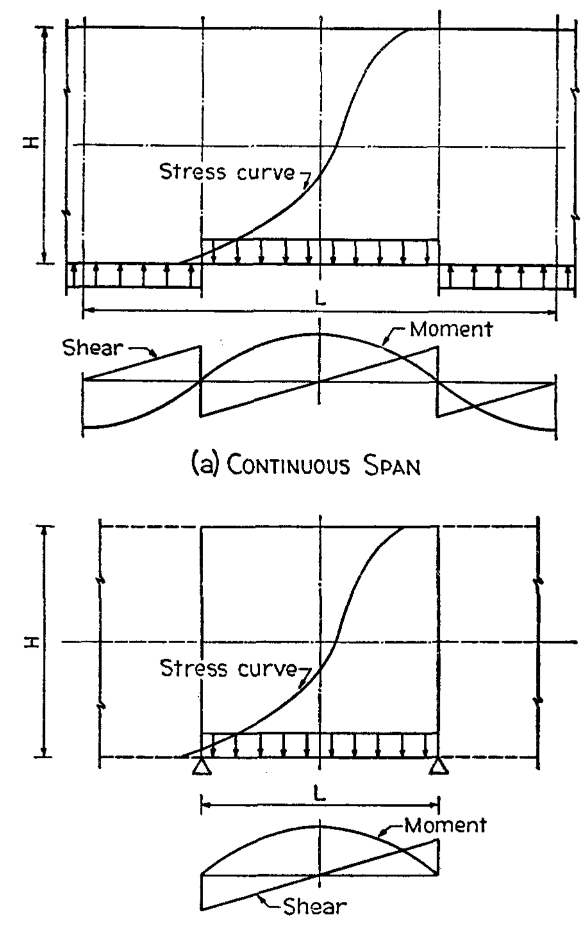

Fig. 6. Comparison of single-span and continuous girders.

be in the same stress condition as is the simply supported beam in Fig. 6(b). Accordingly, it will be assumed that the stress curves for moment are alike at mid-span of the two beams in Fig. 6.

Note particularly that the ratio of H/L is twice as great for the single-span beam in Fig. 6(b) as for the continuous beam in Fig. 6(a). Consequently, the curves for the continuous beam presented in Fig. 7 may be used for single-span beams when the β-ratio for the latter is computed as $H/2L$. The general rule for single-span beams is to compute $\beta = H/2L$ and to select design data for that ratio and $\epsilon = 1/2$ from Figs. 4 and 7.

As indicated in Figs. 6(b) and 7, the reactions for the single-span deep girder are applied at points a distance of L apart. In an actual structure, the length of the supports is not negligible and L should then be taken as the distance between centerlines of supports. When ϵ is too great, say, more than $1/10$, the design procedure outlined may not be applicable.

The stress-distribution curves in Fig. 7 show that tensile steel for center-span moments is most effective when placed as close to the bottom as possible, so the data for d_0 in Fig. 5 are of no significance for single spans. The coefficient for the resultant, T, of the tensile stresses is selected from Fig. 4 for single-span beams with span L for $\beta = H/2L$ and $\epsilon = 1/2$, after which the coefficient is multiplied by $w(2L)$.

Fig. 7 shows for $\beta = 1/2$ a comparison of stress-distribution curves for straight-line assumption and for deep-beam design. It is seen that the difference is not appreciable, which means that the ordinary shallow-beam design is applicable up to, say, $H = 2L/5$ for continuous beams and up to $H = 4L/5$ for single-span beams. This shows that deep-beam design is confined to a much smaller field for single spans than for multiple spans so that the importance of having an exact method of deep-girder design is less important for single than for continuous spans.

Section 4. Continuous Girder with Concentrated Load at Bottom and at Center of all Spans

The drawing inserted in Fig. 8 represents a continuous beam having a span of L measured between centerlines of supports and having concentrated load at the center of all spans. Both reactions and loads are applied at the bottom edge. Both loads and reactions are equal to P and are assumed to be distributed over a length of C, the ratio of C/L being denoted as ϵ.

For $\epsilon = 1/2$, the loading and the stresses are identical with those for the continuous beam in Fig. 7. Stress distribution for $\epsilon = 1/5$, $1/10$, $1/20$ are shown in Fig. 8, in which the curves are complete for $\epsilon = 1/5$, whereas only compressive stresses are given for $1/10$ and $1/20$. Three ratios of β are included in each group just as for uniform load. Coefficients for maximum tensile stress are recorded on the curves in Fig. 8.

The stresses in Fig. 8 are for the section at mid-span where there is compression at top and tension at bottom. The same curves apply at centerline of support if the words "tension" and "compression" are transposed in Fig. 8. The general character of the stress-distribution curves in Fig. 8 is the same as for the uniform load in Figs. 2 and 3. The horizontal distances to the curves in Fig. 8 multiplied by P/Lb are stresses per sq.in.

The horizontal stress right at the load or reaction for $\beta = 1$ is seen to be equal to the vertical stress due to a load P applied on a surface with area of Cb. For example, when $\epsilon = 1/20$, unit stress $= \dfrac{20P}{Lb} = \dfrac{20P\epsilon}{Cb} = \dfrac{P}{Cb}$. These stresses increase slightly with decreasing value of β. They are compression when the force is directed inward but tension when it is directed outward. The stress distribution, except at the loaded edge, approximates the straight-line distribution when $\beta = 1/2$. The distribution curve undergoes a radical change

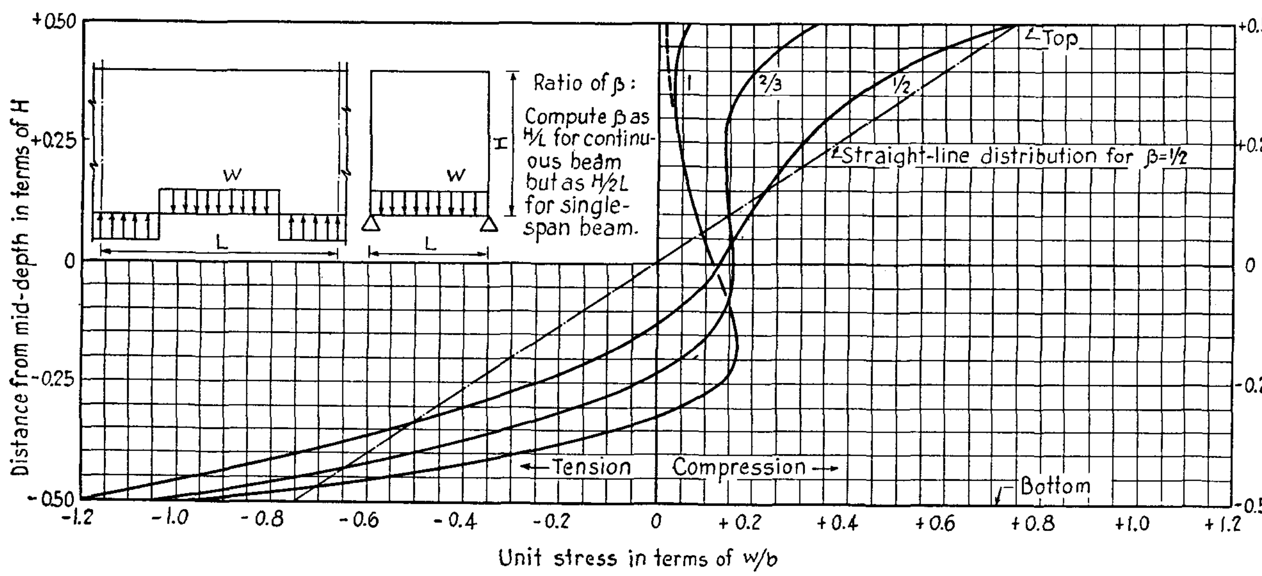

Fig. 7. Moment stresses in simply supported single-span girders.

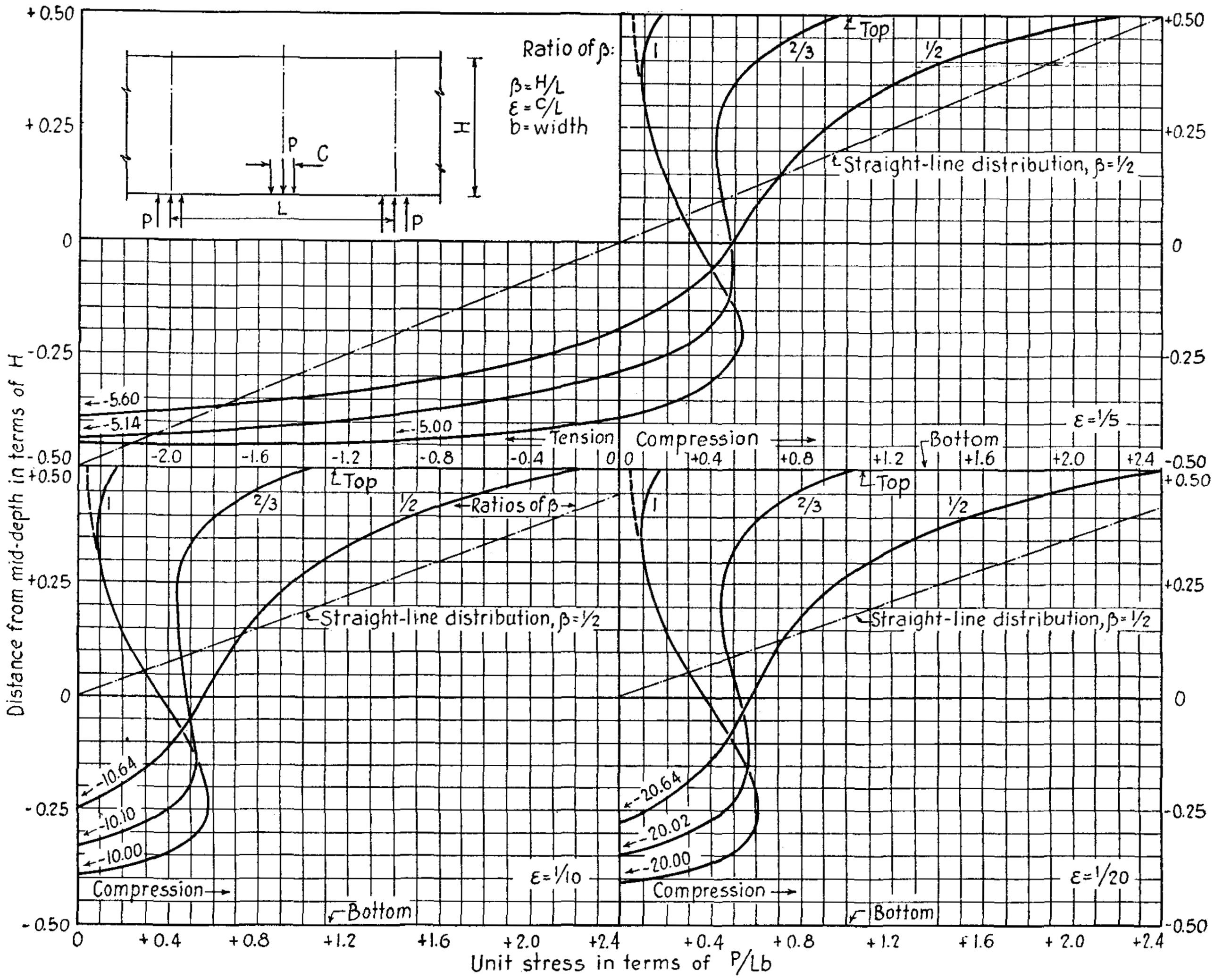

Fig. 8. Moment stresses at mid-span of girders having concentrated loading at bottom edge.

when β increases from ½ to 1, but further increase has practically no effect upon the stresses.

From curves like those in Fig. 8, Dischinger determined the resultant T of all the tensile stresses, and the distance d_0 from the resultant to the loaded edge. The comments made about d_0 and T in Section 2 for uniform load apply also in connection with concentrated load for d_0 in Fig. 9 and T shown in Fig. 10.

The curves for $\epsilon = ½$ are identical in Figs. 4 and 10 except that the ordinates are twice as large in Fig. 10 as in Fig. 4. This is because P used in connection with Fig. 10 is one-half of wL used in connection with Fig. 4, as seen by comparison of drawings in Figs. 7 and 8.

Fig. 10 shows that the ordinary type of design procedure stops being applicable for β-values approximating ⅖, and that for values of β larger than 0.7 the T-values for deep beams are practically constant and independent of the height-to-span ratio. Whatever extra depth is added above $0.7L$ has evidently little or no effect upon T. The same general conclusions

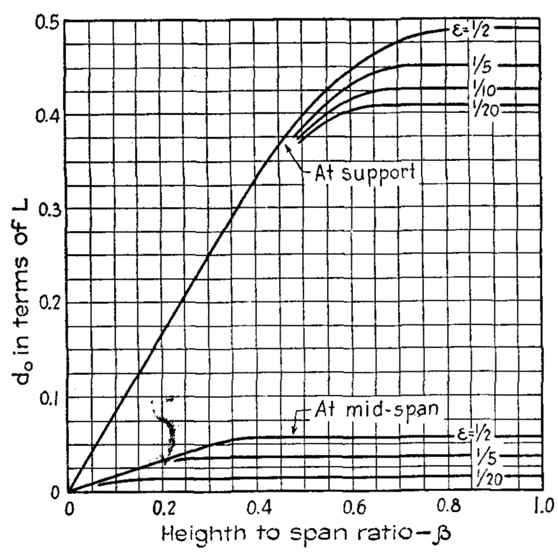

Fig. 9. Distance, d_0, from resultant of tensile stresses to bottom edge of girders having concentrated loading at mid-span.

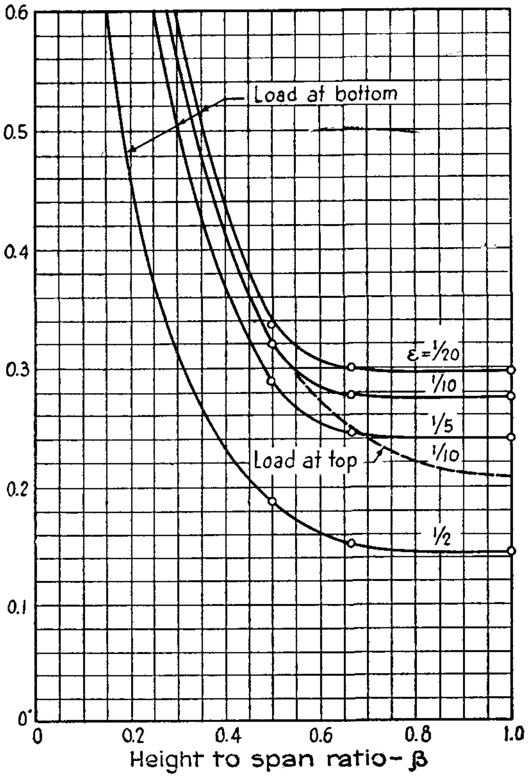

Fig. 10. Resultant, T, of tensile stresses in girders having concentrated loading at mid-span.

may be justified by observations of the curves in Fig. 9 which are so similar to those in Fig. 5 that no further comment is needed.

The moment, M, for the loading in Fig. 8 is $M = \frac{1}{8}PL(1-\epsilon)$ both at mid-span and at centerline of support. All the data discussed in this section pertain exclusively to the case with load and reaction acting upon the same edge of the beam. If the load is raised so that it acts upon the beam at any other elevation, the stresses are different from those described in this section.

Section 5. Continuous Girder with Concentrated Load at Top and at Center of Spans

Fig. 11 shows stresses in a continuous girder with load applied at top and reaction applied at bottom. Data are made available only for the value of $\epsilon = \frac{1}{10}$, and the curves in Fig. 11 are reproduced from a Dischinger figure and are therefore less accurate than curves elsewhere in this article, which are reproduced from tables in Dischinger's original publication.

Comparison of Figs. 8 and 11, for $\epsilon = \frac{1}{10}$, shows that the stress curves are nearly alike for $\beta = \frac{1}{2}$ but that the similarity is reduced as β increases. For $\beta = 1$ in Fig. 11 the stress changes direction three times and there are three neutral axes, a phenomenon of unusual nature. Stresses to the left of the vertical base line are compression and stresses to the right are tension.

According to Dischinger it gives conservative values for continuous girders with load at the top edge, to select the value of T from Fig. 10 which is made for the case with load applied at the bottom. In placing the tensile reinforcement consideration should be given to shape and distribution of stress curves in Fig. 11.

Section 6. Single-Span Girder with Concentrated Load at Center of Span

No data are available for single spans with load at mid-span, but the conclusions reached for uniform load may be applied with a fair degree of accuracy.

It is apparent by comparing Fig. 6(a) with the drawing inserted in Fig. 8 that the continuous beam has zero moment at all sections midway between the load and centerlines of supports. As a result it is to be expected that there is a fairly close similarity between moments at mid-span of the continuous beam with span length L and moments at mid-span of a single-span girder with half that span length, both girders having the same height, H.

Proceeding on basis of this assumption regarding the single-span girder, the design procedure is as follows. For a single span with length, L, and height, H, com-

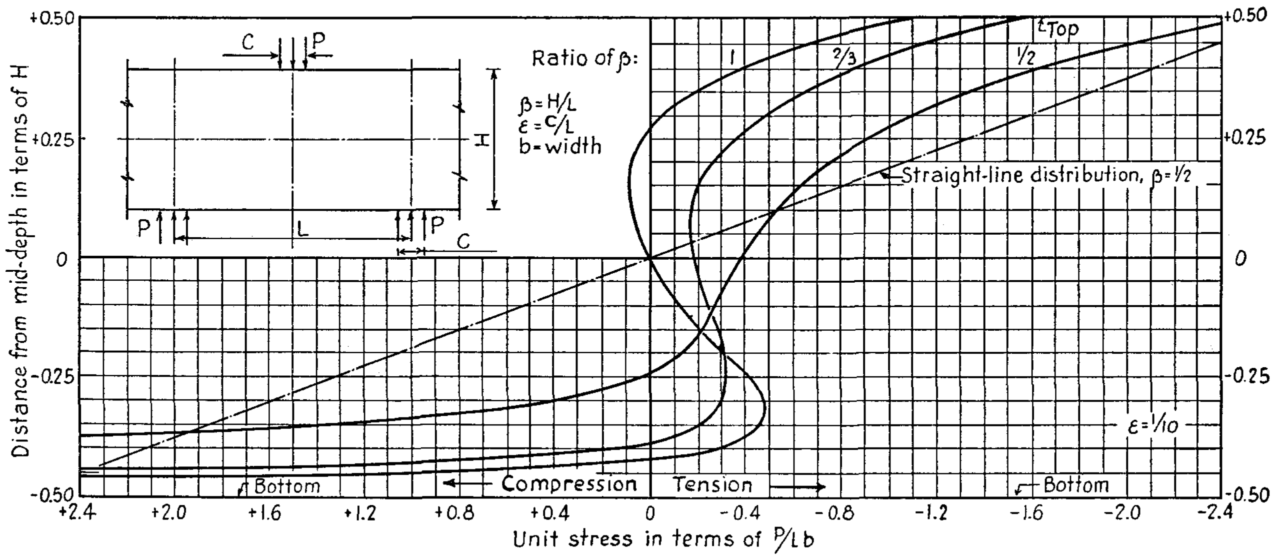

Fig. 11. Moment stresses at centerline of support of girders having concentrated loading at top edge.

pute $\beta = H/2L$ and $\epsilon = C/2L$. Enter Fig. 10 with these two values, select the proper coefficient from the curves, and compute the resultant tensile force as $T = \text{coef.} \times P$, P being the load. Place the tensile steel required close to the bottom of the beam since this is where it is most effective. Use the solid curves in Fig. 10 for load applied at the bottom, but the dash-line curve may be used for $\epsilon = 1/10$ when the load is at the the top. It gives conservative results to use the solid curves for all cases regardless of the elevation at which the concentrated load is applied.

Section 7. End-Spans

Dischinger did not report any investigation regarding moment stresses in end spans, and yet some modification is required since these moments are greater than those in interior spans. One way to make a reasonable allowance for increase in end-span moments is to determine relative moments in an ordinary continuous beam on knife-edge supports and to multiply the interior moments by ratios of the computed continuous beam moments.

For illustration, refer to Table 1A in the *Handbook** and assume the ratio of uniform live to uniform dead load is equal to one. The moment coefficients are 0.113 at the first interior support and 0.094 at the second interior support. The ratio is $0.113/0.094 = 1.20$, so it may be anticipated that moment stresses and tensile steel area for deep beams are 20 per cent larger at first interior than at second interior support. Similarly, for moments at mid-span, the ratio of moment coefficients in first and second span is $0.085/0.056 = 1.52$, so the increase in moment stress and in tensile steel area may be taken as 52 per cent.

No data are available from which the stress-distribution curve can be derived for the section at the first interior support, so the distribution of stress must be based on judgment. As to arrangement and position of tensile bars in the section at the first interior support it seems simplest and most reasonable to conform to the data provided for interior spans. Consequently, bar details may be made similar at all interior supports, the main difference being that the steel area is increased at the first interior support.

Section 8. Shear and Bond

Shear in flexural members is evaluated in the conventional procedure by computing unit shear as $v = V/bjd$ and ascertaining that v does not exceed some allowable value which is derived experimentally. Actually, beams do not fail in shear but in diagonal tension. The conventional concept of shear, the method of deriving the formula for v, and the accepted allowable stresses cannot be extended from shallow to deep beams without first having undergone special study.

For illustration, consider the two beams in Fig. 12 and trace the development of shear failure in them. In the shallow beam with nominal reinforcement near the ends, a tensile crack begins at the bottom as indicated and as the load increases, the crack extends upward and inward at an angle of approximately 45 deg., or if more heavily reinforced for tensile stresses, diagonal cracks may begin at or near the neutral axis and progress either way as the load is increased until they connect with tensile cracks at the bottom. As indicated by the arrows the crack is caused by tensile stresses perpendicular to the crack, which gives rise to the expression "diagonal tension". If the diagonal cracking is kept within bounds by such means as stirrups or efficient anchorage of the longitudinal bars, failure may be postponed until the tensile steel yields or the concrete is crushed, but if stirrups and anchorage are insufficient, the beam may fail suddenly by shearing through the uncracked concrete above the neutral axis and splitting along the longitudinal bars.

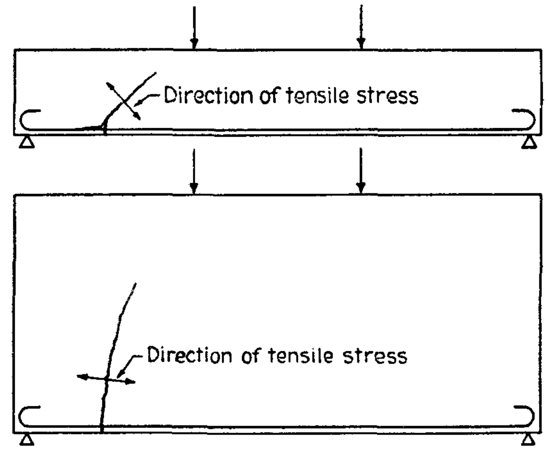

Fig. 12. Diagonal tension in shallow and deep girders compared.

In the deep beam in Fig. 12, the conditions appear to be different. Assuming that the tensile steel percentage is relatively small, the crack starts out vertically and remains practically vertical. The tensile stresses, being perpendicular to the crack, are nearly horizontal, which means that they can be carried by the longitudinal steel and that vertical stirrups are virtually useless. In other words, the diagonal tension that is characteristic of the shallow beam turns more and more into plain horizontal tension the deeper the beam becomes.

The foregoing discussion is presented for the purpose of illustrating that without modification the customary shear investigation is not applicable to girders with relatively large height-to-span ratios. No studies are known which present procedures for investigation of shear in girders with height-to-span ratio varying all the way from the smallest to the largest ratios encountered in practice. In the absence of such data it is tentatively recommended that the unit shear be computed as $8V/7bd$ and that a stress of $v(1+5\beta)/3$ be allowed, v being the allowable shearing stress for shallow beams. The allowable stress shall be modified only when $\beta > 2/5$. According to this procedure the customary shear investigation remains unchanged up to $\beta = 2/5$, and when β increases beyond that value the only change is that the allowable v-stress is increased. It equals $2v$ when $\beta = 1$. This is believed to give con-

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servative results*. The procedure proposed is in a manner of speaking a compromise worked out to take into account several characteristics and is in no way to be construed as being a true picture of the actual theoretical stress conditions, which are as yet unknown.

The allowable bond stresses in the ACI Code, 1941, are twice as large as the allowable shearing stresses on the concrete. The formulas for bond and shear, however, are alike in the code except for the factors of Σo and b being interchanged. Therefore, bond stress is critical only when Σo is smaller than $b/2$, and this will occur rarely in deep beams. For this reason there is no necessity for raising the allowable stress for bond, so the code values of $0.04f'_c$ for ordinary anchorage and $0.06f'_c$ for special anchorage need not be modified. The bond stress may be computed as $8V/7d\Sigma o$.

Section 9. Numerical Examples

EXAMPLE 1. Design a typical interior span of a continuous girder for which the data are as follows:

Length of support, $C = 3$ ft.
Length of span, $L = 30$ ft.
Height of girder, $H = 15$ ft.
Width of girder and support, $b = 1$ ft. 3 in.
Uniform load, $w = 15{,}000$ lb. per lin.ft.

The characteristic ratios used in the stress curves are $\epsilon = C/L = 3/30 = 1/10$ and $\beta = H/L = 15/30 = 1/2$. Assuming that the concrete remains uncracked, select stress coefficients from Figs. 2 and 3 and compute extreme fiber stresses as $f = \text{coef.} \times w/b$. The load is $w = 15{,}000/12 = 1{,}250$ lb. per lin.in. Plus indicates compression and minus tension.

$$\text{Mid-span}\begin{cases}\text{Top} & : +1.07\times 1{,}250/15 = +\ 89 \text{ p.s.i.}\\ \text{Bottom} & : -1.31\times 1{,}250/15 = -109 \text{ p.s.i.}\end{cases}$$

$$\text{Support}\begin{cases}\text{Top} & : -1.25\times 1{,}250/15 = -104 \text{ p.s.i.}\\ \text{Bottom} & : +9.32\times 1{,}250/15 = +777 \text{ p.s.i.}\end{cases}$$

The tensile stresses of 109 and 104 p.s.i. are so small that the concrete, which is assumed to have a 3,000-lb. compressive strength and a corresponding modulus of rupture of approximately 500 p.s.i., will probably remain uncracked, in which case no reinforcement will be needed.

On the other hand, suppose it is required to provide steel stressed to $f_s = 20{,}000$ p.s.i., designed to take all the tension developed in the concrete in accordance with Figs. 2 and 3. The resultant of all concrete tensile stresses, T, is computed by use of the coefficients in Fig. 4: $T = \text{coef.} \times wL$, in which w is in lb. per lin.ft. when L is in ft. Select coefficients for $\epsilon = 1/10$ and $\beta = 1/2$ in Fig. 4.

$$\text{Mid-span}\begin{cases}T = 0.12\times 15{,}000\times 30 = 54{,}000 \text{ lb.}\\ A_s = 2.70 \text{ sq.in.}\end{cases}$$

$$\text{Support}\begin{cases}T = 0.23\times 15{,}000\times 30 = 104{,}000 \text{ lb.}\\ A_s = 5.20 \text{ sq.in.}\end{cases}$$

The distance of the resultants T above the bottom is taken from Fig. 5 for $\epsilon = 1/10$ and $\beta = 1/2$, the distance being $d_0 = \text{coefficient} \times L$.

Mid-span: $d_0 = 0.06\times 30 = 1.8$ ft.
Support : $d_0 = 0.33\times 30 = 9.9$ ft.

It is seen by inspection of the curves for $\epsilon = 1/10$ and $\beta = 1/2$ in Figs. 2 and 3 that the maximum tensile stresses occur at top and bottom. The computed values of d_0 will therefore be disregarded and the tensile reinforcement placed as close to the top and bottom faces as allowed by protection requirements.

A brief check calculation is made to ascertain how the tensile steel areas determined compare with those obtained on basis of the customary straight-line distribution of stresses.

The moments in a typical interior span uniformly loaded are as follows.

$$\text{Mid-span}: \frac{1}{24}wL^2(1-\epsilon^2) = \frac{1}{24}(15{,}000\times 30)30(1-0.1^2) = 560{,}000 \text{ ft.lb.}$$

$$\text{Support}\ : \frac{1}{24}wL^2(1-\epsilon)(2-\epsilon) = \frac{1}{.24}(15{,}000\times 30)\,30(1-0.1)(2-0.1) = 960{,}000 \text{ ft.lb.}$$

Use the approximate value of $j = 0.93$, which gives $jdf_s = 0.93(180-4)20{,}000 = 3{,}280{,}000$, and compute the tensile steel areas, $A_s = M/jdf_s$.

Mid-span: $560{,}000\times 12/3{,}280{,}000 = 2.1$ sq.in.
Support : $960{,}000\times 12/3{,}280{,}000 = 3.5$ sq.in.

This is approximately 30 per cent less steel than computed by the procedure for deep girders. On basis of the gross section of $bH = 15(15\times 12) = 2{,}700$ sq.in., the percentage of tensile steel is $2.70/2{,}700 = 0.0010$ at mid-span and $5.20/2{,}700 = 0.0019$ at support. This is seen to be considerably less than the usual minimum percentage of 0.005 required for ordinary beams and girders. The cross-sectional area of the deep girders discussed here is so large that it would require unreasonably large amounts of steel to bring the ratio up to 0.005 for tension at both critical sections. Whether another but smaller minimum ratio should be adopted is debatable.

Total end shear is

$$V = \frac{1}{2}wL(1-\epsilon) = \frac{1}{2}\times 15{,}000\times 30(1-0.1) = 203{,}000 \text{ lb.}$$

The calculation of shear is made in accordance with the recommendations proposed in Section 8.

$$v = \frac{8V}{7bd} = \frac{8\times 203{,}000}{7\times 15(180-4)} = 88 \text{ p.s.i.}$$

For a 3,000-lb. concrete and reinforcement having special anchorage, the allowable shearing stress on the concrete is $0.03f'_c = 0.03\times 3{,}000 = 90$ p.s.i. In accordance with Section 8, for $\beta > 2/5$, the allowable stress is

*In the experiments reported in "Shear Tests of Reinforced Concrete Beams" by Slater, Lord and Zipprodt, No. 314 of *Technologic Papers* of the Bureau of Standards, the specimens were designed to force failure to occur in the web of beams with relatively high depth-to-span ratios, most of the beams having a ratio of 0.32. The standard cross section was I-shaped because with such high shearing strengths as could be developed it was impossible to get sufficient encasement for longitudinal reinforcement except by making the flanges wider than the web.

increased to $90(1+5\beta)/3 = 90(1+5\times0.5)/3 = 105$ p.s.i. Special anchorage means that all longitudinal bars are to be made continuous or to be hooked in a region of compression. If the 105-lb. stress is exceeded, stirrups or bent bars should be used, but in any case nominal vertical reinforcement should be provided for the purpose of supporting the top bars.

Total bearing pressure on the support is $\frac{1}{2}wL = \frac{1}{2}\times15{,}000\times30 = 225{,}000$ lb., and the area of the bearing surface is $b(12C) = 15(12\times3) = 540$ sq.in., so the unit pressure is $225{,}000/540 = 417$ p.s.i. The ACI Code, 1941, allows a bearing pressure of $0.25f'_c = 750$ p.s.i.

In this example the arrangement of reinforcement is essentially the same as for ordinary shallow beams. As the depth-to-span ratio increases, however, the dissimilarity between deep- and shallow-girder design becomes more and more pronounced. For illustration, Figs. 4 and 5 illustrate that when β is greater than $\frac{1}{2}$, the area of the tensile steel and its distance above the bottom both remain practically constant.

Suppose the interior beam considered in the foregoing is one of four or more continuous beams with the same span lengths. The procedure of design to be illustrated for end span is discussed in Section 7. Assume as in Section 7 that the spans are equal and the live and dead load are also equal, and select moment coefficients from Table 1A in the *Handbook*. The ratio of positive moment coefficient at mid-span of exterior span to that at interior span is $0.085/0.056 = 1.52$. The ratio of negative moment coefficient at first interior support to that at typical interior supports is $0.113/0.094 = 1.20$. The area of steel in the end span may then be taken as follows:

Mid-span: $A_s = 1.52\times2.70 = 4.10$ sq.in.
Support : $A_s = 1.20\times5.20 = 6.24$ sq.in.

The end shear in the exterior span at the interior support may be taken as 20 per cent greater than for a typical interior span. This makes the maximum unit shearing stress equal to $1.20\times88 = 106$ p.s.i. which is only 1 p.s.i. more than allowed.

The bond stress at the point of maximum end shear computed for four $1\frac{1}{4}$-in. square bars is:

$$u = \frac{8V}{7d\Sigma o} = \frac{8\times1.20\times203{,}000}{7\times176\times20} = 80 \text{ p.s.i.}$$

The allowable bond stress is 120 p.s.i. for ordinary anchorage and 180 p.s.i. for special anchorage.

A girder as deep as 15 ft. ought to be considered and reinforced as a concrete wall. If the face of it is exposed to view so that appearance is of importance, secondary or auxiliary reinforcement should be provided, that is, small bars placed relatively close together should be used in an amount equal to approximately 0.25 per cent horizontally and 0.15 per cent vertically.

EXAMPLE 2. Design a single-span deep girder for which the following data are given:

Length of support, $C = 2$ ft.
Length of span, $L = 20$ ft.
Height of girder, $H = 20$ ft.
Width of girder and support, $b = 1$ ft. 3 in.
Uniform load, $w = 30{,}000$ lb. per lin.ft.

In single-span girders, the length of the support is disregarded and the reaction is assumed to be concentrated in the centerline of the support. It has been demonstrated in connection with Fig. 6 that the moment stresses at mid-span of a single-span girder as in Fig. 6(b) are the same as those at mid-span of the continuous girder in Fig. 6(a). Therefore, the continuous girder has the following characteristic ratios: $\epsilon = \frac{1}{2}$ and $\beta = H/2L = 20/2\times20 = \frac{1}{2}$. Using these ratios, select coefficients from Fig. 7 and Fig. 4.

The extreme fiber stress is $f = \text{coef.} \times w/b$ in which w inserted in lb. per in. equals $30{,}000/12 = 2{,}500$. Plus indicates compression and minus tension.

Top : $+0.75\times2{,}500/15 = +125$ p.s.i.
Bottom: $-1.20\times2{,}500/15 = -200$ p.s.i.

A tensile stress of 200 p.s.i. is probably too small to crack a 3,000-lb. concrete, but assume that steel must be provided to take all tensile stresses in the concrete which, from Fig. 4 gives

$T = 0.095w(2L) = 0.095\times30{,}000\times2\times20 = 114{,}000$ lb.
and $A_s = T/f_s = 114{,}000/20{,}000 = 5.70$ sq.in.

The theoretical position of T measured from the bottom of the girder may be determined by use of a coefficient selected from Fig. 5, but Fig. 7 shows that the tensile steel is most effective when placed as close to the bottom as permissible. Note that the steel ratio based on the gross area is only $p = 5.70/3{,}600 = 0.0016$, whereas 0.005 is the minimum ratio allowed for shallow beams.

Using the customary shallow-beam procedure, with $j = 0.93$, gives

$$A_s = \frac{30{,}000\times20^2\times12}{8\times0.93(240-4)20{,}000} = 4.10 \text{ sq.in.}$$

In this case the ordinary shallow-beam design results in 28 per cent less steel area than does the deep-beam procedure. The end shear is

$$V = \frac{1}{2}w(L-C) = \frac{1}{2}\times30{,}000(20-2) = 270{,}000 \text{ lb.}$$

$$\text{The unit shear is } v = \frac{8V}{7bd} = \frac{8\times270{,}000}{7\times15\times236} = 87 \text{ p.s.i.}$$

The reaction to be used for the purpose of checking bearing pressure is computed as

$$R = \frac{1}{2}w(L+C) = \frac{1}{2}\times30{,}000(20+2) = 330{,}000 \text{ lb.}$$

The unit bearing pressure is $330{,}000/(15\times2\times12) = 920$ p.s.i. The unit pressure allowed for a 3,000-lb. concrete is $0.25f'_c = 0.25\times3{,}000 = 750$ p.s.i. The balance of $(920-750)15\times2\times12 = 61{,}000$ lb. must be transmitted into the support by means of dowels stressed to 16,000 p.s.i. as allowed for column design using intermediate grade bars.

The bond stress on four 1-in. round bars ($\Sigma o = 12.6$) is

$$u = \frac{8V}{7d\Sigma o} = \frac{8\times270{,}000}{7\times236\times12.6} = 104 \text{ p.s.i.}$$

With special anchorage, 180 p.s.i. is allowed.

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