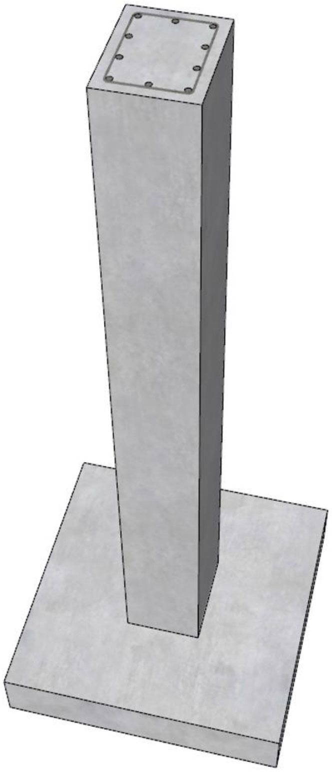
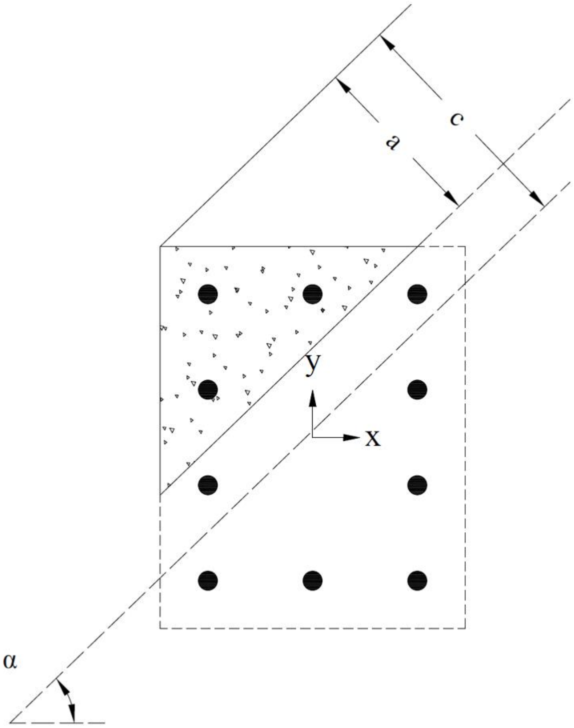
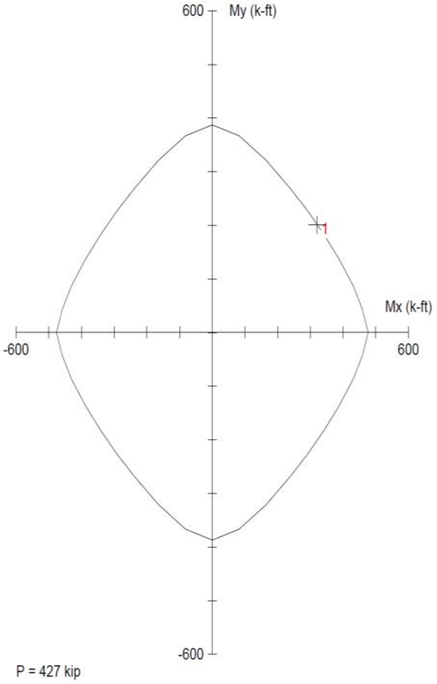
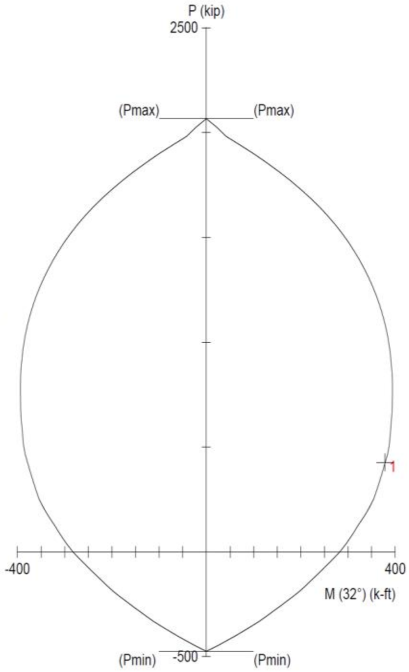
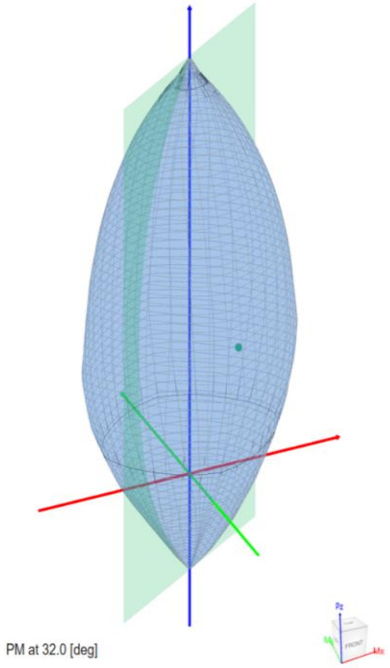
# Combined Axial Force and Biaxial Bending Interaction Diagram - Rectangular Reinforced Concrete Column (ACI 318-14)











# Combined Axial Force and Biaxial Bending Interaction Diagram - Rectangular Reinforced Concrete Column (ACI 318-14)

Biaxial bending of columns occurs when the loading causes bending simultaneously about both principal axes. The commonly encountered case of such loading occurs in corner columns. Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load.

A uniaxial interaction diagram defines the load-moment strength along a single plane of a section under an axial load P and a uniaxial moment M. the biaxial bending resistance of an axially loaded column can be represented schematically as a surface formed by a series of uniaxial interaction curves drawn radially from the P axis. Data for these intermediate curves are obtained by varying the angle of the neutral axis (for assumed strain configurations) with respect to the major axes.

The difficulty associated with the determination of the strength of reinforced columns subjected to combined axial load and biaxial bending is primarily an arithmetic one. The bending resistance of an axially loaded column about a particular skewed axis is determined through iterations involving simple but lengthy calculations. These extensive calculations are compounded when optimization of the reinforcement or cross-section is sought.

This example demonstrates the determination of the biaxial flexural strength of a rectangular reinforced concrete column at a particular nominal axial strength ($P_n$ = 426 kips) with a moment ratio of biaxial bending moments in the X and Y direction ($M_{nx}/M_{ny}$ = 1.60). The figure below shows the reinforced concrete rectangular column cross section in consideration. We will compare the calculated values of the column biaxial bending strength with the values from the reference and the exact values from spColumn engineering software program from StructurePoint. The steps to develop the three-dimensional failure surface (interaction diagram) using spColumn will be shown in detail as well.

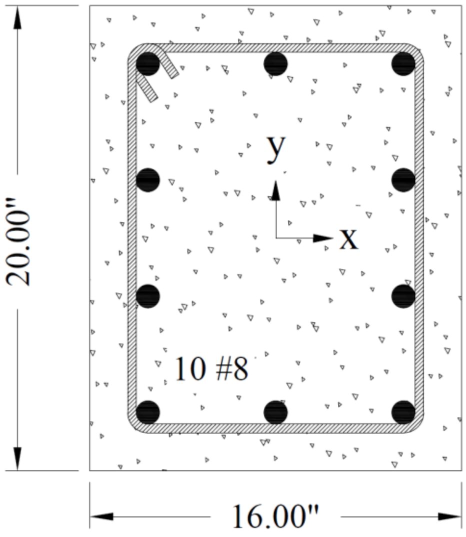


Figure 1 – Reinforced Concrete Column Cross-Section

# Contents

## Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

## Reference

Reinforced Concrete Design, 8th Edition, 2018, Wang et. al., Oxford University Press, Example 10.20.1

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association

spColumn Engineering Software Program Manual v6.50, StructurePoint, 2019

## Design Data

$f_c' = 6000$ psi

$f_y = 60000$ psi

Cover = 2.5 in.

Column dimensions and reinforcement locations are shown in following figure.

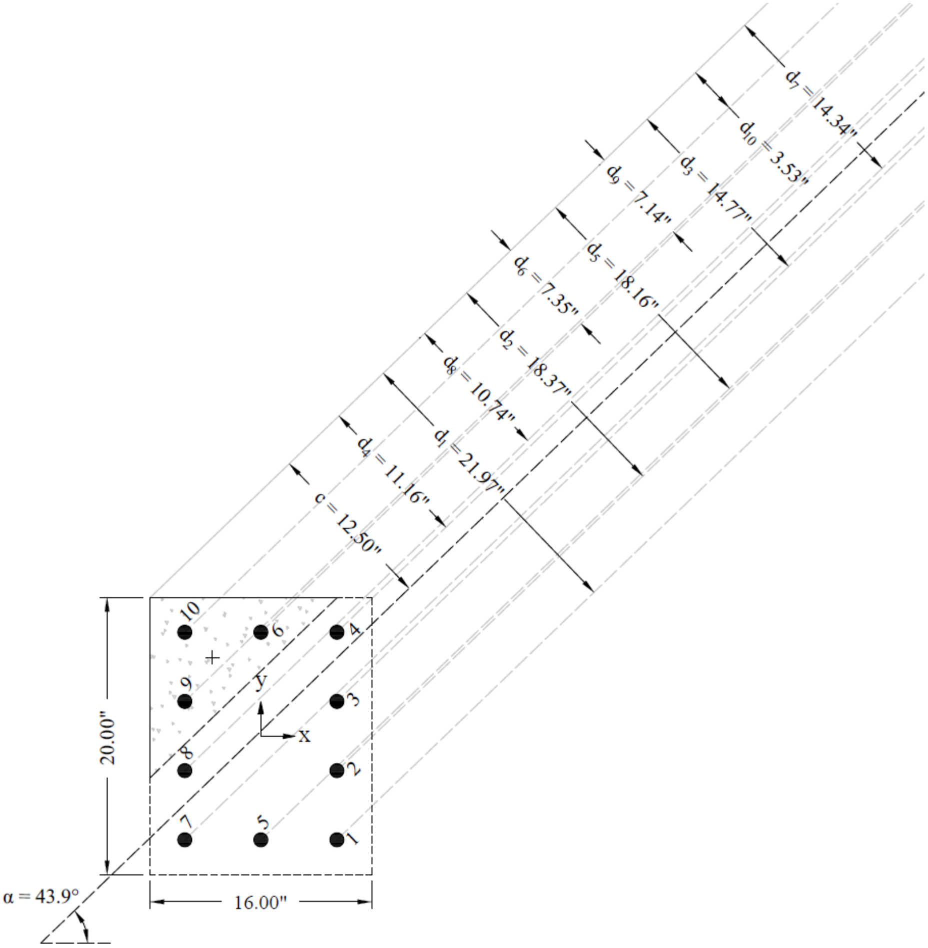


Figure 2 – Reinforced Concrete Column Cross-Section and Reinforcement Locations

**Solution**

In a reinforced concrete column, the determination of biaxial moment strength for a given axial force involves a trial-and-error process for calculating the neutral axis depth and angle α (Figure 2). The reference shows that using this process for the given column section leads to a value of the neutral axis depth of c = 12.5 in. and an angle of α = 43.9º. In this example the last iteration will be shown for illustration of the procedure as a representative calculation of the complete trial and error procedure.

The steps to calculate biaxial flexural strength of a rectangular reinforced concrete column for a given nominal axial strength and moment ratio of biaxial bending moments are as follows:

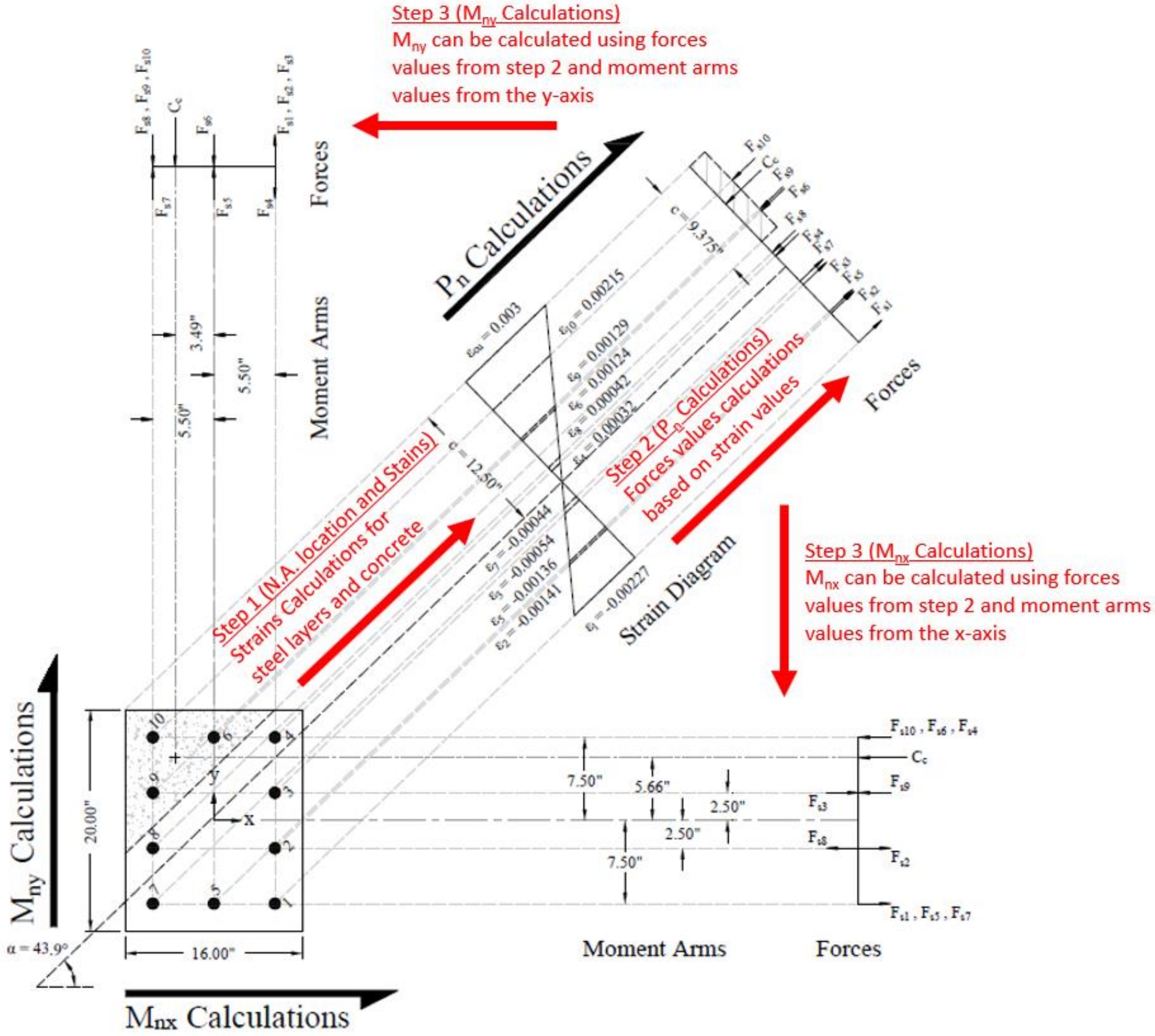
1. Assuming a value for the angle of the neutral axis (α) and the neutral axis depth (c) and calculating the strain values in each reinforcement layer
2. Calculating the forces values in the concrete ( $C_c$ )and reinforcement layers ( $F_{si}$ )
3. Calculating $P_n$ and $M_{nx}/M_{ny}$ using the following equations

$$P_n = C_c + \sum F_s$$

$$M_{ny} = C_c \times \left( \frac{b}{2} - \bar{x}_c \right) + \sum_{i=1}^{n=10} \left( F_{si} \times \left( \frac{b}{2} - x_i \right) \right)$$

$$M_{nx} = C_c \times \left( \frac{h}{2} - \bar{y}_c \right) + \sum_{i=1}^{n=10} \left( F_{si} \times \left( \frac{h}{2} - y_i \right) \right)$$

The procedure above should be repeated until the calculated $P_n$ , and $M_{nx}/M_{ny}$ are equal to the given $P_n$ , and $M_{nx}/M_{ny}$ . The following figure demonstrates the procedure explained above:



Step 3 ($M_{ny}$ Calculations)
$M_{ny}$ can be calculated using forces values from step 2 and moment arms values from the y-axis

Step 1 (N.A. location and Stains)
Strains Calculations for steel layers and concrete

Step 2 ($P_n$ Calculations)
Forces values calculations based on strain values

Step 3 ($M_{nx}$ Calculations)
$M_{nx}$ can be calculated using forces values from step 2 and moment arms values from the x-axis

Figure 3 – Biaxial Flexural Strength Calculation Methods for a Reinforced Concrete Column.

## 1. Concrete Column Biaxial Strength Calculations

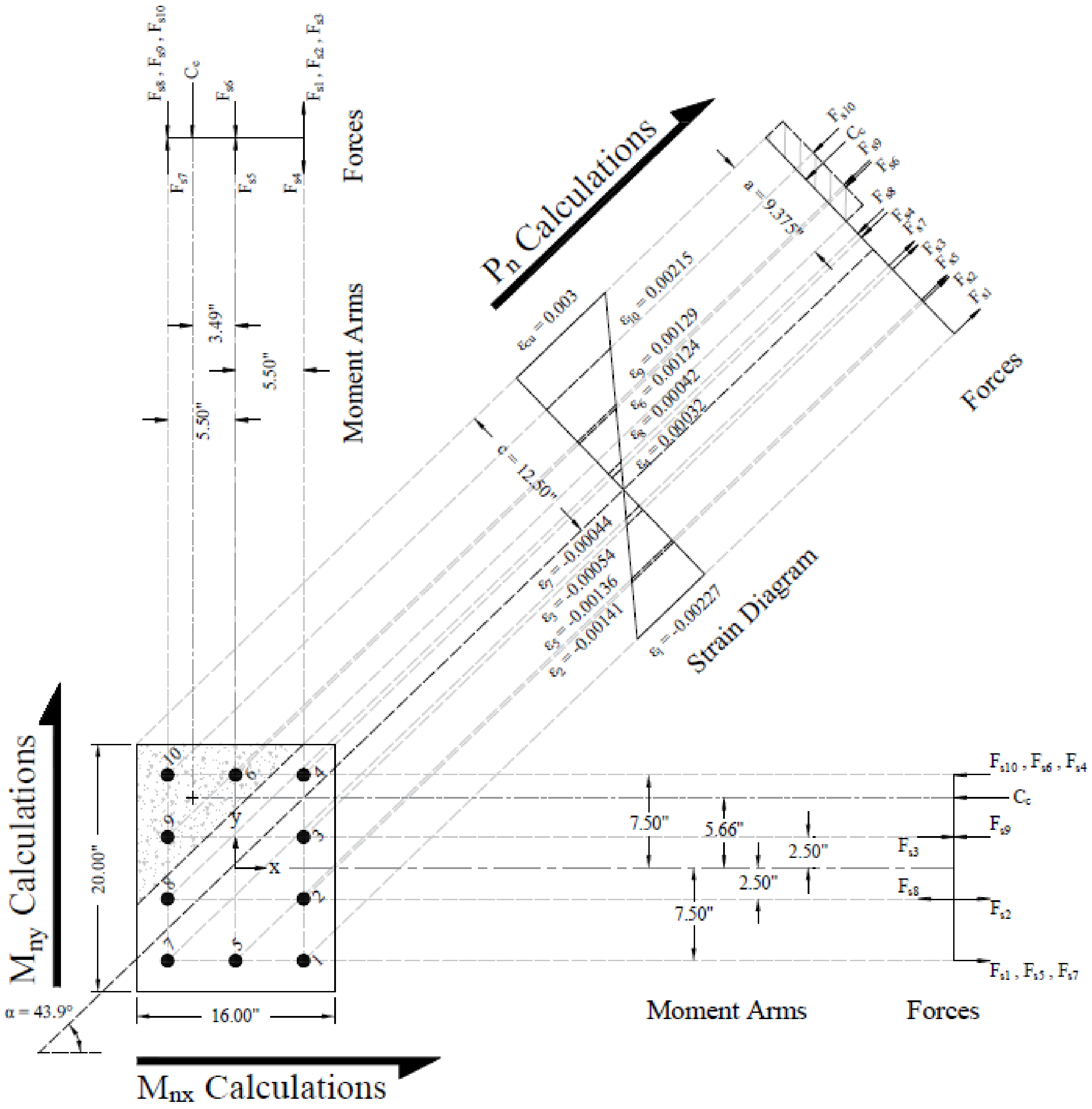


Figure 4 – Strains, Forces, and Moment Arms Diagram

## 1.1. Location of Neutral Axis and Concrete Compression Force

Use and iterative procedure to determine the nominal moment capacities for a point where the nominal axial load capacity, $P_n$, = 426 kips and $M_{nx}/M_{ny}$ = 1.60.

Several previous trials are conducted to determine the c value and the angle α. The following shows the last trial:

Try c = 12.50 in. and α = 43.9°, where *c* is the distance from the fiber of maximum compressive strain to the neutral axis and α is the angle of the neutral axis. ***ACI 318-14 (22.2.2.4.2)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

$$\varepsilon_{s1} = (c - d_1) \times \frac{\varepsilon_{cu}}{c} = (12.50 - 21.97) \times \frac{0.003}{12.50} = -0.00227 \text{ (Tension)} > \varepsilon_y \rightarrow \text{reinforcement has yielded}$$

$$\phi = 0.65 + 0.25 \times \left( \frac{\varepsilon_{s1} - \varepsilon_y}{0.005 - \varepsilon_y} \right) = 0.65 + 0.25 \times \left( \frac{0.00227 - 0.00207}{0.005 - 0.00207} \right) = 0.673$$ ***ACI 318-14 (Table 21.2.2)***

$$a = \beta_1 \times c = 0.75 \times 12.50 = 9.375 \text{ in.}$$ ***ACI 318-14 (22.2.2.4.1)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-14 (22.2.2.1)***

Where:

*a* = Depth of equivalent rectangular stress block ***ACI 318-14 (Table 22.2.2.4.3)***

$$\beta_1 = 0.85 - \frac{0.05 \times \left( f_c' \times 4000 \right)}{1000} = 0.85 - \frac{0.05 \times (6000 - 4000)}{1000} = 0.75$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$C_c = 0.85 \times f_c' \times A_{comp} = 0.85 \times 6000 \times 87.96 = 448.57 \text{ kip (Compression)}$$ ***ACI 318-14 (22.2.2.4.1)***

Where:

$$A_{comp} = \frac{a^2}{2 \times \sin(\alpha) \times \cos(\alpha)} = \frac{(9.375)^2}{2 \times \sin(43.9°) \times \cos(43.9°)} = 87.96 \text{ in.}^2$$

$\bar{x} = 3.49$ in. $\quad \bar{y} = 5.66$ in. (see the following figure)

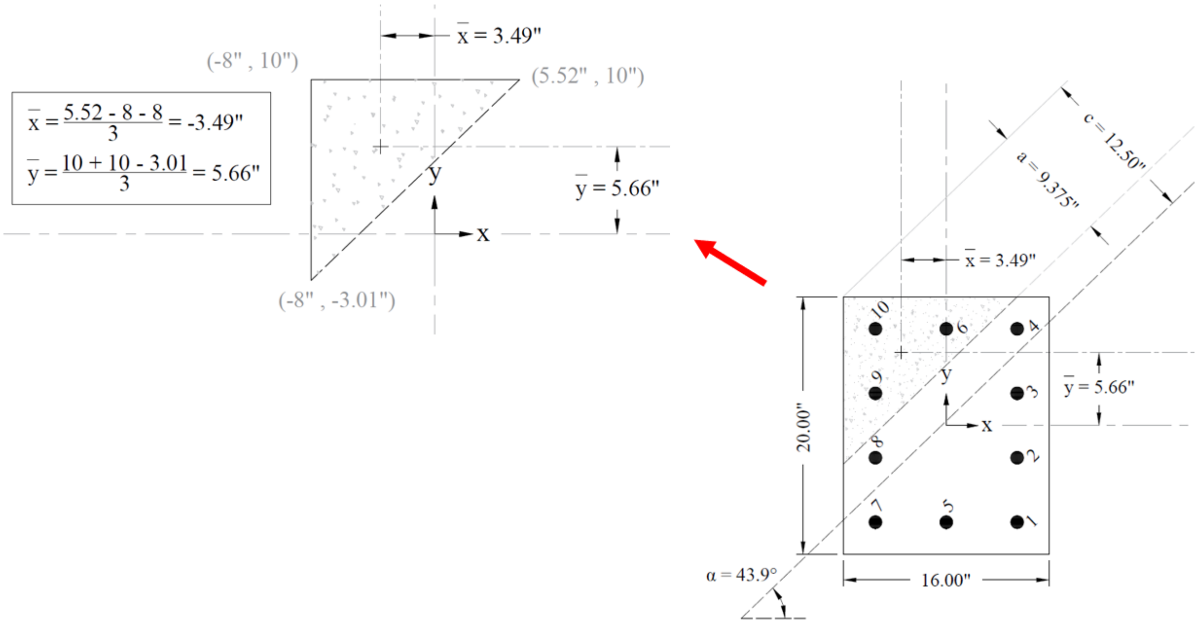


Figure 5 – Cracked Concrete Column Section Centroid Calculations

## 1.2. Strains and Forces Determination in Reinforcement Layers

For the 1st reinforcement layer:

$\varepsilon_{s1} = -0.00227$ (Tension) > $\varepsilon_y$ → reinforcement has yielded

$\therefore\ f_{s1} = f_y = -60000$ psi

$\text{F}_{s1} = f_{s1} \times A_{s1} = -60000 \times (1 \times 0.79) = -47.4$ kip (Tension)

For the 9th reinforcement layer:

$\varepsilon_{s9} = (c - d_9) \times \frac{\varepsilon_{cu}}{c} = (12.50 - 7.138) \times \frac{0.003}{12.50} = 0.00129$ (Compression) < $\varepsilon_y$ → reinforcement has not yielded

$\therefore\ f_{s9} = \varepsilon_{s9} \times E_s = 0.00129 \times 29000000 = 37320$ psi

The area of the reinforcement in this layer is included in the area used to compute $C_c$ (a = 9.38 in. > $d_9$ = 7.14 in.). As a result, it is necessary to subtract $0.85 f_c'$ from $f_{s9}$ before computing $F_{s9}$:

$\text{F}_{s9} = f_{s9} \times A_{s9} = (35851 - 0.85 \times 6000) \times (1 \times 0.79) = 25.45$ kip (Compression)

The same procedure shown above can be repeated to calculate the forces in the remaining reinforcement layers, results are summarized in the following table:

Table 1 - Strains, internal force resultants and Moments

| Layer | d, in. | ε, in./in. | $f_s$, psi | $F_s$, kip | $C_c$, kip | Moment arm (x), in. | $M_y$, kip-ft | Moment arm (y), in. | $M_x$, kip-ft |
|---|---|---|---|---|---|---|---|---|---|
| Concrete | --- | 0.00300 | --- | --- | 448.57 | 3.49 | 130.46 | 5.66 | 211.58 |
| 1 | 21.97 | -0.00227 | -60000 | -47.40 | --- | -5.50 | 21.73 | -7.50 | 29.63 |
| 2 | 18.37 | -0.00141 | -40841 | -32.26 | --- | -5.50 | 14.79 | -2.50 | 6.72 |
| 3 | 14.77 | -0.00054 | -15764 | -12.45 | --- | -5.50 | 5.71 | 2.50 | -2.59 |
| 4 | 11.16 | 0.00032 | 9312 | 7.36 | --- | -5.50 | -3.37 | 7.50 | 4.60 |
| 5 | 18.16 | -0.00136 | -39373 | -31.10 | --- | 0.00 | 0.00 | -7.50 | 19.44 |
| 6 | 7.35 | 0.00124 | 35851 | 24.29* | --- | 0.00 | 0.00 | 7.50 | 15.18 |
| 7 | 14.34 | -0.00044 | -12827 | -10.13 | --- | 5.50 | -4.64 | -7.50 | 6.33 |
| 8 | 10.74 | 0.00042 | 12250 | 9.68 | --- | 5.50 | 4.44 | -2.50 | -2.02 |
| 9 | 7.14 | 0.00129 | 37320 | 25.45* | --- | 5.50 | 11.66 | 2.50 | 5.30 |
| 10 | 3.53 | 0.00215 | 60000 | 43.37* | --- | 5.50 | 19.88 | 7.50 | 27.11 |
| Axial Force and Biaxial Bending Moments Capacities | | | $P_n$, kip | 425.38 | | $M_{ny}$, kip-ft | 200.64 | $M_{nx}$, kip-ft | 321.27 |
| | | | $\phi P_n$, kip | 286.28 | | $\phi M_{ny}$, kip-ft | 135.03 | $\phi M_{nx}$, kip-ft | 216.21 |

* The area of the reinforcement in this layer has been included in the area used to compute $C_c$. As a result, $0.85 f_c'$ is subtracted from $f_s$ in the computation of $F_s$.

### 1.3. Calculation of $P_n$, $M_{nx}$ and $M_{ny}$

$$P_n = C_c + \sum F_s$$

(+) = Compression (-) = Tension

$$\phi P_n = \phi \times P_n = 0.673 \times P_n$$

$$M_{ny} = C_c \times \left( \frac{b}{2} - \bar{x}_c \right) + \sum_{i=1}^{n=10} \left( F_{si} \times \left( \frac{b}{2} - x_i \right) \right)$$

(+) = Counter Clockwise (-) = Clockwise

$$\phi M_{ny} = \phi \times M_{ny} = 0.673 \times M_{ny}$$

$$M_{nx} = C_c \times \left( \frac{h}{2} - \bar{y}_c \right) + \sum_{i=1}^{n=10} \left( F_{si} \times \left( \frac{h}{2} - y_i \right) \right)$$

(+) = Counter Clockwise (-) = Clockwise

$$\phi M_{nx} = \phi \times M_{nx} = 0.673 \times M_{nx}$$

Since the calculated $P_n$, and $M_{nx}/M_{ny}$ are equal to the given $P_n$, and $M_{nx}/M_{ny}$ ($P_n$, = 426 kip and $M_{nx}/M_{ny}$ = 321/201 = 1.60), the assumptions that c = 12.5 in. and α = 43.9º are verified as correct.

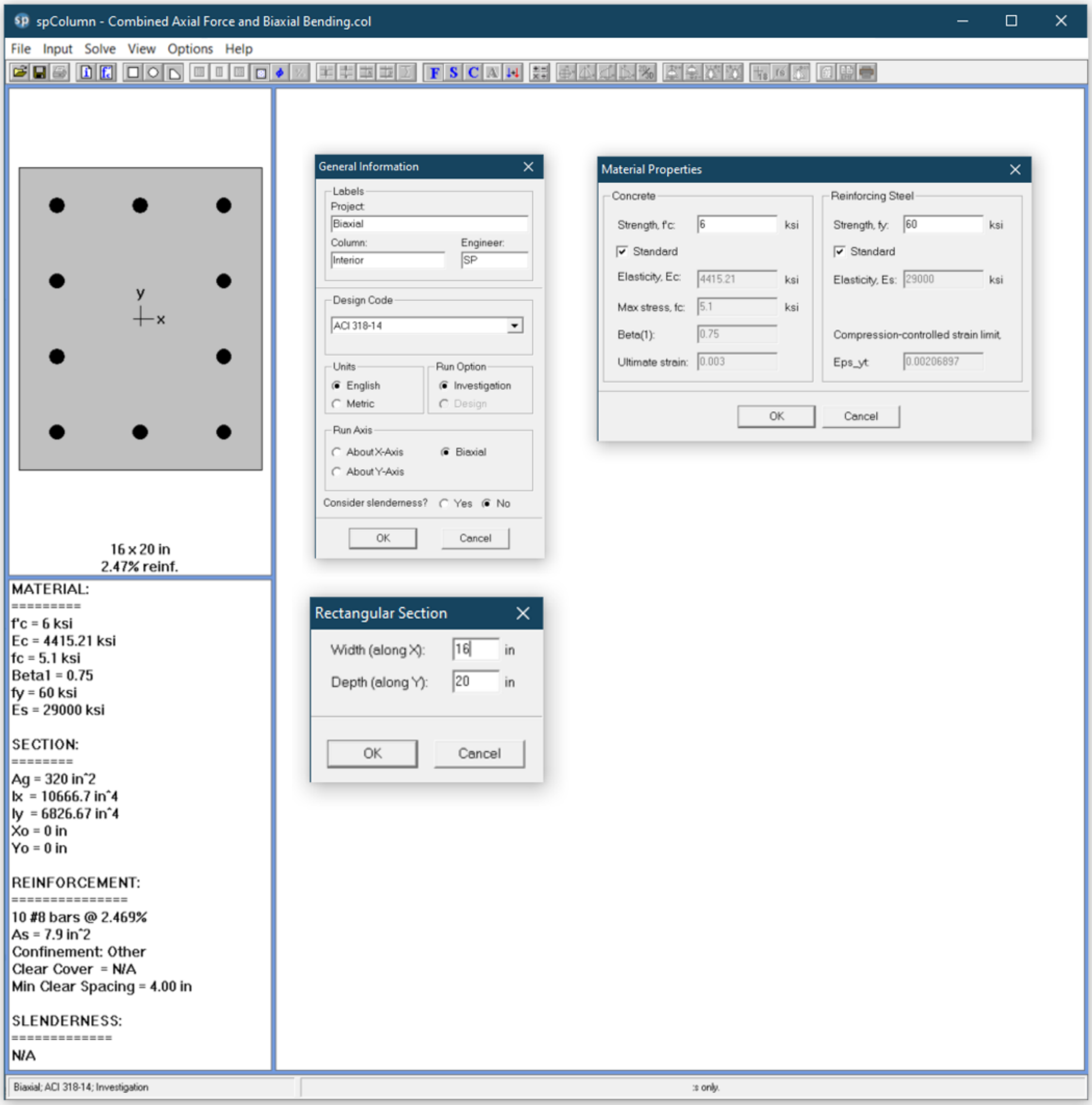
## 2. Column Biaxial Bending Interaction Diagram – spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with **“biaxial”** option for “Run Axis” using the ACI 318-14.

For biaxial runs, the values of maximum compressive axial load capacity and maximum tensile load capacity are computed. These two values set the range within which the moment capacities are computed for a predetermined number of axial load values. For each level of axial load, the section is rotated in 10-degree increments from 0 degrees to 360 degrees and the $M_x$ and $M_y$ moment capacities are computed. Thus, for each level of axial load, an $M_x$-$M_y$ contour is developed. Repeating this for the entire range of axial loads, the three-dimensional failure surface is computed. A three-dimensional visualization of the resulting entire nominal and factored failure surface is provided to support enhanced understanding of the section capacity.

The **“biaxial”** feature allows the user to investigate the P-M interaction diagrams, the $M_x$-$M_y$ moment contour plots, as well as the 3D failure surface for even the most irregular column and shear wall sections quickly, simply, and accurately.

In lieu of using program shortcuts, spColumn model editor was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.



General Information

Labels
Project: Biaxial
Column: Interior
Engineer: SP

Design Code: ACI 318-14

Units: ● English ○ Metric
Run Option: ● Investigation ○ Design
Run Axis: ○ About X-Axis ○ About Y-Axis ● Biaxial
Consider slenderness? ○ Yes ● No

Material Properties

Concrete
Strength, f'c: 6 ksi
☑ Standard
Elasticity, Ec: 4415.21 ksi
Max stress, fc: 5.1 ksi
Beta(1): 0.75
Ultimate strain: 0.003

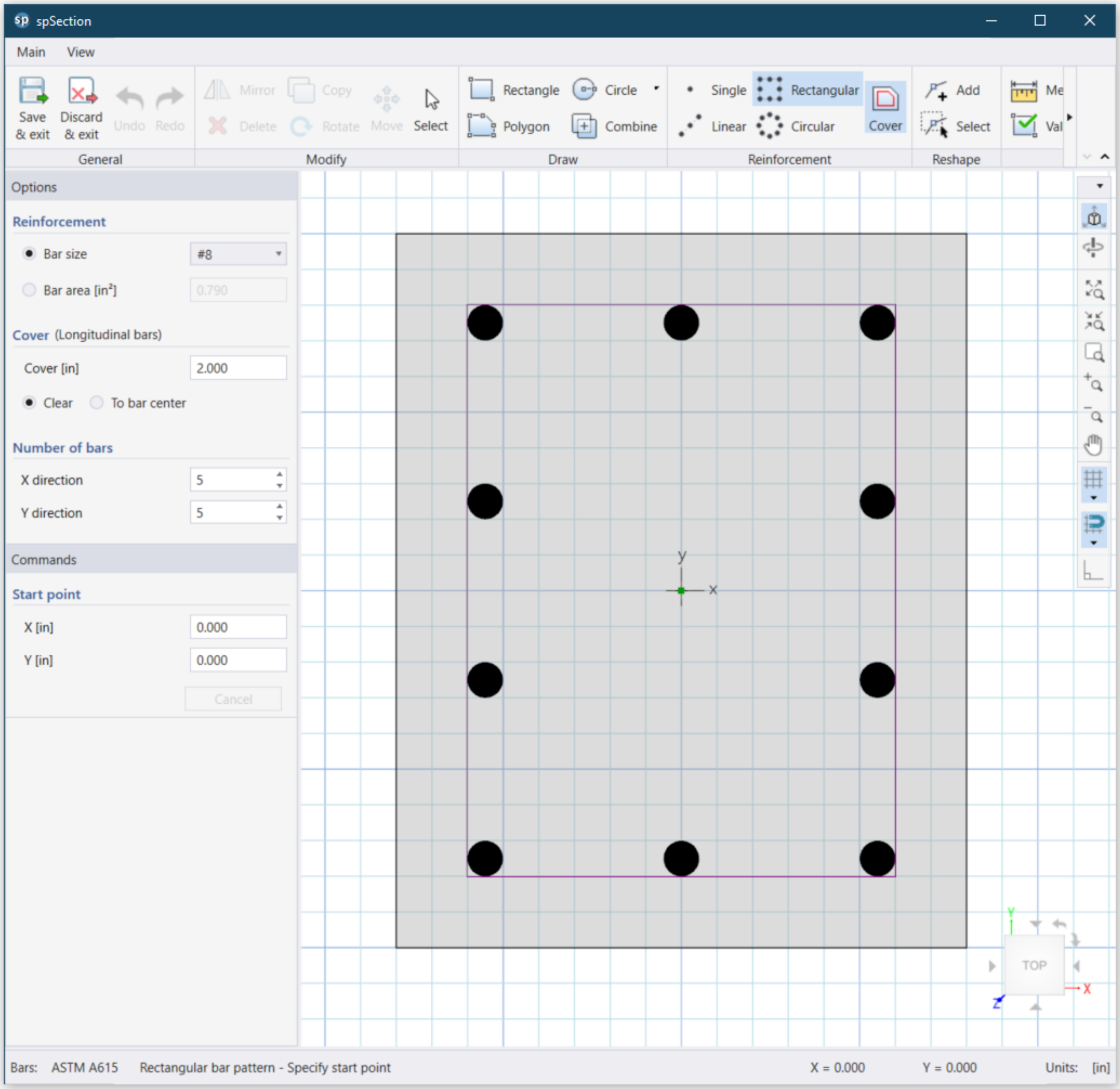
Reinforcing Steel
Strength, fy: 60 ksi
☑ Standard
Elasticity, Es: 29000 ksi
Compression-controlled strain limit,
Eps_yt: 0.00206897

Rectangular Section
Width (along X): 16 in
Depth (along Y): 20 in

```
MATERIAL:
=========
f'c = 6 ksi
Ec = 4415.21 ksi
fc = 5.1 ksi
Beta1 = 0.75
fy = 60 ksi
Es = 29000 ksi

SECTION:
========
Ag = 320 in^2
Ix  = 10666.7 in^4
Iy  = 6826.67 in^4
Xo = 0 in
Yo = 0 in

REINFORCEMENT:
===============
10 #8 bars @ 2.469%
As = 7.9 in^2
Confinement: Other
Clear Cover  = N/A
Min Clear Spacing = 4.00 in

SLENDERNESS:
=============
N/A
```

Biaxial; ACI 318-14; Investigation

Figure 6 – Generating spColumn Model



Options

Reinforcement

● Bar size: #8

○ Bar area [in²]: 0.790

Cover (Longitudinal bars)

Cover [in]: 2.000

● Clear ○ To bar center

Number of bars

X direction: 5

Y direction: 5

Commands

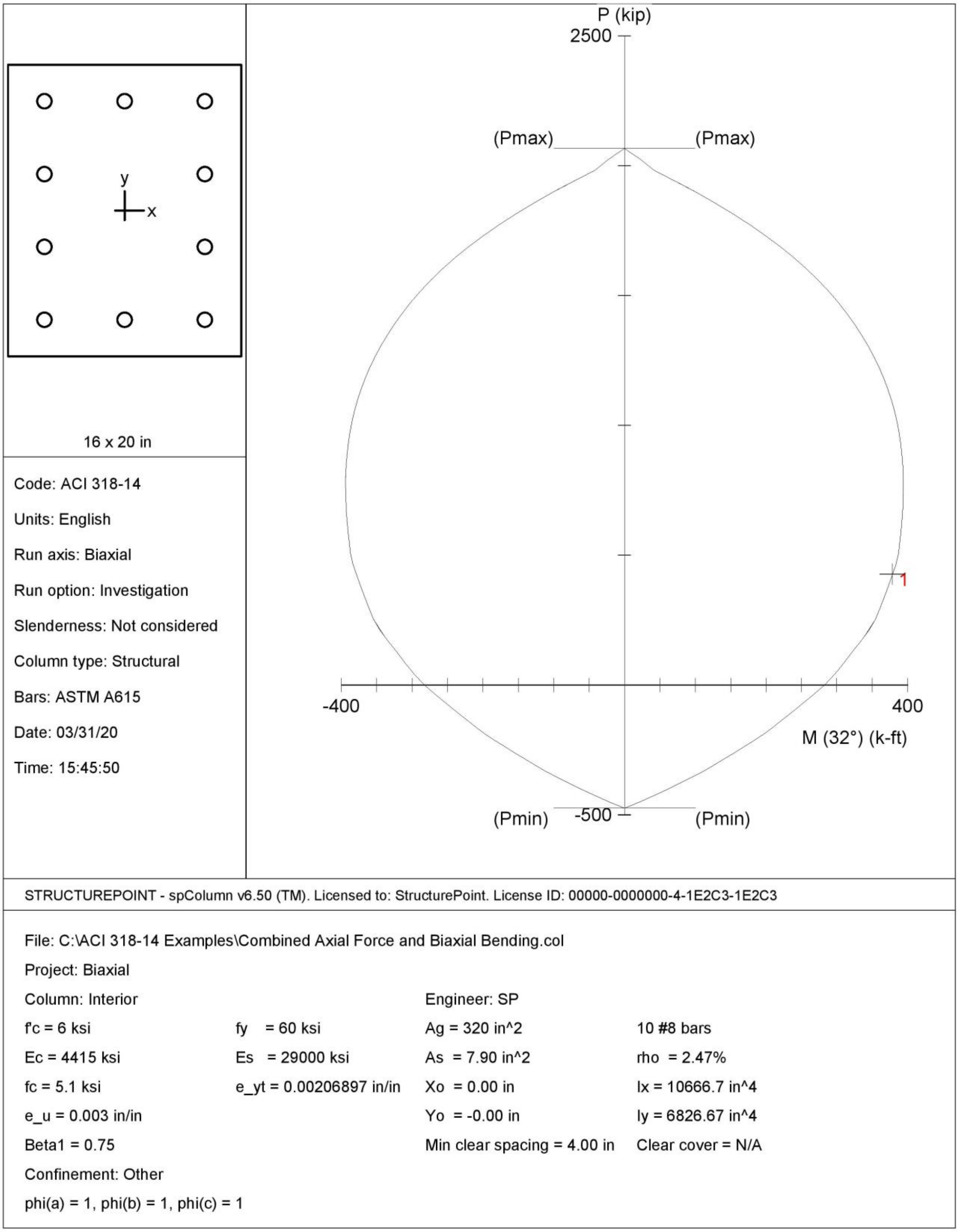
Start point

X [in]: 0.000

Y [in]: 0.000

Bars: ASTM A615 Rectangular bar pattern - Specify start point X = 0.000 Y = 0.000 Units: [in]

Figure 7 – spColumn Model Editor (spSection)



Code: ACI 318-14

Units: English

Run axis: Biaxial

Run option: Investigation

Slenderness: Not considered

Column type: Structural

Bars: ASTM A615

Date: 03/31/20

Time: 15:45:50

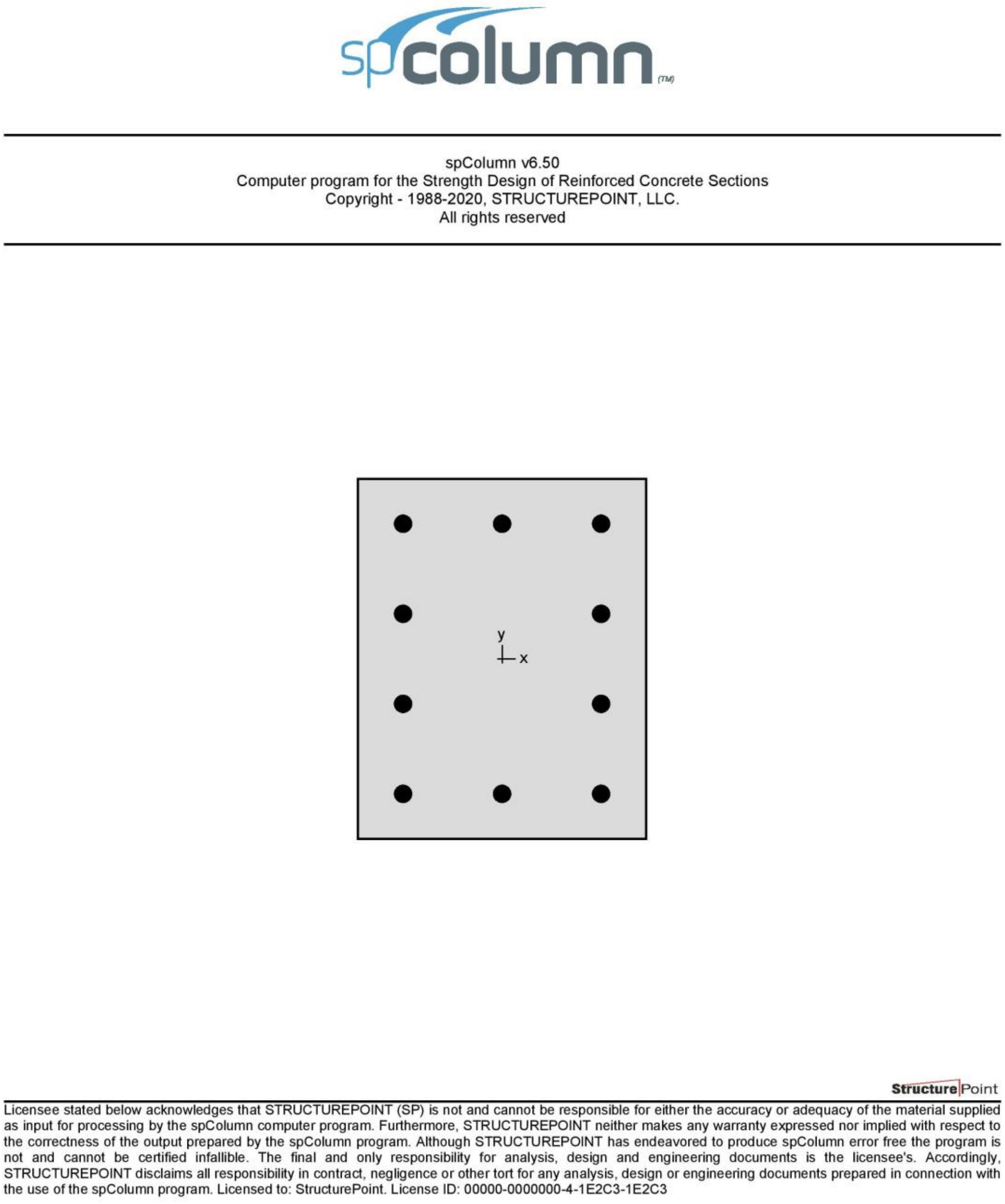
STRUCTUREPOINT - spColumn v6.50 (TM). Licensed to: StructurePoint. License ID: 00000-0000000-4-1E2C3-1E2C3

File: C:\ACI 318-14 Examples\Combined Axial Force and Biaxial Bending.col

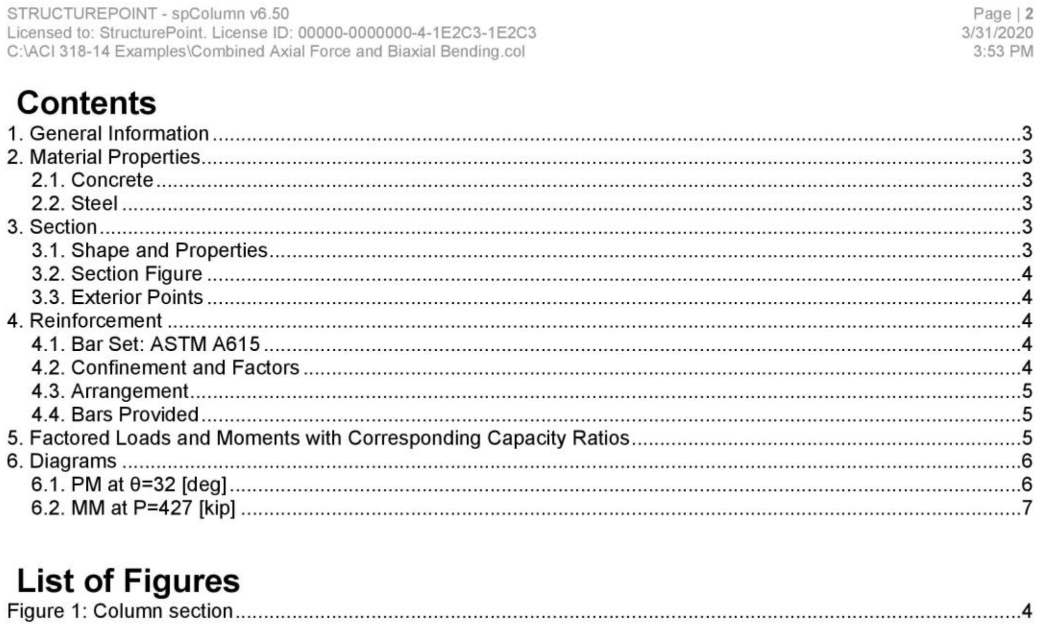
Project: Biaxial

|  |  |  |  |
|---|---|---|---|
| Column: Interior |  | Engineer: SP |  |
| f'c = 6 ksi | fy = 60 ksi | Ag = 320 in^2 | 10 #8 bars |
| Ec = 4415 ksi | Es = 29000 ksi | As = 7.90 in^2 | rho = 2.47% |
| fc = 5.1 ksi | e_yt = 0.00206897 in/in | Xo = 0.00 in | Ix = 10666.7 in^4 |
| e_u = 0.003 in/in |  | Yo = -0.00 in | Iy = 6826.67 in^4 |
| Beta1 = 0.75 |  | Min clear spacing = 4.00 in | Clear cover = N/A |
| Confinement: Other |  |  |  |
| phi(a) = 1, phi(b) = 1, phi(c) = 1 |  |  |  |

Figure 8 – Column Section Interaction Diagram at 32º (spColumn)



spColumn v6.50
Computer program for the Strength Design of Reinforced Concrete Sections



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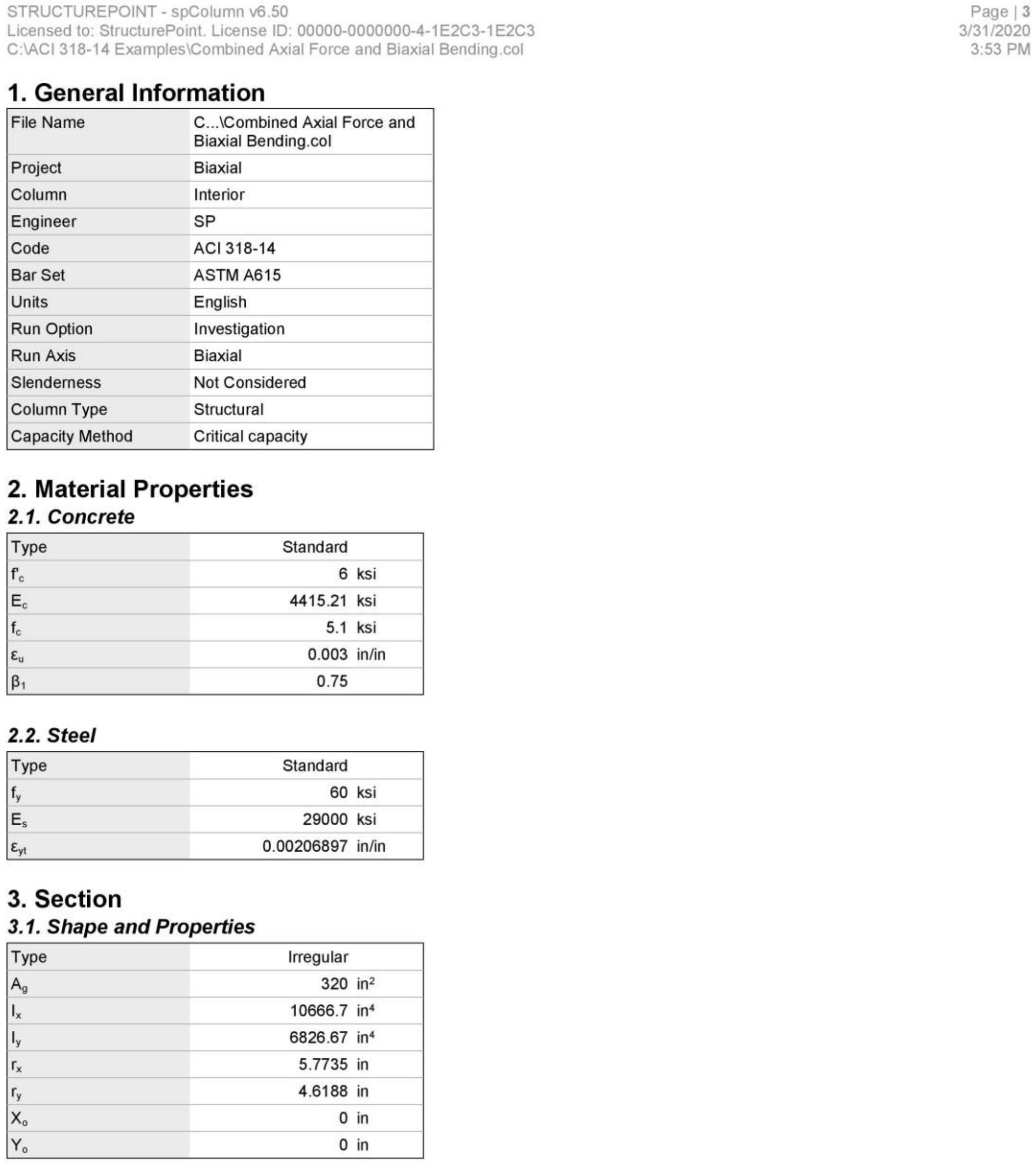
## Contents

## List of Figures



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# 1. General Information

| | |
|---|---|
| File Name | C...\Combined Axial Force and Biaxial Bending.col |
| Project | Biaxial |
| Column | Interior |
| Engineer | SP |
| Code | ACI 318-14 |
| Bar Set | ASTM A615 |
| Units | English |
| Run Option | Investigation |
| Run Axis | Biaxial |
| Slenderness | Not Considered |
| Column Type | Structural |
| Capacity Method | Critical capacity |

# 2. Material Properties

## 2.1. Concrete

| | | |
|---|---|---|
| Type | Standard | |
| $f'_c$ | 6 | ksi |
| $E_c$ | 4415.21 | ksi |
| $f_c$ | 5.1 | ksi |
| $\varepsilon_u$ | 0.003 | in/in |
| $\beta_1$ | 0.75 | |

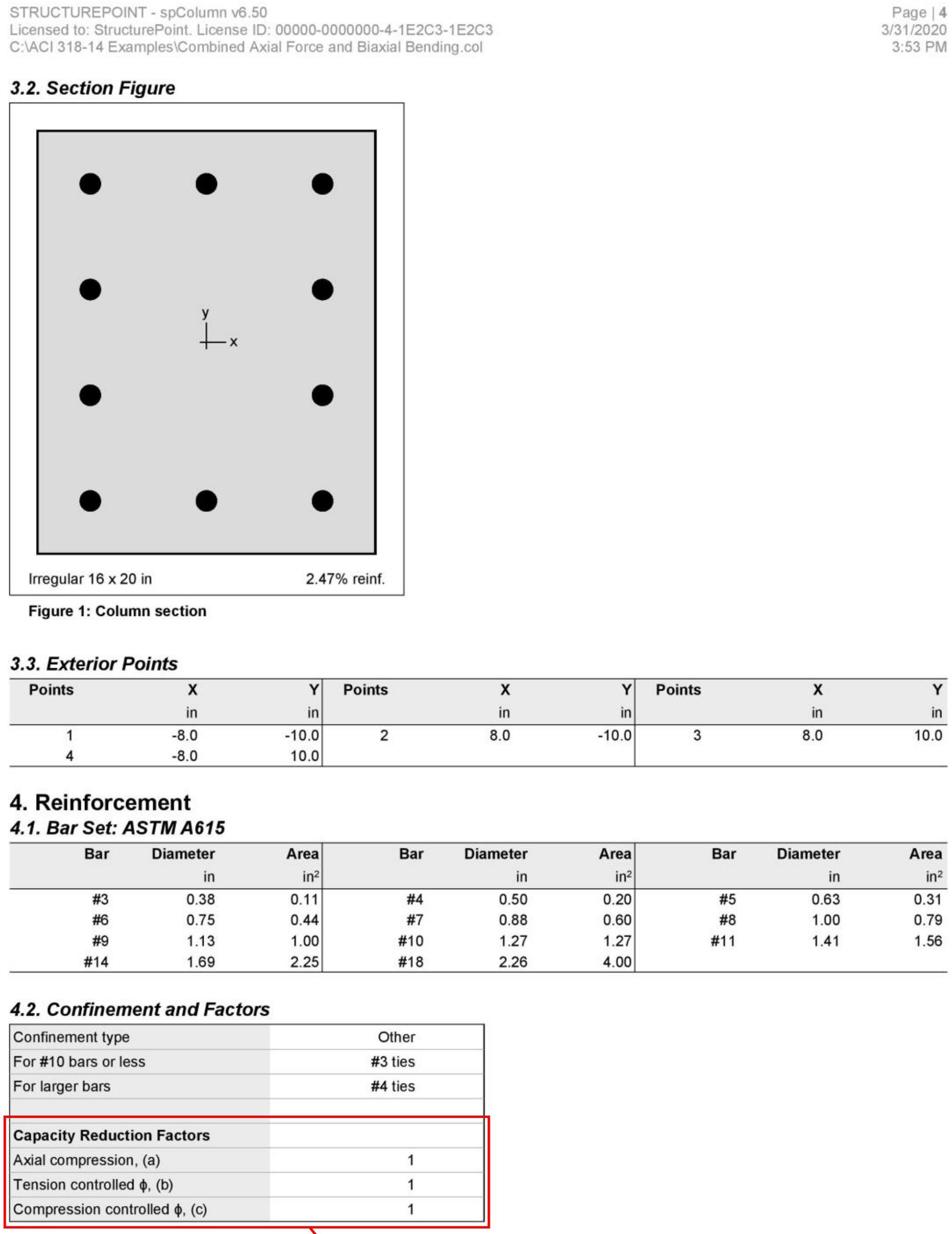
## 2.2. Steel

| | | |
|---|---|---|
| Type | Standard | |
| $f_y$ | 60 | ksi |
| $E_s$ | 29000 | ksi |
| $\varepsilon_{yt}$ | 0.00206897 | in/in |

# 3. Section

## 3.1. Shape and Properties

| | | |
|---|---|---|
| Type | Irregular | |
| $A_g$ | 320 | in² |
| $I_x$ | 10666.7 | in⁴ |
| $I_y$ | 6826.67 | in⁴ |
| $r_x$ | 5.7735 | in |
| $r_y$ | 4.6188 | in |
| $X_o$ | 0 | in |
| $Y_o$ | 0 | in |



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### 3.2. Section Figure

Irregular 16 x 20 in | 2.47% reinf.

**Figure 1: Column section**

### 3.3. Exterior Points

| Points | X | Y | Points | X | Y | Points | X | Y |
|---|---|---|---|---|---|---|---|---|
| | in | in | | in | in | | in | in |
| 1 | -8.0 | -10.0 | 2 | 8.0 | -10.0 | 3 | 8.0 | 10.0 |
| 4 | -8.0 | 10.0 | | | | | | |

## 4. Reinforcement

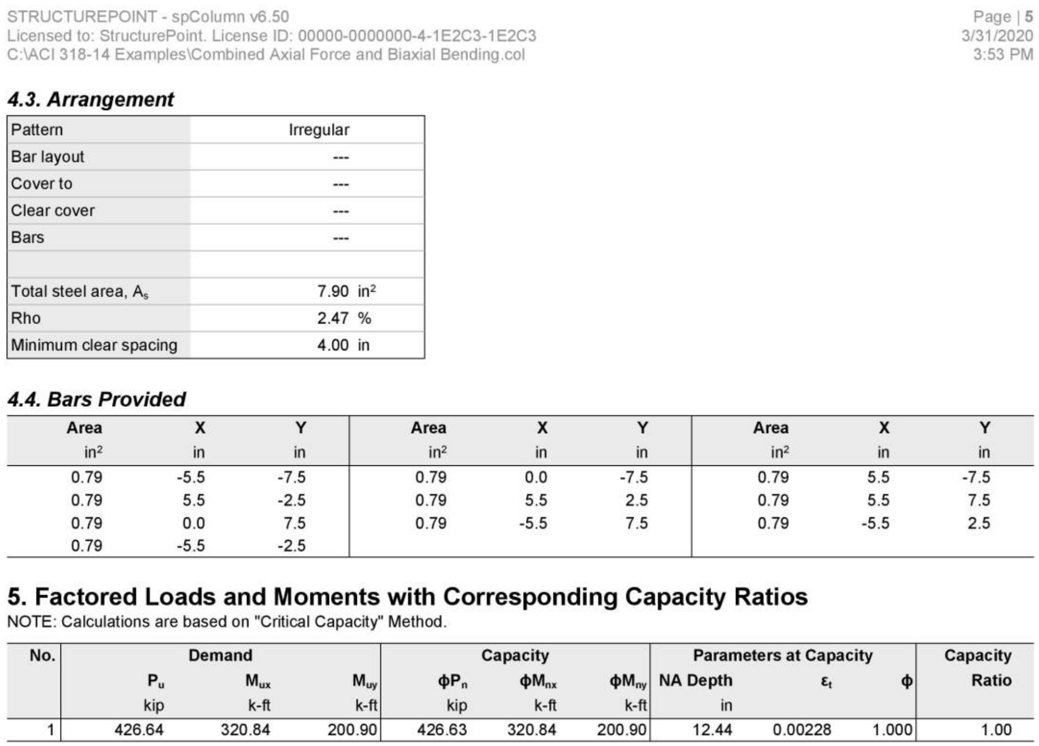
### 4.1. Bar Set: ASTM A615

| Bar | Diameter | Area | Bar | Diameter | Area | Bar | Diameter | Area |
|---|---|---|---|---|---|---|---|---|
| | in | in² | | in | in² | | in | in² |
| #3 | 0.38 | 0.11 | #4 | 0.50 | 0.20 | #5 | 0.63 | 0.31 |
| #6 | 0.75 | 0.44 | #7 | 0.88 | 0.60 | #8 | 1.00 | 0.79 |
| #9 | 1.13 | 1.00 | #10 | 1.27 | 1.27 | #11 | 1.41 | 1.56 |
| #14 | 1.69 | 2.25 | #18 | 2.26 | 4.00 | | | |

### 4.2. Confinement and Factors

| | |
|---|---|
| Confinement type | Other |
| For #10 bars or less | #3 ties |
| For larger bars | #4 ties |
| | |
| **Capacity Reduction Factors** | |
| Axial compression, (a) | 1 |
| Tension controlled ϕ, (b) | 1 |
| Compression controlled ϕ, (c) | 1 |

Capacity reduction factors set to 1.0 in spColumn input for illustration purposes



### *4.3. Arrangement*

| | |
|---|---|
| Pattern | Irregular |
| Bar layout | --- |
| Cover to | --- |
| Clear cover | --- |
| Bars | --- |
| | |
| Total steel area, $A_s$ | 7.90 in² |
| Rho | 2.47 % |
| Minimum clear spacing | 4.00 in |

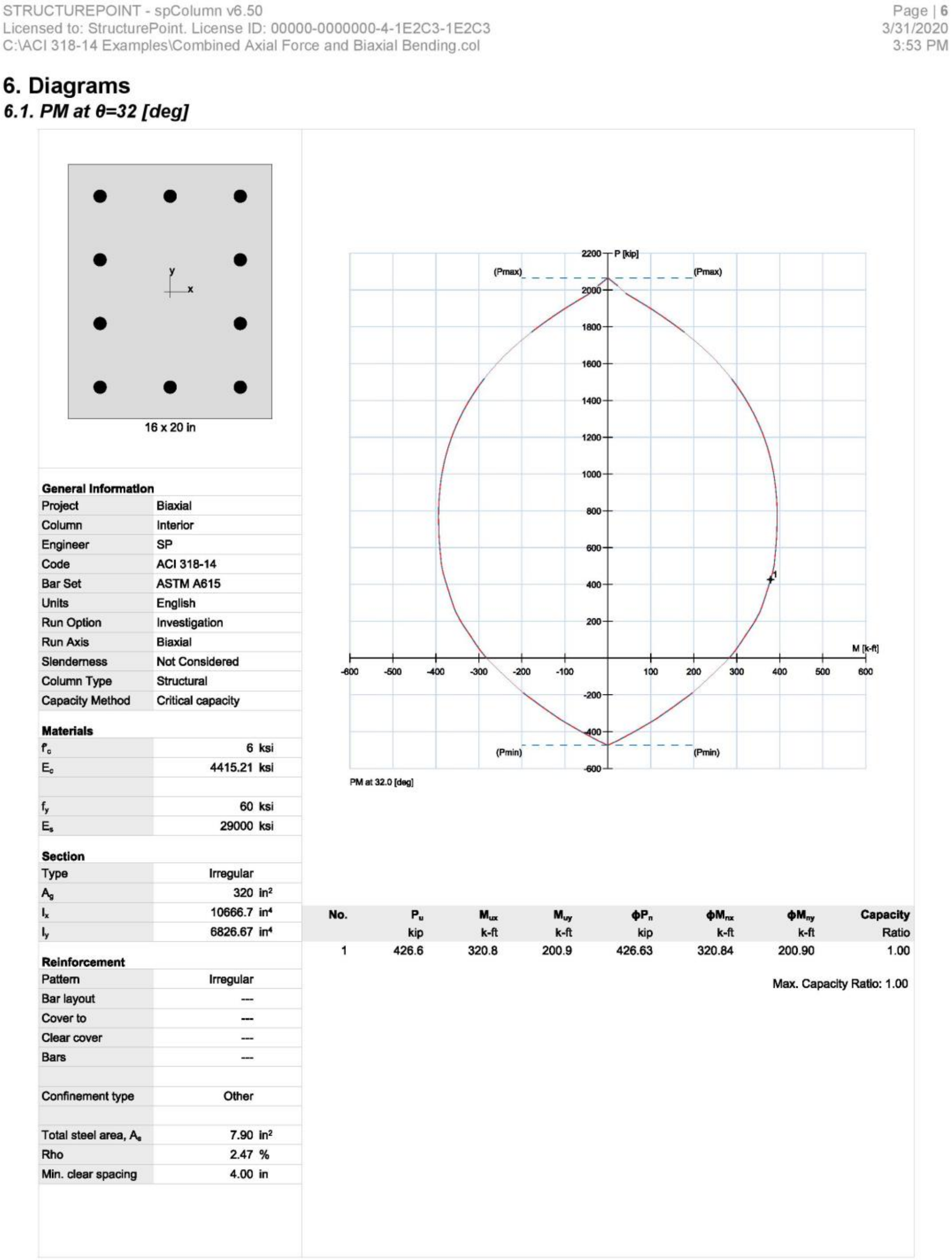
### *4.4. Bars Provided*

| Area | X | Y | Area | X | Y | Area | X | Y |
|---|---|---|---|---|---|---|---|---|
| in² | in | in | in² | in | in | in² | in | in |
| 0.79 | -5.5 | -7.5 | 0.79 | 0.0 | -7.5 | 0.79 | 5.5 | -7.5 |
| 0.79 | 5.5 | -2.5 | 0.79 | 5.5 | 2.5 | 0.79 | 5.5 | 7.5 |
| 0.79 | 0.0 | 7.5 | 0.79 | -5.5 | 7.5 | 0.79 | -5.5 | 2.5 |
| 0.79 | -5.5 | -2.5 | | | | | | |

## 5. Factored Loads and Moments with Corresponding Capacity Ratios

NOTE: Calculations are based on "Critical Capacity" Method.

| No. | Demand | | | Capacity | | | Parameters at Capacity | | | Capacity |
|---|---|---|---|---|---|---|---|---|---|---|
| | $P_u$ | $M_{ux}$ | $M_{uy}$ | $\phi P_n$ | $\phi M_{nx}$ | $\phi M_{ny}$ | NA Depth | $\varepsilon_t$ | $\phi$ | Ratio |
| | kip | k-ft | k-ft | kip | k-ft | k-ft | in | | | |
| 1 | 426.64 | 320.84 | 200.90 | 426.63 | 320.84 | 200.90 | 12.44 | 0.00228 | 1.000 | 1.00 |



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## 6. Diagrams

### *6.1. PM at θ=32 [deg]*

**General Information**

| | |
|---|---|
| Project | Biaxial |
| Column | Interior |
| Engineer | SP |
| Code | ACI 318-14 |
| Bar Set | ASTM A615 |
| Units | English |
| Run Option | Investigation |
| Run Axis | Biaxial |
| Slenderness | Not Considered |
| Column Type | Structural |
| Capacity Method | Critical capacity |

**Materials**

| | |
|---|---|
| $f'_c$ | 6 ksi |
| $E_c$ | 4415.21 ksi |
| | |
| $f_y$ | 60 ksi |
| $E_s$ | 29000 ksi |

**Section**

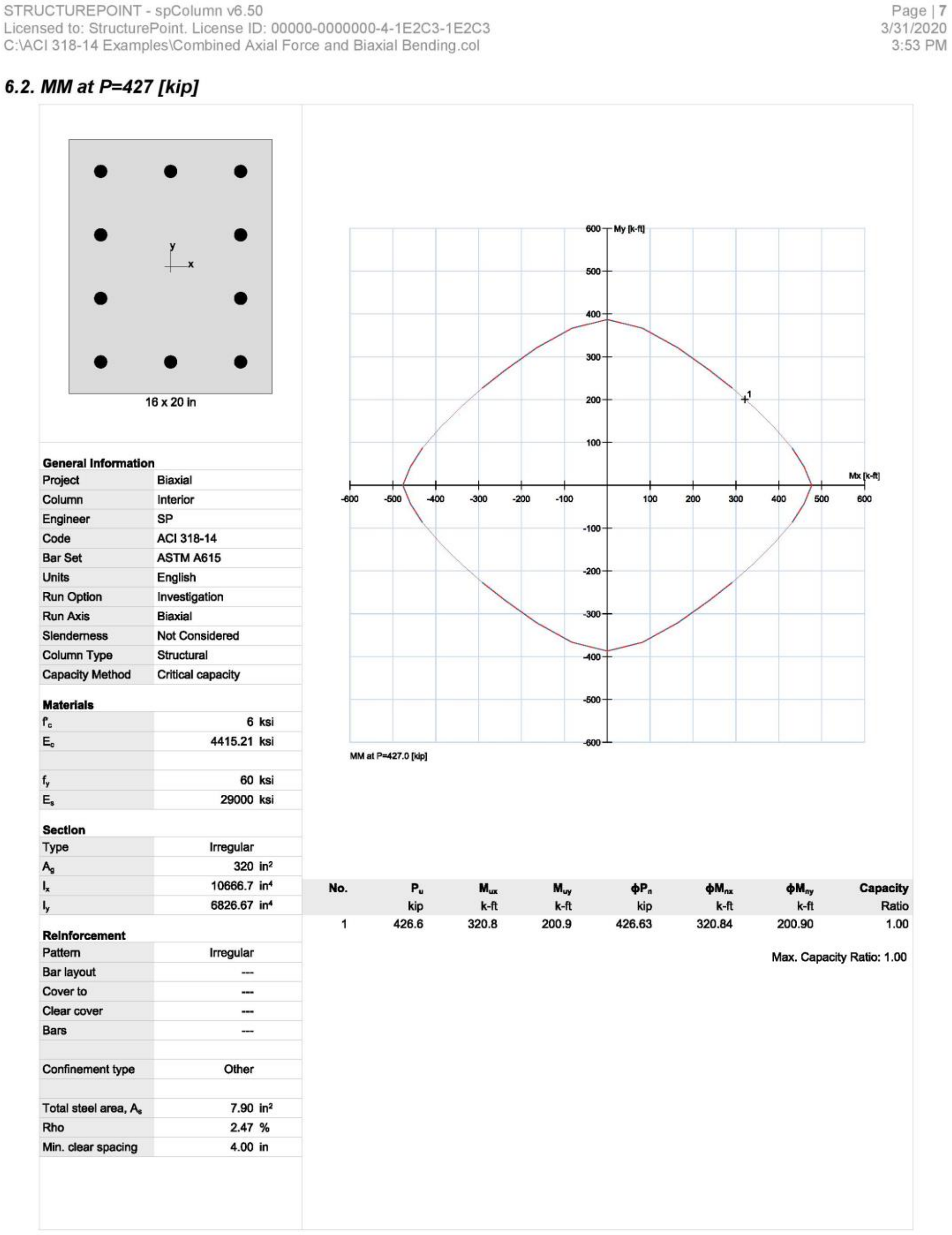
| | |
|---|---|
| Type | Irregular |
| $A_g$ | 320 in² |
| $I_x$ | 10666.7 in⁴ |
| $I_y$ | 6826.67 in⁴ |

**Reinforcement**

| | |
|---|---|
| Pattern | Irregular |
| Bar layout | --- |
| Cover to | --- |
| Clear cover | --- |
| Bars | --- |
| | |
| Confinement type | Other |
| | |
| Total steel area, $A_s$ | 7.90 in² |
| Rho | 2.47 % |
| Min. clear spacing | 4.00 in |

| No. | $P_u$ kip | $M_{ux}$ k-ft | $M_{uy}$ k-ft | $\phi P_n$ kip | $\phi M_{nx}$ k-ft | $\phi M_{ny}$ k-ft | Capacity Ratio |
|---|---|---|---|---|---|---|---|
| 1 | 426.6 | 320.8 | 200.9 | 426.63 | 320.84 | 200.90 | 1.00 |

Max. Capacity Ratio: 1.00



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### 6.2. MM at P=427 [kip]

**General Information**

| | |
|---|---|
| Project | Biaxial |
| Column | Interior |
| Engineer | SP |
| Code | ACI 318-14 |
| Bar Set | ASTM A615 |
| Units | English |
| Run Option | Investigation |
| Run Axis | Biaxial |
| Slenderness | Not Considered |
| Column Type | Structural |
| Capacity Method | Critical capacity |

**Materials**

| | |
|---|---|
| $f'_c$ | 6 ksi |
| $E_c$ | 4415.21 ksi |
| | |
| $f_y$ | 60 ksi |
| $E_s$ | 29000 ksi |

**Section**

| | |
|---|---|
| Type | Irregular |
| $A_g$ | 320 in² |
| $I_x$ | 10666.7 in⁴ |
| $I_y$ | 6826.67 in⁴ |

**Reinforcement**

| | |
|---|---|
| Pattern | Irregular |
| Bar layout | --- |
| Cover to | --- |
| Clear cover | --- |
| Bars | --- |
| | |
| Confinement type | Other |
| | |
| Total steel area, $A_s$ | 7.90 in² |
| Rho | 2.47 % |
| Min. clear spacing | 4.00 in |

| No. | $P_u$<br>kip | $M_{ux}$<br>k-ft | $M_{uy}$<br>k-ft | $\phi P_n$<br>kip | $\phi M_{nx}$<br>k-ft | $\phi M_{ny}$<br>k-ft | Capacity<br>Ratio |
|---|---|---|---|---|---|---|---|
| 1 | 426.6 | 320.8 | 200.9 | 426.63 | 320.84 | 200.90 | 1.00 |

Max. Capacity Ratio: 1.00

## 3. Summary and Comparison of Design Results

Table 2 - Comparison of Results

| Parameter | Reference | Hand | spColumn |
|---|---|---|---|
| c, in. | 12.50 | 12.50 | 12.50 |
| $d_1$, in. | 21.97 | 21.97 | 21.97 |
| $\varepsilon_{s1}$, in./in. | 0.00227 | 0.00227 | 0.00228 |
| $P_n$, kip | 426.0 | 425.4 | 426.6 |
| $M_{nx}$, kip-ft | 321.0 | 321.3 | 320.8 |
| $M_{ny}$, kip-ft | 201.0 | 200.6 | 200.9 |

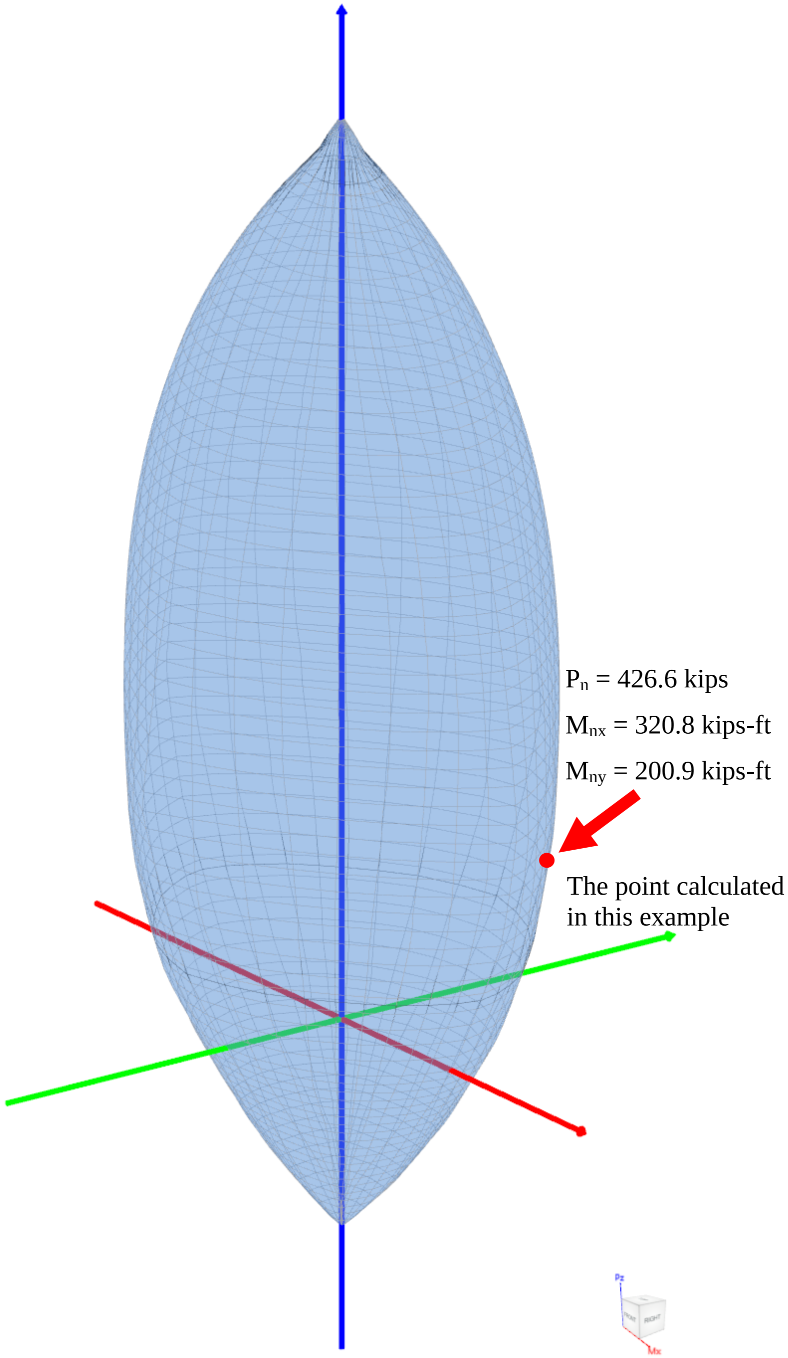
In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.

## 4. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In most building design calculations, such as the examples shown for flat plate or flat slab concrete floor systems, all building columns may be subjected to biaxial bending ($M_x$ and $M_{y)}$ due to lateral effects and unbalanced moments from both directions of analysis. This requires an investigation of the column P-$M_x$-$M_y$ interaction diagram in two directions simultaneously (axial force interaction with biaxial bending).

This example shows the calculations needed to obtain one point on the three-dimensional failure surface (biaxial $M_x$-$M_y$ interaction diagram). Generating the three-dimensional failure surface (interaction diagram) for a column section subjected to a combined axial force and biaxial bending moments is tedious and challenging for engineers and the use of a computer aid can save time and eliminate errors. StucturePoint's spColumn program can, quickly, simply and accurately generate the three-dimensional failure surface (interaction diagram) for all commonly encountered column, beam or wall sections in addition to highly complex and irregular cross-sections.



$P_n$ = 426.6 kips

$M_{nx}$ = 320.8 kips-ft

$M_{ny}$ = 200.9 kips-ft

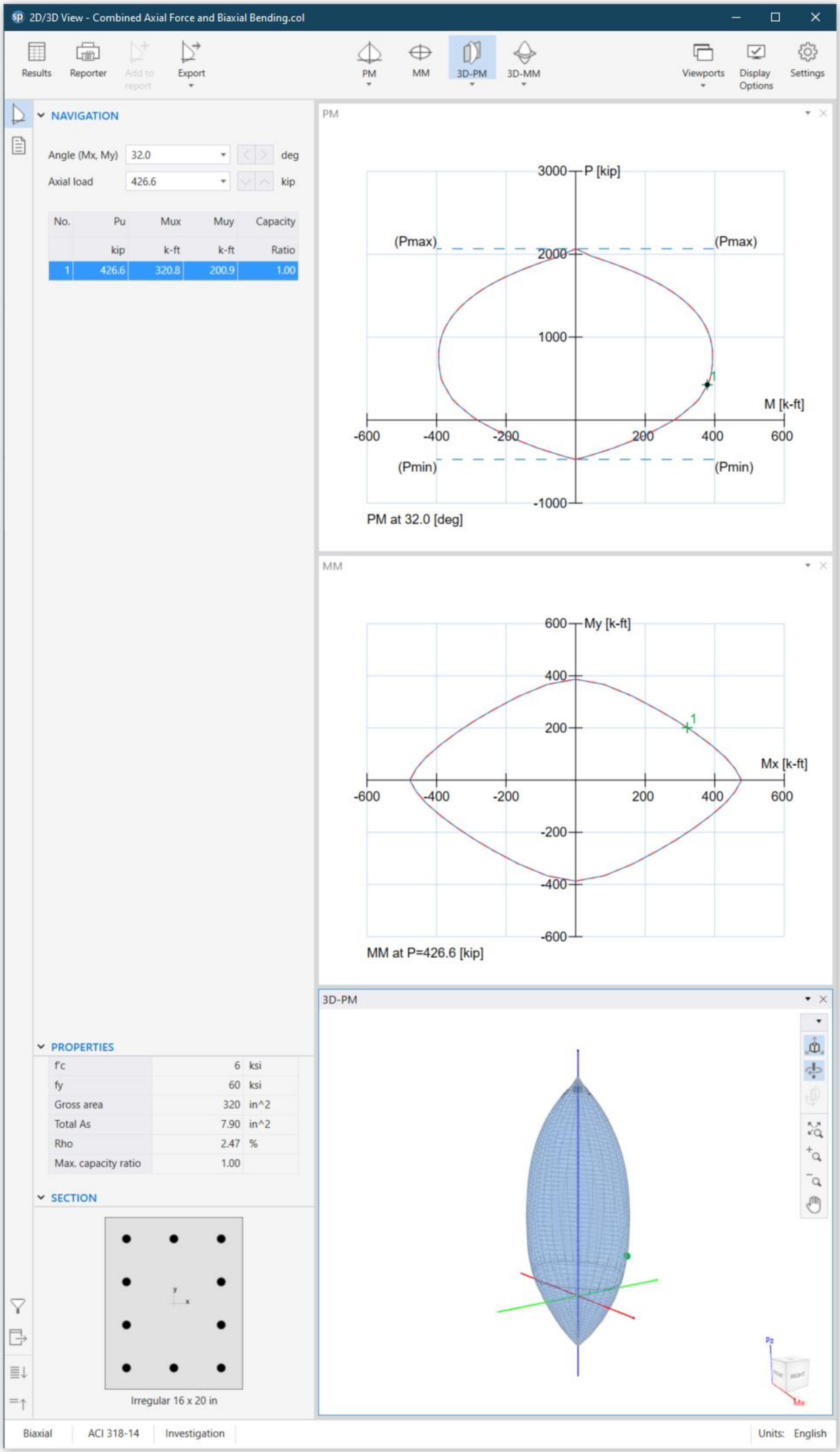
The point calculated in this example

Figure 9 – Nominal Interaction Diagram in Two Directions (Biaxial) (spColumn)

The spColumn 2D/3D viewer is a very powerful tool especially for investigating interaction diagrams (failure surfaces) for columns and walls sections subjected to a combined axial force and biaxial bending moments. The viewer allows the user to view and analyze 2D interaction diagrams and contours along with 3D failure surfaces in a multi viewport environment. The following figure shows three views of:

1. P-M interaction diagram cut at angle of 32º
2. Mx-My interaction diagram cut at axial load of 426 kip in compression
3. A 3D failure surface (interaction diagram showing the point calculated in this example.

Figures 12 and 13 show 3D visualization of failure surface with a horizontal and vertical plane cut, respectively.



2D/3D View - Combined Axial Force and Biaxial Bending.col

Angle (Mx, My) 32.0 deg

Axial load 426.6 kip

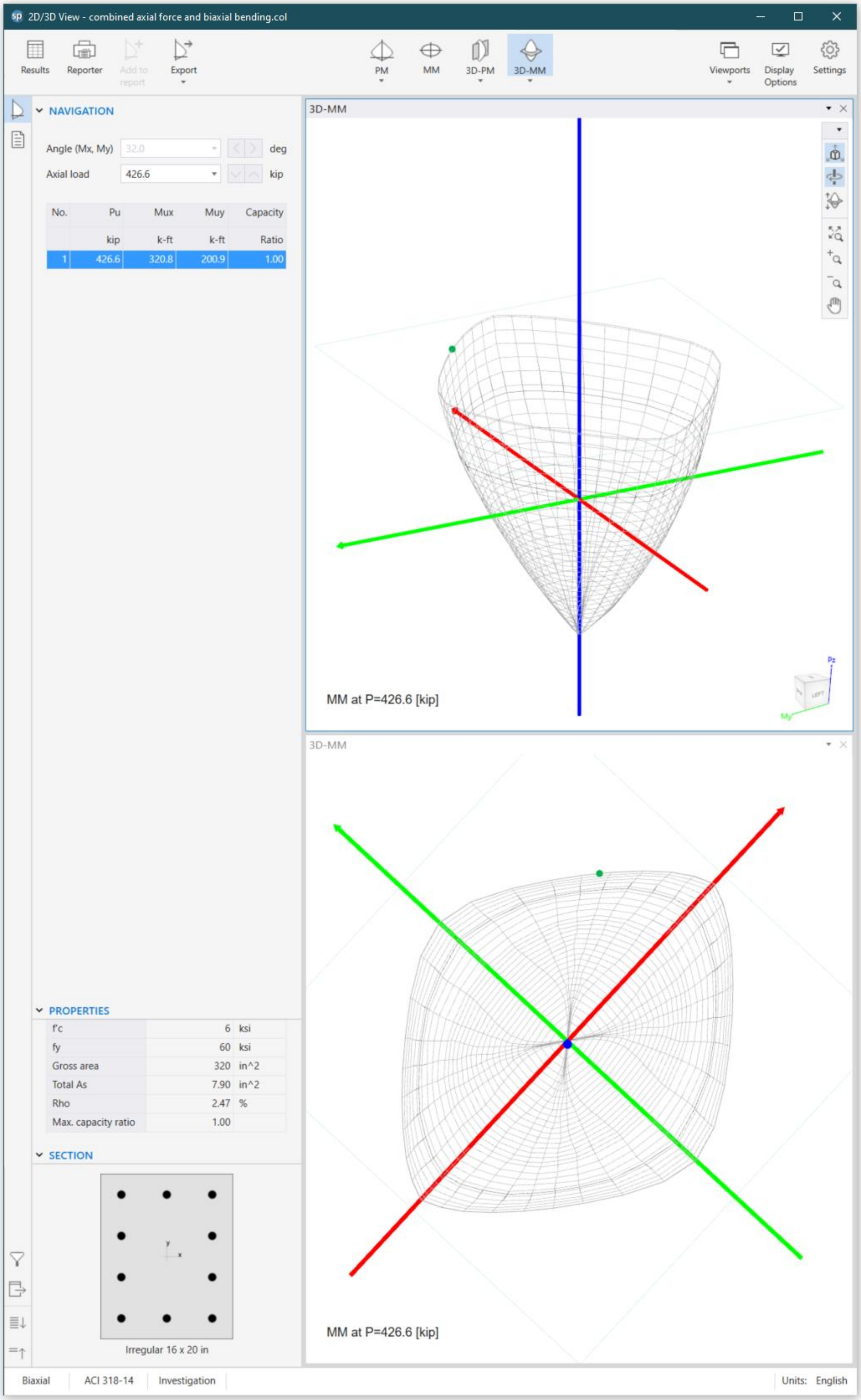
| No. | Pu<br>kip | Mux<br>k-ft | Muy<br>k-ft | Capacity<br>Ratio |
|---|---|---|---|---|
| 1 | 426.6 | 320.8 | 200.9 | 1.00 |

| | | |
|---|---|---|
| f'c | 6 | ksi |
| fy | 60 | ksi |
| Gross area | 320 | in^2 |
| Total As | 7.90 | in^2 |
| Rho | 2.47 | % |
| Max. capacity ratio | 1.00 | |

Irregular 16 x 20 in

Biaxial | ACI 318-14 | Investigation | Units: English

Figure 10 – 2D/3D Biaxial Interaction Diagram Viewer (spColumn)



2D/3D View - combined axial force and biaxial bending.col

NAVIGATION

Angle (Mx, My) 32.0 deg

Axial load 426.6 kip

| No. | Pu | Mux | Muy | Capacity |
|---|---|---|---|---|
| | kip | k-ft | k-ft | Ratio |
| 1 | 426.6 | 320.8 | 200.9 | 1.00 |

PROPERTIES

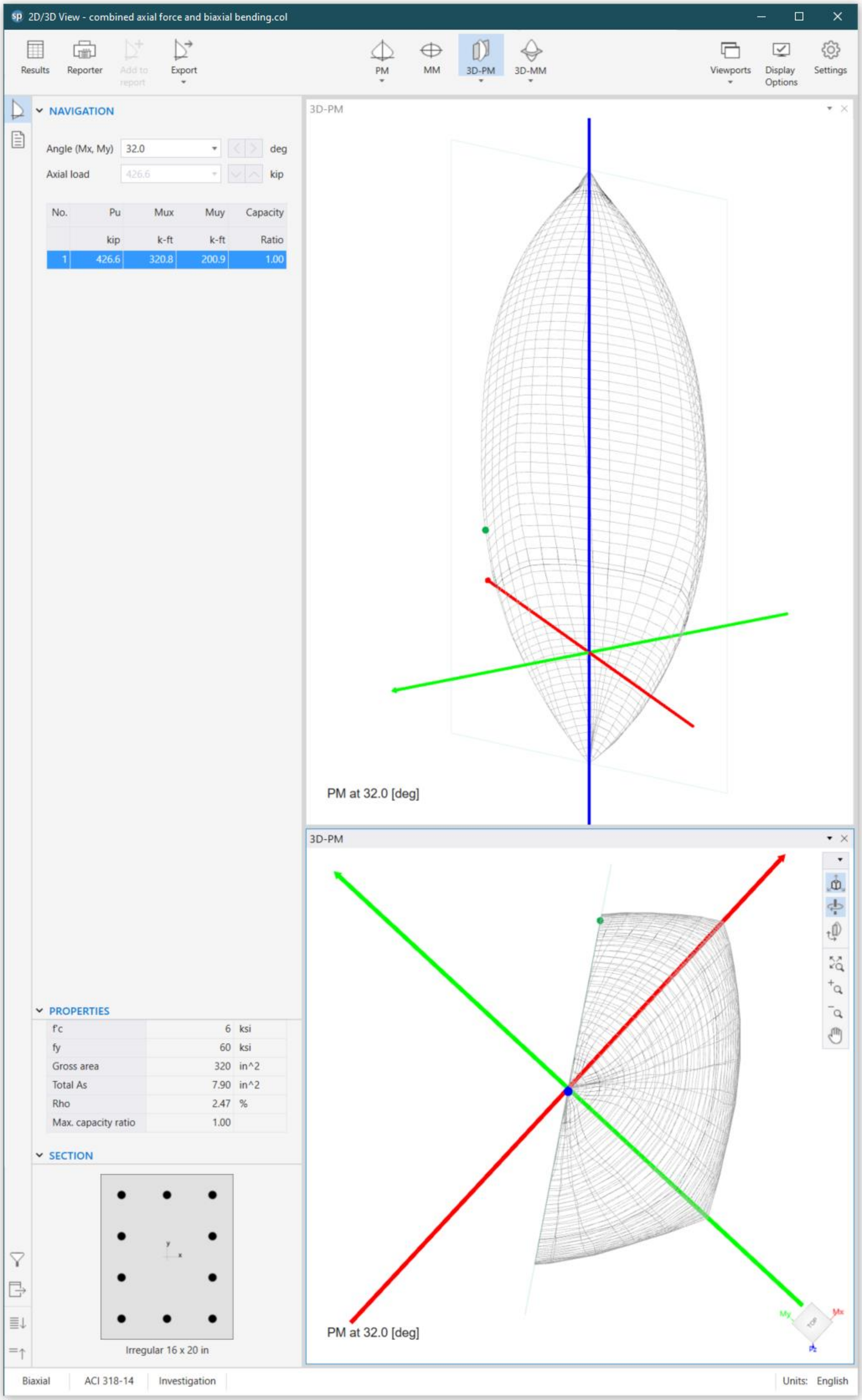
| | | |
|---|---|---|
| f'c | 6 | ksi |
| fy | 60 | ksi |
| Gross area | 320 | in^2 |
| Total As | 7.90 | in^2 |
| Rho | 2.47 | % |
| Max. capacity ratio | 1.00 | |

SECTION

Irregular 16 x 20 in

Biaxial | ACI 318-14 | Investigation | Units: English

Figure 11 – 3D Visualization of Failure Surface with a Horizontal Plane Cut (spColumn)



2D/3D View - combined axial force and biaxial bending.col

NAVIGATION

Angle (Mx, My) 32.0 deg

Axial load 426.6 kip

| No. | Pu | Mux | Muy | Capacity |
|---|---|---|---|---|
| | kip | k-ft | k-ft | Ratio |
| 1 | 426.6 | 320.8 | 200.9 | 1.00 |

PROPERTIES

| | | |
|---|---|---|
| f'c | 6 | ksi |
| fy | 60 | ksi |
| Gross area | 320 | in^2 |
| Total As | 7.90 | in^2 |
| Rho | 2.47 | % |
| Max. capacity ratio | 1.00 | |

SECTION

Irregular 16 x 20 in

Biaxial | ACI 318-14 | Investigation | Units: English

Figure 12 – 3D Visualization of Failure Surface with a Vertical Plane Cut (spColumn)