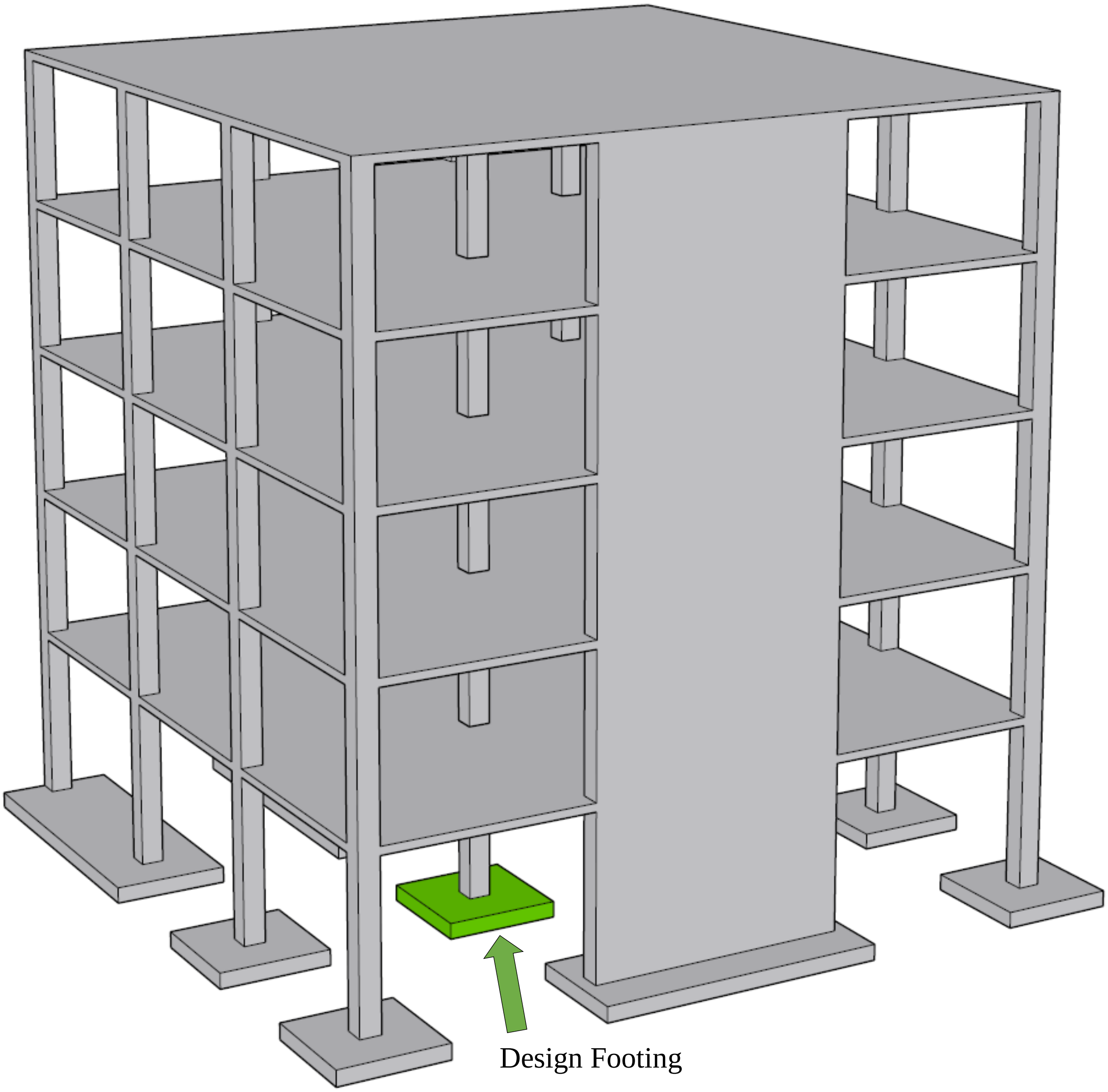
Reinforced Concrete Spread Footing (Isolated Footing) Analysis and Design



Reinforced Concrete Spread Footing (Isolated Footing) Analysis and Design

A square spread footing supports an 18 in. square column supporting a service dead load of 400 kips and a service live load of 270 kips. The column is built with 5000 psi concrete and has eight #9 Grade 60 longitudinal bars. Design a spread footing using 3000 psi normal weight concrete and Grade 60 bars. It is quite common for the strength of the concrete in the footing to be lower than that in the column. Dowels may be required to carry some of the column load across the column-footing interface. The top of the footing will be covered with 6 in. of fill with a density of 120 lb/ft^3 and a 6 in. basement floor. The basement floor loading is 100 psf. The allowable soil bearing pressure is 6000 psi. Using load resistance factors from ACI Code, the hand solution will be used for a comparison with the finite element analysis and design results of the engineering software program spMats.

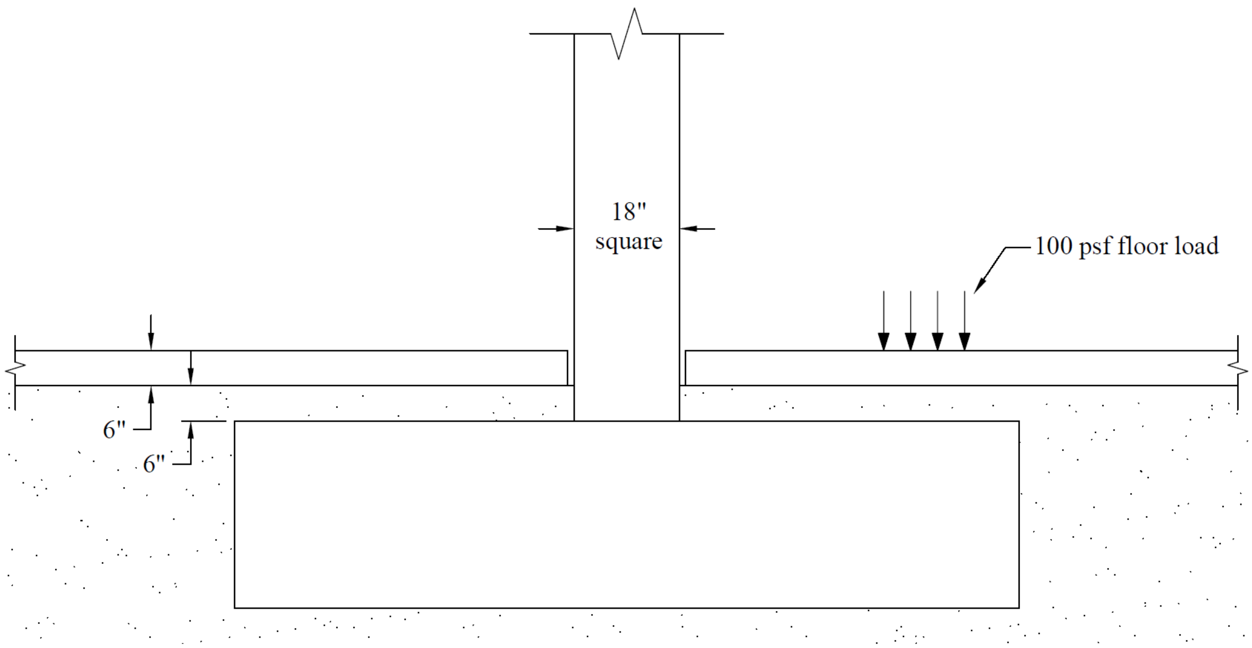


Figure 1 – Reinforced Concrete Spread Footing

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

Reinforced Concrete Mechanics and Design, 7th Edition, 2016, James Wight, Pearson, Example 15-2

spMats Engineering Software Program Manual v8.50, StucturePoint LLC., 2016

Design Data

For column

f_c' = 4,000 psi normal weight concrete

f_y = 60,000 psi (8 #9 longitudinal reinforcement)

For footing

f_c' = 3,000 psi normal weight concrete

f_y = 60,000 psi

For loading:

Dead load, D = 400 kips

Live load, L = 270 kips

Floor load, w_{floor} = 100 psf

For fill:

Depth = 6 in.

Density = 120 lb/ft^3

Allowable bearing pressure on the soil, $q_{allowable}$ = 6,000 psi

1. Loads and Load Combinations

The following load combinations are applicable for this example since dead and live load are only considered:

The total factored axial load on the column:

$$P_u = \text{Greater of} \begin{bmatrix} 1.4 \times P_D \\ 1.2 \times P_D + 1.6 \times P_L \end{bmatrix} = \text{Greater of} \begin{bmatrix} 1.4 \times 400 \\ 1.2 \times 400 + 1.6 \times 270 \end{bmatrix} = \text{Greater of} \begin{bmatrix} 560 \\ 912 \end{bmatrix} = 912 \text{ kips}$$

The strength reduction factors:

For flexure: ϕ_f = 0.65-0.90 (function of the extreme-tension layer of bars strain) ***ACI 318-14 (21.2.1)***

For shear: ϕ_v = 0.75 ***ACI 318-14 (21.2.1)***

2. Foundation Shear Strength and Thickness

2.1. Preliminary Foundation Sizing

Assume footing thickness, h = 32 in.

The net soil pressure is calculated as follows:

$$q_n = q_{allowable} - weight_{footing} - weight_{fill} - weight_{basement\ floor} - weight_{floor\ load}$$

$$q_n = 6000 - \frac{32}{12} \times 150 - \frac{6}{12} \times 120 - \frac{6}{12} \times 150 - 100 = 5370 \text{ psf}$$

$$A_{g,required} = \frac{P_{service}}{q_n} = \frac{400 + 270}{5370/1000} = 125 \text{ ft}^2 = 11.18 \text{ ft} \times 11.18 \text{ ft}$$

Try 11 ft 2 in. square by 32 in. thick.

The factored net soil pressure is calculated as follows:

$$q_{n,u} = \frac{P_u}{A_g} = \frac{912}{11.17^2} = 7310 \text{ psf}$$

This value will be used for the following shear and flexural strength design calculations and to arrive at the minimum required footing thickness.

2.2. Two-Way Shear Strength

The thickness of a spread footing is commonly governed by two-way shear strength. The average depth shall be the average of the effective depths in the two orthogonal directions. ***ACI 318-14 (22.6.2.1)***

Assuming a bar size of #8, the average depth can be found as follows:

$$d_{avg}=\frac{\left(h-\text{cover}-\frac{d_b}{2}\right)+\left(h-\text{cover}-d_b-\frac{d_b}{2}\right)}{2}=\frac{\left(32-3-\frac{1}{2}\right)+\left(32-3-1-\frac{1}{2}\right)}{2}=28\text{ in.}$$

$$V_u=q_{n,u}\times\text{Tributary Area for Shear}=\frac{7310}{1000}\times\left(11.17^2-\left(\frac{46}{12}\right)^2\right)=805\text{ kips}$$

$$v_u=\frac{V_u}{b_o\times d}=\frac{805}{(4\times 46)\times 28}=0.156\text{ ksi}=156\text{ psi}$$

Where b_o is the perimeter of critical section for two-way shear in footings.

The design shear strength for interior square column:

$$\phi v_c=\text{Least of}\begin{bmatrix}\phi\times 4\times\lambda\times\sqrt{f_c^{'}}\\ \phi\times\left(2+\frac{4}{\beta}\right)\times\lambda\times\sqrt{f_c^{'}}\\ \phi\times\left(2+\frac{\alpha_s\times d}{b_o}\right)\times\lambda\times\sqrt{f_c^{'}}\end{bmatrix}$$ ***ACI 318-14 (22.6.5.2)***

$$\phi v_c=\text{Least of}\begin{bmatrix}0.75\times 4\times 1\times\sqrt{3000}\\ 0.75\times\left(2+\frac{4}{1}\right)\times 1\times\sqrt{3000}\\ 0.75\times\left(2+\frac{40\times 28}{4\times 46}\right)\times 1\times\sqrt{3000}\end{bmatrix}=\text{Least of}\begin{bmatrix}164\\246\\332\end{bmatrix}\text{psi}=164\text{ psi}>v_u\rightarrow o.k.$$

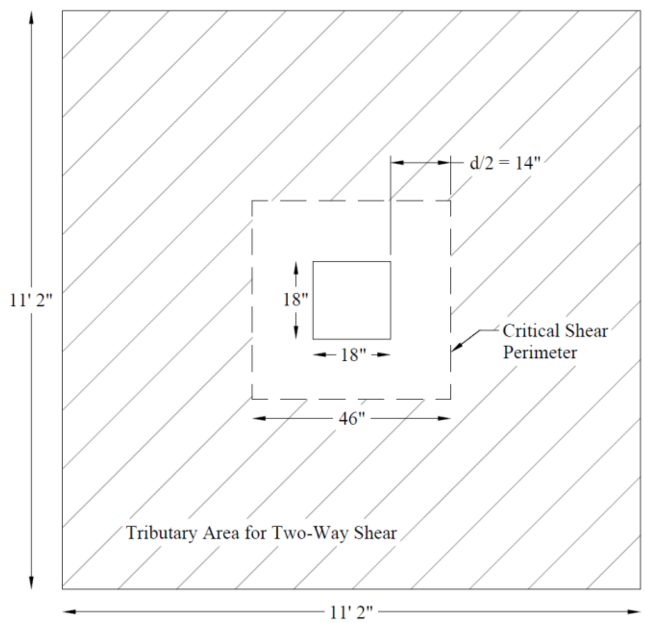


Figure 2 – Critical Section for Two-Way Shear

2.3. One-Way Shear Strength

One-way shear check is performed even though it is seldom critical.

$$V_u = q_{n,u} \times \text{Tributary Area for Shear} = \frac{7310}{1000} \times \left(11.17 \times \frac{30}{12}\right) = 204 \text{ kips}$$

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f_c^{'}} \times b_w \times d$$ ***ACI 318-14 (22.5.5.1)***

$$\phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{3000} \times (11.17 \times 12) \times 28 / 1000 = 308 \text{ kips}$$

$V_u < \phi V_c$ → o.k.

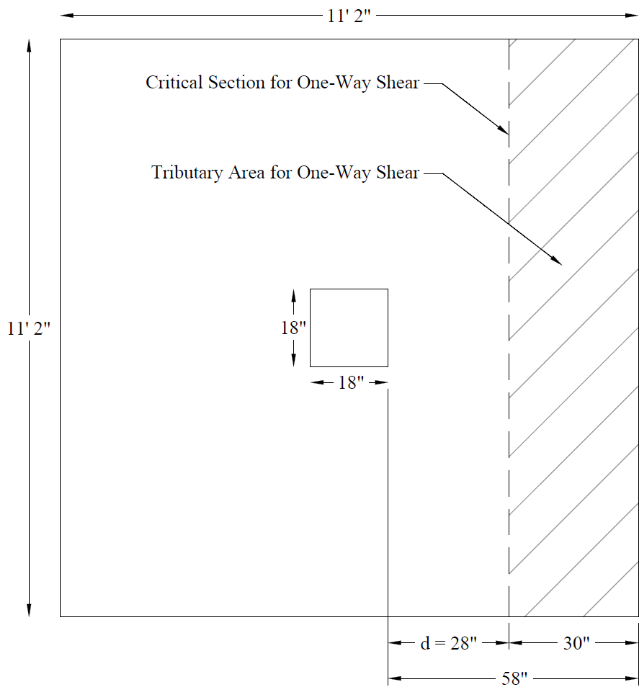


Figure 3 – Critical Section for One-Way Shear

3. Footing Flexural Strength and Reinforcement

The factored moment at the critical section (at the face of the column) is calculated as follows:

$$M_u = \frac{7310}{1000} \times \left(11.17 \times \frac{(58/12)^2}{2}\right) = 954 \text{ kips-ft}$$

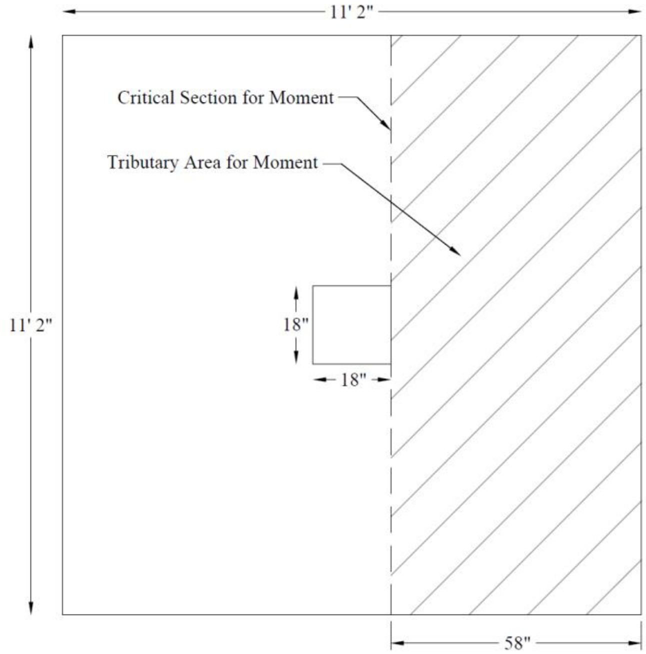


Figure 4 – Critical Section for Moment

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the footing section (*jd*). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and *jd* will be taken equal to 0.976*d*. The assumptions will be verified once the area of steel in finalized.

Assume $jd = 0.976 \times d = 27.3$ in.

$$A_s = \frac{M_u}{\varphi f_y jd} = \frac{954 \times 12000}{0.9 \times 60000 \times 27.3} = 7.76 \text{ in.}^2$$

Recalculate $'a'$ for the actual $A_s = 7.76 \text{ in.}^2 \rightarrow a = \dfrac{A_s f_y}{0.85 f'_c b} = \dfrac{7.76 \times 60000}{0.85 \times 3000 \times (11.17 \times 12)} = 1.36$ in.

$$c = \frac{a}{\beta_1} = \frac{1.36}{0.85} = 1.60 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003 = \left(\frac{0.003}{1.60}\right) \times 28 - 0.003 = 0.049 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{954 \times 12000}{0.9 \times 60000 \times (28 - 1.36/2)} = 7.76 \text{ in.}^2$$

$$A_{s,\min} = \text{Greater of} \begin{Bmatrix} \dfrac{0.0018 \times 60,000}{f} \\ 0.0014 \end{Bmatrix} \times b \times h$$ ***ACI 318-14 (7.6.1.1)***

$$A_{s,\min} = 0.0018 \times (11.17 \times 12) \times 32 = 7.72 \text{ in.}^2 < A_s$$

$$\therefore \; A_s = 7.76 \text{ in.}^2$$

$$s_{\max} = \text{ lesser of } \begin{Bmatrix} 3h \\ 18 \text{ in.} \end{Bmatrix} = \text{lesser of } \begin{Bmatrix} 3 \times 32 = 96 \text{ in.} \\ 18 \text{ in.} \end{Bmatrix} = 18 \text{ in.}$$ ***ACI 318-14 (7.7.2.3)***

Providing 11#8 bars with A_s = 8.69 in.^{2} satisfies strength requirements. Since the footing is square and d_{avg} is used for the flexural design in the x-direction. The same design (11#8) is used in the y-direction.

4. Reinforcement Bar Development Length

Flexural reinforcement must be properly developed in a concrete foundation in order for the foundation to perform as intended in accordance with the strength design method. The concept of the development length is stated as follows: minimum lengths of reinforcement must be provided beyond the locations of peak stress (critical sections) in the reinforcement in order to fully develop the bars.

Clear spacing between bars being developed = 18 in. – 1 in. = 17 in. > 2d_b = 2 in.

Clear cover = 3 in. > d_b = 1 in.

Thus, Case 1 in Table 25.4.2.2 from ACI 318-14 can be used as follows:

$$l_d = \left(\frac{1}{20} \times \frac{f_y}{\lambda \sqrt{f_c'}} \times \Psi_t \Psi_e \right) \times d_b \rightarrow \text{for No.7 and larger bars}$$ ***ACI 318-14 (Table 25.4.2.2)***

$$l_d = \left(\frac{1}{20} \times \frac{60,000}{1.0 \times \sqrt{3,000}} \times 1.0 \times 1.0 \right) \times 1.0 = 54.8 \text{ in.}$$

Where:

$\lambda = 1.0$ (Light weight modification factor: normal weight concrete) ***ACI 318-14 (Table 25.4.2.4)***

$\Psi_t = 1.0$ (Casting position modification factor: less than 12 in. of fresh concrete placed below horizontal reinforcement) ***ACI 318-14 (Table 25.4.2.4)***

$\Psi_e = 1.0$ (Epoxy modification factor: uncoated or zinc-coated reinforcement) ***ACI 318-14 (Table 25.4.2.4)***

The provided bar length is equal to:

$$l_{d,provided} = \frac{11.17 \times 12}{2} - \frac{18}{2} - 3 = 55 \text{ in.} \geq l_d = 54.8 \text{ in.} \rightarrow \underline{o.k.}$$

5. Column-Footing Joint Design

5.1. Maximum Bearing Load - Top of Footing

$$\phi B_{n,\,footing} = \text{Lesser of} \begin{bmatrix} (a)\ \phi \times 0.85 \times f_c^{'} \times A_1 \times \sqrt{A_2 / A_1} \\ (b)\ \phi \times 0.85 \times f_c^{'} \times A_1 \times 2 \end{bmatrix}$$

ACI 318-14 (Table 22.8.3.2)

$$\phi B_{n,\,footing} = \text{Lesser of} \begin{bmatrix} (a)\ 0.65 \times 0.85 \times 3,000 \times 18^2 \times \sqrt{(11.17 \times 12)^2 / 18^2} \\ (b)\ 65 \times 0.85 \times 3,000 \times 18^2 \times 2 \end{bmatrix}$$

$$\phi B_{n,\,footing} = \text{Lesser of} \begin{bmatrix} (a)\ 4000 \text{ kips} \\ (b)\ 1070 \text{ kips} \end{bmatrix} = 1070 \text{ kips} > P_u = 912 \text{ kips} \rightarrow o.k.$$

Where:

$\phi = 0.65$ for bearing ***ACI 318-14 (Table 21.2.1)***

A_1 = Loaded area for consideration of bearing strength.

A_2 = Area of the lower base of a right pyramid or cone formed by extending lines out from the sides of the bearing area at a slope of 2 horizontal to 1 vertical to the point where the first such line intersects an edge.

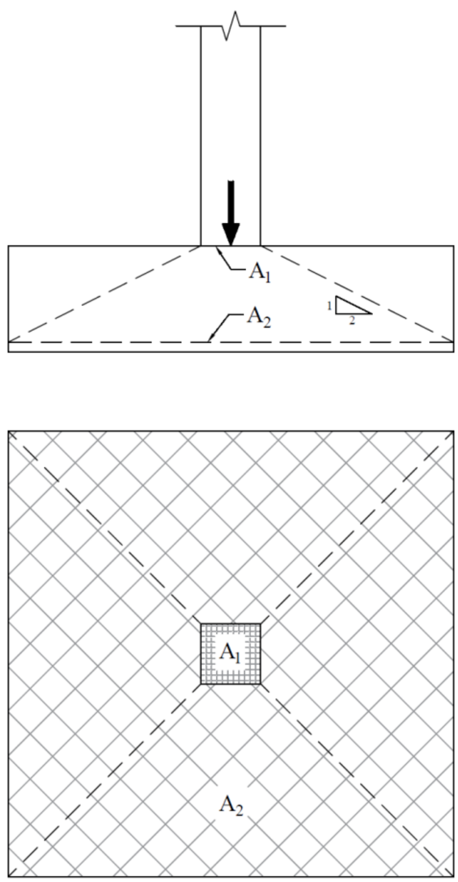


Figure 5 – Definition of A_1 and A_2

5.2. Allowable Bearing Load - Base of Column

$$\phi B_{n,column} = \phi \times 0.85 \times f_c^{'} \times A_1$$ **_ACI 318-14 (Table 22.8.3.2(c))_**

$$\phi B_{n,column} = 0.65 \times 0.85 \times 3,000 \times 18^2 = 895 \text{ kips} < P_u \rightarrow \text{Dowels are needed to transfer the excess load}$$

Where:

$\phi = 0.65$ for compression-controlled tied columns **_ACI 318-14 (Table 21.2.2)_**

$$A_{s,dowels\ required} = \frac{P_u - \phi B_{n,column}}{\phi\ f_y} = \frac{912 - 895}{0.65 \times\ 60,000} = 0.44 \text{ in.}^2$$

$$A_{s,dowels\ required} = 0.44 \text{ in.}^2 < 0.005 \times A_g = 0.005 \times 18^2 = 1.62 \text{ in.}^2$$ **_ACI 318-14 (16.3.4.1)_**

Thus, $A_{s,dowels\ required} = 1.62 \text{ in.}^2$

Providing 4#6 dowels with A_s = 1.76 in.^{2}, dowel each corner bar. The dowel must extend into the footing a distance equal to the compression-development length for a #6 bar in 3000 psi concrete, or 16 in. the bars will be extended down to the level of the main footing steel and hooked 90^o. The hooks will be tied to the main steel to hold the dowels in place. From ACI code section 25.5.5.4, the dowels must extend into the column a distance equal to the greater of a compression splice for the dowels (23 in.) or the compression-development length of the #9 column bars for f_c' = 5000 psi (20 in.). More information about this procedure can be found in the reference.

Use four #6 dowels, dowel each corner bar. Extend dowels 23 in. into column.

6. Spread Footing Analysis and Design – spMats Software

spMats uses the Finite Element Method for the structural modeling, analysis, and design of reinforced concrete slab systems or mat foundations subject to static loading conditions.

The slab, mat, or footing is idealized as a mesh of rectangular elements interconnected at the corner nodes. The same mesh applies to the underlying soil with the soil stiffness concentrated at the nodes. Slabs of irregular geometry can be idealized to conform to geometry with rectangular boundaries. Even though slab and soil properties can vary between elements, they are assumed uniform within each element.

For illustration and comparison purposes, the following figures provide a sample of the input modules and results obtained from an spMats model created for the reinforced concrete spread footing in this example.

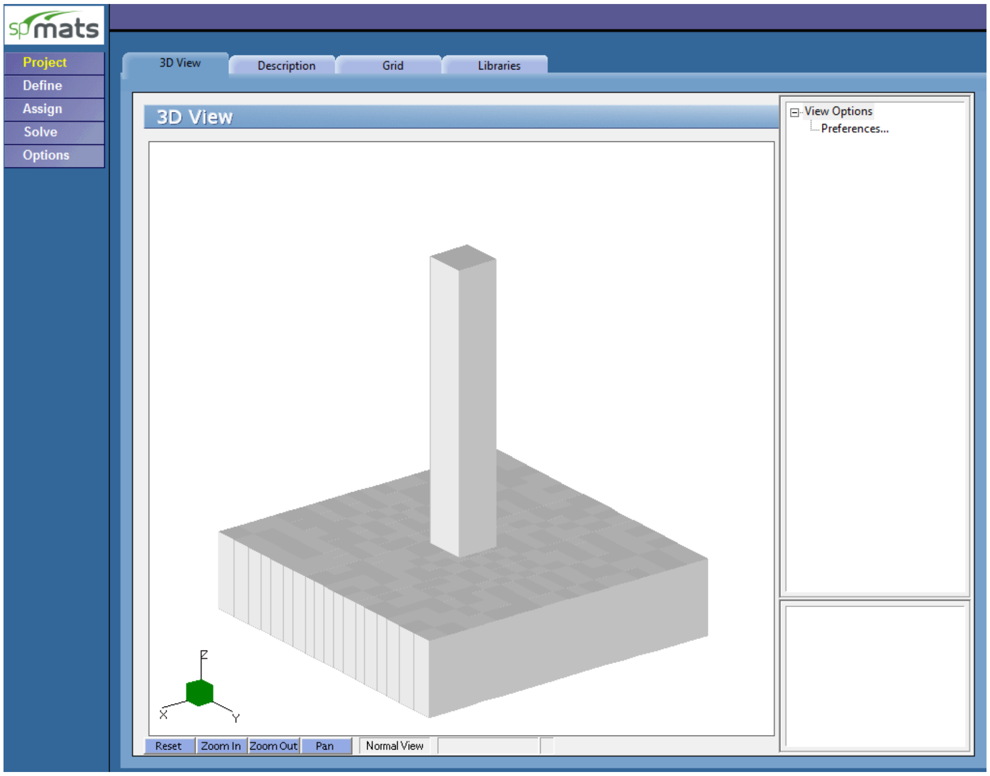


Figure 6 – 3D View for Spread Footing Foundation Model (spMats)

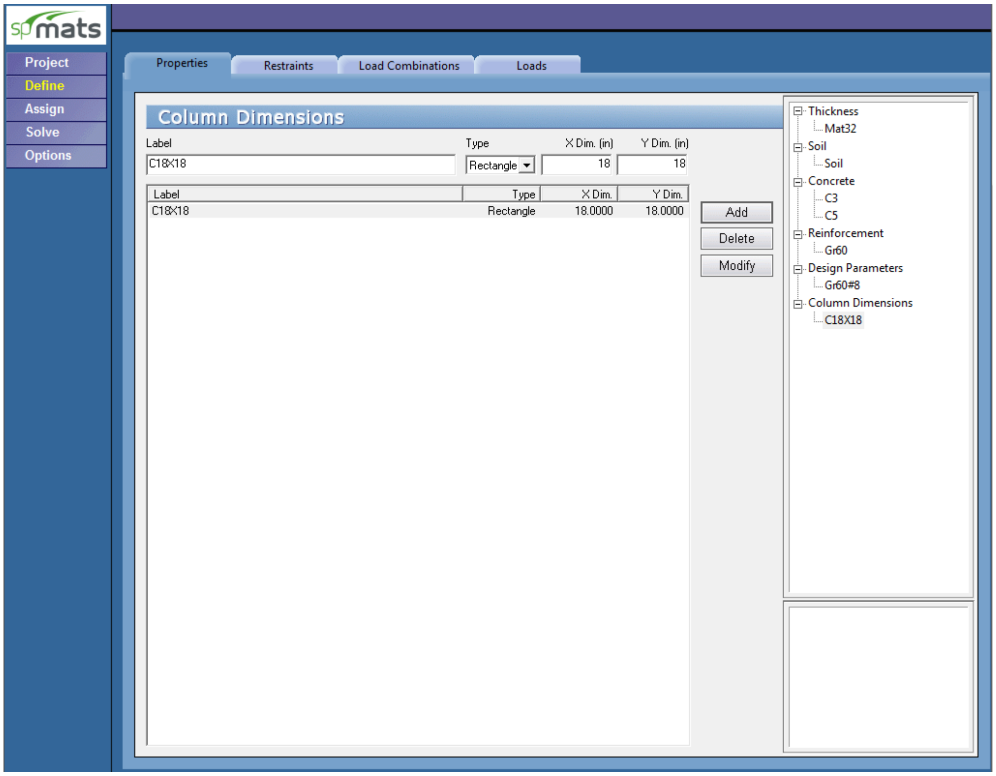


Figure 7 –Defining Column (spMats)

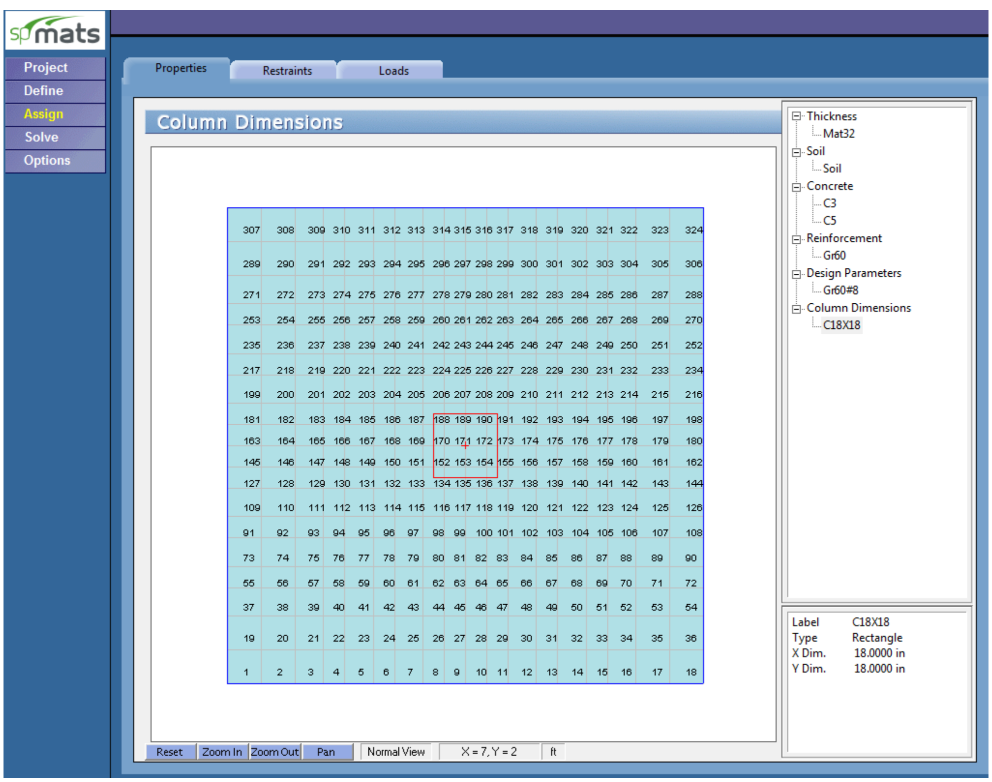


Figure 8 – Assigning Column (spMats)

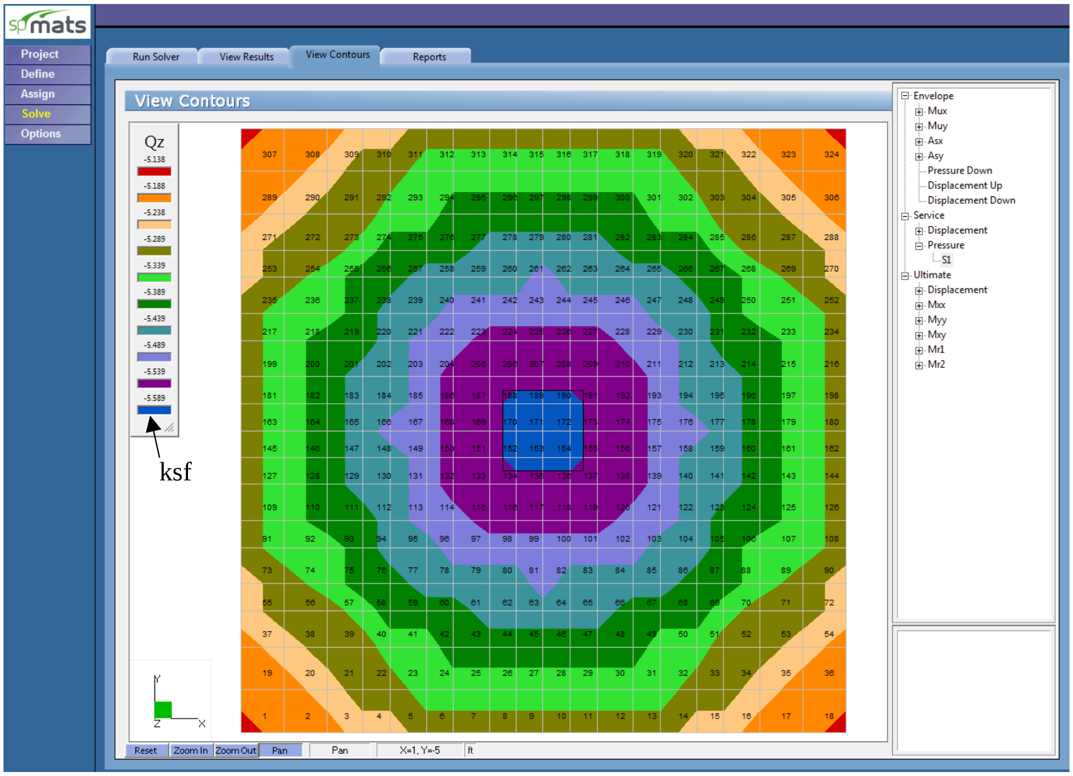


Figure 9 – Footing Soil Pressure Contour (spMats)

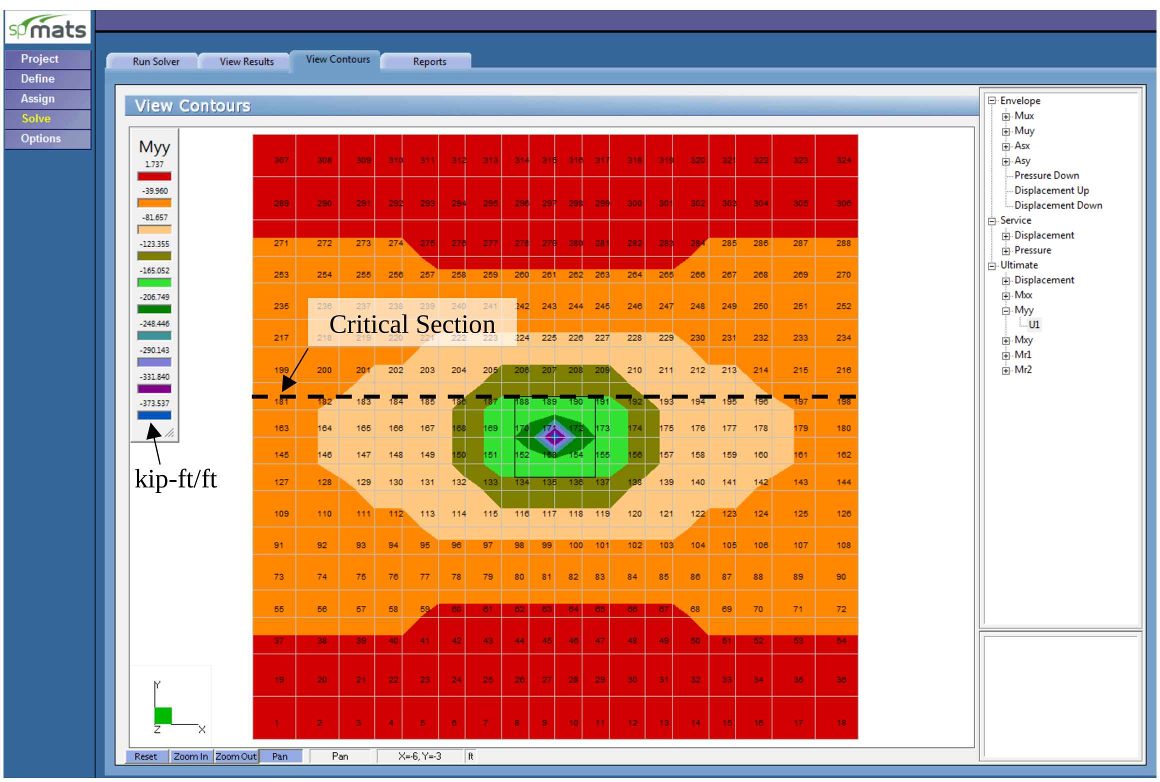


Figure 10 – Footing Moment Contour along Y-Axis (spMats)

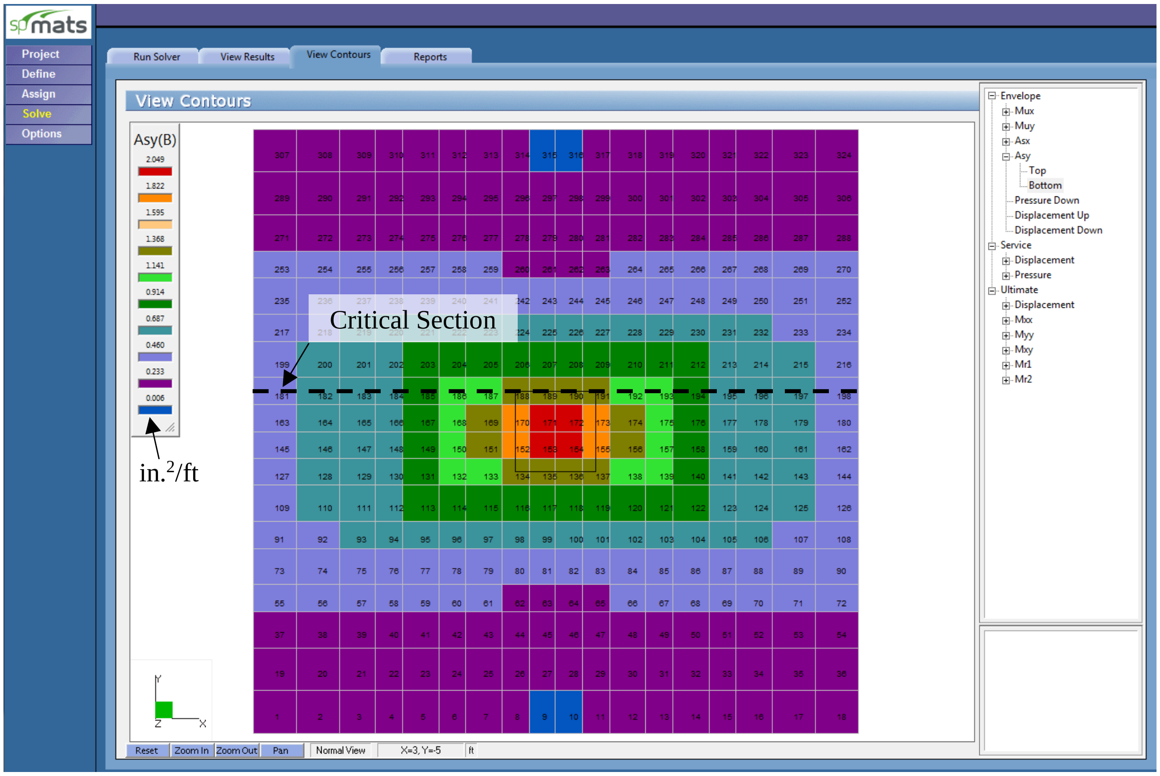


Figure 11 – Footing Required Reinforcement along Y-Axis (spMats)

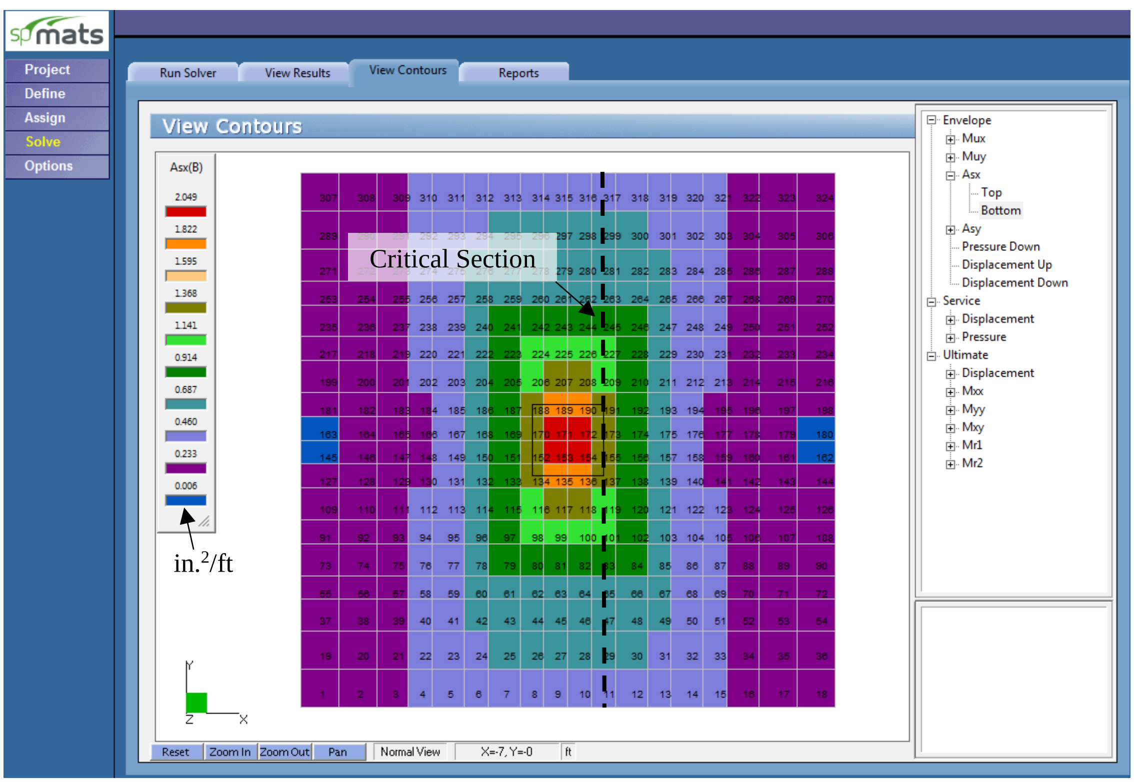


Figure 12 – Footing Required Reinforcement along X-Axis (spMats)

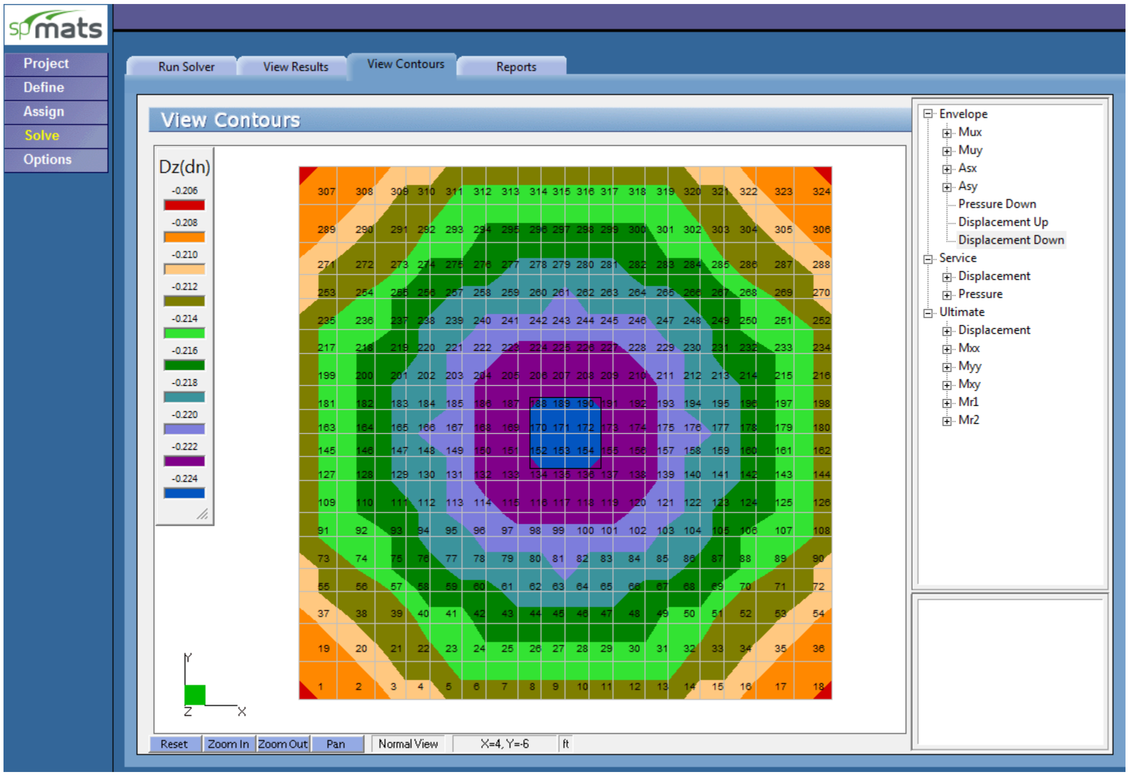


Figure 13 – Footing Vertical Displacement Contour (spMats)

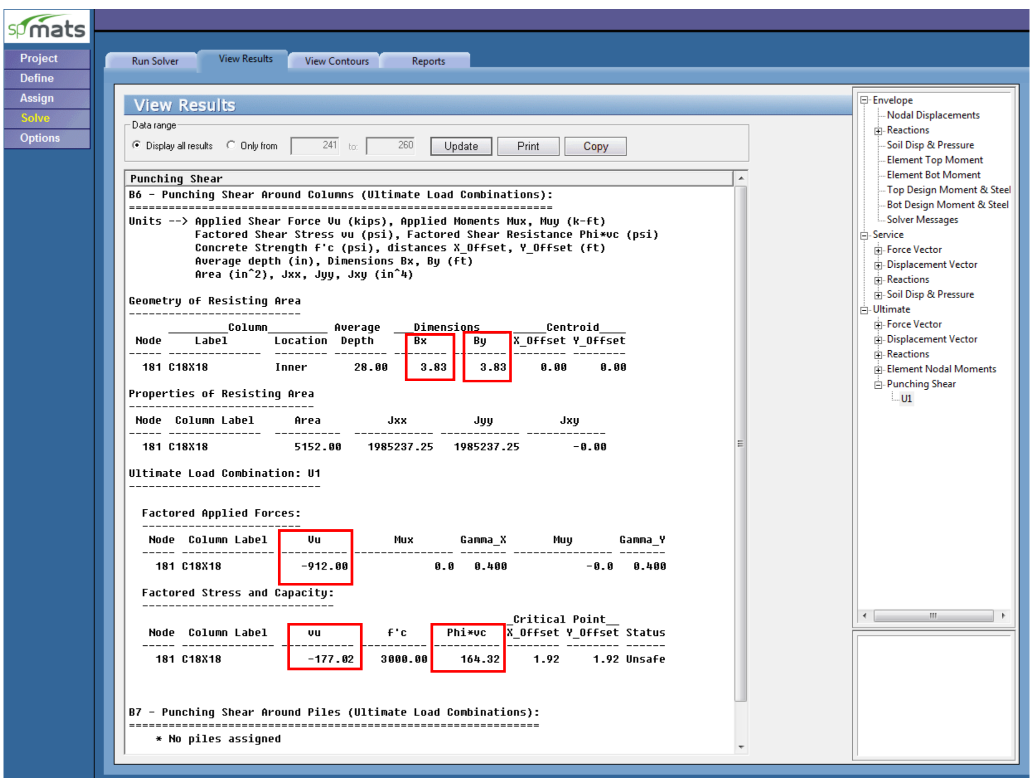


Figure 14 – Two-Way (Punching) Shear Check around the Column (spMats)

7. Design Results Comparison and Conclusions

Table 1 – Comparison of Pile Cap Analysis and Design Results (Flexural)						
Solution	Qz, ksf	v_u*, psi	φv_c, psi	M_{yy}, (kips-ft)	M_u†, (kips-ft)	$A_{s,required}$, (in.^{2})
Hand	5.37	156	164	954	954	7.76
Reference	5.37	156	164	954	954	7.97
spMats	5.39	177	164	947	1,084	8.89

* spMats conservatively consider the entire footing cross-sectional area as tributary area for two-way shear
† spMats consider two-way action in calculating the design moment and required area of steel

The results of all the hand calculations and the reference used illustrated above are in good agreement with the automated results obtained from the spMats FEA except for two-way factored shear stress and required area of steel.

In practice, flexural reinforcement is generally provided in the orthogonal directions of the footing system and not in the principal directions. Therefore, the Principal of Minimum Resistance is used by spMats to obtain values for the design moments (M_{ux} or M_{uy}), which include the effects of the twisting moment (M_{xy}) in addition to the bending moment (M_{xx} or M_{yy}) as shown in the following figure.

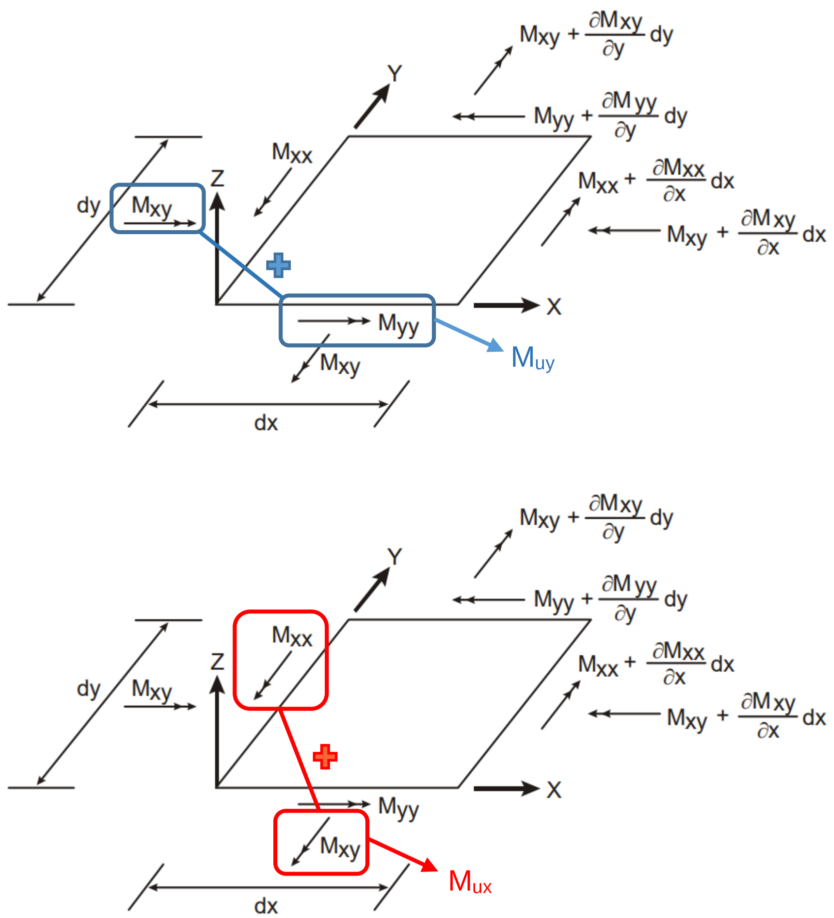


Figure 15 – Element Nodal and Design Moments (spMats)