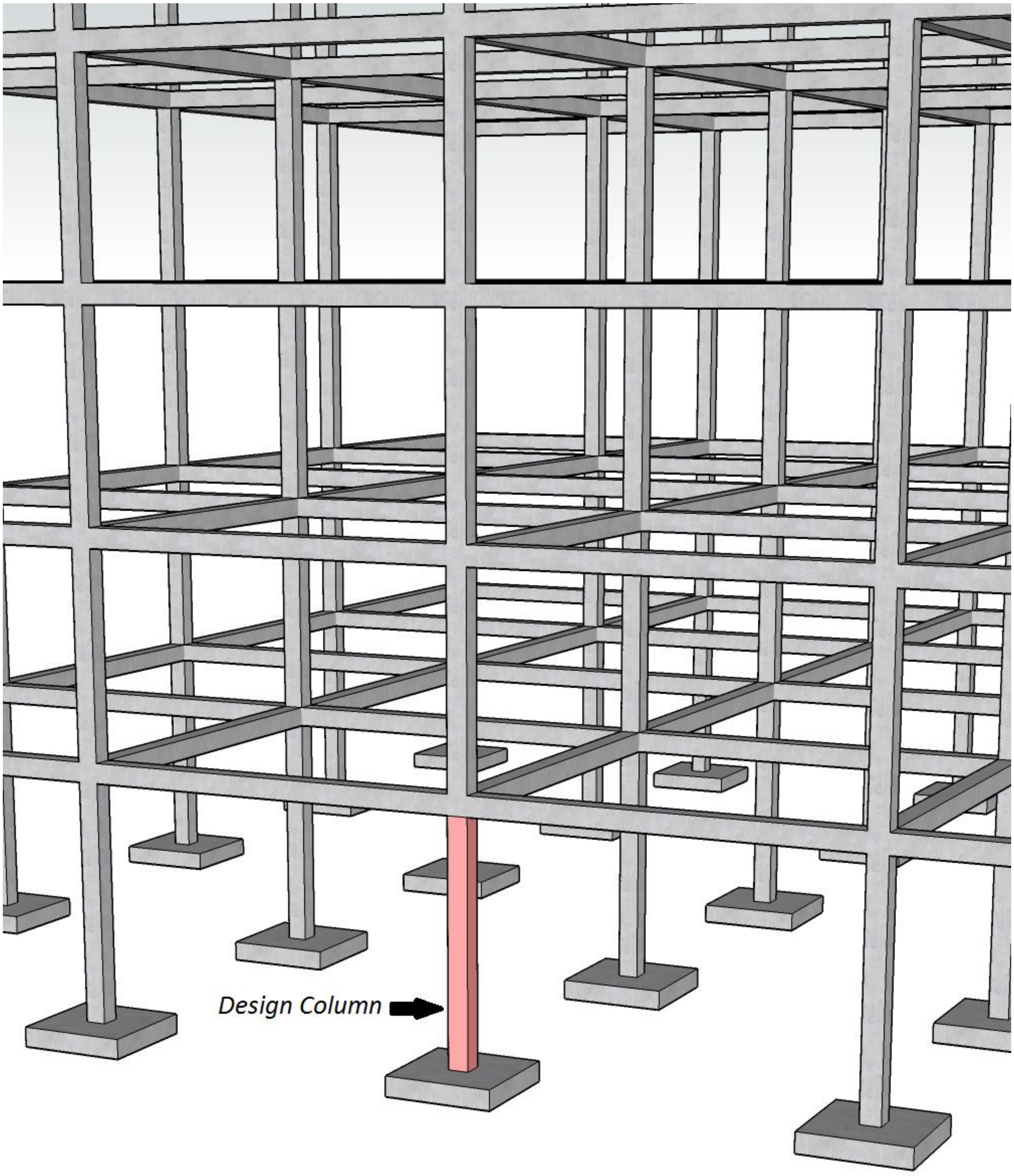
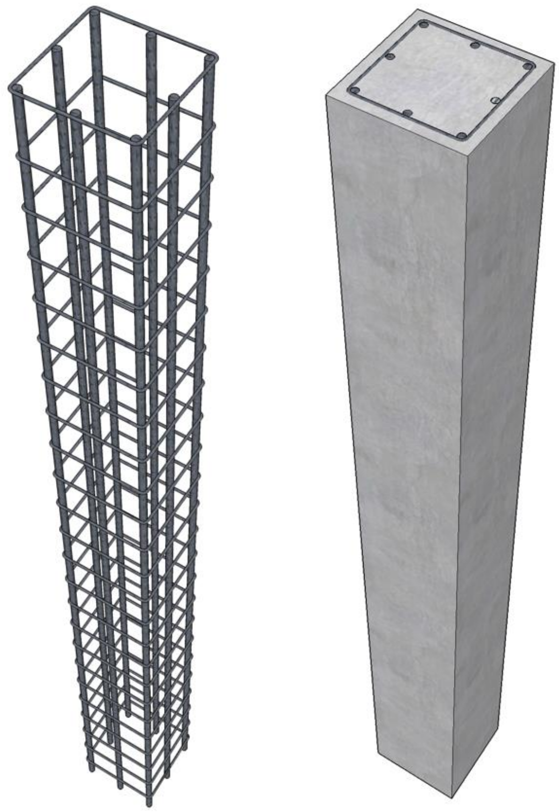
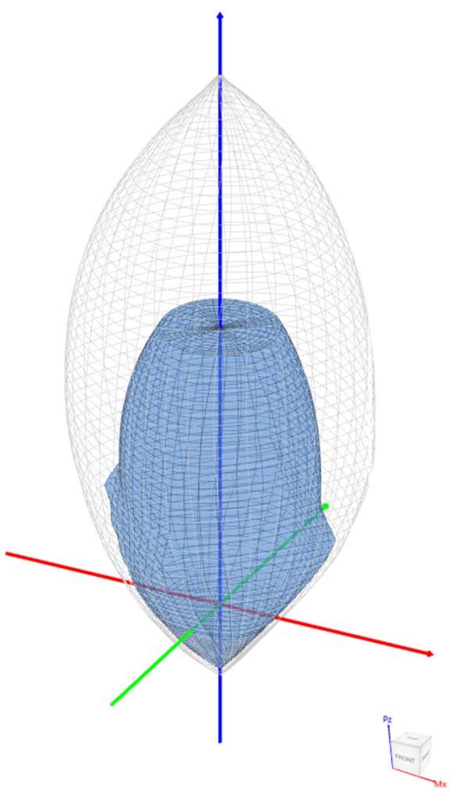
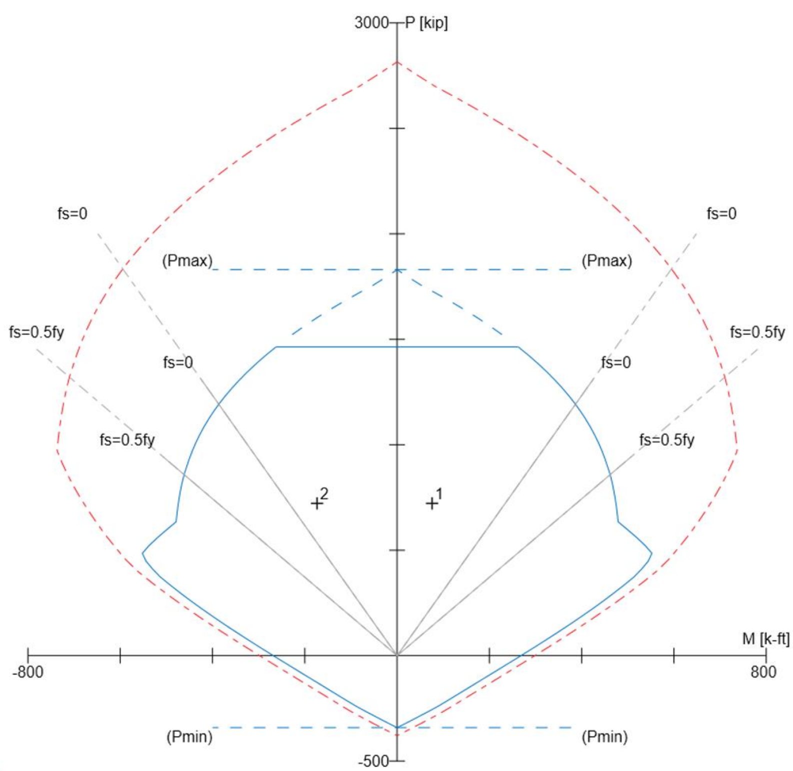
Slenderness Effects for Concrete Columns in Sway Frame - Moment Magnification Method (ACI 318-19)








3000
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(Pmax)
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2
1
M [k-ft]
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-500

Slenderness Effects for Concrete Columns in Sway Frame - Moment Magnification Method (ACI 318-19)

Evaluate slenderness effect for columns in a sway frame multistory reinforced concrete building by designing the first story exterior column. The clear height of the first story is 13 ft-4 in., and is 10 ft-4in. for all of the other stories. Lateral load effects on the building are governed by wind forces. Compare the calculated results with exact values from spColumn engineering software program from StructurePoint.

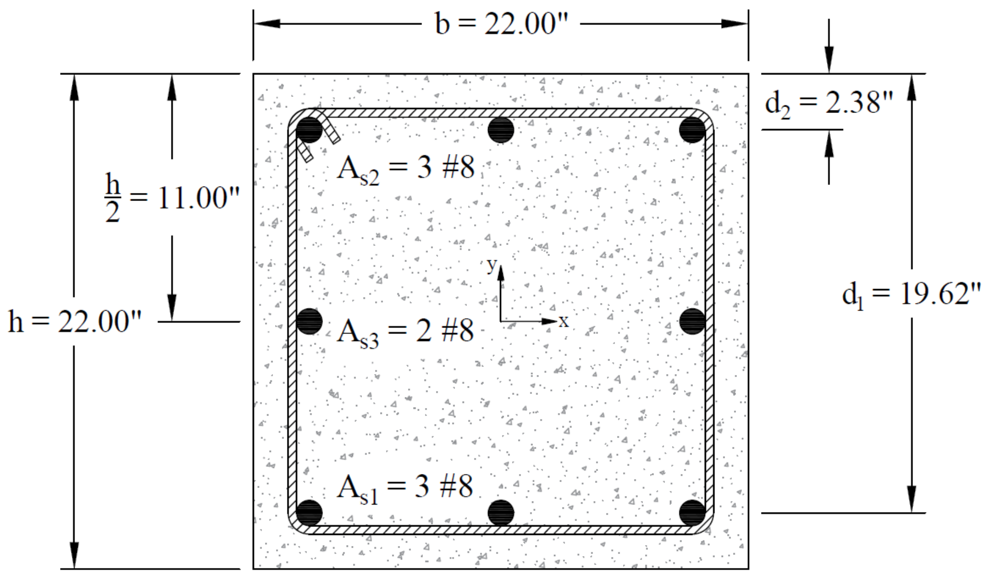

Figure 1 – Reinforced Concrete Column Cross-Section

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Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

References

- Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association, Example 11-2
- spColumn Engineering Software Program Manual v10.00, STRUCTUREPOINT, 2021
- "Slender Concrete Column Design in Sway Frames – Moment Magnification Method (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022
- "Slenderness Effects for Columns in Non-Sway Frame – Moment Magnification Method (ACI 318-19)" Design Example, STRUCTUREPOINT, 2022

Design Data

f_c' = 6,000 psi for columns in the bottom two stories

= 4,000 psi elsewhere

f_y = 60,000 psi

Slab thickness = 7 in.

Exterior Columns = 22 in. x 22 in.

Interior Columns = 24 in. x 24 in.

Beams = 24 in. x 20 in. x 24 ft

Superimposed dead load = 30 psf

Roof live load = 30 psf

Floor live load = 50 psf

Wind loads computed according to ASCE 7-10

Total building loads in the first story from structural analysis:

D = 17,895 kip

L = 1,991 kip

L_r = 270 kip

W = 0 kip, wind loads in the story cause compression in some columns and tension in others and thus would cancel out.

1. Factored Axial Loads and Bending Moments

1.1. Service loads

Table 1 - Exterior column service loads			
Load Case	Axial Load, kip	Bending Moment, ft-kip	
		Top	Bottom
Dead, D	622.4	34.8	17.6
Live, L	73.9	15.4	7.7
Roof Live, L_r	8.6	0.0	0.0
Wind, W (N-S)	-48.3	17.1	138.0
Wind, W (S-N)	48.3	-17.1	-138.0

1.2. Load Combinations – Factored Loads

ASCE 7-10 (2.3.2)

Table 2 - Exterior column factored loads									
ASCE 7-10 Reference	No.	Load Combination	Axial Load, kip	Bending Moment, ft-kip		$M_{Top,ns}$ ft-kip	$M_{Bottom,ns}$ ft-kip	$M_{Top,s}$ ft-kip	$M_{Bottom,s}$ ft-kip
				Top	Bottom				
2.3.2-1	1	1.4D	871.4	48.7	24.6	48.7	24.6	---	---
2.3.2-2	2	1.2D + 1.6L + 0.5L_r	869.4	66.4	33.4	66.4	33.4	---	---
2.3.2-3	3	1.2D + 0.5L + 1.6 L_r	797.6	49.5	25.0	49.5	25.0	---	---
	4	1.2D + 1.6L_r + 0.8W	722.0	55.4	131.5	41.8	21.1	13.7	110.4
	5	1.2D + 1.6L_r - 0.8W	799.3	28.1	-89.3	41.8	21.1	-13.7	-110.4
2.3.2-4	6	1.2D + 0.5L + 0.5L_r + 1.6W	710.9	76.8	245.8	49.5	25.0	27.4	220.8
	7	1.2D + 0.5L + 0.5L_r - 1.6W	865.4	22.1	-195.8	49.5	25.0	-27.4	-220.8
2.3.2-6	8	0.9D + 1.6W	482.9	58.7	236.6	31.3	15.8	27.4	220.8
	9	0.9D - 1.6W	637.4	4.0	-205.0	31.3	15.8	-27.4	-220.8

2. Slenderness Effects and Sway or Nonsway Frame Designation

Columns and stories in structures are considered as non-sway frames if the increase in column end moments due to second-order effects does not exceed 5% of the first-order end moments, or the stability index for the story (Q) does not exceed 0.05. ***ACI 318-19 (6.6.4.3)***

$\sum P_u$ is the total vertical load in the first story corresponding to the lateral loading case for which $\sum P_u$ is greatest (without the wind loads, which would cause compression in some columns and tension in others and thus would cancel out). ***ACI 318-19 (6.6.4.4.1 and R6.6.4.3)***

V_{us} is the factored horizontal story shear in the first story corresponding to the wind loads, and Δ_o is the first-order relative deflection between the top and bottom of the first story due to V_u. ***ACI 318-19 (6.6.4.4.1 and R6.6.4.3)***

From Table 2, load combinations (2.3.2-4 No. 5 and 6) provide the greatest value of $\sum P_u$.

$$\Sigma P_u = 1.2 \times D + 0.5 \times L + 0.5 \times L_r = 1.2 \times 17{,}895 + 0.5 \times 1{,}991 + 0.5 \times 270 = 22{,}605 \text{ kip}$$ ***ASCE 7-10 (2.3.2-4)***

$$V_{us} = 1.6 \times V_s = 1.6 \times 302.6 = 484.2 \text{ kip}$$ ***ASCE 7-10 (2.3.2-6)***

$$\Delta_o = 1.6 \times \Delta = 1.6 \times (0.28 - 0) = 0.45 \text{ in.}$$

$$Q = \frac{\Sigma P_u \times \Delta_o}{V_{us} \times l_c} = \frac{22{,}605 \times 0.45}{484.2 \times (15 \times 12 - 20/2)} = 0.12 > 0.05$$ ***ACI 318-19 (Eq. 6.6.4.4.1)***

Thus, the frame at the first story level is considered sway.

3. Determine Slenderness Effects

$$I_{column} = 0.7 \times \frac{c^4}{12} = 0.7 \times \frac{22^4}{12} = 13,665 \text{ in.}^4$$ **ACI 318-19 (Table 6.6.3.1.1(a))**

$$E_c = 57,000 \times \sqrt{f_c'} = 57,000 \times \sqrt{6000} = 4,415 \text{ ksi}$$ **ACI 318-19 (19.2.2.1.b)**

For the column below level 2:

$$\frac{E_c \times I_{column}}{l_c} = \frac{4,415 \times 13,665}{15 \times 12 - 20/2} = 355 \times 10^3 \text{ in.kip}$$

For the column above level 2:

$$\frac{E_c \times I_{column}}{l_c} = \frac{4,415 \times 13,665}{12 \times 12} = 419 \times 10^3 \text{ in.kip}$$

For beams framing into the columns:

$$\frac{E_b \times I_{beam}}{l_b} = \frac{3,605 \times 5,600}{24 \times 12} = 70 \times 10^3 \text{ in.kip}$$

Where:

$$E_b = 57,000 \times \sqrt{f_c'} = 57,000 \times \sqrt{4000} = 3,605 \text{ ksi}$$ **ACI 318-19 (19.2.2.1.b)**

$$I_{beam} = 0.35 \times \frac{b \times h^3}{12} = 0.35 \times \frac{24 \times 20^3}{12} = 5,600 \text{ in.}^4$$ **ACI 318-19 (Table 6.6.3.1.1(a))**

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{355 + 419}{70} = 11.040$$ **ACI 318-19 (Figure R6.2.5.1)**

$$\Psi_B = 0.0 \text{ (Column essentially fixed at base)}$$ **ACI 318-19 (Figure R6.2.5.1)**

Using Figure R6.2.5.1 from ACI 318-19 → k = 1.693 as shown in the figure below for the exterior columns with one beam framing into them in the directions of analysis.

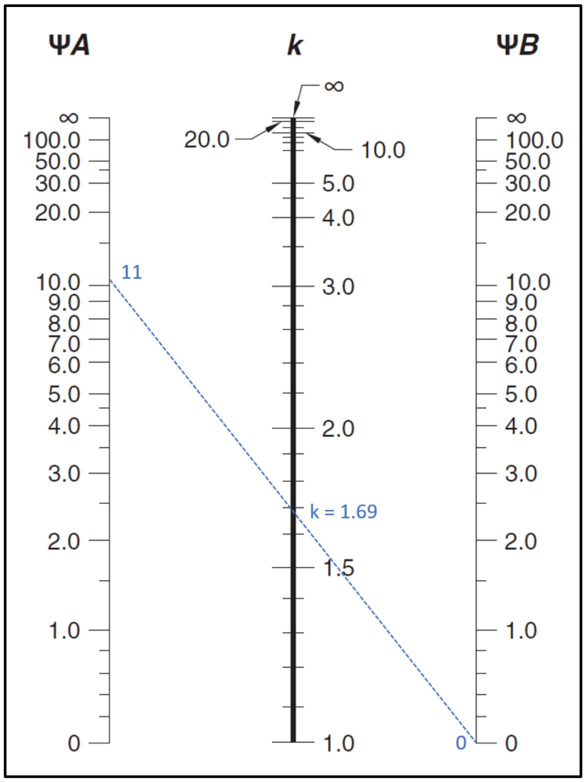


Figure 2 – Effective Length Factor (k) Calculations for Exterior Columns with One Beam Framing into them in the Direction of Analysis (Sway Frame)

$$\frac{k \times l_u}{r} = \frac{1.693 \times 13.333}{6.35} = 42.65 > 22 \quad \rightarrow \text{Consider Slenderness}$$ ***ACI 318-19 (6.2.5.1a)***

Where:

$$r = \text{radius of gyration} = (a)\ \sqrt{\frac{I_g}{A_g}} \quad or \quad \text{(b) } 0.3 \times c_1$$ ***ACI 318-19 (6.2.5.2)***

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{c_1^{\ 2}}{12}} = \sqrt{\frac{22^2}{12}} = 6.35 \text{ in.}$$

4. Moment Magnification at Ends of Compression Member

A detailed calculation for load combination 4 (gravity plus wind) is shown below to illustrate the procedure. Table 3 summarizes the magnified moment computations for the exterior columns.

$$M_2 = M_{2ns} + \delta_s M_{2s}$$ ***ACI 318-19 (6.6.4.6.1b)***

Where:

$$\delta_s = \text{ moment magnifier} = \begin{Bmatrix} \text{(a)} & \dfrac{1}{1-Q} \\ \text{(b)} & \dfrac{1}{1-\dfrac{\Sigma P_u}{0.75\Sigma P_c}} \\ \text{(c)} & \text{Second-order elastic analysis} \end{Bmatrix}$$ ***ACI 318-19 (6.6.4.6.2)***

ACI 318-19 (6.6.4.6.2(b)) will be used for comparison purposes with results obtained from spColumn model. However, (a) and (c) can also be used to calculate the moment magnifier.

$\sum P_u$ is the summation of all the factored vertical loads in the first story, and $\sum P_c$ is the summation of the critical buckling load for all sway-resisting columns in the first story.

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ ***ACI 318-19 (6.6.4.4.2)***

Where:

$$(EI)_{eff} = \begin{Bmatrix} \text{(a)} & \dfrac{0.4E_cI_g}{1+\beta_{ds}} \\ \text{(b)} & \dfrac{0.2E_cI_g + E_sI_{se}}{1+\beta_{ds}} \\ \text{(c)} & \dfrac{E_cI}{1+\beta_{ds}} \end{Bmatrix}$$ ***ACI 318-19 (6.6.4.4.4)***

There are three options for calculating the effective flexural stiffness of slender concrete columns $(EI)_{eff}$. The second equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

$$I_{column} = \frac{c^4}{12} = \frac{22^4}{12} = 19{,}521 \text{ in.}^4$$ ***ACI 318-19 (Table 6.6.3.1.1(a))***

$$E_c = 57{,}000 \times \sqrt{f'_c} = 57{,}000 \times \sqrt{6000} = 4{,}415 \text{ ksi}$$ ***ACI 318-19 (19.2.2.1.b)***

β_{ds} is the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination. The maximum factored sustained shear in this example is equal to zero leading to $\beta_{ds} = 0$. ***ACI 318-19 (6.6.3.1.1)***

For exterior columns with one beam framing into them in the direction of analysis (12 columns):

With 8-#8 reinforcement equally distributed on all sides and 22 in. x 22 in. column section → I_{se} = 352.6 in.^{4}.

$$(EI)_{eff} = \frac{0.2E_cI_g + E_sI_{se}}{1+\beta_{ds}}$$ ***ACI 318-19 (6.6.4.4.4(b))***

$$\left(EI\right)_{eff} = \frac{0.2\times 4,415\times 19,521+29,000\times 352.6}{1+0} = 27.5\times 10^{6} \text{ kip-in.}^{2}$$

k = 1.693 (calculated previously).

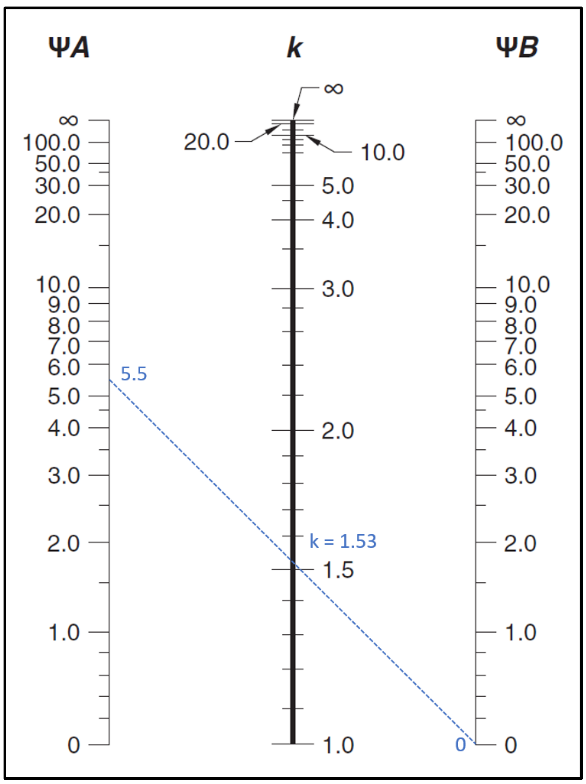
$$P_{c1} = \frac{\pi^{2}\times 27.5\times 10^{6}}{\left(1.693\times 13.333\right)^{2}} = 3,694.04 \text{ kip}$$

<u>For exterior columns with two beams framing into them in the direction of analysis (4 columns):</u>

$$\Psi_{A} = \frac{\left(\sum \frac{EI}{l_{c}}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{355+419}{70+70} = 5.520$$ ***<u>ACI 318-19 (Figure R6.2.5.1)</u>***

$\Psi_{B} = 0.0$ (Column essentially fixed at base) ***<u>ACI 318-19 (Figure R6.2.5.1)</u>***

Using Figure R6.2.5.1 from ACI 318-19 → k = 1.527 as shown in the figure below for the exterior columns with two beams framing into them in the directions of analysis.



<u>Figure 3 – Effective Length Factor (k) Calculations for Exterior Columns with Two Beams Framing into them in the Direction of Analysis</u>

$$P_{c2} = \frac{\pi^{2}\times 27.5\times 10^{6}}{\left(1.527\times 13.333\times 12\right)^{2}} = 4,540.86 \text{ kip}$$

For interior columns (8 columns):

$$I_{column} = 0.7 \times \frac{c^4}{12} = 0.7 \times \frac{24^4}{12} = 19,354 \text{ in.}^4$$ **ACI 318-19 (Table 6.6.3.1.1(a))**

$$E_c = 57,000 \times \sqrt{f_c'} = 57,000 \times \sqrt{6000} = 4,415 \text{ ksi}$$ **ACI 318-19 (19.2.2.1.b)**

For the column below level 2:

$$\frac{E_c \times I_{column}}{l_c} = \frac{4,415 \times 19,354}{15 - 20/2} = 503 \times 10^3 \text{ in.kip}$$

For the column above level 2:

$$\frac{E_c \times I_{column}}{l_c} = \frac{4,415 \times 19,354}{12} = 593 \times 10^3 \text{ in.kip}$$

For beams framing into the columns:

$$\frac{E_b \times I_{beam}}{l_b} = \frac{3,605 \times 5,600}{24} = 70 \times 10^3 \text{ in.kip}$$

Where:

$$E_b = 57,000 \times \sqrt{f_c'} = 57,000 \times \sqrt{4000} = 3,605 \text{ ksi}$$ **ACI 318-19 (19.2.2.1.b)**

$$I_{beam} = 0.35 \times \frac{b \times h^4}{12} = 0.35 \times \frac{24 \times 20^4}{12} = 5,600 \text{ in.}^4$$ **ACI 318-19 (Table 6.6.3.1.1(a))**

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{503 + 593}{70 + 70} = 7.818$$ **ACI 318-19 (Figure R6.2.5.1)**

$\Psi_B = 0.0$ (Column essentially fixed at base) **ACI 318-19 (Figure R6.2.5.1)**

Using Figure R6.2.5.1 from ACI 318-19 → k = 1.614 as shown in the figure below for the interior columns.

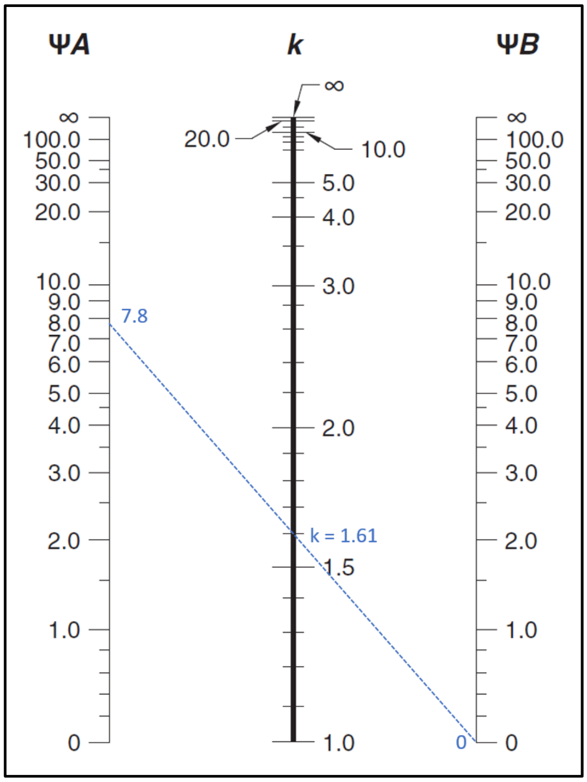


Figure 4 – Effective Length Factor (k) Calculations for Interior Columns

With 8-#8 reinforcement equally distributed on all sides and 24 in. x 24 in. column section → I_{se} = 439.1 in.^{4}.

$$\left(EI\right)_{eff} = \frac{0.2E_cI_g + E_sI_{se}}{1+\beta_{ds}}$$ **ACI 318-19 (6.6.4.4.4(b))**

$$\left(EI\right)_{eff} = \frac{0.2\times 4,415\times 27,648 + 29,000\times 439.1}{1+0} = 37.1\times 10^6 \text{ kip-in.}^2$$

$$P_{c3} = \frac{\pi^2\times 37.1\times 10^6}{\left(1.614\times 13.333\times 12\right)^2} = 5,497.82 \text{ kip}$$

$$\Sigma P_c = n_1\times P_{c1} + n_2\times P_{c2} + n_3\times P_{c3}$$

$$\Sigma P_c = 12\times 3,694.04 + 4\times 4,540.86 + 8\times 5,497.82 = 106,474.52 \text{ kip}$$

For load combination 4:

$$\Sigma P_u = 1.2\times D + 1.6\times L_r = 1.2\times 17,895 + 1.6\times 270 = 21,906.00 \text{ kip}$$ **ASCE 7-10 (2.3.2-3)**

$$\delta_s = \frac{1}{1-\dfrac{\Sigma P_u}{0.75\times\Sigma P_c}}$$ **ACI 318-19 (6.6.4.6.2(b))**

$$\delta_s = \frac{1}{1 - \dfrac{21{,}906}{0.75 \times 106{,}474.52}} = 1.378$$

$$\delta_s M_{Top,s} = 1.378 \times 13.70 = 18.88 \text{ ft.kip}$$

$$M_{Top\_2^{nd}} = M_{Top,ns} + \delta_s M_{Top,s} = 41.80 + 18.88 = 60.68 \text{ ft.kip}$$ ***ACI 318-19 (6.6.4.6.1)***

$$\delta_s M_{Bottom,s} = 1.378 \times 110.40 = 152.13 \text{ ft.kip}$$

$$M_{Bottom\_2^{nd}} = M_{Bottom,ns} + \delta_s M_{Bottom,s} = 21.10 + 152.13 = 173.23 \text{ ft.kip}$$ ***ACI 318-19 (6.6.4.6.1)***

$$M_{2\_2^{nd}} = max\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Bottom\_2^{nd}} = 173.23 \text{ ft.kip} \rightarrow M_{2\_1^{st}} = M_{Bottom\_1^{st}} = 131.50 \text{ ft.kip}$$

$$M_{1\_2^{nd}} = min\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Top\_2^{nd}} = 60.68 \text{ ft.kip} \rightarrow M_{1\_1^{st}} = M_{Top\_1^{st}} = 55.50 \text{ ft.kip}$$

P_u = 722.0 kip

A summary of the moment magnification factors and magnified moments for the exterior column for all load combinations using both equation options ACI 318-19 (6.6.4.4.4(a)) and (6.6.4.4.4(b)) to calculate $(EI)_{eff}$ is provided in the table below for illustration and comparison purposes. Note: The designation of M_1 and M_2 is made based on the second-order (magnified) moments and not based on the first-order (unmagnified) moments.

Table 3 - Factored Axial loads and Magnified Moments for Exterior Column

No.	Load Combination	Axial Load, kip	Using ACI 6.6.4.4.4(a)			Using ACI 6.6.4.4.4(b)		
			δ_s	M_1, ft-kip	M_2, ft-kip	δ_s	M_1, ft-kip	M_2, ft-kip
1	1.4D	871.4	---	24.6	48.7	---	24.6	48.7
2	1.2D + 1.6L + 0.5L_r	869.4	---	33.4	66.4	---	33.4	66.4
3	1.2D + 0.5L + 1.6 L_r	797.6	---	25.0	49.5	---	25.0	49.5
4	1.2D + 1.6L_r + 0.8W	722.0	1.27	59.2	161.6	1.38	60.7	173.2
5	1.2D + 1.6L_r - 0.8W	799.3	1.27	24.4	-119.4	1.38	22.9	-131.0
6	1.2D + 0.5L + 0.5L_r + 1.6W	710.9	1.28	84.7	308.5	1.39	87.7	332.9
7	1.2D + 0.5L + 0.5L_r - 1.6W	865.4	1.28	14.3	-258.5	1.39	11.3	-282.9
8	0.9D + 1.6W	482.9	1.19	63.8	277.9	1.25	65.6	292.4
9	0.9D - 1.6W	637.4	1.19	-1.2	-246.3	1.25	-3.0	-260.8

5. Moment Magnification along Length of Compression Member

In sway frames, second-order effects shall be considered along the length of columns. It shall be permitted to account for these effects using ***ACI 318-19 (6.6.4.5)*** (Nonsway frame procedure), where C_m is calculated using M_1 and M_2 from ***ACI 318-19 (6.6.4.6.1)*** as follows: ***ACI 318-19 (6.6.4.6.4)***

$$M_{c2} = \delta M_2$$ ***ACI 318-19 (6.6.4.5.1)***

Where:

M_2 = the second-order factored moment.

$$\delta = \text{ magnification factor} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0$$ ***ACI 318-19 (6.6.4.5.2)***

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ ***ACI 318-19 (6.6.4.4.2)***

Where:

$$(EI)_{eff} = \left\{ \begin{array}{ll} \text{(a)} & \dfrac{0.4E_cI_g}{1+\beta_{dns}} \\ \text{(b)} & \dfrac{0.2E_cI_g + E_sI_{se}}{1+\beta_{dns}} \\ \text{(c)} & \dfrac{E_cI}{1+\beta_{dns}} \end{array} \right\}$$ ***ACI 318-19 (6.6.4.4.4)***

There are three options for calculating the effective flexural stiffness of slender concrete columns $(EI)_{eff}$. The second equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

$$I_{column} = \frac{c^4}{12} = \frac{22^4}{12} = 19{,}521 \text{ in.}^4$$ ***ACI 318-19 (Table 6.6.3.1.1(a))***

$$E_c = 57{,}000 \times \sqrt{f'_c} = 57{,}000 \times \sqrt{6000} = 4{,}415 \text{ ksi}$$ ***ACI 318-19 (19.2.2.1.b)***

β_{dns} is the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination. ***ACI 318-19 (6.6.4.4.4)***

For load combination 4:

$$P_{u,sustained} = 1.2 \times 622.4 = 746.9 \text{ kip}$$

$$P_u = 1.2 \times 622.4 + 1.6 \times 8.6 + 0.8 \times -48.3 = 722 \text{ kip}$$

$$\beta_{dns} = \frac{P_{u,sustained}}{P_u} = \frac{746.9}{722} = 1.03 > 1.00 \rightarrow \therefore \beta_{dns} = 1.0$$

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{355 + 419}{70} = 11.040 \text{ (Calculated previously)}$$ ***ACI 318-19 (Figure R6.2.5.1)***

$\Psi_B = 0.0$ (Column essentially fixed at base) ***ACI 318-19 (Figure R6.2.5.1)***

Using Figure R6.2.5.1(a) from ACI 318-19 → k = 0.690 as shown in the figure below for the exterior column.

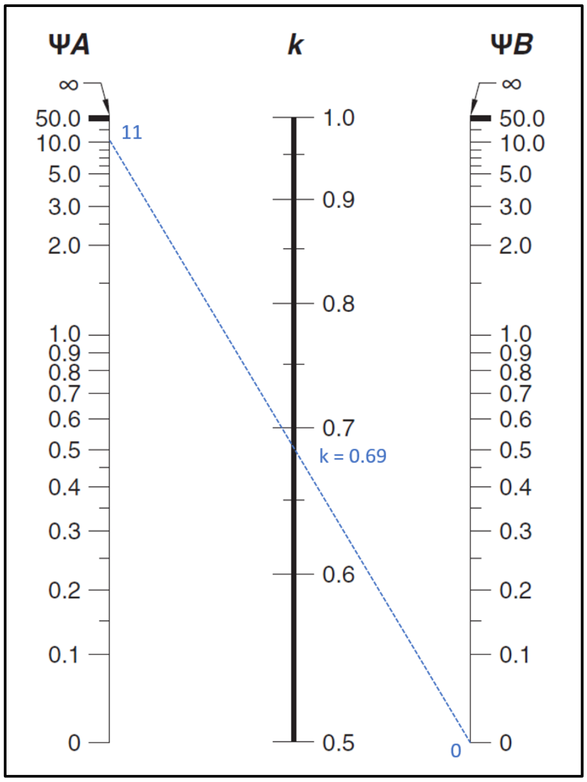


Figure 5 – Effective Length Factor (k) Calculations for Exterior Column (Nonsway)

With 8-#8 reinforcement equally distributed on all sides and 22 in. x 22 in. column section → I_{se} = 352.6 in.^{4}.

$$(EI)_{eff} = \frac{0.2E_c I_g + E_s I_{se}}{1 + \beta_{dns}}$$ ***ACI 318-19 (6.6.4.4.4(b))***

$$(EI)_{eff} = \frac{0.2 \times 4,415 \times 19,521 + 29,000 \times 352.6}{1+1} = 13.7 \times 10^6 \text{ kip-in.}^2$$

$$P_c = \frac{\pi^2 \times 13.7 \times 10^6}{(0.690 \times 13.333 \times 12)^2} = 11{,}119.57 \text{ kip}$$

For load combination 4:

$$P_u = 1.2 \times 622.4 + 1.6 \times 8.6 + 0.8 \times -48.3 = 722 \text{ kip}$$ **ASCE 7-10 (2.3.2-3)**

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2}$$ **ACI 318-19 (6.6.4.5.3a)**

$$M_2 = M_{2\_2^{nd}} = 173.23 \text{ ft.kip (as concluded from section 4)}$$ **ACI 318-19 (6.6.4.6.4)**

$$M_1 = M_{1\_2^{nd}} = 60.68 \text{ ft.kip (as concluded from section 4)}$$ **ACI 318-19 (6.6.4.6.4)**

Since the column is bent in double curvature, M_1/M_2 is positive. **ACI 318-19 (6.6.4.5.3)**

$$C_m = 0.6 - 0.4\left(\frac{60.68}{173.23}\right) = 0.460$$

$$\delta = \frac{C_m}{1 - \frac{P_u}{0.75 P_c}} \geq 1.0$$ **ACI 318-19 (6.6.4.5.2)**

$$\delta = \frac{0.460}{1 - \frac{722}{0.75 \times 11{,}119.57}} = 0.503 < 1.00 \rightarrow \ \delta = 1.00$$

$$M_{\min} = P_u (0.6 + 0.03h)$$ **ACI 318-19 (6.6.4.5.4)**

Where P_u = 722 kip, and h = the section dimension in the direction being considered = 22 in.

$$M_{\min} = 722\left(\frac{0.6 + 0.03 \times 22}{12}\right) = 75.81 \text{ ft.kip}$$

$$M_1 = 60.68 \text{ ft.kip} < M_{\min} = 75.81 \text{ ft.kip} \rightarrow M_1 = 75.81 \text{ ft.kip}$$ **ACI 318-19 (6.6.4.5.4)**

$$M_{c1} = \delta M_1$$ **ACI 318-19 (6.6.4.5.1)**

$$M_{c1} = 1.00 \times 75.81 = 75.81 \text{ ft.kip}$$

$$M_2 = 173.23 \text{ ft.kip} > M_{2,\min} = 75.81 \text{ ft.kip} \rightarrow M_2 = 173.23 \text{ ft.kip}$$ **ACI 318-19 (6.6.4.5.4)**

$$M_{c2} = \delta M_2$$ **ACI 318-19 (6.6.4.5.1)**

$$M_{c2} = 1.00 \times 173.23 = 173.23 \text{ ft.kip}$$

M_{c1} and M_{c2} will be considered separately to ensure proper comparison of resulting magnified moments against negative and positive moment capacities of unsymmetrical sections as can be seen in the following figure.

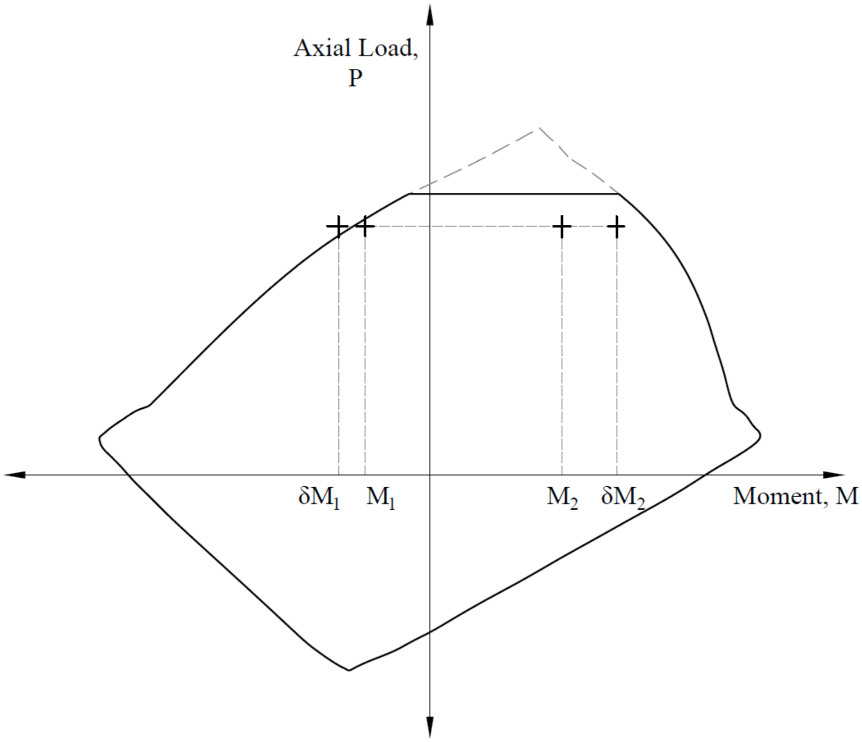


Figure 6 – Column Interaction Diagram for Unsymmetrical Section

A summary of the moment magnification factors and magnified moments for the exterior column for all load combinations using both equation options ACI 318-19 (6.6.4.4.4(a)) and (6.6.4.4.4(b)) to calculate $(EI)_{eff}$ is provided in the table below for illustration and comparison purposes.

Table 4 - Factored Axial loads and Magnified Moments along Exterior Column Length

No.	Load Combination	Axial Load, kip	Using ACI 6.6.4.4.4(a)			Using ACI 6.6.4.4.4(b)		
			δ	M_{c1}, ft-kip	M_{c2}, ft-kip	δ	M_{c1}, ft-kip	M_{c2}, ft-kip
1	1.4D	871.4	1.00	91.5	91.5	1.00	91.5	91.5
2	1.2D + 1.6L + 0.5L_r	869.4	1.00	91.3	91.3	1.00	91.3	91.3
3	1.2D + 0.5L + 1.6 L_r	797.6	1.00	83.7	83.7	1.00	83.7	83.7
4	1.2D + 1.6L_r + 0.8W	722.0	1.00	75.8	161.6	1.00	75.8	173.2
5	1.2D + 1.6L_r - 0.8W	799.3	1.00	83.9	-119.4	1.00	83.9	-131.0
6	1.2D + 0.5L + 0.5L_r + 1.6W	710.9	1.00	84.7	308.5	1.00	87.7	333.0
7	1.2D + 0.5L + 0.5L_r - 1.6W	865.4	1.00	90.9	-258.5	1.00	90.9	-283.0
8	0.9D + 1.6W	482.9	1.00	63.8	277.9	1.00	65.6	292.4
9	0.9D - 1.6W	637.4	1.00	66.9	-246.3	1.00	66.9	-260.8

For column design ACI 318 requires the second-order moment to first-order moment ratios should not exceed 1.40. If this value is exceeded, the column design needs to be revised. ***ACI 318-19 (6.2.5.3)***

Table 5 - Second-Order Moment to First-Order Moment Ratios					
No.	Load Combination	Using ACI 6.6.4.4.4(a)		Using ACI 6.6.4.4.4(b)	
		$M_{c1}/M_{1(1st)}$	$M_{c2}/M_{2(1st)}$	$M_{c1}/M_{1(1st)}$	$M_{c2}/M_{2(1st)}$
1	1.4D	1.00*	1.00*	1.00*	1.00*
2	1.2D + 1.6L + 0.5L_r	1.00*	1.00*	1.00*	1.00*
3	1.2D + 0.5L + 1.6 L_r	1.00*	1.00*	1.00*	1.00*
4	1.2D + 1.6L_r + 0.8W	1.00*	1.23	1.00*	1.32
5	1.2D + 1.6L_r - 0.8W	1.00*	1.34	1.00*	1.40 < 1.47
6	1.2D + 0.5L + 0.5L_r + 1.6W	1.10	1.26	1.14	1.36
7	1.2D + 0.5L + 0.5L_r - 1.6W	1.00*	1.32	1.00*	1.40 < 1.45
8	0.9D + 1.6W	1.09	1.18	1.12	1.24
9	0.9D - 1.6W	1.00*	1.20	1.00*	1.27

* Cutoff value of M_{min} is applied to $M_{1(1st)}$ and $M_{2(1st)}$ in order to avoid unduly large ratios in cases where $M_{1(1st)}$ and $M_{2(1st)}$ moments are smaller than M_{min}.

6. Column Design

Based on the factored axial loads and magnified moments considering slenderness effects, the capacity of the assumed column section (22 in. x 22 in. with 8-#8 bars distributed all sides equal) will be checked and confirmed to finalize the design. A column interaction diagram will be generated using strain compatibility analysis, the detailed procedure to develop column interaction diagram can be found in "[Interaction Diagram – Tied Reinforced Concrete Column Design Strength (ACI 318-19)]" example.

The axial compression capacity ϕP_n for all load combinations will be set equals to P_u, then the moment capacity ϕM_n associated to ϕP_n will be compared with the magnified applied moment M_u. The design check for load combination #4 is shown below for illustration. The rest of the checks for the other load combinations are shown in the following Table.

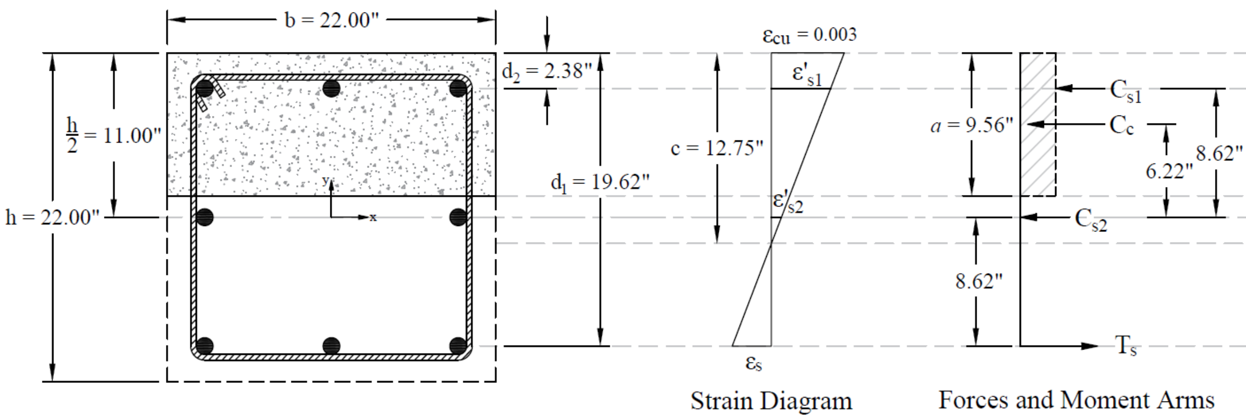


Figure 7 – Strains, Forces, and Moment Arms (Load Combination 4)

The following procedure is used to determine the nominal moment capacity by setting the design axial load capacity, ϕP_n, equal to the applied axial load, P_u and iterating on the location of the neutral axis.

6.1. c, *a*, and strains in the reinforcement

Try $c = 12.75$ in.

Where c is the distance from the fiber of maximum compressive strain to the neutral axis.

ACI 318-19 (22.2.2.4.2)

$a = \beta_1 \times c = 0.75 \times 12.75 = 9.563$ in. ***ACI 318-19 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times (f'_c - 4000)}{1000} = 0.85 - \frac{0.05 \times (6000 - 4000)}{1000} = 0.75$$ ***ACI 318-19 (Table 22.2.2.4.3)***

$\varepsilon_{cu} = 0.003$ ***ACI 318-19 (22.2.2.1)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.003}{c} = (19.625 - 12.75) \times \frac{0.003}{12.75} = 0.00162 \text{ (Tension)} < \varepsilon_y$$

$\therefore$ tension reinforcement has not yielded

$\therefore \phi = 0.65$ ***ACI 318-19 (Table 21.2.2)***

$$\varepsilon'_{s1} = (c - d_2) \times \frac{0.003}{c} = (12.75 - 2.375) \times \frac{0.003}{12.75} = 0.00244 \text{ (Compression)} > \varepsilon_y$$

$$\varepsilon'_{s2} = \left(c - \frac{h}{2}\right) \times \frac{0.003}{c} = (12.75 - 11) \times \frac{0.003}{12.75} = 0.00041 \text{ (Compression)} < \varepsilon_y$$

6.2. Forces in the concrete and steel

$C_c = 0.85 \times f'_c \times a \times b = 0.85 \times 6,000 \times 9.563 \times 22 = 1073 \text{ kip}$ ***ACI 318-19 (22.2.2.4.1)***

$$f_s = \varepsilon_s \times E_s = 0.00162 \times 29,000,000 = 46,907 \text{ psi}$$

$$\mathrm{T}_s = f_y \times A_{s1} = 46,912 \times (3 \times 0.79) = 111.2 \text{ kip}$$

Since $\varepsilon'_{s1} > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$$\therefore f'_{s1} = f_y = 60,000 \text{ psi}$$

Since $\varepsilon'_{s2} < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f'_{s2} = \varepsilon'_{s2} \times E_s = 0.00041 \times 29,000,000 = 11,944 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area (ab) used to compute C_c. As a result, it is necessary to subtract $0.85f_c'$ from f_s' before computing C_s:

$$\mathrm{C}_{s1} = \left(f'_{s1} - 0.85 f'_c\right) \times A'_{s1} = (60,000 - 0.85 \times 6,000) \times (3 \times 0.79) = 130.1 \text{ kip}$$

$$\mathrm{C}_{s2} = \left(f'_{s2} - 0.85 f'_c\right) \times A'_{s2} = (11,941 - 0.85 \times 6,000) \times (2 \times 0.79) = 18.9 \text{ kip}$$

6.3. ϕP_n and ϕM_n

$$P_n = C_c + C_{s1} + C_{s2} - T_s = 1,073 + 130.1 + 18.9 - 111.2 = 1,110.8 \text{ kip}$$

$$\phi P_n = 0.65 \times 1,111 = 722.0 \text{ kip} = P_u$$

The assumption that c = 12.75 in. is correct.

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_{s1} \times \left(\frac{h}{2} - d_2\right) + C_{s2} \times \left(\frac{h}{2} - \frac{h}{2}\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 1,073 \times \left(\frac{22}{2} - \frac{9.563}{2}\right) + 130.1 \times \left(\frac{22}{2} - 2.375\right) + 18.9 \times \left(\frac{22}{2} - \frac{22}{2}\right) + 111.2 \times \left(19.625 - \frac{22}{2}\right) = 729.44 \text{ kip.ft}$$

$$\phi M_n = 0.65 \times 729 = 474.14 \text{ kip.ft} < M_u = M_{c2} = 173.2 \text{ kip.ft}$$

Table 6 – Exterior Column Axial and Moment Capacities

No.	P_u, kip	$M_u = M_{2(2nd)}$, ft-kip	c, in.	$\varepsilon_t = \varepsilon_s$	ϕ	ϕP_n, kip	ϕM_n, kip.ft
1	871.4	91.5	14.85	0.00097	0.65	871.4	459.4
2	869.4	91.3	14.82	0.00097	0.65	869.4	459.7
3	797.6	83.7	13.75	0.00128	0.65	797.6	468.2
4	722.0	173.2	12.75	0.00162	0.65	722.0	474.1
5	799.3	-131.0	13.78	0.00127	0.65	799.3	468.0
6	710.9	333.0	12.61	0.00167	0.65	710.9	474.9
7	865.4	-283.0	14.76	0.00099	0.65	865.4	460.2
8	482.9	292.4	7.41	0.00495	0.89	482.9	552.9
9	637.4	-260.8	11.68	0.00204	0.65	637.4	478.8

Therefore, since $\phi M_n > M_u$ for all $\phi P_n = P_u$, use 22 x 22 in. column with 8-#8 bars.

7. Column Interaction Diagram - spColumn Software

spColumn is a StructurePoint software program that performs the analysis and design of reinforced concrete sections subjected to axial force combined with uniaxial or biaxial bending. Using the provisions of the Strength Design Method and Unified Design Provisions, slenderness considerations are used for moment magnification due to second order effect (P-Delta) for sway and non-sway frames.

For this column section, investigation mode is used, service loads are defined, and slenderness effects are considered using ACI 318-19 provisions. The model input parameters, results, and report (for load combination #4) are shown below.

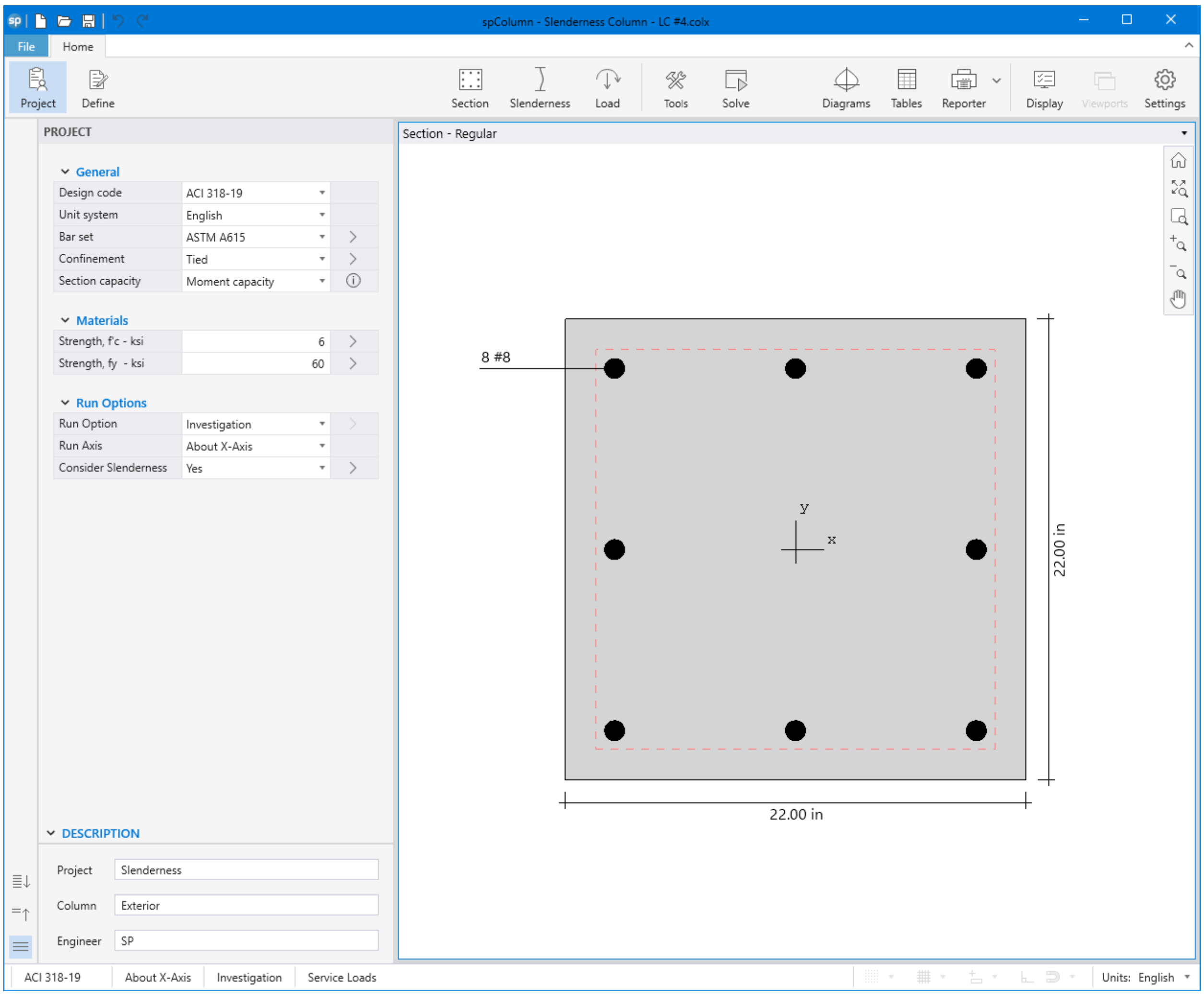


Figure 8 – spColumn Interface

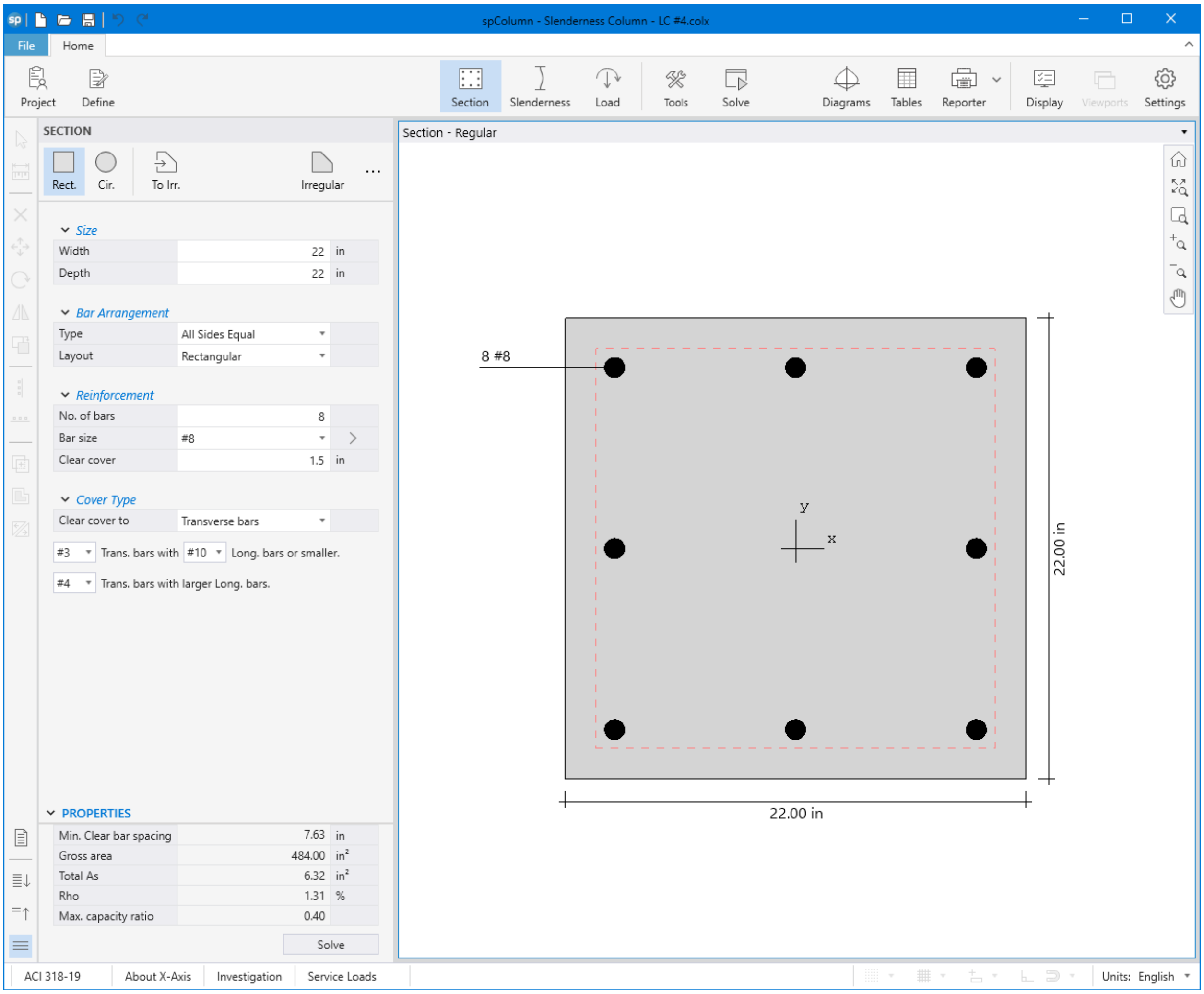


Figure 9 – spColumn Model Editor

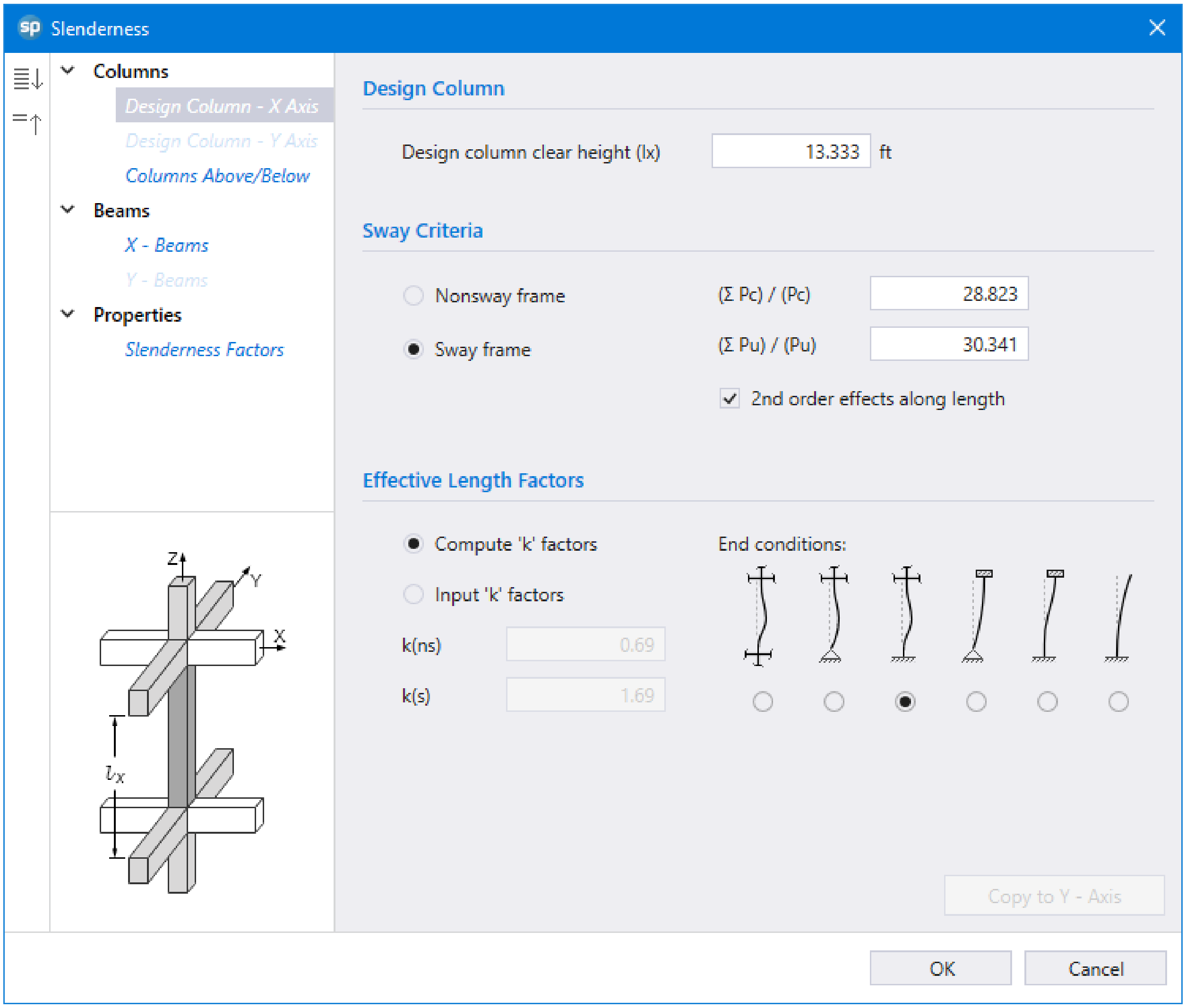


Figure 10 – Defining Slenderness – Load Combination #4 (spColumn)

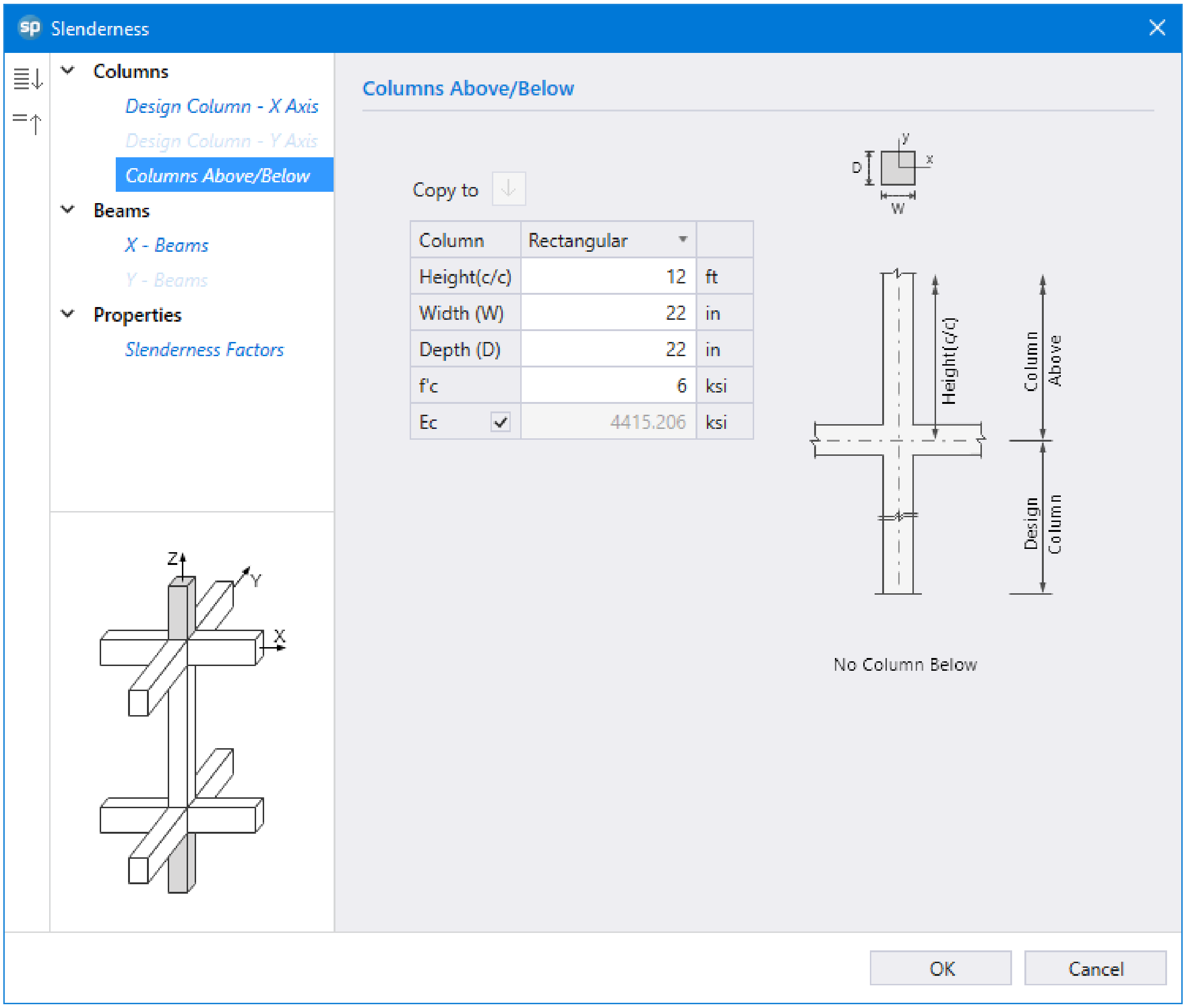


Figure 11 – Defining Columns Above / Below (spColumn)

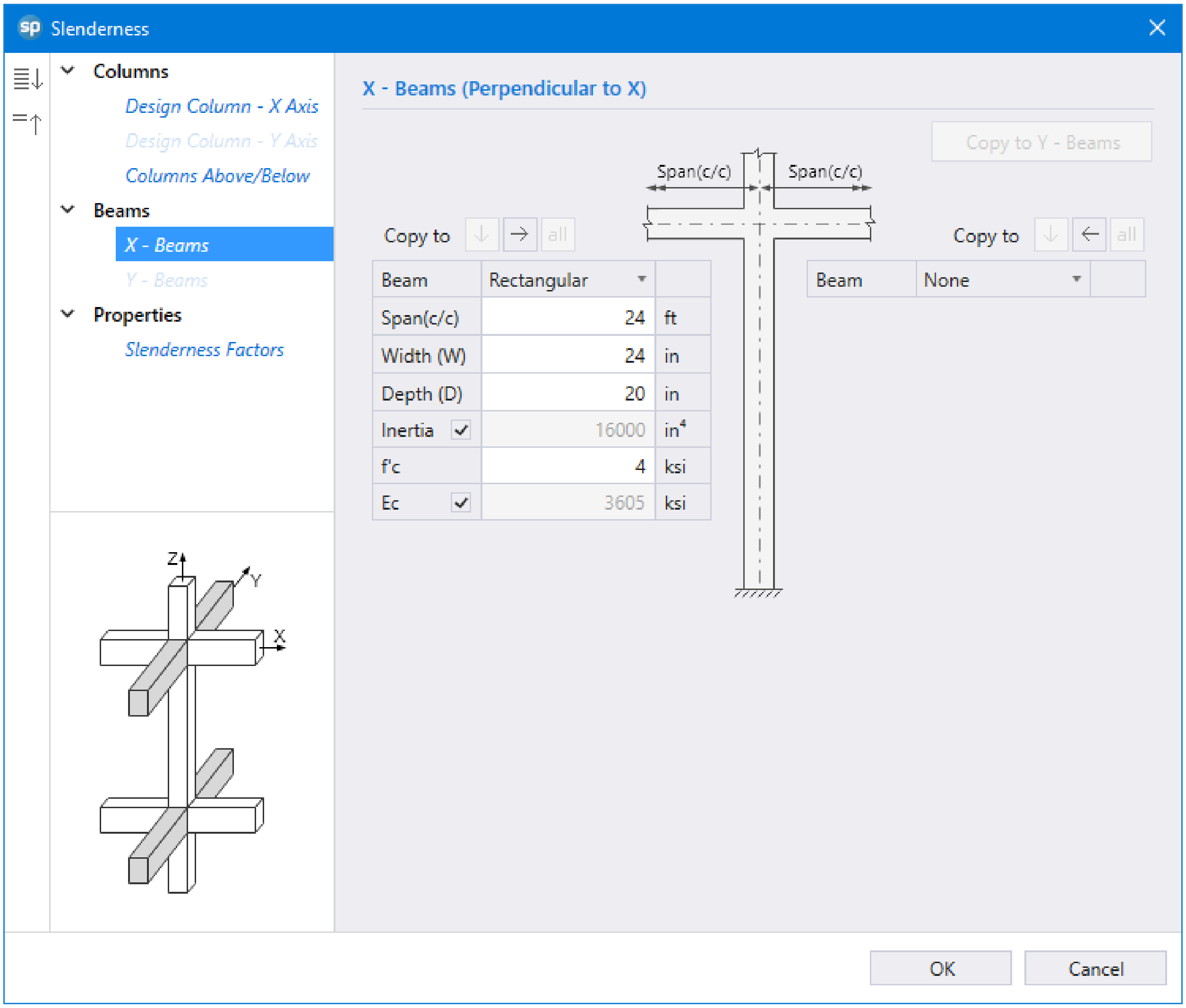


Figure 12 – Defining Beams in X-Direction (spColumn)

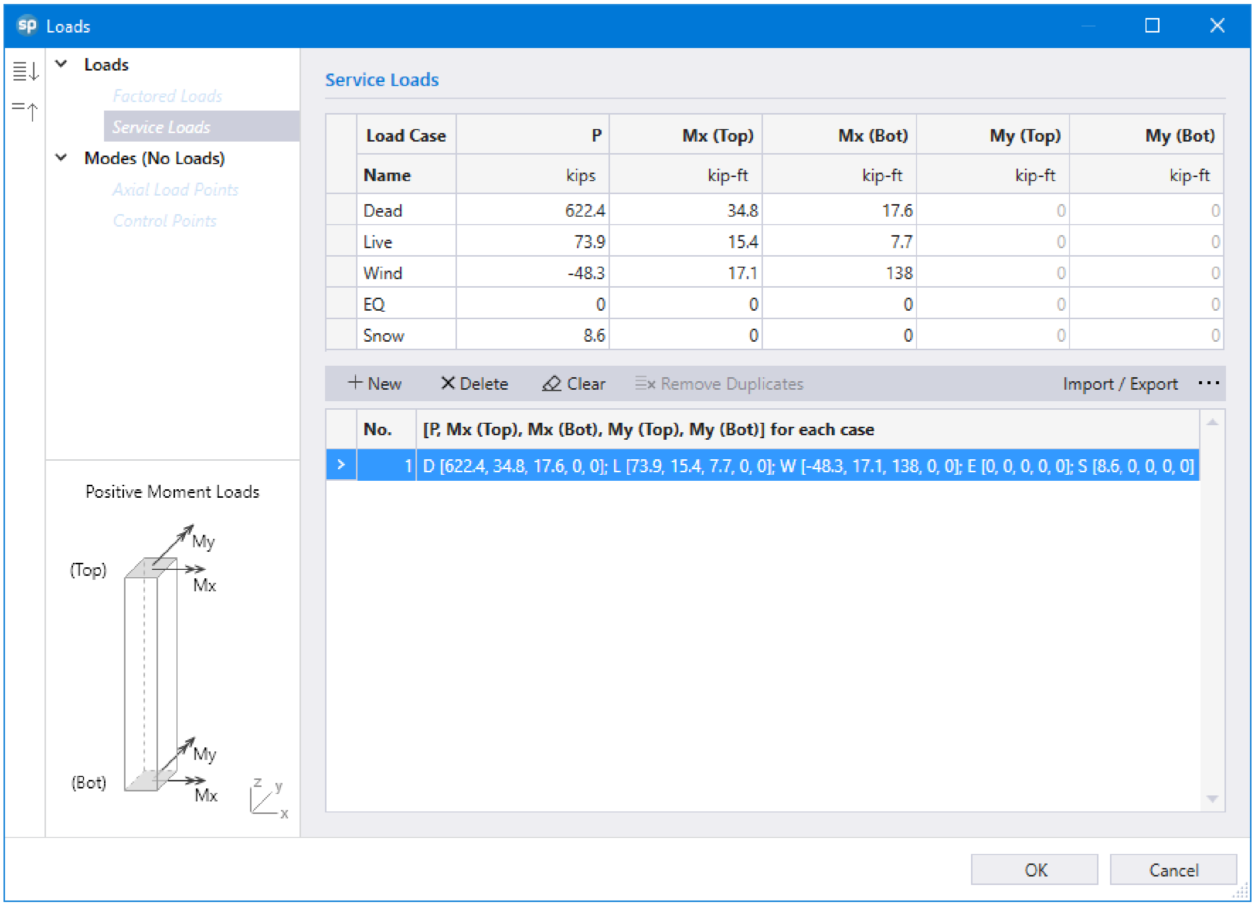


Figure 13 – Defining Loads / Modes (spColumn)

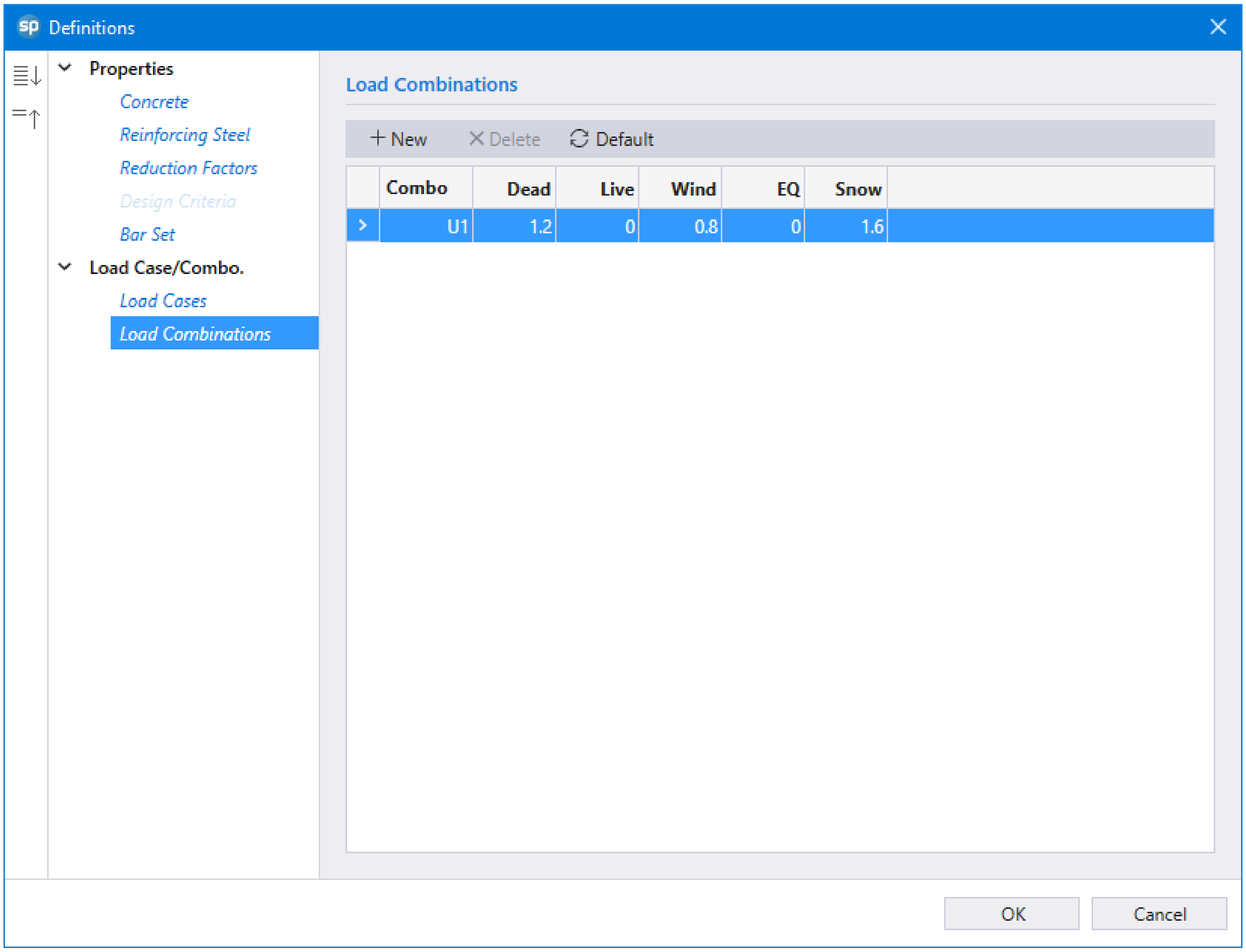


Figure 14 – Defining Load Combination #4 (spColumn)

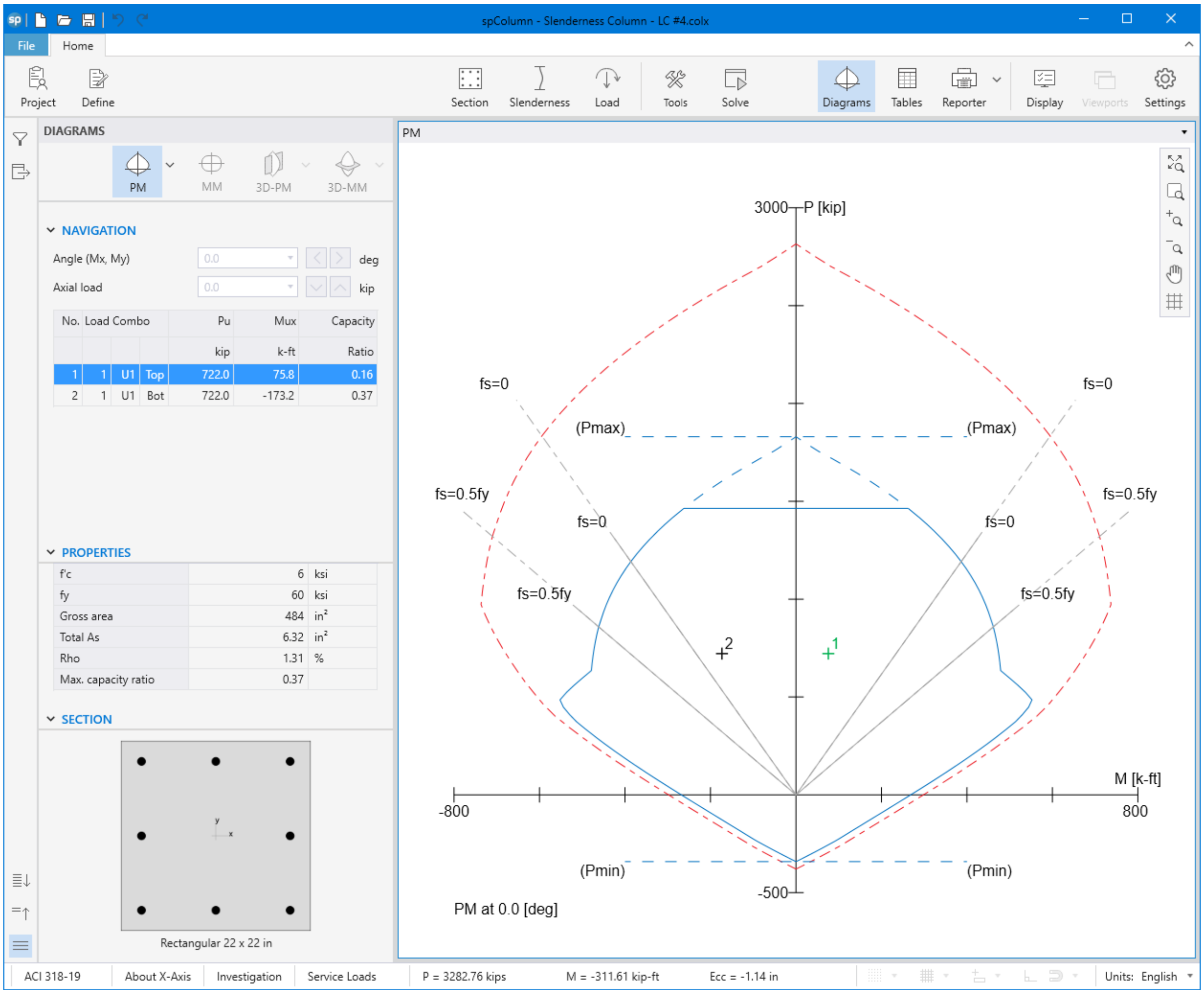
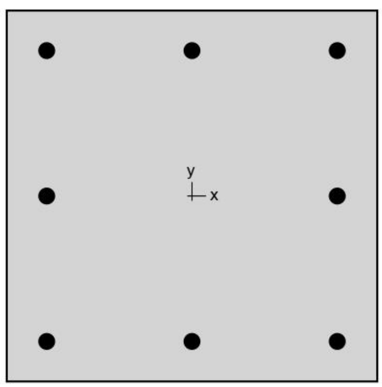


Figure 15 – Column Section Interaction Diagram about the X-Axis – Design Check for Load Combination #4 (spColumn)

spColumn v10.00 (TM)
Computer program for the Strength Design of Reinforced Concrete Sections



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List of Figures

Figure 1: Column section 4

1. General Information

File Name	E:\StructurePo...\Slenderness Column - LC #4.colx
Project	Slenderness
Column	Exterior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Considered
Column Type	Structural
Capacity Method	Moment capacity

2. Material Properties

2.1. Concrete

Type	Standard	
f'_c	6	ksi
E_c	4415.21	ksi
f_c	5.1	ksi
ε_u	0.003	in/in
β_1	0.75	

2.2. Steel

Type	Standard	
f_y	60	ksi
E_s	29000	ksi
ε_{ty}	0.00206897	in/in

3. Section

3.1. Shape and Properties

Type	Rectangular	
Width	22	in
Depth	22	in
A_g	484	in²
I_x	19521.3	in⁴
I_y	19521.3	in⁴
r_x	6.35085	in
r_y	6.35085	in
X_o	0	in
Y_o	0	in

STRUCTUREPOINT - spColumn v10.00 (TM)
Licensed to: StructurePoint, LLC. License ID: 00000-0000000-4-20FC1-20FC1
E:\StructurePoint\spColumn\Slenderness Column - LC #4.colx

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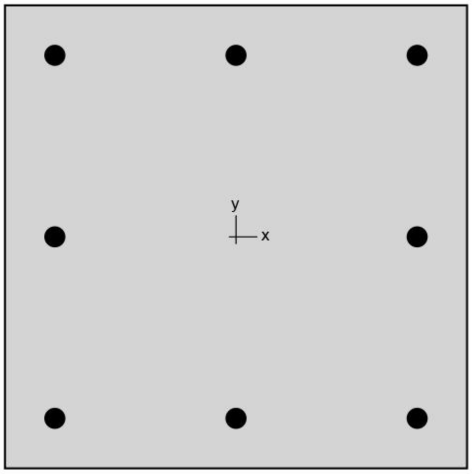
3.2. Section Figure



Rectangular 22 x 22 in 1.31% reinf.

Figure 1: Column section

4. Reinforcement

4.1. Bar Set: ASTM A615

Bar	Diameter	Area	Bar	Diameter	Area	Bar	Diameter	Area
	in	in²		in	in²		in	in²
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

4.2. Confinement and Factors

Confinement type	Tied
For #10 bars or less	#3 ties
For larger bars	#4 ties
Capacity Reduction Factors	
Axial compression, (a)	0.8
Tension controlled ϕ, (b)	0.9
Compression controlled ϕ, (c)	0.65

4.3. Arrangement

Pattern	All sides equal
Bar layout	Rectangular
Cover to	Transverse bars
Clear cover	1.5 in
Bars	8 #8

Total steel area, A_s	6.32 in²
Rho	1.31 %
Minimum clear spacing	7.63 in

5. Loading

5.1. Load Cases

Case	Type	Sustained Load
		%
A	Dead	100
B	Live	0
C	Wind	0
D	EQ	0
E	Snow	0

5.2. Load Combinations

Combination	Dead	Live	Wind	EQ	Snow
U1	1.200	0.000	0.800	0.000	1.600

5.3. Service Loads

No.	Load Case	Axial Load	Mx @ Top	Mx @ Bottom	My @ Top	My @ Bottom
		kip	k-ft	k-ft	k-ft	k-ft
1	Dead	622.40	34.80	17.60	0.00	0.00
1	Live	73.90	15.40	7.70	0.00	0.00
1	Wind	-48.30	17.10	138.00	0.00	0.00
1	EQ	0.00	0.00	0.00	0.00	0.00
1	Snow	8.60	0.00	0.00	0.00	0.00

6. Slenderness

6.1. Sway Criteria

X-Axis	Sway column
2nd order effects along length	Considered
$\sum P_c$	28.82 x P_c
$\sum P_u$	30.34 x P_u

6.2. Columns

Column	Axis	Height	Width	Depth/Dia.	I	f'c	Ec
		ft	in	in	in⁴	ksi	ksi
Design	X	13.333	22	22	19521.3	6	4415.21
Above	X	12	22	22	19521.3	6	4415.21
Below	X	(no column specified...)					

6.3. X - Beams

Beam	Length	Width	Depth	I	f'c	Ec
	ft	in	in	in⁴	ksi	ksi
Above Left	24	24	20	16000	4	3605
Above Right	(no beam specified...)					
Below Left	Rigid beam					
Below Right	Rigid beam					

7. Moment Magnification

7.1. General Parameters

Factors	Code defaults
Stiffness reduction factor, ϕ_K	0.75
Cracked section coefficients, cI(beams)	0.35
Cracked section coefficients, cI(columns)	0.7
$0.2 E_c I_g + E_s I_{se}$ (X-axis)	2.75e+007 kip-in²
Minimum eccentricity, $e_{x\ min}$	1.26 in

7.2. Effective Length Factors

Axis	Ψ_{top}	Ψ_{bottom}	k (Nonsway)	k (Sway)	kl_u/r
X	11.040	0.000	0.690	1.693	42.65

7.3. Magnification Factors: X - axis

Load Combo	At Ends					Along Length					
	$\sum P_u$	P_c	$\sum P_c$	β_{ds}	δ_s	P_u	$k'l_u/r$	P_c	β_{dns}	C_m	δ
	kip	kip	kip			kip		kip			
1 U1	21906.20	3694.49	106486.42	0.000	1.378	722.00	(N/A)	11126.81	1.000	0.460	1.000

8. Factored Moments

NOTE: Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

8.1. X - axis

Load Combo			1st Order				2nd Order			Ratio
			M_{ns}	M_s	M_u	M_{min}		M_i	M_c	2nd/1st
			k-ft	k-ft	k-ft	k-ft		k-ft	k-ft	
1	U1	Top	41.76	13.68	55.44	75.81	M_1=	60.61	75.81	1.000
1	U1	Bot	-21.12	-110.40	-131.52	-75.81	M_2=	-173.25	-173.25	1.317

9. Control Points

About Point	P	X-Moment	Y-Moment	NA Depth	d_t Depth	ε_t	ϕ
	kip	k-ft	k-ft	in	in		
X @ Max compression	1830.0	0.00	0.00	63.24	19.63	-0.00207	0.65000
X @ Allowable comp.	1464.0	262.54	0.00	23.99	19.63	-0.00055	0.65000
X @ f_s = 0.0	1192.0	386.45	0.00	19.63	19.63	0.00000	0.65000
X @ f_s = 0.5 f_y	858.6	461.68	0.00	14.59	19.63	0.00103	0.65000
X @ Balanced point	632.2	478.99	0.00	11.61	19.63	0.00207	0.65000
X @ Tension control	476.1	554.68	0.00	7.30	19.63	0.00507	0.90000
X @ Pure bending	0.0	268.27	0.00	2.60	19.63	0.01962	0.90000
X @ Max tension	-341.3	0.00	0.00	0.00	19.63	9.99999	0.90000
-X @ Max compression	1830.0	0.00	0.00	63.24	19.63	-0.00207	0.65000
-X @ Allowable comp.	1464.0	-262.54	0.00	23.99	19.63	-0.00055	0.65000
-X @ f_s = 0.0	1192.0	-386.45	0.00	19.63	19.63	0.00000	0.65000
-X @ f_s = 0.5 f_y	858.6	-461.68	0.00	14.59	19.63	0.00103	0.65000
-X @ Balanced point	632.2	-478.99	0.00	11.61	19.63	0.00207	0.65000
-X @ Tension control	476.1	-554.68	0.00	7.30	19.63	0.00507	0.90000
-X @ Pure bending	0.0	-268.27	0.00	2.60	19.63	0.01962	0.90000
-X @ Max tension	-341.3	0.00	0.00	0.00	19.63	9.99999	0.90000

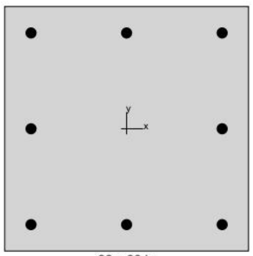
10. Factored Loads and Moments with Corresponding Capacity Ratios

NOTE: Calculations are based on "Moment Capacity" Method.
Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

No.	Load Combo			Demand P_u kip	Demand M_{ux} k-ft	Capacity ϕP_n kip	Capacity ϕM_{nx} k-ft	Parameters at Capacity NA Depth in	ε_t	ϕ	Capacity Ratio
1	1	U1	Top	722.00	75.81	722.00	474.14	12.75	0.00162	0.650	0.16
2	1	U1	Bot	722.00	-173.25	722.00	-474.14	12.75	0.00162	0.650	0.37

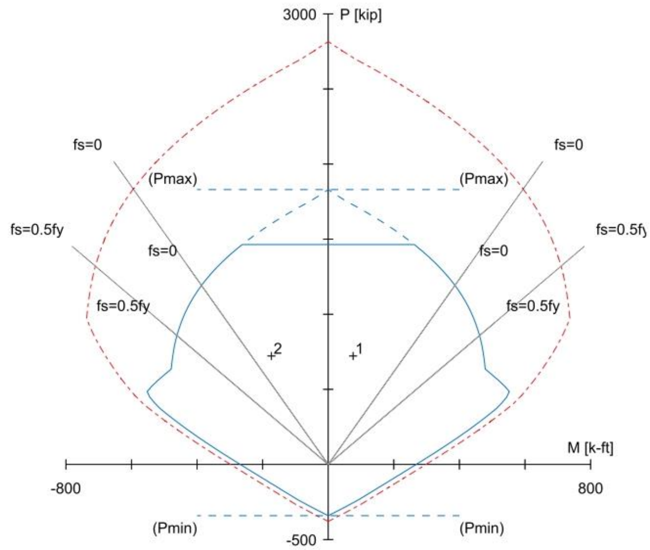
11. Diagrams

11.1. PM at θ=0 [deg]



22 x 22 in

General Information	
Project	Slenderness
Column	Exterior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	X - axis
Slenderness	Considered
Column Type	Structural
Capacity Method	Moment capacity
Materials	
f'_c	6 ksi
E_c	4415.21 ksi
f_y	60 ksi
E_s	29000 ksi
Section	
Type	Rectangular
Width	22 in
Depth	22 in
A_g	484 in²
I_x	19521.3 in⁴
I_y	19521.3 in⁴
Reinforcement	
Pattern	All sides equal
Bar layout	Rectangular
Cover to	Transverse bars
Clear cover	1.5 in
Bars	8 #8
Confinement type	Tied
Total steel area, A_s	6.32 in²
Rho	1.31 %
Min. clear spacing	7.63 in



PM at 0.0 [deg]

No.	Load Combo			P_u	M_{ux}	ϕP_n	ϕM_{nx}	Capacity
				kip	k-ft	kip	k-ft	Ratio
2	1	U1	Bot	722.0	-173.2	722.00	-474.14	0.37
1	1	U1	Top	722.0	75.8	722.00	474.14	0.16

Max. Capacity Ratio: 0.37

8. Summary and Comparison of Design Results

Table 7 - Factored Axial loads and Magnified Moments at Column Ends Comparison										
No.	P_u, kip		k_s		δ_s		$M_{1(2nd)}$, ft-kip		$M_{2(2nd)}$, ft-kip	
	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn
1	871.4	871.4	1.693	1.693	1.46	1.46	24.6	24.6	48.7	48.7
2	869.4	869.4	1.693	1.693	1.45	1.45	33.4	33.4	66.4	66.4
3	797.6	797.6	1.693	1.693	1.40	1.40	25.0	25.0	49.5	49.5
4	722.0	722.0	1.693	1.693	1.38	1.38	60.7	60.6	173.2	173.3
5	799.3	799.3	1.693	1.693	1.38	1.38	22.9	22.9	-131.0	-131.0
6	710.9	710.9	1.693	1.693	1.39	1.40	87.7	87.6	332.9	332.9
7	865.4	865.4	1.693	1.693	1.39	1.40	11.3	11.3	-282.9	-283.0
8	482.9	482.9	1.693	1.693	1.25	1.25	65.6	65.6	292.4	292.4
9	637.4	637.4	1.693	1.693	1.25	1.25	-3.0	-3.0	-260.8	-260.7

Table 8 - Magnified Moments along Column Length to First-Order Moment Ratios Comparison												
No.	k_{ns}		δ		M_{c1}, ft-kip		M_{c2}, ft-kip		$M_{c1}/M_{1(1st)}$		$M_{c2}/M_{2(1st)}$	
	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn
1	0.690	0.690	1.00	1.00	91.5	91.5	91.5	91.5	1.00	1.00	1.00	1.00
2	0.690	0.690	1.00	1.00	91.3	91.3	91.3	91.3	1.00	1.00	1.00	1.00
3	0.690	0.690	1.00	1.00	83.7	83.7	83.7	83.7	1.00	1.00	1.00	1.00
4	0.690	0.690	1.00	1.00	75.8	75.8	173.2	173.3	1.00	1.00	1.32	1.32
5	0.690	0.690	1.00	1.00	83.9	83.9	-131.0	-131.0	1.00	1.00	1.47	1.47
6	0.690	0.690	1.00	1.00	87.7	87.6	333.0	332.9	1.14	1.14	1.36	1.36
7	0.690	0.690	1.00	1.00	90.9	90.9	-283.0	-283.0	1.00	1.00	1.45	1.45
8	0.690	0.690	1.00	1.00	65.6	65.6	292.4	292.4	1.12	1.12	1.24	1.24
9	0.690	0.690	1.00	1.00	66.9	66.9	-260.8	260.7	1.00	1.00	1.27	1.27

Table 9 - Design Parameters Comparison										
No.	c, in.		$\varepsilon_t = \varepsilon_s$		ϕ		ϕP_n, kip		ϕM_n, ft-kip	
	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn	Hand	spColumn
1	14.85	14.85	0.00097	0.00097	0.65	0.65	871.4	871.4	459.4	459.4
2	14.82	14.82	0.00097	0.00097	0.65	0.65	869.4	869.4	459.7	459.7
3	13.75	13.75	0.00128	0.00128	0.65	0.65	797.6	797.6	468.2	468.2
4	12.75	12.75	0.00162	0.00162	0.65	0.65	722.0	722.0	474.1	474.1
5	13.78	13.78	0.00127	0.00127	0.65	0.65	799.3	799.3	468.0	468.0
6	12.61	12.61	0.00167	0.00167	0.65	0.65	710.9	710.9	474.9	474.9
7	14.76	14.76	0.00099	0.00099	0.65	0.65	865.4	865.4	460.2	460.2
8	7.41	7.41	0.00495	0.00495	0.89	0.89	482.9	482.9	552.9	552.9
9	11.68	11.68	0.00204	0.00204	0.65	0.65	637.4	637.4	478.8	478.8

In all of the hand calculations illustrated above, the results are in precise agreement with the automated exact results obtained from the spColumn program.

9. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames.

ACI 318 provides multiple options for calculating values of k, $(EI)_{eff}$, δ_s, and δ leading to variability in the determination of the adequacy of a column section. Engineers must exercise judgment in selecting suitable options to match their design condition. The spColumn program utilizes the exact methods whenever possible and allows user to override the calculated values with direct input based on their engineering judgment wherever it is permissible.

In load combinations 5 and 7, M_u including second-order effects exceeds 1.4 M_u due to first-order effects (see Table 5). This indicates that in this building, the weight of the structure is high in proportion to its lateral stiffness leading to excessive $P\Delta$ effect (secondary moments are more than 25 percent of the primary moments). The $P\Delta$ effects will eventually introduce singularities into the solution to the equations of equilibrium, indicating physical structural instability. It was concluded in the literature that the probability of stability failure increases rapidly when the stability index Q exceeds 0.2, which is equivalent to a secondary-to-primary moment ratio of 1.25. The maximum value of the stability coefficient θ (according to ASCE/SEI 7) which is close to stability coefficient Q (according to ACI 318) is 0.25. The value 0.25 is equivalent to a secondary-to-primary moment ratio of 1.33. Hence, the upper limit of 1.4 on the secondary-to-primary moment ratio was selected by the ACI 318.

ACI 318 provides three equation options to calculate the effective stiffness modulus $(EI)_{eff}$ as was discussed previously in this document. Equation 6.6.4.4.4(b) is more accurate than equation 6.6.4.4(a) but is more difficult to use because I_{se} is not known until reinforcement is chosen. spColumn uses equation 6.6.4.4.4(b) since an iterative procedure is used to select the optimum reinforcement configuration.

As can be seen in Table 5 of this example, exploring the impact of other code permissible equation options provides the engineer added flexibility in decision making regarding design. For load combinations 5 and 7 resolving the stability concern may be viable through a frame analysis providing values for V_{us} and Δ_o to calculate magnification factor δ_s and may allow the proposed design to be acceptable. Creating a complete model with detailed lateral loads and load combinations to account for second order effects may not be warranted for all cases of slender column design nor is it disadvantageous to have a higher margin of safety when it comes to column slenderness and frame stability considerations.