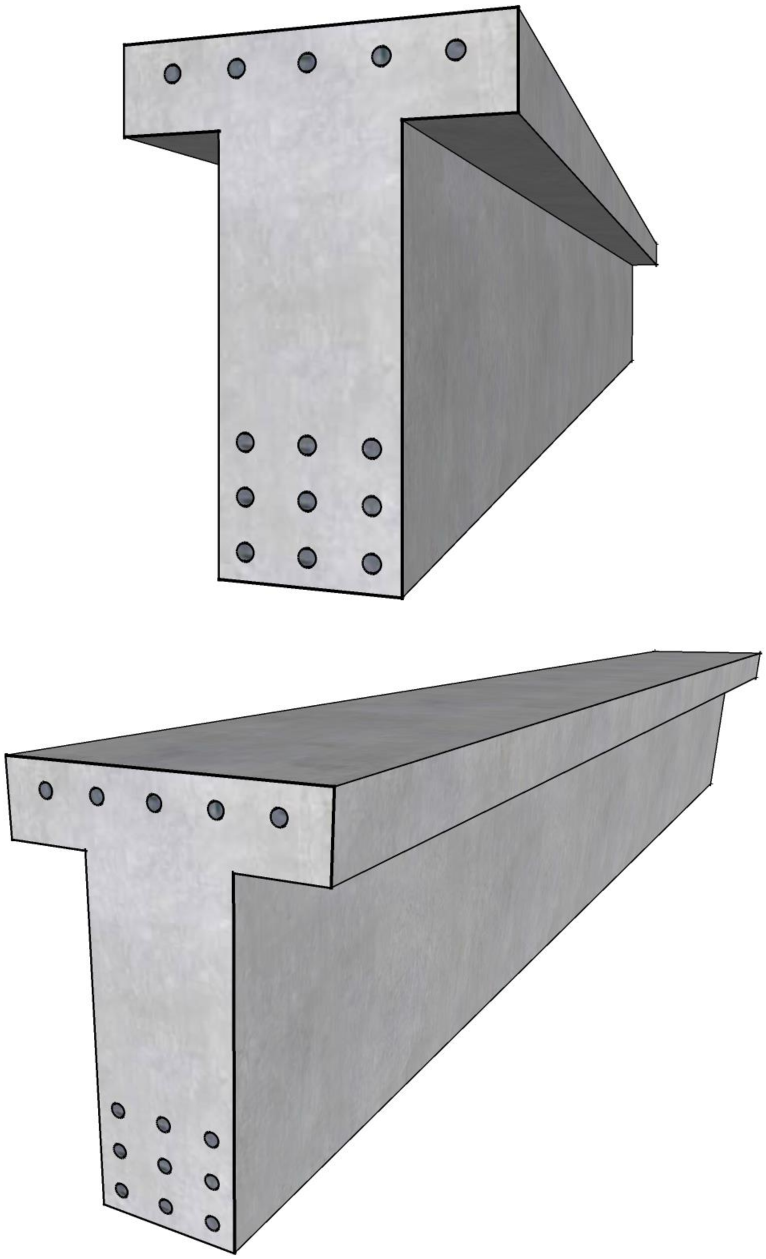
Moment Strength of Flanged Reinforced Concrete Beam (ACI 318-14)



Version: October-05-2021

Moment Strength of Flanged Reinforced Concrete Beam (ACI 318-14)

Determining the flexural design strength of the isolated T-section shown in Figure 1 in accordance with the ACI 318-14 code. Tension reinforcement is 9-#11 with three bars in each of the three layers. The T-section is also reinforced with 5-#8 bars as compression reinforcement and #4 stirrups. Concrete compressive strength f_c' is 4,000 psi and reinforcement yield strength f_y is 60,000 psi. Compare the calculated values in the Reference and the hand calculations with values obtained by spBeam engineering software program from StructurePoint.

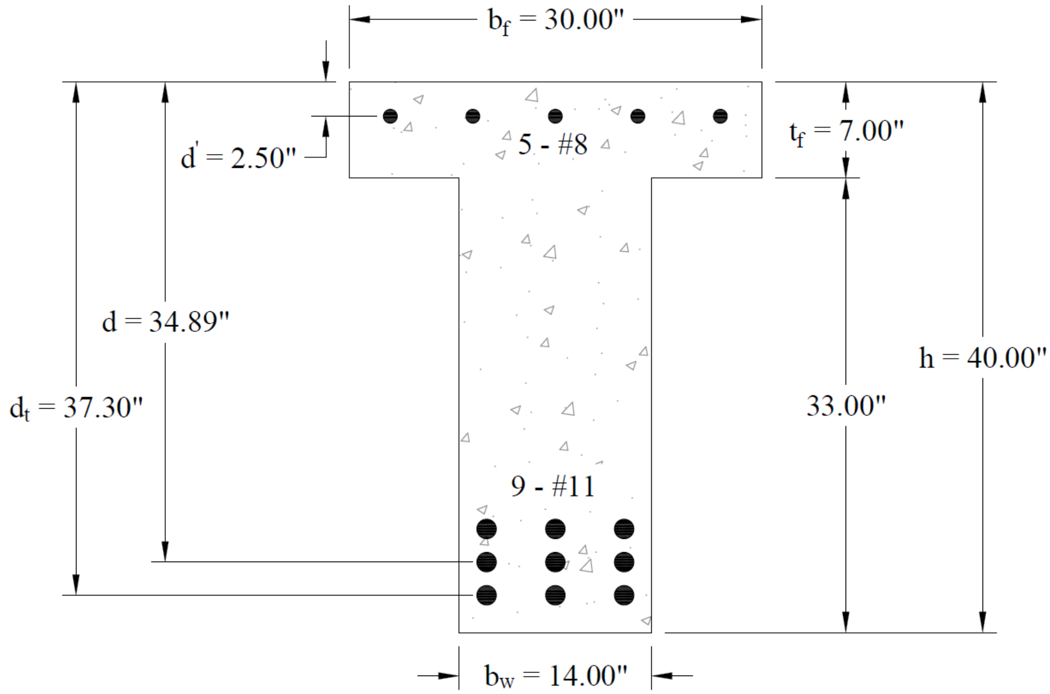


Figure 1 – Flanged Reinforced Concrete Beam Cross-Section

Contents

Code

- Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

[1] Reinforced Concrete Design, 8th Edition, 2017, Chu-Kia Wang, Charles G. Salmon, Jose A. Pincheira, Gustavo J. Parra-Montesinos, Oxford University Press, Example 4.4.4.

[2] spBeam Engineering Software Program Manual v5.50, StructurePoint LLC., 2018.

Design Data

f_c'= 4,000 psi normal weight concrete

f_y = 60,000 psi

Beam overall height, h = 40 in. (Including flange thickness)

Flange thickness, t_f = 7 in.

Flange width, b_f = 30 in.

Clear cover = 1.5 in. (For tension reinforcement)

Tension reinforcement (9-#11), $A_s = 9\times1.56 \text{ in.}^2 = 14.04 \text{ in.}^2$

Compression reinforcement (5-#8), $A_s = 5\times0.79 \text{ in.}^2 = 3.95 \text{ in.}^2$, centroid of compression reinforcement is located 2.5 in. from top of the flange.

Solution

1. Effective Flange Width

Determining the effective flange width following ACI 318-14 (6.3.2.2), the effective flange width b_f will be:

$b_f \leq 4b_w$ ***ACI 318-14 (6.3.2.2)***

$b_f = 30 \text{ in.} \leq 4b_w = 4\times14 \text{ in.} = 56 \text{ in.}$

$t_f \geq 0.5b_w$ ***ACI 318-14 (6.3.2.2)***

$t_f = 7 \text{ in.} \leq 0.5b_w = 0.5\times14 \text{ in.} = 7 \text{ in.}$

Therefore, flange width and flange thickness are satisfactory to ACI 318-14 (6.3.2.2).

2. Flanged Section Analysis

Determining the effective depth and calculating the stress block depth, assuming rectangular section behavior.

For a clear cover of 1.5 in. and #4 stirrups:

$$d = h - \text{ clear cover} - \text{stirrup diameter} - 1.5 \times \text{bar diameters} - \text{ spacing between layers}$$

$$d = 40\text{ in.} - 1.5\text{ in.} - 0.5\text{ in.} - 1.5 \times 1.41\text{ in.} - 1\text{ in.} = 34.89\text{ in.}$$

Rectangular section behavior is assumed where the stress block depth "a" is less than the flange thickness (a<t_f) and yielding of the reinforcement is expected.

$$a = \frac{A_s \times f_y - A_s^{'} \times \left(f_y - 0.85 \times f_c^{'}\right)}{0.85 \times f_c^{'} \times b_f}$$

$$a = \frac{14.04\text{ in.}^2 \times 60\text{ ksi} - 3.95\text{ in.}^2 \times \left(60\text{ ksi} - 0.85 \times 4\text{ ksi}\right)}{0.85 \times 4\text{ ksi} \times 30\text{ in.}} = 6.07\text{ in.} < t_f = 7\text{ in.}$$

The stress in the compression steel has been adjusted to account for the area of displaced concrete in compression. Verifying that for the calculated stress block depth, the compression steel reinforcement located at the flange yields as assumed.

Since f_c'= 4,000 psi:

$\beta_1 = 0.85$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$c = \frac{a}{\beta_1} = \frac{6.07\text{ in.}}{0.85} = 7.14\text{ in.}$$

$$\varepsilon_s^{'} = 0.003 \times \frac{c - d^{'}}{c} = 0.003 \times \frac{7.14\text{ in.} - 2.5\text{ in.}}{7.14\text{ in.}} = 0.00195 < \varepsilon_y = 0.00207$$

Compression reinforcement does not yield; therefore, equilibrium needs to be checked.

The internal forces are:

$$T = A_s \times f_y = 14.04\text{ in.}^2 \times 60\text{ ksi} = 842.40\text{ kips}$$

$$C_c = 0.85 \times f_c^{'} \times b_f \times a = 0.85 \times 4\text{ ksi} \times 30\text{ in.} \times 6.07\text{ in.} = 619.14\text{ kips}$$

$$\varepsilon_s^{'} = 0.003 \times \frac{c - d^{'}}{c} = 0.003 \times \frac{7.14\text{ in.} - 2.5\text{ in.}}{7.14\text{ in.}} = 0.00195$$

$$C_s = A_s^{'} \times \left(\varepsilon_s^{'} \times E_s - 0.85 \times f_c^{'}\right) = 3.95\text{ in.}^2 \times \left(0.00195\text{ in./in.} \times 29000\text{ ksi} - 0.85 \times 4\text{ ksi}\right) = 209.89\text{ kips}$$

$\left[C = 619.14\text{ kips} + 209.89\text{ kips} = 829.03\text{ kips}\right] \neq \left[T = 842.40\text{ kips}\right]$, equilibrium is not satisfied.

Many iterations have been carried out to find the depth of compression block at which equilibrium is satisfied. The depth of the stress block "a" found to be a = 6.18 in.

$$c = \frac{a}{\beta_1} = \frac{6.18\text{ in.}}{0.85} = 7.27\text{ in.}$$

The internal forces are:

$$T = A_s \times f_y = 14.04 \text{ in.}^2 \times 60 \text{ ksi} = 842.40 \text{ kips}$$

$$C_c = 0.85 \times f_c^{'} \times b_f \times a = 0.85 \times 4 \text{ ksi} \times 30 \text{ in.} \times 6.18 \text{ in.} = 630.36 \text{ kips}$$

$$\varepsilon_s^{'} = 0.003 \times \frac{c - d^{'}}{c} = 0.003 \times \frac{7.27 \text{ in.} - 2.5 \text{ in.}}{7.27 \text{ in.}} = 0.00197$$

$$C_s = A_s^{'} \times \left(\varepsilon_s^{'} \times E_s - 0.85 \times f_c^{'}\right) = 3.95 \text{ in.}^2 \times \left(0.00197 \text{ in./in.} \times 29000 \text{ ksi} - 0.85 \times 4 \text{ ksi}\right) = 212.23 \text{ kips}$$

$\left[C = 630.36 \text{ kips} + 212.23 \text{ kips} = 842.59 \text{ kips}\right] \cong \left[T = 842.40 \text{ kips}\right]$, equilibrium is satisfied.

3. Nominal Flexural Strength

The nominal moment strength M_n can be calculated by taking moments with respect to the centroid of the tension steel.

$$M_n = C_c \times \left(d - \frac{a}{2}\right) + C_s \times \left(d - d^{'}\right)$$

$$M_n = \left[630.36 \text{ kips} \times \left(34.89 \text{ in.} - \frac{6.18 \text{ in.}}{2}\right) + 212.23 \text{ kips} \times \left(34.89 \text{ in.} - 2.5 \text{ in.}\right)\right] \times \frac{1 \text{ ft}}{12 \text{ in.}}$$

$$M_n = 1670.45 \text{ ft-kips} + 572.84 \text{ ft-kips} = 2243.29 \text{ ft-kips}$$

4. Design Flexural Strength

In order to calculate the net tensile strain ε_t at the extreme tension steel. the distance d_t to the extreme tension steel is:

$$d_t = d + \frac{1.41 \text{ in.}}{2} + 1 \text{ in.} + \frac{1.41 \text{ in.}}{2} = 34.89 \text{ in.} + 2.41 \text{ in.} = 37.3 \text{ in.}$$

$$\varepsilon_t = 0.003 \times \frac{d_t - c}{c} = 0.003 \times \frac{37.3 \text{ in.} - 7.27 \text{ in.}}{7.27 \text{ in.}} = 0.0123 > 0.005$$

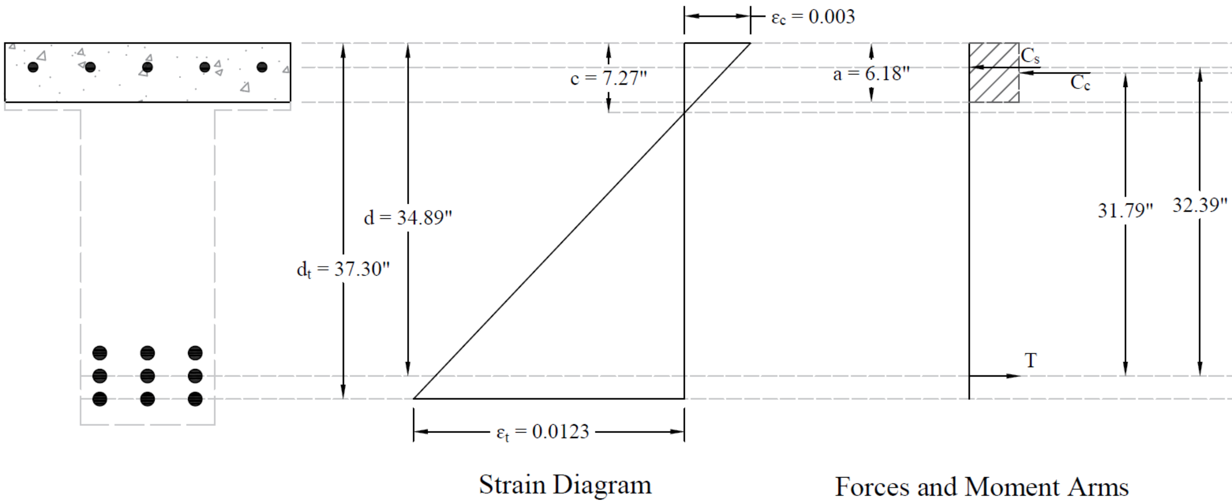


Figure 2 – Strain diagram and internal forces

Therefore: ϕ = 0.90 (function of the extreme-tension layer of bars strain) ***ACI 318-14 (21.2.1)***

$$\phi M_n = 0.9 \times 2243.29 \text{ ft-kips} = 2018.96 \text{ ft-kips}$$

As the compression reinforcement has significant impact on the ductility of beams, the net tensile strain ε_t nearly doubled compared to that of the same section without compression steel. therefore, the addition of compression steel, results in marginal contribution on the flexural strength.

5. Moment Strength of Flanged Reinforced Concrete Beam – spBeam Software

spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest American (ACI 318) and Canadian (CSA A23.3) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

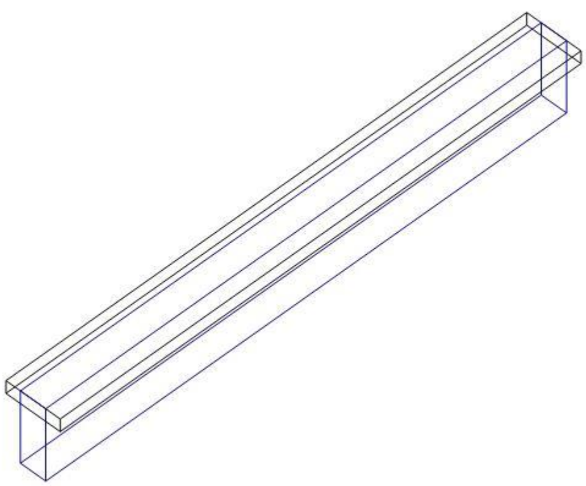
Beam analysis and design requires engineering judgment in most situations to properly simulate the behavior of the targeted beam and take into account important design considerations such as: designing the beam as rectangular or T-shaped sections; using the effective flange width or the center-to-center distance between the beam and the adjacent beams. Regardless which of these options is selected, spBeam provide users with options and flexibility to:

1. Design the beam as a rectangular cross-section or a T-shaped section.
2. Use the effective or full beam flange width.
3. Include the flanges effects in the deflection calculations.
4. Invoke moment redistribution to lower negative moments
5. Using gross (uncracked) or effective (cracked) moment of inertia
6. Design the beam as singly or doubly reinforced section.

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the beam covered in this design example.

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the beam covered in this design example.

spBeam v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams and One-way Slab Systems



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Contents

1. Input Echo

1.1. General Information

File Name	...\Strength of T Section (Investigation) 4.4....
Project	Strength of T Section (Investigation) 4.4.4 - ACI318 -14
Frame	Example 4.4.4
Engineer	SP
Code	ACI 318-14
Reinforcement Database	ASTM A615
Mode	Investigation
Number of supports =	2
Floor System	One-Way/Beam

1.2. Solve Options

Live load pattern ratio = 0%
Deflections are based on gross section properties.
Long-term deflections are NOT calculated.
Compression reinforcement calculations selected.
Default incremental rebar design selected.
Moment redistribution NOT selected.
Effective flange width calculations selected.
Rigid beam-column joint NOT selected.
Torsion analysis and design NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	150	lb/ft³
f'_c	4	ksi
E_c	3834.3	ksi
f_r	0.47434	ksi

1.3.2. Concrete: Columns

w_c	150	lb/ft³
f'_c	4	ksi
E_c	3834.3	ksi
f_r	0.47434	ksi

1.3.3. Reinforcing Steel

f_y	60	ksi
f_{yt}	60	ksi
E_s	29000	ksi
Epoxy coated bars	No	

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#3	0.38	0.11	0.38	#4	0.50	0.20	0.67

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Size	Db	Ab	Wb	Size	Db	Ab	Wb
	in	in²	lb/ft		in	in²	lb/ft
#5	0.63	0.31	1.04	#6	0.75	0.44	1.50
#7	0.88	0.60	2.04	#8	1.00	0.79	2.67
#9	1.13	1.00	3.40	#10	1.27	1.27	4.30
#11	1.41	1.56	5.31	#14	1.69	2.25	7.65
#18	2.26	4.00	13.60				

1.5. Span Data

1.5.1. Slabs

Span	Loc	L1	t	wL	wR	bE_{ff}	H_{min}
		ft	in	ft	ft	in	in
1	Int	24.000	7.00	1.250	1.250	30.00	0.00

1.5.2. Ribs and Longitudinal Beams

Span	Ribs			Beams		Span
	b	h	Sp	b	h	H_{min}
	in	in	in	in	in	in
1	0.00	0.00	0.00	14.00	40.00	18.00

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	in	in	ft	in	in	ft	
1	28.00	28.00	0.000	28.00	28.00	0.000	100
2	28.00	28.00	0.000	28.00	28.00	0.000	100

1.6.2. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kip/in	kip-in/rad		
1	0	0	Pinned	Pinned
2	0	0	Pinned	Pinned

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.200	1.600

1.7.2. Line Loads

Case/Patt	Span	Wa	La	Wb	Lb
		lb/ft	ft	lb/ft	ft
Live	1	17500.00	0.000	17500.00	24.000

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#3	#4	#3	#4

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar spacing	in	1.00	18.00	1.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	in	3.00		3.00	

There is NOT more than 12 in of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#3	#3	#4	#4	#4	#4
Bar spacing	in	1.00	18.00	1.00	18.00	3.00	18.00
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	in	3.00		3.00			
Layer dist.	in	1.00		1.00			
No. of legs						2	2
Side cover	in					1.50	
1st Stirrup	in					3.00	

There is NOT more than 12 in of concrete below top bars.

1.9. Reinforcing Bars

1.9.1. Top Bars

Span	Left			Continuous		Right		
	Bars	Length	Cover	Bars	Cover	Bars	Length	Cover
		ft	in		in		ft	in
1				5-#8	2.00			

1.9.2. Bottom Bars

Span	Continuous		Discontinuous			
	Bars	Cover	Bars	Length	Start	Cover
		in		ft	ft	in
1	3-#11	2.00				
	3-#11	4.41				
	3-#11	6.82				

1.9.3. Transverse Reinforcement

Span	Stirrups (2 legs each unless otherwise noted)
1	65-#4 [3L] @ 4.0

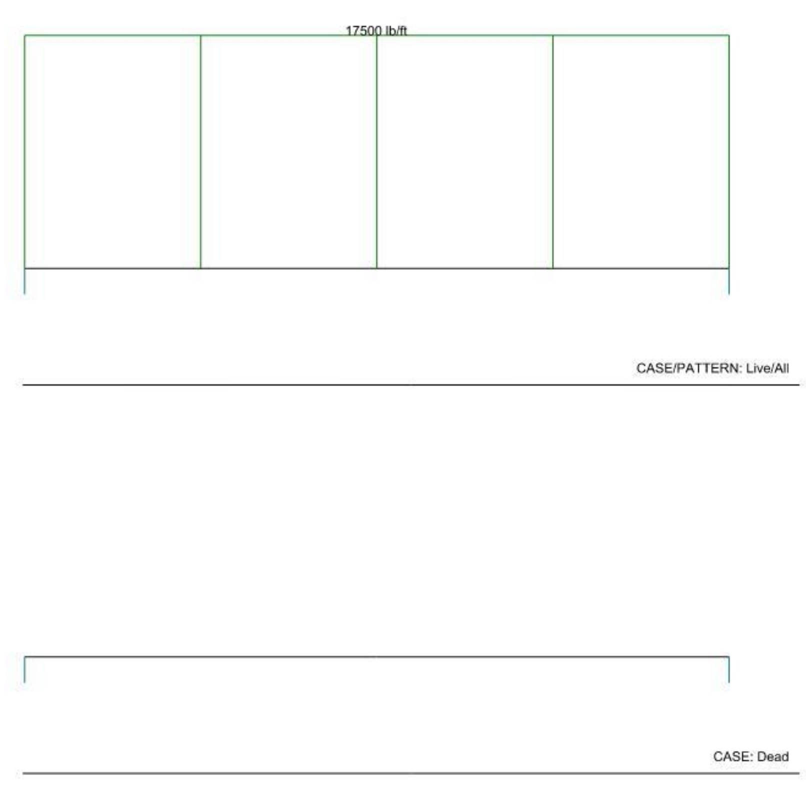
2. Design Results

2.1. Flexural Capacity

Span	Top						Bottom				
	x	$A_{s,top}$	ΦMn-	Mu-	Comb Pat	Status	$A_{s,bot}$	ΦMn+	Mu+	Comb Pat	Status
	ft	in²	k-ft	k-ft			in²	k-ft	k-ft		
1	0.000	3.95	-685.90	0.00	U1 All	---	14.04	2018.19	0.00	U1 All	---
	1.167	3.95	-685.90	0.00	U1 All	OK	14.04	2018.19	372.95	U1 All	OK
	8.750	3.95	-685.90	0.00	U1 All	OK	14.04	2018.19	1867.91	U1 All	OK
	12.000	3.95	-685.90	0.00	U1 All	OK	14.04	2018.19	2015.78	U1 All	OK
	12.125	3.95	-685.90	0.00	U1 All	OK	14.04	2018.19	2015.78	U1 All	OK
	15.250	3.95	-685.90	0.00	U1 All	OK	14.04	2018.19	1867.91	U1 All	OK
	22.833	3.95	-685.90	0.00	U1 All	OK	14.04	2018.19	372.95	U1 All	OK
	24.000	3.95	-685.90	0.00	U1 All	---	14.04	2018.19	0.00	U1 All	---

3. Diagrams

3.1. Loads



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Project: Strength of T Section (Investigation) 4.4.4 - ACI318 -14

Frame: Example 4.4.4

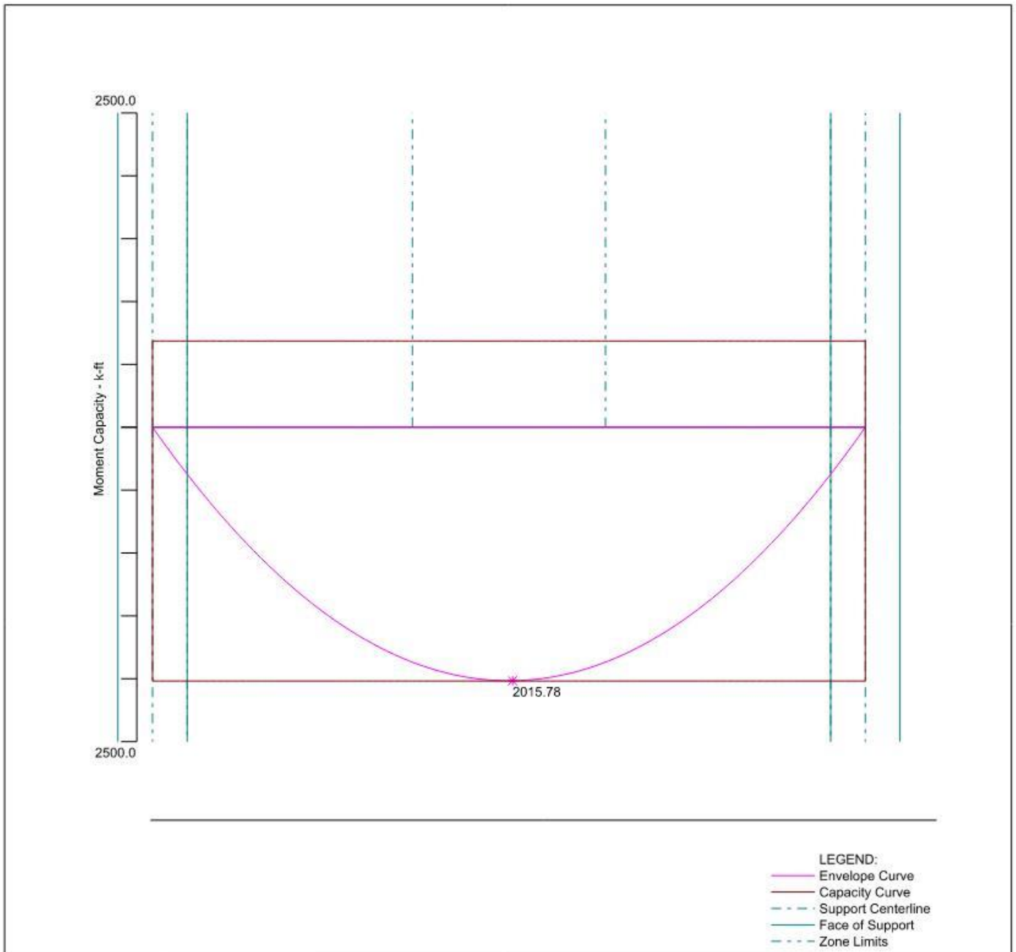
Engineer: SP

Code: ACI 318-14

Date: 10/02/21

Time: 13:12:13

3.2. Moment Capacity



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Project: Strength of T Section (Investigation) 4.4.4 - ACI318 -14

Frame: Example 4.4.4

Engineer: SP

Code: ACI 318-14

Date: 10/02/21

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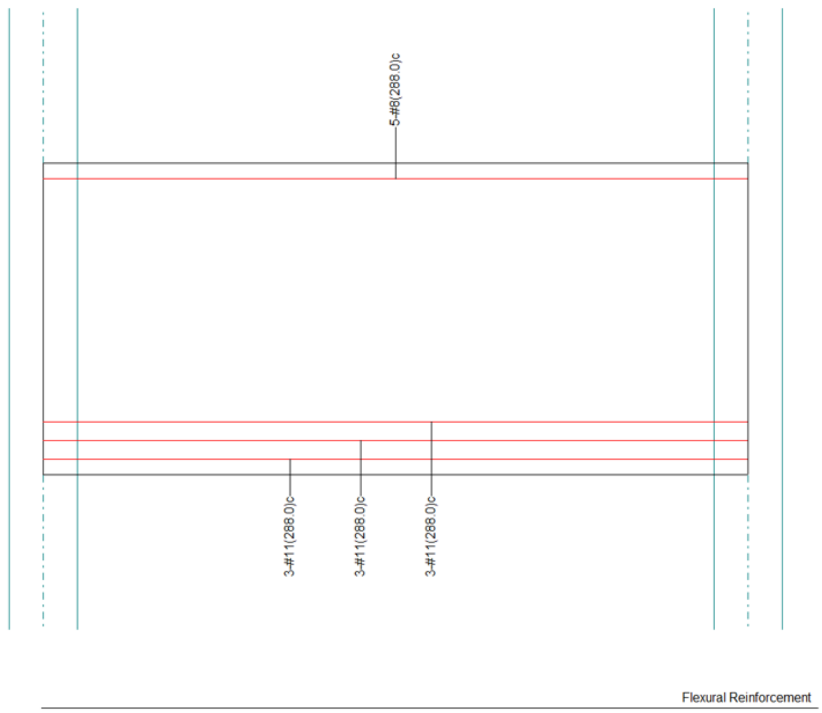
3.3. Reinforcement



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Project: Strength of T Section (Investigation) 4.4.4 - ACI318 -14

Frame: Example 4.4.4

Engineer: SP

Code: ACI 318-14

Date: 10/02/21

Time: 13:12:13

4. Comparison of Analysis Results

Table 1 - Comparison of Results				
Method	**$A_{s,provided}$**, in.^{2}	**$A'_{s,provided}$**, in.^{2}	**b_f**, in.	**ϕM_n**, kip-ft
Reference	14.04	3.95	30	2024.00
Hand	14.04	3.95	30	2018.96
spBeam	14.04	3.95	30	2018.19

In the hand calculations illustrated above, the results are in precise agreement with the automated exact results obtained from the spBeam program, which is attributed to the satisfactory number of iterations that were performed until exact solution was found. On the other side, reference made assumptions and avoided iterations to get the exact solution for simplicity. As performing iterations by hand can be tedious, utilizing spBeam program would lead to a potential economic and time savings.