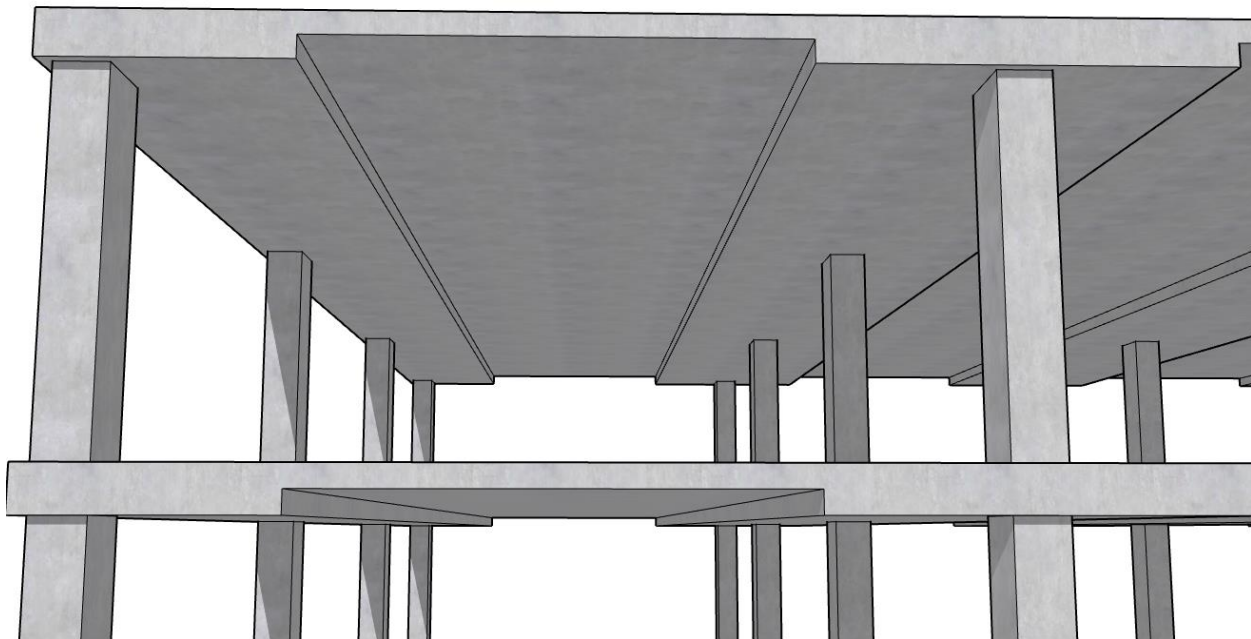
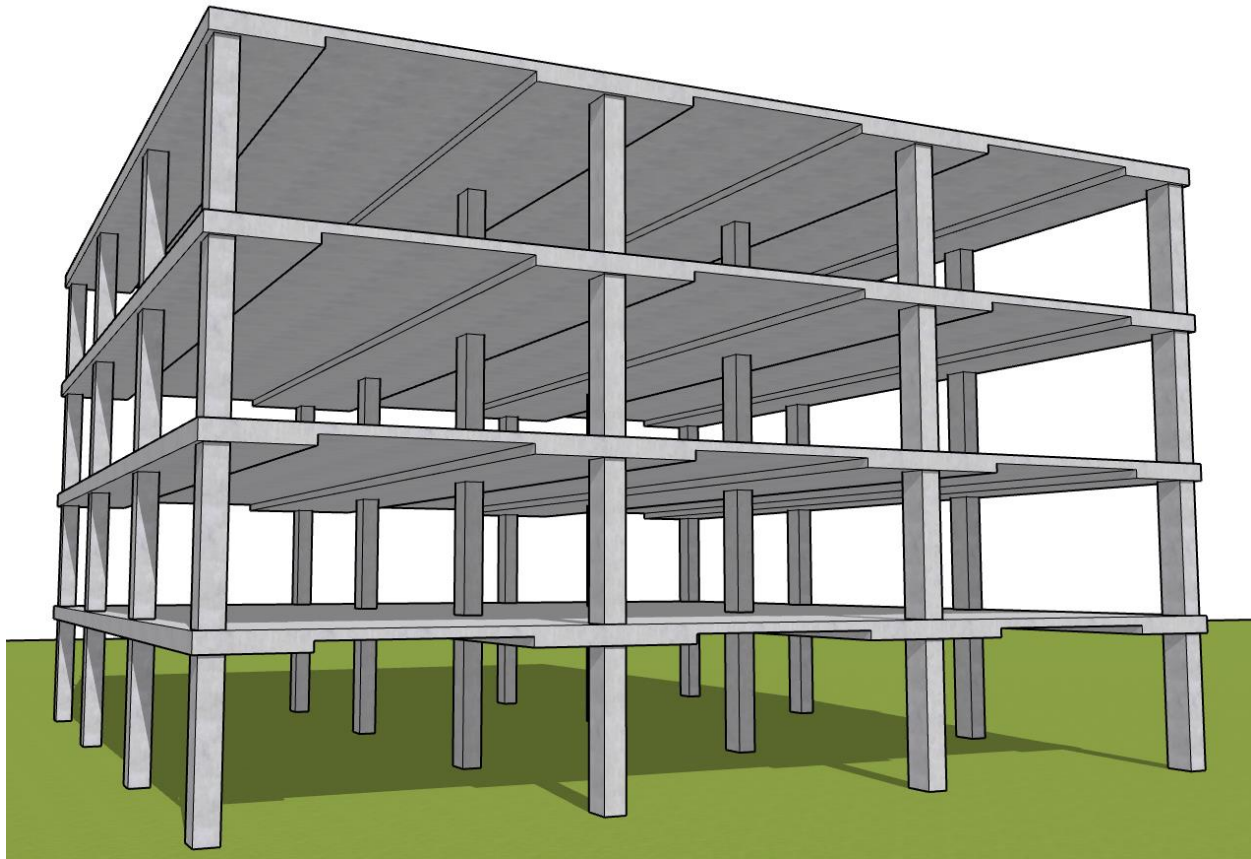


Two-Way Concrete Floor with Slab Bands – Transverse Bands Analysis & Design (CAC Design Handbook)



Two-Way Concrete Floor with Slab Bands – Transverse Bands Analysis & Design (CAC Design Handbook)

Slab bands are thickened portions extended along columns centerlines in one direction of the slab to increase the nominal strength of the concrete floor at the critical section around the columns. This system is considered more economical compared to slabs with drop panels due to the savings in the formwork and labor cost. Slab bands are sometimes viewed as continuous extension of drop panels between supports or a support and another slab band. In U.S. standards like ACI-318, slab bands are modeled as a system of wide and shallow beams in one direction.

The concrete floor system with slab bands shown below is for an intermediate floor to be designed considering loads described in design data below. The lateral loads are independently resisted by shear walls. The use of flat plate system will be checked first. If the use of flat plate is not adequate, the use of a slab system with slab bands will be investigated. The analysis procedure “Elastic Frame Method (EFM)” prescribed in [CSA A23.3-14](#) is illustrated in detail in this example (Example #3 from the CAC Design Handbook). The EFM hand solution is also used for a comprehensive comparison with results from the Reference using the Direct Design Method (DDM). The EFM hand solution results are further compared with the output from the engineering software program [spSlab](#). Explanation of the EFM is available in [StructurePoint Video Tutorials](#) page. A table comparing the three two-way slab analysis methods is provided at the end of this document.

This example will examine floor design strips with slab bands perpendicular to the direction of analysis (Transverse Bands). Floor design strips with slab bands parallel to the direction of analysis (Longitudinal Bands) are covered in detail in ([Two-Way Concrete Floor with Slab Bands – Longitudinal Bands Analysis & Design \(CAC Design Handbook\)](#)) design example.

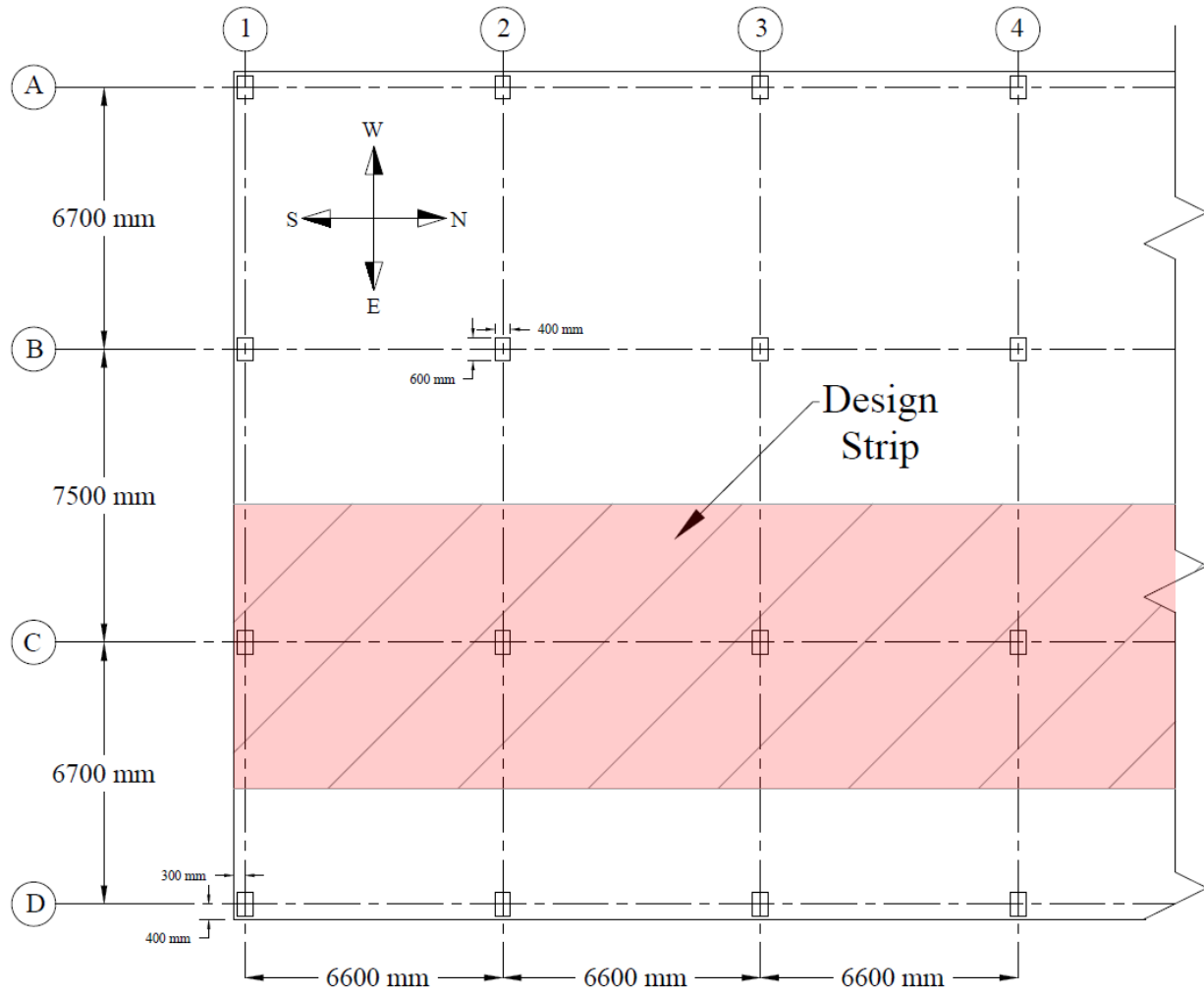


Figure 1 - Two-Way Flat Plate Concrete Floor System

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Code

Design of Concrete Structures (CSA A23.3-14)

Reference

CAC Concrete Design Handbook, 4th Edition, Cement Association of Canada

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association

Design Data

Floor-to-Floor Height = 3 m (provided by architectural drawings)

Superimposed Dead Load, $SDL = 1 \text{ kN/m}^2$ for framed partitions, wood studs plaster 2 sides
 $= 1 \text{ kN/m}^2$ for mechanical services

Live Load, $LL = 3.6 \text{ kN/m}^2$ for Residential floors

$f'_c = 25 \text{ MPa}$ (for slabs and columns)

$f'_y = 400 \text{ MPa}$

Column Dimensions = 400 mm x 600 mm

Solution

1. Preliminary Member Sizing

1.1. Preliminary Member Sizing For Slabs Without Slab Bands

1.1.1. Slab minimum thickness - Deflection

CSA A23.3-14 (13.2)

Minimum member thickness and depths from CSA A23.3-14 will be used for preliminary sizing.

Using CSA A23.3-14 minimum slab thickness for two-way construction without interior beams in *Section 13.2.3*.

Exterior Panels (N-S Direction Governs):

$$h_{s,\min} = 1.1 \times \frac{l_n (0.6 + f_y / 1000)}{30} = 1.1 \times \frac{6200(0.6 + 400 / 1000)}{30} = 227 \text{ mm} \quad \text{CSA A23.3-14 (13.2.3)}$$

But not less than 120 mm.

CSA A23.3-14 (13.2.1)

Where l_n = length of clear span in the long direction = 6600 – 400 = 6200 mm

Interior Panels (E-W Direction Governs):

$$h_{s,\min} = \frac{l_n (0.6 + f_y / 1000)}{30} = \frac{6900(0.6 + 400 / 1000)}{30} = 230 \text{ mm} \quad \text{CSA A23.3-14 (13.2.3)}$$

But not less than 120 mm.

CSA A23.3-14 (13.2.1)

Where l_n = length of clear span in the long direction = 7500 – 600 = 6900 mm

Try 250 mm slab for all panels (self-weight = 5.89 kN/m²)

1.1.2. Slab one way shear strength

Evaluate the average effective depth (Figure 2):

$$d_t = t_{slab} - c_{clear} - d_b - \frac{d_b}{2} = 250 - 25 - 16 - \frac{16}{2} = 201 \text{ mm}$$

$$d_l = t_{slab} - c_{clear} - \frac{d_b}{2} = 250 - 25 - \frac{16}{2} = 217 \text{ mm}$$

$$d_{avg} = \frac{d_t + d_l}{2} = \frac{201 + 217}{2} = 209 \text{ mm}$$

Where:

c_{clear} = 20 mm for 15M steel bar

CSA A23.3-14 (Annex A, Table 17)

Note that the reference used 25 mm as clear cover, in this example the clear cover used is 25 mm to be consistent with reference.

$d_b = 16$ mm for 15M steel bar

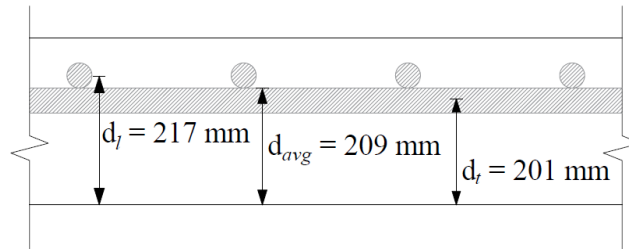


Figure 2 - Two-Way Flat Concrete Floor System

Load Combination 1:

Factored dead load, $w_{df} = 1.4 \times (5.89 + 1 + 1) = 11.05$ kN/m² CSA A23.3-14 (Annex C. Table C.1 a)

Total factored load $w_f = 11.05$ kN/m²

Load Combination 2:

Factored dead load, $w_{df} = 1.25 \times (5.89 + 1 + 1) = 9.86$ kN/m²

Factored live load, $w_{lf} = 1.5 \times 3.6 = 5.40$ kN/m² CSA A23.3-14 (Annex C. Table C.1 a)

Total factored load $w_f = w_{df} + w_{lf} = 15.26$ kN/m² (Controls)

Check the adequacy of slab thickness for beam action (one-way shear) CSA A23.3-14 (13.3.6)

At an interior column:

The critical section for one-way shear is extending in a plane across the entire width and located at a distance, d_v from the face of support or concentrated load (see Figure 3). CSA A23.3-14 (13.3.6.1)

Consider a 1 m. wide strip.

$$\text{Tributary area for one-way shear is } A_{Tributary} = \left(\frac{\left[\left(\frac{6000}{2} \right) - \left(\frac{400}{2} \right) - 188 \right] \times (1000)}{1000^2} \right) = 2.91 \text{ m}^2$$

$$V_f = w_f \times A_{Tributary} = 15.26 \times 2.91 = 44.44 \text{ kN}$$

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v \quad \text{CSA A23.3-14 (Eq. 11.6)}$$

Where:

$\lambda = 1$ for normal weight concrete CSA A23.3-14 (8.6.5)

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm CSA A23.3-14 (11.3.6.2)

$$d_v = \text{Max} (0.9d_{avg}, 0.72h) = \text{Max} (0.9 \times 209, 0.72 \times 250) = \text{Max} (188, 180) = 188 \text{ mm} \quad \text{CSA A23.3-14 (3.2)}$$

$$\sqrt{f'_c} = 5 \text{ MPa} < 8 \text{ MPa} \quad \text{CSA A23.3-14 (11.3.4)}$$

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{25} \times 1000 \times \frac{188}{1000} = 128.3 \text{ kN} > V_f$$

Slab thickness of 250 mm is adequate for one-way shear.

1.1.3. Slab two-way shear strength

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 4):

$$\text{Shear perimeter: } b_0 = 2 \times (600 + 400 + 2 \times 209) = 2836 \text{ mm} \quad \text{CSA A23.3-14 (13.3.3)}$$

Tributary area for two-way shear is

$$A_{\text{Tributary}} = \left(\frac{7.5 + 6.7}{2} \times 6.6 \right) - \left(\frac{600 + 209}{1,000} \times \frac{400 + 209}{1,000} \right) = 46.86 - 0.49 = 46.37 \text{ m}^2$$

The factored resisting shear stress, V_r shall be the smallest of:

CSA A23.3-14 (13.3.4.1)

$$\text{a) } v_r = v_c = \left(1 + \frac{2}{\beta_c} \right) 0.19 \lambda \phi_c \sqrt{f'_c} \quad \text{CSA A23.3-14 (Eq. 13.5)}$$

$$v_r = \left(1 + \frac{2}{1.5} \right) \times 0.19 \times 0.65 \times \sqrt{25} = 1.44 \text{ MPa}$$

$$\text{Where } \beta_c = \frac{600}{400} = 1.5 \text{ (ratio of long side to short side of the column)} \quad \text{CSA A23.3-14 (13.3.4.1)}$$

$$\text{b) } v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19 \right) \lambda \phi_c \sqrt{f'_c} \quad \text{CSA A23.3-14 (Eq. 13.6)}$$

$$v_r = \left(\frac{4 \times 209}{2836} + 0.19 \right) \times 1 \times 0.65 \times \sqrt{25} = 1.58 \text{ MPa}$$

$$\text{c) } v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{25} = 1.24 \text{ MPa} \quad \text{CSA A23.3-14 (Eq. 13.7)}$$

$$v_{f,ave} = \frac{V_f}{b_o d} = \frac{15.26 \times \left(\frac{7.5 + 6.7}{2} \times 6.6 \right)}{2836 \times 209} \times 1,000 = 1.206 \text{ MPa}$$

Determine the shear stress due to the factored direct shear:

$$(v_f)_{avr} = \frac{V_f}{b_o d} = \frac{15.26 \times \left(\frac{7.5 + 6.7}{2} \times 6.6 \right)}{2836 \times 209} \times 1,000 = 1.206 \text{ MPa}$$

For an interior column, multiply this value with 1.20 in order to account for the effect of unbalanced moment.

$$1.20 \times (v_f)_{avr} = 1.20 \times 1.206 = 1.45 \text{ MPa} > v_r = 1.24 \text{ MPa (No Good)}$$

CAC Concrete Design Handbook 4th Edition (5.2.3)

Slab thickness of 250 mm is **NOT** adequate for two-way shear.

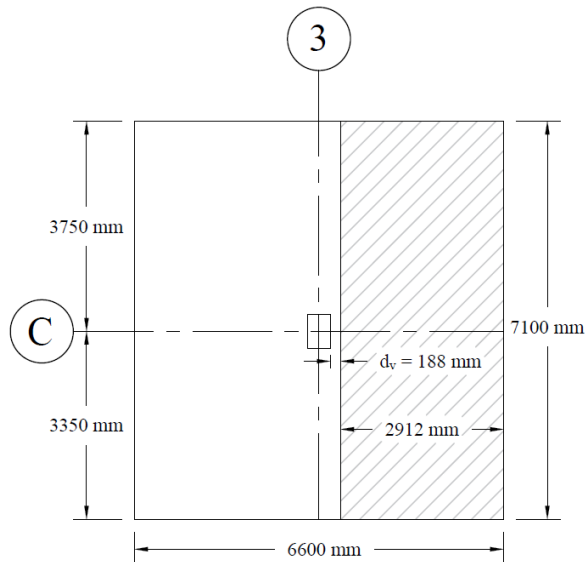


Figure 3 – Critical Section for One-Way Shear

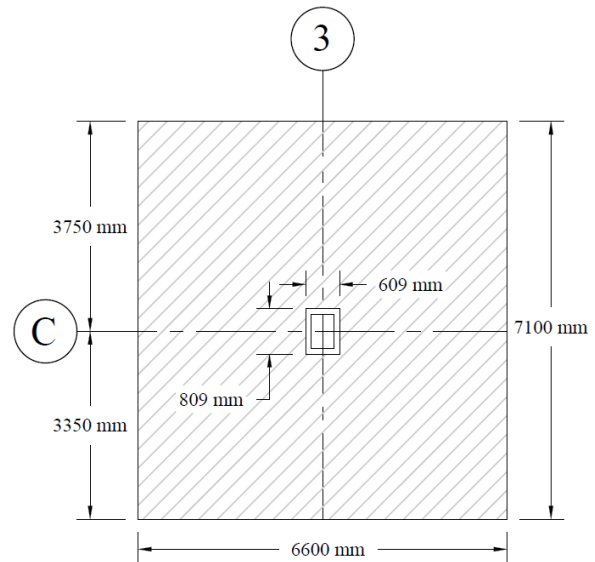


Figure 4 – Critical Section for Two-Way Shear

In this case, four options could be used: 1) to increase the slab thickness, 2) to increase column's cross sectional dimensions or cut the spacing between columns (reducing span lengths), however, this option is assumed to be not permissible in this example due to architectural limitations, 3) to use headed shear reinforcement, or 4) to use drop panels or slab bands. In this example, slab bands will be used to achieve an economical design.

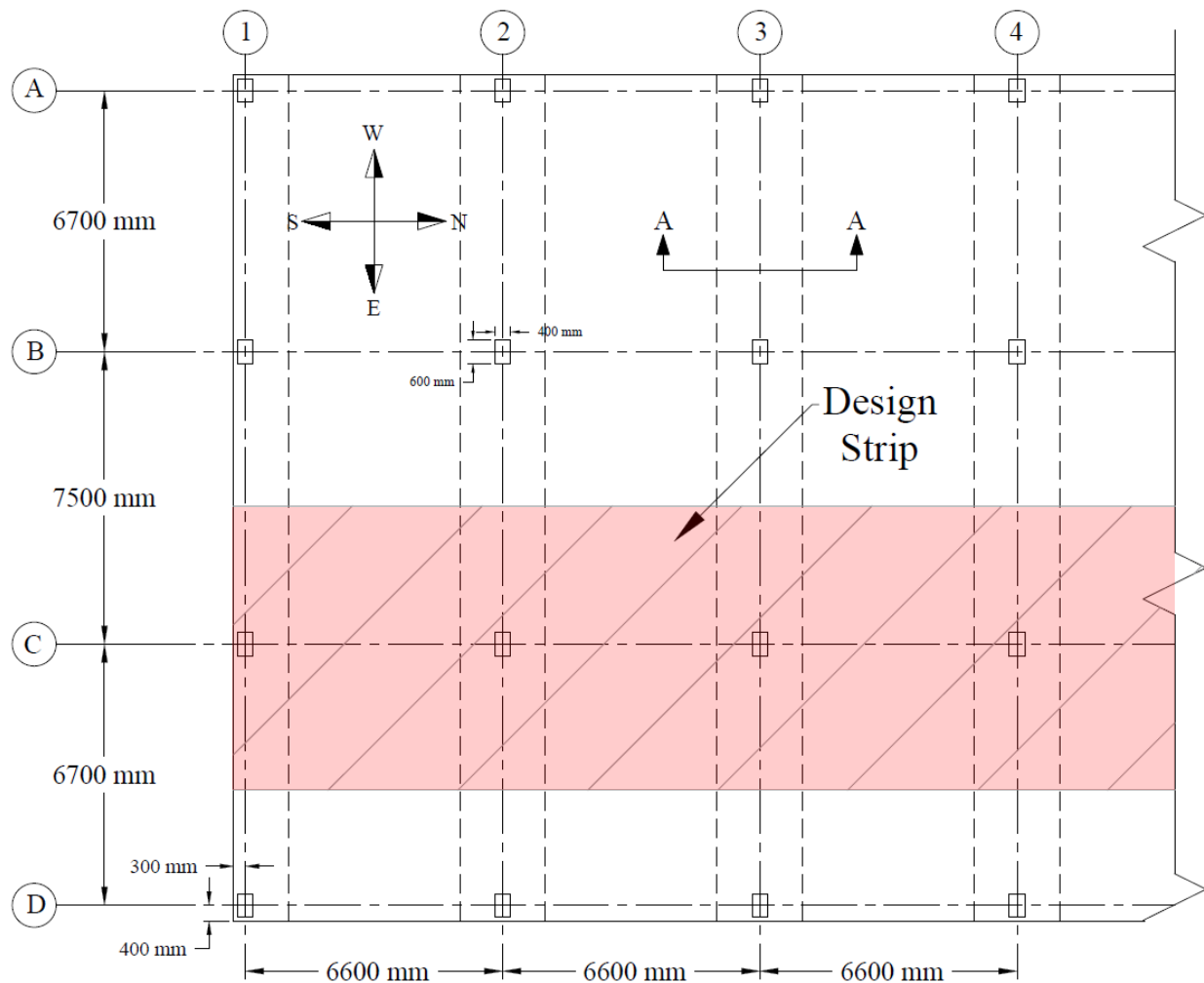


Figure 5 – Two-Way Slab with Slab Bands

1.2. Preliminary Member Sizing For Slab With Slab Bands

For slabs with changes in thickness and subjected to bending in two directions, it is necessary to check shear at multiple sections as defined in the CSA A23.3-14. The critical sections for two-way action shall be located with respect to:

- 1) Perimeter of the concentrated load or reaction area. CSA A.23.3-14 (13.3.3.1)
- 2) Changes in slab thickness, such as edges of slab bands. CSA A.23.3-14 (13.3.3.2)

1.2.1. Slab band minimum thickness (E-W direction) – Deflection

Minimum member thickness and depths from CSA A23.3-14 will be used for preliminary sizing.

Determine the slab band thickness by using CSA A23.3-14 minimum slab thickness for slab bands per *Clause 13.2.6*.

End span (Governs):

$$h_{band,min} = \frac{l_n}{18} = \frac{6100}{18} = 339 \text{ mm} \quad \text{CSA A23.3-14 (13.2.3)}$$

But not less than 120 mm. CSA A23.3-14 (13.2.1)

Where l_n = length of clear span in the long direction = 6700 – 600 = 6100 mm

Interior span:

$$h_{band,min} = \frac{l_n}{21} = \frac{6900}{21} = 329 \text{ mm} \quad \text{CSA A23.3-14 (13.2.3)}$$

But not less than 120 mm. CSA A23.3-14 (13.2.1)

Where l_n = length of clear span in the long direction = 7500 – 600 = 6900 mm

Try $h_{band} = 350$ mm slab bands for all panels

1.2.2. Slab minimum thickness (E-W direction) – Deflection

Determine the slab thickness by using CSA A23.3-14 minimum slab thickness for slabs with drop panels.

CSA A23.3-14 (13.2.4)

By definition a slab band is an extended drop panel. However, as a drop panel, the slab band is very deep. The difference between the band thickness and the slab thickness, Δ_h , is likely to exceed the slab thickness. Since, for the purposes of **Equation 13.2** in **CSA A23.3-14**, Δ_h cannot be taken larger than the slab thickness, a preliminary estimate of slab thickness is based on **Equation 13.2** with Δ_h equal to h_s . In the spanning direction of the slab band the term x_d/l_n would take its maximum value of 0.25.

Interior Panel (E-W Direction):

$$h_{s,min} = \frac{(0.6 + f_y / 1,000)}{1 + \frac{2x_d}{l_n}} \times \left(\frac{l_n}{30} \right) \quad \text{CSA A23.3-14 (13.2.4)}$$

$$h_{s,min} = \frac{(0.6 + 0.4)}{1 + 2 \times 0.25} \times \left(\frac{7500 - 600}{30} \right) = 153.3 \text{ mm}$$

But not less than 120 mm. CSA A23.3-14 (13.2.1)

The N-S direction shall be checked in order to determine slab thickness.

Try $h_s = 160$ mm slab for all panels.

Self-weight for slab section without slab bands = $24 \text{ kN/m}^3 \times 0.160 \text{ m} = 3.84 \text{ kN/m}^2$

Self-weight for slab section with slab bands = $24 \text{ kN/m}^3 \times 0.350 \text{ m} = 8.40 \text{ kN/m}^2$

1.2.3. Slab Band Width

The slab band width is assumed to extend in each direction from the centerline of support one-sixth the span length measured from center-to-center of supports in that direction.

$$l_{sb} = \frac{6.6}{6} + \frac{6.6}{6} = 2.2 \text{ m}$$

1.2.4. Slab shear strength – one way shear

For critical section at distance d_v from the edge of the column (slab section with slab band):

Evaluate the average effective depth:

$$d_t = h_{band} - c_{clear} - d_b - \frac{d_b}{2} = 350 - 25 - 16 - \frac{16}{2} = 301 \text{ mm}$$

$$d_l = h_{band} - c_{clear} - \frac{d_b}{2} = 350 - 25 - \frac{16}{2} = 317 \text{ mm}$$

$$d_{avg} = \frac{d_t + d_l}{2} = \frac{301 + 317}{2} = 309 \text{ mm}$$

Where:

$$c_{clear} = 20 \text{ mm}$$

CSA A23.3-14 (Annex A. Table 17)

Note that the reference used 25 mm as clear cover, in this example the clear cover used is 25 mm to be consistent with reference.

$$d_b = 16 \text{ mm for 15M steel bar}$$

$$\text{Factored dead load} \rightarrow w_{df} = 1.25 \times \left(\left[8.40 \times \frac{2.2}{6.6} + 3.84 \times \frac{6.6 - 2.2}{6.6} \right] + 1 + 1 \right) = 9.20 \text{ kN/m}^2$$

$$\text{Factored live load} \rightarrow w_f = 1.5 \times 3.6 = 5.40 \text{ kN/m}^2$$

CSA A23.3-14 (Annex C. Table C.1 a)

$$\text{Total factored load} \rightarrow w_f = 9.20 + 5.40 = 14.60 \text{ kN/m}^2$$

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior column

CSA A23.3-14 (13.3.6)

Consider a 1 m wide strip. The critical section for one-way shear is located at a distance d_v , from the edge of the column (see Figure 6)

$$\text{Tributary area for one-way shear is } A_{Tributary} = \left(\frac{\left[\left(\frac{6600}{2} \right) - \left(\frac{400}{2} \right) - 278 \right] \times (1000)}{1000^2} \right) = 2.82 \text{ m}^2$$

Where:

$$d_v = \text{Max} \left\{ \begin{matrix} 0.9d \\ 0.72h \end{matrix} \right\} = \text{Max} \left\{ \begin{matrix} 0.9(309) \\ 0.72(350) \end{matrix} \right\} = \text{Max} \left\{ \begin{matrix} 278 \\ 252 \end{matrix} \right\} = 278 \text{ mm}$$

CSA A23.3-14 (3.2)

$$V_f = w_f \times A_{Tributary} = 14.60 \times 2.82 = 41.20 \text{ kN}$$

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v \quad \text{CSA A23.3-14 (Eq. 11.6)}$$

Where $\lambda = 1$ for normal weight concrete

$$\beta = 0.21 \text{ for slabs with overall thickness not greater than 350 mm} \quad \text{CSA A23.3-14 (11.3.6.2)}$$

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{25} \times 1000 \times \frac{278}{1000} = 189.8 \text{ kN} > V_f$$

Slab band thickness of 350 mm is adequate for one-way shear for the first critical section (from the edge of the column).

For critical section at the edge of the slab band (slab section without slab band):

Evaluate the average effective depth:

$$d_t = h_s - c_{clear} - d_b - \frac{d_b}{2} = 160 - 25 - 16 - \frac{16}{2} = 111 \text{ mm}$$

$$d_l = h_s - c_{clear} - \frac{d_b}{2} = 160 - 25 - \frac{16}{2} = 127 \text{ mm}$$

$$d_{avg} = \frac{d_t + d_l}{2} = \frac{111 + 127}{2} = 119 \text{ mm}$$

Where:

$$c_{clear} = 20 \text{ mm} \quad \text{CSA A23.3-14 (Annex A, Table 17)}$$

Note that the reference used 25 mm as clear cover, in this example the clear cover used is 25 mm to be consistent with reference.

$$d_b = 16 \text{ mm for 15M steel bar}$$

Check the adequacy of slab thickness for beam action (one-way shear) from the edge of the interior slab band. CSA A23.3-14 (13.3.6)

Consider a 1 m wide strip. The critical section for one-way shear is located at a distance, d_v from the face of the interior slab band (see Figure 6)

$$\text{Tributary area for one-way shear is } A_{Tributary} = \left(\frac{\left[\left(\frac{6600}{2} \right) - \left(\frac{2200}{2} \right) - 107 \right] \times (1000)}{1000^2} \right) = 2.09 \text{ m}^2$$

Where:

$$d_v = \text{Max} \left\{ \begin{array}{l} 0.9d \\ 0.72h \end{array} \right\} = \text{Max} \left\{ \begin{array}{l} 0.9(119) \\ 0.72(160) \end{array} \right\} = \text{Max} \left\{ \begin{array}{l} 107 \\ 115 \end{array} \right\} = 115 \text{ mm} \quad \text{CSA A23.3-14 (3.2)}$$

$$V_f = w_f \times A_{Tributary} = 14.6 \times 2.09 = 30.50 \text{ kN}$$

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v$$

CSA A23.3-14 (Eq. 11.6)

Where $\lambda = 1$ for normal weight concrete

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm

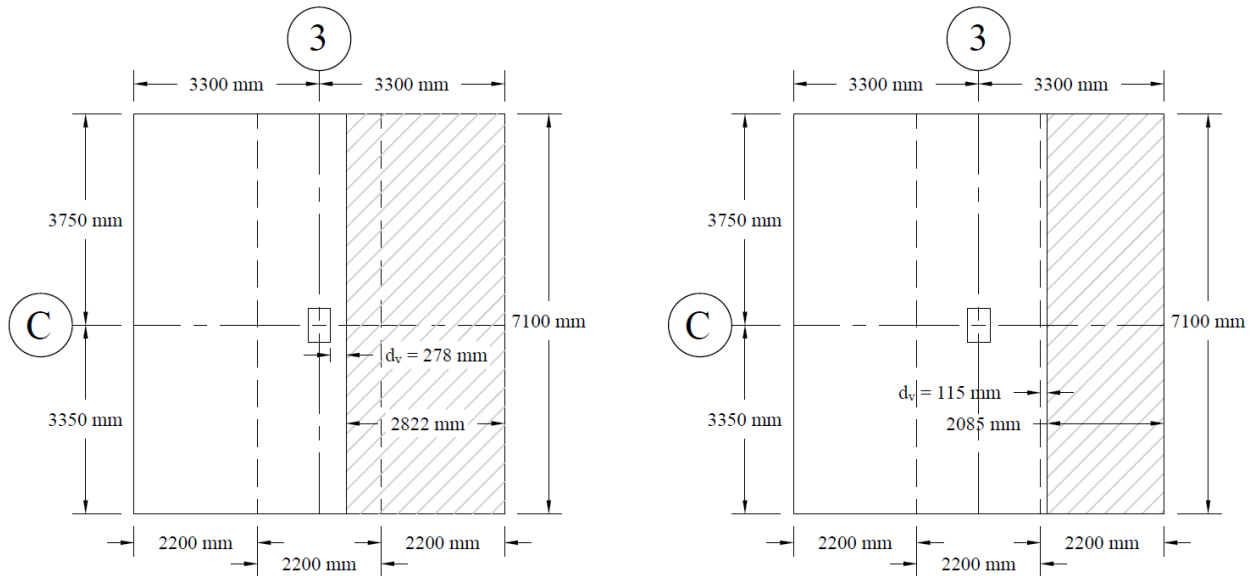
CSA A23.3-14 (11.3.6.2)

$$\sqrt{f'_c} = 5 \text{ MPa} < 8 \text{ MPa}$$

CSA A23.3-14 (11.3.4)

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{25} \times 1000 \times \frac{115}{1000} = 78 \text{ kN} > V_f$$

Slab thickness of 160 mm is adequate for one-way shear for the second critical section (from the edge of the slab band).



Critical Section from the Edge of the Column

Critical Section from the Edge of the Slab Band

Figure 6 – Critical Sections for One-Way Shear

1.2.5. Slab shear strength – two-way shear

For critical section at distance $d/2$ from the edge of the column (slab section with slab band):

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 7):

$$\begin{aligned} \text{Tributary area for two-way shear is } A_{\text{Tributary}} &= (7.5/2 + 6.7/2) \times (6.6) - (0.6 + 0.309) \times (0.4 + 0.309) \\ &= 46.22 \text{ m}^2 \end{aligned}$$

$$V_f = w_f \times A_{\text{Tributary}} = 14.6 \times 46.22 = 675 \text{ kN}$$

$$b_o = 2 \times (600 + 309) + 2 \times (400 + 309) = 3236 \text{ mm}$$

CSA A23.3-14 (13.3.3)

$$v_f = \frac{V_f}{b_o d} = \frac{675 \times 1000}{3236 \times 309} = 0.675 \text{ MPa}$$

The factored resisting shear stress, v_r shall be the smallest of :

CSA A23.3-14 (13.3.4.1)

$$a) \quad v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c} \quad \text{CSA A23.3-14 (Eq. 13.5)}$$

$$v_r = \left(1 + \frac{2}{1.5}\right) \times 0.19 \times 0.65 \times \sqrt{25} = 1.44 \text{ MPa}$$

Where $\beta_c = \frac{600}{400} = 1.5$ (ratio of long side to short side of the column) CSA A23.3-14 (13.3.4.1)

$$b) \quad v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c} \quad \text{CSA A23.3-14 (Eq. 13.6)}$$

$$v_r = \left(\frac{4 \times 309}{3236} + 0.19\right) \times 1 \times 0.65 \times \sqrt{25} = 1.86 \text{ MPa}$$

$$c) \quad v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{25} = 1.24 \text{ MPa (Governs)} \quad \text{CSA A23.3-14 (Eq. 13.7)}$$

$$\frac{v_r}{v_{f,ave}} = \frac{1.240}{0.675} = 1.84 > 1.20 \quad \text{CAC Concrete Design Handbook 4th Edition (5.2.3)}$$

Slab band thickness of 350 mm is adequate for two-way shear for the critical section (from the edge of the column).

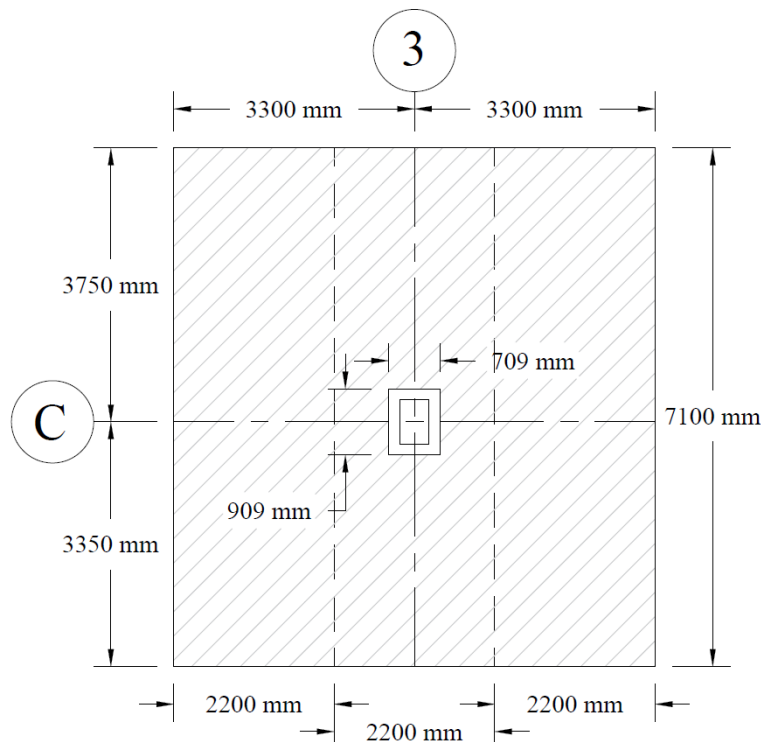


Figure 7 – Critical Section for Two-Way Shear

While the preliminary sizes determined above and summarized in the Figure 8 leads to a more optimal design, we will proceed with the dimensions provided in the reference example (Example #3 of CAC Design Handbook) for comparison purposes (see Figure 9 below).

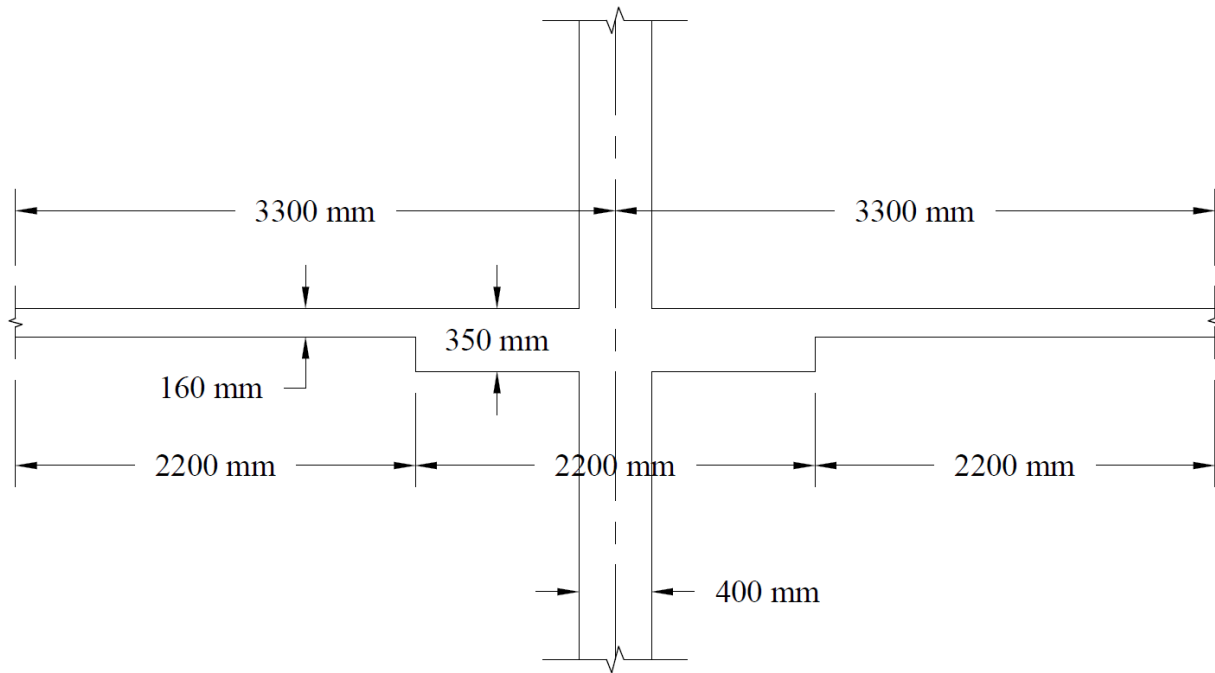


Figure 8 – Section A-A (As Calculated Above)

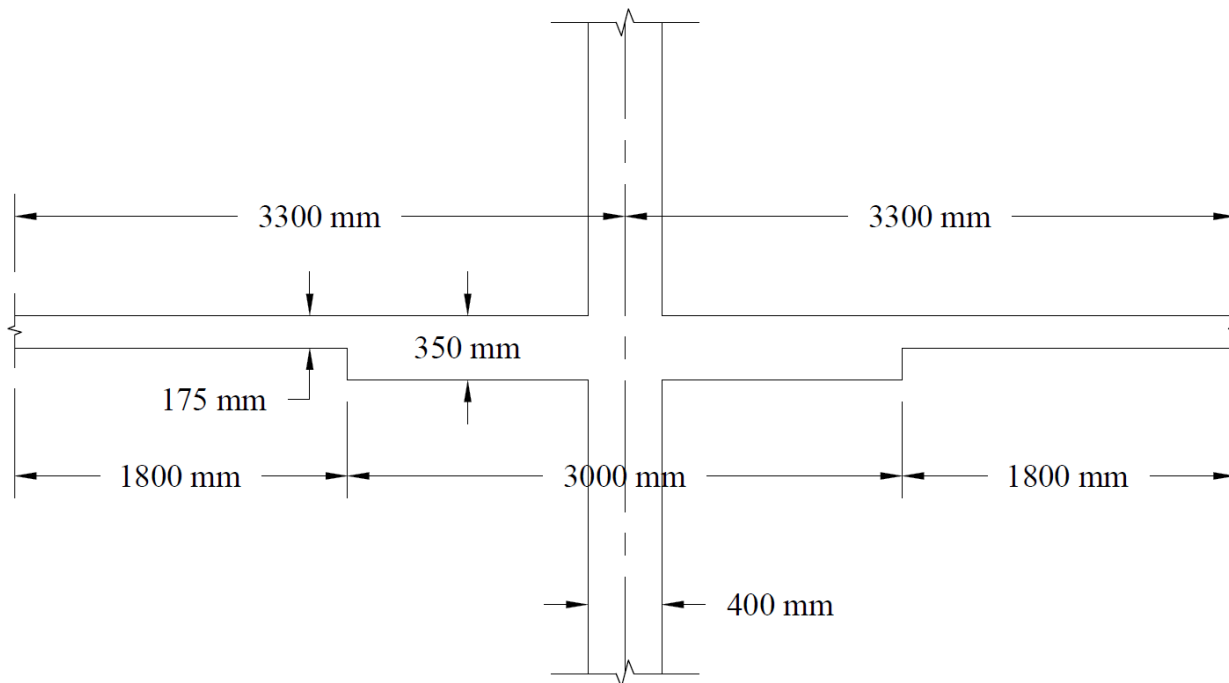


Figure 9 – Section A-A (As Calculated by the Reference)

1.3. Preliminary Member Sizing for Columns

Check the adequacy of column dimensions for axial load:

Tributary area for interior column for live load, superimposed dead load, and self-weight of the slab is

$$A_{Tributary} = \frac{7.5+6.7}{2} \times 6.6 = 46.86 \text{ m}^2$$

Tributary area for interior column for self-weight of additional slab thickness due to the presence of the slab band is

$$A_{Tributary} = \frac{7.5+6.7}{2} \times 3 = 21.3 \text{ m}^2$$

Assuming five story building

$$P_f = n \times w_f \times A_{Tributary} = 5 \times (13.05 \times 46.86 + 5.15 \times 21.3) = 3606 \text{ kN}$$

Assume 600 mm x 400 mm column with 12 – 30M vertical bars with design axial strength, $P_{r,max}$ of

$$P_{r,max} = (0.2 + 0.002h)P_{ro} \leq 0.80P_{ro} \quad (\text{For tied column along full length}) \quad \text{CSA A23.3-14 (Eq. 10.9)}$$

$$P_{ro} = \alpha_1 \phi_c f'_c (A_g - A_{st} - A_t - A_p) + \phi_s f_y A_{st} + \phi_\alpha F_y A_t - f_{pr} A_p \quad \text{CSA A23.3-14 (Eq. 10.11)}$$

$$P_{ro} = 0.81 \times 0.65 \times 25 \times (600 \times 400 - 12 \times 700) + 0.85 \times 400 \times (12 \times 700) + 0 = 5904 \text{ kN}$$

$$P_{r,max} = (0.2 + 0.002 \times 600) \times 5904 \leq 0.80 \times 5904$$

$$= 8266 \leq 4723$$

$$= 4723 \text{ kN} > P_f = 3606 \text{ kN}$$

Where:

$$\alpha_1 = 0.85 - 0.0015 f'_c = 0.85 - 0.0015 \times 25 = 0.81 > 0.67 \quad \text{CSA A23.3-14 (Eq. 10.1)}$$

Column dimensions of 600 mm x 400 mm are adequate for axial load.

2. Two-Way Slab Analysis and Design

CSA A23.3 states that a regular slab system may be designed using any procedure satisfying conditions of equilibrium and compatibility with the supports, provided that it is shown that the factored resistance at every section is at least equal to the effects of the factored loads and that all serviceability conditions, including specified limits on deflections, are met. CSA A23.3-14 (13.5.1)

CSA A23.3 permits the use of Plastic Plate Theory Method (PPTM), Theorems of Plasticity Method (TPM), Direct Design Method (DDM) and Elastic Frame Method (EFM); known as Equivalent Frame Method in the ACI; for the gravity load analysis of orthogonal frames. The following sections outline a brief description of DDM, a detailed hand solution using EFM and an automated solution using spSlab software respectively.

2.1. Direct Design Method (DDM)

Two-way slabs satisfying the limits in CSA A23.3-14 (13.9) are permitted to be designed in accordance with the DDM.

2.1.1. Direct design method limitations

There shall be a minimum of three continuous spans in each direction (3 spans) CSA A23.3-14 (13.9.1.2)

Successive span lengths centre-to-centre of supports in each direction shall not differ by more than one-third of the longer span ($(7500-6700)/6700 = 0.12 < 0.33$) CSA A23.3-14 (13.9.1.3)

All loads shall be due to gravity only and uniformly distributed over an entire panel (Loads are uniformly distributed over the entire panel) CSA A23.3-14 (13.9.1.4)

The factored live load shall not exceed twice the factored dead load (factored live-to-dead load ratio of $0.71 < 2.0$) CSA A23.3-14 (13.9.1.4)

Since all the criteria are met, Direct Design Method is utilized in the CAC Design Handbook.

Even though this system meets all the limitations of the DDM, based on engineering judgment, DDM is not recommended to be used with floor systems with slab bands since the generic moment distribution factors used in DDM might, in some cases as this example, underestimate the negative moment values since these factors were derived based on a two-way slab systems without beams between interior supports (Flat Plate). The stiffer the supports (due to the presence of drop panels and slab bands) the more moments the supports will carry. The EFM takes into consideration detailed geometry of the cross section and the slab-beam distribution factors are calculated exactly. This calculation can be tedious and complicated to be done by hand for slab systems with different thicknesses but computer aids such as spSlab or spMats can be utilized. There are design aids tables that can be utilized for simplifying hand calculation. However, the available tables are only applicable for flat plates and some special cases of slabs with drop panels. There are no design aid tables for two-way slabs with slab bands, slabs with beams between all supports, or two-way joist (waffle) slabs. For these systems, using the available design aid tables might in some cases also underestimate the moment values at the supports.

Detailed illustration of analysis and design of a two-way flat plate concrete slab system using DDM can be found in “[Two-Way Flat Plate Concrete Slab Floor Analysis and Design \(CSA A23.3-14\)](#)” example available in the [design examples](#) page in [StructurePoint](#) website. This example focuses on the analysis of two-way slabs with slab bands using EFM.

2.1.2. Design moments

- a. Calculate the total factored static moment:

$$M_o = \frac{w_f \ell_{2a} \ell_n^2}{8} \quad \text{CSA A23.3-14 (13.9.1.4)}$$

Distribute the total factored moment, M_o , in an interior and end span: CSA A23.3-14 (13.9.3.1 & 13.9.3.2)

Table 1 - Distribution of M_o along the span		
Location		Total Design Strip Moment, M_{DES} (kN.m)
Exterior Span	Exterior Negative	$0.26 \times M_o = 34.8$
	Positive	$0.52 \times M_o = 69.6$
	Interior Negative	$0.70 \times M_o = 93.68$
Interior Span	Positive	$0.35 \times M_o = 46.8$

- b. Calculate the column strip moments. CSA A23.3-14 (13.11.2)

That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips. CSA A23.3-14 (13.11.3.1)

Table 2 - Lateral Distribution of the Total Design Strip Moment, M_{DES}				
Location		Total Design Strip Moment, (kN.m)	Column Strip Moment, (kN.m)	Moment in Two Half Middle Strips, (kN.m)
Exterior Span	Exterior Negative*	34.8	$1.00 \times M_{DES} = 34.8$	$0.00 \times M_{DES} = 0.0$
	Positive	69.6	$0.6 \times M_{DES} = 41.8$	$0.4 \times M_{DES} = 27.8$
	Interior Negative*	93.68	$0.8 \times M_{DES} = 74.94$	$0.2 \times M_{DES} = 18.7$
Interior Span	Positive	46.8	$0.6 \times M_{DES} = 28.1$	$0.4 \times M_{DES} = 18.7$

* All negative moments are at face of support.

Figure 10 – Sample Calculations Using DDM from “Two-Way Flat Plate Concrete Slab Floor Analysis and Design”
Design Example

2.2. Elastic Frame Method (EFM)

EFM (also known as Equivalent Frame Method in the ACI 318) is the most comprehensive and detailed procedure provided by the CSA A23.3 for the analysis and design of two-way slab systems where these systems may, for purposes of analysis, be considered a series of plane frames acting longitudinally and transversely through the building. Each frame shall be composed of equivalent line members intersecting at member centrelines, shall follow a column line, and shall include the portion of slab bounded laterally by the centreline of the panel on each side. CSA A23.3-14 (13.8.1.1)

Probably the most frequently used method to determine design moments in regular two-way slab systems is to consider the slab as a series of two-dimensional frames that are analyzed elastically. When using this analogy, it is essential that stiffness properties of the elements of the frame be selected to properly represent the behavior of the three-dimensional slab system.

In a typical frame analysis it is assumed that at a beam-column connection all members meeting at the joint undergo the same rotation. For uniform gravity loading this reduced restraint is accounted for by reducing the effective stiffness of the column by either Clause 13.8.2 or Clause 13.8.3. CSA A23.3-14 (N.13.8)

Each floor and roof slab with attached columns may be analyzed separately, with the far ends of the columns considered fixed. CSA A23.3-14 (13.8.1.2)

The moment of inertia of column and slab-beam elements at any cross-section outside of joints or column capitals shall be based on the gross area of concrete at that section. CSA A23.3-14 (13.8.2.5)

An equivalent column shall be assumed to consist of the actual columns above and below the slab-beam plus an attached torsional member transverse to the direction of the span for which moments are being determined. CSA A23.3-14 (13.8.2.5)

2.2.1. Elastic frame method limitations

In EFM, live load shall be arranged in accordance with 13.8.4 which requires:

- slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. CSA A23.3-14 (13.8.4)
- Complete analysis must include representative interior and exterior equivalent elastic frames in both the longitudinal and transverse directions of the floor. CSA A23.3-14 (13.8.1.1)
- Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. CSA A23.3-14 (3.1a)
- For slab systems with beams between supports, the relative effective stiffness of beams in the two directions is not less than 0.2 or greater than 5.0. CSA A23.3-14 (3.1b)
- Column offsets are not greater than 20% of the span (in the direction of offset) from either axis between centerlines of successive columns. CSA A23.3-14 (3.1c)

The reinforcement is placed in an orthogonal grid. CSA A23.3-14 (3.1d)

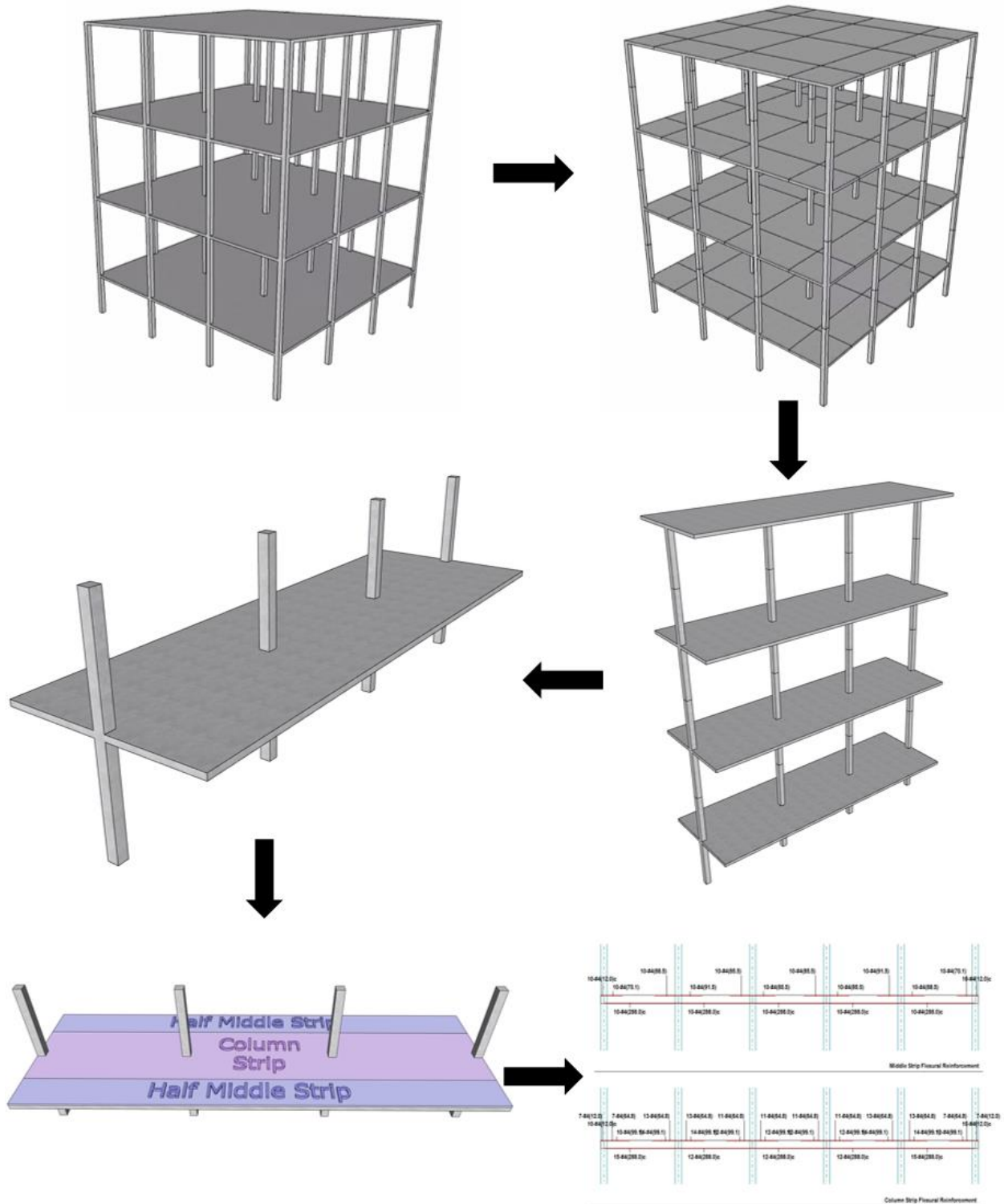


Figure 11 – Elastic (Equivalent) Frame Methodology

2.2.2. Frame members of elastic frame

Determine moment distribution factors and fixed-end moments for the elastic frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors k , carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at [Appendix 20A of PCA Notes on ACI 318-11](#). Note that the available tables are limited to flat plate and slab with drop panels systems, literature showed that these tables can be used for other systems for simplicity to an extent. This point will be discussed later in this document. These calculations are shown below.

- a. Flexural stiffness of slab-beams at both ends, K_{sb}

Table A5 in the PCA Notes handbook with drop panel thickness equals to slab thickness will be used to calculate the flexural stiffness of slab-beams. This table has been adopted in this example and is deemed to represent the most comparable system for the analysis of two-way slab with transverse slab bands.

$$\frac{c_{N1}}{\ell_1} = \frac{400}{6600} = 0.061, \quad \frac{c_{N2}}{\ell_2} = \frac{600}{7100} = 0.085$$

For $c_{F1} = c_{N2}$, stiffness factors, $k_{NF} = k_{FN} = 7.98$

[PCA Notes on ACI 318-11 \(Table A5\)](#)

$$\text{Thus, } K_{sb} = k_{NF} \frac{E_{cs} I_s}{\ell_1} = 7.98 \frac{E_{cs} I_s}{\ell_1}$$

[PCA Notes on ACI 318-11 \(Table A5\)](#)

$$K_{sb} = 7.98 \times 24,986 \times \frac{3.17 \times 10^9}{6600} \times 10^{-3} = 95.8 \times 10^6 \text{ N.m}$$

$$\text{where, } I_s = \frac{\ell_s h^3}{12} = \frac{6600(175)^3}{12} = 3.17 \times 10^9 \text{ mm}^4$$

$$E_{cs} = (3300\sqrt{f'_c} + 6900) \left(\frac{\gamma_c}{2300} \right)^{1.5}$$

[CSA A23.3-14\(8.6.2.2\)](#)

$$E_{cs} = (3300\sqrt{25} + 6900) \left(\frac{2402.8}{2300} \right)^{1.5} = 24,986 \text{ MPa}$$

Carry-over factor COF = 0.660

[PCA Notes on ACI 318-11 \(Table A5\)](#)

$$\text{Fixed-end moment FEM} = 0.0996 w_u \ell_2 \ell_1^2$$

[PCA Notes on ACI 318-11 \(Table A5\)](#)

b. Flexural stiffness of column members at both ends, K_c .

Referring to **Table A7, Appendix 20A**,

For the Top Column (Above):

$$t_a = 350 - \frac{175}{2} = 262.5 \text{ mm}, t_b = \frac{175}{2} = 87.5 \text{ mm}$$

$$\frac{t_a}{t_b} = \frac{262.5}{87.5} = 3$$

$$H = 3 \text{ m} = 3000 \text{ mm}, H_c = 3000 \text{ mm} - 350 = 2650 \text{ mm}$$

$$\frac{H}{H_c} = \frac{3000}{2650} = 1.132$$

Thus, $k_{AB} = 6.02$ and $C_{AB} = 0.536$ by interpolation.

$$K_{c,top} = \frac{6.02 E_{cc} I_c}{\ell_c}$$

PCA Notes on ACI 318-11 (Table A7)

$$K_{c,top} = 6.02 \times 24,986 \times \frac{3.20 \times 10^9}{3000 \times 1000} = 160.4 \times 10^6 \text{ N.m}$$

$$\text{Where } I_c = \frac{b \times h^3}{12} = \frac{600(400)^3}{12} = 3.20 \times 10^9 \text{ mm}^4$$

$$E_{cc} = (3300 \sqrt{f'_c} + 6900) \left(\frac{\gamma_c}{2300} \right)^{1.5}$$

CSA A23.3-14(8.6.2.2)

$$E_{cc} = (3300 \sqrt{25} + 6900) \left(\frac{2402.8}{2300} \right)^{1.5} = 24,986 \text{ MPa}$$

$$\ell_c = 3.00 \text{ m} = 3000 \text{ mm}$$

For the Bottom Column (Below):

$$\frac{t_b}{t_a} = \frac{87.5}{262.5} = 0.33$$

$$\frac{H}{H_c} = \frac{3000}{2650} = 1.132$$

Thus, $k_{BA} = 4.99$ and $C_{BA} = 0.641$ by interpolation.

$$K_c = \frac{4.99 E_{cc} I_c}{\ell_c}$$

PCA Notes on ACI 318-11 (Table A7)

$$K_{c,bottom} = 4.99 \times 24,986 \times \frac{3.20 \times 10^9}{3000 \times 1000} = 132.9 \times 10^6 \text{ N.m}$$

c. Torsional stiffness of torsional members, K_t

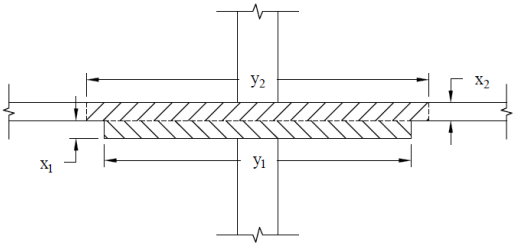
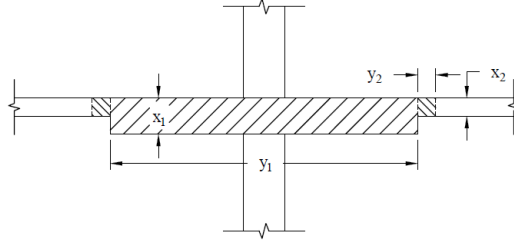
$$K_t = \sum \left[\frac{9E_{cs}C}{\ell_t \left(1 - \frac{c_2}{\ell_t}\right)^3} \right] \quad \text{CSA A23.3-14 (13.8.2.8)}$$

For Interior Columns:

$$K_{t_{int}} = \frac{9 \times 24,986 \times 40.0 \times 10^9}{7100 \times \left(1 - \frac{600}{7100}\right)^3} \times 10^{-3} = 164.9 \times 10^7 \text{ N.m}$$

Where:

$$C = \sum \left(1 - 0.63 \frac{x}{y} \right) \left(\frac{x^3 y}{3} \right) \quad \text{CSA A23.3-14 (13.8.2.9)}$$

$x_1 = 175 \text{ mm}$ $y_1 = 3000 \text{ mm}$ $C_1 = 5.16 \times 10^9$	$x_2 = 175 \text{ mm}$ $y_2 = 3350 \text{ mm}$ $C_2 = 5.79 \times 10^9$	$x_1 = 350 \text{ mm}$ $y_1 = 3000 \text{ mm}$ $C_1 = 39.72 \times 10^9$	$x_2 = 175 \text{ mm}$ $y_2 = 175 \text{ mm}$ $C_2 = 0.116 \times 10^9$
$\Sigma C = 5.16 \times 10^9 + 5.79 \times 10^9 = 11 \times 10^9 \text{ mm}^4$		$\Sigma C = 39.72 \times 10^9 + 2 \times 0.116 \times 10^9 = 40 \times 10^9 \text{ mm}^4$	
			

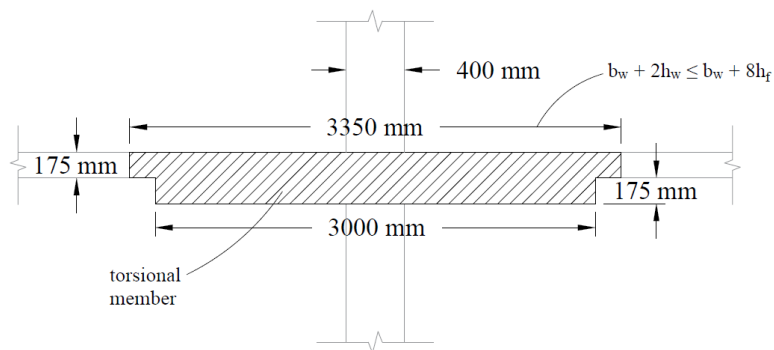


Figure 12 – Attached Torsional Member at Interior Column

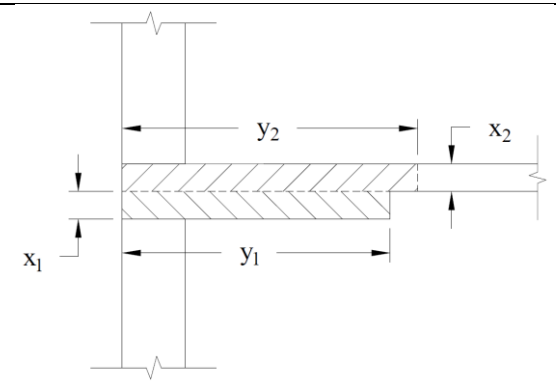
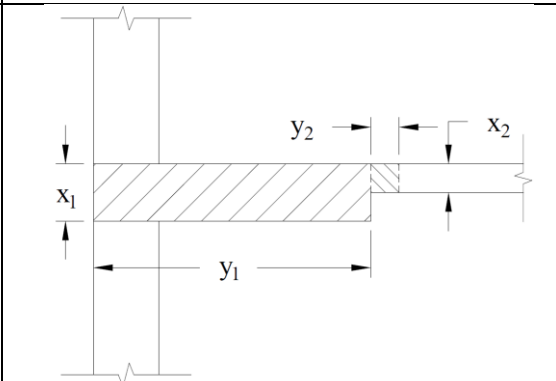
For Exterior Columns:

$$K_{i_ext} = \frac{9 \times 24,986 \times 22.7 \times 10^9}{7100 \times \left(1 - \frac{600}{7100}\right)^3} \times 10^{-3} = 93.66 \times 10^7 \text{ N.m}$$

Where:

$$C = \sum \left(1 - 0.63 \frac{x}{y}\right) \left(\frac{x^3 y}{3}\right)$$

CSA A23.3-14 (13.8.2.9)

$x_1 = 175 \text{ mm}$ $y_1 = 1800 \text{ mm}$ $C_1 = 3.02 \times 10^9$	$x_2 = 175 \text{ mm}$ $y_2 = 1975 \text{ mm}$ $C_2 = 3.33 \times 10^9$	$x_1 = 350 \text{ mm}$ $y_1 = 1800 \text{ mm}$ $C_1 = 22.57 \times 10^9$	$x_2 = 175 \text{ mm}$ $y_2 = 175 \text{ mm}$ $C_2 = 0.116 \times 10^9$
$\Sigma C = 3.02 \times 10^9 + 3.33 \times 10^9 = 6.35 \times 10^9 \text{ mm}^4$		$\Sigma C = 22.57 \times 10^9 + 0.116 \times 10^9 = 22.7 \times 10^9 \text{ mm}^4$	
			

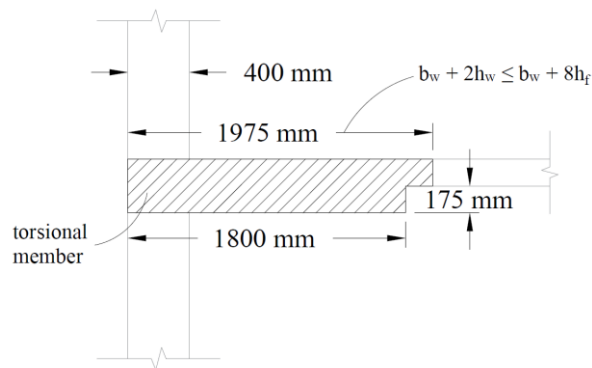


Figure 13 – Attached Torsional Member at Exterior Column

d. Equivalent column stiffness, K_{ec}

Where $\sum K_t$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

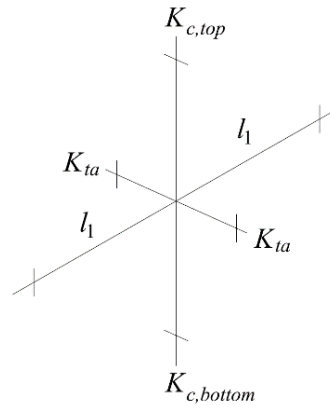


Figure 14 – Equivalent Column Stiffness

For Interior Columns:

$$K_{ec_int} = \frac{(160.4 \times 10^6 + 132.9 \times 10^6)(2 \times 1.6 \times 10^9)}{(160.4 \times 10^6 + 132.9 \times 10^6) + (2 \times 1.6 \times 10^9)} = 269.3 \times 10^6 \text{ N.m}$$

For Exterior Columns:

$$K_{ec_ext} = \frac{(160.4 \times 10^6 + 132.9 \times 10^6)(2 \times 93.66 \times 10^7)}{(160.4 \times 10^6 + 132.9 \times 10^6) + (2 \times 93.66 \times 10^7)} = 253.6 \times 10^6 \text{ N.m}$$

e. Slab-beam joint distribution factors, DF

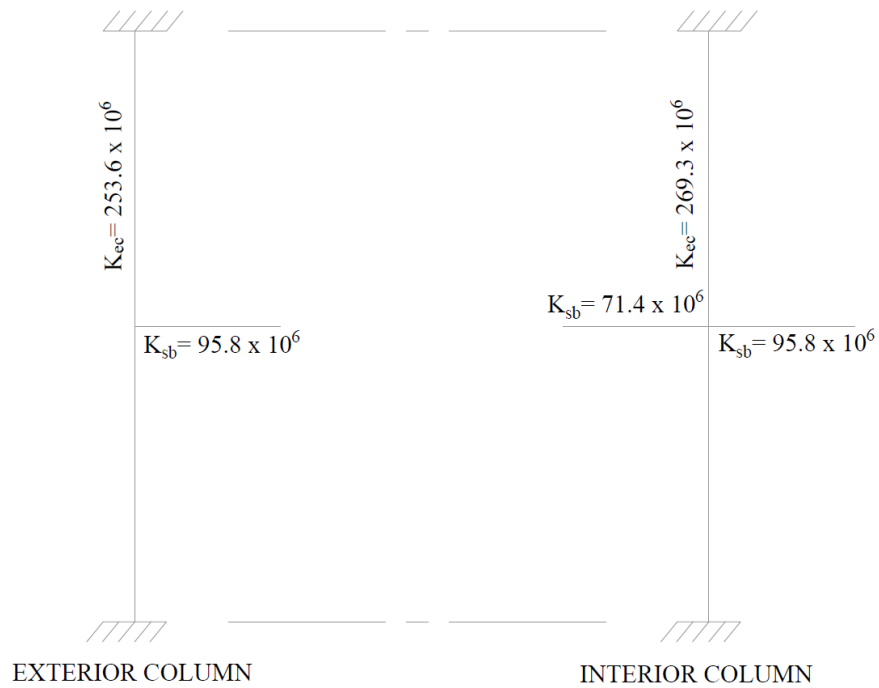


Figure 15 – Slab and Column Stiffness

At exterior joint:

$$DF = \frac{95.8 \times 10^6}{(95.8 \times 10^6 + 253.6 \times 10^6)} = 0.274$$

At interior joint:

$$DF_{Ext} = \frac{95.8 \times 10^6}{(95.8 \times 10^6 + 95.8 \times 10^6 + 269.3 \times 10^6)} = 0.208$$

$$DF_{Int} = \frac{95.8 \times 10^6}{(95.8 \times 10^6 + 95.8 \times 10^6 + 269.3 \times 10^6)} = 0.208$$

COF for slab-beam = 0.660 for Interior Span

= 0.660 for Exterior Span

2.2.3. Elastic frame analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. CSA A23.3-14 (13.8.4.2)

$$\frac{L}{D} = \frac{3.6}{(4.12 + 1.87 + 2)} = 0.45 < \frac{3}{4}$$

- a. Factored load and Fixed-End Moments (FEM's).

$$\text{Factored dead load, } w_{df} = 1.25 \times \left(175 \times 2400 + (350 - 175) \times 2400 \times \frac{3}{6.6} + 2 \right)$$

$$w_{df} = 1.25 \times (4.2 + 1.87 + 2) = 10 \text{ kN/m}^2$$

$$\text{Factored live load, } w_{lf} = 1.5 \times 3.6 = 5.4 \text{ kN/m}^2$$

$$\text{Total factored load } q_u = w_f = w_{df} + w_{lf} = 15.4 \text{ kN/m}^2$$

$$\begin{aligned} \text{FEM's for slab-beams} &= m_{NF} q_u \ell_2 \ell_1^2 && \text{PCA Notes on ACI 318-11 (Table A1)} \\ &= 0.0996 \times 15.4 \times 7.1 \times 6.6^2 = 474.3 \text{ kN.m (For interior and exterior spans)} \end{aligned}$$

- b. Moment distribution. Computations are shown in Table 1. Counterclockwise rotational moments acting on the member ends are taken as positive.

Positive span moment is determined from the following equation (For positive moment span 2-3):

$$M_u \text{ (midspan)} = M_o - \frac{M_{uL} + M_{uR}}{2}$$

$$M_u^+ = \frac{(15.4 \times 7.1) \times 6.6^2}{8} - \frac{(481.5 + 481.5)}{2} = 113.8 \text{ kN.m}$$

Where M_o is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, the maximum positive moment for a uniformly distributed load and variable end moments can be calculated using any design aid as follows (For positive moment in span 1-2):

$$M_u^+ = \frac{(15.4 \times 7.1) \times 6.6^2}{8} - \frac{(541.3 + 354.5)}{2} + \frac{(541.3 - 354.5)^2}{2 \times (15.4 \times 7.1) \times 6.6^2} = 151.1 \text{ kN.m}$$

Table 1 – Moment Distribution for Elastic Frame						
Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.274	0.208	0.208	0.208	0.208	0.274
COF	0.66	0.66	0.66	0.66	0.66	0.66
FEM	474.30	-474.30	474.30	-474.30	474.30	-474.30
Dist	-129.96	0.00	0.00	0.00	0.00	129.96
CO	0.00	-85.77	0.00	0.00	85.77	0.00
Dist	0.00	17.84	17.84	-17.84	-17.84	0.00
CO	11.77	0.00	-11.77	11.77	0.00	-11.77
Dist	-3.23	2.45	2.45	-2.45	-2.45	3.23
CO	1.62	-2.13	-1.62	1.62	2.13	-1.62
Dist	-0.44	0.78	0.78	-0.78	-0.78	0.44
CO	0.51	-0.29	-0.51	0.51	0.29	-0.51
Dist	-0.14	0.17	0.17	-0.17	-0.17	0.14
CO	0.11	-0.09	-0.11	0.11	0.09	-0.11
Dist	-0.03	0.04	0.04	-0.04	-0.04	0.03
CO	0.03	-0.02	-0.03	0.03	0.02	-0.03
Dist	-0.01	0.01	0.01	-0.01	-0.01	0.01
CO	0.01	-0.01	-0.01	0.01	0.01	-0.01
Dist	0.00	0.00	0.00	0.00	0.00	0.00
M, kN.m	354.50	-541.30	481.50	-481.50	541.30	-354.50
Midspan M, kN.m	151.10		113.82		151.10	

2.2.4. Design moments

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 14. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than $0.175\ell_1$ from the centers of supports.

CSA A23.3-14 (13.8.5.1)

$$\frac{400}{2} = 200 \text{ mm} < 0.175 \times 6600 = 1155 \text{ mm (use face of supporting location)}$$

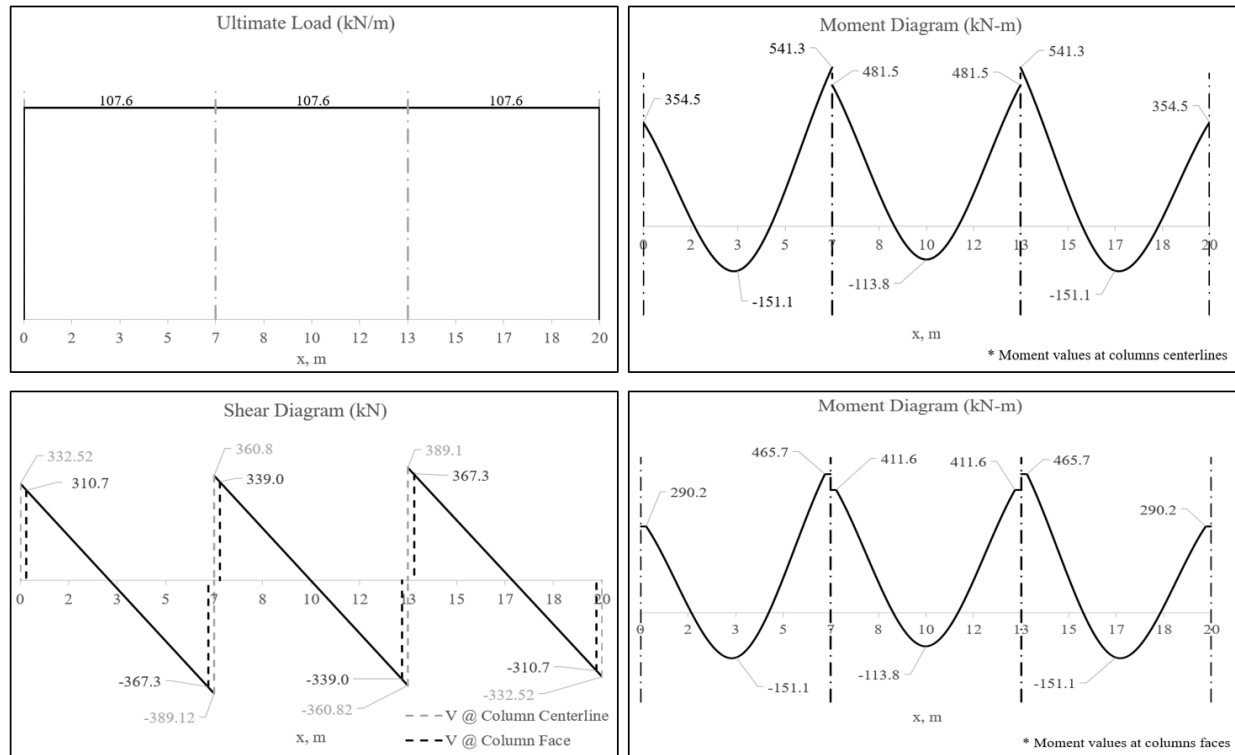


Figure 16 - Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)

2.2.5. Distribution of design moments

After the negative and positive moments have been determined for the slab-beam strip, the CSA code permits the distribution of the moments at critical sections to the column strips, beams (if any), and middle strips in accordance with the DDM for EFM. **CSA A23.3-14 (13.11.2.5)**

- For negative moment at an interior column in width b_b , the multiplication factor should be taken not less than 5% to 15% (10% will be used to be consistent with spSlab) with the remaining negative moment assumed evenly distributed over the entire frame width.
- For negative moment at an exterior column, the column strip should resist 100% of the total frame strip moment.
- For positive moment at all spans where $(l_1/l_2) \geq 1.0$, the column strip should resist 50% to 60% of the total frame strip moment.

For positive moment at all spans where $(l_1/l_2) < 1.0$, the column strip should resist $50(l_1/l_2)\%$ to $60(l_1/l_2)\%$ of the total frame strip moment.

$$\frac{l_1}{l_2} = \frac{6.6}{7.1} = 0.93 < 1.00 \rightarrow \text{the column strip should resist } 50(l_1/l_2)\% \text{ to } 60(l_1/l_2)\% \text{ of the total frame}$$

strip moment. \rightarrow use $55(l_1/l_2)\%$

Distribution of factored moments at critical sections is summarized in Table 2.

Table 2 - Distribution of factored moments						
		Slab-beam Strip	Column Strip		Middle Strip	
		Moment (kN.m)	Percent	Moment (kN.m)	Percent	Moment (kN.m)
End Span	Exterior Negative	290.2	100.0	290.2	0.0	0.0
	Positive	151.1	51.1	77.3	48.9	73.8
	Interior Negative	465.7	55.0	256.1 (144.0)*	45.0	209.6
Interior Span	Negative	411.6	55.0	226.4 (127.2)*	45.0	185.2
	Positive	113.8	51.1	58.2	48.9	55.6

* Negative moment at an interior column in width b_b .

2.2.6. Flexural reinforcement requirements

- a. Determine the flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – exterior negative location is provided below.

Reinforcement for the total factored negative moment transferred to the exterior columns shall be placed within a band width b_b . Temperature and shrinkage reinforcement determined as specified in clause 7.8.1 shall be provided in that section of the slab outside of the band region defined by b_b or as required by clause 13.10.9. **CSA A23.3-14 (13.10.3)**

$$M_r = 290.2 \text{ kN.m}$$

$$\text{Use } d = 350 - (25 + 16/2) = 317 \text{ mm}$$

In this example, jd will be assumed to be taken equal to $0.966d$. The assumptions will be verified once the area of steel is finalized.

$$\text{Assume } jd = 0.966 \times d = 306.2 \text{ mm}$$

$$\text{Column strip width, } b = \min\left(\frac{6600}{2}, \frac{7100}{2}\right) = 3300 \text{ mm}$$

$$\text{Middle strip width, } b = 7100 - 3300 = 3800 \text{ mm}$$

$$A_s = \frac{M_f}{\phi_s f_y jd} = \frac{290.2}{0.85 \times 400 \times 0.966 \times 317} = 2787 \text{ mm}^2$$

$$\alpha_1 = 0.85 - 0.0015 f'_c = 0.81 > 0.67$$

CSA A23.3-14 (10.1.7)

$$\text{Recalculate 'a' for the actual } A_s = 2787 \text{ mm}^2 \rightarrow a = \frac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \frac{0.85 \times 2787 \times 400}{0.65 \times 0.81 \times 25 \times 3300} = 21.75 \text{ mm}$$

$$A_s = \frac{M_f}{\phi_s f_y \left(d - \frac{a}{2} \right)} = \frac{290.2}{0.85 \times 400 \times \left(317 - \frac{21.75}{2} \right)} = 2788 \text{ mm}^2$$

$$jd = d - \frac{a}{2} = 0.966 \times d$$

Therefore, the assumption that jd equals to $0.966d$ is valid.

$$A_{s,\min} = 0.002A_g = 0.002 \times 350 \times 3300 = 2310 \text{ mm}^2$$

CSA A23.3-14 (7.8.1)

$$\therefore A_{s,\text{req}} = 2788 \text{ mm}^2$$

Reinforcement for the total factored negative moment transferred to the exterior columns shall be placed within a band width b_b . CSA A23.3-14 (13.10.3)

For the part of the slab inside of the band region:

Provide 14 - 15M bars ($2800 \text{ mm}^2 > 2788 \text{ mm}^2$)

Temperature and shrinkage reinforcement determined as specified in clause 7.8.1 shall be provided in that section of the slab outside of the band region defined by b_b or as required by clause 13.10.9 (including middle strip and the remaining part of the column strip outside the band region). CSA A23.3-14 (13.10.3)

For the remaining part of the slab outside of the band region:

$$A_{s,\min} = 0.002A_g = 0.002 \times 350 \times (7100 - 1650) = 3815 \text{ mm}^2$$

CSA A23.3-14 (7.8.1)

Provide 20 - 15M bars ($4000 \text{ mm}^2 > 3815 \text{ mm}^2$)

For middle strip:

$$A_{s,\min} = 0.002A_g = 0.002 \times 350 \times 3800 = 2660 \text{ mm}^2$$

CSA A23.3-14 (7.8.1)

Provide 14 - 15M bars ($2800 \text{ mm}^2 > 2660 \text{ mm}^2$)

For the remaining part of the column strip outside of the band region:

$$(20 - 15M) - (14 - 15M) = (6 - 15M)$$

Total Reinforcement in the column Strip:

$$(14 - 15M) + (6 - 15M) = (20 - 15M)$$

Maximum spacing:

CSA A23.3-14 (13.10.4)

Negative reinforcement in the band defined by b_b : $1.5h_s = 525 \text{ mm} \leq 250 \text{ mm}$

$$s_{\max} = 250 \text{ mm} > s_{\text{provided}} = 1650/14 = 118 \text{ mm}$$

Remaining negative moment reinforcement: $3h_s = 1050 \text{ mm} \leq 500 \text{ mm}$

$$s_{\max} = 500 \text{ mm} > s_{\text{provided}} = (7100 - 1650)/20 = 272.5 \text{ mm}$$

Based on the procedure outlined above, values for all span locations are given in Table 3.

Table 3 - Required Slab Reinforcement for Flexure								
Span Location		M_r (kN.m)	b (m)	d (mm)	A_s Req'd for flexure (mm ²)	Min A_s (mm ²)	Reinforcement Provided	A_s Prov. for flexure (mm ²)
End Span								
Column Strip	Exterior Negative	290.2	3300	317	2788.0	2310	20 - 15M* (14 - 15M)**	4000
	Positive	77.3	3300	142	1678.5	1155	9 - 15M	1800
	Interior Negative	256.1 (144.0) [†]	3300	317	2452.2	2310	13 - 15M (7 - 15M)**	2600
Middle Strip	Exterior Negative	0.0	3800	317	0	2660	14 - 15M*	2800
	Positive	73.8	3800	142	1588.8	1330	8 - 15M	1600
	Interior Negative	209.6	3800	317	1986.9	2660	14 - 15M	2800
Interior Span								
Column Strip	Negative	226.4 (127.2) [†]	3300	317	2157.9	2310	12 - 15M (6 - 15M)**	2400
	Positive	58.2	3300	142	1248.3	1155	7 - 15M	1400
Middle Strip	Negative	185.2	3800	317	1751.1	2660	14 - 15M	2800
	Positive	55.6	3800	142	1185.1	1330	8 - 15M	1600
[†] Negative moment at an interior column in width b_b . * The reinforcement is selected to meet CSA A23.3-14 provision 13.10.3. ** Reinforcement in width b_b .								

- b. Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure
 When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of unbalanced moment given by γ_f shall be transferred by flexural reinforcement placed within a width b_b . CSA A23.3-14 (13.10.2)

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_r$

$$\gamma_f = \frac{1}{1 + (2/3) \times \sqrt{b_1 / b_2}} \quad \text{CSA A23.3-14 (13.10.2)}$$

Where

b_1 = Width of the critical section for shear measured in the direction of the span for which moments are determined according to CSA A23.3-14, clause 13 (see Figure 15).

b_2 = Width of the critical section for shear measured in the direction perpendicular to b_1 according to CSA A23.3-14, clause 13 (see Figure 15).

b_b = Effective slab width = $c_2 + 3 \times h_s$ CSA A23.3-14 (3.2)

$$b_b = 600 + 3 \times 350 = 1650 \text{ mm}$$

For Exterior Column

$$b_1 = 100 + 400 + \frac{317}{2} = 658.5 \text{ mm}$$

$$b_2 = 600 + 317 = 917 \text{ mm}$$

$$\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right) \times \sqrt{\frac{658.5}{917}}} = 0.639$$

For Interior Column

$$b_1 = 400 + 317 = 717 \text{ mm}$$

$$b_2 = 600 + 317 = 917 \text{ mm}$$

$$\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right) \times \sqrt{\frac{717}{917}}} = 0.629$$

Repeat the same procedure in section 2.2.6.a to calculate the additional reinforcement required for the unbalanced moment as shown in the following table:

Table 4 - Additional Slab Reinforcement required for moment transfer between slab and column									
Span Location		M_u^* (kN.m)	γ_r	$\gamma_r M_u$ (kN.m)	Effective slab width, b_b (mm)	d (mm)	A_s req'd within b_b (mm ²)	A_s prov. For flexure within b_b (mm ²)	Add'l Reinf.
End Span									
Column Strip	Exterior Negative	354.5	0.639	226.5	1650	317	2233	2800	-
	Interior Negative	59.8	0.629	37.6	1650	317	352	1400	-

* M_u is taken at the centerline of the support in Elastic Frame Method solution.

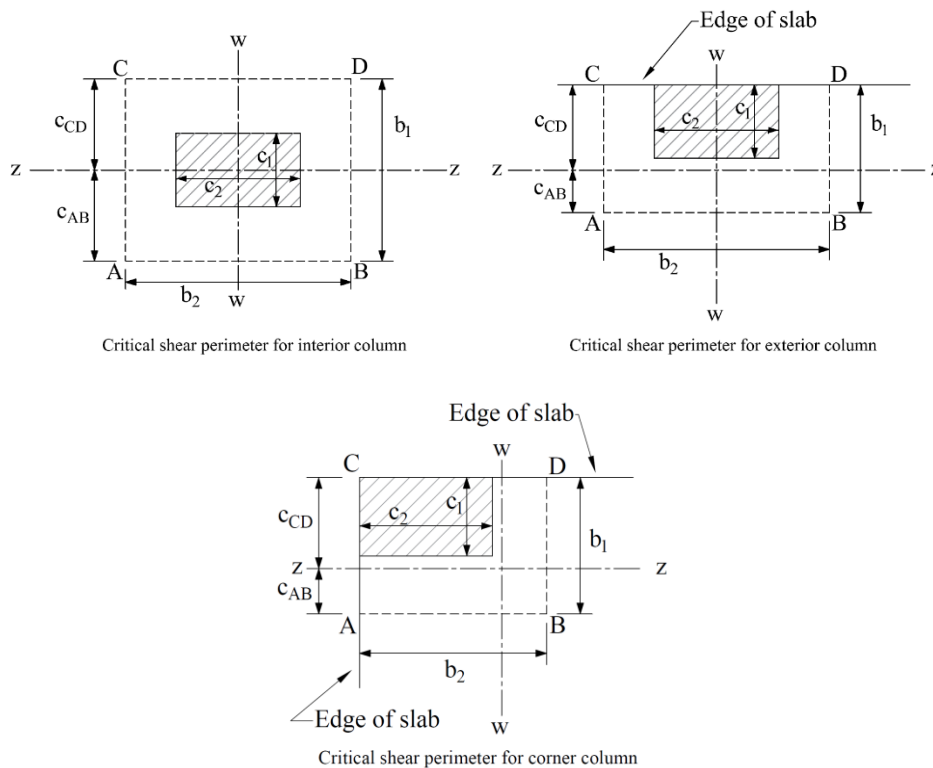


Figure 17 - Critical Shear Perimeters for Columns

2.2.7. Factored moments in columns

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the support columns above and below the slab-beam in proportion to the relative stiffness of the support columns. Detailed calculations regarding this topic (including column design for axial load and biaxial moments) can be found in “[Two-Way Flat Slab \(Drop Panels\) Concrete Floor Analysis and Design \(CSA A23.3-14\)](#)” example available in the [design examples](#) page in [StructurePoint](#) website.

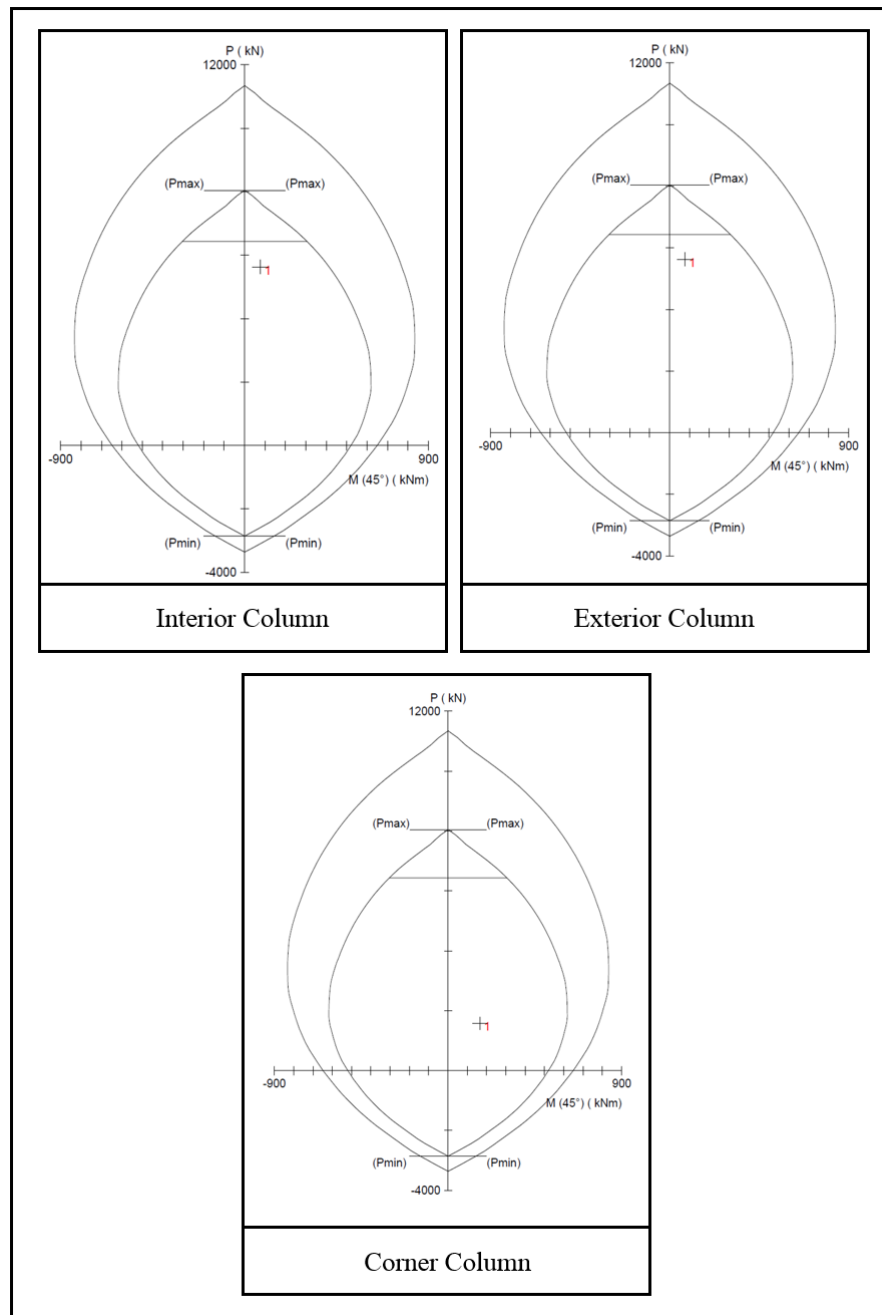


Figure 18 - Sample Calculations of Column Design from “Two-Way Flat Slab (Drop Panels) Concrete Floor Analysis and Design” Design Example

3. Two-Way Slab Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with CSA A23.3-14 clause 13.

3.1. One-Way (Beam action) Shear Strength

CSA A23.3-14 (13.3.6)

One-way shear is critical at a distance d_v from the face of the column and slab band. Figure 17 and 18 show the factored shear forces (V_r) at the critical sections around each column and slab band. In members without shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$V_r = V_c + V_s + V_p = V_c \quad , \quad (V_s = V_p = 0) \quad \text{CSA A23.3-14 (Eq. 11.4)}$$

Where:

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b d_v \quad \text{CSA A23.3-14 (Eq. 11.5)}$$

3.1.1. At distance d_v from the supporting column

$$h = 350 \text{ mm}$$

$$d = 350 - 25 - \frac{16}{2} = 317 \text{ mm}$$

$$d_v = \text{Max} (0.9d, 0.72h) = \text{Max} (0.9 \times 317, 0.72 \times 350) = 285.3 \text{ mm} \quad \text{CSA A23.3-14 (3.2)}$$

$\lambda = 1$ for normal weight concrete

$$\beta = 0.21 \text{ for slabs with overall thickness not greater than } 350 \text{ mm} \quad \text{CSA A23.3-14 (11.3.6.2)}$$

$$V_c = 0.65 \times 1.0 \times 0.21 \times \sqrt{25} \times 7100 \times \frac{285.3}{1000} = 1382 \text{ kN} > V_f$$

Because $V_c \geq V_f$ at all the critical sections, the slab has adequate one-way shear strength.

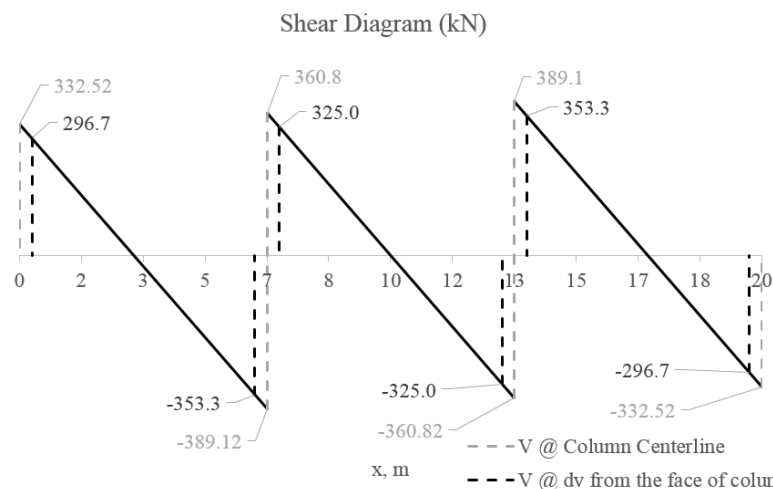


Figure 19 – One-way shear at critical sections (at distance d_v from the face of the supporting column)

3.1.2. At the face of the slab band

$$h = 350 \text{ mm}$$

$$d = 175 - 25 - \frac{16}{2} = 142 \text{ mm}$$

$$d_v = \text{Max} (0.9d, 0.72h) = \text{Max} (0.9 \times 142, 0.72 \times 175) = 127.8 \text{ mm} \quad \text{CSA A23.3-14 (3.2)}$$

$\lambda = 1$ for normal weight concrete

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm CSA A23.3-14 (11.3.6.2)

$$V_c = 0.65 \times 1.0 \times 0.21 \times \sqrt{25} \times 7100 \times \frac{127.8}{1000} = 619.3 \text{ kN} > V_f$$

Because $V_c \geq V_f$ at all the critical sections, the slab has adequate one-way shear strength.

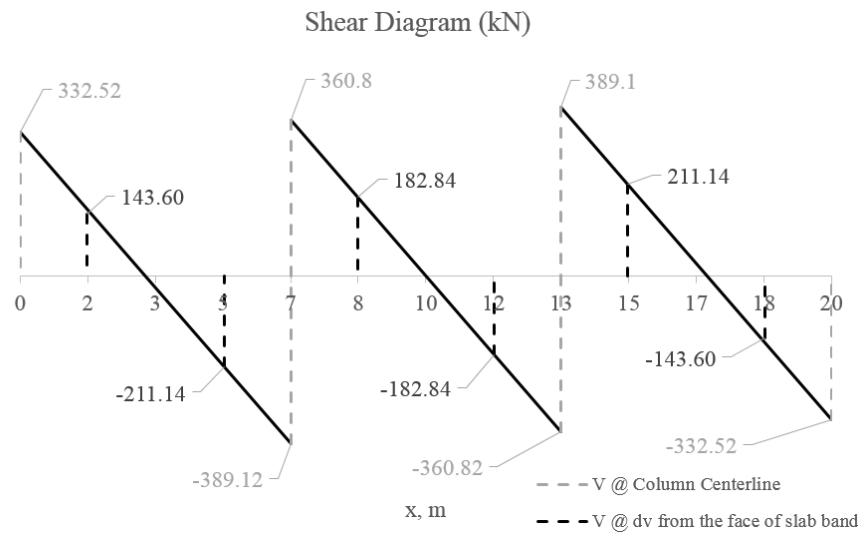


Figure 20 – One-way shear at critical sections (at distance d_v from the face of the slab band)

3.2. Two-Way (Punching) Shear Strength

CSA A23.3-14 (13.3.2)

Two-way shear is critical on a rectangular section located at $d/2$ away from the face of the column as shown in Figure 15.

a. Exterior column:

The factored shear force (V_f) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section ($d/2$ away from column face).

$$V_f = 332.5 - 15.4(0.6585 \times 0.917) = 323.2 \text{ kN}$$

The factored unbalanced moment used for shear transfer, M_{unb} , is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$M_{\text{unb}} = M_u - V_f \left(\frac{b_1 - c_{AB} - c_1 / 2 - 100 \text{ mm}}{1000 \text{ mm}} \right)$$

$$M_{\text{unb}} = 354.5 - 323.2 \left(\frac{658.5 - 194.1 - 400 / 2 - 100}{1000} \right) = 301.4 \text{ kN.m}$$

For the exterior column in Figure 15, the location of the centroidal axis z-z is:

$$c_{AB} = e = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{2 \times (658.5 \times 317 \times 658.5 / 2)}{2 \times 658.5 \times 317 + 917 \times 317} = 194.1 \text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2 \left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d) \left(\frac{b_1}{2} - c_{AB} \right)^2 \right) + b_2 d c_{AB}^2$$

$$J_c = 2 \left(\frac{658.5 \times 317^3}{12} + \frac{317 \times 658.5^3}{12} + (658.5 \times 317) \left(\frac{658.5}{2} - 194.1 \right)^2 \right) + 617 \times 317 \times (194.1)^2 = 37.16 \times 10^9 \text{ mm}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.639 = 0.361$$

CSA A23.3-14 (Eq. 13.8)

The length of the critical perimeter for the exterior column:

$$b_o = 2 \times 658.5 + 917 = 2234 \text{ mm}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{\text{unb}} e}{J}$$

CSA A23.3-14 (Eq. 13.9)

$$v_f = \frac{323.2 \times 1000}{2234 \times 317} + \frac{0.361 \times (301.4 \times 10^6) \times 194.1}{37.16 \times 10^9}$$

$$v_f = 0.456 + 0.568 = 1.025 \text{ MPa}$$

The factored resisting shear stress, v_r shall be the smallest of :

CSA A23.3-14 (13.3.4.1)

$$a) \quad v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c} = \left(1 + \frac{2}{0.67}\right) 0.19 \times 0.65 \times \sqrt{25} = 2.470 \text{ MPa}$$

$$\text{Where } \beta_c = c_1/c_2 = 400/600 = 0.67$$

$$b) \quad v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c} = \left(\frac{3 \times 317}{2234} + 0.19\right) \times 1 \times 0.65 \times \sqrt{25} = 2.001 \text{ MPa}$$

$$\text{Where } \alpha_s = 3 \text{ for edge columns}$$

$$c) \quad v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{25} = 1.235 \text{ MPa}$$

$$v_c = \min(2.470, 2.001, 1.235) = 1.235 \text{ MPa}$$

CSA A23.3 requires multiplying the value of v_c by $1300/(1000+d)$ if the effective depth used in the two-way shear calculations exceeds 300 mm.

CSA A23.3-14 (13.3.4.3)

$$v_c = \left(\frac{1300}{1000 + 317}\right) \times 1.235 = 1.219 \text{ MPa}$$

Since ($v_r = 1.219 \text{ MPa} \geq v_f = 1.025 \text{ MPa}$) at the critical section, the slab with slab band has adequate two-way shear strength at this joint.

b. Interior column:

$$V_f = (360.8 + 389.1) - 15.4 \times (0.717 \times 0.917) = 739.8 \text{ kN}$$

$$M_{umb} = (541.3 - 481.5) - 739.8 \times 0 = 59.8 \text{ kN.m}$$

For the interior column in Figure 15, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{717}{2} = 358.5 \text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2 \left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d) \left(\frac{b_1}{2} - c_{AB} \right)^2 \right) + 2 b_2 d c_{AB}^2$$

$$J_c = 2 \left(\frac{717 \times 317^3}{12} + \frac{317 \times 717^3}{12} + (717 \times 317) \left(\frac{717}{2} - 358.5 \right)^2 \right) + 2 \times 917 \times 317 \times (358.5)^2 = 98.00 \times 10^9 \text{ mm}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.629 = 0.371$$

CSA A23.3-14 (Eq. 13.8)

The length of the critical perimeter for the interior column:

$$b_o = 2 \times (717 + 917) = 3268 \text{ mm}$$

$$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{unb} e}{J}$$

CSA A23.3-14 (Eq.13.9)

$$v_f = \frac{739.8 \times 1000}{3268 \times 317} + \frac{0.371 \times (59.8 \times 10^6) \times 358.5}{98.00 \times 10^9}$$

$$v_f = 0.714 + 0.081 = 0.795 \text{ MPa}$$

The factored resisting shear stress, V_r shall be the smallest of :

CSA A23.3-14 (13.3.4.1)

$$a) \quad v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c} = \left(1 + \frac{2}{0.67}\right) 0.19 \times 0.65 \times \sqrt{25} = 2.470 \text{ MPa}$$

$$b) \quad v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c} = \left(\frac{4 \times 317}{3268} + 0.19\right) \times 1 \times 0.65 \times \sqrt{25} = 1.879 \text{ MPa}$$

$$c) \quad v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{25} = 1.235 \text{ MPa}$$

$$v_c = \min(1.441, 1.993, 1.235) = 1.235 \text{ MPa}$$

CSA A23.3 requires multiplying the value of v_c by $1300/(1000+d)$ if the effective depth used in the two-way shear calculations exceeds 300 mm.

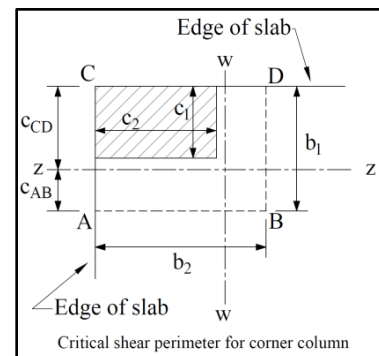
CSA A23.3-14 (13.3.4.3)

$$v_c = \left(\frac{1300}{1000 + 317}\right) \times 1.235 = 1.219 \text{ MPa}$$

Since ($v_r = 1.219 \text{ MPa} \geq v_f = 0.795 \text{ MPa}$) at the critical section, the slab with slab band has adequate two-way shear strength at this joint.

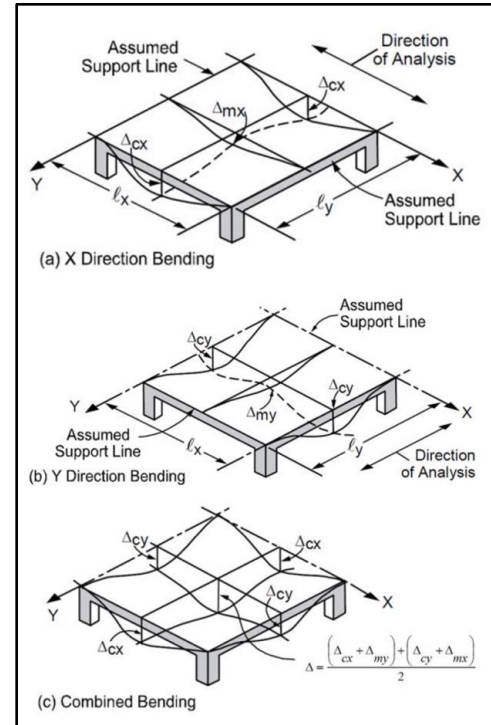
c. Corner column:

In this example, interior equivalent elastic frame strip was selected where it only have exterior and interior supports (no corner supports are included in this strip). Detailed calculations for two-way (punching) shear check around corner supports can be found in “[Two-Way Flat Plate Concrete Slab Floor Analysis and Design \(CSA A23.3-14\)](#)” example available in the [design examples](#) page in [StructurePoint](#) website.



4. Two-Way Slab Deflection Control (Serviceability Requirements)

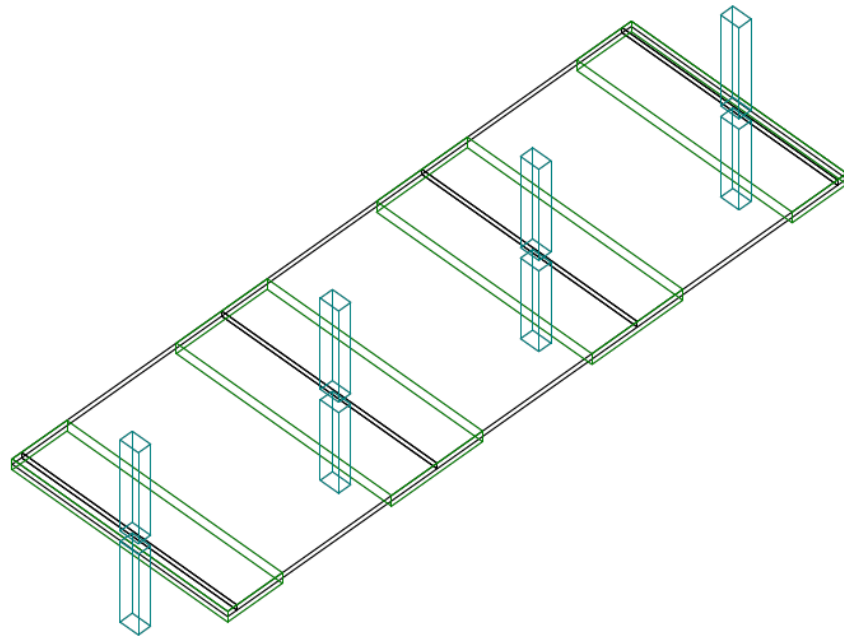
Since the slab thickness was selected based on the minimum slab thickness equations in CSA A23.3-14, the deflection calculations are not required. Detailed calculations of immediate and time-dependent deflections can be found in “[Two-Way Flat Plate Concrete Slab Floor Analysis and Design \(CSA A23.3-14\)](#)” example available in the [design examples](#) page in [StructurePoint](#) website.



5. spSlab Software Solution

[spSlab](#) program utilizes the Elastic (Equivalent) Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems. [spSlab](#) uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept.

[spSlab](#) Program models the equivalent elastic frame as a design strip. The design strip is, then, separated by [spSlab](#) into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips. The graphical and text results are provided below for both input and output of the [spSlab](#) model.



spSlab v5.50. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-2C6B6

File: C:\CSA A23.3-14 Examples\Slab with Slab Bands.slb

Project: CAC 4th Edition - Section 5.7 - Example 3

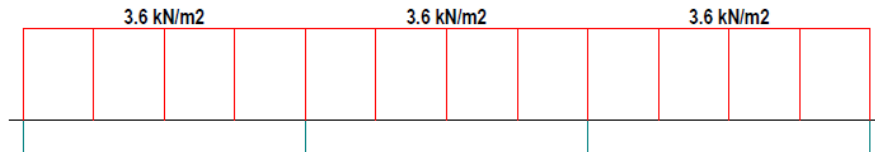
Frame: Interior

Engineer: SP

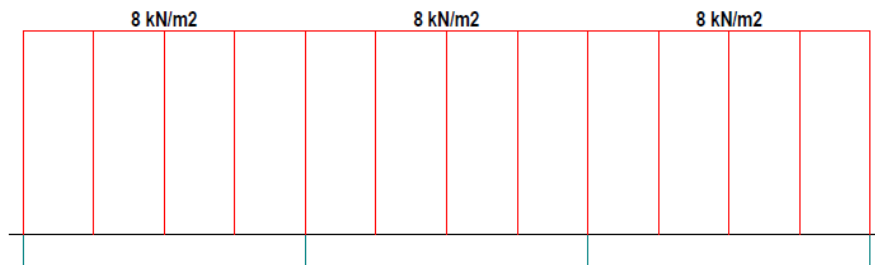
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Date: 12/24/19

Time: 10:35:34



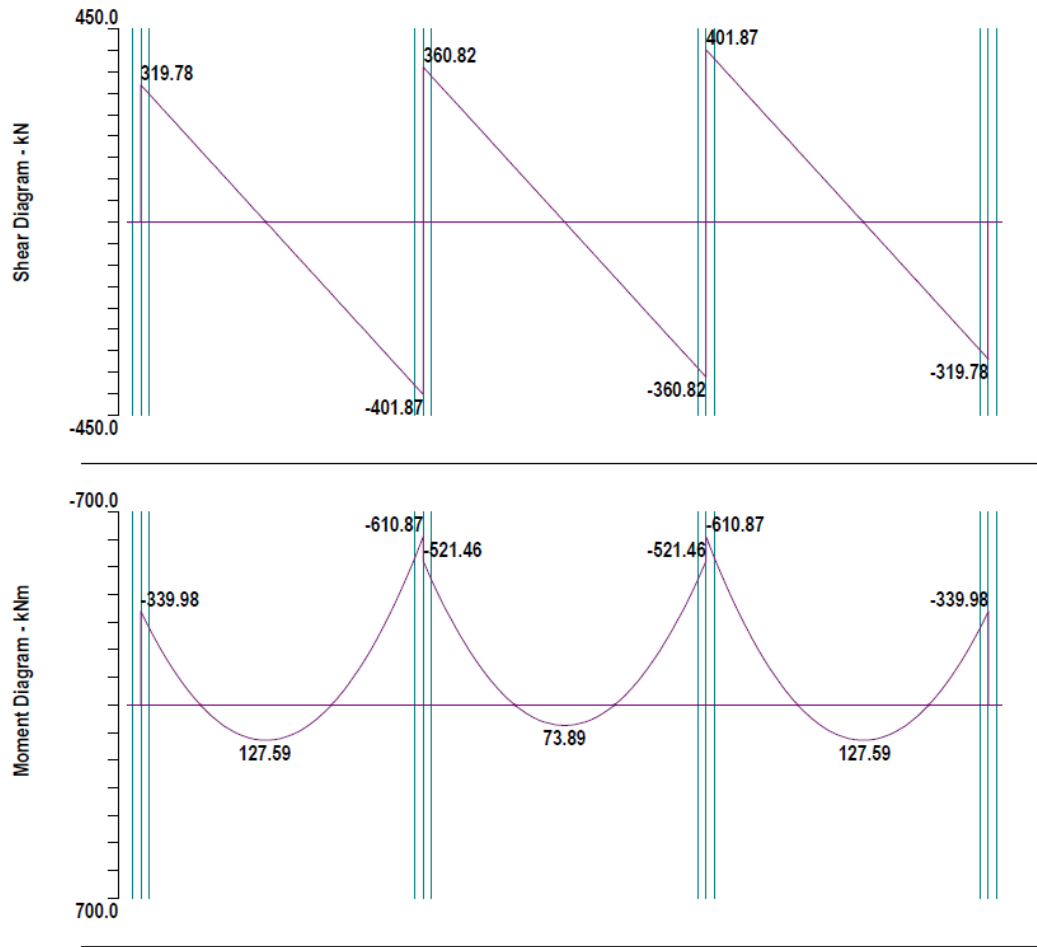
CASE/PATTERN: Live/All



CASE: Dead

spSlab v5.50. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-2C6B6

File: C:\CSA A23.3-14 Examples\Slab with Slab Bands.slb
Project: CAC 4th Edition - Section 5.7 - Example 3
Frame: Interior
Engineer: SP
Code: CSA A23.3-14
Date: 12/24/19
Time: 10:36:40



LEGEND:
Envelope

spSlab v5.50. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-2C6B6

File: C:\CSA A23.3-14 Examples\Slab with Slab Bands.slb

Project: CAC 4th Edition - Section 5.7 - Example 3

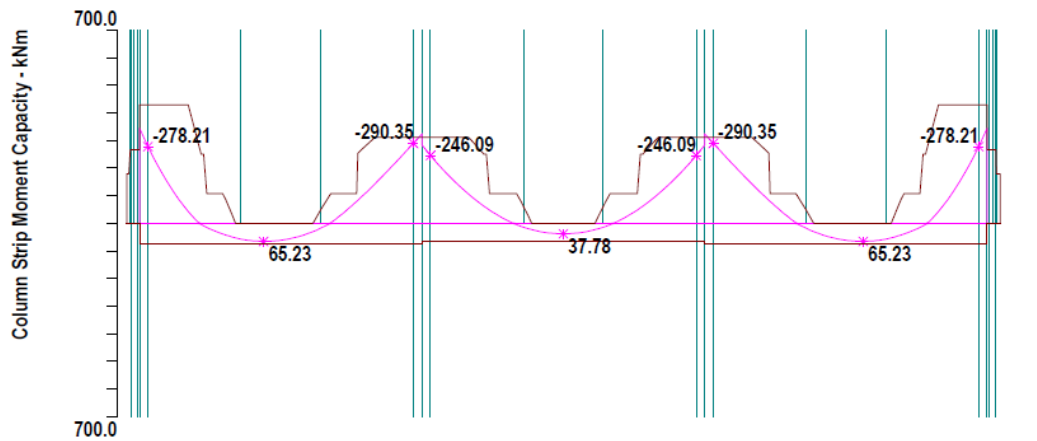
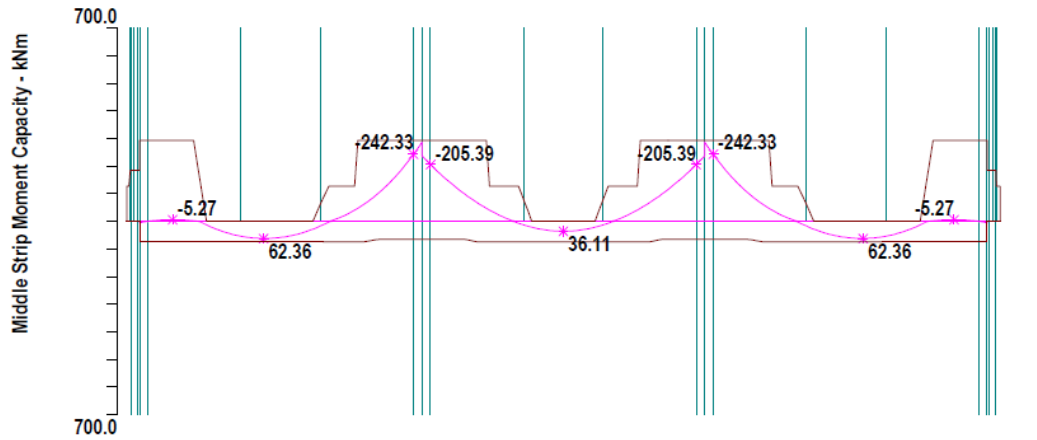
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 12/24/19

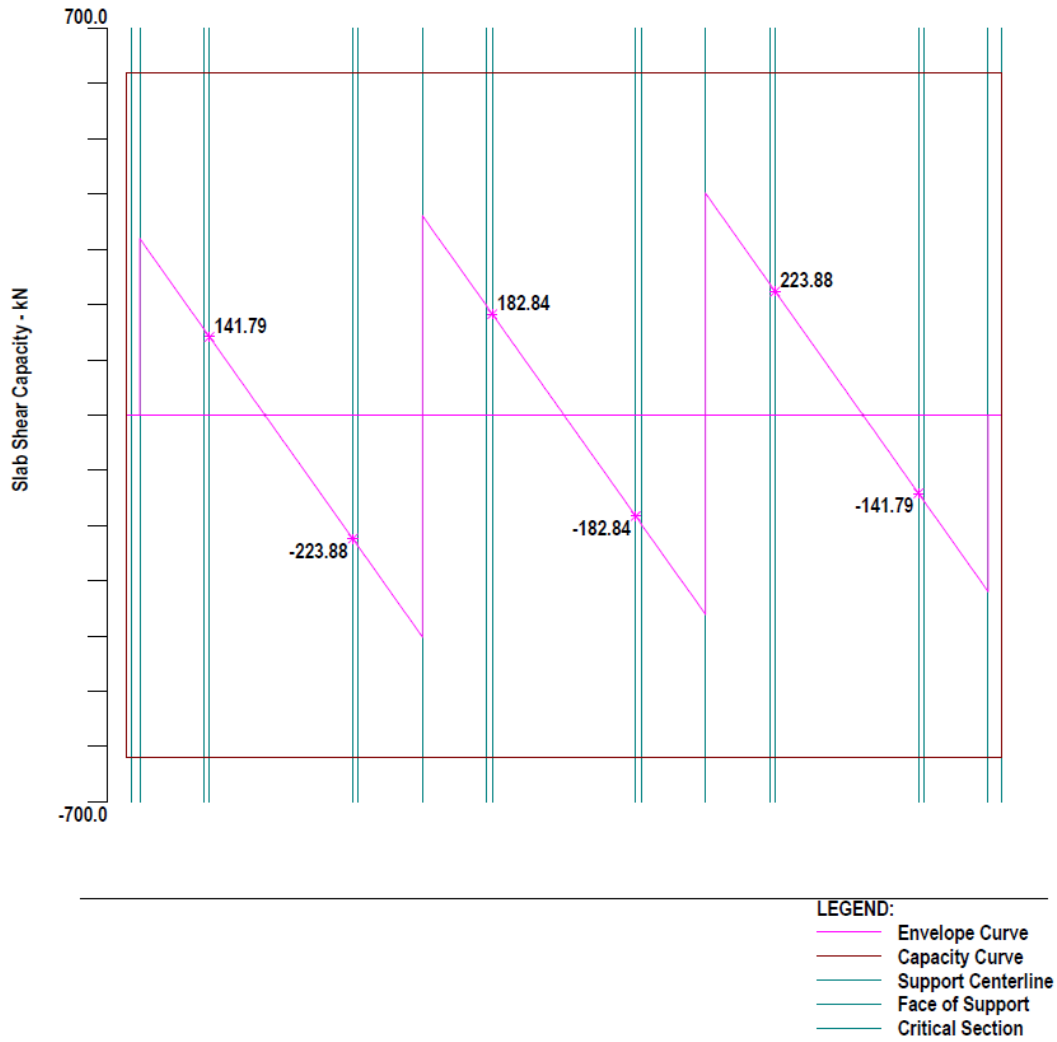
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LEGEND:
— Envelope Curve
— Capacity Curve
— Support Centerline
— Face of Support
— Zone Limits

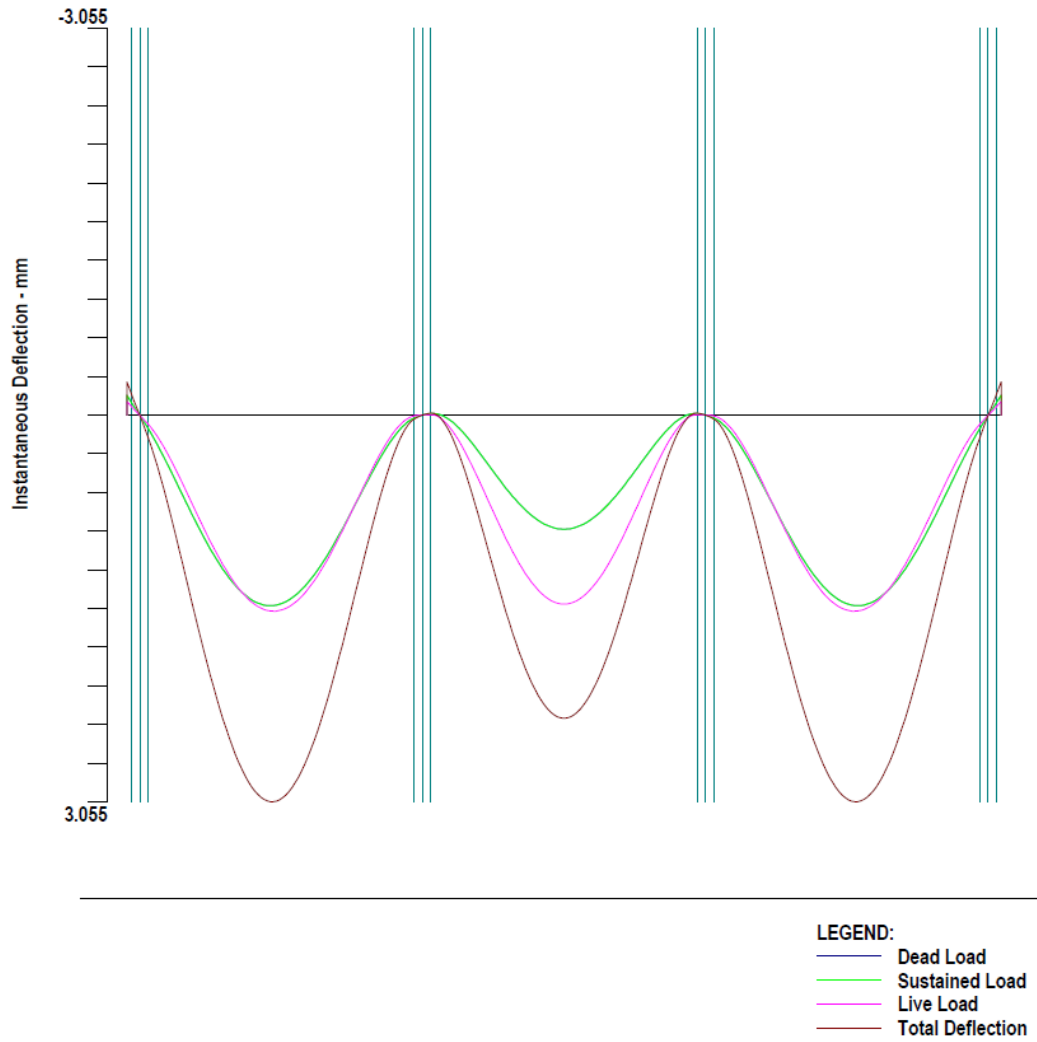
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File: C:\CSA A23.3-14 Examples\Slab with Slab Bands.slb
 Project: CAC 4th Edition - Section 5.7 - Example 3
 Frame: Interior
 Engineer: SP
 Code: CSA A23.3-14
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File: C:\CSA A23.3-14 Examples\Slab with Slab Bands.slb
 Project: CAC 4th Edition - Section 5.7 - Example 3
 Frame: Interior
 Engineer: SP
 Code: CSA A23.3-14
 Date: 12/24/19
 Time: 10:41:17



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File: C:\CSA A23.3-14 Examples\Slab with Slab Bands.slb

Project: CAC 4th Edition - Section 5.7 - Example 3

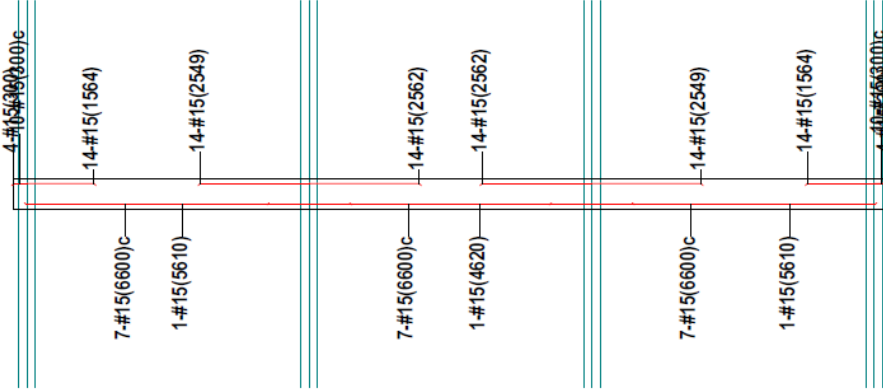
Frame: Interior

Engineer: SP

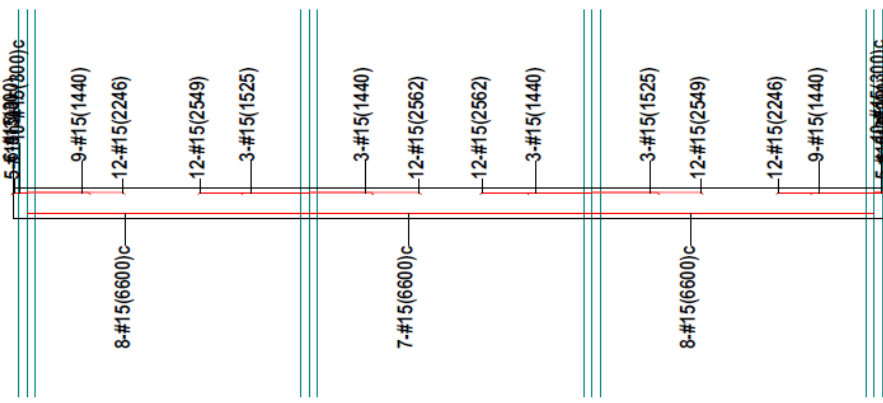
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Middle Strip Flexural Reinforcement



Column Strip Flexural Reinforcement

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File: C:\CSA A23.3-14 Examples\Slab with Slab Bands.slb

Project: CAC 4th Edition - Section 5.7 - Example 3

Frame: Interior

Engineer: SP

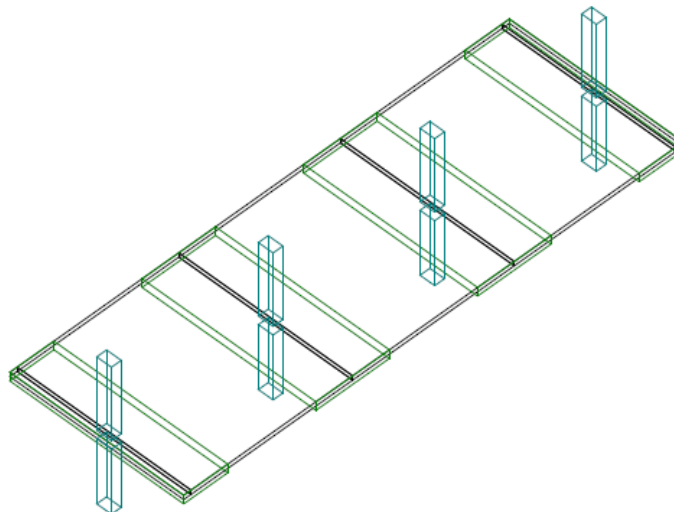
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Date: 12/24/19

Time: 11:30:15



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A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems
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1. Input Echo

1.1. General Information

File Name	C:\CSA A23.3-14 Examp...\Slab with Slab Bands.slb
Project	CAC 4th Edition - Section 5.7 - Example 3
Frame	Interior
Engineer	SP
Code	CSA A23.3-14
Reinforcement Database	CSA G30.18
Mode	Design
Number of supports =	4 + Left cantilever + Right cantilever
Floor System	Two-Way
Slab Bands	Transverse

1.2. Solve Options

Live load pattern ratio = 0%
Minimum free edge distance for punching shear = 5 times slab effective depth.
Circular critical section around circular supports used (if possible).
Deflections are based on cracked section properties.
In negative moment regions, I_g and M_{cr} DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Combined M-V-T reinforcement design NOT selected.
User-defined slab strip widths NOT selected.
User-defined distribution factors NOT selected.
One-way shear in drop panel NOT selected.
Distribution of shear to strips NOT selected.
Beam T-section design NOT selected.
Longitudinal beam contribution in negative reinforcement design over support NOT selected.
Transverse beam contribution in negative reinforcement design over support NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Bands

w_c	2402.8 kg/m ³
f_c	25 MPa
E_c	24986 MPa
f_r	1.5 MPa
Precast concrete	No

1.3.2. Concrete: Columns

w_c	2402.8 kg/m ³
f_c	25 MPa
E_c	24986 MPa
f_r	1.5 MPa
Precast concrete	No

1.3.3. Reinforcing Steel

f_y	400 MPa
f_{yt}	400 MPa
E_s	200000 MPa
Epoxy coated bars	No

1.4. Reinforcement Database

Size	Db mm	Ab mm ²	Wb kg/m	Size	Db mm	Ab mm ²	Wb kg/m
#10	11	100	1	#15	16	200	2
#20	20	300	2	#25	25	500	4
#30	30	700	5	#35	36	1000	8
#45	44	1500	12	#55	56	2500	20

1.5. Span Data

1.5.1. Slabs

Notes:

Deflection check required for panels where code-specified H_{min} for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

Span	Loc	L1 m	t mm	wL m	wR m	L2L m	L2R m	H_{min} mm
1	Int	0.300	175	3.750	3.350	7.500	6.700	--- LC *i
2	Int	6.600	175	3.750	3.350	7.500	6.700	133
3	Int	6.600	175	3.750	3.350	7.500	6.700	133
4	Int	6.600	175	3.750	3.350	7.500	6.700	133
5	Int	0.300	175	3.750	3.350	7.500	6.700	--- RC *i

1.6. Support Data

1.6.1. Columns

Support	c1a mm	c2a mm	Ha m	c1b mm	c2b mm	Hb m	Red %
1	400	600	3.000	400	600	3.000	100
2	400	600	3.000	400	600	3.000	100
3	400	600	3.000	400	600	3.000	100
4	400	600	3.000	400	600	3.000	100

1.6.2. Transverse Slab Bands

Supports	b mm	h mm	Ecc mm
1	1800	350	600
2	3000	350	0
3	3000	350	0
4	1800	350	-600

1.6.3. Boundary Conditions

Support	Spring		Far End	
	K_z kN/mm	K_{ry} kN-mm/rad	Above	Below
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case Type	Dead DEAD	Live LIVE
U1	1.250	1.500

1.7.2. Area Loads

Case/Patt	Span	Wa kN/m ²
Dead	2	8.00
	3	8.00
	4	8.00
Live	2	3.60
	3	3.60
	4	3.60

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#15	#15	#15	#15
Bar spacing	mm	25	500	25	500
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	mm	25		25	

There is NOT more than 300 mm of concrete below top bars.

2. Design Results*

*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

2.1. Strip Widths and Distribution Factors

Notes:

*Used for bottom reinforcement. **Used for top reinforcement.

Span	Strip	Width			Moment Factor		
		Left ** m	Right ** m	Bottom * m	Left ** m	Right ** m	Bottom * m
1	Column	3.30	3.30	3.30	1.000	1.000	0.550
	Middle	3.80	3.80	3.80	0.000	0.000	0.450
2	Column	3.30	3.30	3.30	1.000	0.545	0.511
	Middle	3.80	3.80	3.80	0.000	0.455	0.489
3	Column	3.30	3.30	3.30	0.545	0.545	0.511
	Middle	3.80	3.80	3.80	0.455	0.455	0.489
4	Column	3.30	3.30	3.30	0.545	1.000	0.511
	Middle	3.80	3.80	3.80	0.455	0.000	0.489
5	Column	3.30	3.30	3.30	1.000	1.000	0.550
	Middle	3.80	3.80	3.80	0.000	0.000	0.450

2.2. Top Reinforcement

Notes:

*3 - Design governed by minimum reinforcement.

*5 - Number of bars governed by maximum allowable spacing.

Span	Strip	Zone	Width m	M _{max} kNm	X _{max} m	A _{s,min} mm ²	A _{s,max} mm ²	A _{s,req} mm ²	Sp _{prov} mm	Bars	
1	Column	Left	3.30	0.00	0.000	1155	10509	0	330	10-#15 *3 *5	
		Midspan	3.30	0.00	0.124	1563	15088	0	330	10-#15 *3	
		Right	3.30	0.00	0.248	1563	15088	0	157	21-#15 *3 *5	
	Middle	Left	3.80	0.00	0.000	1330	12101	0	380	10-#15 *3 *5	
		Midspan	3.80	0.00	0.124	1800	17374	0	380	10-#15 *3	
		Right	3.80	0.00	0.248	1800	17374	0	271	14-#15 *3	
	2	Column	Left	3.30	278.21	0.200	2310	23460	2669	157	21-#15
			Midspan	3.30	0.00	3.300	0	10509	0	0	---
			Right	3.30	290.35	6.400	2310	23460	2790	220	15-#15
Middle		Left	3.80	5.27	0.778	2660	27014	49	271	14-#15 *3	
		Midspan	3.80	0.00	3.300	0	12101	0	0	---	
		Right	3.80	242.33	6.400	2660	27014	2305	271	14-#15 *3	
3		Column	Left	3.30	246.09	0.200	2310	23460	2351	220	15-#15
			Midspan	3.30	0.00	3.300	0	10509	0	0	---
			Right	3.30	246.09	6.400	2310	23460	2351	220	15-#15
	Middle	Left	3.80	205.39	0.200	2660	27014	1946	271	14-#15 *3	
		Midspan	3.80	0.00	3.300	0	12101	0	0	---	
		Right	3.80	205.39	6.400	2660	27014	1946	271	14-#15 *3	
	4	Column	Left	3.30	290.35	0.200	2310	23460	2790	220	15-#15
			Midspan	3.30	0.00	3.300	0	10509	0	0	---
			Right	3.30	278.21	6.400	2310	23460	2669	157	21-#15
Middle		Left	3.80	242.33	0.200	2660	27014	2305	271	14-#15 *3	
		Midspan	3.80	0.00	3.300	0	12101	0	0	---	
		Right	3.80	5.27	5.822	2660	27014	49	271	14-#15 *3	
5		Column	Left	3.30	0.00	0.052	1563	15088	0	157	21-#15 *3 *5
			Midspan	3.30	0.00	0.176	1563	15088	0	330	10-#15 *3
			Right	3.30	0.00	0.300	1155	10509	0	330	10-#15 *3 *5
	Middle	Left	3.80	0.00	0.052	1800	17374	0	271	14-#15 *3	
		Midspan	3.80	0.00	0.176	1800	17374	0	380	10-#15 *3	
		Right	3.80	0.00	0.300	1330	12101	0	380	10-#15 *3 *5	

2.3. Top Bar Details

NOTES:

* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

Span	Strip	Left				Continuous		Right			
		Bars	Length m	Bars	Length m	Bars	Length m	Bars	Length m	Bars	Length m
1	Column	---	---	---	---	10-#15	0.30	6-#15	0.30	5-#15	0.30
	Middle	---	---	---	---	10-#15	0.30	4-#15	0.30	---	---
2	Column	12-#15	2.25	9-#15	1.44	---	---	12-#15	2.55	3-#15 *	1.53

Span Strip	Bars	Left		Continuous		Right				
		Length m	Bars	Length m	Bars	Length m	Bars	Length m	Bars	Length m
Middle	14-#15	1.56	---	---	---	14-#15	2.55	---	---	---
3 Column	12-#15	2.56	3-#15 *	1.44	---	12-#15	2.56	3-#15 *	1.44	---
Middle	14-#15	2.56	---	---	---	14-#15	2.56	---	---	---
4 Column	12-#15	2.55	3-#15 *	1.53	---	12-#15	2.25	9-#15	1.44	---
Middle	14-#15	2.55	---	---	---	14-#15	1.56	---	---	---
5 Column	6-#15	0.30	5-#15	0.30	10-#15	0.30	---	---	---	---
Middle	4-#15	0.30	---	---	10-#15	0.30	---	---	---	---

2.4. Top Bar Development Lengths

Span Strip	Bars	Left		Continuous		Right				
		DevLen mm	Bars	DevLen mm	Bars	DevLen mm	Bars	DevLen mm	Bars	DevLen mm
1 Column	---	---	---	10-#15	300.00	6-#15	300.00	5-#15	300.00	---
Middle	---	---	---	10-#15	300.00	4-#15	300.00	---	---	---
2 Column	12-#15	300.00	9-#15	300.00	---	12-#15	417.00	3-#15	417.00	---
Middle	14-#15	300.00	---	---	---	14-#15	369.19	---	---	---
3 Column	12-#15	351.47	3-#15	351.47	---	12-#15	351.47	3-#15	351.47	---
Middle	14-#15	311.68	---	---	---	14-#15	311.68	---	---	---
4 Column	12-#15	417.00	3-#15	417.00	---	12-#15	300.00	9-#15	300.00	---
Middle	14-#15	369.19	---	---	---	14-#15	300.00	---	---	---
5 Column	6-#15	300.00	5-#15	300.00	10-#15	300.00	---	---	---	---
Middle	4-#15	300.00	---	---	10-#15	300.00	---	---	---	---

2.5. Band Reinforcement at Supports

NOTES:

<C> Total Strip, Banded Strip, <S> Remaining Strip

Support	Width <C> mm	Width mm	Width <S> mm	A _s <C> mm ²	A _s mm ²	A _s <S> mm ²	Bars <C>	Bars 	Bars <S>
1	3300	1200	2100	4200	2800	1400	21-#15	14-#15	7-#15
2	3300	1650	1650	3000	1800	1200	15-#15	9-#15	6-#15
3	3300	1650	1650	3000	1800	1200	15-#15	9-#15	6-#15
4	3300	1200	2100	4200	2800	1400	21-#15	14-#15	7-#15

2.6. Bottom Reinforcement

Notes:

*3 - Design governed by minimum reinforcement.

*5 - Number of bars governed by maximum allowable spacing.

Span Strip	Width m	M _{max} kNm	X _{max} m	A _{s,min} mm ²	A _{s,max} mm ²	A _{s,req} mm ²	Sp _{prov} mm	Bars
1 Column	3.30	0.00	0.124	0	10509	0	0	---
Middle	3.80	0.00	0.124	0	12101	0	0	---
2 Column	3.30	65.23	2.896	1155	10509	1405	413	8-#15
Middle	3.80	62.36	2.896	1330	12101	1334	475	8-#15 *5

Span Strip	Width m	M _{max} kNm	X _{max} m	A _{s,min} mm ²	A _{s,max} mm ²	A _{s,req} mm ²	Sp _{Prov} mm	Bars
3 Column	3.30	37.78	3.300	1155	10509	800	471	7-#15 *3 *5
3 Middle	3.80	36.11	3.300	1330	12101	762	475	8-#15 *3 *5
4 Column	3.30	65.23	3.704	1155	10509	1405	413	8-#15
4 Middle	3.80	62.36	3.704	1330	12101	1334	475	8-#15 *5
5 Column	3.30	0.00	0.176	0	10509	0	0	---
5 Middle	3.80	0.00	0.176	0	12101	0	0	---

2.7. Bottom Bar Details

Span Strip	Long Bars			Short Bars		
	Bars	Start m	Length m	Bars	Start m	Length m
1 Column	---			---		
1 Middle	---			---		
2 Column	8-#15	0.00	6.60	---		
2 Middle	7-#15	0.00	6.60	1-#15	0.00	5.61
3 Column	7-#15	0.00	6.60	---		
3 Middle	7-#15	0.00	6.60	1-#15	0.99	4.62
4 Column	8-#15	0.00	6.60	---		
4 Middle	7-#15	0.00	6.60	1-#15	0.99	5.61
5 Column	---			---		
5 Middle	---			---		

2.8. Bottom Bar Development Lengths

Span Strip	Long Bars		Short Bars	
	Bars	DevLen mm	Bars	DevLen mm
1 Column	---		---	
1 Middle	---		---	
2 Column	8-#15	393.89	---	
2 Middle	7-#15	373.89	1-#15	373.89
3 Column	7-#15	300.00	---	
3 Middle	7-#15	300.00	1-#15	300.00
4 Column	8-#15	393.89	---	
4 Middle	7-#15	373.89	1-#15	373.89
5 Column	---		---	
5 Middle	---		---	

2.9. Flexural Capacity

Span Strip	x m	A _{s,top} mm ²	Top				Bottom				
			ΦM _{n-} kNm	M _{u-} kNm	Comb Pat	Status	A _{s,bot} mm ²	ΦM _{n+} kNm	M _{u+} kNm	Comb Pat	Status
1 Column	0.000	4200	-179.37	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK

Span Strip	x m	Top					Bottom				
		A _{s,top} mm ²	ΦM _{n-} kNm	M _{u-} kNm	Comb Pat	Status	A _{s,bot} mm ²	ΦM _{n+} kNm	M _{u+} kNm	Comb Pat	Status
Middle	0.087	4200	-267.73	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.150	4200	-267.73	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.161	4200	-267.73	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.248	4200	-267.73	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.300	4200	-267.73	0.00	U1 All	---	0	0.00	0.00	U1 All	---
	0.000	2800	-126.15	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.087	2800	-185.06	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.150	2800	-185.06	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.161	2800	-185.06	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.248	2800	-185.06	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
0.300	2800	-185.06	0.00	U1 All	---	0	0.00	0.00	U1 All	---	
2 Column	0.000	4200	-429.27	-344.97	U1 All	---	1600	73.85	0.00	U1 All	---
	0.200	4200	-429.27	-278.21	U1 All	OK	1600	73.85	0.00	U1 All	OK
	1.140	4200	-429.27	-43.28	U1 All	OK	1600	73.85	0.00	U1 All	OK
	1.440	2400	-251.03	0.00	U1 All	OK	1600	73.85	3.63	U1 All	OK
	1.946	2400	-108.23	0.00	U1 All	OK	1600	73.85	38.48	U1 All	OK
	2.246	0	0.00	0.00	U1 All	OK	1600	73.85	52.36	U1 All	OK
	2.370	0	0.00	0.00	U1 All	OK	1600	73.85	56.64	U1 All	OK
	2.896	0	0.00	0.00	U1 All	OK	1600	73.85	65.23	U1 All	OK
	3.300	0	0.00	0.00	U1 All	OK	1600	73.85	61.28	U1 All	OK
	4.051	0	0.00	0.00	U1 All	OK	1600	73.85	29.75	U1 All	OK
4.230	1029	-48.26	0.00	U1 All	OK	1600	73.85	17.61	U1 All	OK	
4.468	2400	-108.23	-1.86	U1 All	OK	1600	73.85	0.00	U1 All	OK	
5.075	2400	-108.23	-80.40	U1 All	OK	1600	73.85	0.00	U1 All	OK	
5.492	3000	-311.40	-142.35	U1 All	OK	1600	73.85	0.00	U1 All	OK	
6.400	3000	-311.40	-290.35	U1 All	OK	1600	73.85	0.00	U1 All	OK	
6.533	3000	-311.40	-312.78	U1 All	---	1600	73.85	0.00	U1 All	---	
6.600	3000	-311.40	-324.00	U1 All	---	1600	73.85	0.00	U1 All	---	
Middle	0.000	2800	-292.75	4.99	U1 All	---	1600	74.30	0.00	U1 All	---
	0.200	2800	-292.75	0.00	U1 All	OK	1600	74.30	0.00	U1 All	OK
	0.778	2800	-292.75	-5.27	U1 All	OK	1600	74.30	0.00	U1 All	OK
	1.264	2800	-292.75	-1.81	U1 All	OK	1600	74.30	0.00	U1 All	OK
	1.564	0	0.00	0.00	U1 All	OK	1600	74.30	12.90	U1 All	OK
	2.370	0	0.00	0.00	U1 All	OK	1600	74.30	54.14	U1 All	OK
	2.896	0	0.00	0.00	U1 All	OK	1600	74.30	62.36	U1 All	OK
	3.300	0	0.00	0.00	U1 All	OK	1600	74.30	58.58	U1 All	OK
	4.051	0	0.00	0.00	U1 All	OK	1600	74.30	28.43	U1 All	OK
	4.230	1355	-63.33	0.00	U1 All	OK	1600	74.30	16.83	U1 All	OK
4.420	2800	-126.15	0.00	U1 All	OK	1600	74.30	2.57	U1 All	OK	
5.236	2800	-292.75	-60.79	U1 All	OK	1600	74.30	0.00	U1 All	OK	
5.610	2800	-292.75	-105.84	U1 All	OK	1400	65.33	0.00	U1 All	OK	
6.400	2800	-292.75	-242.33	U1 All	OK	1400	65.33	0.00	U1 All	OK	
6.600	2800	-292.75	-286.87	U1 All	---	1400	65.33	0.00	U1 All	---	
3 Column	0.000	3000	-311.40	-284.23	U1 All	---	1400	64.99	0.00	U1 All	---
	0.200	3000	-311.40	-246.09	U1 All	OK	1400	64.99	0.00	U1 All	OK
	1.089	3000	-311.40	-105.49	U1 All	OK	1400	64.99	0.00	U1 All	OK
	1.440	2400	-251.03	-62.84	U1 All	OK	1400	64.99	0.00	U1 All	OK
	2.211	2400	-108.23	0.00	U1 All	OK	1400	64.99	4.59	U1 All	OK
	2.370	1314	-61.17	0.00	U1 All	OK	1400	64.99	13.57	U1 All	OK
	2.562	0	0.00	0.00	U1 All	OK	1400	64.99	22.55	U1 All	OK
	3.300	0	0.00	0.00	U1 All	OK	1400	64.99	37.78	U1 All	OK
	4.038	0	0.00	0.00	U1 All	OK	1400	64.99	22.55	U1 All	OK

Span Strip	x m	Top					Bottom					
		A _{s,top} mm ²	ΦM _{n-} kNm	M _{u-} kNm	Comb Pat	Status	A _{s,bot} mm ²	ΦM _{n+} kNm	M _{u+} kNm	Comb Pat	Status	
Middle	4.230	1314	-61.17	0.00	U1 All	OK	1400	64.99	13.57	U1 All	OK	
	4.389	2400	-108.23	0.00	U1 All	OK	1400	64.99	4.59	U1 All	OK	
	5.160	2400	-251.03	-62.84	U1 All	OK	1400	64.99	0.00	U1 All	OK	
	5.511	3000	-311.40	-105.49	U1 All	OK	1400	64.99	0.00	U1 All	OK	
	6.400	3000	-311.40	-246.09	U1 All	OK	1400	64.99	0.00	U1 All	OK	
	6.600	3000	-311.40	-284.23	U1 All	---	1400	64.99	0.00	U1 All	---	
	0.000	2800	-292.75	-237.23	U1 All	---	1400	65.33	0.00	U1 All	---	
	0.200	2800	-292.75	-205.39	U1 All	OK	1400	65.33	0.00	U1 All	OK	
	0.990	2800	-292.75	-99.11	U1 All	OK	1400	65.33	0.00	U1 All	OK	
	1.290	2800	-292.75	-66.88	U1 All	OK	1600	74.30	0.00	U1 All	OK	
	2.251	2800	-126.15	0.00	U1 All	OK	1600	74.30	6.70	U1 All	OK	
	2.370	1729	-80.05	0.00	U1 All	OK	1600	74.30	12.97	U1 All	OK	
	2.562	0	0.00	0.00	U1 All	OK	1600	74.30	21.56	U1 All	OK	
	3.300	0	0.00	0.00	U1 All	OK	1600	74.30	36.11	U1 All	OK	
	4.038	0	0.00	0.00	U1 All	OK	1600	74.30	21.56	U1 All	OK	
	4.230	1729	-80.05	0.00	U1 All	OK	1600	74.30	12.97	U1 All	OK	
	4.349	2800	-126.15	0.00	U1 All	OK	1600	74.30	6.70	U1 All	OK	
	5.310	2800	-292.75	-66.88	U1 All	OK	1600	74.30	0.00	U1 All	OK	
	5.610	2800	-292.75	-99.11	U1 All	OK	1400	65.33	0.00	U1 All	OK	
	6.400	2800	-292.75	-205.39	U1 All	OK	1400	65.33	0.00	U1 All	OK	
6.600	2800	-292.75	-237.23	U1 All	---	1400	65.33	0.00	U1 All	---		
4 Column	0.000	3000	-311.40	-324.00	U1 All	---	1600	73.85	0.00	U1 All	---	
	0.067	3000	-311.40	-312.78	U1 All	---	1600	73.85	0.00	U1 All	---	
	0.200	3000	-311.40	-290.35	U1 All	OK	1600	73.85	0.00	U1 All	OK	
	1.108	3000	-311.40	-142.35	U1 All	OK	1600	73.85	0.00	U1 All	OK	
	1.525	2400	-108.23	-80.40	U1 All	OK	1600	73.85	0.00	U1 All	OK	
	2.132	2400	-108.23	-1.86	U1 All	OK	1600	73.85	0.00	U1 All	OK	
	2.370	1029	-48.26	0.00	U1 All	OK	1600	73.85	17.61	U1 All	OK	
	2.549	0	0.00	0.00	U1 All	OK	1600	73.85	29.75	U1 All	OK	
	3.300	0	0.00	0.00	U1 All	OK	1600	73.85	61.28	U1 All	OK	
	3.704	0	0.00	0.00	U1 All	OK	1600	73.85	65.23	U1 All	OK	
	4.230	0	0.00	0.00	U1 All	OK	1600	73.85	56.64	U1 All	OK	
	4.354	0	0.00	0.00	U1 All	OK	1600	73.85	52.36	U1 All	OK	
	4.654	2400	-108.23	0.00	U1 All	OK	1600	73.85	38.48	U1 All	OK	
	5.160	2400	-251.03	0.00	U1 All	OK	1600	73.85	3.63	U1 All	OK	
	5.460	4200	-429.27	-43.28	U1 All	OK	1600	73.85	0.00	U1 All	OK	
	6.400	4200	-429.27	-278.21	U1 All	OK	1600	73.85	0.00	U1 All	OK	
	6.600	4200	-429.27	-344.97	U1 All	---	1600	73.85	0.00	U1 All	---	
	Middle	0.000	2800	-292.75	-286.87	U1 All	---	1400	65.33	0.00	U1 All	---
		0.200	2800	-292.75	-242.33	U1 All	OK	1400	65.33	0.00	U1 All	OK
		0.990	2800	-292.75	-105.84	U1 All	OK	1400	65.33	0.00	U1 All	OK
1.364		2800	-292.75	-60.79	U1 All	OK	1600	74.30	0.00	U1 All	OK	
2.180		2800	-126.15	0.00	U1 All	OK	1600	74.30	2.57	U1 All	OK	
2.370		1355	-63.33	0.00	U1 All	OK	1600	74.30	16.83	U1 All	OK	
2.549		0	0.00	0.00	U1 All	OK	1600	74.30	28.43	U1 All	OK	
3.300		0	0.00	0.00	U1 All	OK	1600	74.30	58.58	U1 All	OK	
3.704		0	0.00	0.00	U1 All	OK	1600	74.30	62.36	U1 All	OK	
4.230		0	0.00	0.00	U1 All	OK	1600	74.30	54.14	U1 All	OK	
5.036		0	0.00	0.00	U1 All	OK	1600	74.30	12.90	U1 All	OK	
5.336		2800	-292.75	-1.81	U1 All	OK	1600	74.30	0.00	U1 All	OK	
5.822		2800	-292.75	-5.27	U1 All	OK	1600	74.30	0.00	U1 All	OK	
6.400		2800	-292.75	0.00	U1 All	OK	1600	74.30	0.00	U1 All	OK	
6.600		2800	-292.75	4.99	U1 All	---	1600	74.30	0.00	U1 All	---	

Span Strip	x m	Top					Bottom				
		A _{s,top} mm ²	ΦM _{n-} kNm	M _{u-} kNm	Comb Pat	Status	A _{s,bot} mm ²	ΦM _{n+} kNm	M _{u+} kNm	Comb Pat	Status
5 Column	0.000	4200	-267.73	0.00	U1 All	---	0	0.00	0.00	U1 All	---
	0.052	4200	-267.73	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.139	4200	-267.73	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.150	4200	-267.73	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.213	4200	-179.37	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.300	4200	-179.37	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
Middle	0.000	2800	-185.06	0.00	U1 All	---	0	0.00	0.00	U1 All	---
	0.052	2800	-185.06	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.139	2800	-185.06	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.150	2800	-185.06	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.213	2800	-126.15	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK
	0.300	2800	-126.15	0.00	U1 All	OK	0	0.00	0.00	U1 All	OK

2.10. Slab Shear Capacity

Span	b mm	d _v mm	β	V _{ratio}	ΦV _c kN	V _u kN	X _u m
1	7100	128	0.210	1.000	619.29	0.00	0.00
2	7100	128	0.210	1.000	619.29	223.88	4.97
3	7100	128	0.210	1.000	619.29	182.84	1.63
4	7100	128	0.210	1.000	619.29	223.88	1.63
5	7100	128	0.210	1.000	619.29	0.00	0.00

2.11. Flexural Transfer of Negative Unbalanced Moment at Supports

Support	Width mm	Width-c mm	d mm	M _{unb} kNm	Comb Pat	γ _r	A _{s,req} mm ²	A _{s,prov} mm ²	Add Bars
1	1200	1200	317	339.98	U1 All	0.639	2176	2800	---
2	1650	1650	317	89.41	U1 All	0.629	529	1800	---
3	1650	1650	317	89.41	U1 All	0.629	529	1800	---
4	1200	1200	317	339.98	U1 All	0.639	2176	2800	---

2.12. Punching Shear Around Columns

2.12.1. Critical Section Properties

Support	Type	b ₁ mm	b ₂ mm	b ₀ mm	d _{avg} mm	CG mm	c _(left) mm	c _(right) mm	A _c mm ²	J _c mm ⁴
1	Rect	658.5	917.0	2234.0	317.0	164.4	464.4	194.1	7.0818e+005	3.7159e+010
2	Rect	717.0	917.0	3268.0	317.0	0.0	358.5	358.5	1.036e+006	9.8001e+010
3	Rect	717.0	917.0	3268.0	317.0	0.0	358.5	358.5	1.036e+006	9.8001e+010
4	Rect	658.5	917.0	2234.0	317.0	-164.4	194.1	464.4	7.0818e+005	3.7159e+010

2.12.2. Punching Shear Results

Support	V _u kN	v _u N/mm ²	M _{unb} kNm	Comb	Patt	γ _v	v _u N/mm ²	ΦV _c N/mm ²
1	314.72	0.444	288.24	U1	All	0.361	0.988	1.219
2	752.56	0.726	-89.41	U1	All	0.371	0.848	1.219
3	752.56	0.726	89.41	U1	All	0.371	0.848	1.219
4	314.72	0.444	-288.24	U1	All	0.361	0.988	1.219

2.13. Integrity Reinforcement at Supports

Notes:

The sum of bottom reinforcement crossing the perimeter of the support on all sides shall not be less than the below listed values.

Support	V _{se} kN	A _{sb} mm ²
1	313.64	1568
2	770.71	3854
3	770.71	3854
4	313.64	1568

2.14. Material TakeOff

2.14.1. Reinforcement in the Direction of Analysis

Top Bars	672.3 kg	<=>	32.95 kg/m	<=>	4.641 kg/m ²
Bottom Bars	480.8 kg	<=>	23.57 kg/m	<=>	3.320 kg/m ²
Stirrups	0.0 kg	<=>	0.00 kg/m	<=>	0.000 kg/m ²
Total Steel	1153.1 kg	<=>	56.52 kg/m	<=>	7.961 kg/m ²
Concrete	37.3 m ³	<=>	1.83 m ³ /m	<=>	0.257 m ³ /m ²

3. Deflection Results: Summary

3.1. Section Properties

3.1.1. Frame Section Properties

Notes:

M+ve values are for positive moments (tension at bottom face).

M-ve values are for negative moments (tension at top face).

Span Zone	M _{+ve}			M _{-ve}		
	I _g mm ⁴	I _{cr} mm ⁴	M _{cr} kNm	I _g mm ⁴	I _{cr} mm ⁴	M _{cr} kNm
1 Left	3.171e+009	0	54.36	3.171e+009	4.6492e+008	-54.36
Midspan	3.171e+009	0	54.36	3.171e+009	7.3441e+008	-54.36
Right	2.5368e+010	0	217.44	2.5368e+010	4.2067e+009	-217.44
2 Left	2.5368e+010	3.6402e+008	217.44	2.5368e+010	4.2067e+009	-217.44
Midspan	3.171e+009	3.6402e+008	54.36	3.171e+009	0	-54.36
Right	2.5368e+010	3.6402e+008	217.44	2.5368e+010	3.5761e+009	-217.44
3 Left	2.5368e+010	3.4299e+008	217.44	2.5368e+010	3.5761e+009	-217.44
Midspan	3.171e+009	3.4299e+008	54.36	3.171e+009	0	-54.36
Right	2.5368e+010	3.4299e+008	217.44	2.5368e+010	3.5761e+009	-217.44
4 Left	2.5368e+010	3.6402e+008	217.44	2.5368e+010	3.5761e+009	-217.44
Midspan	3.171e+009	3.6402e+008	54.36	3.171e+009	0	-54.36
Right	2.5368e+010	3.6402e+008	217.44	2.5368e+010	4.2067e+009	-217.44
5 Left	2.5368e+010	0	217.44	2.5368e+010	4.2067e+009	-217.44
Midspan	3.171e+009	0	54.36	3.171e+009	7.3441e+008	-54.36
Right	3.171e+009	0	54.36	3.171e+009	4.6492e+008	-54.36

3.1.2. Frame Effective Section Properties

Span Zone	Weight	Load Level					
		Dead		Sustained		Dead+Live	
		M _{max} kNm	I _e mm ⁴	M _{max} kNm	I _e mm ⁴	M _{max} kNm	I _e mm ⁴
1 Right	1.000	0.00	2.5368e+010	0.00	2.5368e+010	0.00	2.5368e+010
Span Avg	----	----	2.5368e+010	----	2.5368e+010	----	2.5368e+010
2 Left	0.250	-176.61	2.5368e+010	-176.61	2.5368e+010	-256.09	1.716e+010
Middle	0.500	66.28	1.9125e+009	66.28	1.9125e+009	96.11	8.7193e+008

Span Zone	Weight	Load Level					
		Dead		Sustained		Dead+Live	
		M _{max} kNm	I _g mm ⁴	M _{max} kNm	I _g mm ⁴	M _{max} kNm	I _g mm ⁴
Right	0.250	-317.34	1.0586e+010	-317.34	1.0586e+010	-460.14	5.8756e+009
Span Avg	----	----	9.9448e+009	----	9.9448e+009	----	6.1948e+009
3 Left	0.250	-270.89	1.4846e+010	-270.89	1.4846e+010	-392.79	7.2727e+009
Middle	0.500	38.39	3.171e+009	38.39	3.171e+009	55.66	2.9774e+009
Right	0.250	-270.89	1.4846e+010	-270.89	1.4846e+010	-392.79	7.2727e+009
Span Avg	----	----	9.0084e+009	----	9.0084e+009	----	5.125e+009
4 Left	0.250	-317.34	1.0586e+010	-317.34	1.0586e+010	-460.14	5.8756e+009
Middle	0.500	66.28	1.9125e+009	66.28	1.9125e+009	96.11	8.7193e+008
Right	0.250	-176.61	2.5368e+010	-176.61	2.5368e+010	-256.09	1.716e+010
Span Avg	----	----	9.9448e+009	----	9.9448e+009	----	6.1948e+009
5 Left	1.000	0.00	2.5368e+010	0.00	2.5368e+010	0.00	2.5368e+010
Span Avg	----	----	2.5368e+010	----	2.5368e+010	----	2.5368e+010

3.1.3. Strip Section Properties at Midspan

Notes:

Load distribution factor, LDL, averages moment distribution factors listed in Design Results.

Ratio refers to proportion of strip to frame deflections under fix-end conditions.

Span	Column Strip			Middle Strip		
	I _g mm ⁴	LDL	Ratio	I _g mm ⁴	LDL	Ratio
1	1.47383e+009	0.775	1.667	1.69714e+009	0.225	0.420
2	1.47383e+009	0.642	1.381	1.69714e+009	0.358	0.669
3	1.47383e+009	0.528	1.136	1.69714e+009	0.472	0.882
4	1.47383e+009	0.642	1.381	1.69714e+009	0.358	0.669
5	1.47383e+009	0.775	1.667	1.69714e+009	0.225	0.420

3.2. Instantaneous Deflections

3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

Span	Direction	Value	Units	Live				Total	
				Dead	Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.16	---	-0.11	-0.11	-0.16	-0.26
		Loc	m	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	mm	1.51	---	1.55	1.55	1.51	3.05
		Loc	m	3.043	---	3.116	3.116	3.043	3.116
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
3	Down	Def	mm	0.90	---	1.49	1.49	0.90	2.40
		Loc	m	3.300	---	3.300	3.300	3.300	3.300
	Up	Def	mm	-0.01	---	0.00	0.00	-0.01	-0.02
		Loc	m	0.272	---	0.200	0.200	0.272	0.200
4	Down	Def	mm	1.51	---	1.55	1.55	1.51	3.05
		Loc	m	3.557	---	3.484	3.484	3.557	3.484
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
5	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.16	---	-0.11	-0.11	-0.16	-0.26
		Loc	m	0.300	---	0.300	0.300	0.300	0.300

3.2.2. Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.16	---	-0.11	-0.11	-0.16	-0.26
		Loc	m	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	mm	1.87	---	2.01	2.01	1.87	3.88
		Loc	m	3.116	---	3.190	3.190	3.116	3.116
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
3	Down	Def	mm	1.05	---	1.70	1.70	1.05	2.75
		Loc	m	3.300	---	3.300	3.300	3.300	3.300
	Up	Def	mm	-0.01	---	0.00	0.00	-0.01	-0.01
		Loc	m	0.200	---	0.200	0.200	0.200	0.200
4	Down	Def	mm	1.87	---	2.01	2.01	1.87	3.88
		Loc	m	3.484	---	3.410	3.410	3.484	3.484
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
5	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.16	---	-0.11	-0.11	-0.16	-0.26
		Loc	m	0.300	---	0.300	0.300	0.300	0.300

3.2.3. Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.16	---	-0.11	-0.11	-0.16	-0.26
		Loc	m	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	mm	1.19	---	1.15	1.15	1.19	2.34
		Loc	m	2.969	---	3.043	3.043	2.969	3.043
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
3	Down	Def	mm	0.78	---	1.31	1.31	0.78	2.09
		Loc	m	3.300	---	3.300	3.300	3.300	3.300
	Up	Def	mm	-0.01	---	0.00	0.00	-0.01	-0.02
		Loc	m	0.272	---	0.200	0.200	0.272	0.200
4	Down	Def	mm	1.19	---	1.15	1.15	1.19	2.34
		Loc	m	3.631	---	3.557	3.557	3.631	3.557
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
5	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.16	---	-0.11	-0.11	-0.16	-0.26
		Loc	m	0.300	---	0.300	0.300	0.300	0.300

3.3. Long-term Deflections

3.3.1. Long-term Column Strip Deflection Factors

Notes:

Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span Zone	M _{-ve}					M _{+ve}				
	A _{s,top} mm ²	b mm	d mm	Rho' %	Lambda	A _{s,bot} mm ²	b mm	d mm	Rho' %	Lambda
1 Right	---	---	---	0.000	2.000	---	---	---	0.000	2.000
2 Midspan	---	---	---	0.000	2.000	---	---	---	0.000	2.000
3 Midspan	---	---	---	0.000	2.000	---	---	---	0.000	2.000
4 Midspan	---	---	---	0.000	2.000	---	---	---	0.000	2.000
5 Left	---	---	---	0.000	2.000	---	---	---	0.000	2.000

3.3.2. Long-term Middle Strip Deflection Factors

Notes:

Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span Zone	M _{-ve}					M _{+ve}				
	A _{s,top} mm ²	b mm	d mm	Rho' %	Lambda	A _{s,bot} mm ²	b mm	d mm	Rho' %	Lambda
1 Right	---	---	---	0.000	2.000	---	---	---	0.000	2.000
2 Midspan	---	---	---	0.000	2.000	---	---	---	0.000	2.000
3 Midspan	---	---	---	0.000	2.000	---	---	---	0.000	2.000
4 Midspan	---	---	---	0.000	2.000	---	---	---	0.000	2.000
5 Left	---	---	---	0.000	2.000	---	---	---	0.000	2.000

3.3.3. Extreme Long-term Column Strip Deflections and Corresponding Locations

Notes:

Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.

Incremental deflections after partitions are installed can be estimated by deflections due to:

- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,

- creep and shrinkage plus live load (cs+l), if live load applied after partitions.

Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
	Up	Def	mm	-0.31	-0.42	-0.42	-0.58
		Loc	m	0.000	0.000	0.000	0.000
2	Down	Def	mm	3.74	5.75	5.75	7.62
		Loc	m	3.116	3.116	3.116	3.116
	Up	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
3	Down	Def	mm	2.09	3.80	3.80	4.84
		Loc	m	3.300	3.300	3.300	3.300
	Up	Def	mm	-0.02	-0.03	-0.03	-0.04
		Loc	m	0.200	0.200	0.200	0.200
4	Down	Def	mm	3.74	5.75	5.75	7.62
		Loc	m	3.484	3.484	3.484	3.484
	Up	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
5	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
		Loc	m	---	---	---	---
	Up	Def	mm	-0.31	-0.42	-0.42	-0.58
		Loc	m	0.300	0.300	0.300	0.300

3.3.4. Extreme Long-term Middle Strip Deflections and Corresponding Locations

Notes:

Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.

Incremental deflections after partitions are installed can be estimated by deflections due to:

- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,

- creep and shrinkage plus live load (cs+l), if live load applied after partitions.

Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
	Up	Def	mm	-0.31	-0.42	-0.42	-0.58
		Loc	m	0.000	0.000	0.000	0.000
2	Down	Def	mm	2.38	3.54	3.54	4.73
		Loc	m	2.969	2.969	2.969	2.969
	Up	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
3	Down	Def	mm	1.55	2.86	2.86	3.64
		Loc	m	3.300	3.300	3.300	3.300
	Up	Def	mm	-0.03	-0.03	-0.03	-0.04
		Loc	m	0.272	0.272	0.272	0.272
4	Down	Def	mm	2.38	3.54	3.54	4.73
		Loc	m	3.631	3.631	3.631	3.631
	Up	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
5	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
	Up	Def	mm	-0.31	-0.42	-0.42	-0.58
		Loc	m	0.300	0.300	0.300	0.300

6. Summary and Comparison of Two-Way Slab Design Results

Table 5 – Summary of Flexural Design Moments				
		Reference (DDM)	Hand (EFM)	spSlab
Exterior Span				
Frame Strip	Exterior Negative	138	354.5	334.0
	Positive	275	151.1	127.6
	Interior Negative	370	541.3	610.9
Interior Span				
Frame Strip	Interior Negative	344	481.5	521.5
	Positive	185	113.8	73.9

The reference used the Direct Design Method (DDM) to calculate the design moments, this method uses generic distribution factors for slabs regardless of the geometric properties of the transverse slab bands. In [spSlab](#) and hand calculations, Elastic Frame Method (EFM) is being used, in this method, the exact geometric properties of the transverse slab bands can be used to perform the analysis and calculate the design moments.

In the hand calculations, the calculations of the moment distribution constants are approximated using the design aids tables for slabs with drop panels since tables for two-way slabs with transverse slab bands are not available. On the other hand, [spSlab](#) calculates the values of these constants taking into account the effect of the transverse slab bands.

Table 6 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution			
		Hand (EFM)	spSlab
Exterior Span			
Column Strip	Exterior Negative *	290.2	278.2
	Positive	77.3	65.2
	Interior Negative *	256.1	290.4
Middle Strip	Exterior Negative *	0.0	0.0
	Positive	73.8	62.4
	Interior Negative *	209.6	242.3
Interior Span			
Column Strip	Interior Negative *	226.4	246.1
	Positive	58.2	37.8
Middle Strip	Interior Negative *	185.2	205.4
	Positive	55.6	36.1

* negative moments are taken at the faces of supports

Table 7 - Comparison of Reinforcement Results with Hand and spSlab Solution							
Span Location		Reinforcement Provided for Flexure		Additional Reinforcement Provided for Unbalanced Moment Transfer		Total Reinforcement Provided	
		Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior Span							
Column Strip	Exterior Negative	20 - 15M	21 - 15M	---	---	20 - 15M	21 - 15M
	Positive	9 - 15M	8 - 15M	n/a	n/a	9 - 15M	8 - 15M
	Interior Negative	13 - 15M	15 - 15M	---	---	13 - 15M	15 - 15M
Middle Strip	Exterior Negative	14 - 15M	14 - 15M	n/a	n/a	14 - 15M	14 - 15M
	Positive	8 - 15M	8 - 15M	n/a	n/a	8 - 15M	8 - 15M
	Interior Negative	14 - 15M	14 - 15M	n/a	n/a	14 - 15M	14 - 15M
Interior Span							
Column Strip	Negative	12 - 15M	15-15M	---	---	12 - 15M	15-15M
	Positive	7 - 15M	7-15M	n/a	n/a	7 - 15M	7-15M
Middle Strip	Negative	14 - 15M	14-15M	---	---	14 - 15M	14-15M
	Positive	8 - 15M	8-15M	n/a	n/a	8 - 15M	8-15M

Span	V_u , kN		ϕV_c , kN	
	Hand	spSlab	Hand	spSlab
Exterior	211.1	223.9	619.3	619.3
Interior	182.8	182.8	619.3	619.3

Support	b_1 , mm		b_2 , mm		b_o , mm		A_c , mm ²		V_f , kN		v_f , N/mm ²	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	658.5	658.5	917	917	2234	2234	7.08×10^5	7.08×10^5	323.2	314.7	0.456	0.444
Interior	717	717	917	917	3268	3268	1.04×10^6	1.04×10^6	739.8	752.6	0.714	0.726

Support	c_{AB} , mm		J_c , $\times 10^9$ mm ⁴		γ_v		M_{unb} , kN.m		v_f , MPa		ϕv_c , MPa	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	194.1	194.1	37.16	37.16	0.361	0.361	301.4	288.2	1.025	0.988	1.219	1.219
Interior	358.5	358.5	98.00	98.00	0.371	0.371	59.8	89.4	0.795	0.848	1.219	1.219

7. Comparison of Two-Way Slab Analysis and Design Methods

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in [CSA A23.3-14 Clauses \(13.8 and 13.9\)](#) for regular two-way slab systems. [CSA A23.3-14 \(13.5.1\)](#)

Direct Design Method (DDM) is an approximate method and is applicable to flat plate concrete floor systems that meet the stringent requirements of [CSA A23.3-14 \(13.9.1\)](#). In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Elastic Frame Method (EFM) has less stringent limitations compared to DDM. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StructurePoint's [spSlab](#) software program solution utilizes the EFM to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as [spMats](#). Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable CSA A23.3-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
13.8.1.1 13.9.1.1	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
13.8.1.1 13.9.1.1	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑	☑	
13.8.1.1 13.9.1.1	Column offset shall not exceed 20% of the span in direction of offset from either axis between centerlines of successive columns	☑	☑	
13.8.1.1 13.9.1.1	The reinforcement is placed in an orthogonal grid.	☑	☑	
13.9.1.2	Minimum of three continuous spans in each direction	☑		
13.9.1.3	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
13.9.1.4	All loads shall be due to gravity only	☑		
13.9.1.4	All loads shall be uniformly distributed over an entire panel (q_f)	☑		
13.9.1.4	Unfactored live load shall not exceed two times the unfactored dead load	☑		
13.10.6	Structural integrity steel detailing	☑	☑	☑
13.10.10	Openings in slab systems	☑	☑	☑
8.2	Concentrated loads	Not permitted	☑	☑
13.8.4.1	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
13.10.2*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
13.8.2	Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)	Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb}).