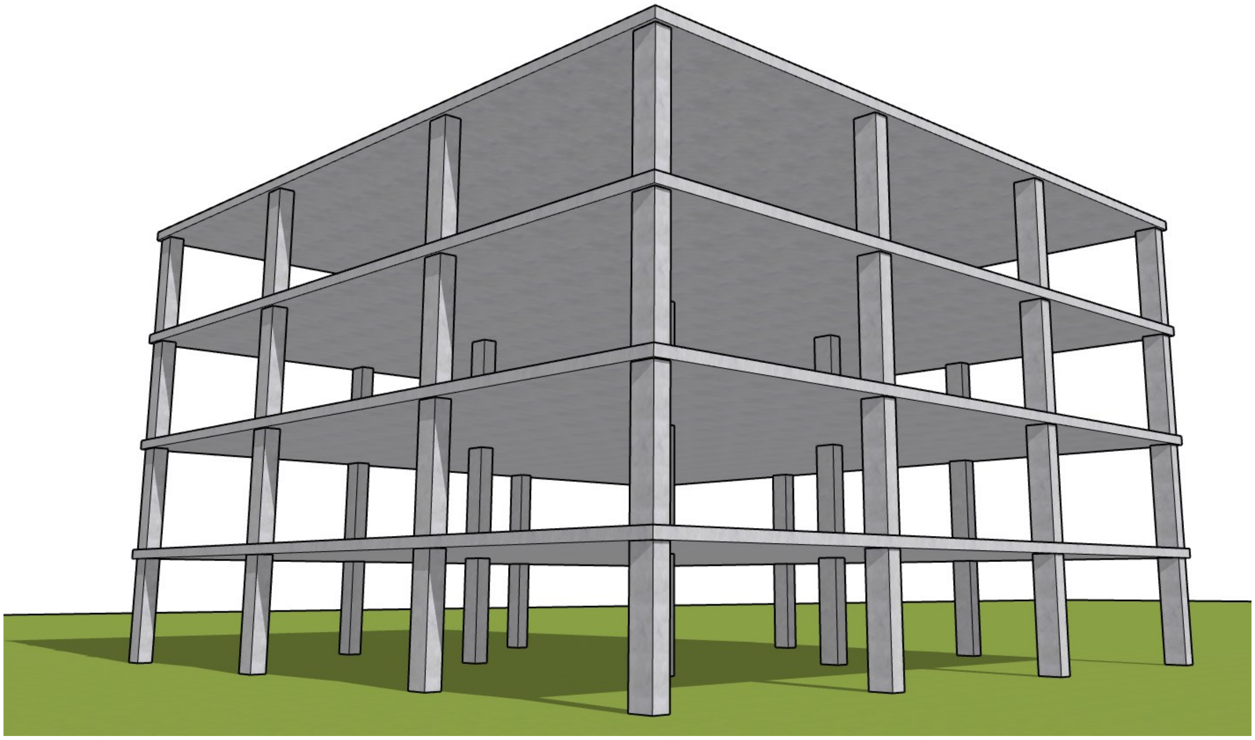
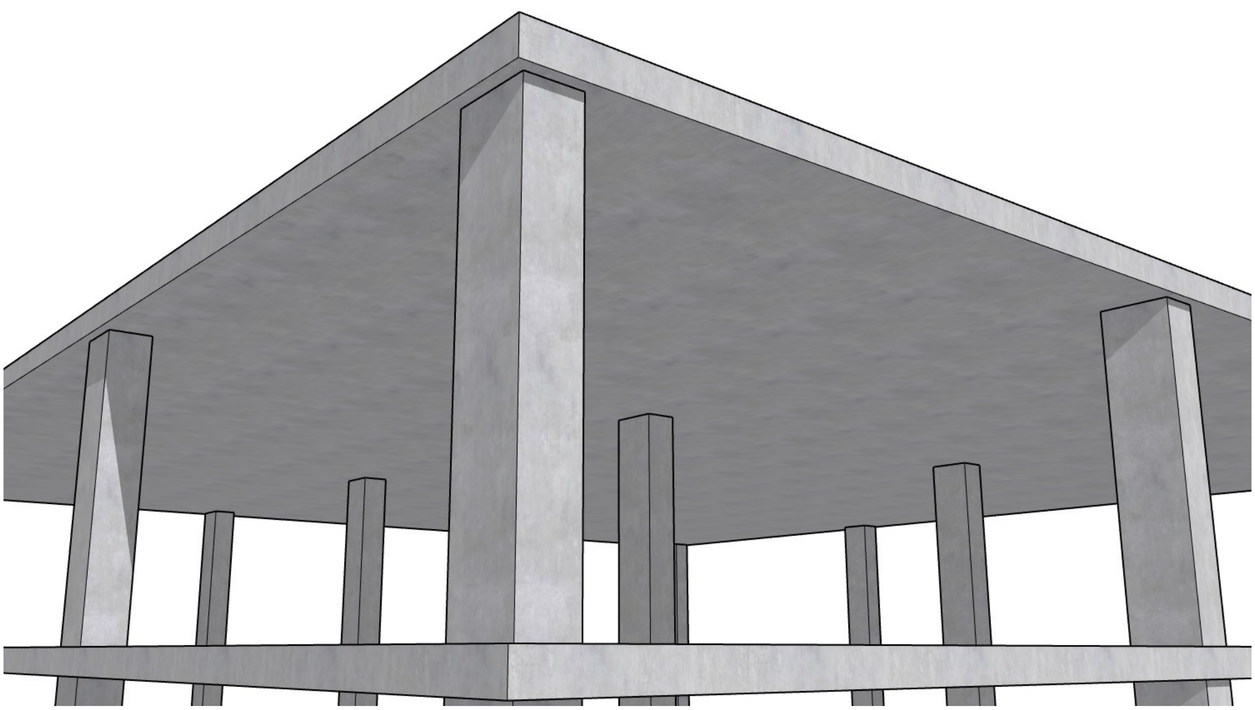
**Two-Way Flat Plate Concrete Floor System Analysis and Design (CAC Design Handbook)**





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**Two-Way Flat Plate Concrete Floor System Analysis and Design (CAC Design Handbook)**

The concrete floor slab system shown below is for an intermediate floor to be designed considering partition weight = 1 kN/m$^2$, and unfactored live load = 1.9 kN/m$^2$ . Flat plate concrete floor system does not use beams between columns or drop panels and it is usually suited for lightly loaded floors with short spans typically for residential and hotel buildings. The lateral loads are independently resisted by shear walls. The analysis procedure "Elastic Frame Method (EFM)" prescribed in ***CSA A23.3-14*** is illustrated in detail in this example (Example #1 from the CAC Design Handbook). The hand solution from EFM is also used for a detailed comparison with the Reference results using Direct Design Method (DDM) and results of the engineering software program spSlab. Explanation of the EFM is available in StructurePoint Video Tutorials page.

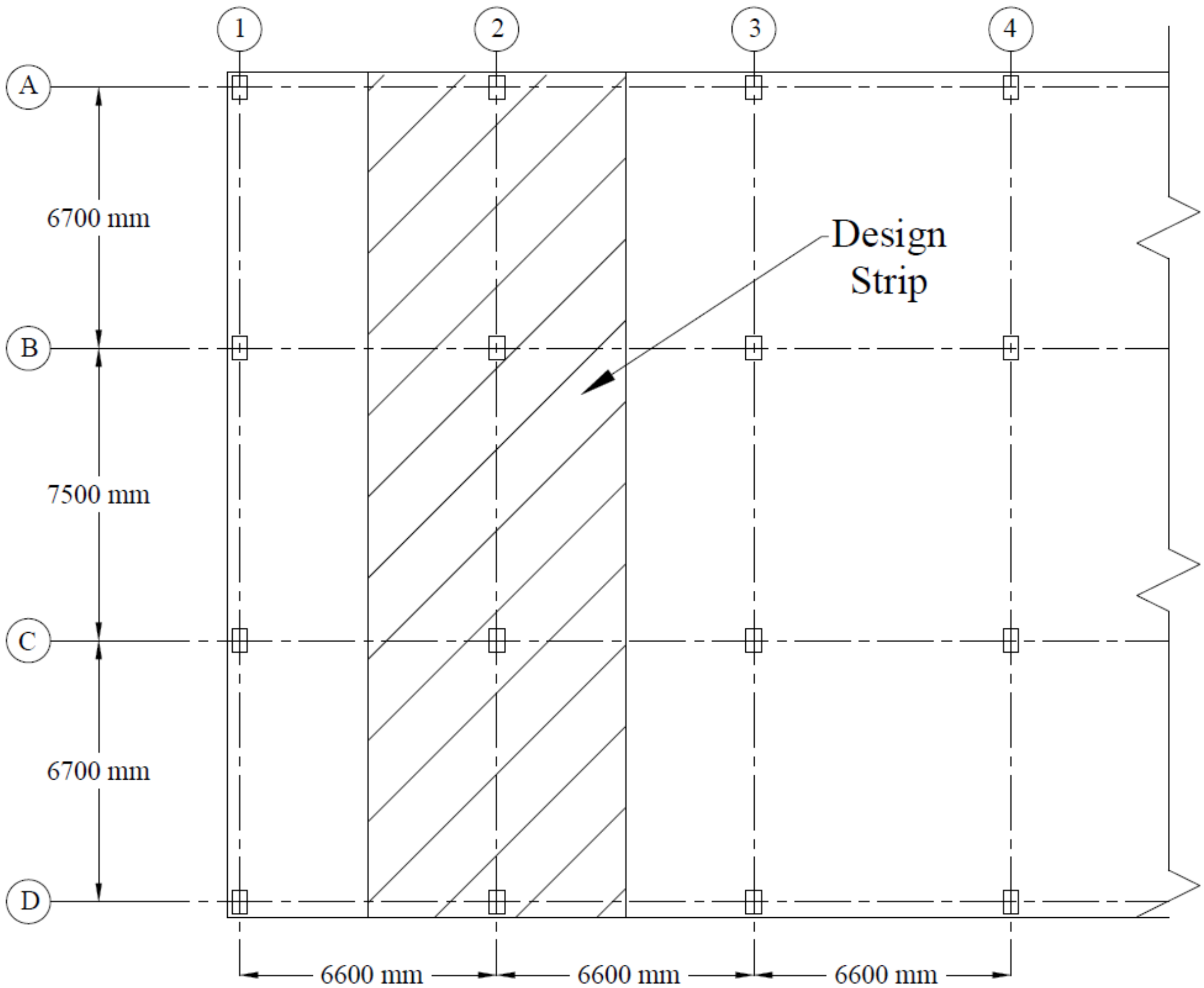



Figure 1 - Two-Way Flat Concrete Floor System

# Contents

**Code**

Design of Concrete Structures (CSA A23.3-14)

**Reference**

CAC Concrete Design Handbook, 4th Edition, Cement Association of Canada

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association

**Design Data**

Floor-to-Floor Height = 3 m (provided by architectural drawings)

Superimposed Dead Load, $\text{SDL} = 1\ \text{kN/m}^2$ for framed partitions, wood studs plaster 2 sides

Live Load, $\text{LL} = 1.9\ \text{kN/m}^2$ for Residential floors

Exterior Cladding Panels Weight = 2.4 kN/m

$\text{f}'_c = 25\ \text{MPa}$ (for slabs)

$\text{f}'_c = 25\ \text{MPa}$ (for columns)

$\text{f}'_y = 400\ \text{MPa}$

Column Dimensions = 400 mm x 600 mm

**Solution**

## 1. Preliminary Member Sizing

### 1.1 Slab minimum thickness - Deflection

***CSA A23.3-14 (13.2)***

Minimum member thickness and depths from CSA A23.3-14 will be used for preliminary sizing.

Using CSA A23.3-14 minimum slab thickness for two-way construction without interior beams in ***Section 13.2.3.***

Exterior Panels (N-S Direction Governs):

$$h_{s,\min} = 1.1 \times \frac{l_n\left(0.6 + f_y / 1000\right)}{30} = 1.1 \times \frac{6200\left(0.6 + 400/1000\right)}{30} = 227\ \text{mm}$$ ***CSA A23.3-14 (13.2.3)***

But not less than 120 mm. ***CSA A23.3-14 (13.2.1)***

Where $l_n$ = length of clear span in the long direction = 6600 – 400 = 6200 mm

Interior Panels (E-W Direction Governs):

$$h_{s,\min} = \frac{l_n\left(0.6 + f_y / 1000\right)}{30} = \frac{6900\left(0.6 + 400/1000\right)}{30} = 230 \text{ mm}$$ **CSA A23.3-14 (13.2.3)**

But not less than 120 mm. **CSA A23.3-14 (13.2.1)**

Where $l_n$ = length of clear span in the long direction = 7500 – 600 = 6900 mm

Try 250 mm slab for all panels (self-weight = 5.89 kN/m$^2$)

1.2. Slab one way shear strength

Evaluate the average effective depth (Figure 2):

$$d_t = t_{slab} - c_{clear} - d_b - \frac{d_b}{2} = 250 - 25 - 16 - \frac{16}{2} = 201 \text{ mm}$$

$$d_l = t_{slab} - c_{clear} - \frac{d_b}{2} = 250 - 25 - \frac{16}{2} = 217 \text{ mm}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{201 + 217}{2} = 209 \text{ mm}$$

Where:

$c_{clear}$ = 20 mm for 15M steel bar **CSA A23.3-14 (Annex A. Table 17)**

Note that the reference used 25 mm as clear cover, in this example the clear cover used is 25 mm to be consistent with reference.

$d_b$ = 16 mm for 15M steel bar

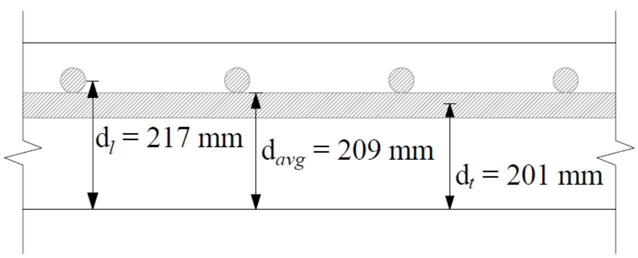



Figure 2 - Two-Way Flat Concrete Floor System

Load Combination 1:

Factored dead load, $w_{df} = 1.4 \times (5.89 + 1) = 9.65 \text{ kN/m}^2$ **CSA A23.3-14 (Annex C. Table C.1 a)**

Total factored load $w_f = 9.65 \text{ kN/m}^2$

Load Combination 2:

Factored dead load, $w_{df} = 1.25 \times (5.89 + 1) = 8.61 \text{ kN/m}^2$

Factored live load, $w_{lf} = 1.5 \times 1.9 = 2.85 \text{ kN/m}^2$ ***CSA A23.3-14 (Annex C. Table C.1 a)***

Total factored load $w_f = w_{df} + w_{lf} = 11.5 \text{ kN/m}^2$ (Controls)

Check the adequacy of slab thickness for beam action (one-way shear) ***CSA A23.3-14 (13.3.6)***

At an interior column:

The critical section for one-way shear is extending in a plane across the entire width and located at a distance, $d_v$ from the face of support or concentrated load (see Figure 3). ***CSA A23.3-14 (13.3.6.1)***

Consider a 1 m. wide strip.

Tributary area for one-way shear is $A_{Tributary} = \left( \dfrac{\left[ \left( \dfrac{7500}{2} \right) - \left( \dfrac{600}{2} \right) - 209 \right] \times (1000)}{1000^2} \right) = 3.26 \text{ m}^2$

$V_f = w_f \times A_{Tributary} = 11.5 \times 3.26 = 37.49 \text{ kN}$

$V_c = \varphi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$ ***CSA A23.3-14 (Eq. 11.6)***

Where:

$\lambda = 1$ for normal weight concrete ***CSA A23.3-14 (8.6.5)***

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm ***CSA A23.3-14 (11.3.6.2)***

$d_v = \text{Max } (0.9 d_{avg}, 0.72h) = \text{Max } (0.9 \times 209, 0.72 \times 250) = \text{Max } (188,180) = 188 \text{ mm}$ ***CSA A23.3-14 (3.2)***

$\sqrt{f'_c} = 5 \text{ MPa} < 8 \text{ MPa}$ ***CSA A23.3-14 (11.3.4)***

$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{25} \times 1000 \times \dfrac{188}{1000} = 128.3 \text{ kN} > V_f$

Slab thickness of 250 mm is adequate for one-way shear.

## 1.3. Slab two-way shear strength

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 4):

Shear prerimeter: $b_0 = 2 \times (600 + 400 + 2 \times 209) = 2836 \text{ mm}$ ***CSA A23.3-14 (13.3.3)***

Tributary area for two-way shear is

$$A_{Tributary} = \left(\frac{7.5+6.7}{2}\times 6.6\right) - \left(\frac{600+209}{1,000}\times\frac{400+209}{1,000}\right) = 46.86 - 0.49 = 46.37\ \text{m}^2$$

The factored resisiting shear stress, $V_r$ shall be the smallest of : ***CSA A23.3-14 (13.3.4.1)***

a) $$v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c}$$ ***CSA A23.3-14 (Eq. 13.5)***

$$v_r = \left(1+\frac{2}{1.5}\right)\times 0.19\times 0.65\times\sqrt{25} = 1.44\ \text{MPa}$$

Where $\beta_c = \frac{600}{400} = 1.5$ (ratio of long side to short side of the column) ***CSA A23.3-14 (13.3.4.1)***

b) $$v_r = v_c = \left(\frac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c}$$ ***CSA A23.3-14 (Eq. 13.6)***

$$v_r = \left(\frac{4\times 209}{2836}+0.19\right)\times 1\times 0.65\times\sqrt{25} = 1.58\ \text{MPa}$$

c) $$v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times 1\times 0.65\times\sqrt{25} = 1.24\ \text{MPa}$$ ***CSA A23.3-14 (Eq. 13.7)***

$$v_{f,ave} = \frac{V_f}{b_o d} = \frac{11.5\times\left(\frac{7.5+6.7}{2}\times 6.6\right)}{2836\times 209}\times 1,000 = 0.909\ \text{MPa}$$

$$\frac{v_r}{v_{f,ave}} = \frac{1.240}{0.909} = 1.36 > 1.20$$ ***CAC Concrete Design Handbook 4th Edition (5.2.3)***

Slab thickness of 250 mm is adequate for two-way shear.

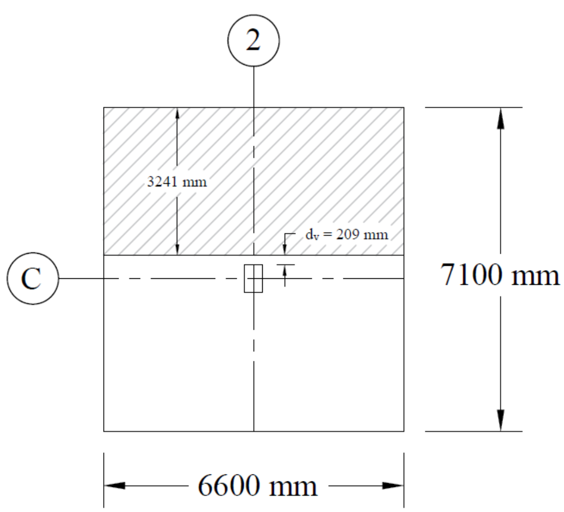



Figure 3 - Critical Section for One-Way

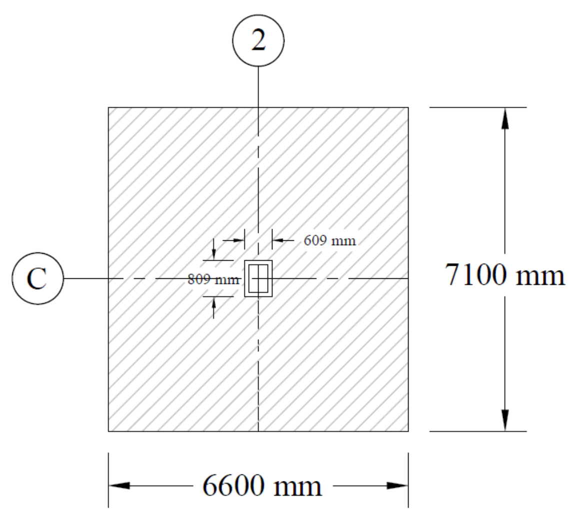



Figure 4 - Critical Section for Two-Way

1.4. Column axial load strength

Check the adequacy of column dimensions for axial load:

Tributary area for interior column is $A_{Tributary} = (7.5 \times 6.6) = 49.5 \text{ m}^2$

$P_f = w_f \times A_{Tributary} = 11.5 \times 49.5 = 569 \text{ kN}$

$P_{r,\max} = (0.2 + 0.002h)P_{ro} \leq 0.80P_{ro}$ (For tied column along full length) **CSA A23.3-14 (Eq. 10.9)**

$P_{ro} = \alpha_1 \ \phi_c f'_c (A_g - A_{st} - A_t - A_p) + \phi_s \ \text{f}_y \ \text{A}_{st} + \phi_\alpha F_y A_t - f_{pr} A_p$ **CSA A23.3-14 (Eq. 10.11)**

$P_{ro} = 0.813 \times \ 0.65 \times \ 25 \times (600 \times 400 - 0) + 0.85 \times 420 \times 0 + 0 = 3{,}170{,}700 \text{ N} = 3170.7 \text{ kN}$

$P_{r,\max} = (0.2 + 0.002 \times 600) \times 3170.7 \leq 0.80 \times 3170.7$

$P_{r,\max} = 4439 \leq 2537$

$P_{r,\max} = 2537 \text{ kN} \ < \ P_f$

$\alpha_1 = 0.85 - 0.0015 f'_c = 0.85 - 0.0015 \times 25 = 0.813 > 0.67$ **CSA A23.3-14 (Eq. 10.1)**

Column dimensions of 600 mm x 400 mm are adequate for axial load.

## 2. Two-Way Slab Analysis and Design

CSA A23.3 states that a regular slab system may be designed using any procedure satisfying conditions of equilibrium and compatibility with the supports, provided that it is shown that the factored resistance at every section is at least equal to the effects of the factored loads and that all serviceability conditions, including specified limits on deflections, are met. **CSA A23.3-14 (13.5.1)**

CSA A23.3 permits the use of Plastic Plate Theory Method (PPTM), Theorems of Plasticity Method (TPM), Direct Design Method (DDM) and Elastic Frame Method (EFM); known as Equivalent Frame Method in the ACI; for the gravity load analysis of orthogonal frames. The following sections outline a brief description of DDM, a detailed hand solution using EFM and an automated solution using spSlab software respectively.

### 2.1. Direct Design Method (DDM)

Two-way slabs satisfying the limits in ***CSA A23.3-14 (13.9)*** are permitted to be designed in accordance with the DDM.

#### 2.1.1. Direct design method limitations

There shall be a minimum of three continuous spans in each direction (3 spans) **CSA A23.3-14 (13.9.1.2)**

Successive span lengths centre-to-centre of supports in each direction shall not differ by more than one- third of the longer span ((7500-6700)/6700 = 0.12 < 0.33) **CSA A23.3-14 (13.9.1.3)**

All loads shall be due to gravity only and uniformly distributed over an entire panel (Loads are uniformly distributed over the entire panel) **CSA A23.3-14 (13.9.1.4)**

The factored live load shall not exceed twice the factored dead load (Service live-to-dead load ratio of 0.28 < 2.0) ***CSA A23.3-14 (13.9.1.4)***

Since all the criteria are met, Direct Design Method can be utilized.

Detailed illustration of analysis and design of flat plate slab using DDM can be found in "Two-Way Flat Plate Concrete Slab Floor Analysis and Design (CSA A23.3-14)" example available in the design examples page in StructurePoint website. This example focuses on the analysis of flat plates using EFM.

**2.1.2. Design moments**

a. Calculate the total factored static moment:

$$M_o = \frac{w_f \ell_{2a} \ell_n^2}{8}$$ ***CSA A23.3-14 (13.9.1.4)***

Distribute the total factored moment, $M_o$, in an interior and end span: ***CSA A23.3-14 (13.9.3.1 &13.9.3.2)***

| **Table 1 - Distribution of $M_o$ along the span** | | |
|---|---|---|
| **Location** | | **Total Design Strip Moment, $M_{DES}$ (kN.m)** |
| Exterior Span | Exterior Negative | $0.26 \times M_o = 34.8$ |
| | Positive | $0.52 \times M_o = 69.6$ |
| | Interior Negative | $0.70 \times M_o = 93.68$ |
| Interior Span | Positive | $0.35 \times M_o = 46.8$ |

b. Calculate the column strip moments. ***CSA A23.3-14 (13.11.2)***

That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips. ***CSA A23.3-14 (13.11.3.1)***

| **Table 2 - Lateral Distribution of the Total Design Strip Moment, $M_{DES}$** | | | | |
|---|---|---|---|---|
| **Location** | | **Total Design Strip Moment, (kN.m)** | **Column Strip Moment, (kN.m)** | **Moment in Two Half Middle Strips, (kN.m)** |
| Exterior Span | Exterior Negative* | 34.8 | $1.00 \times M_{DES} = 34.8$ | $0.00 \times M_{DES} = 0.0$ |
| | Positive | 69.6 | $0.6 \times M_{DES} = 41.8$ | $0.4 \times M_{DES} = 27.8$ |
| | Interior Negative* | 93.68 | $0.8 \times M_{DES} = 74.94$ | $0.2 \times M_{DES} = 18.7$ |
| Interior Span | Positive | 46.8 | $0.6 \times M_{DES} = 28.1$ | $0.4 \times M_{DES} = 18.7$ |
| * All negative moments are at face of support. | | | | |

Figure 5 – Sample Calculations Using DDM from "Two-Way Flat Plate Concrete Slab Floor Analysis and Design" Design Example

## 2.2. Elastic Frame Method (EFM)

EFM (also known as Equivalent Frame Method in the ACI 318) is the most comprehensive and detailed procedure provided by the CSA A23.3 for the analysis and design of two-way slab systems where these systems may, for purposes of analysis, be considered a series of plane frames acting longitudinally and transversely through the building. Each frame shall be composed of equivalent line members intersecting at member centrelines, shall follow a column line, and shall include the portion of slab bounded laterally by the centreline of the panel on each side. ***CSA A23.3-14 (13.8.1.1)***

Probably the most frequently used method to determine design moments in regular two-way slab systems is to consider the slab as a series of two-dimensonal frames that are analyzed elastically. When using this analogy, it is essential that stiffness properties of the elements of the frame be selected to properly represent the behavior of the three-dimensional slab system.

In a typical frame analysis it is assumed that at a beam-column cconnection all members meeting at the joint undergo the same rotaion. For uniform gravity loading this reduced restraint is accounted for by reducing the effective stiffness of the column by either Clause 13.8.2 or Clause 13.8.3. ***CSA A23.3-14 (N.13.8)***

Each floor and roof slab with attached columns may be analyzed separately, with the far ends of the columns considered fixed. ***CSA A23.3-14 (13.8.1.2)***

The moment of inertia of column and slab-beam elements at any cross-section outside of joints or column capitals shall be based on the gross area of concrete at that section. ***CSA A23.3-14 (13.8.2.5)***

An equivalent column shall be assumed to consist of the actual columns above and below the slab-beam plus an attached torsional member transverse to the direction of the span for which moments are being determined.
***CSA A23.3-14 (13.8.2.5)***

### 2.2.1. Elastic frame method limitations

In EFM, live load shall be arranged in accordance with 13.8.4 which requires:

- slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***CSA A23.3-14 (13.8.4)***
- Complete analysis must include representative interior and exterior equivalent elastic frames in both the longitudinal and transverse directions of the floor. ***CSA A23.3-14 (13.8.1.1)***
- Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***CSA A23.3-14 (3.1a)***
- For slab systems with beams between sypports, the relative effective stiffness of beams in the two directions is not less than 0.2 or greater than 5.0. ***CSA A23.3-14 (3.1b)***
- Column offsets are not greater than 20% of the span (in the direction of offset) from either axis between centerlines of successive columns. ***CSA A23.3-14 (3.1c)***

The reinforcement is placed in an orthogonal grid. ***CSA A23.3-14 (3.1d)***

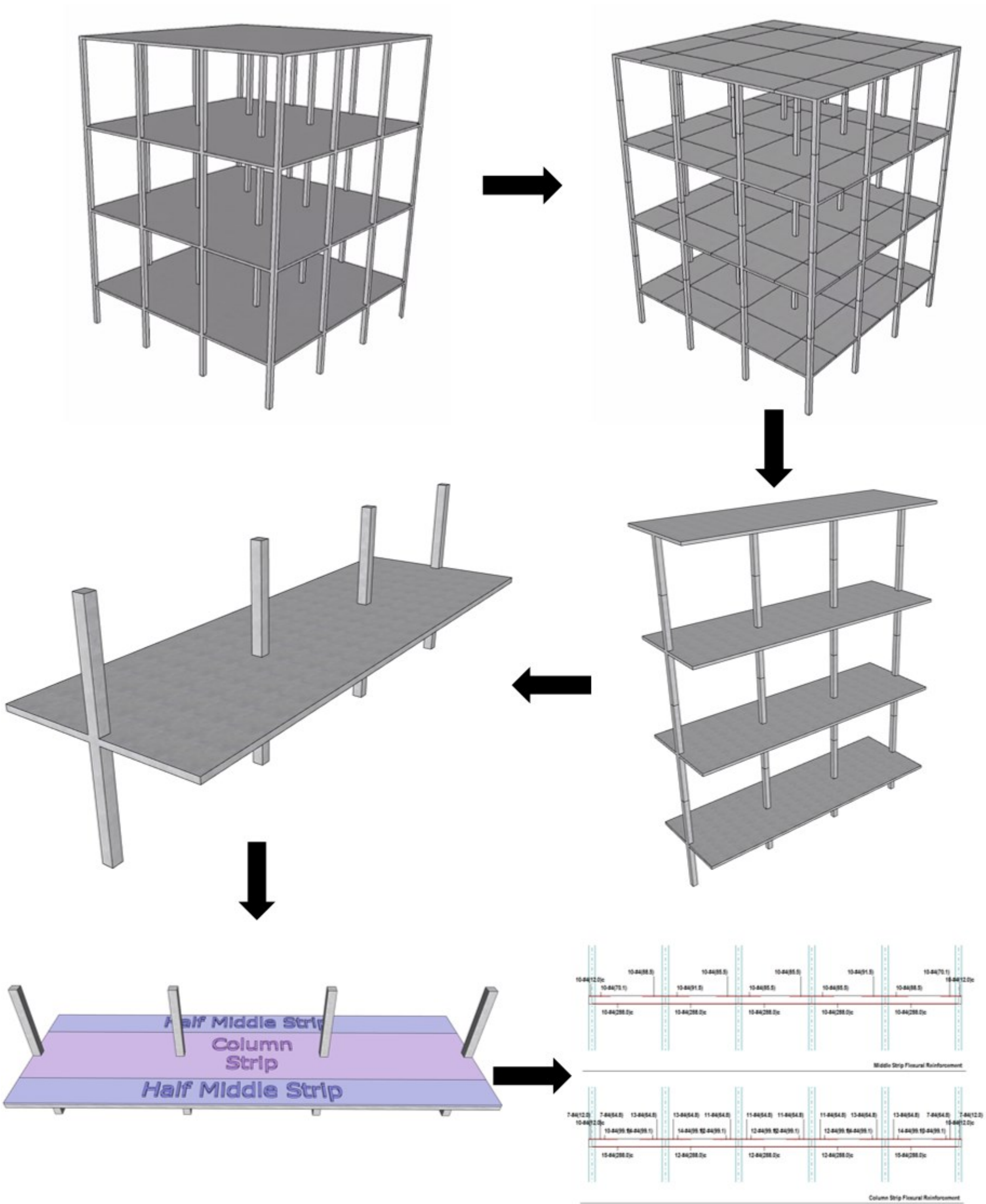



Figure 6 – Equivalent Frame Methodology

### 2.2.2. Frame members of elastic frame

Determine moment distribution factors and fixed-end moments for the elastic frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors $k$, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a. Flexural stiffness of slab-beams at both ends, $K_{sb}$

For Interior Span:

$$\frac{c_{N1}}{\ell} \quad \frac{600}{)} = 0.080 \;,\quad \frac{c_{N2}}{\ell} \quad \frac{600}{)} = 0.061$$

For $c_{F1} = c_{N2}$, stiffness factors, $k_{NF} = k_{FN} = 4.09$ ***PCA Notes on ACI 318-11 (Table A1)***

Thus, $K_{sb} = k_{NF}\frac{E_{cs}I_s}{\ell} \quad 4.09\frac{E_{cs}I_s}{\ell}$ ***PCA Notes on ACI 318-11 (Table A1)***

$$K_{sb} = 4.09\times 24,986\times\frac{8.6\times10^9}{7500}\times10^{-3} = 117.2\times10^6\ \text{N.m}$$

where, $I_s = \frac{\ell}{12} \quad \frac{00(250)^3}{12} = 8.6\times10^9\ \text{mm}^4$

$$E_{cs} = (3300\sqrt{f_c'}+6900)\left(\frac{\gamma_c}{2300}\right)^{1.5}$$ ***CSA A23.3-14(8.6.2.2 )***

$$E_{cs} = (3300\sqrt{25}+6900)\left(\frac{2402.8}{2300}\right)^{1.5} = 24,986\ \text{MPa}$$

Carry-over factor COF = 0.50 ***PCA Notes on ACI 318-11 (Table A1)***

Fixed-end moment FEM $= 0.0843 w_u \ell \ \ell$ ***PCA Notes on ACI 318-11 (Table A1)***

For Exterior Span:

$$\frac{c_{N1}}{\ell} \quad \frac{600}{)} = 0.090 \;,\quad \frac{c_{N2}}{\ell} \quad \frac{600}{)} = 0.061$$

For $c_{F1} = c_{N2}$, stiffness factors, $k_{NF} = k_{FN} = 4.10$ ***PCA Notes on ACI 318-11 (Table A1)***

Thus, $K_{sb} = k_{NF}\frac{E_{cs}I_s}{\ell} \quad 4.10\frac{E_{cs}I_s}{\ell}$ ***PCA Notes on ACI 318-11 (Table A1)***

$$K_{sb} = 4.10\times 24,986\times\frac{8.6\times10^9}{6,700}\times10^{-3} = 131.5\times10^6\ \text{N.m}$$

Carry-over factor COF = 0.51 ***PCA Notes on ACI 318-11 (Table A1)***

Fixed-end moment FEM $= 0.0843 w_u \ell \ \ell$ ***PCA Notes on ACI 318-11 (Table A1)***

b. Flexural stiffness of column members at both ends, $K_c$

Referring to ***Table A7, Appendix 20A***, $t_a = 125$ mm, $t_b = 125$ mm,

$\mathrm{H} = 3.00\text{ m} = 3000\text{ mm},\ \mathrm{t} = 250\text{ mm},\ \mathrm{H_c} = 2750\text{ mm},\ \frac{t_a}{t_b} = 1,\ \frac{H}{H_c} = 1.09$

Thus, $k_{AB} = k_{BA} = 4.99$ by interpolation.

$$K_c = \frac{4.99E_{cc}I_c}{\ell}$$

***PCA Notes on ACI 318-11 (Table A7)***

$$K_c = 4.99 \times 24,986 \times \frac{7.20 \times 10^9}{3,000} \times 10^{-3} = 299 \times 10^6 \text{ N.m}$$

Where $I_c = \frac{b \times h^3}{12} = \frac{400(600)^3}{12} = 7.20 \times 10^9 \text{ mm}^4$

$$E_{cc} = (3,300\sqrt{f_c^{'}} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5}$$

***CSA A23.3-14(8.6.2.2)***

$$E_{cc} = (3300\sqrt{25} + 6900)\left(\frac{2402.8}{2300}\right)^{1.5} = 24,986 \text{ MPa}$$

$\ell$ m = 3000 mm

c. Torsional stiffness of torsional members, $K_t$

$$K_t = \frac{9E_{cs}C}{\ell_t\left(1-\frac{c_2}{\ell_t}\right)^3}$$

***CSA A23.3-14(13.8.2.8)***

$$K_t = \frac{9 \times 24,986 \times 2.30 \times 10^9}{6,600 \times \left(1 - \frac{600}{6,600}\right)^3} \times 10^{-3} = 104.3 \times 10^6 \text{N.m}$$

Where $C = \sum\left(1 - 0.63\frac{x}{y}\right)\left(\frac{x^3 y}{3}\right)$

***CSA A23.3-14(13.8.2.9)***

$$C = \left(1 - 0.63 \times \frac{250}{600}\right)\left(\frac{250^3 \times 600}{3}\right) = 2.30 \times 10^9 \text{ mm}^4$$

$c_2 = 600$ mm, and $l_t = 6.6$ m $= 6600$ mm

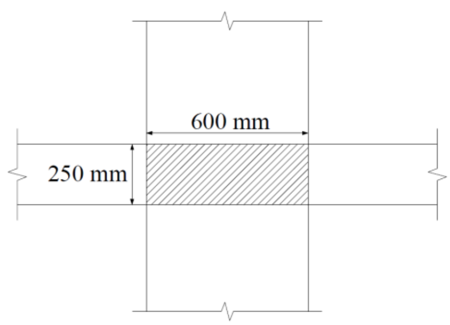



Figure 7 - Torsional Member

d. Equivalent column stiffness, $K_{ec}$

$$K_{ec} = \frac{\sum K_c \times \sum K_t}{\sum K_c + \sum K_t}$$

$$K_{ec} = \frac{(2 \times 299) \times (2 \times 104.3)}{(2 \times 299) + (2 \times 104.3)} \times 10^6$$

$K_{ec} = 154.65 \times 10^6$ N.m

Where $\sum K_t$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

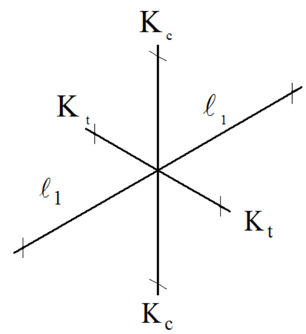



Figure 8 - Column and Edge of Slab

e. Slab-beam joint distribution factors, *DF*

At exterior joint,

$$DF = \frac{131.5}{(131.5 + 154.65)} = 0.46$$

At interior joint,

$$DF_{Ext} = \frac{131.5}{131.5 + 117.2 + 154.65} = 0.33$$

$$DF_{Int} = \frac{117.2}{(131.5 + 117.2 + 154.65)} = 0.29$$

COF for slab-beam $= 0.50$ for Interior Span

$= 0.51$ for Exterior Span

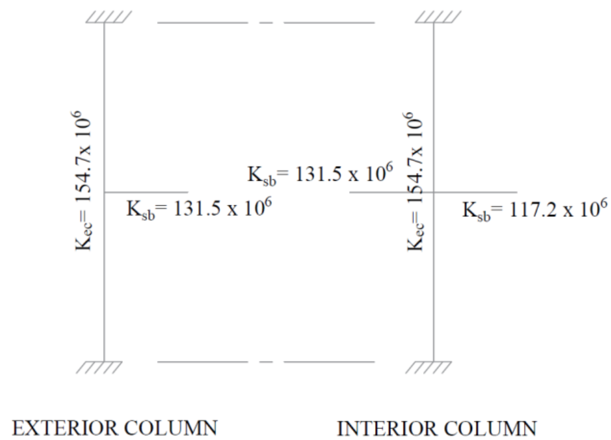



Figure 9 - Slab and Column Stiffness

### 2.2.3. Elastic frame analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. ***CSA A23.3-14 (13.8.4.2)***

$$\frac{L}{D} = \frac{1.9}{(5.89 + 1)} = 0.28 < \frac{3}{4}$$

a. Factored load and Fixed-End Moments (FEM's).

Factored dead load, $w_{df} = 1.25 \times (5.89 + 1) = 8.61$ kN/m$^2$

Factored live load, $w_{lf} = 1.5 \times 1.9 = 2.85$ kN/m$^2$

Total factored load $q_u = w_f = w_{df} + w_{lf} = 11.5$ kN/m$^2$

FEM's for slab-beams $= m_{NF} q_u \ell\ \ell$ ***PCA Notes on ACI 318-11 (Table A1)***

$= 0.0840 \times 11.5 \times 6.6 \times 7.5^2 = 358.6$ kN.m (For Interior Span)

$= 0.0841 \times 11.5 \times 6.6 \times 6.7^2 = 286.5$ kN.m (For Exterior Span)

b. Moment distribution. Computations are shown in Table 1. Counterclockwise rotational moments acting on the member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_u\,(\text{midspan}) = M_o - \frac{M_{uL} + M_{uR}}{2}$$

Where $M_o$ is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example.

Positive moment in span 1-2:

$$+M_u = (9.8\times 4.2)\frac{5.5^2}{8} - \frac{(64.1+119.7)}{2} = 63.8 \text{ kN.m}$$

Positive moment span 2-3:

$$+M_u = (9.8\times 4.2)\frac{5.5^2}{8} - \frac{(108.5+108.5)}{2} = 47.2 \text{ kN.m}$$

**Table 1 – Moment Distribution for Elastic Frame**


| Joint | 1 | 2 | | 3 | | 4 |
|---|---|---|---|---|---|---|
| Member | 1_2 | 2_1 | 2_3 | 3_2 | 3_4 | 4_3 |
| DF | 0.460 | 0.330 | 0.290 | 0.290 | 0.330 | 0.460 |
| COF | 0.510 | 0.510 | 0.500 | 0.500 | 0.510 | 0.510 |
| FEM | 286.50 | -286.50 | 358.60 | -358.60 | 286.50 | -286.50 |
| Dist | -131.79 | -23.79 | -20.91 | 20.91 | 23.79 | 131.79 |
| CO | -12.13 | -67.21 | 10.46 | -10.46 | 67.21 | 12.13 |
| Dist | 5.58 | 18.73 | 16.46 | -16.46 | -18.73 | -5.58 |
| CO | 9.55 | 2.85 | -8.23 | 8.23 | -2.85 | -9.55 |
| Dist | -4.39 | 1.78 | 1.56 | -1.56 | -1.78 | 4.39 |
| CO | 0.91 | -2.24 | -0.78 | 0.78 | 2.24 | -0.91 |
| Dist | -0.42 | 1.00 | 0.88 | -0.88 | -1.00 | 0.42 |
| CO | 0.51 | -0.21 | -0.44 | 0.44 | 0.21 | -0.51 |
| Dist | -0.23 | 0.22 | 0.19 | -0.19 | -0.22 | 0.23 |
| CO | 0.11 | -0.12 | -0.10 | 0.10 | 0.12 | -0.11 |
| Dist | -0.05 | 0.07 | 0.06 | -0.06 | -0.07 | 0.05 |
| CO | 0.04 | -0.03 | -0.03 | 0.03 | 0.03 | -0.04 |
| Dist | -0.02 | 0.02 | 0.02 | -0.02 | -0.02 | 0.02 |
| CO | 0.01 | -0.01 | -0.01 | 0.01 | 0.01 | -0.01 |
| Dist | 0.00 | 0.01 | 0.01 | -0.01 | -0.01 | 0.00 |
| CO | 0.01 | 0.00 | -0.01 | 0.01 | 0.00 | -0.01 |
| Dist | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| M, kN.m | 154.20 | -355.40 | 357.70 | -357.70 | 355.40 | -154.20 |
| Midspan M, kN.m | 171.09 | | 175.94 | | 171.09 | |

### 2.2.4. Design moments

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 8. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than $0.175\ell$ from the centers of supports.
***CSA A23.3-14 (13.8.5.1)***

$$\frac{600}{2} = 300 \text{ mm} < 0.175 \times 6700 = 1172.5 \text{ mm (use face of supporting location)}$$

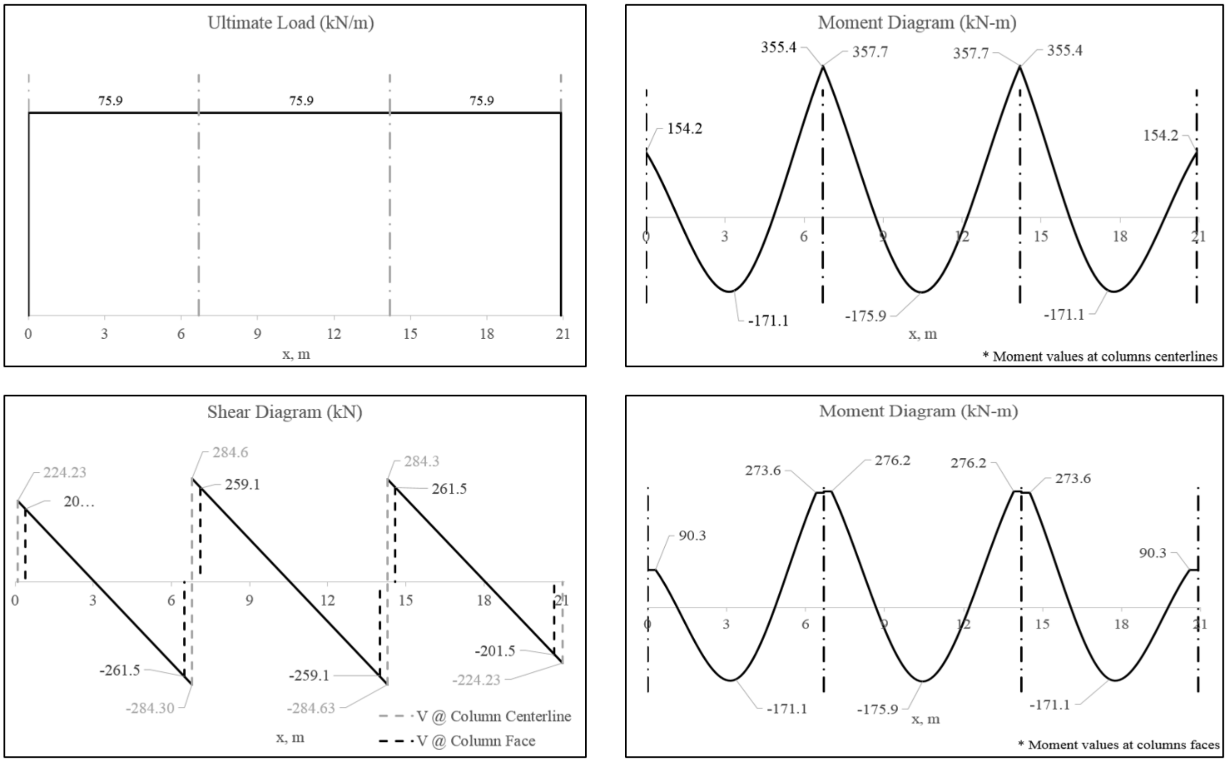



Figure 10 - Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)

### 2.2.5. Distribution of design moments

After the negative and positive moments have been determined for the slab-beam strip, the CSA code permits the distribution of the moments at critical sections to the column strips, beams (if any), and middle strips in accordance with the DDM. ***CSA A23.3-14 (13.11.2.2)***

Distribution of factored moments at critical sections is summarized in Table 2.

**Table 2 - Distribution of factored moments**

| | | Slab-beam Strip | Column Strip | | Middle Strip | |
|---|---|---|---|---|---|---|
| | | Moment (kN.m) | Percent | Moment (kN.m) | Percent | Moment (kN.m) |
| End Span | Exterior Negative | 90.3 | 100 | 90.3 | 0 | 0.00 |
| | Positive | 171.1 | 60 | 102.7 | 40 | 68.4 |
| | Interior Negative | 273.6 | 80 | 218.9 | 20 | 54.7 |
| Interior Span | Negative | 276.2 | 80 | 221.0 | 20 | 55.2 |
| | Positive | 175.9 | 60 | 105.5 | 40 | 70.4 |

### 2.2.6. Flexural reinforcement requirements

a. Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – exterior negative location is provided below.

Reinforcement for the total factored negative moment transferred to the exterior columns shall be placed within a band width $b_b$. Temperature and shrinkage reinforcment determined as specified in clause 7.8.1 shall be provided in that section of the slab outside of the band region defined by $b_b$ or as required by clause 13.10.9. ***CSA A23.3-14 (13.10.3)***

$M_r = 90.3 \text{ kN.m}$

Use average $d_l = 217$ mm

In this example, $jd$ will be assumed to be taken equal to $0.977d$. The assumptions will be verified once the area of steel in finalized.

Assume $jd = 0.977 \times d = 212 \text{ mm}$

Column strip width, $b = 6,600/2 = 3,300 \text{ mm}$

Middle strip width, $b = 6,600 - 3,300 = 3,300 \text{ mm}$

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{90.3}{0.85 \times 400 \times 0.977 \times 217} = 1,253 \text{ mm}^2$$

$\alpha_1 = 0.85 - 0.0015 f_c' = 0.81 > 0.67$ ***CSA A23.3-14 (10.1.7)***

Recalculate '$a$' for the actual $A_s = 1253 \text{ mm}^2 \rightarrow a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \dfrac{0.85 \times 1253 \times 400}{0.65 \times 0.81 \times 25 \times 3,300} = 9.78 \text{ mm}$

$$jd = d - \frac{a}{2} = 0.977d$$

Therefore, the assumption that $jd$ equals to $0.977d$ is valid.

$\therefore A_{s,req} = 1,253 \text{ mm}^2$

Reinforcement for the total factored negative moment transferred to the exterior columns shall be placed within a band width $b_b$. ***CSA A23.3-14 (13.10.3)***

For the part of the slab inside of the band region:

Provide 7 - 15M bars (1,400 mm$^2$ > 1,253 mm$^2$)

Temperature and shrinkage reinforcement determined as specified in clause 7.8.1 shall be provided in that section of the slab outside of the band region defined by $b_b$ or as required by clause 13.10.9 (including middle strip and the remaining part of the column strip outside the band region). ***CSA A23.3-14 (13.10.3)***

For the remaining part of the slab outside of the band region:

$$A_{s,\min} = 0.002 A_g = 0.002 \times 250 \times (6{,}600 - 1{,}150) = 2{,}725 \text{ mm}^2$$ **_CSA A23.3-14 (7.8.1)_**

Provide 14 - 15M bars (2,800 mm$^2$ > 2,725 mm$^2$)

For middle strip:

$$A_{s,\min} = 0.002 A_g = 0.002 \times 250 \times (3{,}300) = 1{,}650 \text{ mm}^2$$ **_CSA A23.3-14 (7.8.1)_**

Provide 9 - 15M bars (1,800 mm$^2$ > 1,650 mm$^2$)

For the remaining part of the column strip outside of the band region:

(14 – 15M) – (9 – 15M) = (5 – 15M)

Total Reinforcement in the column Strip:

(7 – 15M) + (5 – 15M) = (12 – 15M)

Maximum spacing: **_CSA A23.3-14 (13.10.4)_**

- Negative reinforcement in the band defined by $b_b$: $1.5 h_s = 375 \text{ mm} \le 250 \text{ mm}$

  $s_{max}$ = 250 mm > $s_{provided}$ = 1150/7 = 164 mm

- Remaining negative moment reinforcement: $3 h_s = 750 \text{ mm} \le 500 \text{ mm}$

  $s_{max}$ = 500 mm > $s_{provided}$ = (6600-1150)/14 = 389 mm

Based on the procedure outlined above, values for all span locations are given in Table 3.

**Table 3 - Required Slab Reinforcement for Flexure [Elastic Frame Method (EFM)]**

| Span Location | | $M_r$ (kN.m) | b (m) | d (mm) | $A_s$ Req'd for flexure (mm$^2$) | Min $A_s$ (mm$^2$) | Reinforcement Provided | $A_s$ Prov. for flexure (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| **End Span** | | | | | | | | |
| Column Strip | Exterior Negative | 90.3 | 3300 | 217 | 1253 | 1650 | 12 - 15M* | 2400 |
| | Positive | 102.7 | 3300 | 217 | 1429 | 1650 | 9 - 15M | 1800 |
| | Interior Negative | 218.9 | 3300 | 217 | 3146 | 1650 | 16 - 15M | 3200 |
| Middle Strip | Exterior Negative | 0.0 | 3300 | 217 | 0 | 1650 | 9 - 15M* | 1800 |
| | Positive | 68.4 | 3300 | 217 | 943 | 1650 | 9 - 15M | 1800 |
| | Interior Negative | 54.7 | 3300 | 217 | 752 | 1650 | 9 - 15M | 1800 |
| **Interior Span** | | | | | | | | |
| Column Strip | Negative | 221.0 | 3300 | 217 | 3176 | 1650 | 16 - 15M | 3200 |
| | Positive | 105.5 | 3300 | 217 | 1497 | 1650 | 9 - 15M | 1800 |
| Middle Strip | Negative | 55.2 | 3300 | 217 | 759 | 1650 | 9 - 15M | 1800 |
| | Positive | 70.4 | 3300 | 217 | 971 | 1650 | 9 - 15M | 1800 |

* the reinforcement is selected to meet CSA A23.3-14 provision 13.10.3 as described previously.

b. Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of unbalanced moment given by $\gamma_f$ shall be transferred by flexural reinforcement placed within a width $b_b$. ***CSA A23.3-14 (13.10.2)***

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_r$

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{b_1/b_2}}$$ ***CSA A23.3-14 (13.10.2)***

Where

$b_1$ = Width width of the critical section for shear measured in the direction of the span for which moments are determined according to CSA A23.3-14, clause 13 (see Figure 9).

$b_2$ = Width of the critical section for shear measured in the direction perpendicular to $b_1$ according to CSA A23.3-14, clause 13 (see Figure 9).

$b_b$ = Effective slab width $= c_2 + 3\times h_s$ ***CSA A23.3-14 (3.2)***

$$b_b = 400 + 3\times 250 = 1150 \text{ mm}$$

For Exterior Column

$$b_1 = 100 + 600 + \frac{217}{2} = 808.5 \text{ mm}$$

$$b_2 = 400 + 217 = 617 \text{ mm}$$

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{808.5/617}} = 0.567$$

For Interior Column

$$b_1 = 600 + 217 = 817 \text{ mm}$$

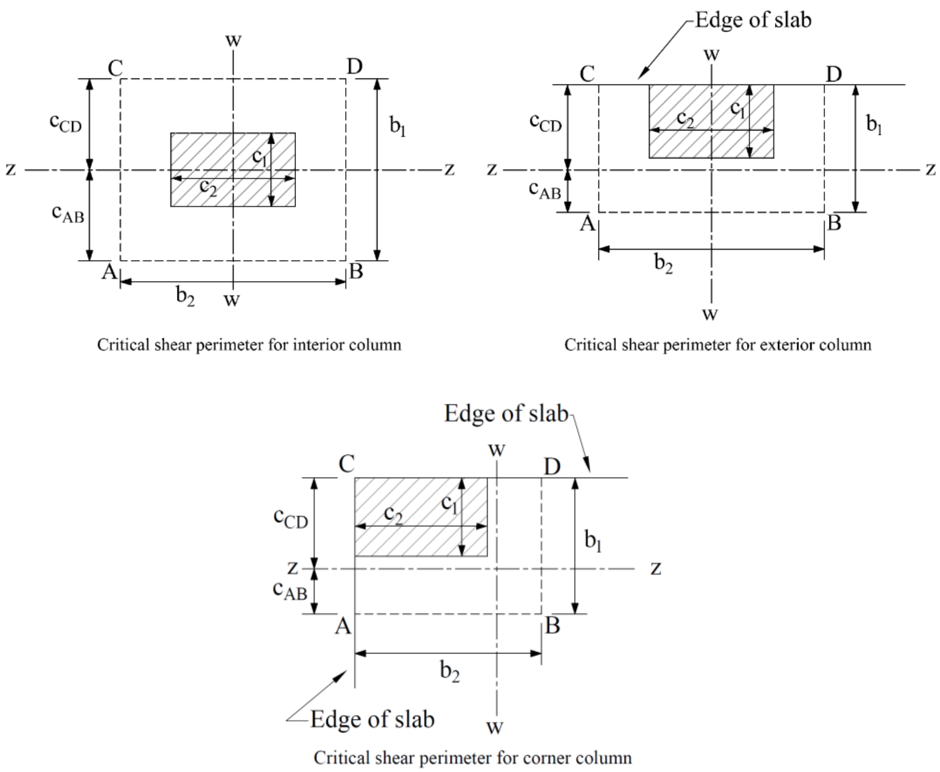
$$b_2 = 400 + 217 = 617 \text{ mm}$$

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{817/617}} = 0.566$$

Repeat the same procedure in section 2.2.6.a to calculate the additional reinforcement required for the unbalanced moment as shown in the following table:

**Table 4 - Additional Slab Reinforcement required for moment transfer between slab and column (EFM)**

| Span Location | | $M_u$* (kN.m) | $\gamma_f$ | $\gamma_f M_u$ (kN.m) | Effective slab width, $b_b$ (mm) | d (mm) | $A_s$ req'd within $b_b$ (mm$^2$) | $A_s$ prov. For flexure within $b_b$ (mm$^2$) | Add'l Reinf. |
|---|---|---|---|---|---|---|---|---|---|
| **End Span** | | | | | | | | | |
| Column Strip | Exterior Negative | 154.2 | 0.567 | 87.5 | 1150 | 217 | 1247 | 1400 | - |
| | Interior Negative | 2.3 | 0.566 | 1.3 | 1150 | 217 | 18 | 1400 | - |
| *$M_u$ is taken at the centerline of the support in Elastic Frame Method solution. | | | | | | | | | |




Critical shear perimeter for interior column

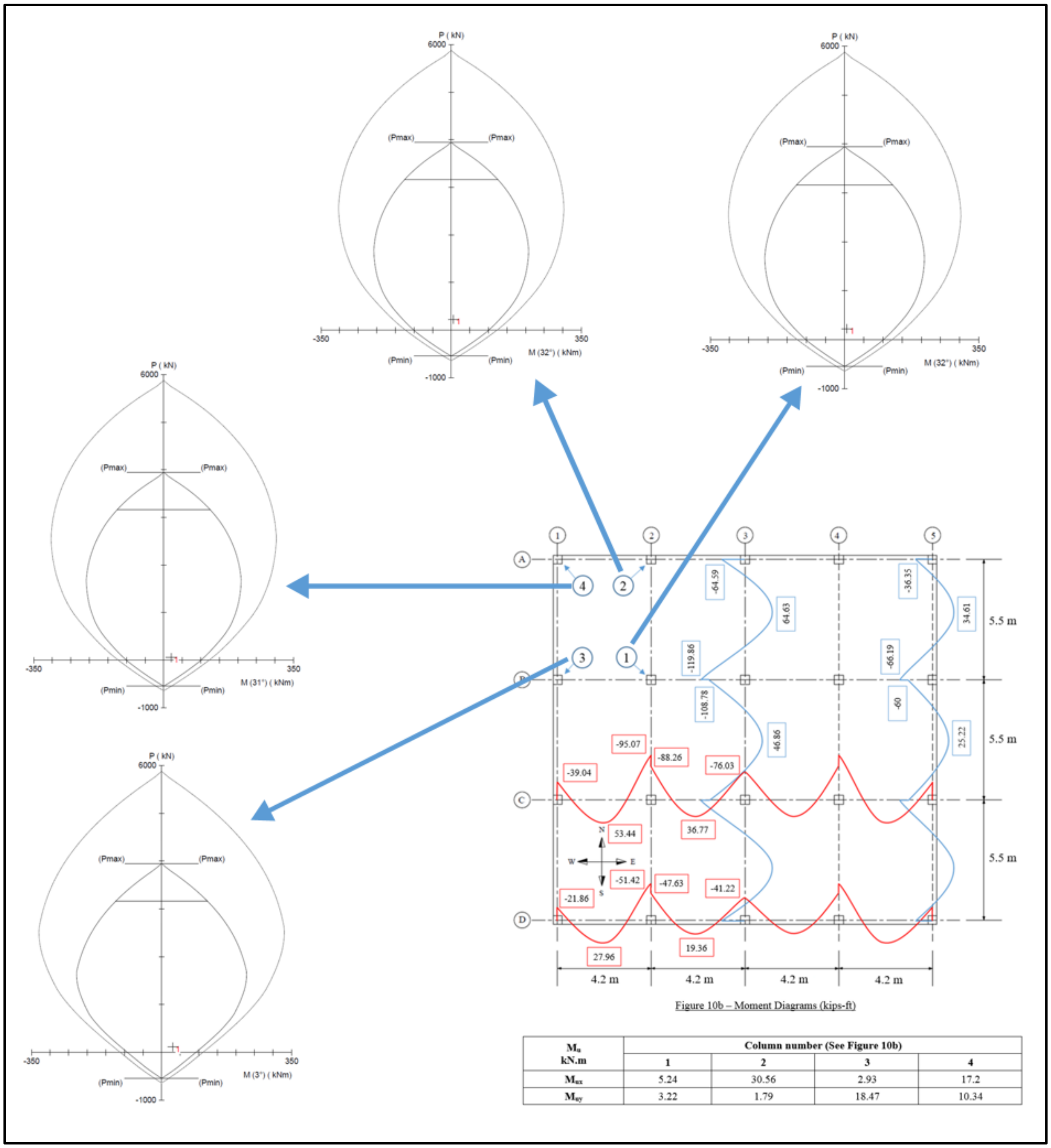
Critical shear perimeter for exterior column

Critical shear perimeter for corner column

Figure 11 - Critical Shear Perimeters for Columns

### 2.2.7. Column design moments

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the support columns above and below the slab-beam in proportion to the relative stiffness of the support columns. Detailed calculations regarding this topic (including column design for axial load and biaxial moments) can be found in "Two-Way Flat Plate Concrete Slab Floor Analysis and Design (CSA A23.3-14)" example available in the design examples page in StructurePoint website.




| $M_u$ kN.m | Column number (See Figure 10b) | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| $M_{ux}$ | 5.24 | 30.56 | 2.93 | 17.2 |
| $M_{uy}$ | 3.22 | 1.79 | 18.47 | 10.34 |

Figure 12 - Sample Calculations of Column Design from "Two-Way Flat Plate Concrete Slab Floor Analysis and Design" Design Example

## 3. Two-Way Slab Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with CSA A23.3-14 clause 13.

### 3.1. One-Way (Beam action) Shear Strength

***CSA A23.3-14 (13.3.6)***

One-way shear is critical at a distance d from the face of the column as shown in Figure 3. Figure 10 shows the factored shear forces ($V_r$) at the critical sections around each column. In members without shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$V_r = V_c + V_s + V_p = V_c \quad , \quad (V_s = V_p = 0)$$ ***CSA A23.3-14 (Eq. 11.4)***

Where:

$$V_c = \varphi_c \lambda \beta \sqrt{f_c^{'}} b_w d_v$$ ***CSA A23.3-14 (Eq. 11.5)***

$\lambda = 1$ for normal weight concrete

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm ***CSA A23.3-14 (11.3.6.2)***

$d_v = \text{Max } (0.9d, 0.72h) = \text{Max } (0.9 \times 217, 0.72 \times 250) = \text{Max } (195,180) = 195 \text{ mm}$ ***CSA A23.3-14 (3.2)***

$\sqrt{f_c^{'}} = \sqrt{25} = 5 \text{ MPa} < 8 \text{ MPa}$ ***CSA A23.3-14 (11.3.4)***

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{25} \times 6600 \times \frac{195}{1000} = 878.40 \text{ kN} > V_f$$

Because $V_r \geq V_f$ at all the critical sections, the slab has adequate one-way shear strength.

Shear forces for the figure below:

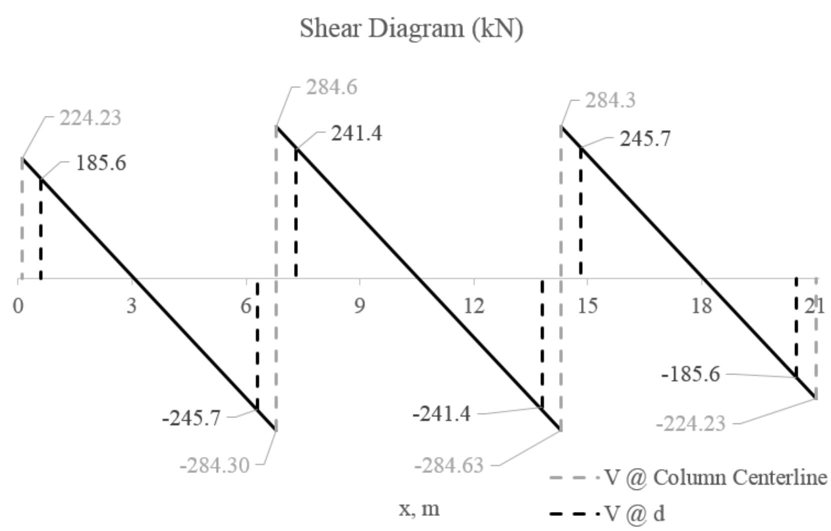



Figure 13 - One-way shear at critical sections (at distance d from the face of the supporting column)

### 3.2. Two-Way (Punching) Shear Strength

***CSA A23.3-14 (13.3.2)***

Two-way shear is critical on a rectangular section located at d/2 away from the face of the column as shown in Figure 9.

**a. Exterior column:**

The factored shear force ($V_f$) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section (*d/2* away from column face).

$$V_f = 224.2 - 11.5(0.808 \times 0.617) = 218.5\,\text{kN}$$

The factored unbalanced moment used for shear transfer, $M_{unb}$, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$\text{M}_{\text{unb}} = 154.2 - 218.5\left(\frac{808.5 - 292.6 - 600/2}{1000}\right) = 107.0\text{ kN.m}$$

For the exterior column in Figure 9, the location of the centroidal axis z-z is:

$$\text{c}_{\text{AB}} = e = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{2 \times (808.5 \times 217 \times 808.5/2)}{2 \times 808.5 \times 217 + 617 \times 217} = 292.6\text{ mm}$$

The polar moment $J_c$ of the shear perimeter is:

$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 d c_{AB}^2$$

$$J_c = 2\left(\frac{808.5 \times 217^3}{12} + \frac{217 \times 808.5^3}{12} + (808.5 \times 217)\left(\frac{808.5}{2} - 292.6\right)^2\right) + 617 \times 217 \times (292.6)^2 = 36.3 \times 10^9\text{ mm}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.567 = 0.433$$

***CSA A23.3-14 (Eq. 13.8)***

The length of the critical perimeter for the exterior column:

$$\text{b}_\text{o} = 2 \times \left(600 + 100 + \frac{217}{2}\right) + (400 + 217) = 2234\,\text{mm}$$

The two-way shear stress ($v_u$) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{unb} e}{J}$$

***CSA A23.3-14 (Eq.13.9)***

$$v_f = \frac{218.5 \times 1000}{2234 \times 217} + \frac{0.433 \times (107 \times 10^6) \times 292.6}{36.3 \times 10^9}$$

$$v_f = 0.451 + 0.373 = 0.824 \text{ MPa}$$

The factored resisiting shear stress, $V_r$ shall be the smallest of : **CSA A23.3-14 (13.3.4.1)**

a) $v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19\lambda\phi_c\sqrt{f'_c} = \left(1 + \frac{2}{1}\right) 0.19 \times 0.65 \times \sqrt{25} = 1.853 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{3 \times 217}{2234} + 0.19\right) \times 1 \times 0.65 \times \sqrt{25} = 1.565 \text{ MPa}$

c) $v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{25} = 1.235 \text{ MPa}$

Since ( $v_r = 1.235 \text{ MPa} \geq v_f = 0.824 \text{ MPa}$ ) at the critical section, the slab has adequate two-way shear strength at this joint.

**b. Interior column:**

$$V_f = 284.6 + 284.3 - 11.5\left(0.817 \times 0.617\right) = 563.1 \text{ kN}$$

$$M_{unb} = 357.7 - 355.4 - 568.9(0) = 2.3 \text{ kN.m}$$

For the interior column in Figure 9, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{817}{2} = 408.5 \text{ mm}$$

The polar moment $J_c$ of the shear perimeter is:

$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + \left(b_1 d\right)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + 2 b_2 d c_{AB}^2$$

$$\text{J}_c = 2\left(\frac{817 \times 217^3}{12} + \frac{217 \times 817^3}{12} + \left(817 \times 217\right)\left(\frac{817}{2} - 408.5\right)^2\right) + 2 \times 617 \times 217 \times (408.5)^2 = 65.8 \times 10^9 \text{ mm}^4$$

$\gamma_v = 1 - \gamma_f = 1 - 0.566 = 0.434$ **CSA A23.3-14 (Eq. 13.8)**

The length of the critical perimeter for the interior column:

$$\text{b}_\text{o} = 2 \times (600 + 217) + 2 \times (400 + 217) = 2868 \text{ mm}$$

$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{unb} e}{J}$ **CSA A23.3-14 (Eq.13.9)**

$$v_f = \frac{563.1 \times 1000}{2868 \times 217} + \frac{0.434 \times (2.30 \times 10^6) \times 408.5}{65.8 \times 10^9}$$

$$v_f = 0.905 + 0.006 = 0.911 \text{ MPa}$$

The factored resisiting shear stress, $V_r$ shall be the smallest of : **_CSA A23.3-14 (13.3.4.1)_**

a) $v_r = v_c = \left(1+\dfrac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\dfrac{2}{1}\right)0.19\times0.65\times\sqrt{25} = 1.853 \text{ MPa}$

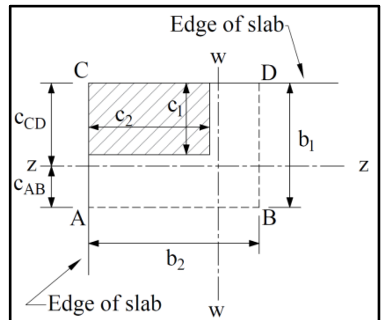
b) $v_r = v_c = \left(\dfrac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\dfrac{4\times217}{2868}+0.19\right)\times1\times0.65\times\sqrt{25} = 1.601 \text{ MPa}$

c) $v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times1\times0.65\times\sqrt{25} = 1.235 \text{ MPa}$

Since ( $v_r = 1.235 \text{ MPa} \geq v_f = 0.911 \text{ MPa}$ ) at the critical section, the slab has adequate two-way shear strength at this joint.

**c. Corner column:**

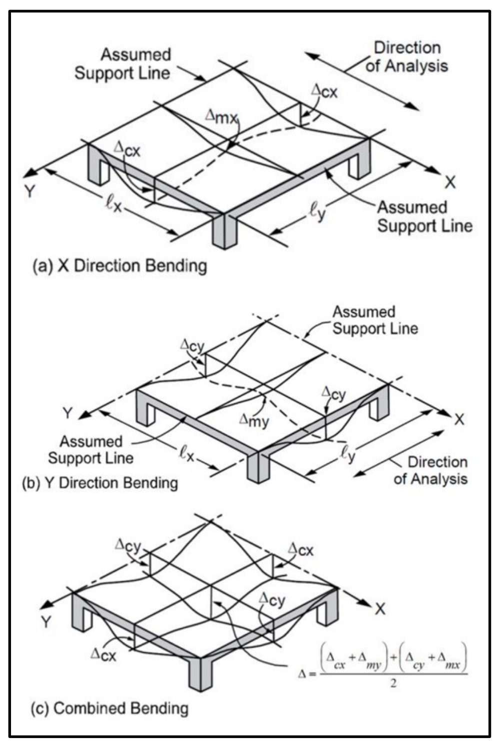
In this example, interior equivalent elastic frame strip was selected where it only have exterior and interior supports (no corner supports are included in this strip). Detailed calculations for two-way (punching) shear check around corner supports can be found in "Two-Way Flat Plate Concrete Slab Floor Analysis and Design (CSA A23.3-14)" example available in the design examples page in StructurePoint website.




Critical shear perimeter for corner column

## 4. Two-Way Slab Deflection Control (Serviceability Requirements)

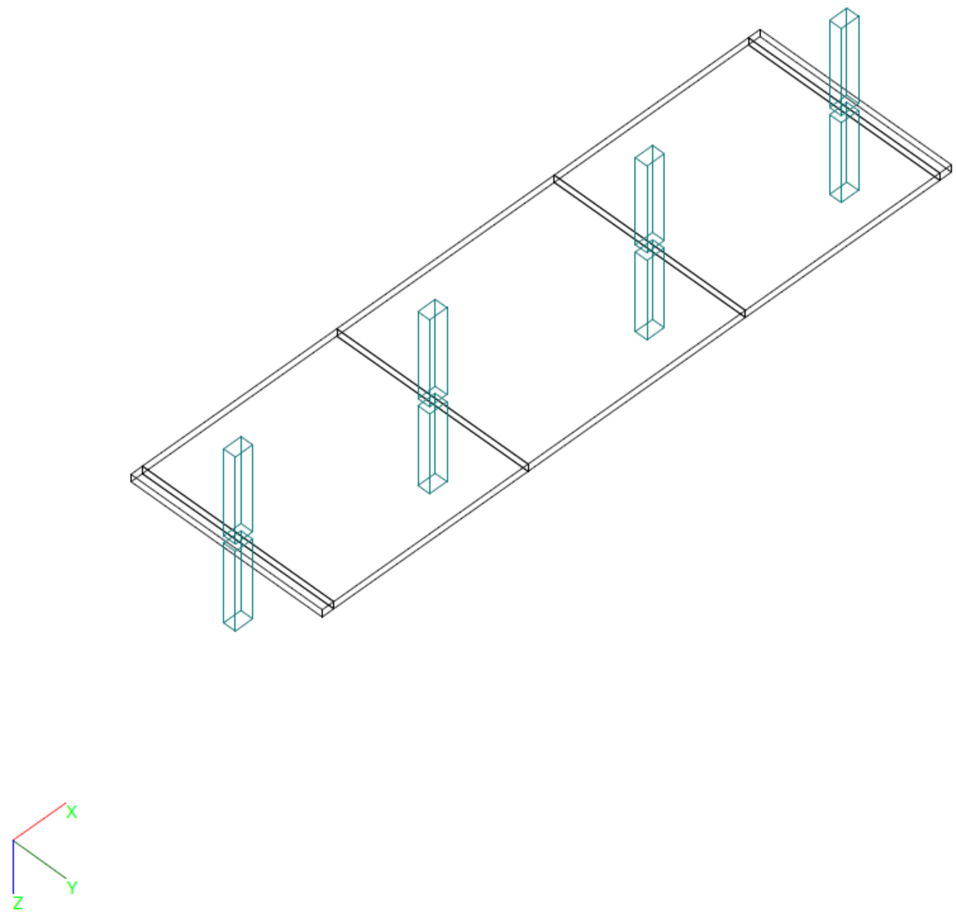
Since the slab thickness was selected based on the minimum slab thickness equations in CSA A23.3-14, the deflection calculations are not required. Detailed calculations of immediate and time-dependent deflections can be found in "Two-Way Flat Plate Concrete Slab Floor Analysis and Design (CSA A23.3-14)" example available in the design examples page in StructurePoint website.



## 5. spSlab Software Solution

spSlab program utilizes the Elastic (Equivalent) Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept.

spSlab Program models the equivalent elastic frame as a design strip. The design strip is, then, separated by spSlab into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips. The graphical and text results are provided below for both input and output of the spSlab model.




spSlab v5.50. Licensed to: StructurePoint. License ID:

File:

Project: CAC 4th Edition - Section 5.7 - Example 1

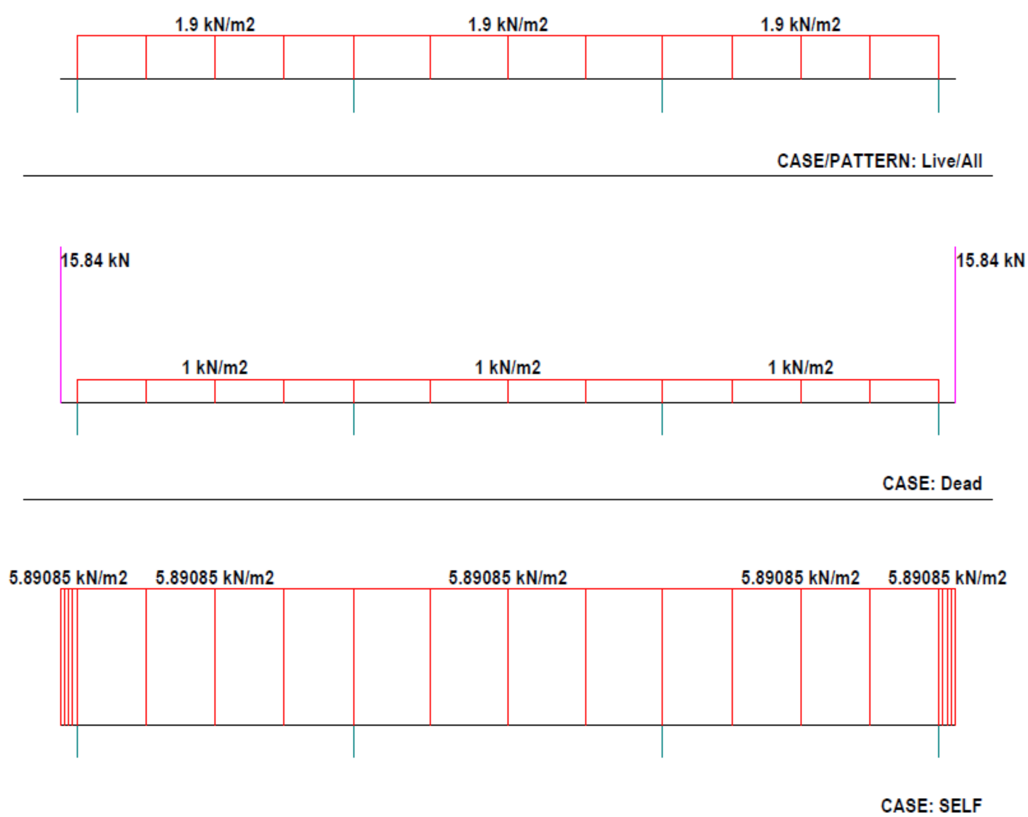
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 03/29/19

Time: 10:20:00




spSlab v5.50. Licensed to: StructurePoint. License ID:

File:

Project: CAC 4th Edition - Section 5.7 - Example 1

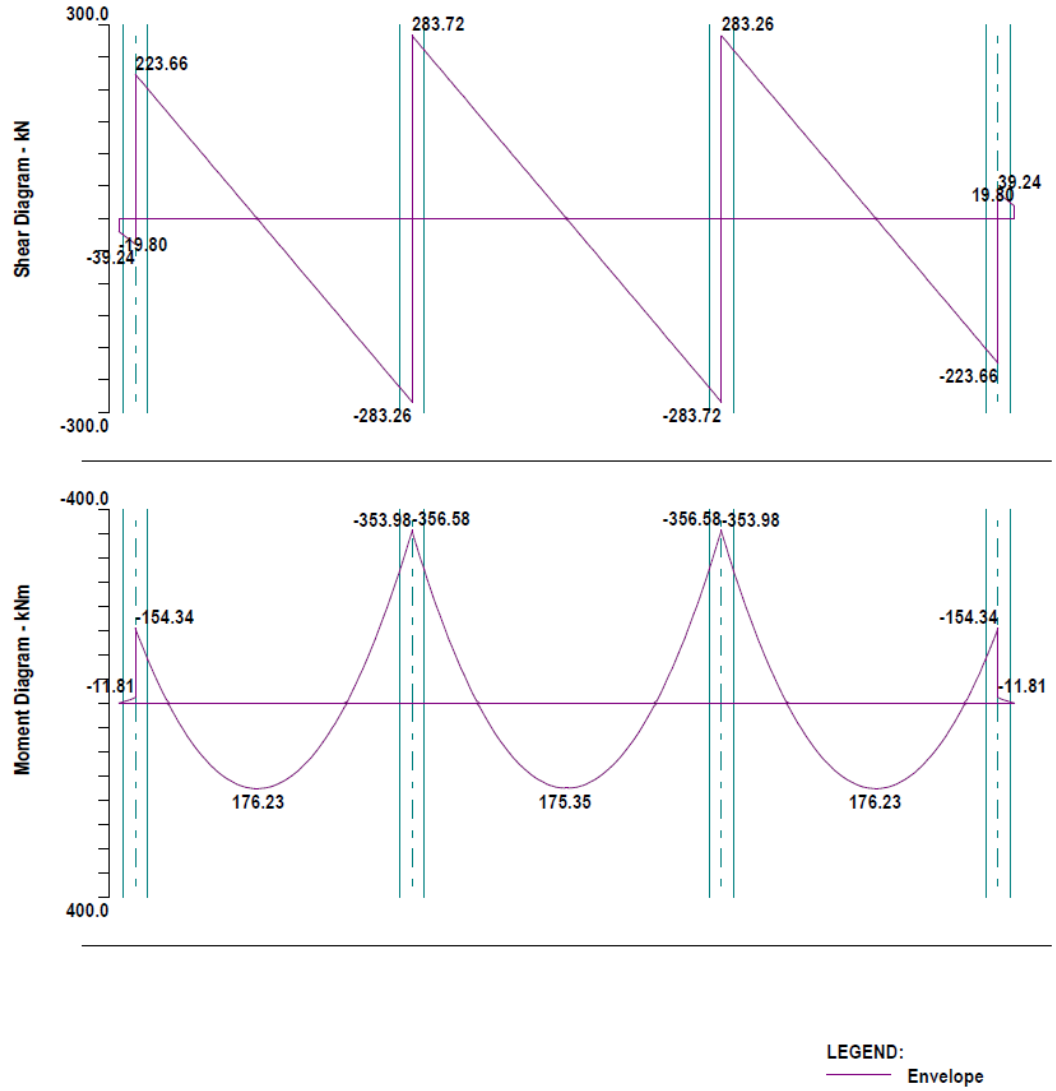
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 03/29/19

Time: 10:45:38




spSlab v5.50. Licensed to: StructurePoint. License ID:

File:

Project: CAC 4th Edition – Section 5.7 – Example 1

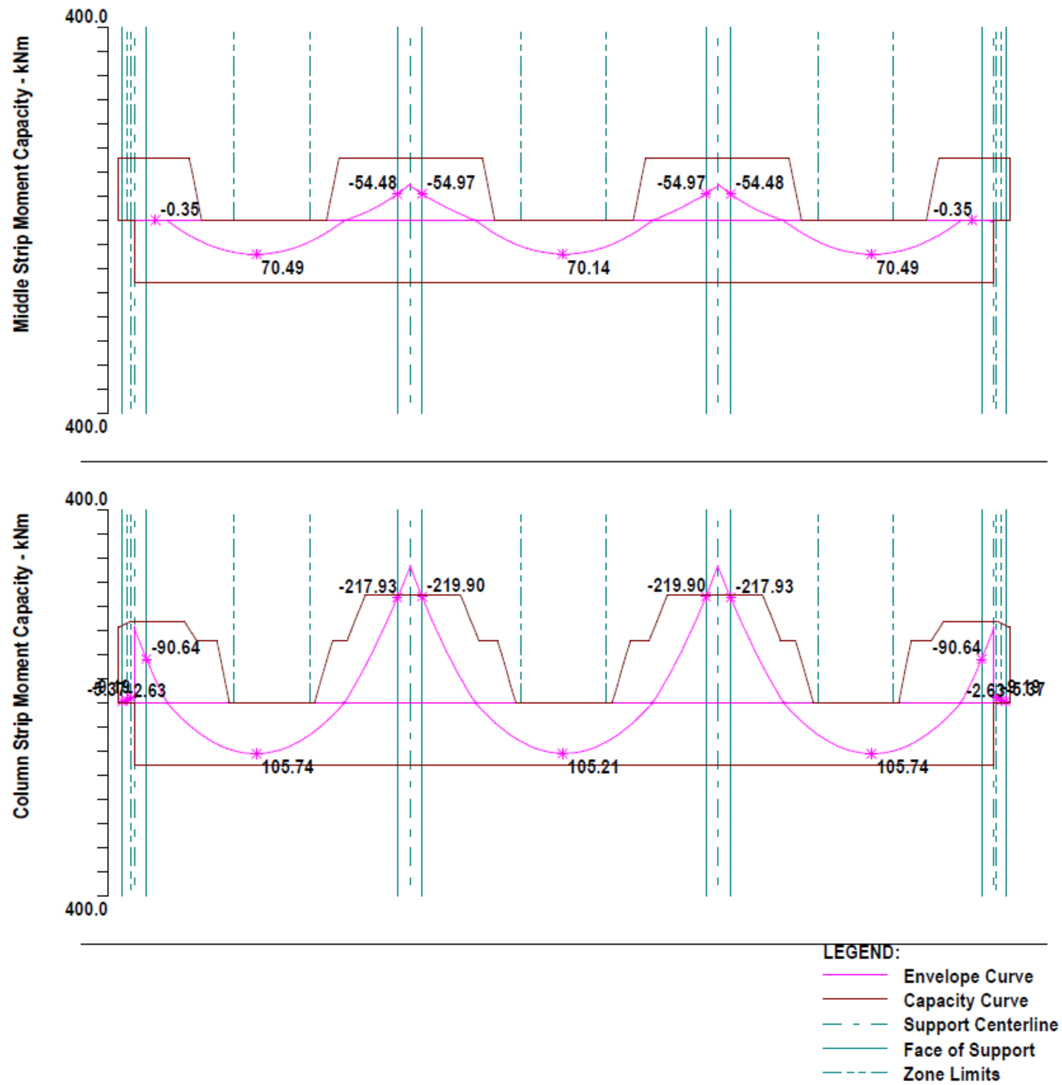
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 03/29/19

Time: 10:46:29




spSlab v5.50. Licensed to: StructurePoint. License ID:

File:

Project: CAC 4th Edition – Section 5.7 – Example 1

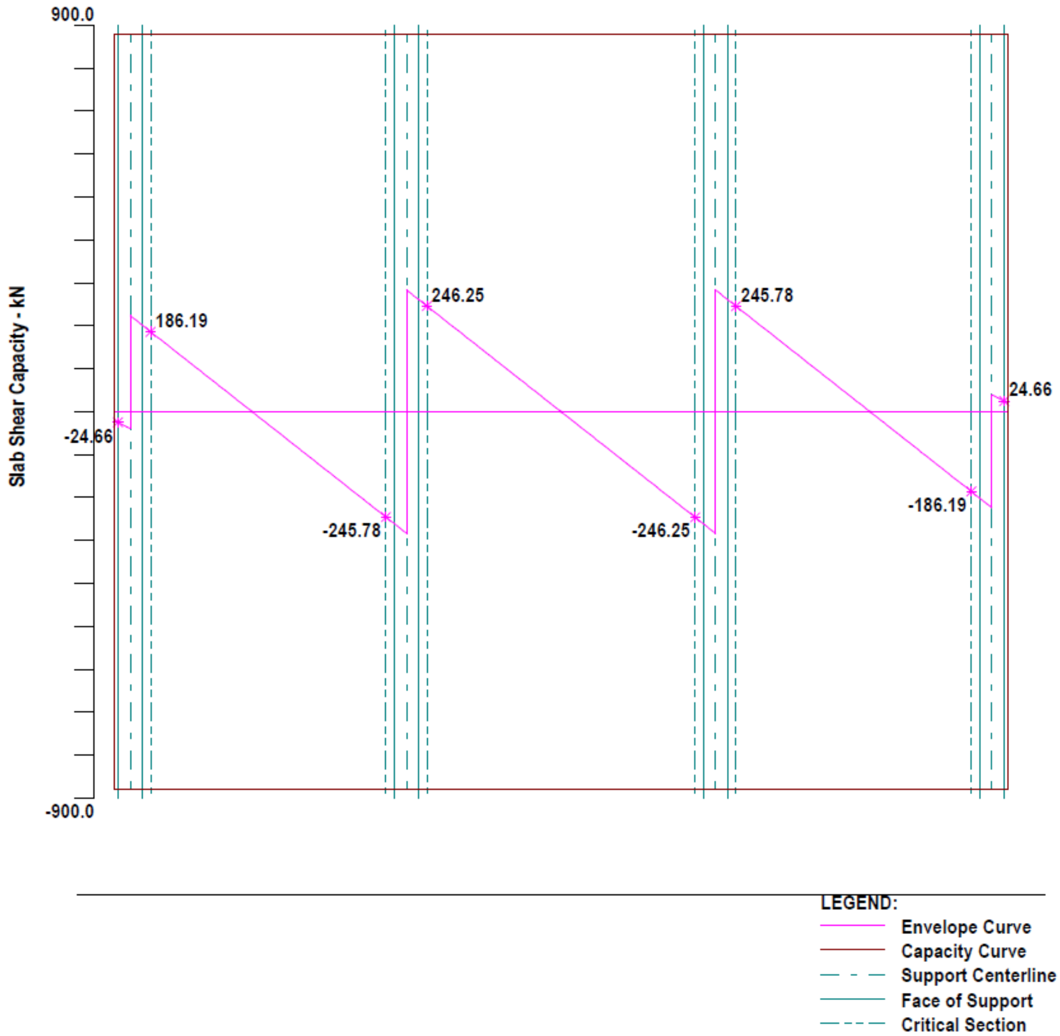
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 03/29/19

Time: 10:47:04




spSlab v5.50. Licensed to: StructurePoint. License ID:

File:

Project: CAC 4th Edition - Section 5.7 - Example 1

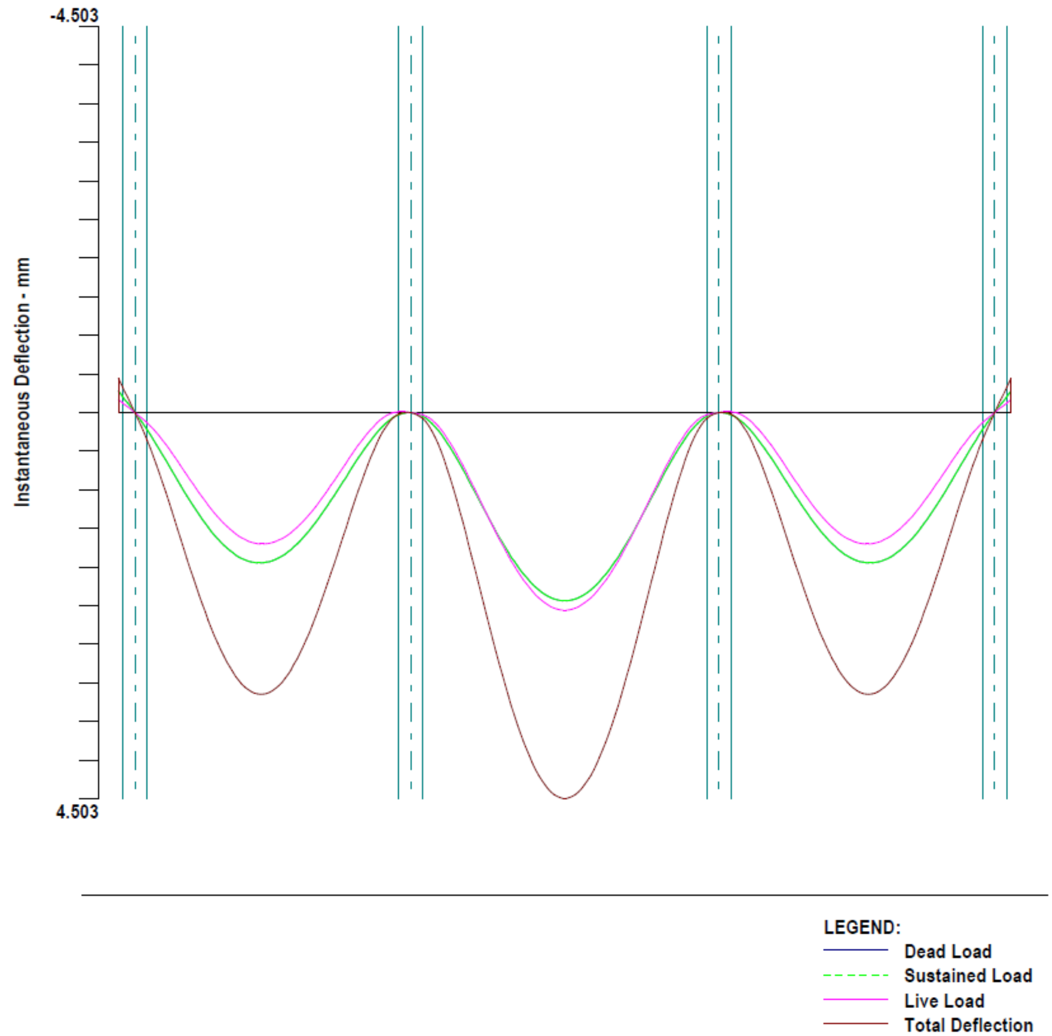
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

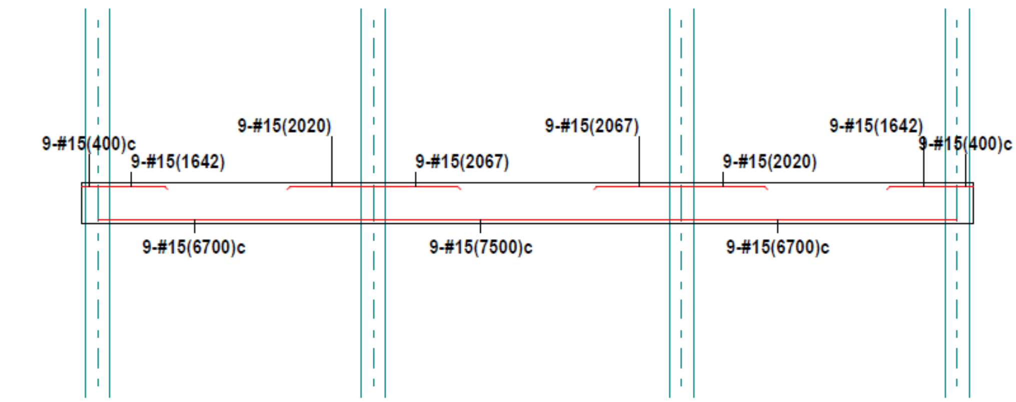
Date: 03/29/19

Time: 10:47:31

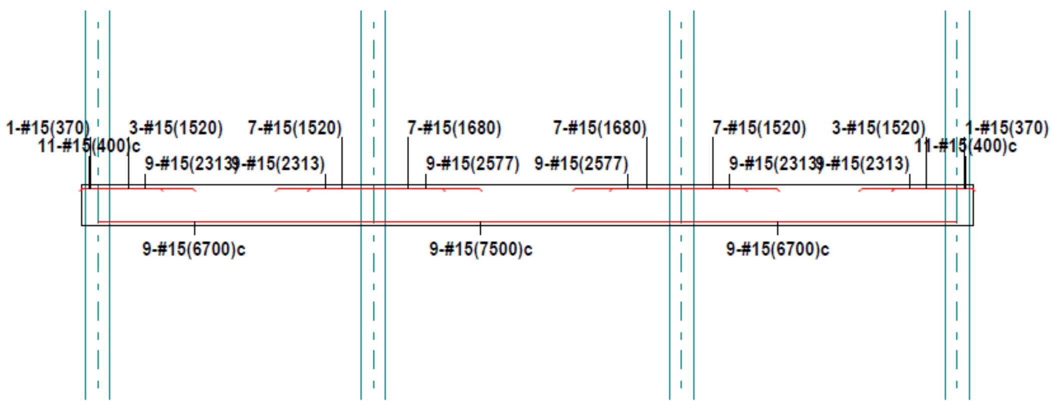



spSlab v5.50. Licensed to: StructurePoint. License ID:

File:
Project: CAC 4th Edition - Section 5.7 - Example 1
Frame: Interior
Engineer: SP
Code: CSA A23.3-14
Date: 03/29/19
Time: 10:47:58




Middle Strip Flexural Reinforcement




Column Strip Flexural Reinforcement

spSlab v5.50. Licensed to: StructurePoint. License ID:

File:

Project: CAC 4th Edition - Section 5.7 - Example 1

Frame: Interior

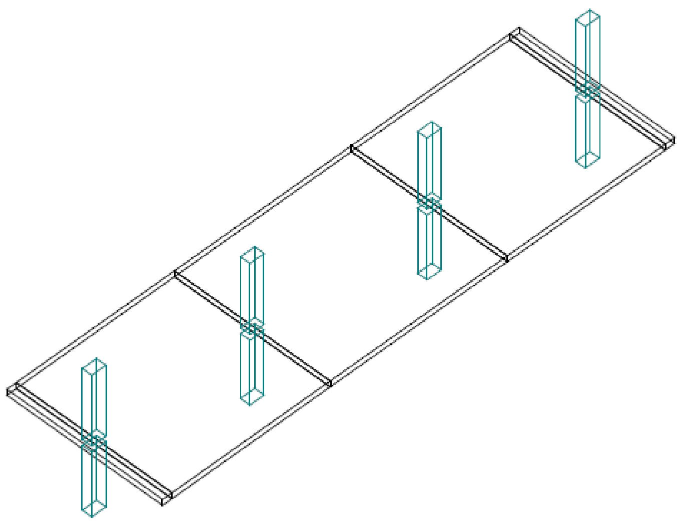
Engineer: SP

Code: CSA A23.3-14

Date: 03/29/19

Time: 10:48:27

spSlab v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems

# Contents

# 1. Input Echo

## 1.1. General Information

| | |
|---|---|
| File Name | |
| Project | CAC 4th Edition - Section 5.7 - Example 1 |
| Frame | Interior |
| Engineer | SP |
| Code | CSA A23.3-14 |
| Reinforcement Database | CSA G30.18 |
| Mode | Design |
| Number of supports = | 4 + Left cantilever + Right cantilever |
| Floor System | Two-Way |

## 1.2. Solve Options

| |
|---|
| Live load pattern ratio = 0% |
| Minimum free edge distance for punching shear = 5 times slab effective depth. |
| Circular critical section around circular supports used (if possible). |
| Deflections are based on cracked section properties. |
| In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available) |
| Long-term deflections are calculated for load duration of 60 months. |
| 0% of live load is sustained. |
| Compression reinforcement calculations NOT selected. |
| Default incremental rebar design selected. |
| Combined M-V-T reinforcement design NOT selected. |
| User-defined slab strip widths NOT selected. |
| User-defined distribution factors NOT selected. |
| One-way shear in drop panel NOT selected. |
| Distribution of shear to strips NOT selected. |
| Beam T-section design NOT selected. |
| Longitudinal beam contribution in negative reinforcement design over support NOT selected. |
| Transverse beam contribution in negative reinforcement design over support NOT selected. |

## 1.3. Material Properties

### 1.3.1. Concrete: Slabs / Beams

| | | |
|---|---|---|
| $w_c$ | 2402.8 | kg/m³ |
| $f'_c$ | 25 | MPa |
| $E_c$ | 24986 | MPa |
| $f_r$ | 1.5 | MPa |
| Precast concrete | No | |

### 1.3.2. Concrete: Columns

| | | |
|---|---|---|
| $w_c$ | 2402.8 | kg/m³ |
| $f'_c$ | 25 | MPa |
| $E_c$ | 24986 | MPa |
| $f_r$ | 1.5 | MPa |
| Precast concrete | No | |

### 1.3.3. Reinforcing Steel

| | |
|---|---|
| $f_y$ | 400 MPa |
| $f_{yt}$ | 400 MPa |
| $E_s$ | 200000 MPa |
| Epoxy coated bars | No |

## 1.4. Reinforcement Database

| Size | Db | Ab | Wb | Size | Db | Ab | Wb |
|---|---|---|---|---|---|---|---|
| | mm | mm² | kg/m | | mm | mm² | kg/m |
| #10 | 11 | 100 | 1 | #15 | 16 | 200 | 2 |
| #20 | 20 | 300 | 2 | #25 | 25 | 500 | 4 |
| #30 | 30 | 700 | 5 | #35 | 36 | 1000 | 8 |
| #45 | 44 | 1500 | 12 | #55 | 56 | 2500 | 20 |

## 1.5. Span Data

### 1.5.1. Slabs

Notes:
Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

| Span | Loc | L1 | t | wL | wR | L2L | L2R | $H_{min}$ |
|---|---|---|---|---|---|---|---|---|
| | | m | mm | m | m | m | m | mm |
| 1 | Int | 0.400 | 250 | 3.300 | 3.300 | 6.600 | 6.600 | --- LC *i |
| 2 | Int | 6.700 | 250 | 3.300 | 3.300 | 6.600 | 6.600 | 227 |
| 3 | Int | 7.500 | 250 | 3.300 | 3.300 | 6.600 | 6.600 | 230 |
| 4 | Int | 6.700 | 250 | 3.300 | 3.300 | 6.600 | 6.600 | 227 |
| 5 | Int | 0.400 | 250 | 3.300 | 3.300 | 6.600 | 6.600 | --- RC *i |

## 1.6. Support Data

### 1.6.1. Columns

| Support | c1a | c2a | Ha | c1b | c2b | Hb | Red % |
|---|---|---|---|---|---|---|---|
| | mm | mm | m | mm | mm | m | |
| 1 | 600 | 400 | 3.000 | 600 | 400 | 3.000 | 100 |
| 2 | 600 | 400 | 3.000 | 600 | 400 | 3.000 | 100 |
| 3 | 600 | 400 | 3.000 | 600 | 400 | 3.000 | 100 |
| 4 | 600 | 400 | 3.000 | 600 | 400 | 3.000 | 100 |

### 1.6.2. Boundary Conditions

| Support | Spring | | Far End | |
|---|---|---|---|---|
| | $K_z$ | $K_{ry}$ | Above | Below |
| | kN/mm | kN-mm/rad | | |
| 1 | 0 | 0 | Fixed | Fixed |
| 2 | 0 | 0 | Fixed | Fixed |
| 3 | 0 | 0 | Fixed | Fixed |
| 4 | 0 | 0 | Fixed | Fixed |

## 1.7. Load Data

### 1.7.1. Load Cases and Combinations

| Case | SELF | Dead | Live |
|---|---|---|---|
| Type | DEAD | DEAD | LIVE |
| U1 | 1.250 | 1.250 | 1.500 |

### 1.7.2. Area Loads

| Case/Patt | Span | Wa |
|---|---|---|
| | | kN/m² |
| SELF | 1 | 5.89 |
| | 2 | 5.89 |
| | 3 | 5.89 |
| | 4 | 5.89 |
| | 5 | 5.89 |
| Dead | 2 | 1.00 |
| | 3 | 1.00 |
| | 4 | 1.00 |
| Live | 2 | 1.90 |
| | 3 | 1.90 |
| | 4 | 1.90 |

### 1.7.3. Point Forces

| Case/Patt | Span | Wa | La |
|---|---|---|---|
| | | kN | m |
| Dead | 1 | 15.84 | 0.000 |
| | 5 | 15.84 | 0.400 |

## *1.8. Reinforcement Criteria*

### 1.8.1. Slabs and Ribs

| | Units | Top Bars | | Bottom Bars | |
|---|---|---|---|---|---|
| | | Min. | Max. | Min. | Max. |
| Bar Size | | #15 | #15 | #15 | #15 |
| Bar spacing | mm | 25 | 500 | 25 | 500 |
| Reinf ratio | % | 0.14 | 5.00 | 0.14 | 5.00 |
| Clear Cover | mm | 25 | | 25 | |

There is NOT more than 300 mm of concrete below top bars.

### 1.8.2. Beams

| | Units | Top Bars | | Bottom Bars | | Stirrups | |
|---|---|---|---|---|---|---|---|
| | | Min. | Max. | Min. | Max. | Min. | Max. |
| Bar Size | | #20 | #35 | #20 | #35 | #10 | #20 |
| Bar spacing | mm | 25 | 457 | 25 | 457 | 152 | 457 |
| Reinf ratio | % | 0.14 | 5.00 | 0.14 | 5.00 | | |
| Clear Cover | mm | 38 | | 38 | | | |
| Layer dist. | mm | 25 | | 25 | | | |
| No. of legs | | | | | | 2 | 6 |
| Side cover | mm | | | | | 38 | |
| 1st Stirrup | mm | | | | | 76 | |

There is NOT more than 300 mm of concrete below top bars.

## 2. Design Results*

*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

### 2.1. Strip Widths and Distribution Factors

Notes:
*Used for bottom reinforcement. **Used for top reinforcement.

| Span | Strip | Width | | | Moment Factor | | |
|---|---|---|---|---|---|---|---|
| | | Left ** | Right ** | Bottom * | Left ** | Right ** | Bottom * |
| | | m | m | m | m | m | m |
| 1 | Column | 3.30 | 3.30 | 3.30 | 1.000 | 1.000 | 0.600 |
| | Middle | 3.30 | 3.30 | 3.30 | 0.000 | 0.000 | 0.400 |
| 2 | Column | 3.30 | 3.30 | 3.30 | 1.000 | 0.800 | 0.600 |
| | Middle | 3.30 | 3.30 | 3.30 | 0.000 | 0.200 | 0.400 |
| 3 | Column | 3.30 | 3.30 | 3.30 | 0.800 | 0.800 | 0.600 |
| | Middle | 3.30 | 3.30 | 3.30 | 0.200 | 0.200 | 0.400 |
| 4 | Column | 3.30 | 3.30 | 3.30 | 0.800 | 1.000 | 0.600 |
| | Middle | 3.30 | 3.30 | 3.30 | 0.200 | 0.000 | 0.400 |
| 5 | Column | 3.30 | 3.30 | 3.30 | 1.000 | 1.000 | 0.600 |
| | Middle | 3.30 | 3.30 | 3.30 | 0.000 | 0.000 | 0.400 |

### 2.2. Top Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

| Span | Strip | Zone | Width | $M_{max}$ | $X_{max}$ | $A_{s,min}$ | $A_{s,max}$ | $A_{s,req}$ | $Sp_{Prov}$ | Bars |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | m | kNm | m | mm² | mm² | mm² | mm | |
| 1 | Column | Left | 3.30 | 2.63 | 0.115 | 1650 | 16059 | 36 | 300 | 11-#15 *3 |
| | | Midspan | 3.30 | 5.37 | 0.215 | 1650 | 16059 | 73 | 300 | 11-#15 *3 |
| | | Right | 3.30 | 9.19 | 0.330 | 1650 | 16059 | 125 | 275 | 12-#15 *3 *5 |
| | Middle | Left | 3.30 | 0.00 | 0.000 | 1650 | 16059 | 0 | 367 | 9-#15 *3 |
| | | Midspan | 3.30 | 0.00 | 0.165 | 1650 | 16059 | 0 | 367 | 9-#15 *3 |
| | | Right | 3.30 | 0.00 | 0.330 | 1650 | 16059 | 0 | 367 | 9-#15 *3 |
| 2 | Column | Left | 3.30 | 90.64 | 0.300 | 1650 | 16059 | 1257 | 275 | 12-#15 *3 |
| | | Midspan | 3.30 | 0.00 | 3.350 | 0 | 16059 | 0 | 0 | --- |
| | | Right | 3.30 | 217.93 | 6.400 | 1650 | 16059 | 3130 | 206 | 16-#15 |
| | Middle | Left | 3.30 | 0.35 | 0.523 | 1650 | 16059 | 5 | 367 | 9-#15 *3 |
| | | Midspan | 3.30 | 0.00 | 3.350 | 0 | 16059 | 0 | 0 | --- |
| | | Right | 3.30 | 54.48 | 6.400 | 1650 | 16059 | 749 | 367 | 9-#15 *3 |
| 3 | Column | Left | 3.30 | 219.90 | 0.300 | 1650 | 16059 | 3160 | 206 | 16-#15 |
| | | Midspan | 3.30 | 0.00 | 3.750 | 0 | 16059 | 0 | 0 | --- |
| | | Right | 3.30 | 219.90 | 7.200 | 1650 | 16059 | 3160 | 206 | 16-#15 |
| | Middle | Left | 3.30 | 54.97 | 0.300 | 1650 | 16059 | 755 | 367 | 9-#15 *3 |
| | | Midspan | 3.30 | 0.00 | 3.750 | 0 | 16059 | 0 | 0 | --- |
| | | Right | 3.30 | 54.97 | 7.200 | 1650 | 16059 | 755 | 367 | 9-#15 *3 |
| 4 | Column | Left | 3.30 | 217.93 | 0.300 | 1650 | 16059 | 3130 | 206 | 16-#15 |

| Span | Strip | Zone | Width | $M_{max}$ | $X_{max}$ | $A_{s,min}$ | $A_{s,max}$ | $A_{s,req}$ | $Sp_{Prov}$ | Bars |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | m | kNm | m | mm² | mm² | mm² | mm | |
| | | Midspan | 3.30 | 0.00 | 3.350 | 0 | 16059 | 0 | 0 | --- |
| | | Right | 3.30 | 90.64 | 6.400 | 1650 | 16059 | 1257 | 275 | 12-#15 *3 |
| | Middle | Left | 3.30 | 54.48 | 0.300 | 1650 | 16059 | 749 | 367 | 9-#15 *3 |
| | | Midspan | 3.30 | 0.00 | 3.350 | 0 | 16059 | 0 | 0 | --- |
| | | Right | 3.30 | 0.35 | 6.177 | 1650 | 16059 | 5 | 367 | 9-#15 *3 |
| 5 | Column | Left | 3.30 | 9.19 | 0.070 | 1650 | 16059 | 125 | 275 | 12-#15 *3 *5 |
| | | Midspan | 3.30 | 5.37 | 0.186 | 1650 | 16059 | 73 | 300 | 11-#15 *3 |
| | | Right | 3.30 | 2.63 | 0.285 | 1650 | 16059 | 36 | 300 | 11-#15 *3 |
| | Middle | Left | 3.30 | 0.00 | 0.070 | 1650 | 16059 | 0 | 367 | 9-#15 *3 |
| | | Midspan | 3.30 | 0.00 | 0.235 | 1650 | 16059 | 0 | 367 | 9-#15 *3 |
| | | Right | 3.30 | 0.00 | 0.400 | 1650 | 16059 | 0 | 367 | 9-#15 *3 |

### *2.3. Top Bar Details*

NOTES:
* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

| Span | Strip | Left Bars | Left Length | Left Bars | Left Length | Continuous Bars | Continuous Length | Right Bars | Right Length | Right Bars | Right Length |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | m | | m | | m | | m | | m |
| 1 | Column | --- | | --- | | 11-#15 | 0.40 | 1-#15 * | 0.37 | --- | |
| | Middle | --- | | --- | | 9-#15 | 0.40 | --- | | --- | |
| 2 | Column | 9-#15 | 2.31 | 3-#15 | 1.52 | --- | | 9-#15 | 2.31 | 7-#15 * | 1.52 |
| | Middle | 9-#15 | 1.64 | --- | | --- | | 9-#15 | 2.02 | --- | |
| 3 | Column | 9-#15 | 2.58 | 7-#15 | 1.68 | --- | | 9-#15 | 2.58 | 7-#15 | 1.68 |
| | Middle | 9-#15 | 2.07 | --- | | --- | | 9-#15 | 2.07 | --- | |
| 4 | Column | 9-#15 | 2.31 | 7-#15 * | 1.52 | --- | | 9-#15 | 2.31 | 3-#15 | 1.52 |
| | Middle | 9-#15 | 2.02 | --- | | --- | | 9-#15 | 1.64 | --- | |
| 5 | Column | 1-#15 * | 0.37 | --- | | 11-#15 | 0.40 | --- | | --- | |
| | Middle | --- | | --- | | 9-#15 | 0.40 | --- | | --- | |

### *2.4. Top Bar Development Lengths*

| Span | Strip | Left Bars | Left DevLen | Left Bars | Left DevLen | Continuous Bars | Continuous DevLen | Right Bars | Right DevLen | Right Bars | Right DevLen |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | mm | | mm | | mm | | mm | | mm |
| 1 | Column | --- | | --- | | 11-#15 | 300.00 | 1-#15 | 300.00 | --- | |
| | Middle | --- | | --- | | 9-#15 | 300.00 | --- | | --- | |
| 2 | Column | 9-#15 | 300.00 | 3-#15 | 300.00 | --- | | 9-#15 | 438.60 | 7-#15 | 438.60 |
| | Middle | 9-#15 | 300.00 | --- | | --- | | 9-#15 | 300.00 | --- | |
| 3 | Column | 9-#15 | 442.82 | 7-#15 | 442.82 | --- | | 9-#15 | 442.82 | 7-#15 | 442.82 |
| | Middle | 9-#15 | 300.00 | --- | | --- | | 9-#15 | 300.00 | --- | |
| 4 | Column | 9-#15 | 438.60 | 7-#15 | 438.60 | --- | | 9-#15 | 300.00 | 3-#15 | 300.00 |
| | Middle | 9-#15 | 300.00 | --- | | --- | | 9-#15 | 300.00 | --- | |
| 5 | Column | 1-#15 | 300.00 | --- | | 11-#15 | 300.00 | --- | | --- | |

| Span | Strip | Left Bars | Left DevLen mm | Left Bars | Left DevLen mm | Continuous Bars | Continuous DevLen mm | Right Bars | Right DevLen mm | Right Bars | Right DevLen mm |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Middle | --- | | --- | | 9-#15 | 300.00 | --- | | --- | |

### 2.5. Band Reinforcement at Supports

NOTES:
<C> Total Strip, <B> Banded Strip, <S> Remaining Strip

| Support | Width <C> mm | Width <B> mm | Width <S> mm | $A_s$ <C> mm² | $A_s$ <B> mm² | $A_s$ <S> mm² | Bars <C> | Bars <B> | Bars <S> |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3300 | 1150 | 2150 | 2400 | 1400 | 1000 | 12-#15 | 7-#15 | 5-#15 |
| 2 | 3300 | 1150 | 2150 | 3200 | 1400 | 1800 | 16-#15 | 7-#15 | 9-#15 |
| 3 | 3300 | 1150 | 2150 | 3200 | 1400 | 1800 | 16-#15 | 7-#15 | 9-#15 |
| 4 | 3300 | 1150 | 2150 | 2400 | 1400 | 1000 | 12-#15 | 7-#15 | 5-#15 |

### 2.6. Bottom Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.

| Span | Strip | Width m | $M_{max}$ kNm | $X_{max}$ m | $A_{s,min}$ mm² | $A_{s,max}$ mm² | $A_{s,req}$ mm² | $Sp_{Prov}$ mm | Bars |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Column | 3.30 | 0.00 | 0.165 | 0 | 16059 | 0 | 0 | --- |
| | Middle | 3.30 | 0.00 | 0.165 | 0 | 16059 | 0 | 0 | --- |
| 2 | Column | 3.30 | 105.74 | 2.978 | 1650 | 16059 | 1472 | 367 | 9-#15 *3 |
| | Middle | 3.30 | 70.49 | 2.978 | 1650 | 16059 | 972 | 367 | 9-#15 *3 |
| 3 | Column | 3.30 | 105.21 | 3.713 | 1650 | 16059 | 1465 | 367 | 9-#15 *3 |
| | Middle | 3.30 | 70.14 | 3.713 | 1650 | 16059 | 967 | 367 | 9-#15 *3 |
| 4 | Column | 3.30 | 105.74 | 3.722 | 1650 | 16059 | 1472 | 367 | 9-#15 *3 |
| | Middle | 3.30 | 70.49 | 3.722 | 1650 | 16059 | 972 | 367 | 9-#15 *3 |
| 5 | Column | 3.30 | 0.00 | 0.235 | 0 | 16059 | 0 | 0 | --- |
| | Middle | 3.30 | 0.00 | 0.235 | 0 | 16059 | 0 | 0 | --- |

### 2.7. Bottom Bar Details

| Span | Strip | Long Bars Bars | Long Bars Start m | Long Bars Length m | Short Bars Bars | Short Bars Start m | Short Bars Length m |
|---|---|---|---|---|---|---|---|
| 1 | Column | --- | | | --- | | |
| | Middle | --- | | | --- | | |
| 2 | Column | 9-#15 | 0.00 | 6.70 | --- | | |
| | Middle | 9-#15 | 0.00 | 6.70 | --- | | |
| 3 | Column | 9-#15 | 0.00 | 7.50 | --- | | |
| | Middle | 9-#15 | 0.00 | 7.50 | --- | | |
| 4 | Column | 9-#15 | 0.00 | 6.70 | --- | | |
| | Middle | 9-#15 | 0.00 | 6.70 | --- | | |
| 5 | Column | --- | | | --- | | |
| | Middle | --- | | | --- | | |

### 2.8. Bottom Bar Development Lengths

| Span | Strip | Long Bars Bars | Long Bars DevLen mm | Short Bars Bars | Short Bars DevLen mm |
|---|---|---|---|---|---|
| 1 | Column | --- | | --- | |
| | Middle | --- | | --- | |
| 2 | Column | 9-#15 | 366.75 | --- | |
| | Middle | 9-#15 | 300.00 | --- | |
| 3 | Column | 9-#15 | 364.85 | --- | |
| | Middle | 9-#15 | 300.00 | --- | |
| 4 | Column | 9-#15 | 366.75 | --- | |
| | Middle | 9-#15 | 300.00 | --- | |
| 5 | Column | --- | | --- | |
| | Middle | --- | | --- | |

### 2.9. Flexural Capacity

| Span | Strip | Top x m | Top $A_{s,top}$ mm² | Top $\Phi M_n$- kNm | Top $M_u$- kNm | Top Comb | Top Pat | Top Status | Bottom $A_{s,bot}$ mm² | Bottom $\Phi M_n$+ kNm | Bottom $M_u$+ kNm | Bottom Comb | Bottom Pat | Bottom Status |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Column | 0.000 | 2200 | -155.90 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.030 | 2200 | -155.90 | -0.63 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.115 | 2257 | -159.76 | -2.63 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.200 | 2313 | -163.58 | -4.95 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.215 | 2323 | -164.23 | -5.37 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.330 | 2400 | -169.43 | -9.19 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.400 | 2400 | -169.43 | -11.81 | U1 | All | --- | 0 | 0.00 | 0.00 | U1 | All | --- |
| | Middle | 0.000 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.115 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.200 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.215 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.330 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.400 | 1800 | -128.51 | 0.00 | U1 | All | --- | 0 | 0.00 | 0.00 | U1 | All | --- |
| 2 | Column | 0.000 | 2400 | -169.43 | -155.86 | U1 | All | --- | 1800 | 128.51 | 0.00 | U1 | All | --- |
| | | 0.300 | 2400 | -169.43 | -90.64 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |
| | | 1.220 | 2400 | -169.43 | 0.00 | U1 | All | OK | 1800 | 128.51 | 37.31 | U1 | All | OK |
| | | 1.520 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 58.91 | U1 | All | OK |
| | | 2.013 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 85.56 | U1 | All | OK |
| | | 2.313 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 96.36 | U1 | All | OK |
| | | 2.435 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 99.56 | U1 | All | OK |
| | | 2.978 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 105.74 | U1 | All | OK |
| | | 3.350 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 102.23 | U1 | All | OK |
| | | 4.265 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 66.84 | U1 | All | OK |
| | | 4.387 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 59.27 | U1 | All | OK |
| | | 4.826 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 26.41 | U1 | All | OK |
| | | 5.180 | 1800 | -128.51 | -9.14 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |
| | | 5.619 | 3200 | -222.51 | -75.92 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |
| | | 6.400 | 3200 | -222.51 | -217.93 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |
| | | 6.475 | 3200 | -222.51 | -233.01 | U1 | All | --- | 1800 | 128.51 | 0.00 | U1 | All | --- |
| | | 6.700 | 3200 | -222.51 | -279.70 | U1 | All | --- | 1800 | 128.51 | 0.00 | U1 | All | --- |
| | Middle | 0.000 | 1800 | -128.51 | 1.52 | U1 | All | --- | 1800 | 128.51 | 0.00 | U1 | All | --- |
| | | 0.300 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |

| Span | Strip | Top x | Top $A_{s,top}$ | Top $\Phi M_n$- | Top $M_u$- | Top Comb Pat | Top Status | Bottom $A_{s,bot}$ | Bottom $\Phi M_n$+ | Bottom $M_u$+ | Bottom Comb Pat | Bottom Status |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | m | mm² | kNm | kNm | | | mm² | kNm | kNm | | |
| | | 0.523 | 1800 | -128.51 | -0.35 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 1.342 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 31.07 | U1 All | OK |
| | | 1.642 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 44.36 | U1 All | OK |
| | | 2.435 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 66.37 | U1 All | OK |
| | | 2.978 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 70.49 | U1 All | OK |
| | | 3.350 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 68.15 | U1 All | OK |
| | | 4.265 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 44.56 | U1 All | OK |
| | | 4.680 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 25.54 | U1 All | OK |
| | | 4.980 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 8.53 | U1 All | OK |
| | | 6.400 | 1800 | -128.51 | -54.48 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 6.700 | 1800 | -128.51 | -74.28 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| 3 | Column | 0.000 | 3200 | -222.51 | -285.27 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| | | 0.240 | 3200 | -222.51 | -232.53 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| | | 0.300 | 3200 | -222.51 | -219.90 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 1.237 | 3200 | -222.51 | -50.81 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 1.680 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 7.95 | U1 All | OK |
| | | 2.134 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 45.95 | U1 All | OK |
| | | 2.577 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 73.98 | U1 All | OK |
| | | 2.715 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 80.89 | U1 All | OK |
| | | 3.713 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 105.21 | U1 All | OK |
| | | 3.750 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 105.21 | U1 All | OK |
| | | 4.785 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 80.89 | U1 All | OK |
| | | 4.923 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 73.98 | U1 All | OK |
| | | 5.366 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 45.95 | U1 All | OK |
| | | 5.820 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 7.95 | U1 All | OK |
| | | 6.263 | 3200 | -222.51 | -50.81 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 7.200 | 3200 | -222.51 | -219.90 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 7.275 | 3200 | -222.51 | -235.73 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| | | 7.500 | 3200 | -222.51 | -285.27 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| | Middle | 0.000 | 1800 | -128.51 | -71.32 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| | | 0.300 | 1800 | -128.51 | -54.97 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 1.767 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 10.63 | U1 All | OK |
| | | 2.067 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 27.27 | U1 All | OK |
| | | 2.715 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 53.93 | U1 All | OK |
| | | 3.713 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 70.14 | U1 All | OK |
| | | 3.750 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 70.14 | U1 All | OK |
| | | 4.785 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 53.93 | U1 All | OK |
| | | 5.433 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 27.27 | U1 All | OK |
| | | 5.733 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 10.63 | U1 All | OK |
| | | 7.200 | 1800 | -128.51 | -54.97 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 7.500 | 1800 | -128.51 | -71.32 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| 4 | Column | 0.000 | 3200 | -222.51 | -279.70 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| | | 0.240 | 3200 | -222.51 | -229.98 | U1 All | --- | 1800 | 128.51 | 0.00 | U1 All | --- |
| | | 0.300 | 3200 | -222.51 | -217.93 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 1.081 | 3200 | -222.51 | -75.92 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 1.520 | 1800 | -128.51 | -9.14 | U1 All | OK | 1800 | 128.51 | 0.00 | U1 All | OK |
| | | 1.874 | 1800 | -128.51 | 0.00 | U1 All | OK | 1800 | 128.51 | 26.41 | U1 All | OK |
| | | 2.313 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 59.27 | U1 All | OK |
| | | 2.435 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 66.84 | U1 All | OK |
| | | 3.350 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 102.23 | U1 All | OK |
| | | 3.722 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 105.74 | U1 All | OK |
| | | 4.265 | 0 | 0.00 | 0.00 | U1 All | OK | 1800 | 128.51 | 99.56 | U1 All | OK |

| Span | Strip | x (m) | $A_{s,top}$ (mm²) | $\Phi M_n-$ (kNm) | $M_u-$ (kNm) | Comb | Pat | Status | $A_{s,bot}$ (mm²) | $\Phi M_n+$ (kNm) | $M_u+$ (kNm) | Comb | Pat | Status |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 4.387 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 96.36 | U1 | All | OK |
| | | 4.687 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 85.56 | U1 | All | OK |
| | | 5.180 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 58.91 | U1 | All | OK |
| | | 5.480 | 2400 | -169.43 | 0.00 | U1 | All | OK | 1800 | 128.51 | 37.31 | U1 | All | OK |
| | | 6.400 | 2400 | -169.43 | -90.64 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |
| | | 6.700 | 2400 | -169.43 | -155.86 | U1 | All | --- | 1800 | 128.51 | 0.00 | U1 | All | --- |
| | Middle | 0.000 | 1800 | -128.51 | -74.28 | U1 | All | --- | 1800 | 128.51 | 0.00 | U1 | All | --- |
| | | 0.300 | 1800 | -128.51 | -54.48 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |
| | | 1.720 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 8.53 | U1 | All | OK |
| | | 2.020 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 25.54 | U1 | All | OK |
| | | 2.435 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 44.56 | U1 | All | OK |
| | | 3.350 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 68.15 | U1 | All | OK |
| | | 3.722 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 70.49 | U1 | All | OK |
| | | 4.265 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 66.37 | U1 | All | OK |
| | | 5.058 | 0 | 0.00 | 0.00 | U1 | All | OK | 1800 | 128.51 | 44.36 | U1 | All | OK |
| | | 5.358 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 31.07 | U1 | All | OK |
| | | 6.177 | 1800 | -128.51 | -0.35 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |
| | | 6.400 | 1800 | -128.51 | 0.00 | U1 | All | OK | 1800 | 128.51 | 0.00 | U1 | All | OK |
| | | 6.700 | 1800 | -128.51 | 1.52 | U1 | All | --- | 1800 | 128.51 | 0.00 | U1 | All | --- |
| 5 | Column | 0.000 | 2400 | -169.43 | -11.81 | U1 | All | --- | 0 | 0.00 | 0.00 | U1 | All | --- |
| | | 0.070 | 2400 | -169.43 | -9.19 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.186 | 2323 | -164.23 | -5.37 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.200 | 2313 | -163.58 | -4.95 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.285 | 2257 | -159.76 | -2.63 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.370 | 2200 | -155.90 | -0.63 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.400 | 2200 | -155.90 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | Middle | 0.000 | 1800 | -128.51 | 0.00 | U1 | All | --- | 0 | 0.00 | 0.00 | U1 | All | --- |
| | | 0.070 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.186 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.200 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.285 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |
| | | 0.400 | 1800 | -128.51 | 0.00 | U1 | All | OK | 0 | 0.00 | 0.00 | U1 | All | OK |

### *2.10. Slab Shear Capacity*

| Span | b (mm) | $d_v$ (mm) | β | $V_{ratio}$ | $\Phi V_c$ (kN) | $V_u$ (kN) | $X_u$ (m) |
|---|---|---|---|---|---|---|---|
| 1 | 6600 | 195 | 0.210 | 1.000 | 879.73 | 24.66 | 0.10 |
| 2 | 6600 | 195 | 0.210 | 1.000 | 879.73 | 245.78 | 6.20 |
| 3 | 6600 | 195 | 0.210 | 1.000 | 879.73 | 246.25 | 7.00 |
| 4 | 6600 | 195 | 0.210 | 1.000 | 879.73 | 245.78 | 0.50 |
| 5 | 6600 | 195 | 0.210 | 1.000 | 879.73 | 24.66 | 0.30 |

### *2.11. Flexural Transfer of Negative Unbalanced Moment at Supports*

| Support | Width (mm) | Width-c (mm) | d (mm) | $M_{unb}$ (kNm) | Comb | Patt | $\gamma_f$ | $A_{s,req}$ (mm²) | $A_{s,prov}$ (mm²) | Add Bars |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1150 | 1150 | 217 | 142.53 | U1 | All | 0.567 | 1166 | 1400 | --- |
| 2 | 1150 | 1150 | 217 | 2.60 | U1 | All | 0.566 | 20 | 1400 | --- |
| 3 | 1150 | 1150 | 217 | 2.60 | U1 | All | 0.566 | 20 | 1400 | --- |
| 4 | 1150 | 1150 | 217 | 142.53 | U1 | All | 0.567 | 1166 | 1400 | --- |

## *2.12. Punching Shear Around Columns*

### 2.12.1. Critical Section Properties

| Support | Type | $b_1$ mm | $b_2$ mm | $b_0$ mm | $d_{avg}$ mm | CG mm | $c_{(left)}$ mm | $c_{(right)}$ mm | $A_c$ mm² | $J_c$ mm⁴ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Rect | 808.5 | 617.0 | 2234.0 | 217.0 | 115.9 | 515.9 | 292.6 | 4.8478e+005 | 3.6328e+010 |
| 2 | Rect | 817.0 | 617.0 | 2868.0 | 217.0 | 0.0 | 408.5 | 408.5 | 6.2236e+005 | 6.5799e+010 |
| 3 | Rect | 817.0 | 617.0 | 2868.0 | 217.0 | 0.0 | 408.5 | 408.5 | 6.2236e+005 | 6.5799e+010 |
| 4 | Rect | 808.5 | 617.0 | 2234.0 | 217.0 | -115.9 | 292.6 | 515.9 | 4.8478e+005 | 3.6328e+010 |

### 2.12.2. Punching Shear Results

| Support | $V_u$ kN | $v_u$ N/mm² | $M_{unb}$ kNm | Comb | Patt | $\gamma_v$ | $v_u$ N/mm² | $\Phi V_c$ N/mm² |
|---|---|---|---|---|---|---|---|---|
| 1 | 258.19 | 0.533 | 112.60 | U1 | All | 0.433 | 0.925 | 1.235 |
| 2 | 561.20 | 0.902 | 2.60 | U1 | All | 0.434 | 0.909 | 1.235 |
| 3 | 561.20 | 0.902 | -2.60 | U1 | All | 0.434 | 0.909 | 1.235 |
| 4 | 258.19 | 0.533 | -112.60 | U1 | All | 0.433 | 0.925 | 1.235 |

## *2.13. Integrity Reinforcement at Supports*

Notes:
The sum of bottom reinforcement crossing the perimeter of the support on all sides shall not be less than the below listed values.

| Support | $V_{se}$ kN | $A_{sb}$ mm² |
|---|---|---|
| 1 | 254.56 | 1273 |
| 2 | 577.31 | 2887 |
| 3 | 577.31 | 2887 |
| 4 | 254.56 | 1273 |

## *2.14. Material TakeOff*

### 2.14.1. Reinforcement in the Direction of Analysis

| | | | | | |
|---|---|---|---|---|---|
| Top Bars | 476.4 kg | <=> | 21.95 kg/m | <=> | 3.326 kg/m² |
| Bottom Bars | 590.6 kg | <=> | 27.22 kg/m | <=> | 4.124 kg/m² |
| Stirrups | 0.0 kg | <=> | 0.00 kg/m | <=> | 0.000 kg/m² |
| Total Steel | 1067.0 kg | <=> | 49.17 kg/m | <=> | 7.450 kg/m² |
| Concrete | 35.8 m³ | <=> | 1.65 m³/m | <=> | 0.250 m³/m² |

# 3. Deflection Results: Summary

## *3.1. Section Properties*

### 3.1.1. Frame Section Properties

Notes:
M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

| Span | Zone | $M_{+ve}$ $I_g$ mm⁴ | $M_{+ve}$ $I_{cr}$ mm⁴ | $M_{+ve}$ $M_{cr}$ kNm | $M_{-ve}$ $I_g$ mm⁴ | $M_{-ve}$ $I_{cr}$ mm⁴ | $M_{-ve}$ $M_{cr}$ kNm |
|---|---|---|---|---|---|---|---|
| 1 | Left | 8.5938e+009 | 0 | 103.13 | 8.5938e+009 | 1.1434e+009 | -103.13 |
| | Midspan | 8.5938e+009 | 0 | 103.13 | 8.5938e+009 | 1.1713e+009 | -103.13 |
| | Right | 8.5938e+009 | 0 | 103.13 | 8.5938e+009 | 1.1926e+009 | -103.13 |
| 2 | Left | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 1.1926e+009 | -103.13 |
| | Midspan | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 0 | -103.13 |
| | Right | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 1.3844e+009 | -103.13 |
| 3 | Left | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 1.3844e+009 | -103.13 |
| | Midspan | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 0 | -103.13 |

| Span | Zone | $M_{+ve}$ $I_g$ mm⁴ | $M_{+ve}$ $I_{cr}$ mm⁴ | $M_{+ve}$ $M_{cr}$ kNm | $M_{-ve}$ $I_g$ mm⁴ | $M_{-ve}$ $I_{cr}$ mm⁴ | $M_{-ve}$ $M_{cr}$ kNm |
|---|---|---|---|---|---|---|---|
| | Right | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 1.3844e+009 | -103.13 |
| 4 | Left | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 1.3844e+009 | -103.13 |
| | Midspan | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 0 | -103.13 |
| | Right | 8.5938e+009 | 1.0435e+009 | 103.13 | 8.5938e+009 | 1.1926e+009 | -103.13 |
| 5 | Left | 8.5938e+009 | 0 | 103.13 | 8.5938e+009 | 1.1926e+009 | -103.13 |
| | Midspan | 8.5938e+009 | 0 | 103.13 | 8.5938e+009 | 1.1713e+009 | -103.13 |
| | Right | 8.5938e+009 | 0 | 103.13 | 8.5938e+009 | 1.1434e+009 | -103.13 |

### 3.1.2. Frame Effective Section Properties

| Span | Zone | Weight | Load Level: Dead $M_{max}$ kNm | Dead $I_e$ mm⁴ | Sustained $M_{max}$ kNm | Sustained $I_e$ mm⁴ | Dead+Live $M_{max}$ kNm | Dead+Live $I_e$ mm⁴ |
|---|---|---|---|---|---|---|---|---|
| 1 | Right | 1.000 | -9.45 | 8.5938e+009 | -9.45 | 8.5938e+009 | -9.45 | 8.5938e+009 |
| | Span Avg | ---- | ---- | 8.5938e+009 | ---- | 8.5938e+009 | ---- | 8.5938e+009 |
| 2 | Left | 0.150 | 97.92 | 8.5938e+009 | 97.92 | 8.5938e+009 | 125.53 | 5.2291e+009 |
| | Middle | 0.700 | 105.51 | 8.0935e+009 | 105.51 | 8.0935e+009 | 135.07 | 4.4034e+009 |
| | Right | 0.150 | -212.42 | 2.2093e+009 | -212.42 | 2.2093e+009 | -271.39 | 1.7799e+009 |
| | Span Avg | ---- | ---- | 7.2859e+009 | ---- | 7.2859e+009 | ---- | 4.1338e+009 |
| 3 | Left | 0.150 | -214.24 | 2.1884e+009 | -214.24 | 2.1884e+009 | -273.43 | 1.7712e+009 |
| | Middle | 0.700 | 105.51 | 8.0941e+009 | 105.51 | 8.0941e+009 | 134.48 | 4.4481e+009 |
| | Right | 0.150 | -214.24 | 2.1884e+009 | -214.24 | 2.1884e+009 | -273.43 | 1.7712e+009 |
| | Span Avg | ---- | ---- | 6.3224e+009 | ---- | 6.3224e+009 | ---- | 3.645e+009 |
| 4 | Left | 0.150 | -212.42 | 2.2093e+009 | -212.42 | 2.2093e+009 | -271.39 | 1.7799e+009 |
| | Middle | 0.700 | 105.51 | 8.0935e+009 | 105.51 | 8.0935e+009 | 135.07 | 4.4034e+009 |
| | Right | 0.150 | 97.92 | 8.5938e+009 | 97.92 | 8.5938e+009 | 125.53 | 5.2291e+009 |
| | Span Avg | ---- | ---- | 7.2859e+009 | ---- | 7.2859e+009 | ---- | 4.1338e+009 |
| 5 | Left | 1.000 | -9.45 | 8.5938e+009 | -9.45 | 8.5938e+009 | -9.45 | 8.5938e+009 |
| | Span Avg | ---- | ---- | 8.5938e+009 | ---- | 8.5938e+009 | ---- | 8.5938e+009 |

### 3.1.3. Strip Section Properties at Midspan

Notes:
Load distirubtion factor, LDL, averages moment distribution factors listed in Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

| Span | Column Strip $I_g$ mm⁴ | Column Strip LDF | Column Strip Ratio | Middle Strip $I_g$ mm⁴ | Middle Strip LDF | Middle Strip Ratio |
|---|---|---|---|---|---|---|
| 1 | 4.29688e+009 | 0.800 | 1.600 | 4.29688e+009 | 0.200 | 0.400 |
| 2 | 4.29688e+009 | 0.750 | 1.500 | 4.29688e+009 | 0.250 | 0.500 |
| 3 | 4.29688e+009 | 0.700 | 1.400 | 4.29688e+009 | 0.300 | 0.600 |
| 4 | 4.29688e+009 | 0.750 | 1.500 | 4.29688e+009 | 0.250 | 0.500 |
| 5 | 4.29688e+009 | 0.800 | 1.600 | 4.29688e+009 | 0.200 | 0.400 |

## *3.2. Instantaneous Deflections*

### 3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

| Span | Direction | Value | Units | Dead | Live Sustained | Live Unsustained | Live Total | Total Sustained | Total Dead+Live |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Down | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| | Up | Def | mm | -0.25 | --- | -0.15 | -0.15 | -0.25 | -0.40 |
| | | Loc | m | 0.000 | --- | 0.000 | 0.000 | 0.000 | 0.000 |

| Span | Direction | Value | Units | Dead | Live Sustained | Live Unsustained | Live Total | Total Sustained | Total Dead+Live |
|---|---|---|---|---|---|---|---|---|---|
| 2 | Down | Def | mm | 1.76 | --- | 1.53 | 1.53 | 1.76 | 3.29 |
| | | Loc | m | 3.052 | --- | 3.127 | 3.127 | 3.052 | 3.052 |
| | Up | Def | mm | --- | --- | -0.01 | -0.01 | --- | 0.00 |
| | | Loc | m | --- | --- | 6.475 | 6.475 | --- | 6.625 |
| 3 | Down | Def | mm | 2.20 | --- | 2.31 | 2.31 | 2.20 | 4.50 |
| | | Loc | m | 3.713 | --- | 3.713 | 3.713 | 3.713 | 3.713 |
| | Up | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| 4 | Down | Def | mm | 1.76 | --- | 1.53 | 1.53 | 1.76 | 3.29 |
| | | Loc | m | 3.648 | --- | 3.573 | 3.573 | 3.648 | 3.648 |
| | Up | Def | mm | 0.00 | --- | -0.01 | -0.01 | 0.00 | 0.00 |
| | | Loc | m | 0.060 | --- | 0.240 | 0.240 | 0.060 | 0.060 |
| 5 | Down | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| | Up | Def | mm | -0.25 | --- | -0.15 | -0.15 | -0.25 | -0.40 |
| | | Loc | m | 0.400 | --- | 0.400 | 0.400 | 0.400 | 0.400 |

### 3.2.2. Extreme Instantaneous Column Strip Deflections and Corresponding Locations

| Span | Direction | Value | Units | Dead | Live Sustained | Live Unsustained | Live Total | Total Sustained | Total Dead+Live |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Down | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| | Up | Def | mm | -0.25 | --- | -0.15 | -0.15 | -0.25 | -0.40 |
| | | Loc | m | 0.000 | --- | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | Down | Def | mm | 2.36 | --- | 2.14 | 2.14 | 2.36 | 4.49 |
| | | Loc | m | 3.127 | --- | 3.201 | 3.201 | 3.127 | 3.127 |
| | Up | Def | mm | --- | --- | 0.00 | 0.00 | --- | 0.00 |
| | | Loc | m | --- | --- | 6.550 | 6.550 | --- | 6.625 |
| 3 | Down | Def | mm | 3.06 | --- | 3.18 | 3.18 | 3.06 | 6.24 |
| | | Loc | m | 3.713 | --- | 3.713 | 3.713 | 3.713 | 3.713 |
| | Up | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| 4 | Down | Def | mm | 2.36 | --- | 2.14 | 2.14 | 2.36 | 4.49 |
| | | Loc | m | 3.573 | --- | 3.499 | 3.499 | 3.573 | 3.573 |
| | Up | Def | mm | --- | --- | 0.00 | 0.00 | --- | 0.00 |
| | | Loc | m | --- | --- | 0.180 | 0.180 | --- | 0.060 |
| 5 | Down | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| | Up | Def | mm | -0.25 | --- | -0.15 | -0.15 | -0.25 | -0.40 |
| | | Loc | m | 0.400 | --- | 0.400 | 0.400 | 0.400 | 0.400 |

### 3.2.3. Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

| Span | Direction | Value | Units | Dead | Live Sustained | Live Unsustained | Live Total | Total Sustained | Total Dead+Live |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Down | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| | Up | Def | mm | -0.25 | --- | -0.15 | -0.15 | -0.25 | -0.40 |
| | | Loc | m | 0.000 | --- | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | Down | Def | mm | 1.17 | --- | 0.94 | 0.94 | 1.17 | 2.10 |
| | | Loc | m | 2.829 | --- | 2.904 | 2.904 | 2.829 | 2.904 |
| | Up | Def | mm | 0.00 | --- | -0.01 | -0.01 | 0.00 | -0.01 |
| | | Loc | m | 6.625 | --- | 6.400 | 6.400 | 6.625 | 6.550 |
| 3 | Down | Def | mm | 1.34 | --- | 1.43 | 1.43 | 1.34 | 2.77 |

| Span | Direction | Value | Units | Dead | Live Sustained | Live Unsustained | Live Total | Total Sustained | Total Dead+Live |
|---|---|---|---|---|---|---|---|---|---|
| | | Loc | m | 3.713 | --- | 3.713 | 3.713 | 3.713 | 3.713 |
| | Up | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| 4 | Down | Def | mm | 1.17 | --- | 0.94 | 0.94 | 1.17 | 2.10 |
| | | Loc | m | 3.871 | --- | 3.796 | 3.796 | 3.871 | 3.796 |
| | Up | Def | mm | 0.00 | --- | -0.01 | -0.01 | 0.00 | -0.01 |
| | | Loc | m | 0.060 | --- | 0.300 | 0.300 | 0.060 | 0.120 |
| 5 | Down | Def | mm | --- | --- | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- | --- | --- |
| | Up | Def | mm | -0.25 | --- | -0.15 | -0.15 | -0.25 | -0.40 |
| | | Loc | m | 0.400 | --- | 0.400 | 0.400 | 0.400 | 0.400 |

### *3.3. Long-term Deflections*

### 3.3.1. Long-term Column Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

| Span | Zone | M+ve $A_{s,top}$ (mm²) | M+ve b (mm) | M+ve d (mm) | M+ve Rho' (%) | M+ve Lambda | M-ve $A_{s,bot}$ (mm²) | M-ve b (mm) | M-ve d (mm) | M-ve Rho' (%) | M-ve Lambda |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Right | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |
| 2 | Midspan | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |
| 3 | Midspan | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |
| 4 | Midspan | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |
| 5 | Left | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |

### 3.3.2. Long-term Middle Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

| Span | Zone | M+ve $A_{s,top}$ (mm²) | M+ve b (mm) | M+ve d (mm) | M+ve Rho' (%) | M+ve Lambda | M-ve $A_{s,bot}$ (mm²) | M-ve b (mm) | M-ve d (mm) | M-ve Rho' (%) | M-ve Lambda |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Right | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |
| 2 | Midspan | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |
| 3 | Midspan | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |
| 4 | Midspan | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |
| 5 | Left | ---- | ---- | ---- | 0.000 | 2.000 | ---- | ---- | ---- | 0.000 | 2.000 |

### 3.3.3. Extreme Long-term Column Strip Deflections and Corresponding Locations

Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

| Span | Direction | Value | Units | cs | cs+lu | cs+l | Total |
|---|---|---|---|---|---|---|---|
| 1 | Down | Def | mm | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- |

| Span | Direction | Value | Units | cs | cs+lu | cs+l | Total |
|---|---|---|---|---|---|---|---|
| | Up | Def | mm | -0.50 | -0.65 | -0.65 | -0.90 |
| | | Loc | m | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | Down | Def | mm | 4.71 | 6.85 | 6.85 | 9.20 |
| | | Loc | m | 3.127 | 3.127 | 3.127 | 3.127 |
| | Up | Def | mm | --- | 0.00 | 0.00 | --- |
| | | Loc | m | --- | 6.625 | 6.625 | --- |
| 3 | Down | Def | mm | 6.11 | 9.30 | 9.30 | 12.35 |
| | | Loc | m | 3.713 | 3.713 | 3.713 | 3.713 |
| | Up | Def | mm | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- |
| 4 | Down | Def | mm | 4.71 | 6.85 | 6.85 | 9.20 |
| | | Loc | m | 3.573 | 3.573 | 3.573 | 3.573 |
| | Up | Def | mm | --- | 0.00 | 0.00 | 0.00 |
| | | Loc | m | --- | 0.060 | 0.060 | 0.060 |
| 5 | Down | Def | mm | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- |
| | Up | Def | mm | -0.50 | -0.65 | -0.65 | -0.90 |
| | | Loc | m | 0.400 | 0.400 | 0.400 | 0.400 |

### 3.3.4. Extreme Long-term Middle Strip Deflections and Corresponding Locations

Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.

Total deflections consist of dead, live, and creep and shrinkage deflections.

| Span | Direction | Value | Units | cs | cs+lu | cs+l | Total |
|---|---|---|---|---|---|---|---|
| 1 | Down | Def | mm | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- |
| | Up | Def | mm | -0.50 | -0.65 | -0.65 | -0.90 |
| | | Loc | m | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | Down | Def | mm | 2.33 | 3.27 | 3.27 | 4.44 |
| | | Loc | m | 2.829 | 2.829 | 2.829 | 2.829 |
| | Up | Def | mm | 0.00 | 0.00 | 0.00 | -0.01 |
| | | Loc | m | 6.625 | 6.625 | 6.625 | 6.625 |
| 3 | Down | Def | mm | 2.68 | 4.10 | 4.10 | 5.44 |
| | | Loc | m | 3.713 | 3.713 | 3.713 | 3.713 |
| | Up | Def | mm | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- |
| 4 | Down | Def | mm | 2.33 | 3.27 | 3.27 | 4.44 |
| | | Loc | m | 3.871 | 3.871 | 3.871 | 3.871 |
| | Up | Def | mm | 0.00 | 0.00 | 0.00 | -0.01 |
| | | Loc | m | 0.060 | 0.120 | 0.120 | 0.060 |
| 5 | Down | Def | mm | --- | --- | --- | --- |
| | | Loc | m | --- | --- | --- | --- |
| | Up | Def | mm | -0.50 | -0.65 | -0.65 | -0.90 |
| | | Loc | m | 0.400 | 0.400 | 0.400 | 0.400 |

## 6. Summary and Comparison of Two-Way Slab Design Results

| Table 5 – Summary of Flexural Design Moments | | | | |
|---|---|---|---|---|
| | | Reference (DDM) | Hand (EFM) | spSlab |
| Exterior Span | | | | |
| Frame Strip | Exterior Negative | 93 | 90.3 | 90.6 |
| | Positive | 185 | 171.1** | 176.2* (170.4)** |
| | Interior Negative | 249 | 273.6 | 272.1 |
| Interior Span | | | | |
| Frame Strip | Interior Negative | 296 | 276.2 | 274.9 |
| | Positive | 160 | 175.9 | 175.4 |

* Maximum positive moment along exterior span (not at midspan)
** Positive moment at the middle of the exterior span

For the Table 5, the negative moments are taken at the supports faces. Note that for the exterior span, the location of the maximum positive moment is not located at the mid span. the hand solution assumed that the maximum positive moment is located at the midspan for simplification. On the other hand, spSlab program provides the exact location of the maximum positive moment.

| Table 6 - Summary of Reinforcement Provided | | | | | |
|---|---|---|---|---|---|
| Span | Support | Strip | Reinforcement Provided* | | |
| | | | Reference** | Hand | spSlab |
| Exterior Span | Exterior Negative | $b_b$ | 7-15M | 7-15M | 7-15M |
| | | rest of Column Strip | 14-15M | 5-15M | 5-15M |
| | | Middle Strip | | 9-15M | 9-15M |
| | | Total | 21-15M | 21-15M | 21-15M |
| | Positive | Column Strip | --- | 9-15M | 9-15M |
| | | Middle Strip | | 9-15M | 9-15M |
| | | Total | 17-15M | 18-15M | 18-15M |
| | Interior Negative | $b_b$ | --- | 7-15M | 7-15M |
| | | rest of Column Strip | | 9-15M | 9-15M |
| | | Middle Strip | | 9-15M | 9-15M |
| | | Total | --- | 25-15M | 25-15M |
| Interior Span | Negative | $b_b$ | 7-15M | 7-15M | 7-15M |
| | | rest of Column Strip | 14-15M | 9-15M | 9-15M |
| | | Middle Strip | | 9-15M | 9-15M |
| | | Total | 21-15M | 25-15M | 25-15M |
| | Positive | Column Strip | --- | 9-15M | 9-15M |
| | | Middle Strip | | 9-15M | 9-15M |
| | | Total | 17-15M | 18-15M | 18-15M |

In Table 6, the reference calculated the reinforcement required and provided based on design strip taking into account the band width $b_b$ and the rest of the design strip. On the other hand, hand and spSlab calculations considered all strips specified by the code (column strip including the band width $b_b$ and middle strip). Reinforcement provided by the reference are based on adjusted moment values using moment redistribution that is applicable for DDM.

Table 5 and table 6 show comparison between the reference results using DDM and hand solution and spSlab results using EFM. Note that the use of DDM is limited and the reference made several simplifying assumptions to make the use of DDM valid for this calculations:

- Exclude the exterior cladding panels weight (2.4 kN/m).
- Exclude the slab projection that supports the cladding panels.
- The use of averaged reinforcement effective depth for shear calculations.
- Using the tributary method and assuming that half of the total load is transferred to the interior column.

The main reason of the slight differences between the reference results and the hand and spSlab results is the use of different analysis techniques (DDM by reference and EFM by hand and spSlab). Additionally, the reinforcement provided by the reference are based on adjusted moment values using moment redistribution that is applicable for DDM.

**Table 7 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution**

| | | Hand (EFM) | spSlab |
|---|---|---|---|
| **Exterior Span** | | | |
| Column Strip | Exterior Negative* | 90.3 | 90.6 |
| | Positive | 102.7 | 107.7 |
| | Interior Negative* | 218.9 | 217.9 |
| Middle Strip | Exterior Negative* | 0.0 | 0 |
| | Positive | 68.4 | 70.5 |
| | Interior Negative* | 54.7 | 54.5 |
| **Interior Span** | | | |
| Column Strip | Interior Negative* | 221.0 | 219.9 |
| | Positive | 105.5 | 105.2 |
| Middle Strip | Interior Negative* | 55.2 | 55.0 |
| | Positive | 70.4 | 70.1 |

* negative moments are taken at the faces of supports

| Table 8 - Comparison of Reinforcement Results with Hand and spSlab Solution | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Span Location** | | **Reinforcement Provided for Flexure** | | **Additional Reinforcement Provided for Unbalanced Moment Transfer*** | | **Total Reinforcement Provided** | |
| | | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** |
| **Exterior Span** | | | | | | | |
| Column Strip | Exterior Negative | 12-15M | 12-15M | --- | --- | 12-15M | 12-15M |
| | Positive | 9-15M | 9-15M | n/a | n/a | 9-15M | 9-15M |
| | Interior Negative | 16-15M | 16-15M | --- | --- | 16-15M | 16-15M |
| Middle Strip | Exterior Negative | 9-15M | 9-15M | n/a | n/a | 9-15M | 9-15M |
| | Positive | 9-15M | 9-15M | n/a | n/a | 9-15M | 9-15M |
| | Interior Negative | 9-15M | 9-15M | n/a | n/a | 9-15M | 9-15M |
| **Interior Span** | | | | | | | |
| Column Strip | Negative | 16-15M | 16-15M | --- | --- | 16-15M | 16-15M |
| | Positive | 9-15M | 9-15M | n/a | n/a | 9-15M | 9-15M |
| Middle Strip | Negative | 9-15M | 9-15M | --- | --- | 9-15M | 9-15M |
| | Positive | 9-15M | 9-15M | n/a | n/a | 9-15M | 9-15M |

* In the EFM, the unbalanced moment ($M_{sc}$, $M_{unb}$) at the support centerline is used to determine the value of the additional reinforcement as compared with DDM using the moments at the face of support.

| Table 9 - Comparison of One-Way (Beam Action) Shear Check Results Using Hand and spSlab Solution | | | | |
|---|---|---|---|---|
| **Span** | **$V_u$ , kN** | | **$\varphi V_c$, kN** | |
| | **Hand** | **spSlab** | **Hand** | **spSlab** |
| Exterior | 245.7 | 245.8 | 878.4 | 879.7 |
| Interior | 241.4 | 246.2 | 878.4 | 879.7 |

| Table 10 - Comparison of Two-Way (Punching) Shear Check Results Using Hand and spSlab Solution | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Support** | **$b_1$, mm** | | **$b_2$, mm** | | **$b_o$, mm** | | **$A_c$, mm$^2$** | | **$V_u$, kN** | | **$v_u$, kN/mm$^2$** | |
| | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** |
| Exterior | 808.5 | 808.5 | 617 | 617 | 2234 | 2234 | 484778 | 484778 | 218.5 | 258.2 | 0.45 | 0.53 |
| Interior | 817 | 817 | 617 | 617 | 2868 | 2868 | 622356 | 622356 | 563.1 | 561.2 | 0.90 | 0.90 |
| | | | | | | | | | | | | |
| **Support** | **$c_{AB}$, mm** | | **$J_c$, x $10^9$ mm$^4$** | | **$\gamma_v$** | | **$M_{unb}$, kN.m** | | **$v_u$, MPa** | | **$\varphi v_c$, MPa** | |
| | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** | **Hand** | **spSlab** |
| Exterior | 292.6 | 292.6 | 36.3 | 36.3 | 0.433 | 0.433 | 107.0 | 112.6 | 0.824 | 0.925 | 1.235 | 1.235 |
| Interior | 408.5 | 408.5 | 65.8 | 65.8 | 0.434 | 0.434 | 2.3 | 2.6 | 0.911 | 0.909 | 1.235 | 1.235 |

Tables 7-10 show detailed comparison between hand solution and spSlab solution. In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the spSlab model except for two-way shear stresses around exterior support (around 11% difference) due to the cladding effect on the small cantilever part which was considered in spSlab model and excluded from the hand calculations for simplification.

The following table shows the effect of including the cladding panels weights and slab projection in the hand solution using EFM on the two-way (punching) shear results.

**Table 11 – The Effect of including the Cladding Panels Weight and Slab Projection on the Punching Shear Results**

| Method of Solution | Slab Projection and Cladding Panels Weights | $V_u$, kips | $M_{ub}$, kip-ft | $v_u$, MPa |
|---|---|---|---|---|
| Hand | Excluded | 218.5 | 107 | 0.824 |
| | Included | 257.7 | 110.4 | 0.916 |
| spSlab | Included | 258.2 | 112.6 | 0.925 |

## 7. Comparison of Two-Way Slab Analysis and Design Methods

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in ***CSA A23.3-14 Clasues (13.8 and 13.9)*** for regular two-way slab systems. ***CSA A23.3-14 (13.5.1)***

Direct Design Method (DDM) is an approximate method and is applicable to flat plate concrete floor systems that meet the stringent requirements of ***CSA A23.3-14 (13.9.1)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Elastic Frame Method (EFM) has less stringent limitations compared to DDM. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StucturePoint's spSlab software program solution utilizes the EFM to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

| Applicable CSA A23.3-14 Provision | Limitations/Applicability | Concrete Slab Analysis Method | | |
|---|---|---|---|---|
| | | DDM (Hand) | EFM (Hand//spSlab) | FEM (spMats) |
| 13.8.1.1<br>13.9.1.1 | Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2. | ☑ | ☑ | |
| 13.8.1.1<br>13.9.1.1 | For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions. | ☑ | ☑ | |
| 13.8.1.1<br>13.9.1.1 | Column offset shall not exceed 20% of the span in direction of offset from either axis between centerlines of successive columns | ☑ | ☑ | |
| 13.8.1.1<br>13.9.1.1 | The reinforcement is placed in an orthogonal grid. | ☑ | ☑ | |
| 13.9.1.2 | Minimum of three continuous spans in each direction | ☑ | | |
| 13.9.1.3 | Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span | ☑ | | |
| 13.9.1.4 | All loads shall be due to gravity only | ☑ | | |
| 13.9.1.4 | All loads shall be uniformly distributed over an entire panel ($q_f$) | ☑ | | |
| 13.9.1.4 | Unfactored live load shall not exceed two times the unfactored dead load | ☑ | | |
| 13.10.6 | Structural integrity steel detailing | ☑ | ☑ | ☑ |
| 13.10.10 | Openings in slab systems | ☑ | ☑ | ☑ |
| 8.2 | Concentrated loads | Not permitted | ☑ | ☑ |
| 13.8.4.1 | Live load arrangement (Load Patterning) | Not required | Required | Engineering judgment required based on modeling technique |
| 13.10.2* | Reinforcement for unbalanced slab moment transfer to column ($M_{sc}$) | Moments @ support face | Moments @ support centerline | Engineering judgment required based on modeling technique |
| 13.8.2 | Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.) | Not permitted | Engineering judgment required | Engineering judgment required |
| Complexity | | Low | Average | Complex to very complex |
| Design time/costs | | Fast | Limited | Unpredictable/Costly |
| Design Economy | | Conservative (see detailed comparison with spSlab output) | Somewhat conservative | Unknown - highly dependent on modeling assumptions:<br>1. Linear vs. non-linear<br>2. Isotropic vs non-isotropic<br>3. Plate element choice<br>4. Mesh size and aspect ratio<br>5. Design & detailing features |
| General (Drawbacks) | | Very limited applications | Limited geometry | Limited guidance non-standard application (user dependent). Required significant engineering judgment |
| General (Advantages) | | Very limited analysis is required | Detailed analysis is required or via software (e.g. spSlab) | Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats) |

* The unbalanced slab moment transferred to the column $M_{sc}$ ($M_{unb}$) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain $M_{sc}$ ($M_{unb}$). In EFM where a frame analysis is used, moments at the column center line are used to obtain $M_{sc}$ ($M_{unb}$).