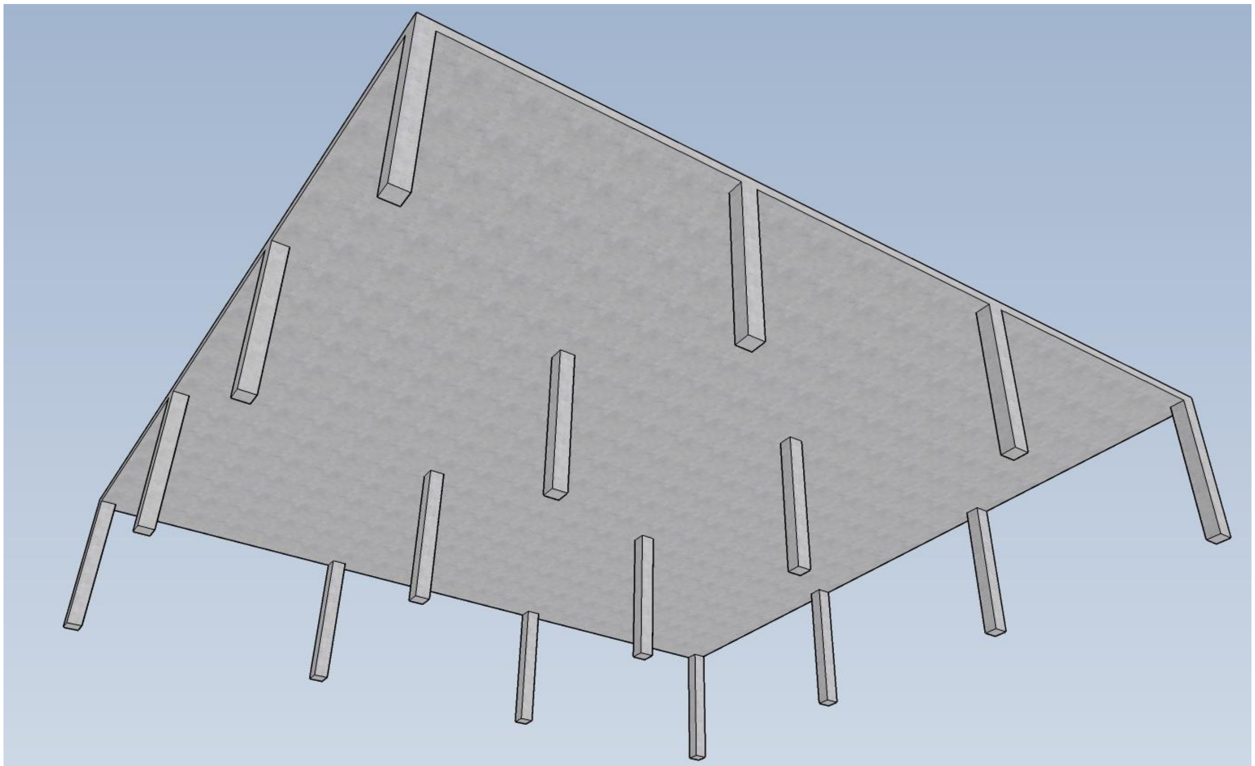
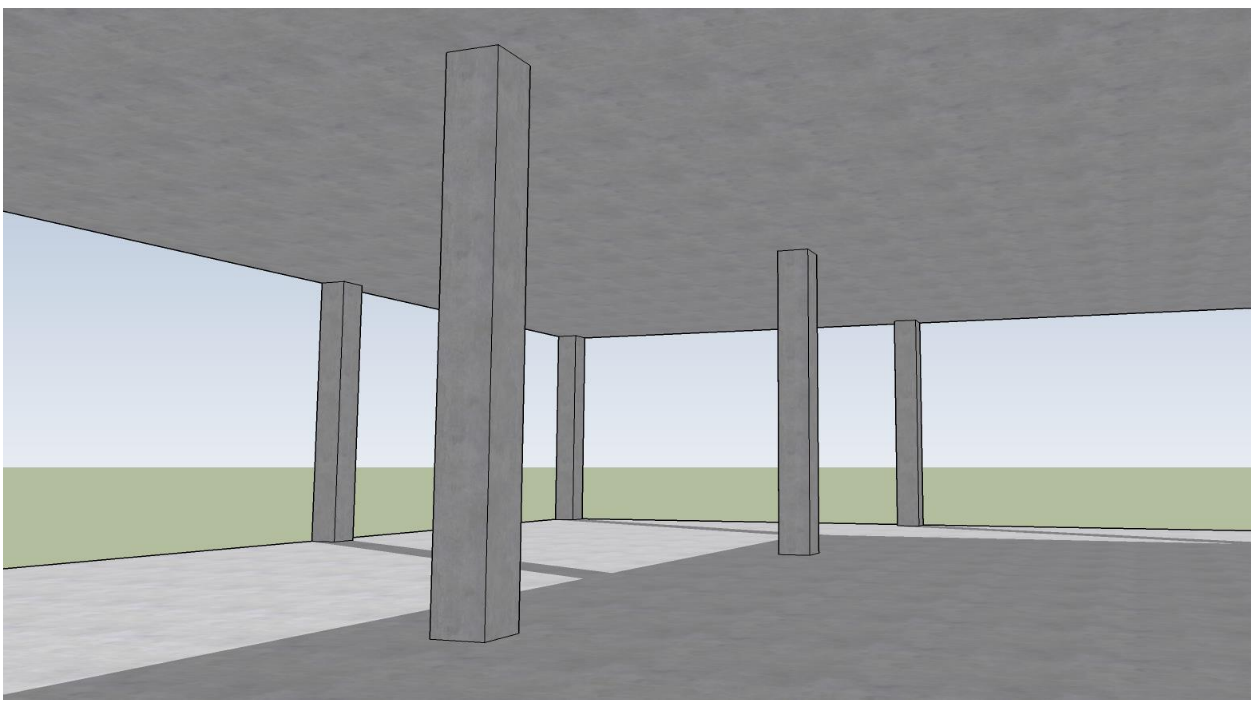
Two-Way Flat Plate Concrete Floor System Analysis and Design





Two-Way Flat Plate Concrete Floor System Analysis and Design

The concrete floor slab system shown below is for an intermediate floor to be designed considering partition weight = 1 kN/m^2, and unfactored live load = 1.9 kN/m^2 . Flat plate concrete floor system does not use beams between columns or drop panels and it is usually suited for lightly loaded floors with short spans typically for residential and hotel buildings. The lateral loads are independently resisted by shear walls. The two analysis procedures prescribed in ***CSA A23.3-14*** Direct Design Method (DDM) and Elastic Frame Method (EFM) are illustrated in detail in this example. The hand solution from EFM is also used for a detailed comparison with the analysis and design results of the engineering software program spSlab.

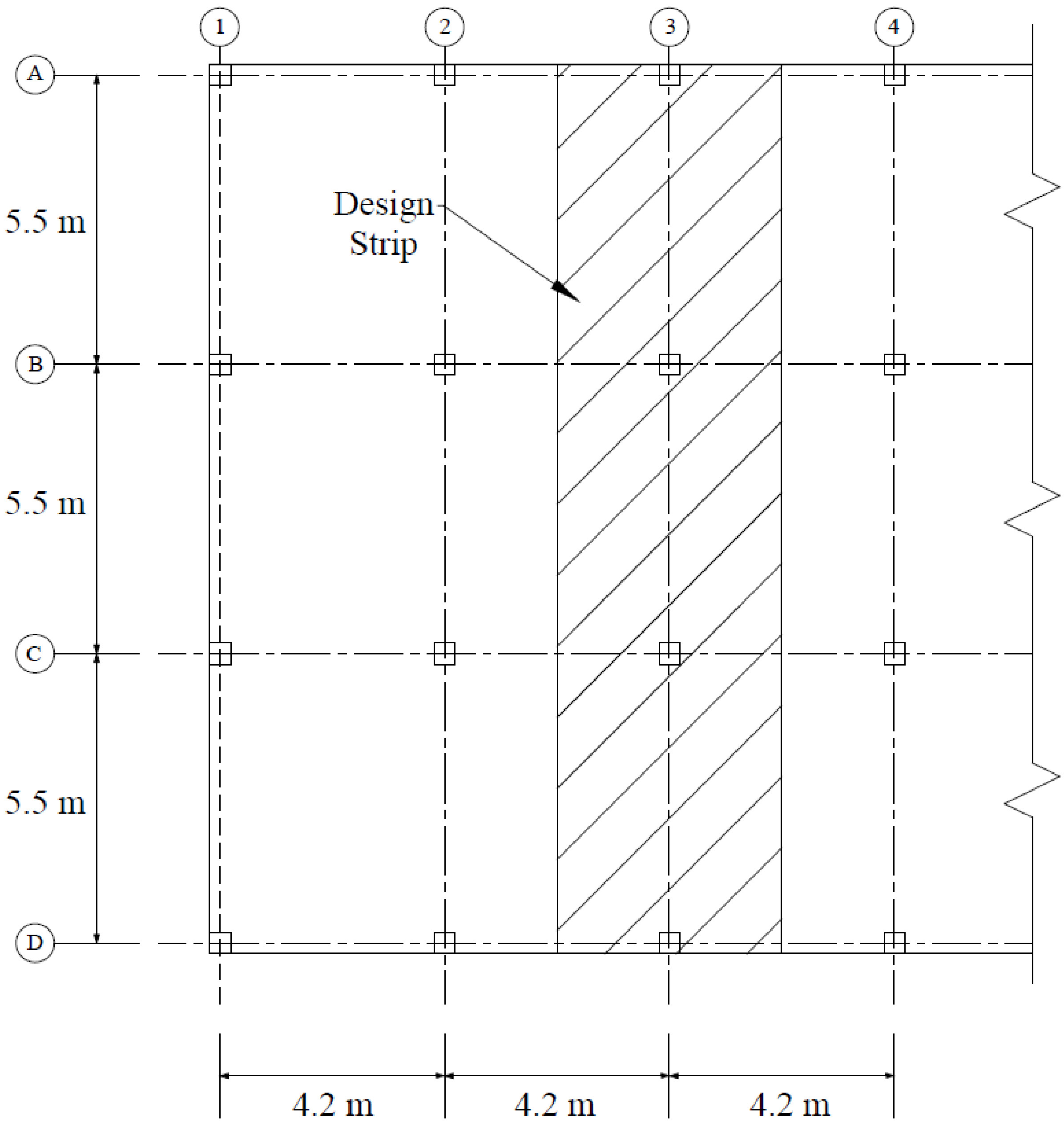


Figure 1 - Two-Way Flat Concrete Floor System

Contents

Code

Design of Concrete Structures (CSA A23.3-14)

Reference

CAC Concrete Design Handbook, 4th Edition, Cement Association of Canada

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association, Example 20.1

Design Data

Floor-to-Floor Height = 2.75 m (provided by architectural drawings)

Superimposed Dead Load, $\text{SDL} = 1 \text{ kN/m}^2$ for framed partitions, wood studs plaster 2 sides

Live Load, $\text{LL} = 1.9 \text{ kN/m}^2$ for Residential floors

$f'_c = 28$ MPa (for slabs)

$f'_c = 42$ MPa (for columns)

$f'_y = 400$ MPa

Required fire resistance rating = 2 hours

Solution

1. Preliminary Member Sizing

a. Slab minimum thickness - Deflection **_CSA A23.3-14 (13.2)_**

In this example deflection will be calculated and checked to satisfy project deflection limits. Minimum member thickness and depths from CSA A23.3-14 will be used for preliminary sizing.

Using CSA A23.3-14 minimum slab thickness for two-way construction without interior beams in ***Section 13.2.3.***

Exterior Panels: $h_{s,\min} = 1.1 \times \dfrac{l_n \left(0.6 + f_y / 1000\right)}{30} = 187 \text{ mm}$ **_CSA A23.3-14 (13.2.3)_**

But not less than 120 mm. **_CSA A23.3-14 (13.2.1)_**

Interior Panels: $h_{s,\min} = \dfrac{l_n \left(0.6 + f_y / 1000\right)}{30} = 170 \text{ mm}$ **_CSA A23.3-14 (13.2.3)_**

But not less than 120 mm. **_CSA A23.3-14 (13.2.1)_**

Where l_n = length of clear span in the long direction = 5500 – 400 = 5100 mm

Try 190 mm slab for all panels ($\text{self-weight} = 4.56 \text{ kN/m}^2$)

b. Slab shear strength – one way shear

Evaluate the average effective depth (Figure 2):

$$d_l = t_{slab} - c_{clear} - d_b - \frac{d_b}{2} = 190 - 20 - 16 - \frac{16}{2} = 146 \text{ mm}$$

$$d_t = t_{slab} - c_{clear} - \frac{d_b}{2} = 190 - 20 - \frac{16}{2} = 162 \text{ mm}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{146 + 162}{2} = 154 \text{ mm}$$

Where:

c_{clear} = 20 mm for 15M steel bar **_CSA A23.3-14 (Annex A. Table 17)_**

d_b = 16 mm for 15M steel bar

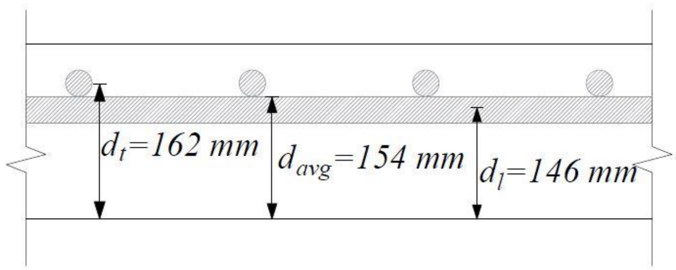


Figure 2 - Two-Way Flat Concrete Floor System

Load Combination 1:

Factored dead load, $w_{df} = 1.4 \times (4.56 + 1) = 7.78 \text{ kN/m}^2$ **_CSA A23.3-14 (Annex C. Table C.1 a)_**

Total factored load $w_f = 7.78 \text{ kN/m}^2$

Load Combination 2:

Factored dead load, $w_{df} = 1.25 \times (4.56 + 1) = 6.95 \text{ kN/m}^2$

Factored live load, $w_{lf} = 1.5 \times 1.9 = 2.85 \text{ kN/m}^2$ **_CSA A23.3-14 (Annex C. Table C.1 a)_**

Total factored load $w_f = 9.8 \text{ kN/m}^2$ (Controls)

Check the adequacy of slab thickness for beam action (one-way shear) **_CSA A23.3-14 (13.3.6)_**

at an interior column:

The critical section for one-way shear is extending in a plane across the entire width and located at a distance, d_v from the face of support or concentrated load (see Figure 3). **_CSA A23.3-14 (13.3.6.1)_**

Consider a 1 m. wide strip.

Tributary area for one-way shear is $A_{Tributary} = \left(\dfrac{\left[\left(\dfrac{5,500}{2} \right) - \left(\dfrac{400}{2} \right) - 139 \right] \times (1,000)}{1,000^2} \right) = 2.41 \text{ m}^2$

$V_f = w_f \times A_{Tributary} = 9.8 \times 2.41 = 23.63 \text{ kN}$

$V_c = \varphi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$ **CSA A23.3-14 (Eq. 11.6)**

Where:

$\lambda = 1$ for normal weight concrete **CSA A23.3-14 (8.6.5)**

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm **CSA A23.3-14 (11.3.6.2)**

$d_v = \text{Max } (0.9 d_{avg}, 0.72h) = \text{Max } (0.9 \times 154, 0.72 \times 190) = 139 \text{ mm}$ **CSA A23.3-14 (3.2)**

$\sqrt{f'_c} = 5.29 \text{ MPa} < 8 \text{ MPa}$ **CSA A23.3-14 (11.3.4)**

$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{28} \times 1,000 \times \dfrac{139}{1,000} = 100.4 \text{ kN } > V_u$

Slab thickness of 190 mm is adequate for one-way shear.

c. Slab shear strength – two-way shear

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 4):

Shear prerimeter: $b_0 = 2 \times (400 + 400 + 2 \times 154) = 2,216 \text{ mm}$ **CSA A23.3-14 (13.3.3)**

Tributary area for two-way shear is $A_{Tributary} = (5.5 \times 4.2) - \left(\dfrac{400 + 154}{1,000} \right)^2 = 22.79 \text{ m}^2$

The factored resisiting shear stress, V_r shall be the smallest of : **CSA A23.3-14 (13.3.4.1)**

a) $v_r = v_c = \left(1 + \dfrac{2}{\beta_c} \right) 0.19 \lambda \phi_c \sqrt{f'_c} = \left(1 + \dfrac{2}{1} \right) 0.19 \times 0.65 \times \sqrt{28} = 1.96 \text{ MPa}$

b) $v_r = v_c = \left(\dfrac{\alpha_s d}{b_o} + 0.19 \right) \lambda \phi_c \sqrt{f'_c} = \left(\dfrac{4 \times 154}{2216} + 0.19 \right) \times 1 \times 0.65 \times \sqrt{28} = 1.61 \text{ MPa}$

c) $v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{28} = 1.31 \text{ MPa}$

$V_{f,ave} = \dfrac{V_f}{b_o d} = \dfrac{223.37 \text{ kN}}{2,216 \times 154} \times 1,000 = 0.655 \text{ MPa}$

$$\frac{V_r}{V_{f,\,ave}} = 2 > 1.2$$ ***CAC Concrete Design Handbook 4th Edition (5.2.3)***

Slab thickness of 190 mm is adequate for two-way shear.

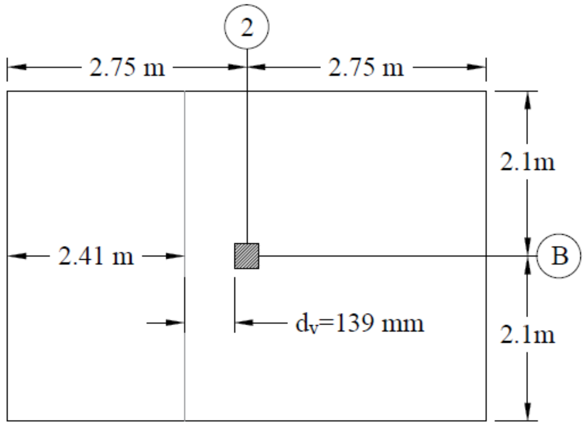


Figure 3 – Critical Section for One-Way

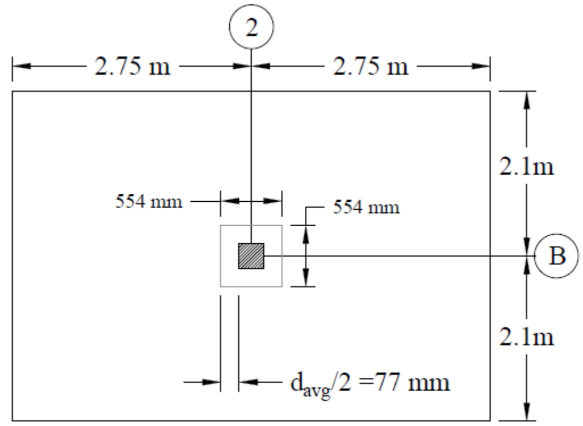


Figure 4 – Critical Section for Two-Way

d. Column dimensions - axial load

Check the adequacy of column dimensions for axial load:

Tributary area for interior column is $A_{Tributary} = (5.5 \times 4.2) = 23.1 \text{ m}^2$

$$P_f = w_f \times A_{Tributary} = 9.8 \times 23.1 = 226.38 \text{ kN}$$

$$P_{r,\max} = (0.2 + 0.002h)P_{ro} \le 0.80 P_{ro} \quad \text{(For tied column along full length)}$$ ***CSA A23.3-14 (Eq. 10.9)***

$$P_{ro} = \alpha_1\ \phi_c\ f'_c (A_g - A_{st} - A_t - A_p) + \phi_s\ f_y\ A_{st} + \phi_a F_y A_t - f_{pr} A_p$$ ***CSA A23.3-14 (Eq. 10.11)***

$$P_{ro} = 0.808 \times\ 0.65 \times\ 28 \times (400 \times 400 - 0) + 0.85 \times 420 \times 0 + 0 = 23{,}528{,}960 \text{ N} = 2{,}352.9 \text{ kN}$$

$$P_{r,\max} = (0.2 + 0.002 \times 400) \times 2{,}352.9 \le 0.80 \times 2{,}352.9$$

$$P_{r,\max} = 1{,}882.32 \text{ kN } < P_f$$

$$\alpha_1 = 0.85 - 0.0015 f'_c = 0.85 - 0.0015 \times 28 = 0.808 > 0.67$$ ***CSA A23.3-14 (Eq. 10.1)***

Column dimensions of $400 \text{ mm} \times 400 \text{ mm}$ are adequate for axial load.

2. Two-Way Slab Analysis and Design

CSA A23.3 states that a regularslab system may be designed using any procedure satisfying conditions of equilibrium and compatibility with the supports, provided that it is shown that the factored resistance at every section is at least equal to the effects of the factored loads and that all serviceability conditions, including specified limits on deflections, are met. ***CSA A23.3-14 (13.5.1)***

CSA A23.3 permits the use of Plastic Plate Theory Method (PPTM), Theorems of Plasticity Method (TPM), Direct Design Method (DDM) and Elastic Frame Method (EFM); known as Equivalent Frame Method in the ACI; for the gravity load analysis of orthogonal frames. The following sections outline the solution per DDM, EFM, and spSlab software respectively.

2.1. Direct Design Method (DDM)

Two-way slabs satisfying the limits in ***CSA A23.3-14 (13.9)*** are permitted to be designed in accordance with the DDM.

2.1.1. Direct design method limitations

There shall be a minimum of three continuous spans in each direction (3 spans) ***CSA A23.3-14 (13.9.1.2)***

Successive span lengths centre-to-centre of supports in each direction shall not differ by more than one- third of the longer span (span lengths are equal) ***CSA A23.3-14 (13.9.1.3)***

All loads shall be due to gravity only and uniformly distributed over an entire panel (Loads are uniformly distributed over the entire panel) ***CSA A23.3-14 (13.9.1.4)***

The factored live load shall not exceed twice the factored dead load (Service live-to-dead load ratio of 0.41 < 2.0) ***CSA A23.3-14 (13.9.1.4)***

Since all the criteria are met, Direct Design Method can be utilized.

2.1.2. Design moments

a. Calculate the total factored static moment:

$$M_o = \frac{w_f \ell_{2a} \ell_n^2}{8} = \frac{9.8 \times 4.2 \times 5.1^2}{8} = 133.82 \text{ kN.m}$$ ***CSA A23.3-14 (13.9.1.4)***

Distribute the total factored moment, M_o, in an interior and end span: ***CSA A23.3-14 (13.9.3.1 &13.9.3.2)***

Table 1 - Distribution of M_o along the span

Location		**Total Design Strip Moment, M_{DES} (kN.m)**
Exterior Span	Exterior Negative	0.26 × M_o = 34.8
	Positive	0.52 × M_o = 69.6
	Interior Negative	0.70 × M_o = 93.68
Interior Span	Positive	0.35 × M_o = 46.8

b. Calculate the column strip moments. ***CSA A23.3-14 (13.11.2)***

That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips. ***CSA A23.3-14 (13.11.3.1)***

Table 2 - Lateral Distribution of the Total Design Strip Moment, M_{DES}				
Location		**Total Design Strip Moment, (kN.m)**	**Column Strip Moment, (kN.m)**	**Moment in Two Half Middle Strips, (kN.m)**
Exterior Span	Exterior Negative*	34.8	$1.00 \times M_{DES} = 34.8$	$0.00 \times M_{DES} = 0.0$
	Positive	69.6	$0.6 \times M_{DES} = 41.8$	$0.4 \times M_{DES} = 27.8$
	Interior Negative*	93.68	$0.8 \times M_{DES} = 74.94$	$0.2 \times M_{DES} = 18.7$
Interior Span	Positive	46.8	$0.6 \times M_{DES} = 28.1$	$0.4 \times M_{DES} = 18.7$
* All negative moments are at face of support.				

2.1.3. Flexural reinforcement requirements

a. Determine flexural reinforcement required for column and middle strips at all critical sections

The following calculation is for the exterior span exterior negative location of the column strip.

Reinforcement for the total factored negative moment transferred to the exterior columns shall be placed within a band width b_b. Temperature and shrinkage reinforcment determined as specified in clause 7.8.1 shall be provided in that sectopm pf the slab outside of the bad region defined by b_b or as required by clause 13.10.9. ***CSA A23.3-14 (13.10.3)***

$M_f = 34.8$ kN.m

Use average $d_{avg} = 154$ mm

In this example, jd will be assumed to taken equal to $0.98d$. The assumptions will be verified once the area of steel in finalized.

Assume $jd = 0.98 \times d = 150.9$ mm

Column strip width, $b = 4,200/2 = 2,100$ mm

Middle strip width, $b = 4,200 - 2,100 = 2,100$ mm

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{34.8 \times 10^6}{0.85 \times 400 \times 0.95 \times 150.9} = 678 \text{ mm}^2$$

$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.80 > 0.67$ ***CSA A23.3-14 (10.1.7)***

$$a = \frac{A_s f_y}{0.9 f'_c b} = \frac{700 \times 400}{0.9 \times 28 \times 2,100} = 5.29 \text{ mm}^2$$

Recalculate 'a' for the actual $A_s = 2834 \text{ mm}^2 \rightarrow a = \frac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \frac{0.85 \times 2834 \times 400}{0.65 \times 0.80 \times 35 \times 4,500} = 11.81$ mm

$$jd = d - a/2 = 0.98d$$

Therefore, the assumption that jd equals to $0.98d$ is valid.

Min $A_s = 0.002 A_g = 0.002 \times 190 \times 2,100 = 798 \text{ mm}^2 > 676.24 \text{ mm}^2$ ***CSA A23.3-14 (7.8.1)***

Provide 4 - 15M bars concentrated within the band b_b (800 mm^2 > 798 mm^2)

Maximum spacing: ***CSA A23.3-14 (13.10.4)***

- Negative reinforcement in the band definded by b_b:

  $1.5h_s = 285 \text{ mm} \leq 250 \text{ mm}$

- Remaining negative moment reinforcement (reinforcement for column strip that is not included in the band b_b):

  $3h_s = 570 \text{ mm} \leq 500 \text{ mm}$

For the exterior negative reinforcements within the band b_b, the maximum spacing is 250 mm. To distribute the bars uniformly, the maximum spacing in the band b_b is applied along the whole column strip.

Provide 9 - 15M bars with $A_s = 200 \text{ mm}^2$ and $s = 2{,}100/9 = 233 \text{ mm} < s_{max}$

Note that the number of bars for this section is governed by the maximum spacing allowed by the code.

Based on the procedure outlined above, values for all span locations are given in Table 3.

Table 3 - Required Slab Reinforcement for Flexure (DDM)

Span Location		M_f (kN.m)	b (m)	d (mm)	A_s Req'd for flexure (mm^2)	Min A_s (mm^2)	Reinforcement Provided	A_s Prov. for flexure (mm^2)
End Span								
Column Strip	Exterior Negative	34.80	2.1	154	678	798	9 – 15 M	1,800
	Positive	41.80	2.1	154	823	798	6 – 15 M	1,200
	Interior Negative	74.94	2.1	154	1,513	798	8 – 15 M	1,600
Middle Strip	Exterior Negative	0.00	2.1	154	0	798	6 – 15 M	1,200
	Positive	27.80	2.1	154	541	798	6 – 15 M	1,200
	Interior Negative	18.70	2.1	154	362	798	6 – 15 M	1,200
Interior Span								
Column Strip	Positive	28.10	2.1	154	548	798	6 – 15 M	1,200
Middle Strip	Positive	18.70	2.1	154	362	798	6 – 15 M	12,00

b. Calculate additional slab reinforcement at columns for moment transfer between slab and column

When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of unbalanced moment given by γ_f shall be transferred by flexural reinforcement placed within a width b_b. ***CSA A23.3-14 (13.10.2)***

$$\gamma_f = 1 - \gamma_v = \frac{1}{1 + \frac{2}{3}\sqrt{b_1/b_2}}$$

Where

b_1 = Width width of the critical section for shear measured in the direction of the span for which moments are determined according to CSA A23.3-14, clause 13 (see Figure 5).

b_2 = Width of the critical section for shear measured in the direction perpendicular to b1 according to CSA A23.3-14, clause 13 (see Figure 5).

b_b = Effective slab width = $c_2 + 3 \times h_s$ ***CSA A23.3-14 (3.2)***

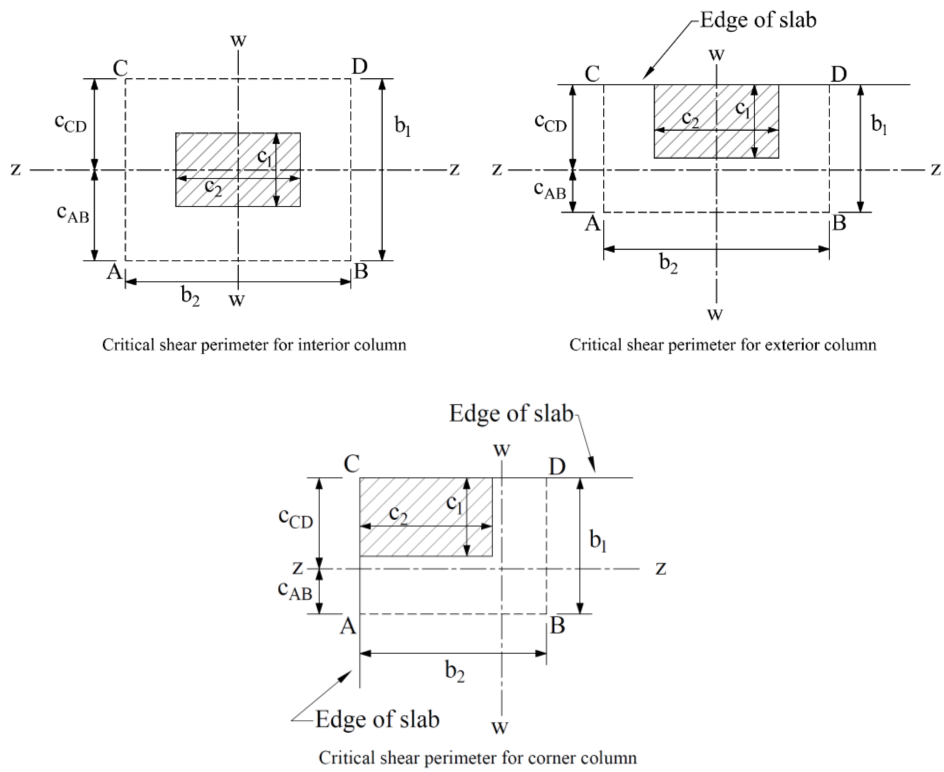


Figure 5 – Critical Shear Perimeters for Columns

Table 4 - Additional Slab Reinforcement required for moment transfer between slab and column (DDM)									
Span Location		**M_f^* (kN.m)**	**γ_f**	**γ_f M_f (kN.m)**	**Effective slab width, b_b (mm)**	**d (mm)**	**A_s req'd within b_b (mm^2)**	**A_s prov. For flexure within b_b (mm^2)**	**Add'l Reinf.**
End Span									
Column Strip	Exterior Negative	34.8	0.62	21.5	970	154	420	1,000	-
	Interior Negative	0.0	0.60	0.0	970	154	0.0	800	-
*M_f is taken at the centerline of the support in Equivalent Frame Method solution.									

2.1.4. Factored moments in columns

a. Interior columns:

$$M_f = 0.07\left(\left(w_{df} + 0.5w_{lf}\right)l_{2a}l_n^2 - w_{df}^{'}l_{2a}^{'}(l_n^{'})^2\right)$$ ***CSA A23.3-14 (13.9.4)***

$$= 0.07\left(\left(6.95 + 0.5\times 2.85\right)4.2\times 5.1^2 - 6.95\times 4.2\times 5.1^2\right) = 10.9 \text{ kN.m}$$

With the same column size and length above and below the slab,

$$M_{column} = \frac{10.9}{2} = 5.45 \text{ kN.m}$$

b. Exterior Columns:

Total exterior negative moment from slab must be transferred directly to the column: $M_f = 34.8$ kN.m With the same column size and length above and below the slab,

$$M_{column} = \frac{34.8}{2} = 17.4 \text{ kN.m}$$

The moments determined above are combined with the factored axial loads (for each story) for design of column sections as shown later in this example.

2.2. Elastic Frame Method (EFM)

EFM (as known as Equivalent Frame Method in the ACI 318) is the most comprehensive and detailed procedure provided by the CSA A23.3 for the analysis and design of two-way slab systems where these systems may, for purposes of analysis, be considered a series of plane frames acting longitudinally and transversely through the building. Each frame shall be composed of equivalent line members intersecting at member centrelines, shall follow a column line, and shall include the portion of slab bounded laterally by the centreline of the panel on each side. ***CSA A23.3-14 (13.8.1.1)***

Probably the most frequently used method to determine design moments in regular two-way slab systems is to consider the slab as a series of two-dimensonal frames that are analyzed elastically. When using this analogy, it is essential that stiffness properties of the elements of the frame be selected to properly represent the behavior of the three-dimensional slab system.

In a typical frame analysis it is assumed that at a beam-column cconnection all members meeting at the joint undergo the same rotaion. For uniform gravity loading this reduced restrtaint is accounted for by reducing the effective stiffness of the column by either Clause 13.8.2 or Clause 13.8.3. ***CSA A23.3-14 (N.13.8)***

Each floor and roof slab with attached columns may be analyzed separately, with the far ends of the columns considered fixed. ***CSA A23.3-14 (13.8.1.2)***

The moment of inertia of column and slab-beam elements at any cross-section outside of joints or column capitals shall be based on the gross area of concrete at that section. ***CSA A23.3-14 (13.8.2.5)***

An equivalent column shall be assumed to consist of the actual columns above and below the slab- beam plus an attached torsional member transverse to the direction of the span for which moments are being determined. ***CSA A23.3-14 (13.8.2.5)***

2.2.1. Elastic frame method limitations

In EFM, live load shall be arranged in accordance with 13.8.4 which requires slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***CSA A23.3-14 (13.8.4)***

Complete analysis must include representative interior and exterior equivalent elastic frames in both the longitudinal and transverse directions of the floor. ***CSA A23.3-14 (13.8.1.1)***

Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***CSA A23.3-14 (3.1a)***

For slab systems with beams between sypports, the relative effective stiffness of beams in the two directions is not less than 0.2 or greater than 2. ***CSA A23.3-14 (3.1b)***

Column offsets are not greater than 20% of the span (in the direction of offset) from either axis between centerlines of successive columns. ***CSA A23.3-14 (3.1c)***

The reinforcement is placed in an orthogonal grid. ***CSA A23.3-14 (3.1d)***

2.2.2. Frame members of elastic frame

Determine moment distribution factors and fixed-end moments for the elastic frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors k, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a. Flexural stiffness of slab-beams at both ends, K_{sb}

$$\frac{c_{N1}}{\ell_1} = \frac{400}{5,500} = 0.073 \ , \quad \frac{c_{N2}}{\ell_2} = \frac{400}{4,200} = 0.095$$

For $c_{F1} = c_{F2}$, stiffness factors, $k_{NF} = k_{FN} = 4.13$ ***PCA Notes on ACI 318-11 (Table A1)***

Thus, $K_{sb} = k_{NF}\dfrac{E_{cs} I_s}{\ell_1} = 4.13\dfrac{E_{cs} I_s}{\ell_1}$ ***PCA Notes on ACI 318-11 (Table A1)***

$$K_{sb} = 4.13 \times 26,739 \times \frac{2.4\times10^9}{5,500} \times 10^{-3} = 48.2\times10^6 \text{ N.m}$$

where, $I_s = \dfrac{\ell_s h^3}{12} = \dfrac{4,200(190)^3}{12} = 2.4\times10^9 \text{ mm}^4$

$$E_{cs} = (3,300\sqrt{f_c^{'}} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5}$$ ***CSA A23.3-14(8.6.2.2)***

$$E_{cs} = (3,300\sqrt{28} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 26,739 \text{ MPa}$$

Carry-over factor COF $= 0.509$ ***PCA Notes on ACI 318-11 (Table A1)***

Fixed-end moment FEM $= 0.0843 w_u \ell_2 \ell_1^{\,2}$ ***PCA Notes on ACI 318-11 (Table A1)***

b. Flexural stiffness of column members at both ends, K_c

Referring to ***Table A7, Appendix 20A***, $t_a = 95$ mm, $t_b = 95$ mm,

$$\mathrm{H} = 2.75 \text{ m} = 2{,}750 \text{ mm},\ \mathrm{t} = 190 \text{ mm},\ \mathrm{H_c} = 2560 \text{ mm},\ \frac{\mathrm{t_a}}{\mathrm{t_b}} = 1,\ \frac{\mathrm{H}}{\mathrm{H_c}} = 1.07$$

Thus, $k_{AB} = k_{BA} = 4.74$ by interpolation.

$$K_c = \frac{4.74 E_{cc} I_c}{\ell_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_c = 4.74 \times 31{,}047 \times \frac{2.13 \times 10^9}{2{,}750} \times 10^{-3} = 114 \times 10^6 \text{ N.m}$$

Where $I_c = \dfrac{c^4}{12} = \dfrac{(400)^4}{12} = 2.13 \times 10^9 \text{ mm}^4$

$$E_{cs} = (3{,}300\sqrt{f_c^{'}} + 6{,}900)\left(\frac{\gamma_c}{2{,}300}\right)^{1.5}$$ ***CSA A23.3-14(8.6.2.2)***

$$E_{cs} = (3{,}300\sqrt{42} + 6{,}900)\left(\frac{2{,}447}{2{,}300}\right)^{1.5} = 31{,}047 \text{ MPa}$$

$\ell_c = 2.75 \text{ m} = 2{,}750 \text{ mm}$

c. Torsional stiffness of torsional members, K_t

$$K_t = \frac{9 E_{cs} C}{\ell_t\left(1 - \dfrac{c_2}{\ell_t}\right)^3}$$ ***CSA A23.3-14(13.8.2.8)***

$$K_t = \frac{9 \times 26{,}739 \times 6.41 \times 10^6}{4{,}200 \times (0.905)^3} = 49.5 \times 10^6 \text{ N.m}$$

Where $C = \sum\left(1 - 0.63\dfrac{x}{y}\right)\left(\dfrac{x^3 y}{3}\right)$ ***CSA A23.3-14(13.8.2.9)***

$$C = \left(1 - 0.63 \times \frac{190}{400}\right)\left(190^3 \times \frac{400}{3}\right) = 6.41 \times 10^8 \text{ mm}^4$$

$c_2 = 400$ mm, and $\ell_2 = 4.2 \text{ m} = 4200 \text{ mm}$

d. Equivalent column stiffness, K_{ec}

$$K_{ec} = \frac{\sum K_c \times \sum K_t}{\sum K_c + \sum K_t}$$

$$K_{ec} = \frac{(2 \times 114)(2 \times 49.5)}{[(2 \times 114) + (2 \times 49.5)]} \times 10^6$$

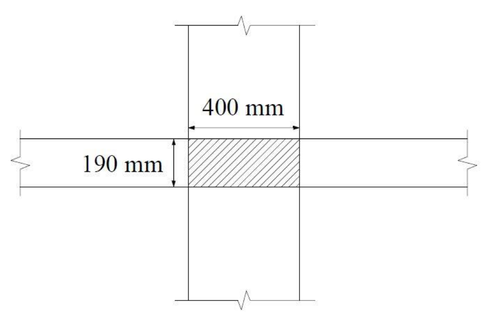


Figure 6 - Torsional Member

$K_{ec} = 69.1 \times 10^6$ N.m

Where $\sum K_t$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

e. Slab-beam joint distribution factors, *DF*

At exterior joint,

$$DF = \frac{48.2}{(48.2 + 69.1)} = 0.41$$

At interior joint,

$$DF = \frac{48.2}{(48.2 + 48.2 + 69.1)} = 0.29$$

COF for slab-beam $= 0.509$

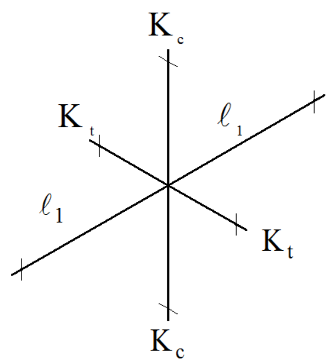


Figure 7 – Column and Edge of Slab

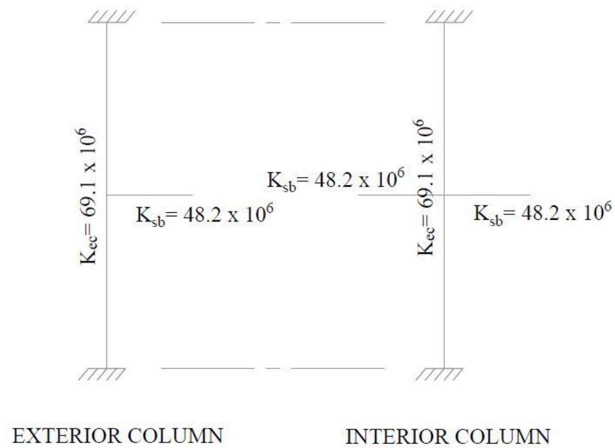


Figure 8 – Slab and Column Stiffness

2.2.3. Elastic frame analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. ***CSA A23.3-14 (13.8.4.2)***

$$\frac{L}{D} = \frac{1.9}{(4.56 + 1)} = 0.34 < \frac{3}{4}$$

a. Factored load and Fixed-End Moments (FEM's).

Factored dead load $w_{df} = 1.25(4.56 + 1) = 6.95 \text{ kN/m}^2$

Factored live load $w_{lf} = 1.5(1.9) = 2.85 \text{ kN/m}^2$

Factored load $q_u = w_f = w_{df} + w_{lf} = 9.8 \text{ kN/m}^2$

FEM's for slab-beams $= m_{NF} q_u \ell_2 \ell_1^{\,2}$ ***PCA Notes on ACI 318-11 (Table A1)***

$= 0.0841 \times (9.8 \times 4.2) \times 5.5^2 = 104.7$ kN.m

b. Moment distribution. Computations are shown in Table 5. Counterclockwise rotational moments acting on the member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_u\,(\text{midspan}) = M_o - \frac{(M_{uL} + M_{uR})}{2}$$

Where M_o is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example.

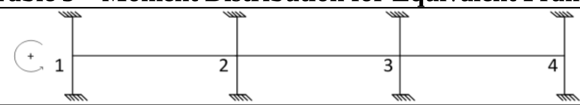
Positive moment in span 1-2:

$$+M_u = (9.8\times4.2)\frac{5.5^2}{8} - \frac{(64.1+119.7)}{2} = 63.8 \text{ kN.m}$$

Positive moment span 2-3:

$$+M_u = (9.8\times4.2)\frac{5.5^2}{8} - \frac{(108.5+108.5)}{2} = 47.2 \text{ kN.m}$$

Table 5 – Moment Distribution for Equivalent Frame



Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.41	0.29	0.29	0.29	0.29	0.41
COF	0.509	0.509	0.509	0.509	0.509	0.509
FEM	+104.7	-104.7	+104.7	-104.7	+104.7	-104.7
Dist	-42.93	0.0	0.0	0.0	0.0	42.93
CO	0.0	-21.85	0.0	0.0	21.85	0.0
Dist	0.0	6.34	6.34	-6.34	-6.34	0.0
CO	3.23	0.0	-3.23	3.23	0.0	-3.23
Dist	-1.32	0.94	0.94	-0.94	-0.94	1.32
CO	0.48	-0.67	-0.48	0.48	0.67	-0.48
Dist	-0.2	0.33	0.33	-0.33	-0.33	0.2
CO	0.17	-0.10	-0.17	0.17	0.1	-0.17
Dist	-0.07	0.08	0.08	-0.08	-0.08	0.07
CO	0.04	-0.04	-0.04	0.04	0.04	-0.04
Dist	-0.02	0.02	0.02	-0.02	-0.02	0.02
CO	0.01	-0.01	-0.01	0.01	0.01	-0.01
Dist	0.00	0.01	0.01	-0.01	-0.01	0.00
CO	0.01	0.00	-0.01	0.01	0.00	-0.01
Dist	0.0	0.0	0.0	0.0	0.0	0.0
Neg. M	64.1	-119.7	108.5	-108.5	119.7	-64.1
M at midspan	63.8		47.2		-86.6	

2.2.4. Design moments

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 9. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than $0.175\ell_1$ from the centers of supports. ***<u>CSA A23.3-14 (13.8.5.1)</u>***

$400 \text{ mm} < 0.175\times5{,}500 = 926.5 \text{ mm}$ (use face of support location)

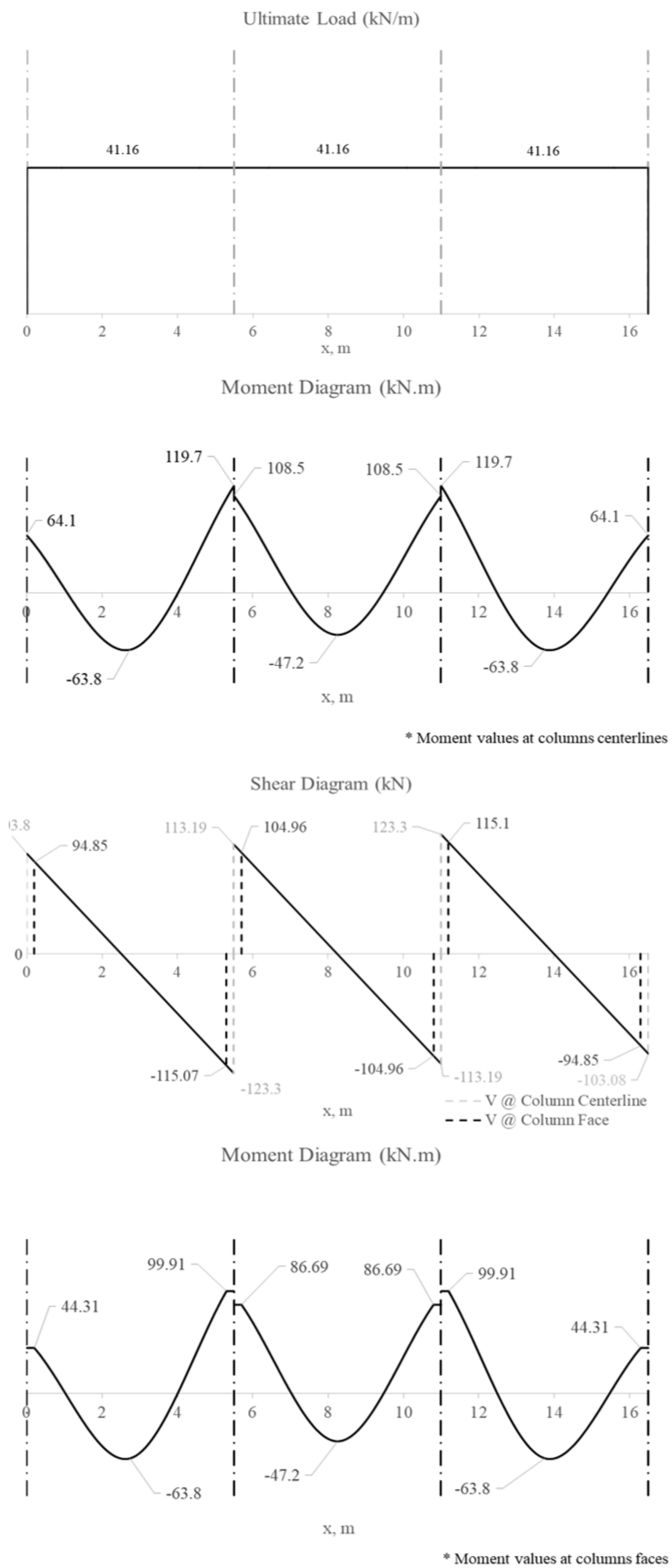


Figure 9 - Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)

2.2.5. Distribution of design moments

After the negative and positive moments have been determined for the slab-beam strip, the CSA code permits the distribution of the moments at critical sections to the column strips, beams (if any), and middle strips in accordance with the DDM. **_CSA A23.3-14 (13.11.2.2)_**

Distribution of factored moments at critical sections is summarized in Table 6.

Table 6 - Distribution of factored moments							
		Slab-beam Strip	Column Strip		Middle Strip		
		Moment (kN.m)	Percent	Moment (kN.m)	Percent	Moment (kN.m)
End Span	Exterior Negative	44.31	100	44.31	0	0.00
	Positive	63.8	60	38.28	40	25.52
	Interior Negative	95.86	80	76.69	20	19.17
Interior Span	Negative	86.69	80	69.35	20	17.34
	Positive	47.2	60	28.32	40	18.88

2.2.6. Flexural reinforcement requirements

a. Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – exterior negative location is provided below.

Reinforcement for the total factored negative moment transferred to the exterior columns shall be placed within a band width b_b. Temperature and shrinkage reinforcment determined as specified in clause 7.8.1 shall be provided in that sectopm pf the slab outside of the bad region defined by b_b or as required by clause 13.10.9. **_CSA A23.3-14 (13.10.3)_**

$M_r = 44.31$ kN.m

Use average d_{avg} = 154 mm

In this example, jd will be assumed to be taken equal to *0.97d*. The assumptions will be verified once the area of steel in finalized.

Assume $jd = 0.97 \times d = 149.4\text{mm}$

Column strip width, $b = 4,200 / 2 = 2,100$ mm

Middle strip width, $b = 4,200 - 2,100 = 2,100$ mm

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{44.31}{0.85 \times 400 \times 0.97 \times 154} = 872 \text{ mm}^2$$

$$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.81 > 0.67$$ **_CSA A23.3-14 (10.1.7)_**

Recalculate 'a' for the actual $A_s = 872 \text{ mm}^2 \rightarrow a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \dfrac{0.85 \times 872 \times 400}{0.65 \times 0.81 \times 28 \times 2,100} = 9.61$ mm

$$jd = d - a/2 = 0.97d$$

Therefore, the assumption that *jd* equals to *0.97d* is valid.

Min $A_s = 0.002A_g = 0.002\times190\times2,100 = 798\ \text{mm}^2 < 872\ \text{mm}^2$ **CSA A23.3-14 (7.8.1)**

Provide 5 - 15M bars (1000 mm^2 > 872 mm^2)

Maximum spacing: **CSA A23.3-14 (13.10.4)**

- Negative reinforcement in the band definded by b_b: $1.5h_s = 285\ \text{mm} \le 250\ \text{mm}$
- Remaining negative moment reinforcement: $3h_s = 570\ \text{mm} \le 500\ \text{mm}$

For the negative reinforcements at the exterior span within the band b_b, the maximum spacing is 250 mm. To distribute the bars uniformly, the maximum spacing in the band b_b is applied along the whole column strip.

Provide 9 - 15M bars with $A_s = 200\ \text{mm}^2$ and $s = 2,100/9 = 233\ \text{mm} < s_{max}$

Note that the number of bars for this section is governed by the maximum spacing allowed by the code.

Based on the procedure outlined above, values for all span locations are given in Table 7.

Table 7 - Required Slab Reinforcement for Flexure [Elastic Frame Method (EFM)]

Span Location		M_r (kN.m)	b (m)	d (mm)	A_s Req'd for flexure (mm^2)	Min A_s (mm^2)	Reinforcement Provided	A_s Prov. for flexure (mm^2)
End Span								
Column Strip	Exterior Negative	44.31	2.1	154	872	798	9 - 15M	1,800
	Positive	38.28	2.1	154	754	798	6 - 15M	1,200
	Interior Negative	76.69	2.1	154	1,548	798	8 - 15M	1,600
Middle Strip	Exterior Negative	0	2.1	154	0	798	6 - 15M	1,200
	Positive	25.52	2.1	154	496	798	6 - 15M	1,200
	Interior Negative	19.17	2.1	154	371	798	6 - 15M	1,200
Interior Span								
Column Strip	Positive	28.32	2.1	154	552	798	6 - 15M	1,200
Middle Strip	Positive	18.88	2.1	154	365	798	6 - 15M	1,200

b. Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of unbalanced moment given by γ_f shall be transferred by flexural reinforcement placed within a width b_b. **CSA A23.3-14 (13.10.2)**

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_r$

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{b_1/b_2}}$$ **CSA A23.3-14 (13.10.2)**

Where

b_1 = Width width of the critical section for shear measured in the direction of the span for which moments are determined according to CSA A23.3-14, clause 13 (see Figure 5).

b_2 = Width of the critical section for shear measured in the direction perpendicular to b1 according to CSA A23.3-14, clause 13 (see Figure 5).

b_b = Effective slab width $= c_2 + 3 \times h_s$ ***CSA A23.3-14 (3.2)***

Table 8 - Additional Slab Reinforcement required for moment transfer between slab and column (EFM)

Span Location		M_u^* (kN.m)	γ_f	$\gamma_f M_u$ (kN.m)	Effective slab width, b_b (mm)	d (mm)	A_s req'd within b_b (mm^2)	A_s prov. For flexure within b_b (mm^2)	Add'l Reinf.
End Span									
Column Strip	Exterior Negative	64.1	0.62	39.6	970	154	778	1,000	-
	Interior Negative	13.2	0.60	7.93	970	154	152	800	-
*M_u is taken at the centerline of the support in Equivalent Frame Method solution.									

2.2.7. Column design moments

The unbalanced moment from the slab-beams at the supports of the equivalent frame are distributed to the support columns above and below the slab-beam in proportion to the relative stiffness of the support columns. Referring to Figure 9, the unbalanced moment at joints 1 and 2 are:

Joint 1= +64.1 kN.m

Joint 2= -119.7 + 108.5 = -11.2 kN.m

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced slab moments (M_{sc}) to the exterior and interior columns are shown in Figure 10a.

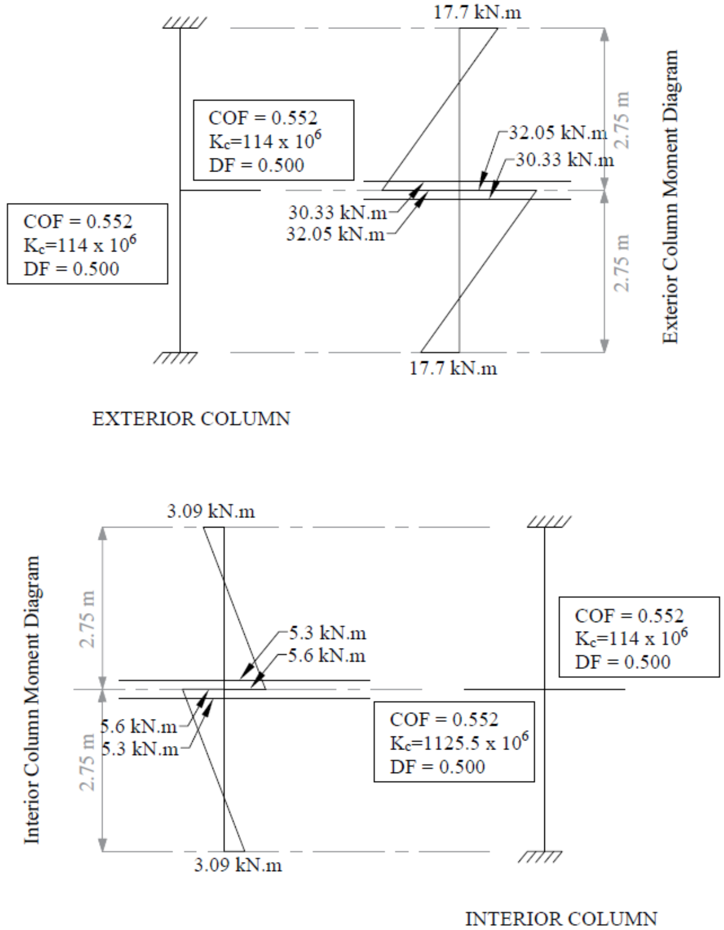


Figure 10a - Column Moments (Unbalanced Moments from Slab-Beam)

In summary:

$$M_{col,Exterior} = 30.33 \text{ kN.m}$$

$$M_{col,Interior} = 5.3 \text{ kN.m}$$

The moments determined above are combined with the factored axial loads (for each story) and factored moments in the transverse direction for design of column sections. Figure 10b shows the moment diagrams in the longitudinal and transverse direction for the interior and exterior equivalent frames. Following the previous procedure, the moment values at the face of interior, exterior, and corner columns from the unbalanced moment values can be obtained. These values are shown in the following table.

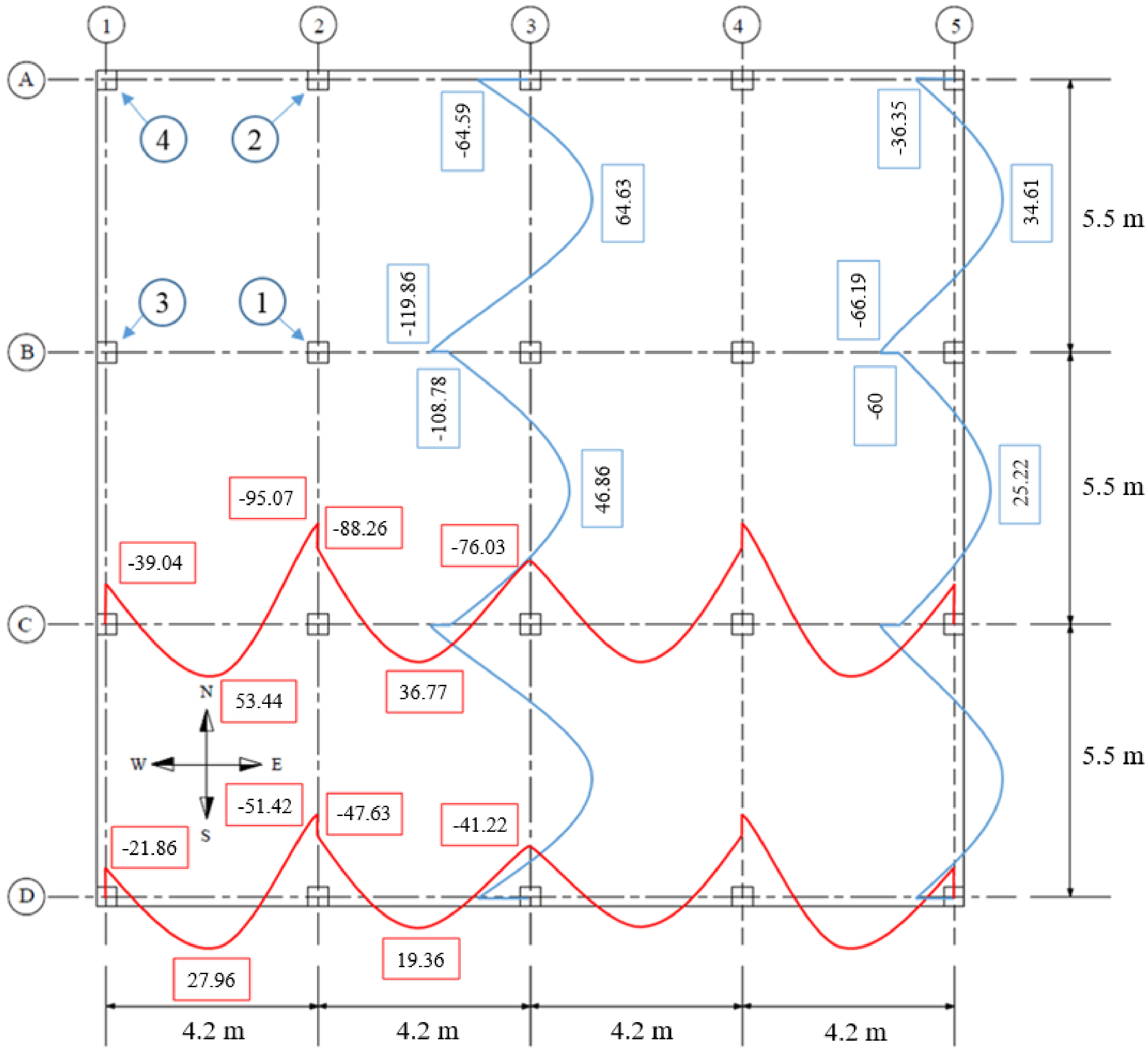


Figure 10b – Moment Diagrams (kips-ft)

M_u kN.m	Column number (See Figure 10b)			
	1	2	3	4
M_{ux}	5.24	30.56	2.93	17.2
M_{uy}	3.22	1.79	18.47	10.34

3. Design of Interior, Edge, and Corner Columns

This section includes the design of interior, edge, and corner columns using spColumn software. The preliminary dimensions for these columns were calculated previously in section one. The reduction of live load will be ignored in this example. However, the detailed procedure to calculate the reduced live loads is explained in the "wide-Module Joist System" example.

3.1. Determination of factored loads

Interior Column (Column #1):

Assume 4 story building

Tributary area for interior column is $A_{Tributary} = (5.5 \times 4.2) = 23.1\ m^2$

$P_u = 4 \times w_f \times A_{Tributary} = 4 \times 9.8 \times 23.1 = 226.38\ \text{kN}$

$M_{u,x}$ = 5.24 kN.m (see the previous Table)

$M_{u,y}$ = 3.22 kN.m (see the previous Table)

Edge (Exterior) Column (Column #2):

Tributary area for interior column is $A_{Tributary} = (5.5/2 \times 14) = 11.55\ \text{m}^2$

$P_u = 4 \times q_u \times A_{Tributary} = 4 \times 9.8 \times 11.5 = 113.19\ \text{kN}$

$M_{u,x}$ = 30.56 kN.m (see the previous Table)

$M_{u,y}$ = 1.79 kN.m (see the previous Table)

Edge (Exterior) Column (Column #3):

Tributary area for interior column is $A_{Tributary} = (5.5 \times 4.2/2) = 11.5\ \text{m}^2$

$P_u = 4 \times q_u \times A_{Tributary} = 4 \times 9.8 \times 1155 = 113.19\ \text{kN}$

$M_{u,x}$ = 2.93 kN.m (see the previous Table)

$M_{u,y}$ = 18.47 kN.m (see the previous Table)

Corner Column (Column #4):

Tributary area for interior column is $A_{Tributary} = (5.5/2 \times 4.2/2) = 5.78\ \text{m}^2$

$P_u = 4 \times q_u \times A_{Tributary} = 4 \times 9.8 \times 5.78 = 56.6\ \text{kN}$

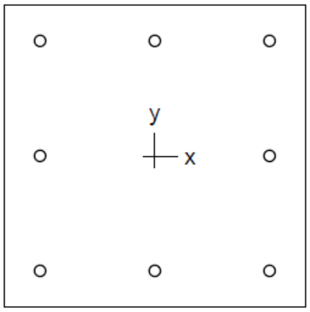
$M_{u,x}$ = 17.2 kN.m (see the previous Table)

$M_{u,y}$ = *10.34* kN.m (see the previous Table)

The factored loads are then input into spColumn to construct the axial load – moment interaction diagram.

3.2. Column Capacity Diagram (Axial-Moment Interaction Diagram)

Interior Column (Column #1):



400 x 400 mm

Code: CSA A23.3-14

Units: Metric

Run axis: Biaxial

Run option: Investigation

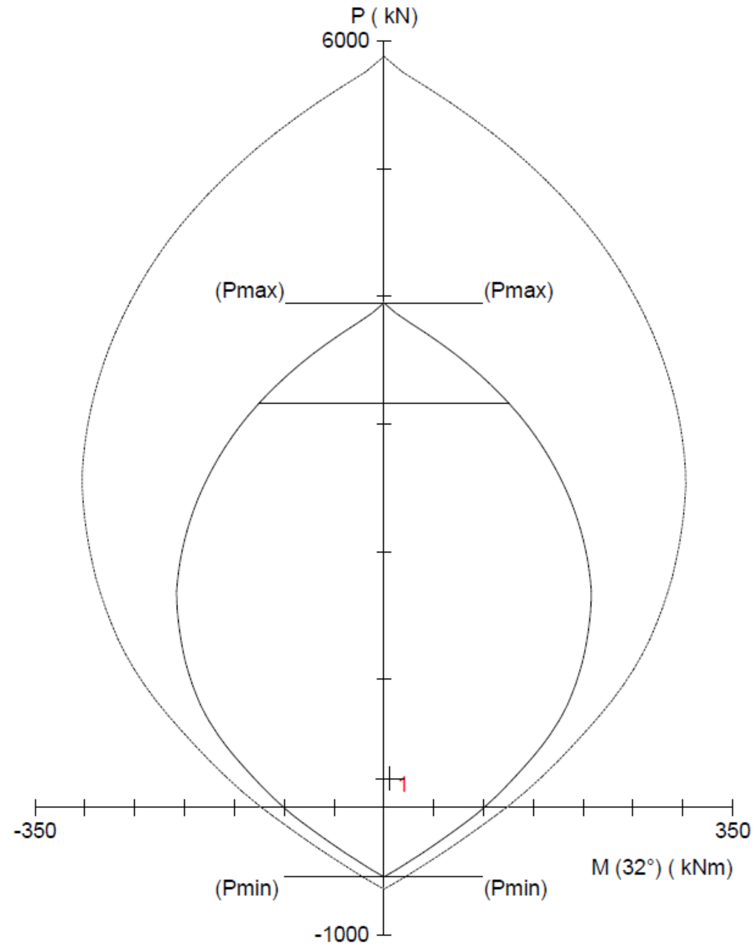
Slenderness: Not considered

Column type: Structural

Bars: CSA G30.18

Date: 06/05/18

Time: 11:18:33



STRUCTUREPOINT - spColumn v6.00 (TM). Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

File: c:\tsda\two-way flat plate system (csa)\column design\interior columns (#1).col

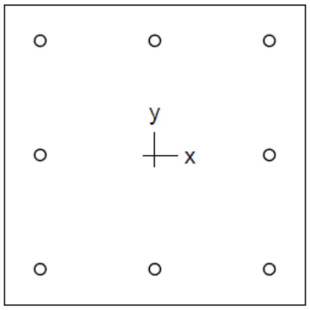
Project: Two-Way Flat Plate System

Column: Interior #1		Engineer: SP	
f'c = 42 MPa	fy = 400 MPa	Ag = 160000 mm^2	8 #15 bars
Ec = 30151 MPa	Es = 200000 MPa	As = 1600 mm^2	rho = 1.00%
fc = 33.054 MPa	e_yt = 0.002 mm/mm	Xo = 0 mm	Ix = 2.13e+009 mm^4
e_u = 0.0035 mm/mm		Yo = 0 mm	Iy = 2.13e+009 mm^4
Beta1 = 0.865		Min clear spacing = 136 mm	Clear cover = 40 mm

Confinement: Tied

phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.65, phi(rho) = 1.00, Min. Dimension(h) = 400 mm

Edge Column (Column #2):



400 x 400 mm

Code: CSA A23.3-14

Units: Metric

Run axis: Biaxial

Run option: Investigation

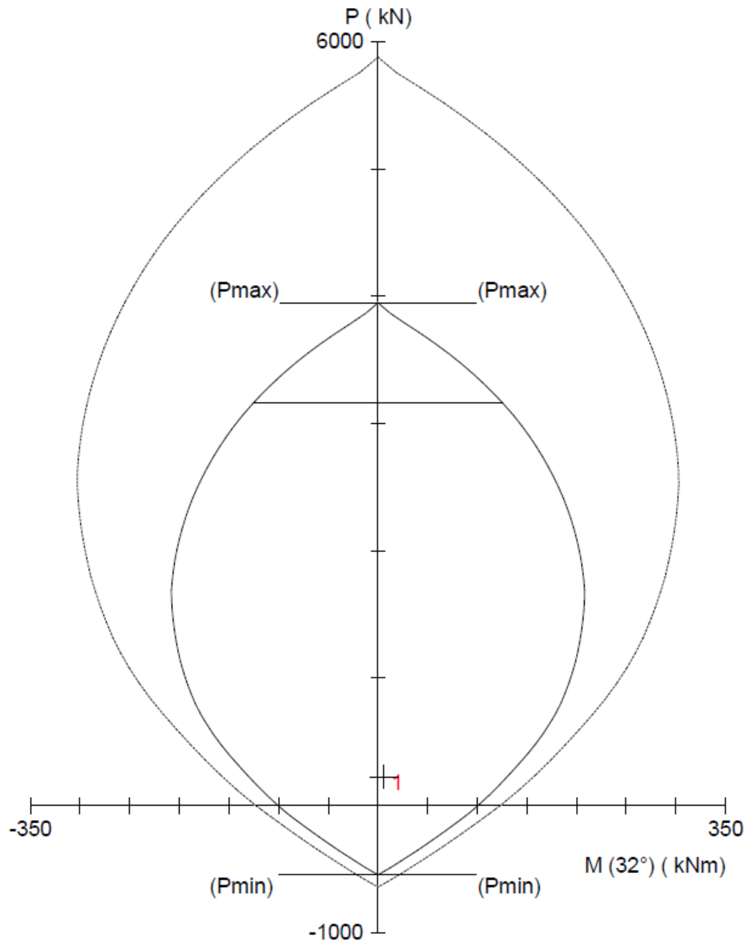
Slenderness: Not considered

Column type: Structural

Bars: CSA G30.18

Date: 06/05/18

Time: 11:22:24



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File: C:\TSDA\Two-Way Flat Plate System (CSA)\Column Design\Edge Column (#2).col

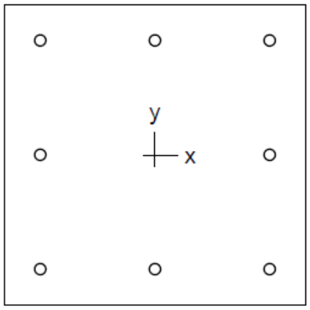
Project: Two-Way Flat Plate System

Column: Edge #2		Engineer: SP	
f'c = 42 MPa	fy = 400 MPa	Ag = 160000 mm^2	8 #15 bars
Ec = 30151 MPa	Es = 200000 MPa	As = 1600 mm^2	rho = 1.00%
fc = 33.054 MPa	e_yt = 0.002 mm/mm	Xo = 0 mm	Ix = 2.13e+009 mm^4
e_u = 0.0035 mm/mm		Yo = 0 mm	Iy = 2.13e+009 mm^4
Beta1 = 0.865		Min clear spacing = 136 mm	Clear cover = 40 mm

Confinement: Tied

phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.65, phi(rho) = 1.00, Min. Dimension(h) = 400 mm

Edge Column (Column #3):



400 x 400 mm

Code: CSA A23.3-14

Units: Metric

Run axis: Biaxial

Run option: Investigation

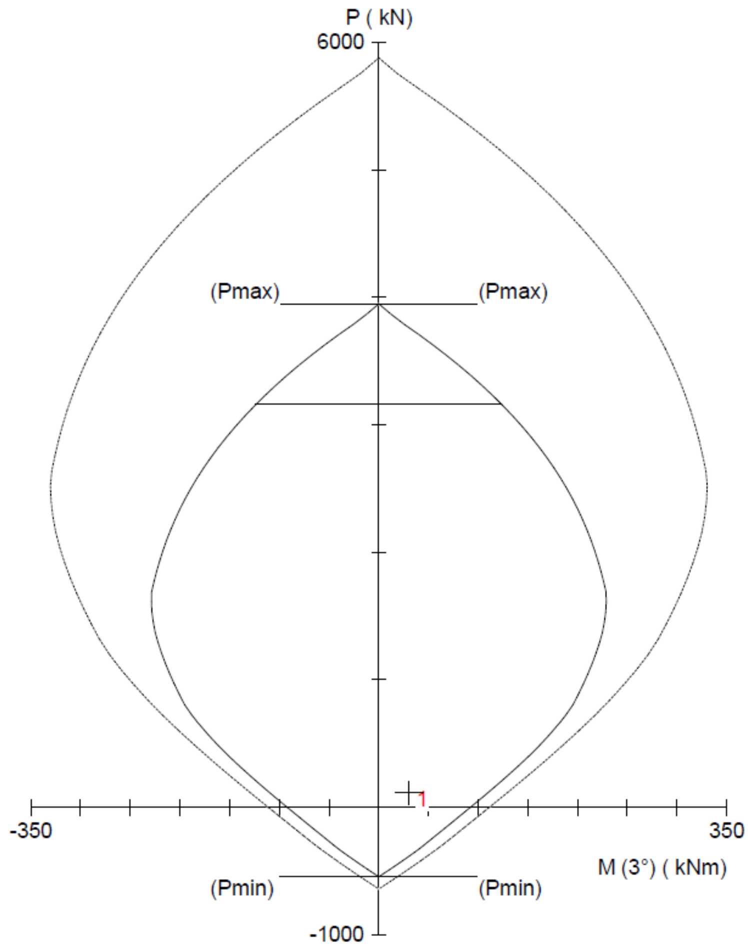
Slenderness: Not considered

Column type: Structural

Bars: CSA G30.18

Date: 06/05/18

Time: 11:24:56



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File: C:\TSDA\Two-Way Flat Plate System (CSA)\Column Design\Edge Column (#3).col

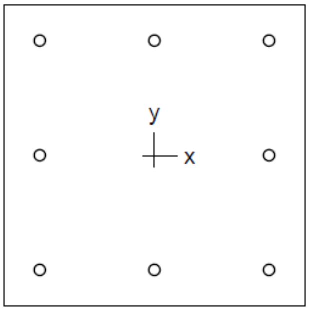
Project: Two-Way Flat Plate System

Column: Edge #3		Engineer: SP	
f'c = 42 MPa	fy = 400 MPa	Ag = 160000 mm^2	8 #15 bars
Ec = 30151 MPa	Es = 200000 MPa	As = 1600 mm^2	rho = 1.00%
fc = 33.054 MPa	e_yt = 0.002 mm/mm	Xo = 0 mm	Ix = 2.13e+009 mm^4
e_u = 0.0035 mm/mm		Yo = 0 mm	Iy = 2.13e+009 mm^4
Beta1 = 0.865		Min clear spacing = 136 mm	Clear cover = 40 mm

Confinement: Tied

phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.65, phi(rho) = 1.00, Min. Dimension(h) = 400 mm

Corner Column (Column #4):



400 x 400 mm

Code: CSA A23.3-14

Units: Metric

Run axis: Biaxial

Run option: Investigation

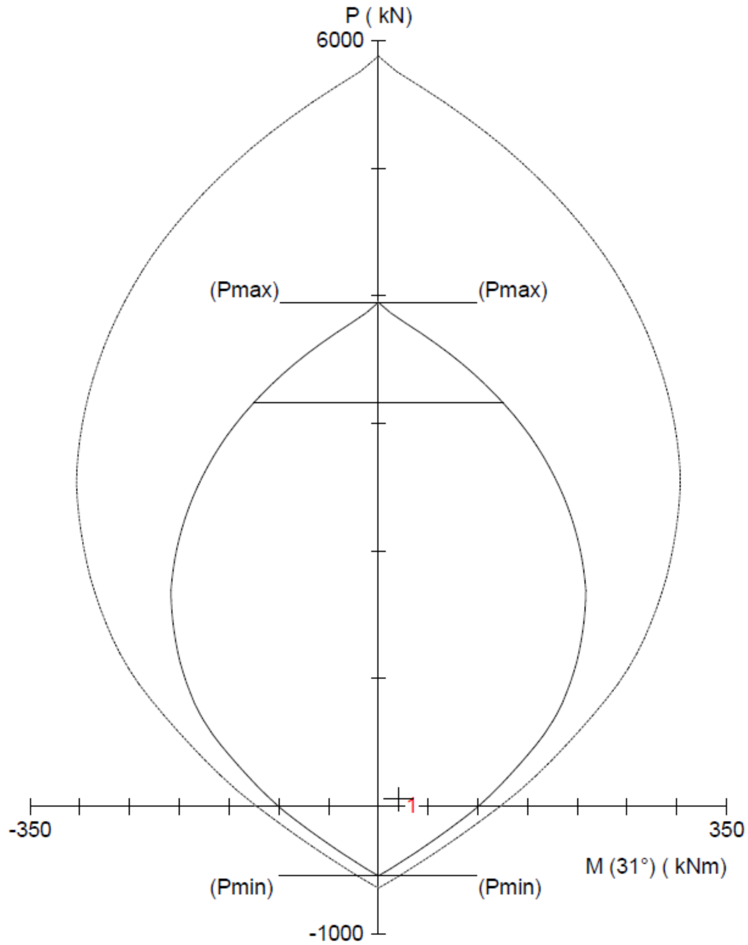
Slenderness: Not considered

Column type: Structural

Bars: CSA G30.18

Date: 06/05/18

Time: 11:28:45



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File: C:\TSDA\Two-Way Flat Plate System (CSA)\Column Design\Corner Column (#4).col

Project: Two-Way Flat Plate System

Column: Corner #4		Engineer: SP	
f'c = 42 MPa	fy = 400 MPa	Ag = 160000 mm^2	8 #15 bars
Ec = 30151 MPa	Es = 200000 MPa	As = 1600 mm^2	rho = 1.00%
fc = 33.054 MPa	e_yt = 0.002 mm/mm	Xo = 0 mm	Ix = 2.13e+009 mm^4
e_u = 0.0035 mm/mm		Yo = 0 mm	Iy = 2.13e+009 mm^4
Beta1 = 0.865		Min clear spacing = 136 mm	Clear cover = 40 mm

Confinement: Tied

phi(a) = 0.8, phi(s) = 0.85, phi(c) = 0.65, phi(rho) = 1.00, Min. Dimension(h) = 400 mm

4. Two-Way Slab Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with CSA A23.3-14 clause 13.

4.1. One-Way (Beam action) Shear Strength

CSA A23.3-14 (13.3.6)

One-way shear is critical at a distance d from the face of the column as shown in Figure 3. Figure 11 shows the factored shear forces (V_r) at the critical sections around each column. In members without shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$V_r = V_c + V_s + V_p = V_c \quad , \quad (V_s = V_p = 0)$$ ***CSA A23.3-14 (Eq. 11.4)***

Where:

$$V_c = \varphi_c \lambda \beta \sqrt{f_c^{'}} b_w d_v$$ ***CSA A23.3-14 (Eq. 11.5)***

$\lambda = 1$ for normal weight concrete

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm ***CSA A23.3-14 (11.3.6.2)***

$d_v = \text{Max}\ (0.9 d_{avg}, 0.72h) = \text{Max}\ (0.9 \times 154, 0.72 \times 190) = 139\ \text{mm}$ ***CSA A23.3-14 (3.2)***

$\sqrt{f_c^{'}} = 5.29\ \text{MPa} < 8\ \text{MPa}$ ***CSA A23.3-14 (11.3.4)***

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{28} \times 4200 \times \frac{139}{1000} = 421.67\ \text{kN} > V_f$$

Because $V_r \geq V_f$ at all the critical sections, the slab has adequate one-way shear strength.

Shear forces for the figure below:

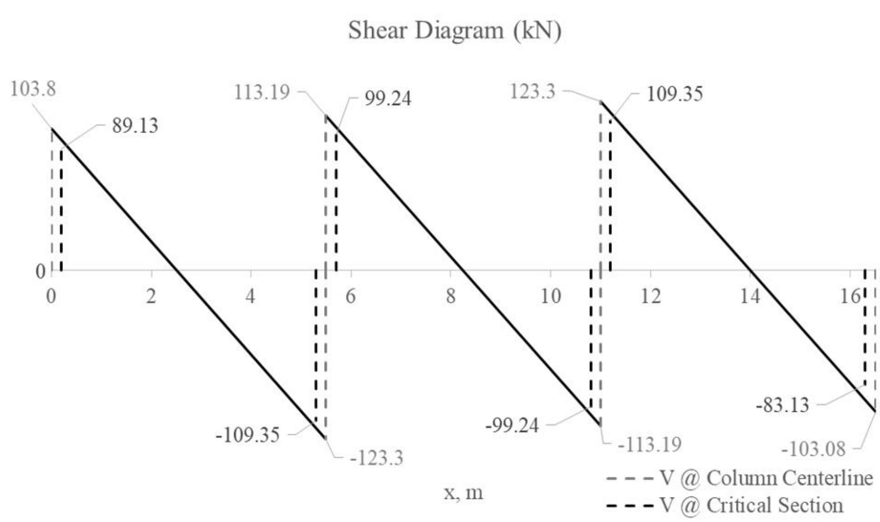


Figure 11 – One-way shear at critical sections (at distance d from the face of the supporting column)

4.2. Two-Way (Punching) Shear Strength

CSA A23.3-14 (13.3.2)

Two-way shear is critical on a rectangular section located at d/2 away from the face of the column as shown in Figure 5.

a. Exterior column:

The factored shear force (V_f) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section (*d/2* away from column face).

$$V_f = 103.8 - 9.8\left(0.477 \times 0.554\right) = 101.21\text{kN}$$

The factored unbalanced moment used for shear transfer, M_{unb}, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$\text{M}_{\text{unb}} = 64.1 - 101.21\left(\frac{477 - 150.9 - 400/2}{1000}\right) = 51.34 \text{ kN.m}$$

For the exterior column in Figure 5, the location of the centroidal axis z-z is:

$$\text{c}_{\text{AB}} = e = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{2\times(447\times154\times447/2)}{2\times447\times154+554\times154} = 150.9 \text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$\text{J}_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + \left(b_1 d\right)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 d c_{AB}^2$$

$$\text{J}_c = 2\left(\frac{477\times154^3}{12} + \frac{154\times477^3}{12} + \left(477\times154\right)\left(\frac{477}{2} - 150.9\right)^2\right) + 554\times154\times(150.9)^2 = 6.15\times10^9 \text{ mm}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.618 = 0.382$$

CSA A23.3-14 (Eq. 13.8)

The length of the critical perimeter for the exterior column:

$$\text{b}_\text{o} = 2\times(400+154/2)+(400+154) = 1508\,\text{mm}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{unb} e}{J}$$

CSA A23.3-14 (Eq.13.9)

$$v_f = \frac{101.21\times1000}{1508\times154} + \frac{0.382\times(51.34\times10^6)\times150.9}{6.15\times10^9} = 0.92 \text{ MPa}$$

The factored resisiting shear stress, V_r shall be the smallest of :

CSA A23.3-14 (13.3.4.1)

a) $v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\frac{2}{1}\right)0.19\times0.65\times\sqrt{28} = 1.96 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{3\times154}{1508}+0.19\right)\times1\times0.65\times\sqrt{28} = 1.71 \text{ MPa}$

c) $v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times1\times0.65\times\sqrt{28} = 1.31 \text{ MPa}$

Since $v_r \geq v_f$ at the critical section, the slab has adequate two-way shear strength at this joint.

b. Interior column:

$$V_f = 113.19+123.3-9.8\left(\frac{554\times554}{10^6}\right) = 233.48 \text{ kN}$$

$$M_{unb} = 119.7-108.5-233.48(0) = 11.2 \text{ kN.m}$$

For the interior column in Figure 5, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{554}{2} = 277 \text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left(\frac{b_1 d^3}{12}+\frac{d b_1^3}{12}+(b_1 d)\left(\frac{b_1}{2}-c_{AB}\right)^2\right)+b_2 d c_{AB}^2$$

$$J_c = 2\left(\frac{554\times154^3}{12}+\frac{154\times554^3}{12}+(554\times154)\left(\frac{554}{2}-277\right)^2\right)+2\times554\times154\times(277)^2 = 1.78\times10^{10} \text{ mm}^4$$

$\gamma_v = 1-\gamma_f = 1-0.60 = 0.40$ ***CSA A23.3-14 (Eq. 13.8)***

The length of the critical perimeter for the interior column:

$$b_o = 2\times(400+154)+2\times(400+154) = 2216 \text{ mm}$$

$v_f = \frac{V_f}{b_o\times d}+\frac{\gamma_v M_{unb} e}{J}$ ***CSA A23.3-14 (Eq.13.9)***

$$v_f = \frac{233.48\times1000}{2216\times154}+\frac{0.4\times(11.2\times10^6)\times277}{1.78\times10^{10}} = 0.75 \text{ MPa}$$

The factored resisiting shear stress, V_r shall be the smallest of : ***CSA A23.3-14 (13.3.4.1)***

a) $v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\frac{2}{1}\right)0.19\times0.65\times\sqrt{28} = 1.96 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{4\times154}{2216} + 0.19\right)\times1\times0.65\times\sqrt{28} = 1.61 \text{ MPa}$

c) $v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times1\times0.65\times\sqrt{28} = 1.31 \text{ MPa}$

Since $v_r \geq v_f$ at the critical section, the slab has adequate two-way shear strength at this joint.

c. Corner column:

In this example, interior equivalent elastic frame strip was selected where it only have exterior and interior supports (no corner supports are included in this strip). However, the two-way shear strength of corner supports usually governs. Thus, the two-way shear strength for the corner column in this example will be checked for educational purposes. Same procedure is used to find the reaction and factored unbalanced moment used for shear transfer at the centroid of the critical section for the corner support for the exterior equivalent elastic frame strip.

$$V_f = 56.56 - 9.8\left(\frac{477\times477}{10^6}\right) = 54.33 \text{ kN}$$

$$\text{M}_{\text{unb}} = 36.35 - 54.33\left(\frac{477 - 119.3 - 400/2}{1,000}\right) = 27.78 \text{ kN.m}$$

For the corner column in Figure 5, the location of the centroidal axis z-z is:

$$\text{c}_{\text{AB}} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{(477\times154\times154/2)}{2\times477\times154} = 119.3 \text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$J_c = \left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 d c_{AB}^2$$

$$\text{J}_c = \left(\frac{447\times154^3}{12} + \frac{447\times554^3}{12} + (447\times154)\left(\frac{447}{2} - 119.3\right)^2\right) + 447\times154\times(119.3)^2 = 3.63\times10^9 \text{ mm}^4$$

$\gamma_v = 1 - \gamma_f = 1 - 0.60 = 0.40$ ***CSA A23.3-14 (Eq.13.8)***

Where:

$$\gamma_f = \frac{1}{1 + (2/3)\times\sqrt{b_1/b_2}}$$

$$\gamma_f = \frac{1}{1 + (2/3)\times\sqrt{477/477}} = 0.60$$

The length of the critical perimeter for the corner column:

$b_o = (400 + 154/2) + (400 + 154/2) = 954 \text{ mm}$

The two-way shear stress (v_u) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{unb} e}{J}$$ **CSA A23.3-14 (Eq.13.9)**

$$v_f = \frac{54.33 \times 1000}{954 \times 154} + \frac{0.4 \times (27.78 \times 10^6) \times 119.3}{3.63 \times 10^9} = 0.74 \text{ MPa}$$

The factored resisiting shear stress, V_r shall be the smallest of : **CSA A23.3-14 (13.3.4.1)**

a) $v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c} = \left(1 + \frac{2}{1}\right) 0.19 \times 0.65 \times \sqrt{28} = 1.96 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c} = \left(\frac{2 \times 154}{954} + 0.19\right) \times 1 \times 0.65 \times \sqrt{28} = 1.76 \text{ MPa}$

c) $v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{28} = 1.31 \text{ MPa}$

Since $v_r \geq v_f$ at the critical section, the slab has adequate two-way shear strength at this joint.

5. Two-Way Slab Deflection Control (Serviceability Requirements)

Since the slab thickness was selected based on the minimum slab thickness equations in CSA A23.3-14, the deflection calculations are not required. However, the calculations of immediate and time-dependent deflections are covered in this section for illustration and comparison with spSlab model results.

5.1. Immediate (Instantaneous) Deflections

When deflections are to be computed, deflections that occur immediately on application of load shall be computed by methods or formulas for elastic deflections, taking into consideration the effects of cracking and reinforcement on member stiffness. Unless deflections are determined by a more comprehensive analysis, immediate deflection shall be computed using elastic deflection equations. **CSA A23.3-14 (9.8.2.2 & 9.8.2.3)**

Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{Full}$) is used to obtain immediate deflections of the two-way slab in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests.

The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the slab. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g$$ **CSA A23.3-14 (Eq.9.1)**

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously in this document. These moments are shown in Figure 12.

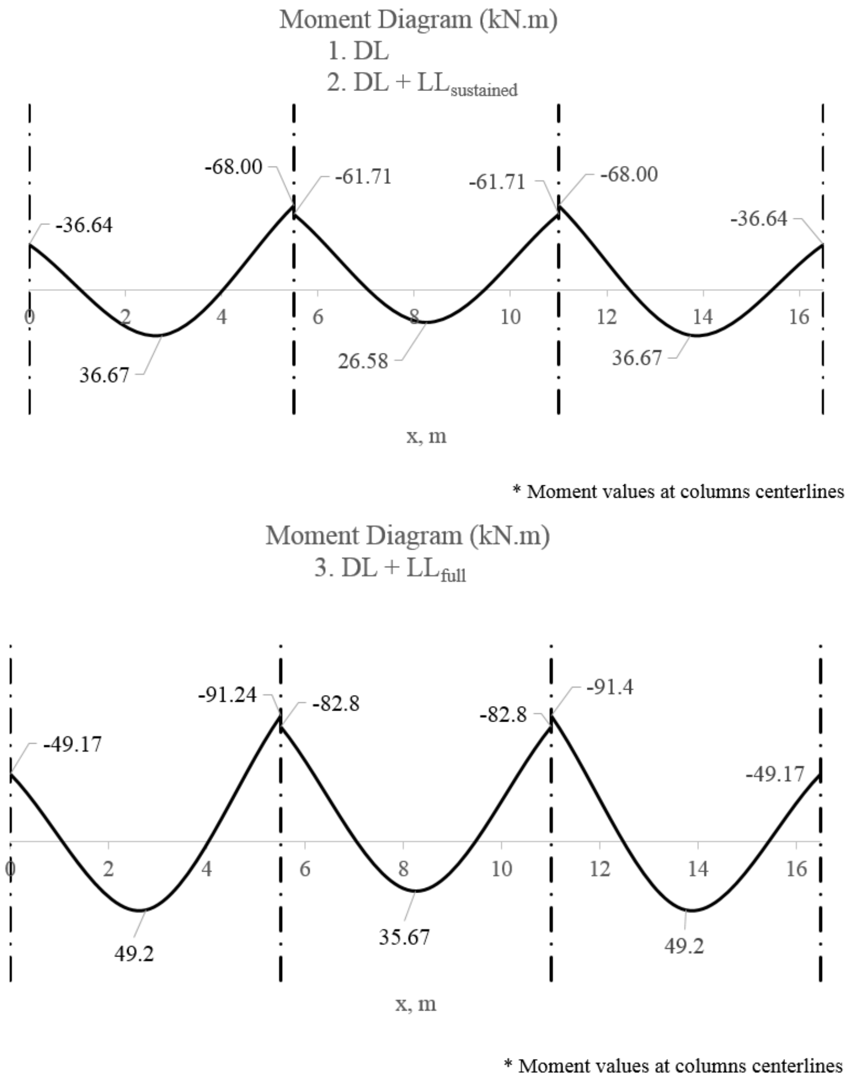


Figure 12 – Maximum Moments for the Three Service Load Levels

M_{cr} = cracking moment.

$$M_{cr} = \frac{f_r I_g}{Y_t} = \frac{(3.17/2)\times(2.4\times10^9)}{95}\times10^{-6} = 40.11\text{ kN.m}$$ **CSA A23.3-14 (Eq.9.2)**

f_r should be taken as half of Eq.8.3 **CSA A23.3-14 (9.8.2.3)**

f_r = Modulus of rapture of concrete.

$$f_r = 0.6\lambda\sqrt{f_c'} = 0.6\times1.0\times\sqrt{28} = 3.17\text{ MPa}$$ **CSA A23.3-14 (Eq.8.3)**

I_g = Moment of inertia of the gross uncracked concrete section

$$I_g = \frac{l_2 h^3}{12} = \frac{4200\ (190)^3}{12} = 2.4\times10^9\ \text{mm}^4$$

$$Y_t = \frac{h}{2} = \frac{190}{2} = 95\,\text{mm}$$

I_{cr} = moment of inertia of the cracked section transformed to concrete.

CAC Concrete Design Handbook 4th Edition (5.2.3)

The calculations shown below are for the design strip (frame strip). The values of these parameters for column and middle strips are shown in Table 9.

As calculated previously, the exterior span frame strip near the interior support is reinforced with 14 – 15 M bars located at 350 mm along the section from the top of the slab. Figure 13 shows all the parameters needed to calculate the moment of inertia of the cracked section transformed to concrete.

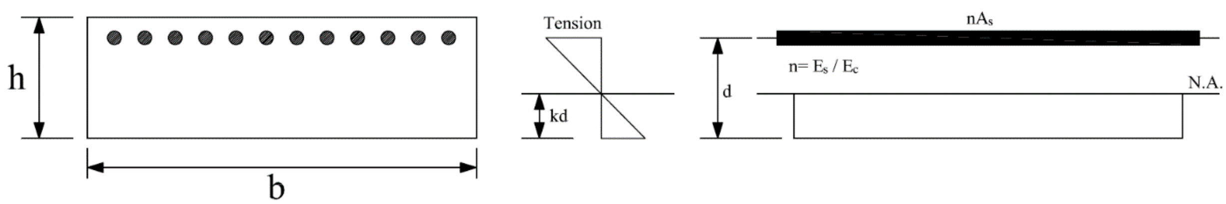


Figure 13 – Cracked Transformed Section

E_{cs} = Modulus of elasticity of slab concrete.

$$E_{cs} = (3{,}300\sqrt{f_c'} + 6{,}900)\left(\frac{\gamma_c}{2{,}300}\right)^{1.5} = (3{,}300\sqrt{28} + 6{,}900)\left(\frac{2{,}447}{2{,}300}\right)^{1.5} = 26{,}739\ \text{MPa}$$ ***CSA A23.3-14(8.6.2.2)***

$$n = \frac{E_s}{E_{cs}} = \frac{200{,}000}{26{,}739} = 7.48$$ ***CAC Concrete Design Handbook 4th Edition (Table 6.2a)***

$$B = \frac{b}{n\ A_s} = \frac{4200}{7.48\times(14\times200)} = 0.2\ \text{mm}^{-1}$$ ***CAC Concrete Design Handbook 4th Edition (Table 6.2a)***

$$kd = \frac{\sqrt{2dB+1}-1}{B} = \frac{\sqrt{2\times154\times0.2+1}-1}{0.2} = 34.52\ \text{mm}$$

CAC Concrete Design Handbook 4th Edition (Table 6.2a)

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$$ ***CAC Concrete Design Handbook 4th Edition (Table 6.2a)***

$$I_{cr} = \frac{4{,}200\times(34.52)^3}{3} + 7.48\times(14\times200)\times(154-34.52)^2 = 3.57\times10^8\ \text{mm}^4$$

The effective moment of inertia procedure is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by Eq. (9.1) in CSA A23.3-14 unless obtained by a more comprehensive analysis.

For continuous prismatic members, the effective moment of inertia may be taken as the weighted average of the values obtained from Eq. (9.1) in CSA A23.3-14 for the critical positive and negative moment sections.

CSA A23.3-14(9.8.2.4)

For the exterior span (span with one end continuous) with service load level ($D+LL_{full}$):

For negative moment section:

$$I_{ec} = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{, since } M_{cr} = 40.11 \text{ kN.m} < M_a = 91.24 \text{ kN.m}$$ ***CSA A23.3-14 (Eq.9.1)***

Where I_{ec} is the effective moment of inertia for the critical negative moment section (near the support).

$$I_{ec} = 3.57\times10^8 + \left(2.4\times10^9 - 3.57\times10^8\right)\left(\frac{40.11}{91.24}\right)^3 = 5.3\times10^8 \text{ mm}^4$$

For positive moment section:

$$M_{cr} = 40.11 \text{ kN.m} < M_a = 49.2 \text{ kN.m}$$

Two of these bars are not continuous and will be conservatively excluded from the calculation of Icr since they might not be adequately developed or tied (10 bars are used).

$$B = \frac{b}{n\,A_s} = \frac{4,200}{7.48\times(10\times200)} = 0.28 \text{ mm}^{-1}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$kd = \frac{\sqrt{2dB+1}-1}{B} = \frac{\sqrt{2\times154\times0.28+1}-1}{0.28} = 29.75 \text{ mm}$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$$ ***PCA Notes on ACI 318-11 (Table 10-2)***

$$I_{cr} = \frac{4200\times(29.75)^3}{3} + 7.48\times(10\times200)\times(154-29.75)^2 = 2.68\times10^8 \text{ mm}^4$$

$$I_{em} = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{, since } M_{cr} = 40.11 \text{ kN.m} < M_a = 49.2 \text{ kN.m}$$ ***CSA A23.3-14 (Eq.9.1)***

$$I_{em} = 2.68\times10^8 + \left(2.4\times10^9 - 2.68\times10^8\right)\left(\frac{40.11}{49.2}\right)^3 = 1.42\times10^9 \text{ mm}^4$$

Where I_{em} is the effective moment of inertia for the critical positive moment section (midspan).

Since midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan section is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.85\, I_{em} + 0.15\, I_{ec} \text{ for one end continuous}$$ ***CSA A23.3-14 (Eq.9.4)***

$$I_{e,avg} = 0.85\,\left(1.42\times10^9\right) + 0.15\,\left(5.3\times10^8\right) = 1.29\times10^9 \text{ mm}^4$$

For the interior span (span with both ends continuous) with service load level (*D+LL_{full}*):

$$I_{ec} = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{ , since } M_{cr} = 41.11 \text{ kN.m} < M_a = 82.8 \text{ kN.m}$$ **CSA A23.3-14 (Eq.9.1)**

$$I_{ec} = 3.57\times10^8 + \left(2.4\times10^9 - 3.57\times10^8\right)\left(\frac{40.11}{82.8}\right)^3 = 5.89\times10^8 \text{ mm}^4$$

$$I_{em} = I_g = 2.4\times10^9 \text{ mm}^4 \text{ , since } M_{cr} = 40.11 \text{ kN.m} > M_a = 35.67 \text{ kN.m}$$

The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.70\, I_{em} + 0.15\, \left(I_{e1} + I_{e2}\right) \text{ for two ends continuous}$$ **CSA A23.3-14 (Eq.9.3)**

$$I_{e,avg} = 0.70\left(2.4\times10^9\right) + 0.15\left(5.89\times10^8 + 5.89\times10^8\right) = 1.86\times10^9 \text{ mm}^4$$

Where:

I_{e1} = The effective moment of inertia for the critical negative moment section at end 1 of continuous beam span.

I_{e2} = The effective moment of inertia for the critical negative moment section at end 2 of continuous beam span.

Table 9 provides a summary of the required parameters and calculated values needed for deflections for exterior and interior equivalent elastic frame. It also provides a summary of the same values for column strip and middle strip to facilitate calculation of panel deflection.

Table 9 – Averaged Effective Moment of Inertia Calculations													
For Frame Strip													
Span	zone	I_g, mm^4 ($\times 10^8$)	I_{cr}, mm^4 ($\times 10^8$)	M_a, kN.m			M_{cr}, kN.m	I_e, mm^4 ($\times 10^8$)			$I_{e,avg}$, mm^4 ($\times 10^8$)		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	24	3.78	-36.64	-36.64	-49.17	40.11	24	24	14.5	21.6	21.6	12.9
	Midspan		2.68	36.67	36.67	49.2		24	24	14.4			
	Right		3.57	-68.00	-68.00	-91.24		7.76	7.76	5.3			
Int	Left		3.57	-61.71	-61.71	-82.80		9.18	9.18	5.89	19.6	19.6	18.6
	Mid		2.68	26.58	26.58	35.67		24	24	24			
	Right		3.57	-61.71	-61.71	-82.80		9.18	9.18	5.89			

Deflections in two-way slab systems shall be calculated taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. For immediate deflections two-way slab systems the midpanel deflection is computed as the sum of deflection at midspan of the column strip or column line in one direction (Δ_{cx} or Δ_{cy}) and deflection at midspan of the middle strip in the orthogonal direction (Δ_{mx} or Δ_{my}). Figure 14 shows the deflection computation for a rectangular panel. The average Δ for panels that have different properties in the two direction is calculated as follows:

$$\Delta = \frac{\left(\Delta_{cx} + \Delta_{my}\right) + \left(\Delta_{cy} + \Delta_{mx}\right)}{2}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)

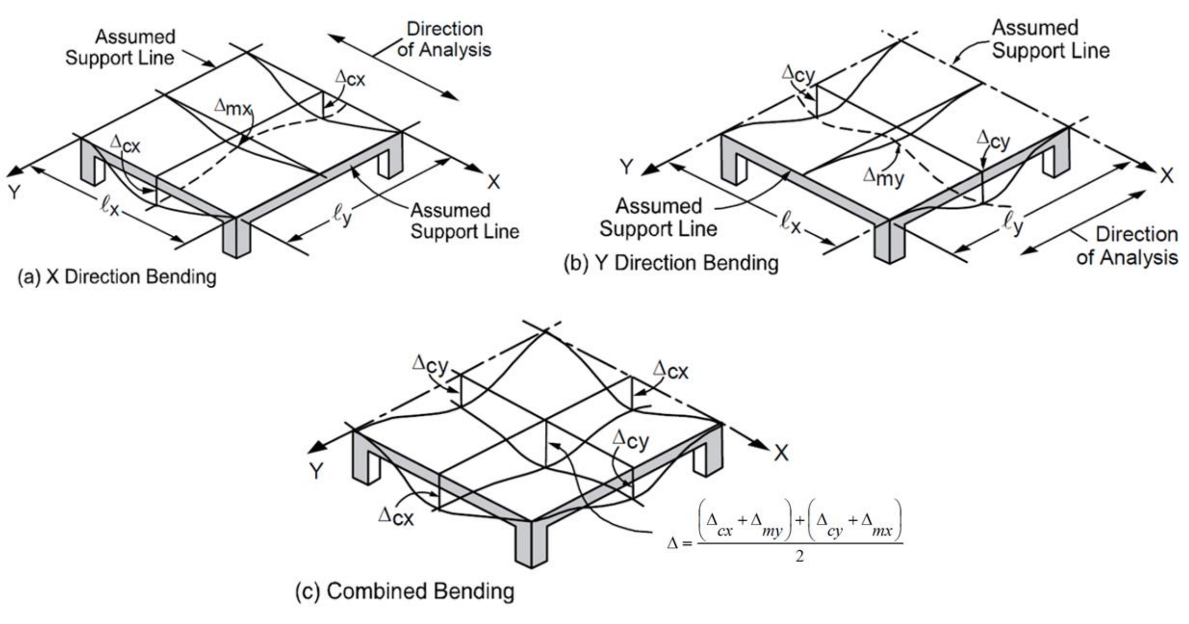


Figure 14 – Deflection Computation for a rectangular Panel

To calculate each term of the previous equation, the following procedure should be used. Figure 15 shows the procedure of calculating the term Δ_{cx}. same procedure can be used to find the other terms.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \frac{I_{frame}}{I_c}$$

$$LDF_c = \frac{LDF^+ + \frac{LDF_l^- + LDF_R^-}{2}}{2}$$

$$\Delta_{frame,fixed} = \frac{wl^4}{384 E_c I_{frame,averaged}}$$

$$\Delta\theta_{c,R} = \theta_{c,R}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame} \qquad \theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}}$$

$$\Delta\theta_{c,L} = \theta_{c,L}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame} \qquad \theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$

Figure 15 – Δ_{cx} calculation procedure

For exterior span - service dead load case:

$$\Delta_{frame,fixed} = \frac{wl^4}{384 E_c I_{frame,averaged}}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 10)***

Where:

$\Delta_{frame,fixed}$ = Deflection of column strip assuing fixed end condition.

$$w = (1 + 24 \times .19)(4.2) = 23.35 \text{ kN/m}$$

$$E_c = (3{,}300\sqrt{f_c'} + 6{,}900)\left(\frac{\gamma_c}{2{,}300}\right)^{1.5}$$ ***CSA A23.3-14(8.6.2.2)***

$$E_c = (3{,}300\sqrt{28} + 6{,}900)\left(\frac{2{,}447}{2{,}300}\right)^{1.5} = 26{,}739 \text{ MPa}$$

$I_{frame,averaged}$ = The averaged effective moment of inertia ($I_{e,avg}$) for the frame strip for service dead load case from Table 9 = 21.6×10^8

$$\Delta_{frame,fixed} = \frac{(23.35)(5{,}500)^4}{384(26{,}739)(21.6 \times 10^8)} \times 10^{-3} = 0.96 \text{ mm}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \frac{I_{frame}}{I_c}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 11)***

Where LDF_c is the load distribution factor for the column strip. The load distribution factor for the column strip can be found from the following equation:

$$LDF_c = \frac{LDF^+ + \frac{LDF_l^- + LDF_R^-}{2}}{2}$$

And the load distribution factor for the middle strip can be found from the following equation:

$$LDF_m = 1 - LDF_c$$

For the end span, LDF for exterior negative region (LDF_L^-), interior negative region (LDF_R^-), and positive region (LDF_L^+) are 1.00, 0.75, and 0.60, respectively (From Table 6 of this document). Thus, the load distribution factor for the column strip for the end span is given by:

$$LDF_c = \frac{0.6 + \frac{1.0 + 0.8}{2}}{2} = 0.75$$

$I_{c,g}$ = The gross moment of inertia (I_g) for the column strip for service dead load = 1.2×10^9 mm^4

$$\Delta_{c,fixed} = 0.75 \times 0.96 \times \frac{2.4\times10^9}{1.2\times10^9} = 1.45 \text{ mm}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 12)***

Where:

$\theta_{c,L}$ = Rotation of the span left support.

$(M_{net,L})_{frame} = 36.64$ kN.m = Net frame strip negative moment of the left support.

K_{ec} = effective column stiffness = 6.91×10^6 N.m (calculated previously).

$$\theta_{c,L} = \frac{36.64\times10^3}{69.1\times10^6} = 0.00053 \text{ rad}$$

$$\Delta\theta_{c,L} = \theta_{c,L}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 14)***

Where:

$\Delta\theta_{c,L}$ = Midspan deflection due to rotation of left support.

$\left(\frac{I_g}{I_e}\right)_{frame}$ = Gross-to-effective moment of inertia ratio for frame strip.

$$\Delta\theta_{c,L} = 0.00053 \times \frac{5,500}{8} \times \frac{2.4\times10^9}{2.16\times10^9} = 0.41 \text{ mm}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}} = \frac{(68-61.71)\times10^3}{69.1\times10^6} = 0.000091 \text{ rad}$$

Where

$\theta_{c,R}$ = rotation of the span right support.

$\left(M_{net,R}\right)_{frame}$ = Net frame strip negative moment of the right support.

$$\Delta\theta_{c,R} = \theta_{c,R}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame} = 0.000091\times\frac{5{,}500}{8}\times\frac{2.4\times10^9}{2.1\times10^9} = 0.07 \text{ mm}$$

Where:

$\Delta\theta_{c,R}$ = Midspan deflection due to rotation of right support.

$$\Delta_{cx} = \Delta_{cx,\,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 9)***

$$\Delta_{cx} = 1.45 + 0.07 + 0.41 = 1.92 \text{ mm}$$

Following the same procedure, Δ_{mx} can be calculated for the middle strip. This procedure is repeated for the equivalent frame in the orthogonal direction to obtain Δ_{cy}, and Δ_{my} for the end and middle spans for the other load levels ($D+LL_{sus}$ *and* $D+LL_{full}$).

Assuming square panel, $\Delta_{cx} = \Delta_{cy}$= 1.92 mm. and $\Delta_{mx} = \Delta_{my}$= 0.96 mm

The average Δ for the corner panel is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2} = \left(\Delta_{cx} + \Delta_{my}\right) = \left(\Delta_{cy} + \Delta_{mx}\right) = 1.92 + 0.96 = 2.88 \text{ mm}$$

Table 10 – Immediate (Instantaneous) Deflections in the x-direction

Column Strip

Span	LDF	D						
		$\Delta_{frame\text{-}fixed}$, mm	$\Delta_{c\text{-}fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.75	0.96	1.45	0.00053	0.000091	0.41	0.07	1.92
Int	0.7	1.06	1.49	-0.00091	-0.00091	-0.08	-0.08	1.33

Span	LDF	$D+LL_{sus}$						
		$\Delta_{frame\text{-}fixed}$, mm	$\Delta_{c\text{-}fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.75	0.96	1.45	0.00053	0.000091	0.41	0.07	1.92
Int	0.7	1.06	1.49	-0.00091	-0.00091	-0.08	-0.08	1.33

Span	LDF	$D+LL_{full}$						
		$\Delta_{frame\text{-}fixed}$, mm	$\Delta_{c\text{-}fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.75	2.16	3.25	0.00071	0.00012	0.91	0.16	4.31
Int	0.7	1.50	2.1	-0.00012	-0.00012	-0.11	-0.11	1.88

Span	LDF	LL
		Δ_{cx}, mm
Ext	0.75	2.39
Int	0.7	0.55

Middle Strip

LDF	D						
	$\Delta_{frame\text{-}fixed}$, mm	$\Delta_{m\text{-}fixed}$, mm	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.25	0.96	0.48	0.00053	0.000091	0.41	0.07	0.96
0.30	1.06	0.64	-0.00009	-0.00009	-0.08	-0.08	0.48

LDF	$D+LL_{sus}$						
	$\Delta_{frame\text{-}fixed}$, mm	$\Delta_{m\text{-}fixed}$, mm	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.25	0.96	0.48	0.00053	0.000091	0.41	0.07	0.96
0.30	1.06	0.64	-0.00009	-0.00009	-0.08	-0.08	0.48

LDF	$D+LL_{full}$						
	$\Delta_{frame\text{-}fixed}$, mm	$\Delta_{m\text{-}fixed}$, mm	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.25	2.16	1.08	0.00071	0.00012	0.91	0.16	2.15
0.30	1.50	0.9	-0.00012	-0.00012	-0.11	-0.11	0.68

LDF	LL
	Δ_{mx}, mm
0.25	1.19
0.30	0.2

From the analysis in the transverse direction the deflection values below are obtained:

For DL loading case:

Δ_{my}

Δ_{cy}

For DL+LL$_{sust}$ loading case:

Δ_{my}

Δ_{cy}

For DL+LL$_{full}$ loading case:

Δ_{my}

Δ_{cy}

These values for the x-direction are shown in Table 10. Then, the total midpanel deflection is calculated by combining the contributions of the column and middle strip deflections from the X and Y directions:

$$\Delta = \frac{\left(\Delta_{cx} + \Delta_{my}\right) + \left(\Delta_{cy} + \Delta_{mx}\right)}{2}$$ ***PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)***

5.2. Time-Dependent (Long-Term) Deflections (Δ_{lt}) (CSA)

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) may be estimated as follows:

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total immediate and long-term deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1 + \lambda_{\Delta}) + [(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}]$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst}$ = Immediate (instantaneous) deflection due to sustained load

Unless values are obtained by a more comprehensive analysis, the total immediate plus long-term deflection for flexural members shall be obtained by multiplying the immediate deflection caused by the sustained load considered by the factor ζ_s, as follows: ***CSA23.3-14 (9.8.2.5)***

$$\xi_s = \left[1 + \frac{s}{1 + 50\rho'}\right]$$ ***CSA23.3-14 (Eq .9.5)***

For the exterior span

s = 2, consider the sustained load duration to be 60 months or more. ***CSA A23.3-14 (9.8.2.5)***

ρ' = 0, conservatively.

$$\frac{s}{1+50\rho'} = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2\times 1.92 = 3.84 \text{ mm}$$

$$\left(\Delta_{total}\right)_{lt} = 1.92\times(1+2)+(4.31-1.92) = 8.16 \text{ mm}$$

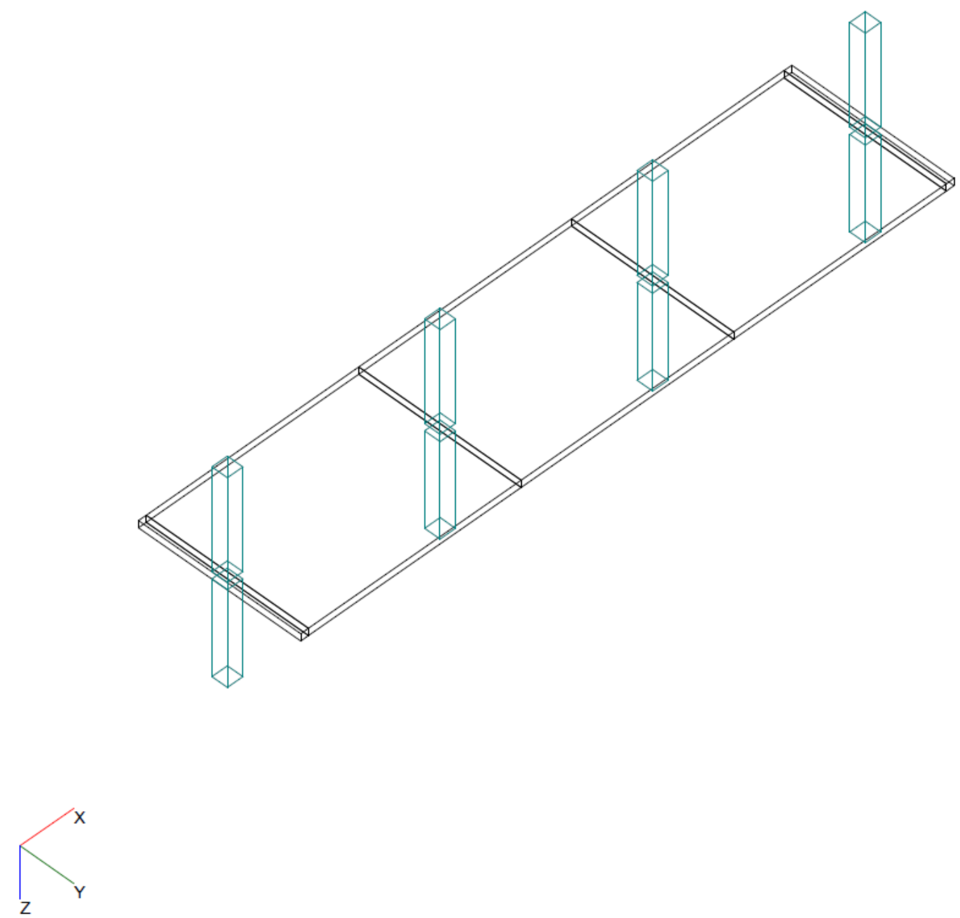
Table 11 shows long-term deflections for the exterior and interior spans for the analysis in the x-direction, for column and middle strips.

Table 11 - Long-Term Deflections					
Column Strip					
Span	**$(\Delta_{sust})_{Inst}$, mm**	**λ_{Δ}**	**Δ_{cs}, mm**	**$(\Delta_{total})_{Inst}$, mm**	**$(\Delta_{total})_{lt}$, mm**
Exterior	1.92	2.000	3.84	4.31	6.37
Interior	1.33	2.000	2.66	1.88	4.64
Middle Strip					
Exterior	0.96	2.000	1.92	2.15	4.06
Interior	0.48	2.000	0.96	0.68	1.65

6. Computer Program Solution

spSlab program utilizes the Elastic (Equivalent) Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept.

spSlab Program models the equivalent elastic frame as a design strip. The design strip is, then, separated by spSlab into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips. The graphical and text results are provided below for both input and output of the spSlab model.



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File: C:\TSDA\Two-Way Flat Plate System (CSA)\TSDA-spSlab-Two-Way Flat Plate Floor-CSA 23.3-14.slb

Project: Two-Way Flat Plate Floor

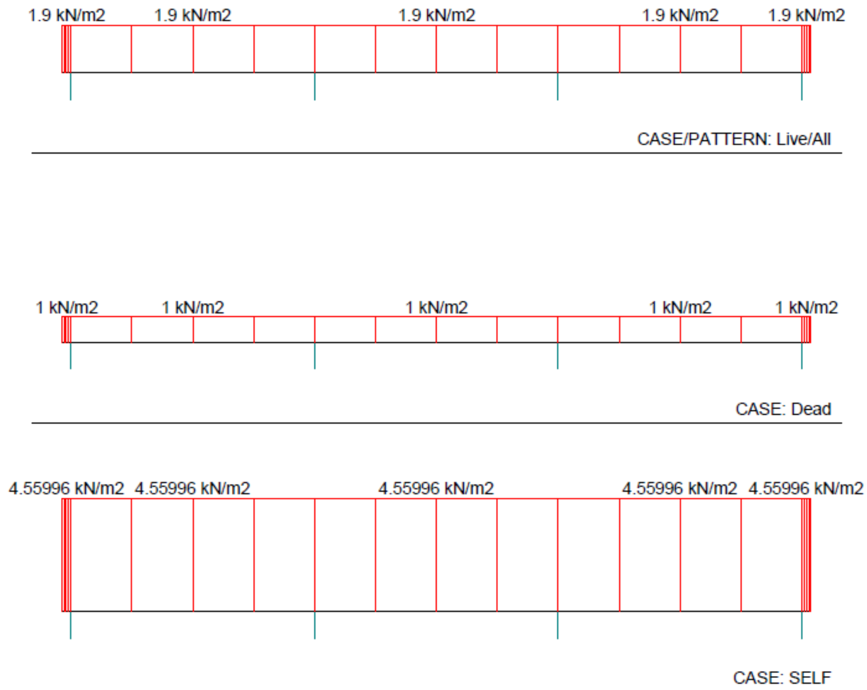
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 06/05/18

Time: 11:57:36



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Project: Two-Way Flat Plate Floor

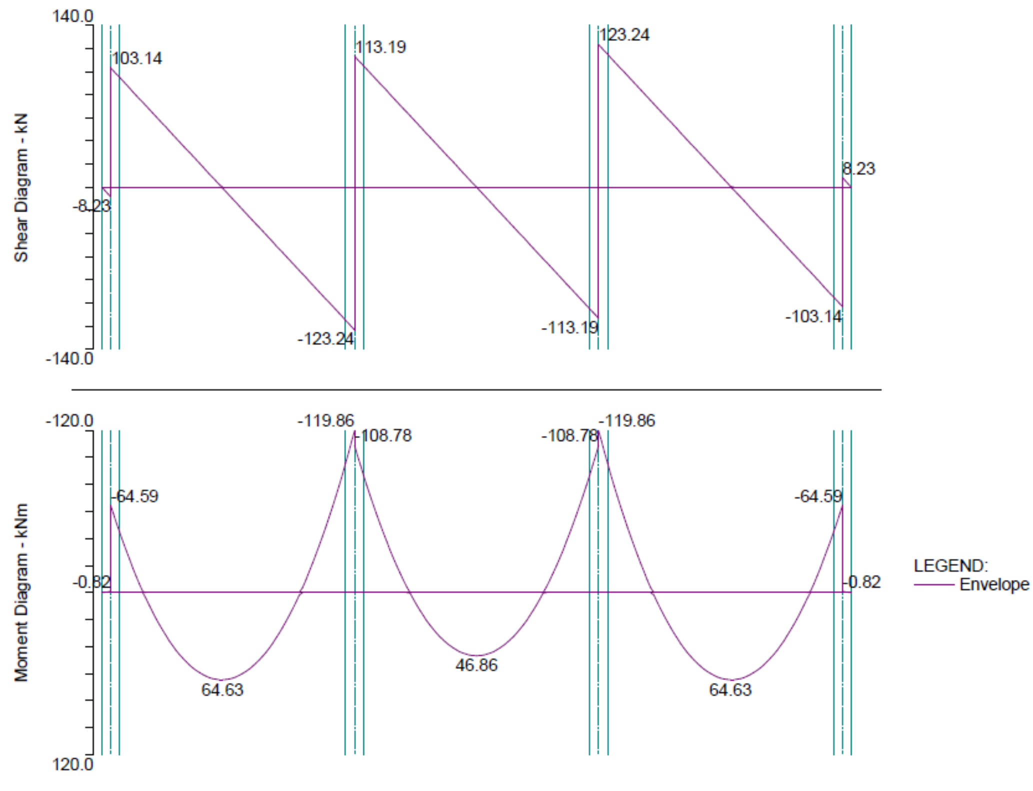
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 06/05/18

Time: 11:57:11



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Project: Two-Way Flat Plate Floor

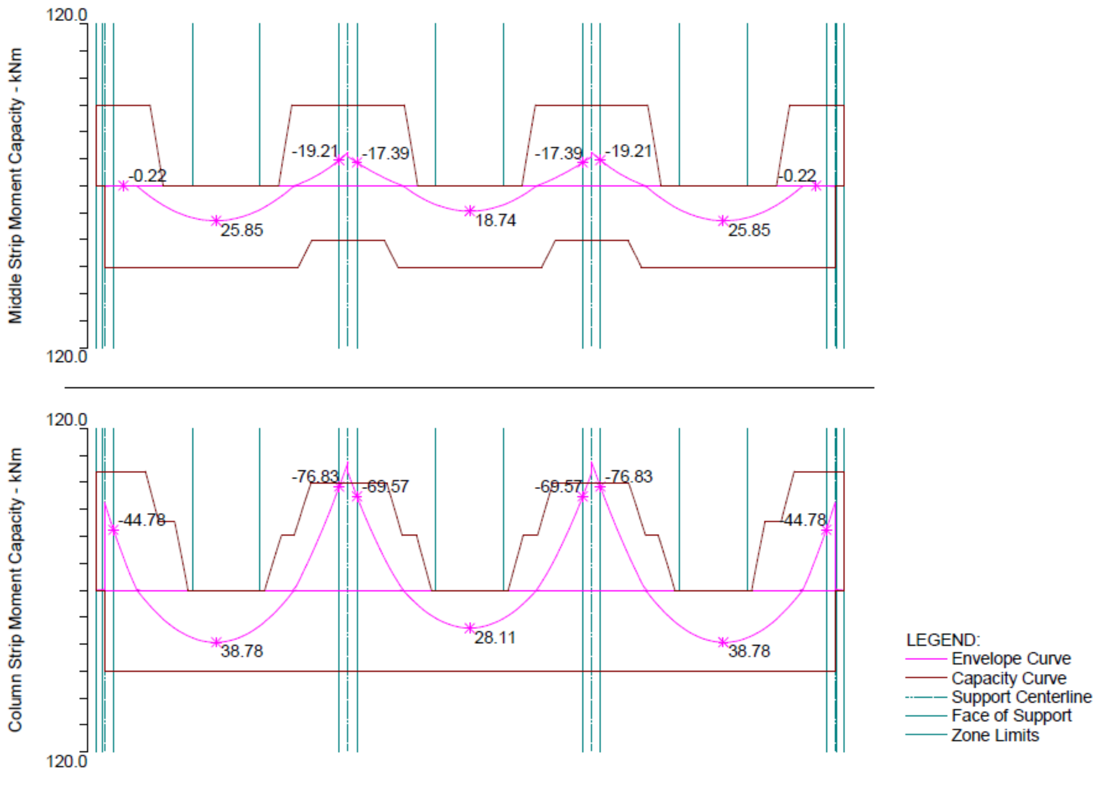
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 06/05/18

Time: 11:58:09



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Project: Two-Way Flat Plate Floor

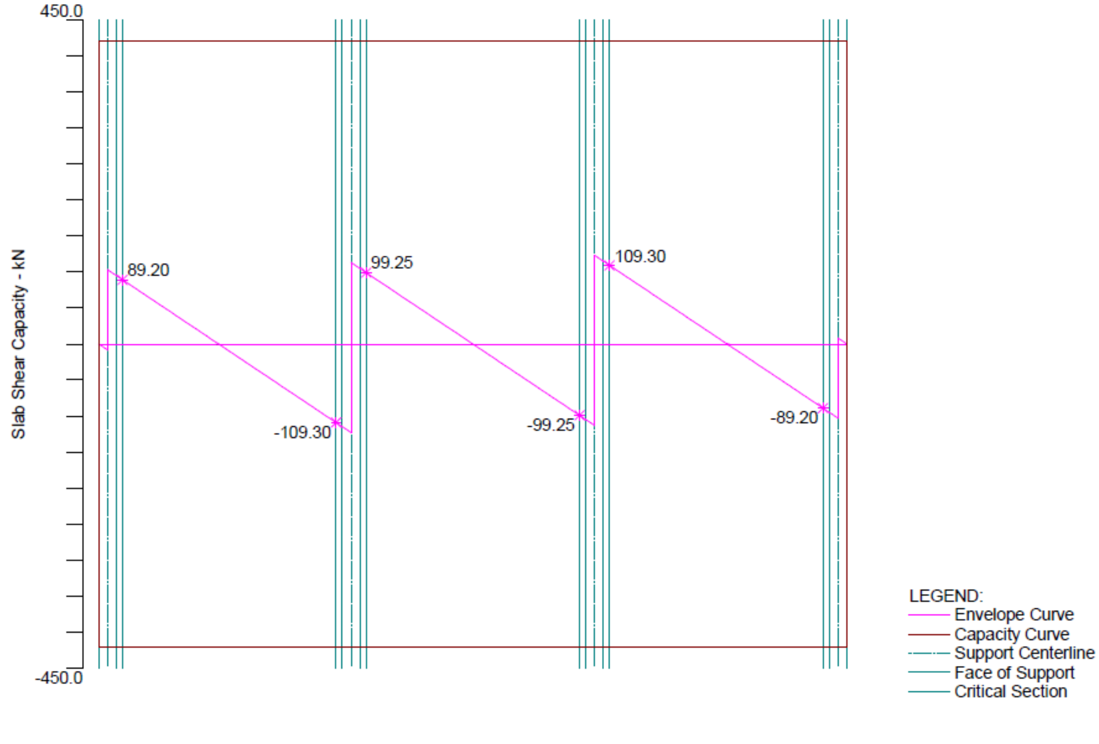
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 06/05/18

Time: 14:12:18



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File: C:\TSDA\Two-Way Flat Plate System (CSA)\TSDA-spSlab-Two-Way Flat Plate Floor-CSA 23.3-14.slb

Project: Two-Way Flat Plate Floor

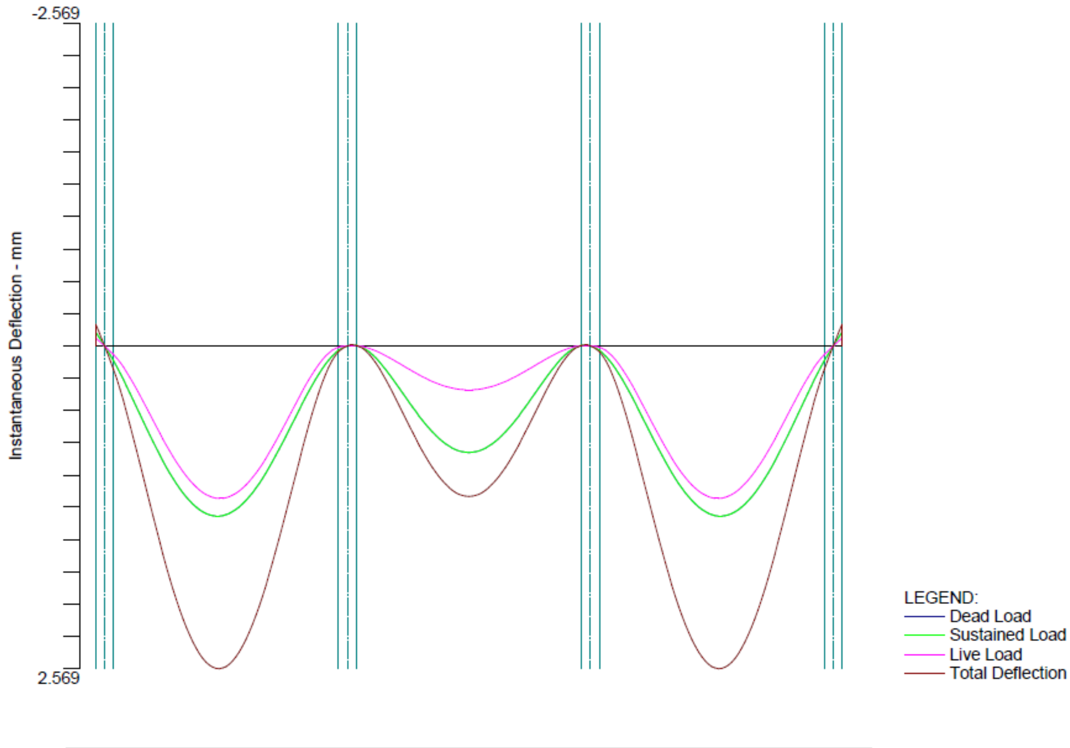
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 06/05/18

Time: 14:11:17



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Project: Two-Way Flat Plate Floor

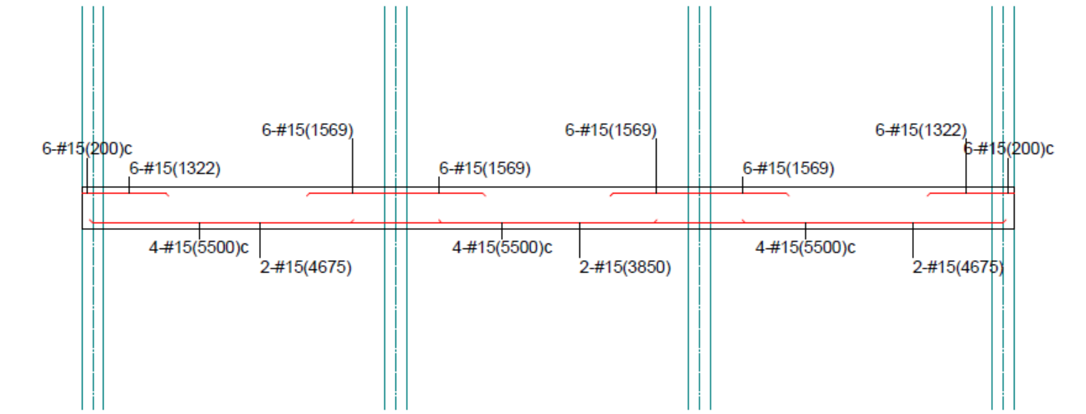
Frame: Interior Frame

Engineer: SP

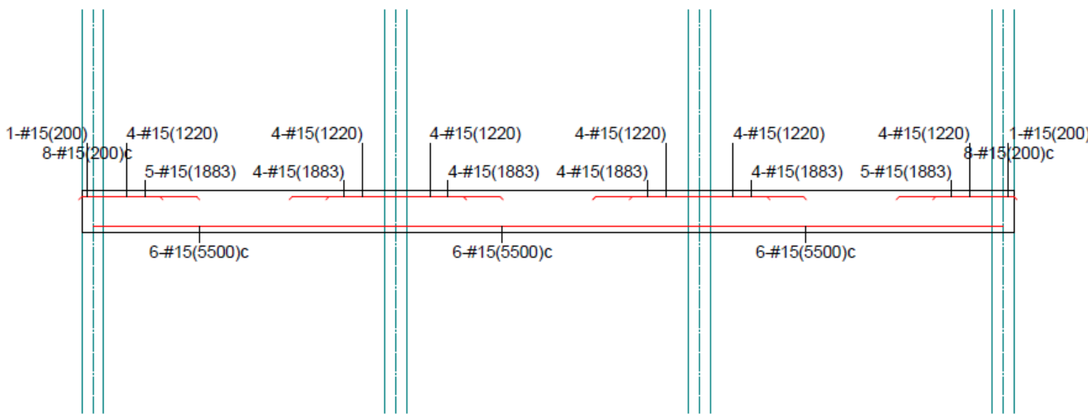
Code: CSA A23.3-14

Date: 06/05/18

Time: 11:59:31



Middle Strip Flexural Reinforcement



Column Strip Flexural Reinforcement

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Project: Two-Way Flat Plate Floor

Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 06/05/18

Time: 12:26:17

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         oo        oo   oo     ooo      oo       oooooo   oooooo
          ooooo    oo   oo       ooo    oo      oo   oo   oo   oo
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          ooooo    oo         oooooo     ooo     ooooo o   ooooo    (TM)

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                           A Computer Program for Analysis, Design, and Investigation of
                            Reinforced Concrete Beams, One-way and Two-way Slab Systems
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==================================================================================================================
[1] INPUT ECHO
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General Information
===================
   File name: C:\TSDA\Two-Way Flat Plate System (CSA)\TSDA-spSlab-Two-Way Flat Plate Floor-CSA 23.3-14.slb
   Project: Two-Way Flat Plate Floor
   Frame: Interior Frame
   Engineer: SP
   Code: CSA A23.3-14
   Reinforcement Database: CSA G30.18
   Mode: Design
   Number of supports = 4 + Left cantilever + Right cantilever
   Floor System: Two-Way

   Live load pattern ratio = 0%
   Minimum free edge distance for punching shear = 5 times slab effective depth.
   Circular critical section around circular supports used (if possible).
   Deflections are based on cracked section properties.
   In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
   Long-term deflections are calculated for load duration of 60 months.
   0% of live load is sustained.
   Compression reinforcement calculations NOT selected.
   Default incremental rebar design selected.
   Combined M-V-T reinforcement design NOT selected.
   User-defined slab strip widths NOT selected.
   User-defined distribution factors NOT selected.
   One-way shear in drop panel NOT selected.
   Distribution of shear to strips NOT selected.
   Beam T-section design NOT selected.
   Longitudinal beam contribution in negative reinforcement design over support NOT selected.
   Transverse beam contribution in negative reinforcement design over support NOT selected.

Material Properties
===================
            Slabs|Beams        Columns
           ------------   ------------
   wc    =       2447.3         2447.3 kg/m3
   f'c   =           28             42 MPa
   Ec    =        26739          31047 MPa
   fr    =       1.5875         3.8884 MPa
   Precast concrete construction is not selected.

   fy    =          400 MPa, Bars are not epoxy-coated
   fyt   =          400 MPa
   Es    =       199950 MPa

Reinforcement Database
======================
   Units: Db (mm), Ab (mm^2), Wb (kg/m)
   Size       Db       Ab       Wb   Size       Db       Ab       Wb
   ---- -------- -------- --------   ---- -------- -------- --------
    #10       11      100        1    #15       16      200        2
    #20       20      300        2    #25       25      500        4
```

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    #30       30      700         5      #35       36      1000         8
    #45       44     1500        12      #55       56      2500        20

Span Data
=========
   Slabs
   -----
   Units: L1, wL, wR, L2L, L2R (m); t, Hmin (mm)
   Span Loc        L1        t        wL        wR       L2L       L2R      Hmin
   ---- ---- -------- -------- -------- -------- -------- -------- --------
      1 Int     0.200      190    2.100    2.100    4.200    4.200       --- LC *i
      2 Int     5.500      190    2.100    2.100    4.200    4.200       187
      3 Int     5.500      190    2.100    2.100    4.200    4.200       170
      4 Int     5.500      190    2.100    2.100    4.200    4.200       187
      5 Int     0.200      190    2.100    2.100    4.200    4.200       --- RC *i
   NOTES:
   Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
   *i - cantilever end span (LC, RC) support condition

Support Data
============
   Columns
   -------
   Units: c1a, c2a, c1b, c2b (mm); Ha, Hb (m)
   Supp       c1a      c2a        Ha         c1b      c2b        Hb  Red%
   ---- -------- -------- --------   -------- -------- -------- ----
      1       400      400    2.750         400      400    2.750   100
      2       400      400    2.750         400      400    2.750   100
      3       400      400    2.750         400      400    2.750   100
      4       400      400    2.750         400      400    2.750   100

   Boundary Conditions
   -------------------
   Units: Kz (kN/mm); Kry (kN-mm/rad)
   Supp    Spring Kz   Spring Kry Far End A Far End B
   ---- ------------ ------------ --------- ---------
      1            0            0   Fixed     Fixed
      2            0            0   Fixed     Fixed
      3            0            0   Fixed     Fixed
      4            0            0   Fixed     Fixed

Load Data
=========
   Load Cases and Combinations
   ---------------------------
   Case     SELF     Dead     Live
   Type     DEAD     DEAD     LIVE
   ---- -------- -------- --------
   U1      1.400    1.400    0.000
   U2      1.250    1.250    1.500
   U3      0.900    0.900    1.500

   Area Loads
   ----------
   Units: Wa (kN/m2)
   Case/Patt Span           Wa
   --------- ---- ------------
   SELF         1         4.56
                2         4.56
                3         4.56
                4         4.56
                5         4.56
   Dead         2         1.00
                3         1.00
                4         1.00
                1         1.00
                5         1.00
   Live         2         1.90
                3         1.90
                4         1.90
                1         1.90
                5         1.90

Reinforcement Criteria
======================
   Slabs and Ribs
   --------------
                 _____Top bars___  __Bottom bars___
                      Min      Max      Min      Max
                  -------  -------  -------  -------
     Bar Size         #15      #15      #15      #15
     Bar spacing       25      375       25      375 mm
     Reinf ratio     0.18     2.00     0.18     2.00 %
     Cover             28                28          mm
     There is NOT more than 300 mm of concrete below top bars.

   Beams
```

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   -----
                 _____Top bars___  __Bottom bars___    ____Stirrups____
                   Min      Max      Min      Max        Min      Max
                 -------  -------  -------  -------    -------  -------
     Bar Size      #20      #35      #20      #35        #10      #20
     Bar spacing    25      457       25      457        152      457 mm
     Reinf ratio  0.14     5.00     0.14     5.00 %
     Cover          38                38          mm
     Layer dist.    25                25          mm
     No. of legs                                           2        6
     Side cover                                           38          mm
     1st Stirrup                                          76          mm
     There is NOT more than 300 mm of concrete below top bars.
```

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spSlab v5.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems

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[2] DESIGN RESULTS*

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*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

Strip Widths and Distribution Factors

Units: Width (m).

Span	Strip	Width Left**	Width Right**	Width Bottom*	Moment Factor Left**	Moment Factor Right**	Moment Factor Bottom*
1	Column	2.10	2.10	2.10	1.000	1.000	0.600
	Middle	2.10	2.10	2.10	0.000	0.000	0.400
2	Column	2.10	2.10	2.10	1.000	0.800	0.600
	Middle	2.10	2.10	2.10	0.000	0.200	0.400
3	Column	2.10	2.10	2.10	0.800	0.800	0.600
	Middle	2.10	2.10	2.10	0.200	0.200	0.400
4	Column	2.10	2.10	2.10	0.800	1.000	0.600
	Middle	2.10	2.10	2.10	0.200	0.000	0.400
5	Column	2.10	2.10	2.10	1.000	1.000	0.600
	Middle	2.10	2.10	2.10	0.000	0.000	0.400

*Used for bottom reinforcement. **Used for top reinforcement.

Top Reinforcement

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Strip	Zone	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Column	Left	2.10	0.08	0.058	798	6468	2	263	8-#15 *3 *5
		Midspan	2.10	0.26	0.107	798	6468	5	263	8-#15 *3 *5
		Right	2.10	0.58	0.165	798	6468	11	233	9-#15 *3 *5
	Middle	Left	2.10	0.00	0.000	798	6468	0	350	6-#15 *3 *5
		Midspan	2.10	0.00	0.083	798	6468	0	350	6-#15 *3 *5
		Right	2.10	0.00	0.165	798	6468	0	350	6-#15 *3 *5
2	Column	Left	2.10	44.78	0.200	798	6468	883	233	9-#15 *5
		Midspan	2.10	0.00	2.750	0	6468	0	0	---
		Right	2.10	76.83	5.300	798	6468	1554	263	8-#15
	Middle	Left	2.10	0.22	0.425	798	6468	4	350	6-#15 *3 *5
		Midspan	2.10	0.00	2.750	0	6468	0	0	---
		Right	2.10	19.21	5.300	798	6468	372	350	6-#15 *3 *5
3	Column	Left	2.10	69.57	0.200	798	6468	1399	263	8-#15 *5
		Midspan	2.10	0.00	2.750	0	6468	0	0	---
		Right	2.10	69.57	5.300	798	6468	1399	263	8-#15 *5
	Middle	Left	2.10	17.39	0.200	798	6468	336	350	6-#15 *3 *5

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C:\TSDA\Two-Way Flat Plate System (CSA)\TSDA-spSlab-Two-Way Flat Plate Floor-CSA 23.3-14.slb Page 2

		Midspan	2.10	0.00	2.750	0	6468	0	0	---
		Right	2.10	17.39	5.300	798	6468	336	350	6-#15 *3 *5
4	Column	Left	2.10	76.83	0.200	798	6468	1554	263	8-#15
		Midspan	2.10	0.00	2.750	0	6468	0	0	---
		Right	2.10	44.78	5.300	798	6468	883	233	9-#15 *5
	Middle	Left	2.10	19.21	0.200	798	6468	372	350	6-#15 *3 *5
		Midspan	2.10	0.00	2.750	0	6468	0	0	---
		Right	2.10	0.22	5.075	798	6468	4	350	6-#15 *3 *5
5	Column	Left	2.10	0.58	0.035	798	6468	11	233	9-#15 *3 *5
		Midspan	2.10	0.26	0.093	798	6468	5	263	8-#15 *3 *5
		Right	2.10	0.08	0.142	798	6468	2	263	8-#15 *3 *5
	Middle	Left	2.10	0.00	0.035	798	6468	0	350	6-#15 *3 *5
		Midspan	2.10	0.00	0.118	798	6468	0	350	6-#15 *3 *5
		Right	2.10	0.00	0.200	798	6468	0	350	6-#15 *3 *5

NOTES:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Top Bar Details
===============

Units: Length (m)

		Left				Continuous		Right			
Span	Strip	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
1	Column	---		---		8-#15	0.20	1-#15	0.20	---	
	Middle	---		---		6-#15	0.20	---		---	
2	Column	5-#15	1.88	4-#15	1.22	---		4-#15	1.88	4-#15*	1.22
	Middle	6-#15	1.32	---		---		6-#15	1.57	---	
3	Column	4-#15	1.88	4-#15*	1.22	---		4-#15	1.88	4-#15*	1.22
	Middle	6-#15	1.57	---		---		6-#15	1.57	---	
4	Column	4-#15	1.88	4-#15*	1.22	---		5-#15	1.88	4-#15	1.22
	Middle	6-#15	1.57	---		---		6-#15	1.32	---	
5	Column	1-#15	0.20	---		8-#15	0.20	---		---	
	Middle	---		---		6-#15	0.20	---		---	

NOTES:
* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

Top Bar Development Lengths
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Units: Length (mm)

		Left				Continuous		Right			
Span	Strip	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
1	Column	---		---		8-#15	300.00	1-#15	300.00	---	
	Middle	---		---		6-#15	300.00	---		---	
2	Column	5-#15	300.00	4-#15	300.00	---		4-#15	377.14	4-#15	377.14
	Middle	6-#15	300.00	---		---		6-#15	300.00	---	
3	Column	4-#15	339.53	4-#15	339.53	---		4-#15	339.53	4-#15	339.53
	Middle	6-#15	300.00	---		---		6-#15	300.00	---	
4	Column	4-#15	377.14	4-#15	377.14	---		5-#15	300.00	4-#15	300.00
	Middle	6-#15	300.00	---		---		6-#15	300.00	---	
5	Column	1-#15	300.00	---		8-#15	300.00	---		---	
	Middle	---		---		6-#15	300.00	---		---	

Band Reinforcement at Supports
==============================

Units: Width (mm), As (mm^2)

Supp	Width<C>	Width<B>	Width<S>	As<C>	As<B>	As<S>	Bars<C>	Bars<B>	Bars<S>
1	2100	970	1130	1800	1000	800	9-#15	5-#15	4-#15
2	2100	970	1130	1600	800	800	8-#15	4-#15	4-#15
3	2100	970	1130	1600	800	800	8-#15	4-#15	4-#15
4	2100	970	1130	1800	1000	800	9-#15	5-#15	4-#15

<C> Total Strip, <B> Banded Strip, <S> Remaining Strip

Bottom Reinforcement
====================

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Strip	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Column	2.10	0.00	0.083	0	6468	0	0	---
	Middle	2.10	0.00	0.083	0	6468	0	0	---
2	Column	2.10	38.78	2.525	798	6468	761	350	6-#15 *3 *5
	Middle	2.10	25.85	2.525	798	6468	503	350	6-#15 *3 *5

3	Column	2.10	28.11	2.750	798	6468	548	350	6-#15	*3 *5
	Middle	2.10	18.74	2.750	798	6468	363	350	6-#15	*3 *5
4	Column	2.10	38.78	2.975	798	6468	761	350	6-#15	*3 *5
	Middle	2.10	25.85	2.975	798	6468	503	350	6-#15	*3 *5
5	Column	2.10	0.00	0.118	0	6468	0	0	---	
	Middle	2.10	0.00	0.118	0	6468	0	0	---	

NOTES:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Bottom Bar Details
==================

Units: Start (m), Length (m)

		Long Bars			Short Bars		
Span	Strip	Bars	Start	Length	Bars	Start	Length
1	Column	---			---		
	Middle	---			---		
2	Column	6-#15	0.00	5.50	---		
	Middle	4-#15	0.00	5.50	2-#15	0.00	4.67
3	Column	6-#15	0.00	5.50	---		
	Middle	4-#15	0.00	5.50	2-#15	0.82	3.85
4	Column	6-#15	0.00	5.50	---		
	Middle	4-#15	0.00	5.50	2-#15	0.82	4.68
5	Column	---			---		
	Middle	---			---		

Bottom Bar Development Lengths
==============================

Units: DevLen (mm)

		Long Bars		Short Bars	
Span	Strip	Bars	DevLen	Bars	DevLen
1	Column	---		---	
	Middle	---		---	
2	Column	6-#15	300.00	---	
	Middle	4-#15	300.00	2-#15	300.00
3	Column	6-#15	300.00	---	
	Middle	4-#15	300.00	2-#15	300.00
4	Column	6-#15	300.00	---	
	Middle	4-#15	300.00	2-#15	300.00
5	Column	---		---	
	Middle	---		---	

Flexural Capacity
=================

Units: x (m), As (mm^2), PhiMn, Mu (kNm)

			Top						Bottom					
Span	Strip	x	AsTop	PhiMn-	Mu-	Comb	Pat	Status	AsBot	PhiMn+	Mu+	Comb	Pat	Status
1	Column	0.000	1800	-88.18	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.058	1800	-88.18	-0.08	U2	All	OK	0	0.00	0.00	U1	All	OK
		0.100	1800	-88.18	-0.23	U2	All	OK	0	0.00	0.00	U1	All	OK
		0.107	1800	-88.18	-0.26	U2	All	OK	0	0.00	0.00	U1	All	OK
		0.165	1800	-88.18	-0.58	U2	All	OK	0	0.00	0.00	U1	All	OK
		0.200	1800	-88.18	-0.82	U2	All	---	0	0.00	0.00	U1	All	---
	Middle	0.000	1200	-60.14	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.058	1200	-60.14	-0.00	U2	All	OK	0	0.00	0.00	U1	All	OK
		0.100	1200	-60.14	-0.00	U2	All	OK	0	0.00	0.00	U1	All	OK
		0.107	1200	-60.14	-0.00	U2	All	OK	0	0.00	0.00	U1	All	OK
		0.165	1200	-60.14	-0.00	U2	All	OK	0	0.00	0.00	U1	All	OK
		0.200	1200	-60.14	-0.00	U2	All	---	0	0.00	0.00	U1	All	---
2	Column	0.000	1800	-88.18	-65.10	U2	All	---	1200	60.14	0.00	U1	All	---
		0.200	1800	-88.18	-44.78	U2	All	OK	1200	60.14	0.00	U1	All	OK
		0.920	1800	-88.18	0.00	U1	All	OK	1200	60.14	7.71	U2	All	OK
		1.220	1000	-50.49	0.00	U1	All	OK	1200	60.14	18.35	U2	All	OK
		1.583	1000	-50.49	0.00	U1	All	OK	1200	60.14	28.25	U2	All	OK
		1.883	0	0.00	0.00	U1	All	OK	1200	60.14	33.98	U2	All	OK
		1.985	0	0.00	0.00	U1	All	OK	1200	60.14	35.42	U2	All	OK
		2.525	0	0.00	0.00	U1	All	OK	1200	60.14	38.78	U2	All	OK
		2.750	0	0.00	0.00	U1	All	OK	1200	60.14	38.05	U2	All	OK
		3.515	0	0.00	0.00	U1	All	OK	1200	60.14	26.20	U2	All	OK
		3.617	0	0.00	0.00	U1	All	OK	1200	60.14	23.52	U2	All	OK
		3.994	800	-40.69	0.00	U1	All	OK	1200	60.14	11.42	U2	All	OK
		4.280	800	-40.69	-0.92	U1	All	OK	1200	60.14	0.58	U1	All	OK

	4.657	1600	-78.98	-25.28	U2 All	OK	1200	60.14	0.00	U1 All	OK
	5.300	1600	-78.98	-76.83	U2 All	OK	1200	60.14	0.00	U1 All	OK
	5.367	1600	-78.98	-82.76	U2 All	---	1200	60.14	0.00	U1 All	---
	5.500	1600	-78.98	-94.95	U2 All	---	1200	60.14	0.00	U1 All	---
Middle	0.000	1200	-60.14	0.51	U2 All	---	1200	60.14	0.00	U1 All	---
	0.200	1200	-60.14	-0.00	U2 All	OK	1200	60.14	0.00	U1 All	OK
	0.425	1200	-60.14	-0.22	U2 All	OK	1200	60.14	0.00	U1 All	OK
	1.022	1200	-60.14	0.00	U1 All	OK	1200	60.14	7.73	U2 All	OK
	1.322	0	0.00	0.00	U1 All	OK	1200	60.14	14.32	U2 All	OK
	1.985	0	0.00	0.00	U1 All	OK	1200	60.14	23.61	U2 All	OK
	2.525	0	0.00	0.00	U1 All	OK	1200	60.14	25.85	U2 All	OK
	2.750	0	0.00	0.00	U1 All	OK	1200	60.14	25.36	U2 All	OK
	3.515	0	0.00	0.00	U1 All	OK	1200	60.14	17.46	U2 All	OK
	3.931	0	0.00	0.00	U1 All	OK	1200	60.14	9.12	U2 All	OK
	4.231	1200	-60.14	0.00	U1 All	OK	1200	60.14	1.34	U2 All	OK
	4.375	1200	-60.14	-1.19	U2 All	OK	1200	60.14	0.00	U1 All	OK
	4.675	1200	-60.14	-5.65	U2 All	OK	800	40.69	0.00	U1 All	OK
	5.300	1200	-60.14	-19.21	U2 All	OK	800	40.69	0.00	U1 All	OK
	5.500	1200	-60.14	-24.91	U2 All	---	800	40.69	0.00	U1 All	---
3 Column	0.000	1600	-78.98	-87.02	U2 All	---	1200	60.14	0.00	U1 All	---
	0.067	1600	-78.98	-81.06	U2 All	---	1200	60.14	0.00	U1 All	---
	0.200	1600	-78.98	-69.57	U2 All	OK	1200	60.14	0.00	U1 All	OK
	0.880	1600	-78.98	-20.06	U2 All	OK	1200	60.14	0.00	U1 All	OK
	1.220	800	-40.69	-1.08	U2 All	OK	1200	60.14	0.16	U1 All	OK
	1.543	800	-40.69	0.00	U1 All	OK	1200	60.14	10.13	U2 All	OK
	1.883	0	0.00	0.00	U1 All	OK	1200	60.14	18.82	U2 All	OK
	1.985	0	0.00	0.00	U1 All	OK	1200	60.14	20.88	U2 All	OK
	2.750	0	0.00	0.00	U1 All	OK	1200	60.14	28.11	U2 All	OK
	3.515	0	0.00	0.00	U1 All	OK	1200	60.14	20.88	U2 All	OK
	3.617	0	0.00	0.00	U1 All	OK	1200	60.14	18.82	U2 All	OK
	3.957	800	-40.69	0.00	U1 All	OK	1200	60.14	10.13	U2 All	OK
	4.280	800	-40.69	-1.08	U2 All	OK	1200	60.14	0.16	U1 All	OK
	4.620	1600	-78.98	-20.06	U2 All	OK	1200	60.14	0.00	U1 All	OK
	5.300	1600	-78.98	-69.57	U2 All	OK	1200	60.14	0.00	U1 All	OK
	5.433	1600	-78.98	-81.06	U2 All	---	1200	60.14	0.00	U1 All	---
	5.500	1600	-78.98	-87.02	U2 All	---	1200	60.14	0.00	U1 All	---
Middle	0.000	1200	-60.14	-21.76	U2 All	---	800	40.69	0.00	U1 All	---
	0.200	1200	-60.14	-17.39	U2 All	OK	800	40.69	0.00	U1 All	OK
	0.825	1200	-60.14	-5.89	U2 All	OK	800	40.69	0.00	U1 All	OK
	1.125	1200	-60.14	-1.50	U2 All	OK	1200	60.14	0.00	U1 All	OK
	1.269	1200	-60.14	0.00	U1 All	OK	1200	60.14	0.67	U2 All	OK
	1.569	0	0.00	0.00	U1 All	OK	1200	60.14	7.25	U2 All	OK
	1.985	0	0.00	0.00	U1 All	OK	1200	60.14	13.92	U2 All	OK
	2.750	0	0.00	0.00	U1 All	OK	1200	60.14	18.74	U2 All	OK
	3.515	0	0.00	0.00	U1 All	OK	1200	60.14	13.92	U2 All	OK
	3.931	0	0.00	0.00	U1 All	OK	1200	60.14	7.25	U2 All	OK
	4.231	1200	-60.14	0.00	U1 All	OK	1200	60.14	0.67	U2 All	OK
	4.375	1200	-60.14	-1.50	U2 All	OK	1200	60.14	0.00	U1 All	OK
	4.675	1200	-60.14	-5.89	U2 All	OK	800	40.69	0.00	U1 All	OK
	5.300	1200	-60.14	-17.39	U2 All	OK	800	40.69	0.00	U1 All	OK
	5.500	1200	-60.14	-21.76	U2 All	---	800	40.69	0.00	U1 All	---
4 Column	0.000	1600	-78.98	-94.95	U2 All	---	1200	60.14	0.00	U1 All	---
	0.133	1600	-78.98	-82.76	U2 All	---	1200	60.14	0.00	U1 All	---
	0.200	1600	-78.98	-76.83	U2 All	OK	1200	60.14	0.00	U1 All	OK
	0.843	1600	-78.98	-25.28	U2 All	OK	1200	60.14	0.00	U1 All	OK
	1.220	800	-40.69	-0.92	U2 All	OK	1200	60.14	0.58	U1 All	OK
	1.506	800	-40.69	0.00	U1 All	OK	1200	60.14	11.42	U2 All	OK
	1.883	0	0.00	0.00	U1 All	OK	1200	60.14	23.52	U2 All	OK
	1.985	0	0.00	0.00	U1 All	OK	1200	60.14	26.20	U2 All	OK
	2.750	0	0.00	0.00	U1 All	OK	1200	60.14	38.05	U2 All	OK
	2.975	0	0.00	0.00	U1 All	OK	1200	60.14	38.78	U2 All	OK
	3.515	0	0.00	0.00	U1 All	OK	1200	60.14	35.42	U2 All	OK
	3.617	0	0.00	0.00	U1 All	OK	1200	60.14	33.98	U2 All	OK
	3.917	1000	-50.49	0.00	U1 All	OK	1200	60.14	28.25	U2 All	OK
	4.280	1000	-50.49	0.00	U1 All	OK	1200	60.14	18.35	U2 All	OK
	4.580	1800	-88.18	0.00	U1 All	OK	1200	60.14	7.71	U2 All	OK
	5.300	1800	-88.18	-44.78	U2 All	OK	1200	60.14	0.00	U1 All	OK
	5.500	1800	-88.18	-65.10	U2 All	---	1200	60.14	0.00	U1 All	---
Middle	0.000	1200	-60.14	-24.91	U2 All	---	800	40.69	0.00	U1 All	---
	0.200	1200	-60.14	-19.21	U2 All	OK	800	40.69	0.00	U1 All	OK
	0.825	1200	-60.14	-5.65	U2 All	OK	800	40.69	0.00	U1 All	OK
	1.125	1200	-60.14	-1.19	U2 All	OK	1200	60.14	0.00	U1 All	OK
	1.269	1200	-60.14	0.00	U1 All	OK	1200	60.14	1.34	U2 All	OK
	1.569	0	0.00	0.00	U1 All	OK	1200	60.14	9.12	U2 All	OK
	1.985	0	0.00	0.00	U1 All	OK	1200	60.14	17.46	U2 All	OK
	2.750	0	0.00	0.00	U1 All	OK	1200	60.14	25.36	U2 All	OK
	2.975	0	0.00	0.00	U1 All	OK	1200	60.14	25.85	U2 All	OK
	3.515	0	0.00	0.00	U1 All	OK	1200	60.14	23.61	U2 All	OK
	4.178	0	0.00	0.00	U1 All	OK	1200	60.14	14.32	U2 All	OK
	4.478	1200	-60.14	0.00	U1 All	OK	1200	60.14	7.73	U2 All	OK
	5.075	1200	-60.14	-0.22	U2 All	OK	1200	60.14	0.00	U1 All	OK
	5.300	1200	-60.14	0.00	U2 All	OK	1200	60.14	0.00	U1 All	OK
	5.500	1200	-60.14	0.51	U2 All	---	1200	60.14	0.00	U1 All	---
5 Column	0.000	1800	-88.18	-0.82	U2 All	---	0	0.00	0.00	U1 All	---

	0.035	1800	-88.18	-0.58	U2	All	OK	0	0.00	0.00	U1	All	OK
	0.093	1800	-88.18	-0.26	U2	All	OK	0	0.00	0.00	U1	All	OK
	0.100	1800	-88.18	-0.23	U2	All	OK	0	0.00	0.00	U1	All	OK
	0.142	1800	-88.18	-0.08	U2	All	OK	0	0.00	0.00	U1	All	OK
	0.200	1800	-88.18	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
Middle	0.000	1200	-60.14	-0.00	U2	All	---	0	0.00	0.00	U1	All	---
	0.035	1200	-60.14	-0.00	U2	All	OK	0	0.00	0.00	U1	All	OK
	0.093	1200	-60.14	-0.00	U2	All	OK	0	0.00	0.00	U1	All	OK
	0.100	1200	-60.14	-0.00	U2	All	OK	0	0.00	0.00	U1	All	OK
	0.142	1200	-60.14	-0.00	U2	All	OK	0	0.00	0.00	U1	All	OK
	0.200	1200	-60.14	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK

Slab Shear Capacity

Units: b, dv (mm), Xu (m), PhiVc, Vu(kN)

Span	b	dv	Beta	Vratio	PhiVc	Vu	Xu
1	4200	139	0.210	1.000	420.46	0.00	0.00
2	4200	139	0.210	1.000	420.46	109.30	5.16
3	4200	139	0.210	1.000	420.46	99.25	5.16
4	4200	139	0.210	1.000	420.46	109.30	0.34
5	4200	139	0.210	1.000	420.46	0.00	0.00

Flexural Transfer of Negative Unbalanced Moment at Supports

Units: Width (mm), Munb (kNm), As (mm^2)

Supp	Width	Width-c	d	Munb	Comb	Pat	GammaF	AsReq	AsProv	Add Bars
1	970	970	154	63.77	U2	All	0.618	802	1000	---
2	970	970	154	11.08	U2	All	0.600	128	800	---
3	970	970	154	11.08	U2	All	0.600	128	800	---
4	970	970	154	63.77	U2	All	0.618	802	1000	---

Punching Shear Around Columns

Critical Section Properties

Units: b1, b2, b0, davg, CG, c(left), c(right) (mm), Ac (mm^2), Jc (mm^4)

Supp	Type	b1	b2	b0	davg	CG	c(left)	c(right)	Ac	Jc
1	Rect	477.0	554.0	1508.0	154.0	126.1	326.1	150.9	2.3223e+005	6.1461e+009
2	Rect	554.0	554.0	2216.0	154.0	0.0	277.0	277.0	3.4126e+005	1.7794e+010
3	Rect	554.0	554.0	2216.0	154.0	0.0	277.0	277.0	3.4126e+005	1.7794e+010
4	Rect	477.0	554.0	1508.0	154.0	-126.1	150.9	326.1	2.3223e+005	6.1461e+009

Punching Shear Results

Units: Vu (kN), Munb (kNm), vu (N/mm^2), Phi*vc (N/mm^2)

Supp	Vu	vu	Munb	Comb	Pat	GammaV	vu	Phi*vc
1	108.78	0.468	50.05	U2	All	0.382	0.938	1.307
2	233.42	0.684	-11.08	U2	All	0.400	0.753	1.307
3	233.42	0.684	11.08	U2	All	0.400	0.753	1.307
4	108.78	0.468	-50.05	U2	All	0.382	0.938	1.307

Integrity Reinforcement at Supports

Units: Vse(kN), Asb(mm^2)

Supp	Vse	Asb
1	101.23	506
2	217.22	1086
3	217.22	1086
4	101.23	506

NOTES: The sum of bottom reinforcement crossing the perimeter of the support on all sides shall not be less than the above listed values.

Material Takeoff

Reinforcement in the Direction of Analysis

Top Bars:	216.3 kg	<=>	12.80 kg/m	<=>	3.047 kg/m^2
Bottom Bars:	300.5 kg	<=>	17.78 kg/m	<=>	4.234 kg/m^2
Stirrups:	0.0 kg	<=>	0.00 kg/m	<=>	0.000 kg/m^2
Total Steel:	516.8 kg	<=>	30.58 kg/m	<=>	7.280 kg/m^2
Concrete:	13.5 m^3	<=>	0.80 m^3/m	<=>	0.190 m^3/m^2

```
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Reinforced Concrete Beams, One-way and Two-way Slab Systems

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[3] DEFLECTION RESULTS

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Section Properties

Frame Section Properties

Units: Ig, Icr (mm^4), Mcr (kNm)

Span	Zone	M+ve Ig	M+ve Icr	M+ve Mcr	M-ve Ig	M-ve Icr	M-ve Mcr
1	Left	2.4007e+009	0.00000	40.12	2.4007e+009	3.5649e+008	-40.12
	Midspan	2.4007e+009	0.00000	40.12	2.4007e+009	3.7765e+008	-40.12
	Right	2.4007e+009	0.00000	40.12	2.4007e+009	3.7765e+008	-40.12
2	Left	2.4007e+009	2.6775e+008	40.12	2.4007e+009	3.7765e+008	-40.12
	Midspan	2.4007e+009	2.6775e+008	40.12	2.4007e+009	0.00000	-40.12
	Right	2.4007e+009	2.6775e+008	40.12	2.4007e+009	3.5649e+008	-40.12
3	Left	2.4007e+009	2.6775e+008	40.12	2.4007e+009	3.5649e+008	-40.12
	Midspan	2.4007e+009	2.6775e+008	40.12	2.4007e+009	0.00000	-40.12
	Right	2.4007e+009	2.6775e+008	40.12	2.4007e+009	3.5649e+008	-40.12
4	Left	2.4007e+009	2.6775e+008	40.12	2.4007e+009	3.5649e+008	-40.12
	Midspan	2.4007e+009	2.6775e+008	40.12	2.4007e+009	0.00000	-40.12
	Right	2.4007e+009	2.6775e+008	40.12	2.4007e+009	3.7765e+008	-40.12
5	Left	2.4007e+009	0.00000	40.12	2.4007e+009	3.7765e+008	-40.12
	Midspan	2.4007e+009	0.00000	40.12	2.4007e+009	3.7765e+008	-40.12
	Right	2.4007e+009	0.00000	40.12	2.4007e+009	3.5649e+008	-40.12

NOTES: M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Frame Effective Section Properties

Units: Ie, Ie,avg (mm^4), Mmax (kNm)

			Load Level					
			Dead		Sustained		Dead+Live	
Span	Zone	Weight	Mmax	Ie	Mmax	Ie	Mmax	Ie
1	Right	1.000	-0.47	2.4007e+009	-0.47	2.4007e+009	-0.63	2.4007e+009
	Span Avg	----	----	2.4007e+009	----	2.4007e+009	----	2.4007e+009
2	Middle	0.850	36.67	2.4007e+009	36.67	2.4007e+009	49.20	1.4240e+009
	Right	0.150	-68.00	7.7618e+008	-68.00	7.7618e+008	-91.24	5.3024e+008
	Span Avg	----	----	2.1570e+009	----	2.1570e+009	----	1.2900e+009
3	Left	0.150	-61.71	9.1792e+008	-61.71	9.1792e+008	-82.80	5.8893e+008
	Middle	0.700	26.58	2.4007e+009	26.58	2.4007e+009	35.67	2.4007e+009
	Right	0.150	-61.71	9.1792e+008	-61.71	9.1792e+008	-82.80	5.8893e+008
	Span Avg	----	----	1.9558e+009	----	1.9558e+009	----	1.8571e+009
4	Left	0.150	-68.00	7.7618e+008	-68.00	7.7618e+008	-91.24	5.3024e+008
	Middle	0.850	36.67	2.4007e+009	36.67	2.4007e+009	49.20	1.4240e+009
	Span Avg	----	----	2.1570e+009	----	2.1570e+009	----	1.2900e+009
5	Left	1.000	-0.47	2.4007e+009	-0.47	2.4007e+009	-0.63	2.4007e+009
	Span Avg	----	----	2.4007e+009	----	2.4007e+009	----	2.4007e+009

Strip Section Properties at Midspan

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Units: Ig (mm^4)

Span	Column Strip Ig	Column Strip LDF	Column Strip Ratio	Middle Strip Ig	Middle Strip LDF	Middle Strip Ratio
1	1.20033e+009	0.800	1.600	1.20033e+009	0.200	0.400
2	1.20033e+009	0.750	1.500	1.20033e+009	0.250	0.500
3	1.20033e+009	0.700	1.400	1.20033e+009	0.300	0.600
4	1.20033e+009	0.750	1.500	1.20033e+009	0.250	0.500
5	1.20033e+009	0.800	1.600	1.20033e+009	0.200	0.400

NOTES: Load distirubtion factor, LDL, averages moment distribution factors listed in [2] Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

Instantaneous Deflections

Extreme Instantaneous Frame Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.11	---	-0.06	-0.06	-0.11	-0.17
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	1.35	---	1.21	1.21	1.35	2.57
		Loc	2.525	---	2.600	2.600	2.525	2.600
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	0.85	---	0.35	0.35	0.85	1.20
		Loc	2.750	---	2.750	2.750	2.750	2.750
	Up	Def	-0.01	---	-0.00	-0.00	-0.01	-0.01
		Loc	0.133	---	0.133	0.133	0.133	0.133
4	Down	Def	1.35	---	1.21	1.21	1.35	2.57
		Loc	2.975	---	2.900	2.900	2.975	2.900
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.11	---	-0.06	-0.06	-0.11	-0.17
		Loc	0.200	---	0.200	0.200	0.200	0.200

Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.11	---	-0.06	-0.06	-0.11	-0.17
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	1.80	---	1.69	1.69	1.80	3.49
		Loc	2.600	---	2.675	2.675	2.600	2.600
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	1.24	---	0.51	0.51	1.24	1.74
		Loc	2.750	---	2.750	2.750	2.750	2.750
	Up	Def	-0.00	---	-0.00	-0.00	-0.00	-0.00
		Loc	0.067	---	0.067	0.067	0.067	0.067
4	Down	Def	1.80	---	1.69	1.69	1.80	3.49
		Loc	2.900	---	2.825	2.825	2.900	2.900
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.11	---	-0.06	-0.06	-0.11	-0.17
		Loc	0.200	---	0.200	0.200	0.200	0.200

Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.11	---	-0.06	-0.06	-0.11	-0.17
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	0.91	---	0.74	0.74	0.91	1.65
		Loc	2.450	---	2.525	2.525	2.450	2.450
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	0.46	---	0.19	0.19	0.46	0.65
		Loc	2.750	---	2.750	2.750	2.750	2.750
	Up	Def	-0.01	---	-0.00	-0.00	-0.01	-0.01
		Loc	0.200	---	0.200	0.200	0.200	0.200

4	Down	Def	0.91	---	0.74	0.74	0.91	1.65
		Loc	3.050	---	2.975	2.975	3.050	3.050
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.11	---	-0.06	-0.06	-0.11	-0.17
		Loc	0.200	---	0.200	0.200	0.200	0.200

Long-term Deflections

Long-term Column Strip Deflection Factors

Time dependant factor for sustained loads = 2.000
Units: Astop, Asbot (mm^2), b, d (mm), Rho' (%), Lambda (-)

		M+ve					M-ve				
Span	Zone	Astop	b	d	Rho'	Lambda	Asbot	b	d	Rho'	Lambda
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Long-term Middle Strip Deflection Factors

Time dependant factor for sustained loads = 2.000
Units: Astop, Asbot (mm^2), b, d (mm), Rho' (%), Lambda (-)

		M+ve					M-ve				
Span	Zone	Astop	b	d	Rho'	Lambda	Asbot	b	d	Rho'	Lambda
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Extreme Long-term Column Strip Deflections and Corresponding Locations

Units: D (mm), x (m)

Span	Direction	Value	cs	cs+lu	cs+l	Total
1	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.22	-0.28	-0.28	-0.39
		Loc	0.000	0.000	0.000	0.000
2	Down	Def	3.60	5.29	5.29	7.10
		Loc	2.600	2.600	2.600	2.600
	Up	Def	---	---	---	---
		Loc	---	---	---	---
3	Down	Def	2.48	2.98	2.98	4.22
		Loc	2.750	2.750	2.750	2.750
	Up	Def	-0.01	-0.01	-0.01	-0.01
		Loc	0.067	0.067	0.067	0.067
4	Down	Def	3.60	5.29	5.29	7.10
		Loc	2.900	2.900	2.900	2.900
	Up	Def	---	---	---	---
		Loc	---	---	---	---
5	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.22	-0.28	-0.28	-0.39
		Loc	0.200	0.200	0.200	0.200

NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Extreme Long-term Middle Strip Deflections and Corresponding Locations

Units: D (mm), x (m)

Span	Direction	Value	cs	cs+lu	cs+l	Total
1	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.22	-0.28	-0.28	-0.39
		Loc	0.000	0.000	0.000	0.000
2	Down	Def	1.82	2.57	2.57	3.48
		Loc	2.450	2.450	2.450	2.450
	Up	Def	---	---	---	---

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                   Loc        ---         ---         ---         ---
     3      Down   Def       0.92        1.11        1.11        1.57
                   Loc      2.750       2.750       2.750       2.750
              Up   Def      -0.02       -0.02       -0.02       -0.03
                   Loc      0.200       0.200       0.200       0.200
     4      Down   Def       1.82        2.57        2.57        3.48
                   Loc      3.050       3.050       3.050       3.050
              Up   Def        ---         ---         ---         ---
                   Loc        ---         ---         ---         ---
     5      Down   Def        ---         ---         ---         ---
                   Loc        ---         ---         ---         ---
              Up   Def      -0.22       -0.28       -0.28       -0.39
                   Loc      0.200       0.200       0.200       0.200

  NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
         Incremental deflections after partitions are installed can be estimated by deflections due to:
         - creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
         - creep and shrinkage plus live load (cs+l), if live load applied after partitions.
         Total deflections consist of dead, live, and creep and shrinkage deflections.
```

7. Summary and Comparison of Two-Way Slab Design Results

Table 12 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution

		Hand (EFM)	spSlab
Exterior Span			
Column Strip	Exterior Negative*	44.31	44.78
	Positive	38.28	38.78
	Interior Negative*	76.69	76.83
Middle Strip	Exterior Negative*	0	0
	Positive	25.52	25.85
	Interior Negative*	19.98	19.21
Interior Span			
Column Strip	Interior Negative*	69.35	69.57
	Positive	28.32	28.11
Middle Strip	Interior Negative*	17.34	17.39
	Positive	18.88	18.74

* negative moments are taken at the faces of supports

Table 13 - Comparison of Reinforcement Results with Hand and spSlab Solution

Span Location		Reinforcement Provided for Flexure		Additional Reinforcement Provided for Unbalanced Moment Transfer*		Total Reinforcement Provided	
		Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior Span							
Column Strip	Exterior Negative	9-15M	9-15M	---	---	9-15M	9-15M
	Positive	6-15M	6-15M	n/a	n/a	6-15M	6-15M
	Interior Negative	8-15M	8-15M	---	---	8-15M	8-15M
Middle Strip	Exterior Negative	6-15M	6-15M	n/a	n/a	6-15M	6-15M
	Positive	6-15M	6-15M	n/a	n/a	6-15M	6-15M
	Interior Negative	6-15M	6-15M	n/a	n/a	6-15M	6-15M
Interior Span							
Column Strip	Positive	6-15M	6-15M	n/a	n/a	6-15M	6-15M
Middle Strip	Positive	6-15M	6-15M	n/a	n/a	6-15M	6-15M

* In the EFM, the unbalanced moment (M_{sc}, M_{unb}) at the support centerline is used to determine the value of the additional reinforcement as compared with DDM using the moments at the face of support.

Table 14 - Comparison of One-Way (Beam Action) Shear Check Results Using Hand and spSlab Solution						
Span	V_u , kN		x_u* , m		φV_c, kN	
	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	109.35	109.3	5.16	5.16	421.67	420.46
Interior	99.24	99.25	5.16	5.16	421.67	420.46
* x_u calculated from the centerline of the left column for each span						

Table 15 - Comparison of Two-Way (Punching) Shear Check Results Using Hand and spSlab Solution												
Support	b_1, mm		b_2, mm		b_o, mm		A_c, mm²		V_u, kN		v_u, kN/mm²	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	477	447	554	554	1508	1508	2.32×10^5	2.32×10^5	101.21	108.78	0.44	0.47
Interior	554	554	554	554	2216	2216	500.00	3.41×10^5	233.48	233.42	0.68	0.68
Support	c_{AB}, mm		J_c, mm⁴		γ_v		M_{unb}, kN.m		v_u, MPa		φv_c, MPa	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	150.9	150.9	6.15×10^9	6.15×10^9	0.38	0.38	51.34	50.05	0.92	0.94	1.31	1.31
Interior	277	277	1.78×10^{10}	1.78×10^{10}	0.4	0.4	11.2	11.08	0.79	0.75	1.31	1.31

Table 16 - Comparison of Immediate Deflection Results Using Hand and spSlab Solution (mm)								
Column Strip								
Span	D		D+LL_{sus}		D+LL_{full}		LL	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	1.92	1.80	1.92	1.80	4.31	3.49	2.39	1.69
Interior	1.33	1.24	1.33	1.24	1.88	1.74	0.55	0.50
Middle Strip								
Span	D		D+LL_{sus}		D+LL_{full}		LL	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	0.96	0.91	0.96	0.91	2.15	1.65	1.19	0.74
Interior	0.48	0.46	0.48	0.46	0.68	0.65	0.20	0.19

Table 17 - Comparison of Time-Dependent Deflection Results Using Hand and spSlab Solution						
Column Strip						
Span	λ_Δ		Δ_{cs}, mm		Δ_{total}, mm	
	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	2.0	2.0	3.84	3.60	8.16	7.10
Interior	2.0	2.0	2.67	2.48	4.55	4.22
Middle Strip						
Span	λ_Δ		Δ_{cs}, mm		Δ_{total}, mm	
	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	2.0	2.0	1.92	1.82	4.06	3.48
Interior	2.0	2.0	0.97	0.92	1.65	1.57

In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the spSlab model except for the deflection were results are differs slightly (See Section 8 for explanation). Excerpts of spSlab graphical and text output are given above for illustration.

8. Deflection Calculation Methods

Deflections calculations in reinforced concrete structures can be very tedious and time consuming because of the difficulty of accounting for the actual end boundary conditions in a building frame. As a result, numerous methods to estimate the deflection and the member stiffness have been presented in literature. It is important to note that these methods can only estimate deflections within an accuracy range of 20% to 40%. It is important for the designer to be aware of this broad range of accuracy, especially in the modeling, design, and detailing of deflection-sensitive members.

spSlab uses elastic analysis (stiffness method) to obtain deflections along the column and middle strips by discretizing the span into 110 elements. It also takes into account the adjacent spans effects, shape effects, supporting members stiffnesses above and below the beam, and cracked section effects based on the applied forces. This level of detail provides the maximum accuracy possible compared with other approximate methods used to calculate deflections. In tables 16 and 17, the deflection values calculated by spSlab is lower than the values calculated by the approximate method recommended by PCA Notes (the method used in the hand solution). This can be expected since the approximate method has a built-in conservatism to accommodate a wide range of applications and conditions. The designer can use spSlab and exploit its numerous features to get a closer deflection estimate and optimize the depth of the slab under consideration.

9. Comparison of Two-Way Slab Analysis and Design Methods

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in ***CSA A23.3-14 Clasues (13.8 and 13.9)*** for regular two-way slab systems. ***CSA A23.3-14 (13.5.1)***

Direct Design Method (DDM) is an approximate method and is applicable to flat plate concrete floor systems that meet the stringent requirements of ***CSA A23.3-14 (13.9.1)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Elastic Frame Method (EFM) has less stringent limitations compared to DDM. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StucturePoint's spSlab software program solution utilizes the EFM to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable CSA A23.3-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
13.8.1.1 13.9.1.1	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
13.8.1.1 13.9.1.1	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑	☑	
13.8.1.1 13.9.1.1	Column offset shall not exceed 20% of the span in direction of offset from either axis between centerlines of successive columns	☑	☑	
13.8.1.1 13.9.1.1	The reinforcement is placed in an orthogonal grid.	☑	☑	
13.9.1.2	Minimum of three continuous spans in each direction	☑		
13.9.1.3	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
13.9.1.4	All loads shall be due to gravity only	☑		
13.9.1.4	All loads shall be uniformly distributed over an entire panel (q_f)	☑		
13.9.1.4	Unfactored live load shall not exceed two times the unfactored dead load	☑		
13.10.6	Structural integrity steel detailing	☑	☑	☑
13.10.10	Openings in slab systems	☑	☑	☑
8.2	Concentrated loads	Not permitted	☑	☑
13.8.4.1	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
13.10.2*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
13.8.2	Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)	Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb}).